Tracing the Dark Matter Sheet in Phase Space

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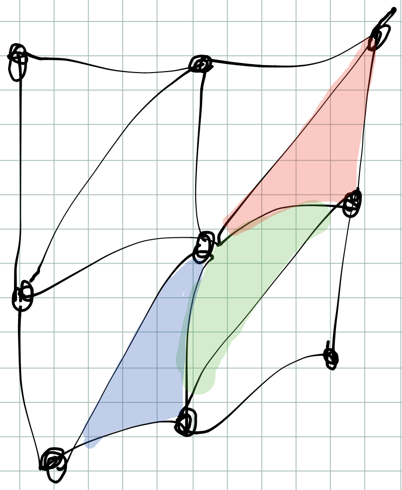
KIPAC / STANFORD



Outline:

- Dark Matter Sheet?
- COSMOLOGICAL N-BODY SIMULATIONS
- Phase Space
- Projecting to configuration space
- How useful is this?
- Collisionless fluids
- New ways to simulate DM & collisionless plasmas.

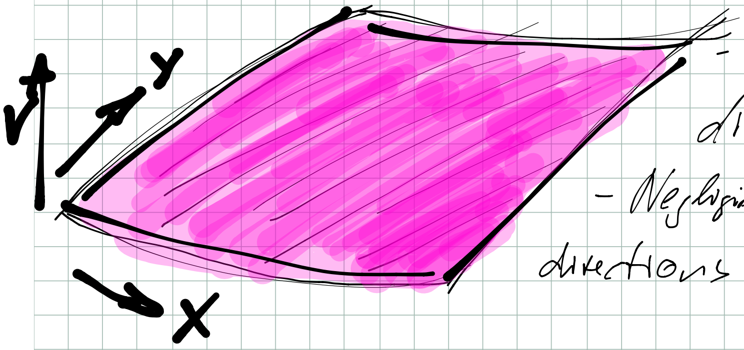
The Dark Matter Sheet?

COOL WIMPS

Dark Matter is commonly hypothesized to originate within seconds after the BIG BANG. If it were moving relativistically today, galaxies and other structures would not exist. We speak of **COLD DARK MATTER**.

Working HYPOTHESIS:

- Weakly interacting massive particle (say ≈ 100 GeV).
- Very cold. Even keV particles would only have $\sim \frac{m}{s}$ speeds today.



- Almost perfectly uniformly distributed initially.
- Negligible extent along velocity directions in phase space.

The Dark Matter Sheet ?

Fluid

$\#$ OF DARK MATTER PARTICLES IN THE MILKY WAY :

$N_{DM} \approx 10^{67} \left(\frac{100 GeV}{m_{DM}} \right) \gg$ $\#$ OF STARS IN THE UNIVERSE

$\gg$ $\#$ OF PARTICLES THAT FIT ON A COMPUTER

USING ALL THE COMPUTERS IN THE WORLD : $\lesssim 10^{17}$ particles

SOLVE VLASOV-POISSON SYSTEM INSTEAD.

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f + \vec{a} \cdot \vec{\nabla}_v f = 0$$

$$\vec{a} = -\vec{\nabla}\phi$$

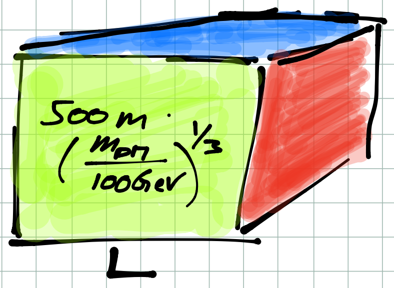
$$\vec{\nabla}^2 \phi = 4\pi G \rho$$

f: distribution function IN PHASE SPACE

ϕ: potential

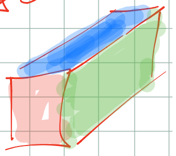
FOR PHASE SPACE ELEMENT TO CONTAIN 10^6 PARTICLES @ MEAN DENSITY IT HAS TO BE LARGER THAN

$$L \sim 500 m \left(\frac{m_{DM}}{100 GeV} \right)^{1/3}$$



The Dark Matter Sheet?

3D MANIFOLD MOVING IN SIX DIMENSIONAL PHASE SPACE



Phase space volume is conserved

$$\Delta V \cdot (\Delta v)^3 = \text{const}$$

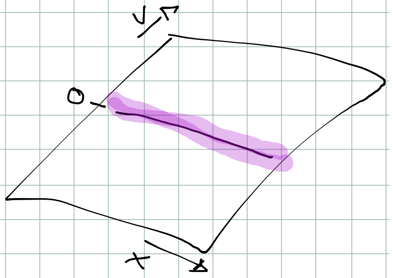
spatial volume volume in velocity space

redshift when $E \approx 100\,\text{GeV}$: $1\,\text{eV}$ @ $z \sim 1000$

$100\,\text{GeV}$ @ $z \sim 10^{14}$

Matter density dropped by a factor $\sim 10^{42}$ since then.

⇒ YES. VERY COLD.

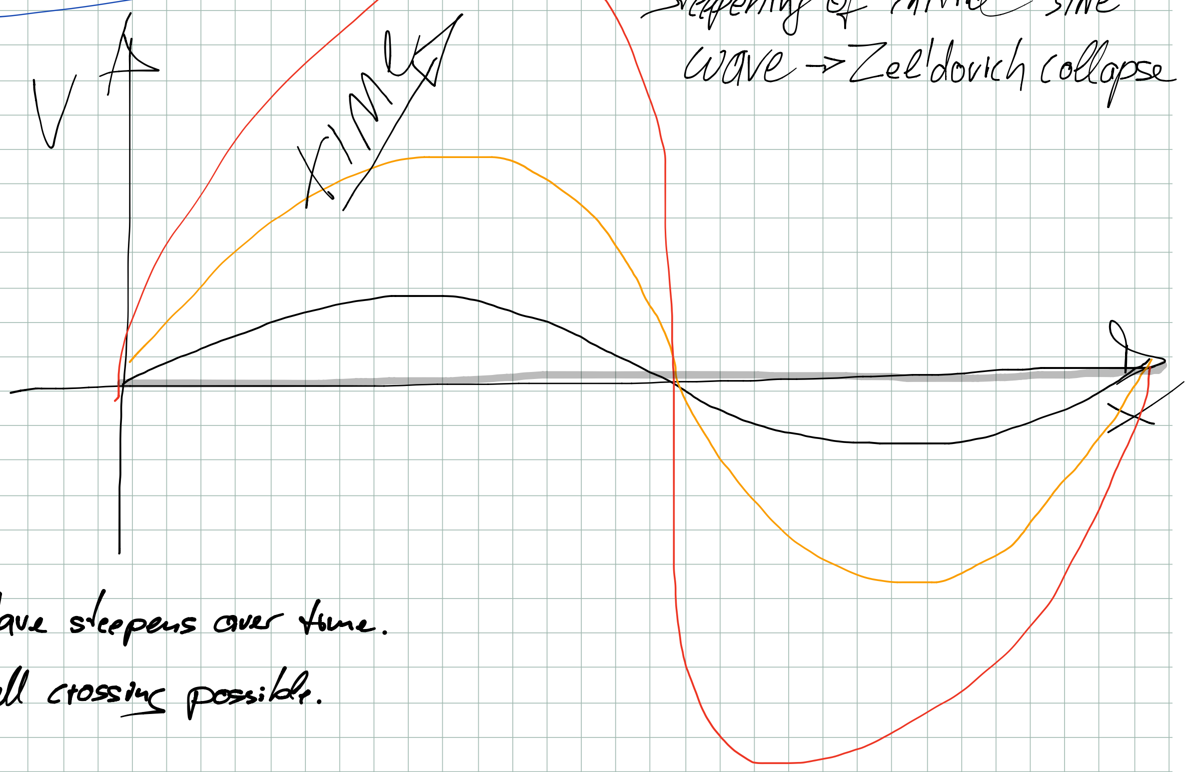


Tiny initial peculiar velocities

⇒ distribution function $f(\vec{x},\vec{v}) = f_0\,\delta(\vec{v})$

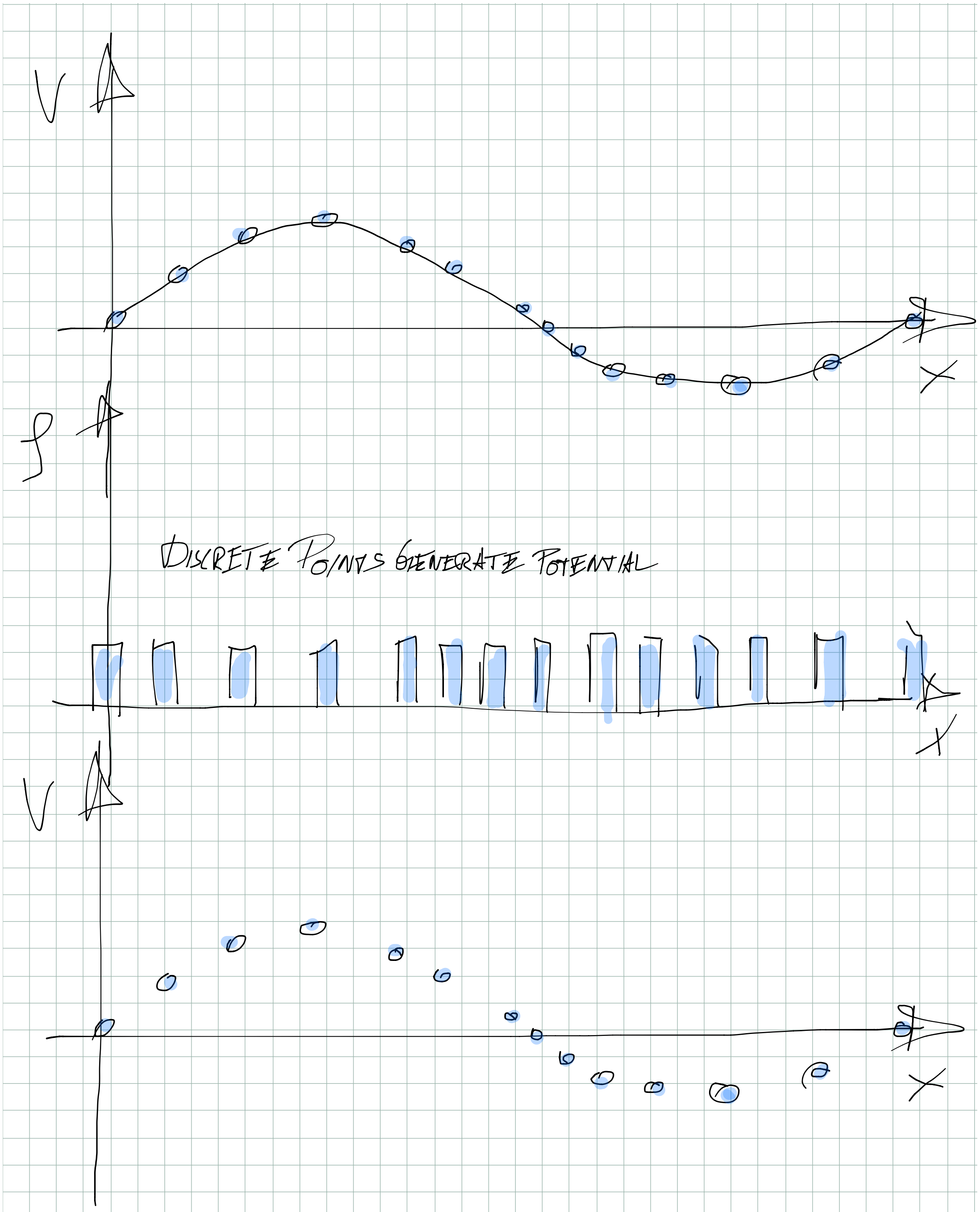
is single valued at every $\vec{x}$ DIRAC DELTA

PHASE SPACE



Wave steepens over time.

Shell crossing possible.

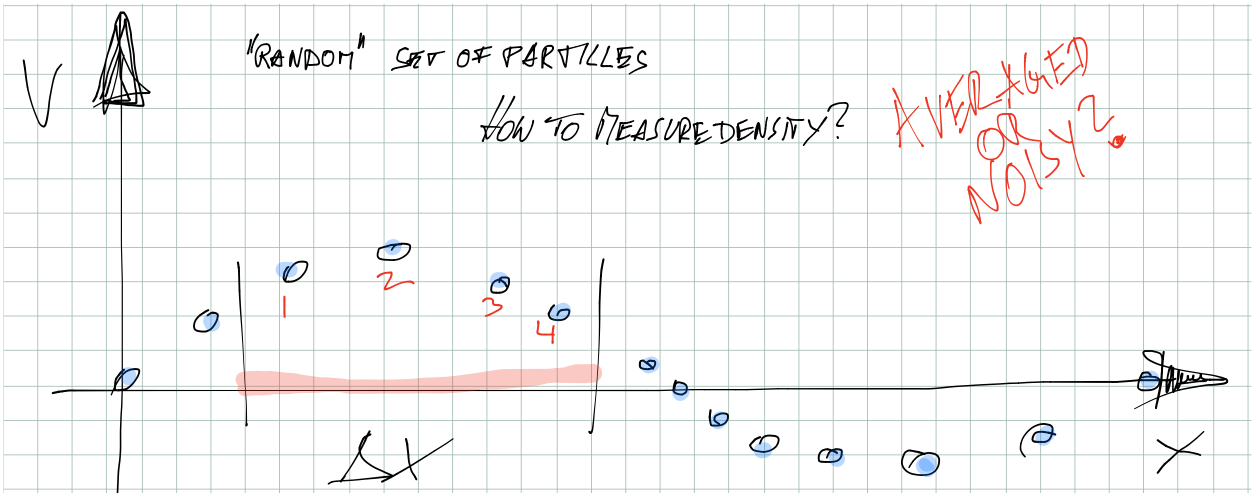


DISCRETE POINTS GENERATE POTENTIAL

"RANDOM" SET OF PARTICLES

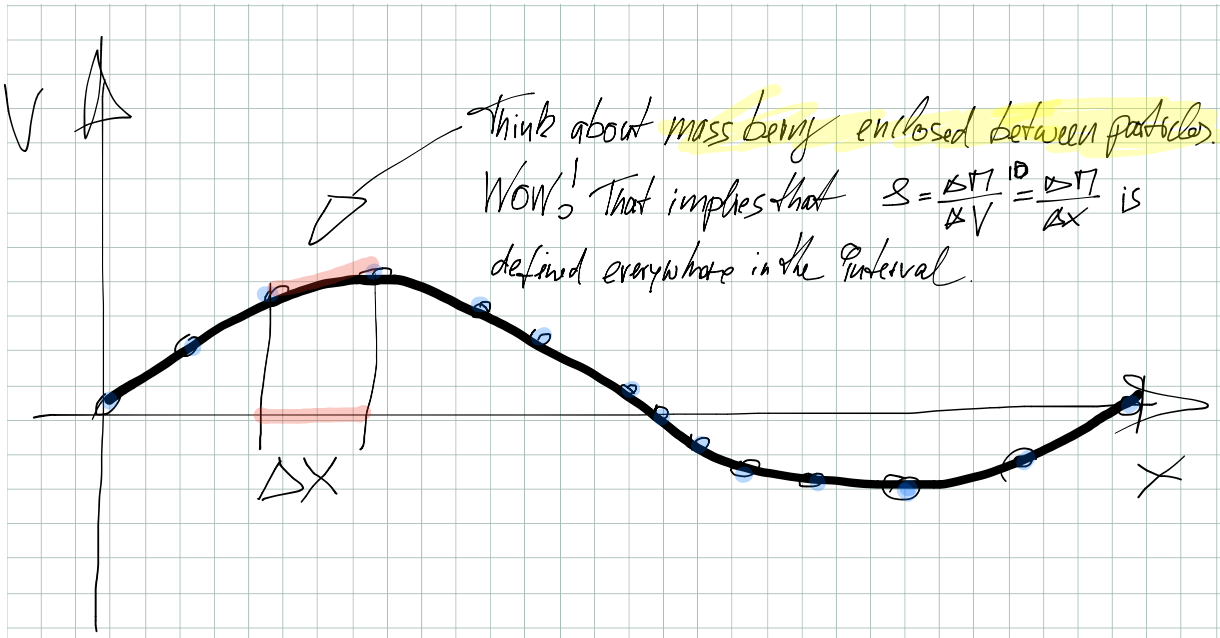
HOW TO MEASURE DENSITY?

AVERAGED OR NOISY?



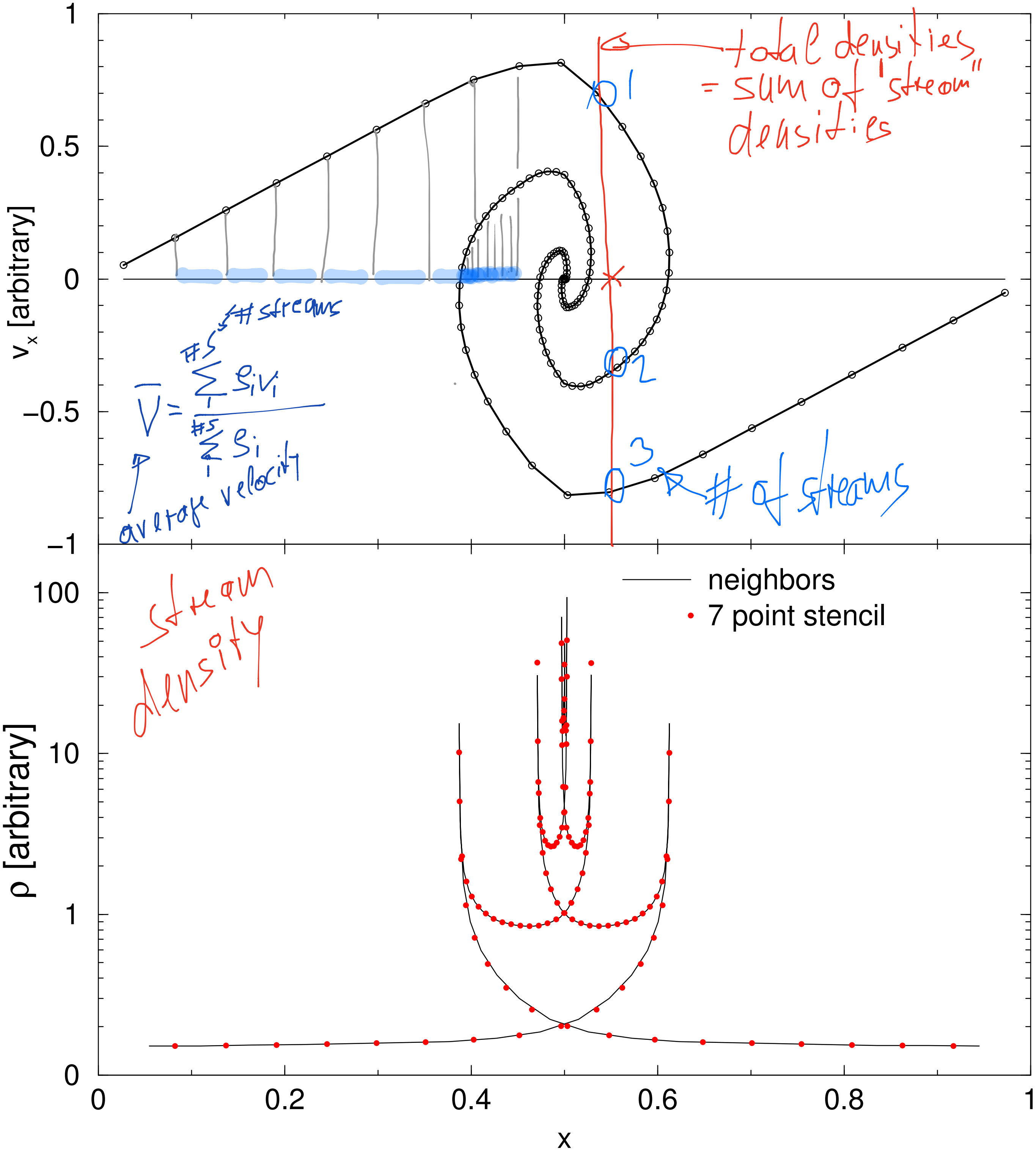
- PICK CONTROL VOLUME Δx & count # of particles: $\frac{N_{COUNT} \cdot m_i}{\Delta x} = \rho$

  ☹ that is an average density.
- Adaptive Kernel smoothing? Still an average...
- VORONOI? Assign every particle minimal volume around it. Not an average but potentially noisy...



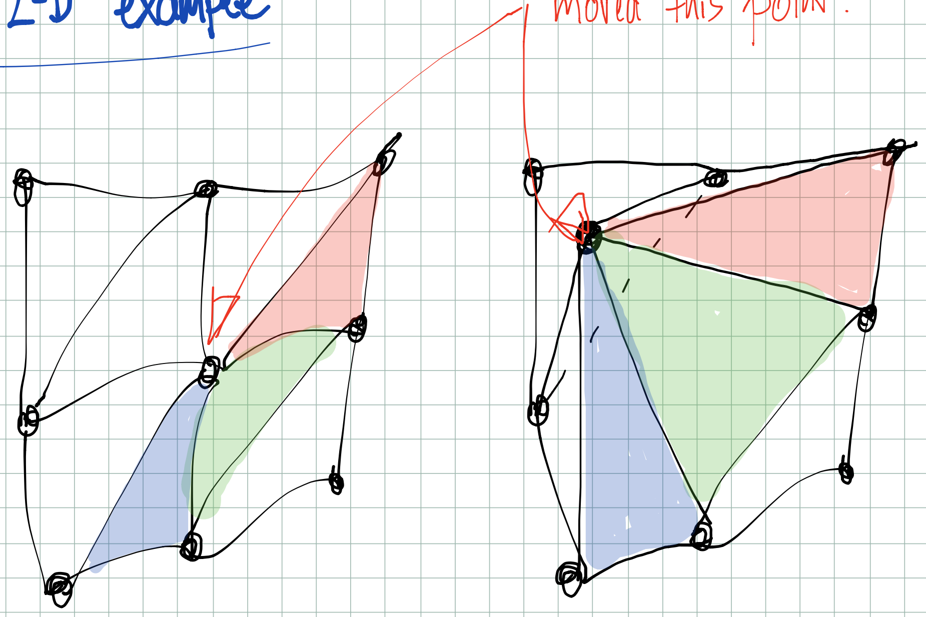
Think about mass being enclosed between particles.

WOW! That implies that $\delta = \frac{\Delta M}{\Delta V} \overset{1D}{=} \frac{\Delta M}{\Delta X}$ is defined everywhere in the interval.



$$\bar{V} = \frac{\sum_{1}^{\#S} \rho_i v_i}{\sum_{1}^{\#S} \rho_i}$$

A 2-D example



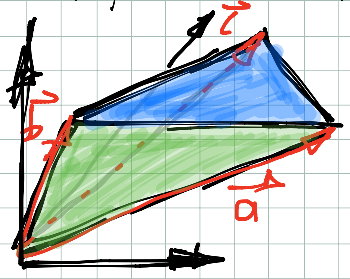
moved this point.

1D → LINE ; 2D → triangle ; 3D → tetrahedron

- just "painting" triangles with a weight $\frac{1}{V} = \frac{2}{\vec{a} \times \vec{b}}$

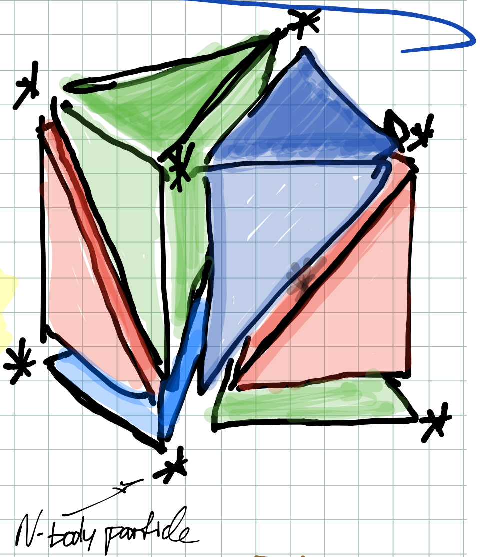
3 dimensional manifold in 6D Phase Space

- Natural tessellation takes unit cube & splits it into six equal size tetrahedra.
- mass per tetrahedron = 1/6 of DM particle mass.

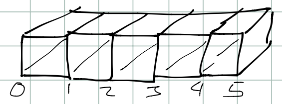


$$V = \frac{|\vec{a}\cdot(\vec{b}\times\vec{c})|}{6}$$

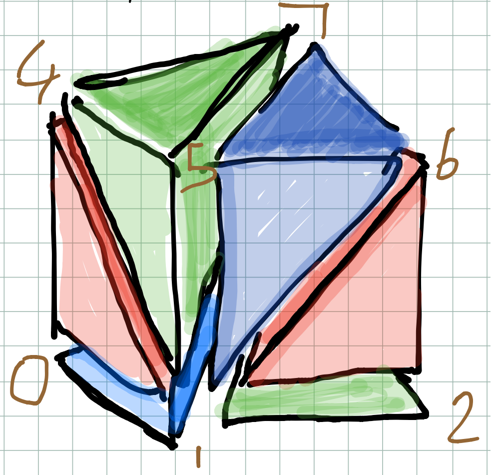
$$\Rightarrow \rho = \frac{m_p}{6V} = \frac{m_p}{|\vec{a}\cdot(\vec{b}\times\vec{c})|}$$



- Number the edges of the cube
- think of lattice
- Looping over

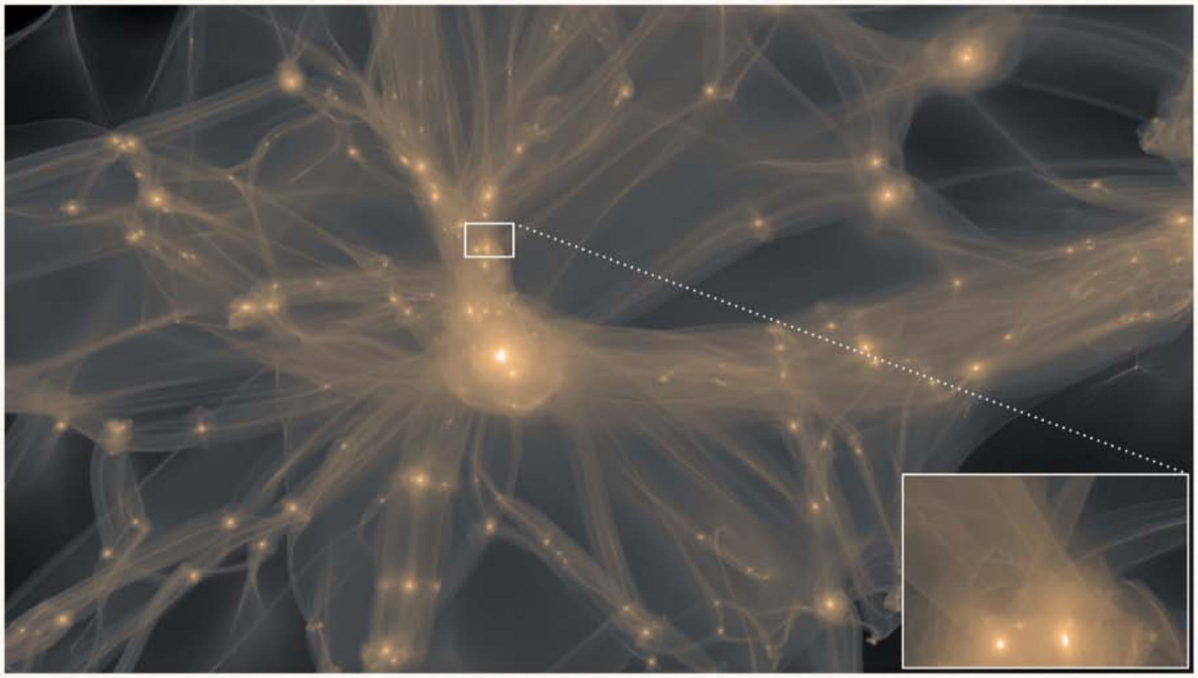


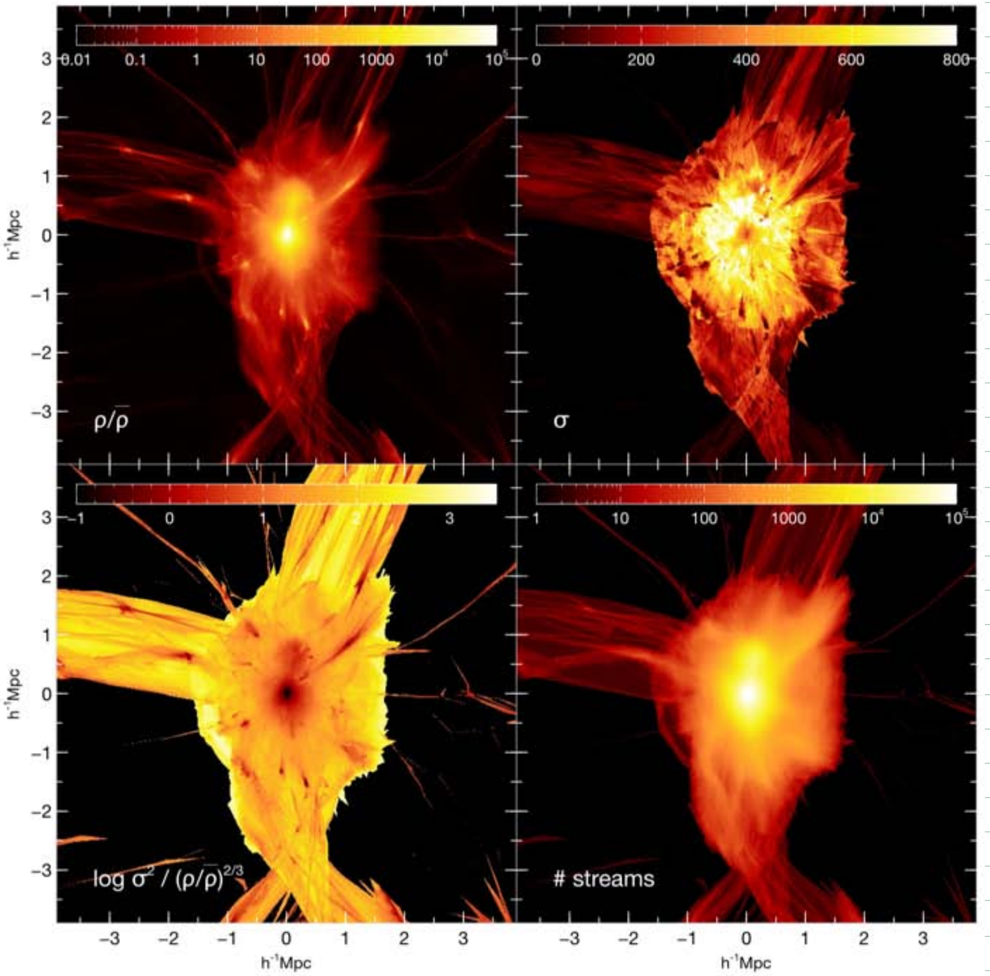
the initial cartesian (LAGRANGIAN) lattice generates the 6N tetrahedra.



A Novel Approach to Visualizing Dark Matter Simulations

Ralf Kaehler, Oliver Hahn and Tom Abel




0.01
0.1
1
10
100
1000
10^4
10^5
0
200
400
600
800
−1
0
1
2
3
1
10
100
1000
10^4
10^5
$\rho/\bar{\rho}$
σ
$\log \sigma^2 / (\rho/\bar{\rho})^{2/3}$
streams
h^{-1}Mpc
−3
−2
−1
0
1
2
3

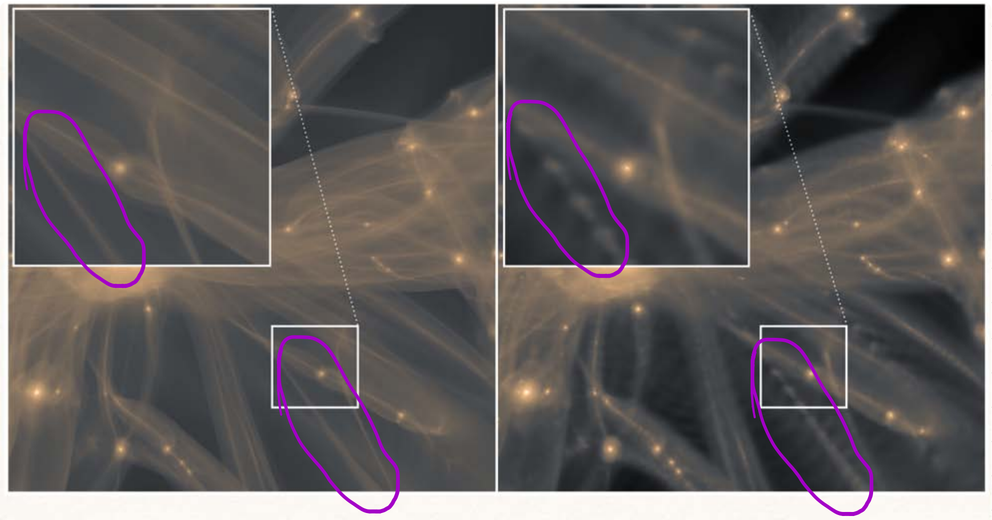


Fig. 9. A direct comparison between our tetrahedral cell-projection approach (left) and a standard SPH adaptive kernel smoothing method. Artifacts due to the poor density estimates in low-density regions are obvious for the SPH method, whereas the tetrahedral approach achieves an overall high image quality, on small and large structures.

Application to same N-body data

New approach gives exact densities

Recovers all caustics

Adaptive Kernel smoothing shows a number of unphysical clumps

ABEL, HAHN & KAEHLER 2011
KAEHLER, HAHN & ABEL 2012

COSMOLOGICAL N-BODY SIMULATIONS

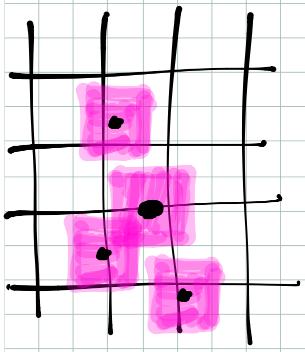
DISCRETIZE PHASE SPACE DISTRIBUTION BY PARTICLES

STANDARD PARTICLE MESH:

Evolve $\dot{\vec{x}}_i = \vec{v}_i$, $\dot{\vec{v}}_i = -\nabla\phi$ for all bodies

DERIVE $\vec{a}$:
- ASSUME PARTICLES ARE CUBES
- DEPOSIT MASS ON GRID $\rightarrow \rho$
- FFT $\rho \rightarrow \tilde{\rho}$
- MULTIPLY GREENSFUNCTION $\rightarrow \tilde{\phi}$

  SOLVING $\nabla^2\phi = 4\pi G\rho$
- FFT BACK $\rightarrow \phi$
- DIFFERENTIATE $\rightarrow \nabla\phi$
- INTERPOLATE TO PARTICLE POSITIONS



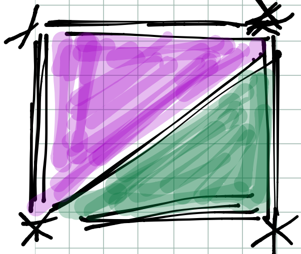
$\Longrightarrow$ SOFTENED ACCELERATIONS @ $\sim 2\Delta x$

TCM: TETRAHEDRON CENTROID MESH

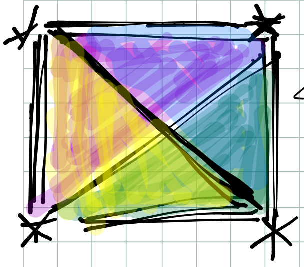
NEW

- NEW WAY TO DO N-BODY SIMULATIONS
- Massless TRACERS moving along characteristics.
- These span tetrahedra.
- FIRST IMPLEMENTATION: Monopole approximation
  → MASS OF TET DEPOSITED AS POINT PARTICLE @ CENTROID LOCATION
  - OTHERWISE IDENTICAL TO A PARTICLE MESH CODE

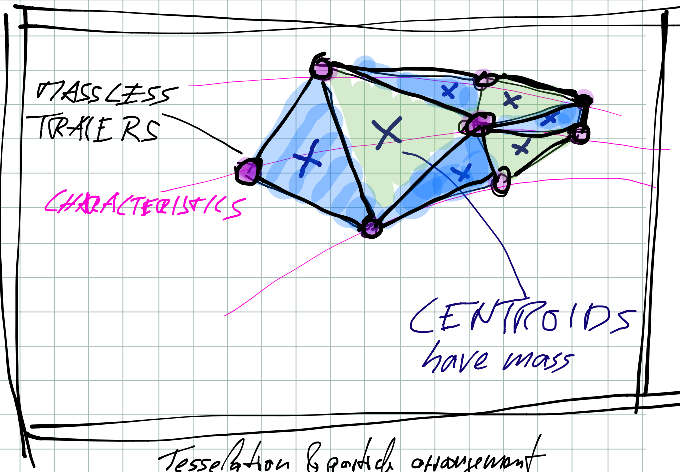
2 TYPES OF PARTICLES



⇐ PROPER TESSELATION
2D : 2 △'s per □
3D : 6 △ per □



⇐ SYMMETRIC VERSION
2D : 4 △'s per □
3D : 8 △ per □



Tesselation & particle arrangement in Tet·PM

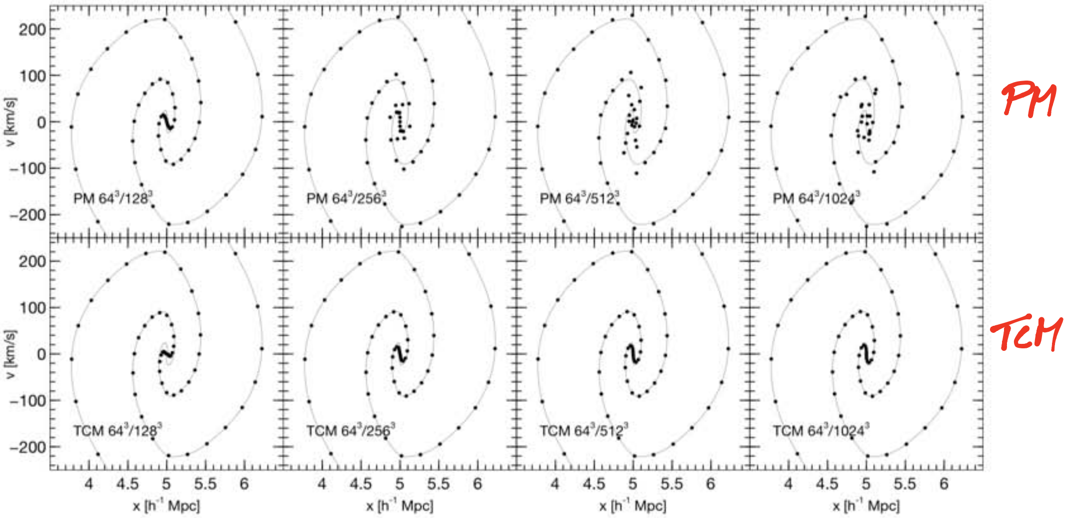


Figure 7. Central region of the axis-parallel plane wave at an expansion factor of 7.7 after shell crossing. Shown are the results for the standard PM method with 64^3 particles (top row) and for the TCM method with 64^3 tracers (bottom row). The left panels show results for a 128^3 mesh to compute forces, the middle panels for 256^3 cells and the right for 512^3, respectively. The thin grey line is the solution obtained with standard PM with 512^3 particles and 512^3 mesh cells. Two body collisions destroy the spiral in the standard PM case for higher force resolutions.

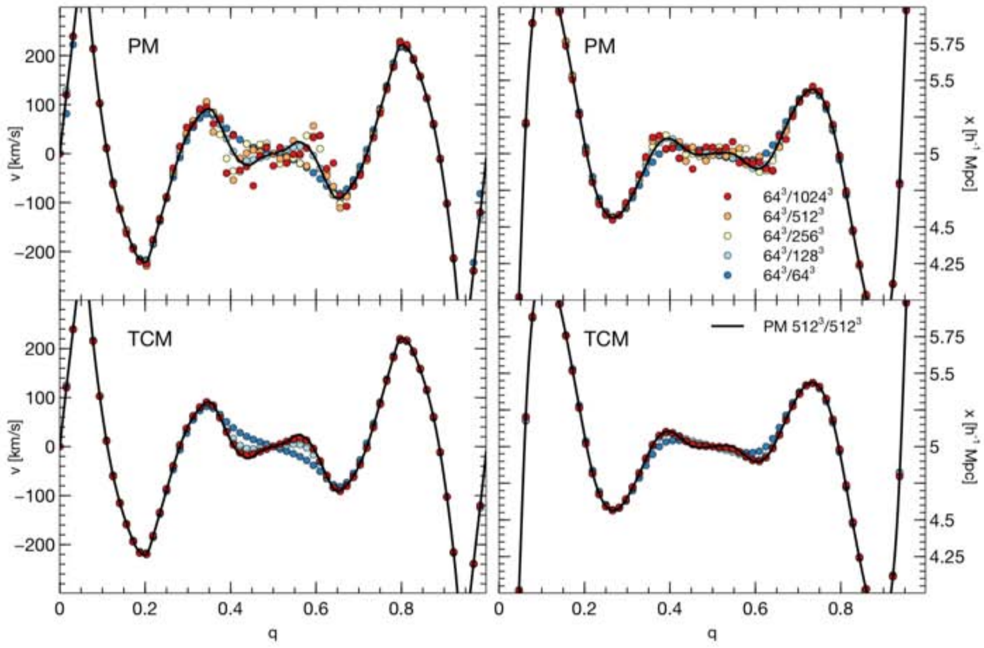
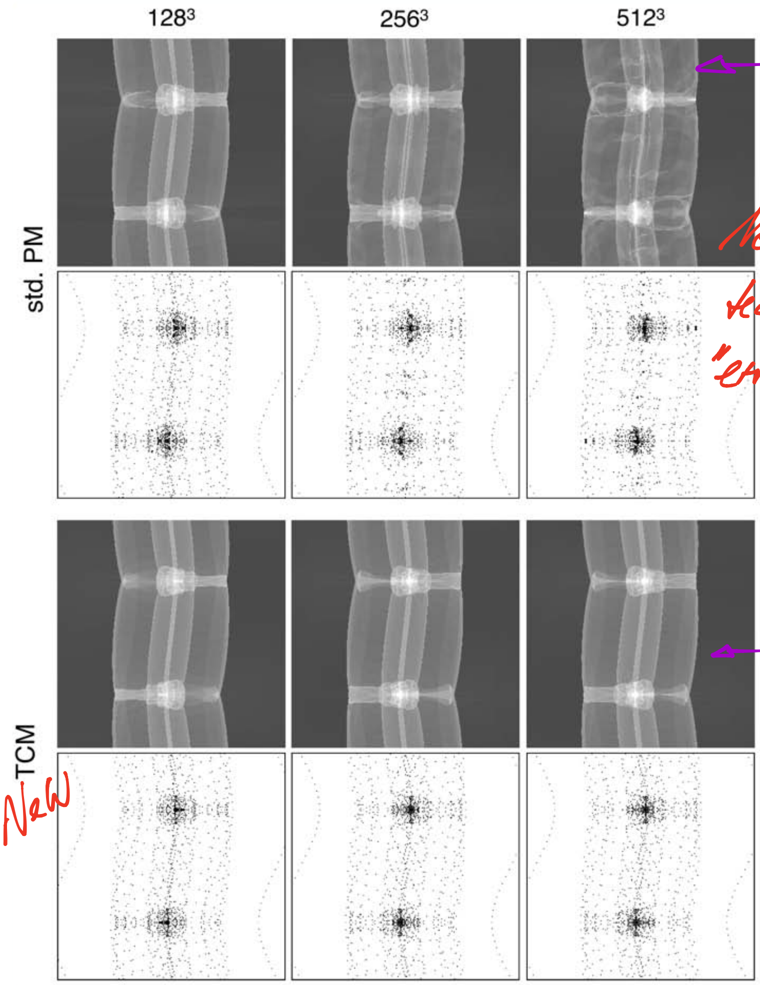


Figure 8. Velocity (left column) and position (right column) as a function of the Lagrangian coordinate in the axis parallel plane wave problem at the same time shown in Figure 7. The top row shows the effect of resolution increase in the standard PM case, the bottom row the respective results for the TCM case. In both cases the particle number is fixed at 64^3 and the resolution of the mesh is varied. The black line is from the high resolution result obtained with 512^3 particles and a 512^3 mesh to compute forces.



New

two body scattering undesirable ...

Note how the new visualization technique helps in spotting "errors" in the N-body integration

Constant mass resolution
Vary force resolution

excellent convergence behavior of our new method.

$\log_{10} \|\nabla\phi\|$

TCM

PM

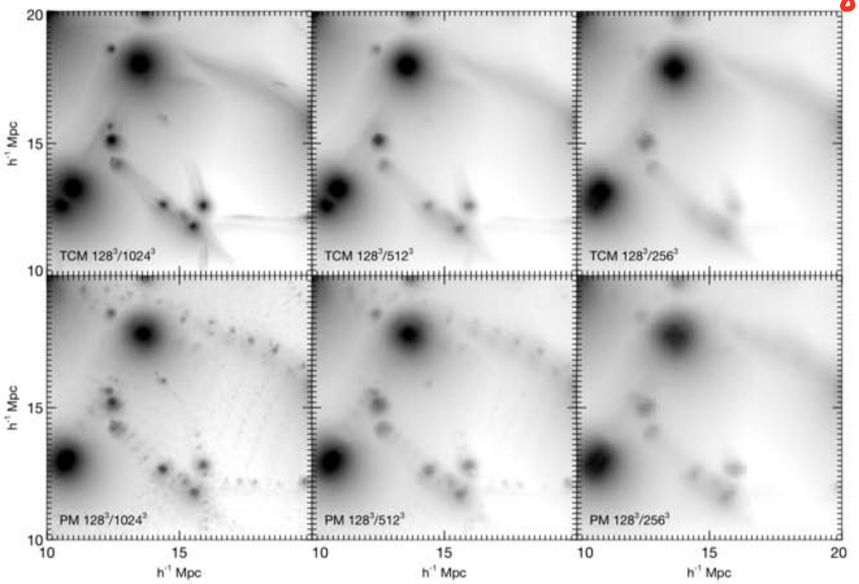


Figure 14. Maximum intensity projections of the magnitude of the gravitational force, $\log_{10} \|\boldsymbol{\nabla}\phi\|$, in a region $10 \times 10 \times 40\, h^{-3}\mathrm{Mpc}^3$ for simulations with fixed mass and varying force resolution. The image resolution directly corresponds to the resolution of the mesh used to compute the gravitational force in each simulation. The force resolution decreases from 1024^3 cells (left column) to 512^3 cells (center) to 256^3 (right column). (Top row:) results for the TCM method, using 128^3 tracer particles. (Bottom row:) results for the standard PM method with 128^3 particles. The artificial fragmentation of the filaments is clearly visible for the standard PM method at high force resolution. No such fragmentation is visible for TCM.

No artificial fragmentation in TCM SIMULATIONS IN HDM SIM

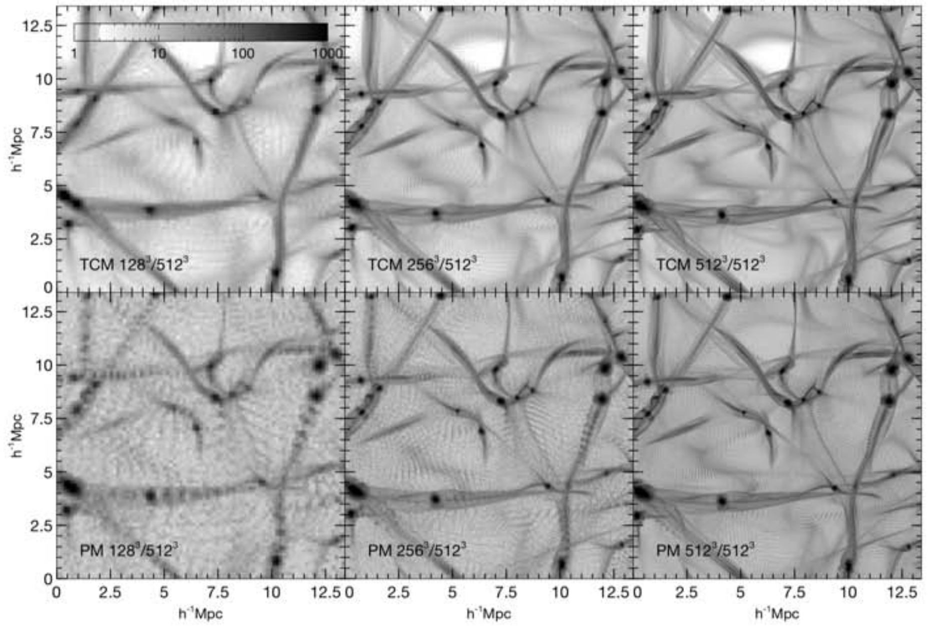
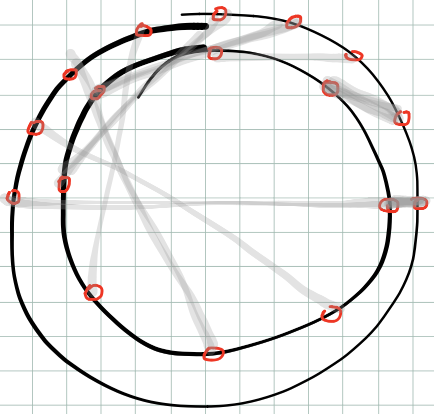
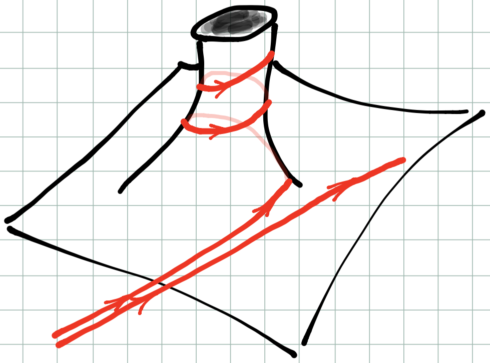


Figure 15. Density fields at fixed force resolution with increasing mass resolution for the two methods. Shown is the maximum density projection of a $13 \times 13 \times 40h^{-1}$Mpc region in the 128^3, 256^3 and 512^3 particle simulations for TCM (top row) and standard PM (bottom row). In all cases a 512^3 mesh was used to compute the forces. CIC density estimates were obtained on a mesh with two times the number of cells compared to the number of particles per linear dimension, i.e. 256^3 for the 128^3 particle runs for example. Compared to the PM simulations, all the beads-on-a-string artificial fragments are gone in TCM.

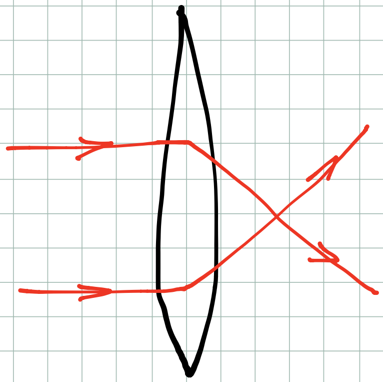
PHASE MISMATCH



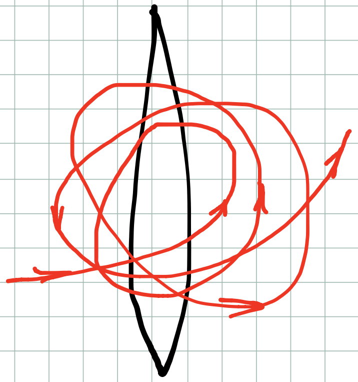
TOPOLOGY/CHAOS

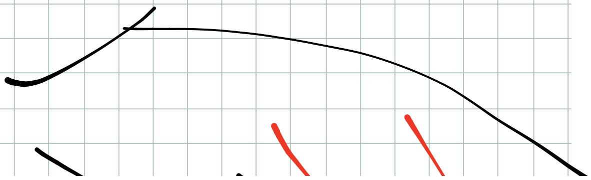


OPTICS

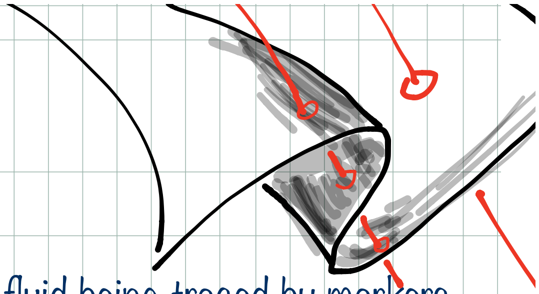


OPTICS GONE MAD





Conclusions:



- thinking of the dark matter distribution as a fluid being traced by markers yields a wonderful new way to study the results of N-body simulations.

- visualization techniques much superior to previous approaches:

  no artificial clumping

  much better contrast

  truthful to caustics

Powerful new techniques to solve the continuum limit.

Refinement in phase space effectively yields a multi resolution technique able to track fine grain phase space structure for dark matter.

New approaches also useful for studying collision-less plasmas.

Serious drawback: TCM has to resolve the sheet which however can grow exponentially in halos.... Refinement will help but not overcome this limitation.