PRACTICAL ANALYTICS

7/19/2012

Tamás Budavári / The Johns Hopkins University

Statistics

- Of numbers
- Of vectors
- Of functions
- Of trees

Statistics

- Description, modeling, inference, machine learning
- Bayesian / Frequentist / Pragmatist ?

	Supervised	Unsupervised
Discrete	Classification	Clustering
Continuous	Regression	Dimensional Reduction

What's Large?

- VOLUME
  - Say >100TB today but tomorrow? Moving target…
- COMPLEXITY
  - The raw dataset are simple unlike their derivatives
- DEFINITION?
  - Large when you cannot apply the "usual" tools

Data

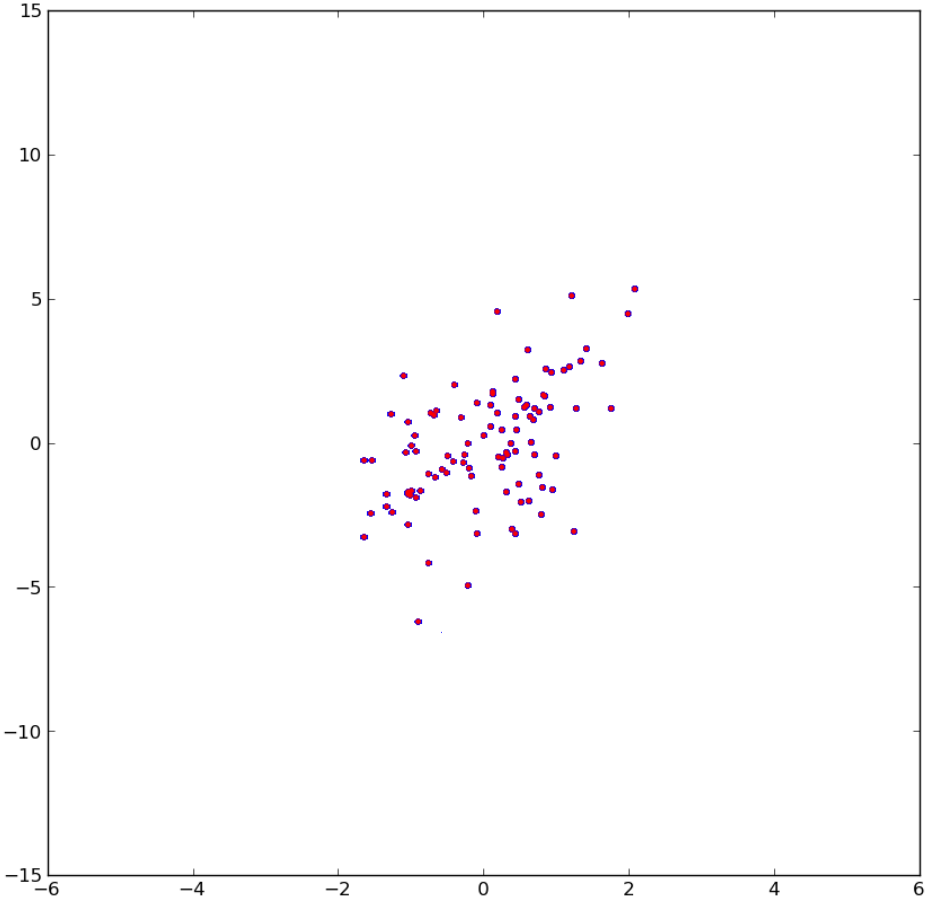
LARGE !!

Data

LARGE !!

Large?

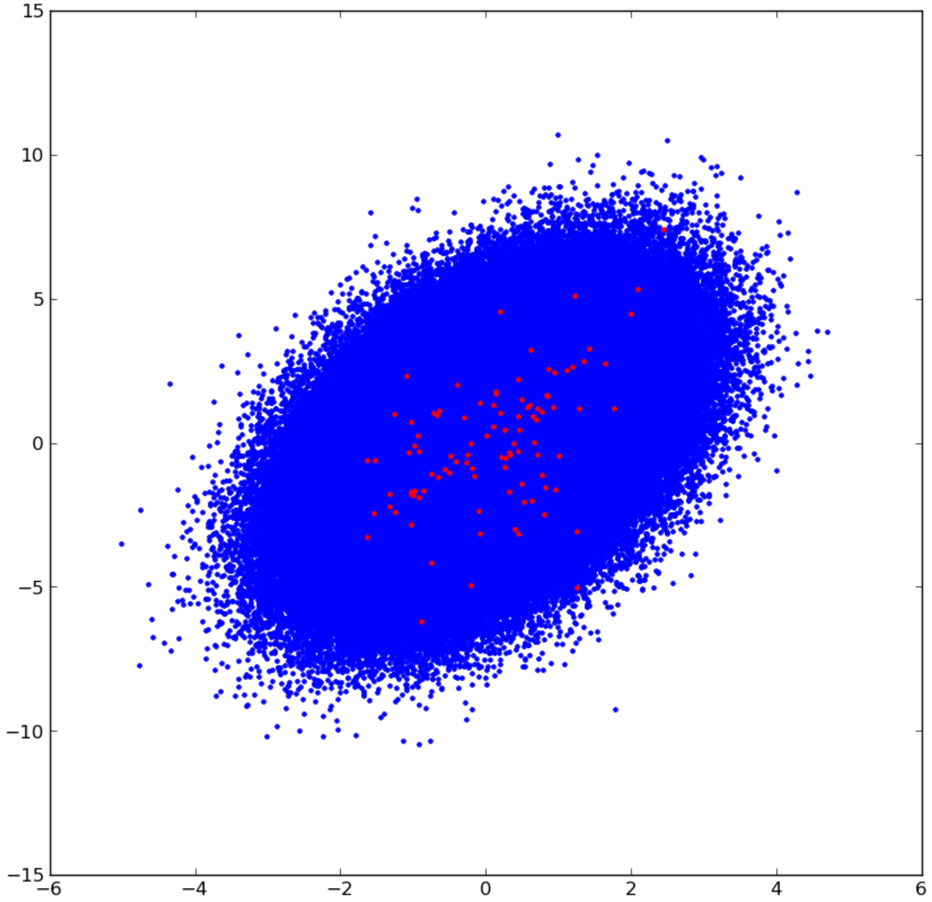
- Sample size



Large?

- Sample size

MORE OUTLIERS!

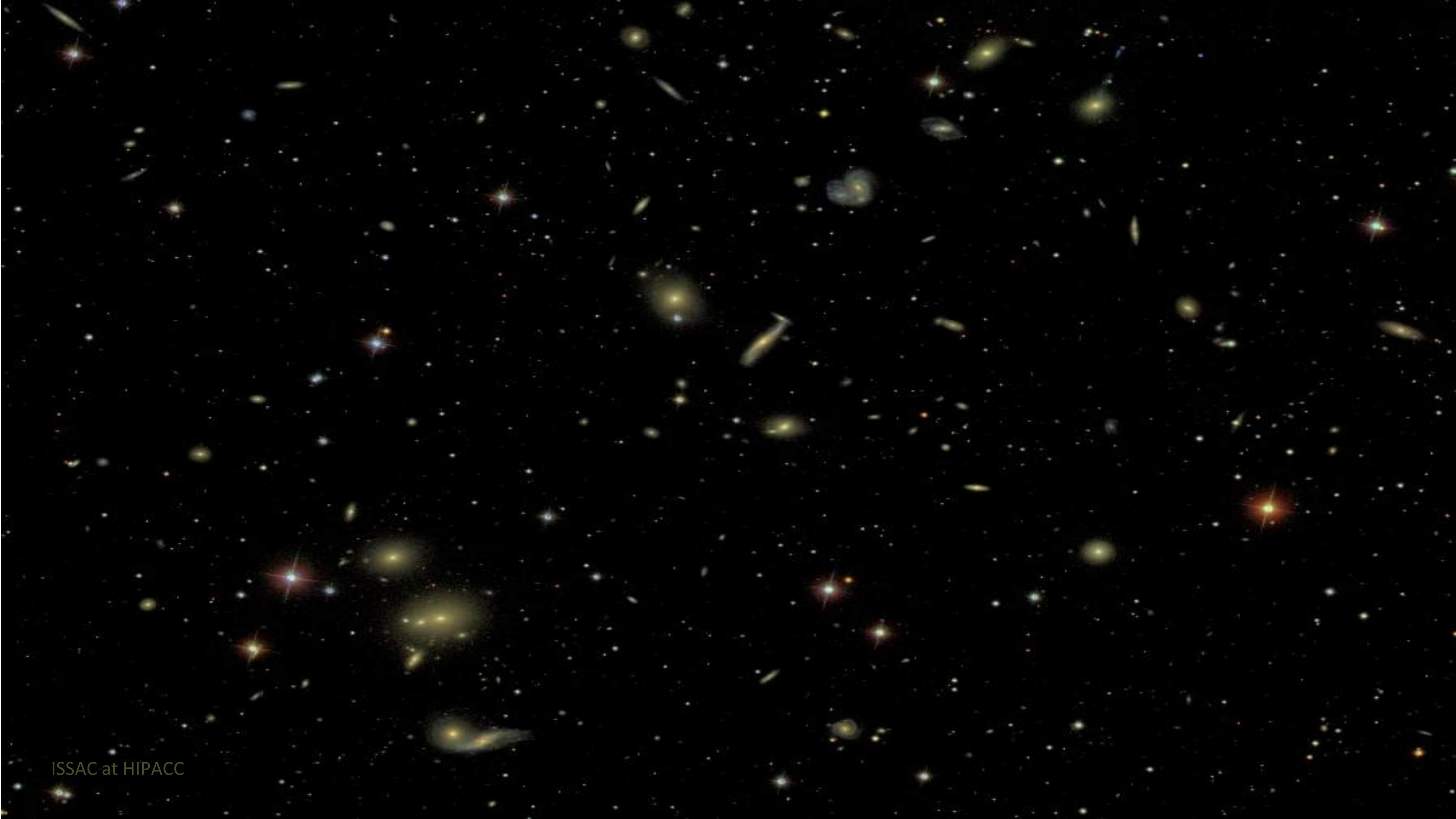


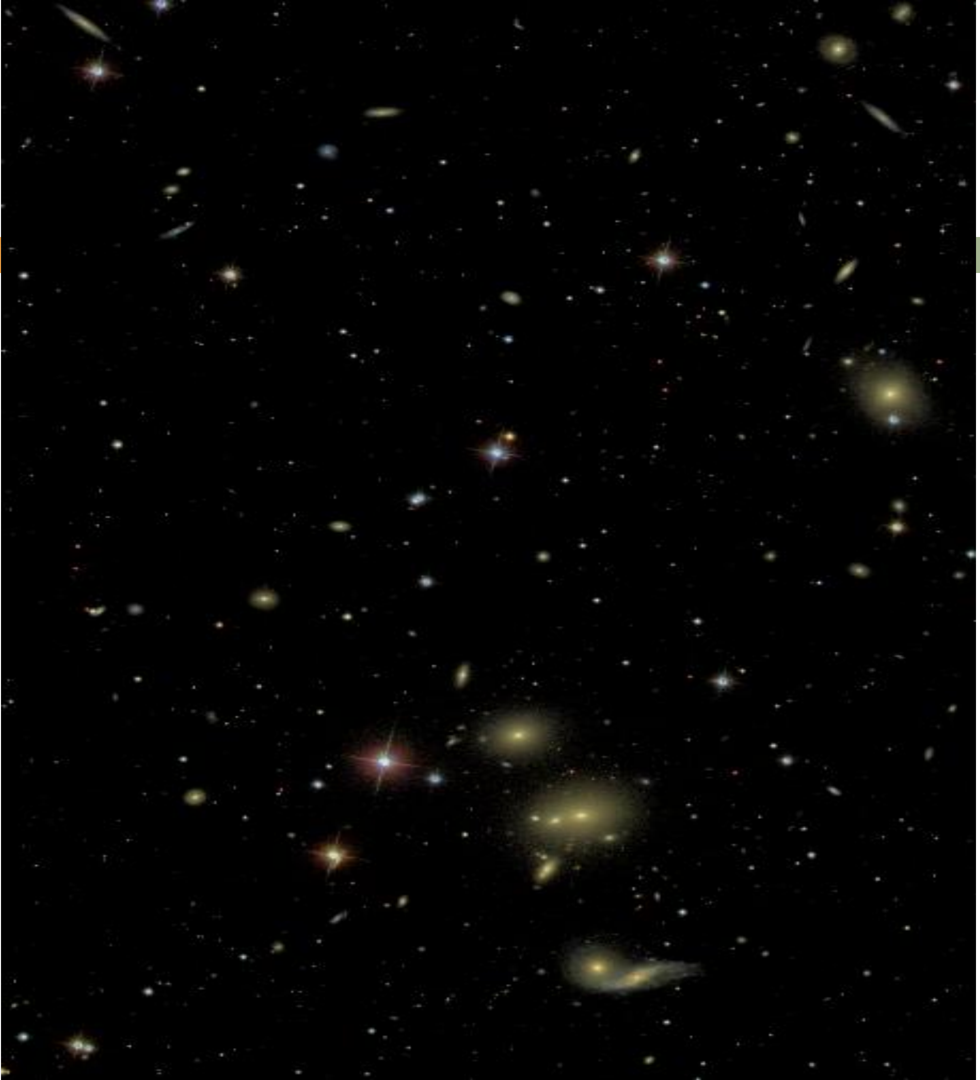
Large?

- Dimensions
  - Ratio of surface/volume grows

all points are lonely in high dimensions

THE CURSE OF DIMENSIONALITY!





Keeping Up?

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- Image processing
- Catalog extraction
  - $O(n)$
- What is difficult?
  - $O(n \log n)$
  - $O(n^2)$, …

Worse w/ Moore's law

Fundamental Challenges

- Cross-identification of sources
  - To assemble multicolor catalogs
- Drop-outs from sky coverage
  - To constrain fluxes not detected
- Constraining physical properties
  - To interpret the data

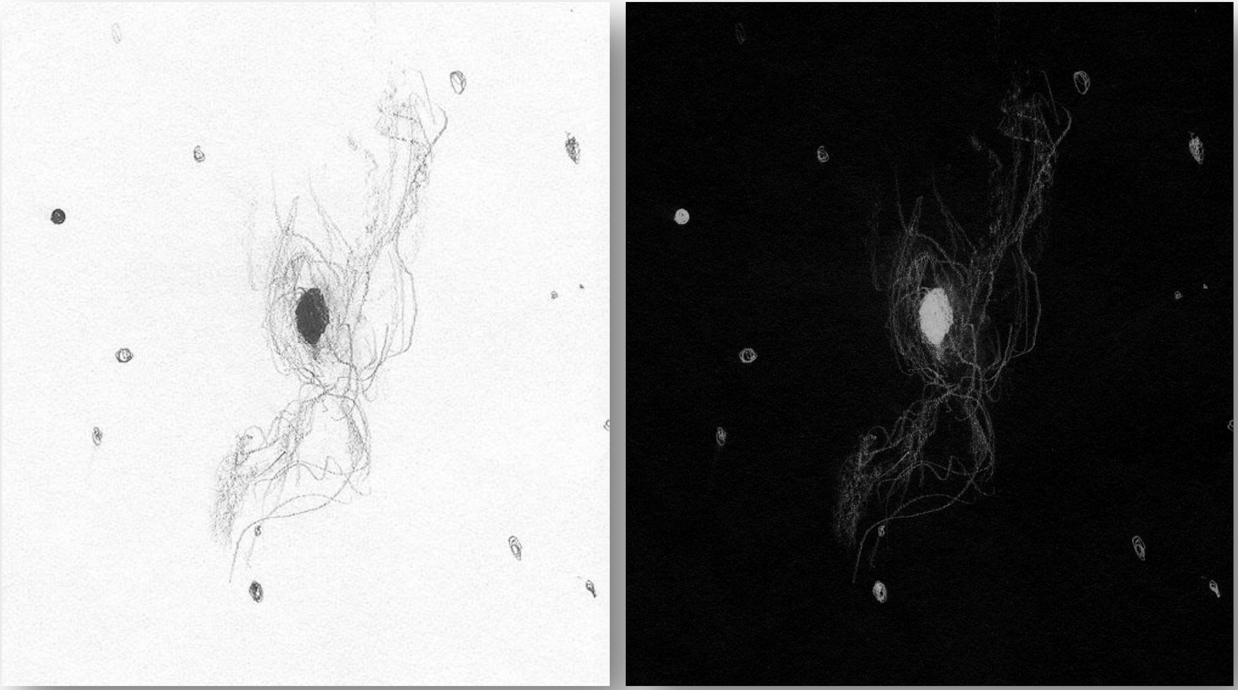
Cross-Identification

From long-tail science to the largest experiments

Recording Observations

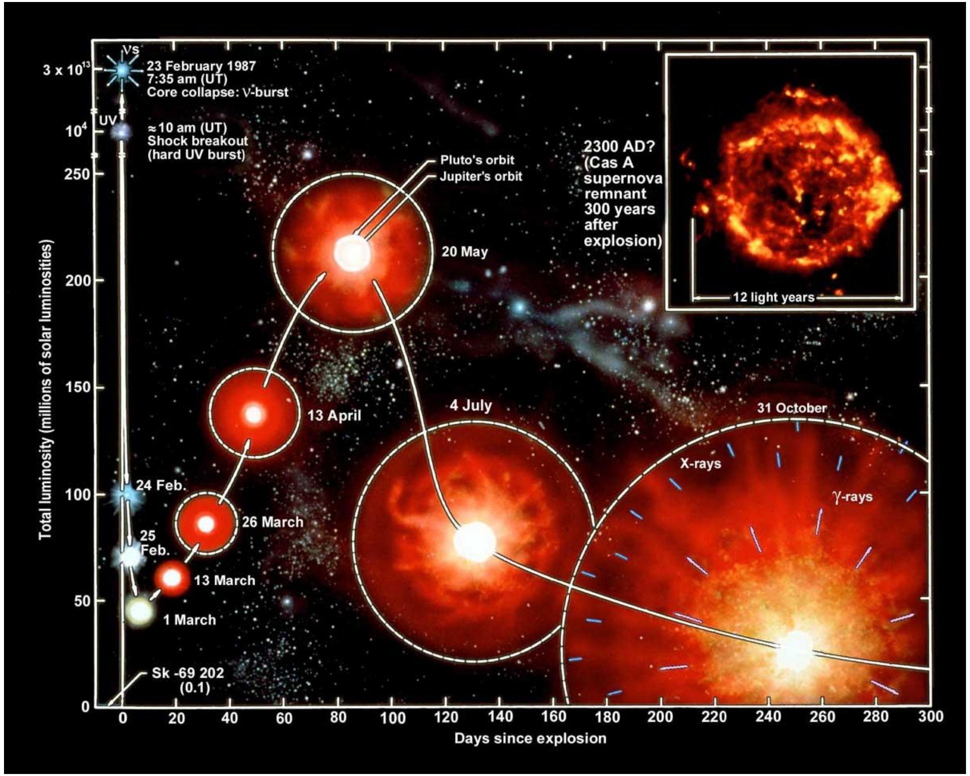
- Astronomers drew it…
- Now kids do it on SkyServer

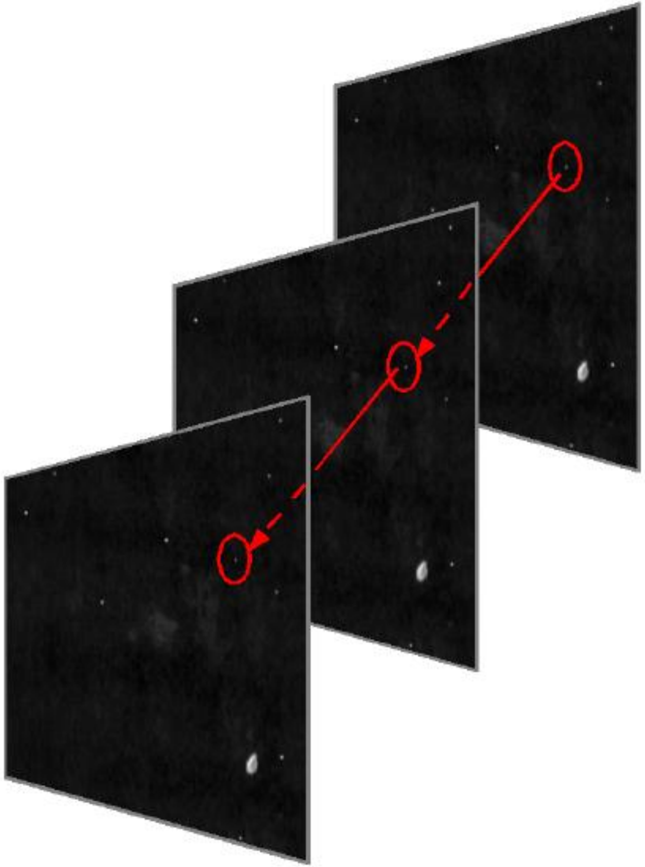
#1 by Haley ⇒



Multicolor Universe

Eventful Universe





Cross-Identification

One of the most fundamental analysis steps

What is the Right Question?

- Cross-identification is a hard problem
  - Computationally, Scientifically & Statistically
  - Need symmetric *n*-way solution
  - Need reliable quality measure

- Same or not?
  - Distance threshold? Maximum likelihood?

Tabletop Astronomy

- Imagine the observed sky has only 6 pixels
  - ***One object***: one die
  - ***Observing***: rolling a die
  - ***Locality***: die is loaded
  - ***Sky***: a bag of dice

Model Comparison: Same or Not?

- ***Crossmatch***: draw two dice with replacement
  - Same or not?

Model Comparison: Same or Not?

- ***Crossmatch***: draw two dice with replacement
  - Same or not?
- Bayes Factor is the ratio of the
  - Likelihood of "Same"
  - Likelihood of "Not"
- Likelihood of a hypothesis?
  - Sum over all possibilities

Model Comparison: Same or Not?

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- Model for loaded dice is matrix of probabilities
  - E.g., loaded toward l =1
  - Etc. for l =2…6

$$P_1(\text{⚀}) = \frac{3}{12},\quad P_1(\text{⚅}) = \frac{1}{12},\quad P_1(\text{⚁}) = \frac{2}{12},\ldots$$

- 2-way case
  - Same: $l_1 = l_2 = l$
  - Not: $l_1 \mathrm{may} \neq l_2$
- *n*-way: same

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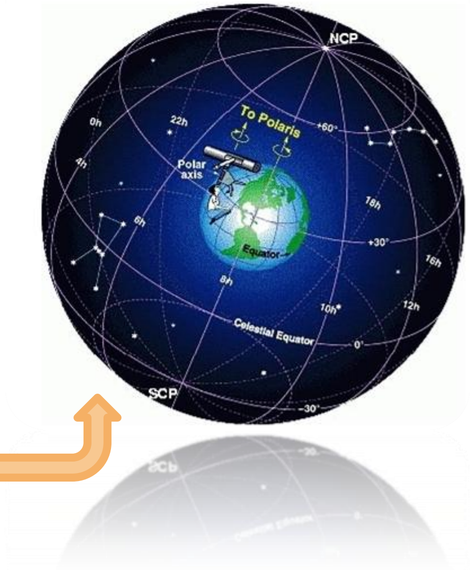
  - Not: $l_1 \,\text{may} \neq l_2$ $\qquad L_{\text{not}} = \left[\frac{1}{6} \sum_{l_1} P_{l_1}(\text{⚀})\right]\left[\frac{1}{6} \sum_{l_2} P_{l_2}(\text{⚃})\right]$
- *n*-way: same

Celestial Sphere

- Continuous functions
- General formalism
  - Accuracy is a density fn on sky



Modeling the Astrometry

- Astrometric precision
  - A simple function
- Where on the sky?
  - Anywhere really…

$$p(\vec{x}|\vec{m}, M)$$

Same or Not?

OR
- The Bayes factor

$$B(H,K|D) = \frac{p(D|H)}{p(D|K)} \qquad D = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$$

SAME
- *H:* all observations of the same object at *m*

NOT
- *K:* might be from separate objects at $\{m_i\}$

Same or Not?

OR
- The Bayes factor

SAME
- *H:* all

NOT
- *K:* mig

Bayes' Rule

$$p(\theta|D,M) = \frac{p(\theta|M)\,p(D|\theta,M)}{p(D|M)}$$

Same or Not?

OR SAME NOT

- The Bayes factor

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- *H:* all observations of the same object at *m*

$$p(D|H) = \int p(\vec{m}|H) \prod_{i=1}^{n} p_i(\vec{x}_i|\vec{m}, H)\, d^3m$$

On the sky

Astrometry

- *K:* might be from separate objects at $\{m_i\}$

Same or Not?

OR
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On the sky

Astrometry

NOT
- *K:* might be from separate objects at $\{m_i\}$

$$p(D|K) = \prod_{i=1}^{n} \left\{ \int p(\vec{m}_i|K)\, p_i(\vec{x}_i|\vec{m}_i, K)\, d^3m_i \right\}$$

Analytic Results

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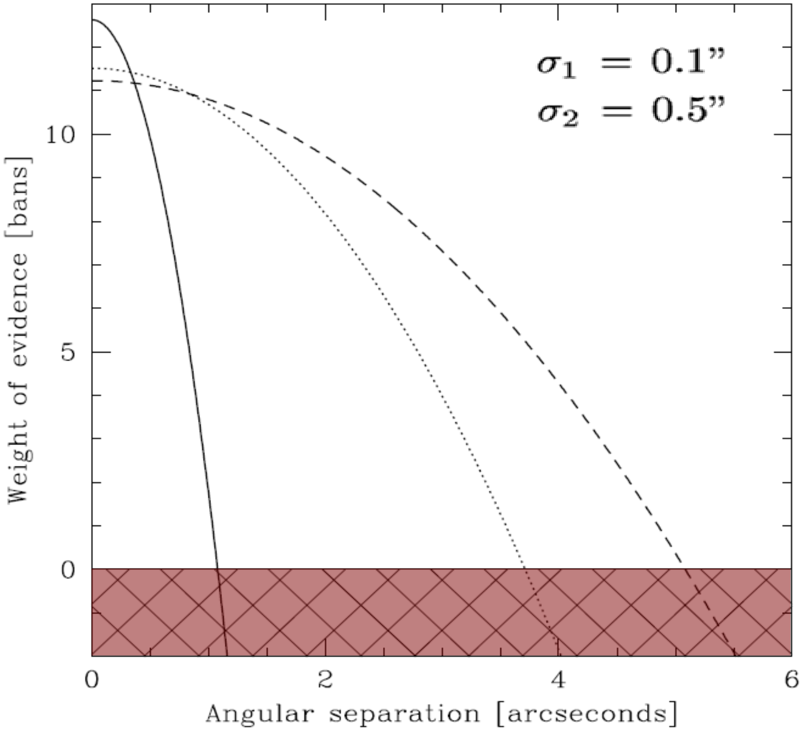
- Normal distribution
  - Flat and spherical
    - Gauss and Fisher

- 2-way results

$$B = \frac{2}{\sigma_1^2 + \sigma_2^2} \exp\left\{ -\frac{\psi^2}{2(\sigma_1^2 + \sigma_2^2)} \right\}$$



Normal Distribution

- Astrometric precision: $$w = 1/\sigma^2$$
- Fisher distribution: $$N(\vec{x}|w,\vec{m}) = \frac{w\,\delta(|\vec{x}|-1)}{4\pi \sinh w}\ \exp\left(w\,\vec{m}\vec{x}\right)$$
  - Analytic results:
$$B(H,K|D) = \frac{\sinh w}{w}\prod_{i=1}^{n}\frac{w_i}{\sinh w_i}, \qquad w = \left|\sum_{i=1}^{n} w_i \vec{x}_i\right|$$
    - For high accuracies: $$= 2^{n-1}\frac{\prod w_i}{\sum w_i}\exp\left\{-\frac{\sum_{i<j} w_i w_j \psi_{ij}^2}{2\sum w_i}\right\}$$

Wikipedia: Interpretation

B	dB	bits	Strength of evidence
< 1:1	< 0		Negative (supports M_2)
1:1 to 3:1	0 to 5	0 to 1.6	Barely worth mentioning
3:1 to 10:1	5 to 10	1.6 to 3.3	Substantial
10:1 to 30:1	10 to 15	3.3 to 5.0	Strong
30:1 to 100:1	15 to 20	5.0 to 6.6	Very strong
>100:1	>20	>6.6	Decisive

Probability of a Match

Same or not?

From Priors to Posteriors

- Bayes factor is the connection

$$\frac{P(H|D)}{P(\bar{H}|D)} = \frac{P(H)\,p(D|H)}{P(\bar{H})\,p(D|\bar{H})}$$

$$\frac{P(H|D)}{P(\bar{H}|D)} = \frac{P(H)}{P(\bar{H})}\;B(H,\bar{H}|D)$$

$$\frac{P(H|D)}{1-P(H|D)} = \frac{P(H)}{1-P(H)}\;B(H,\bar{H}|D)$$

$$P(H|D) = \left[1 + \frac{1-P(H)}{B\,P(H)}\right]^{-1}$$

From Priors to Posteriors

- Posterior probability from prior & Bayes factor

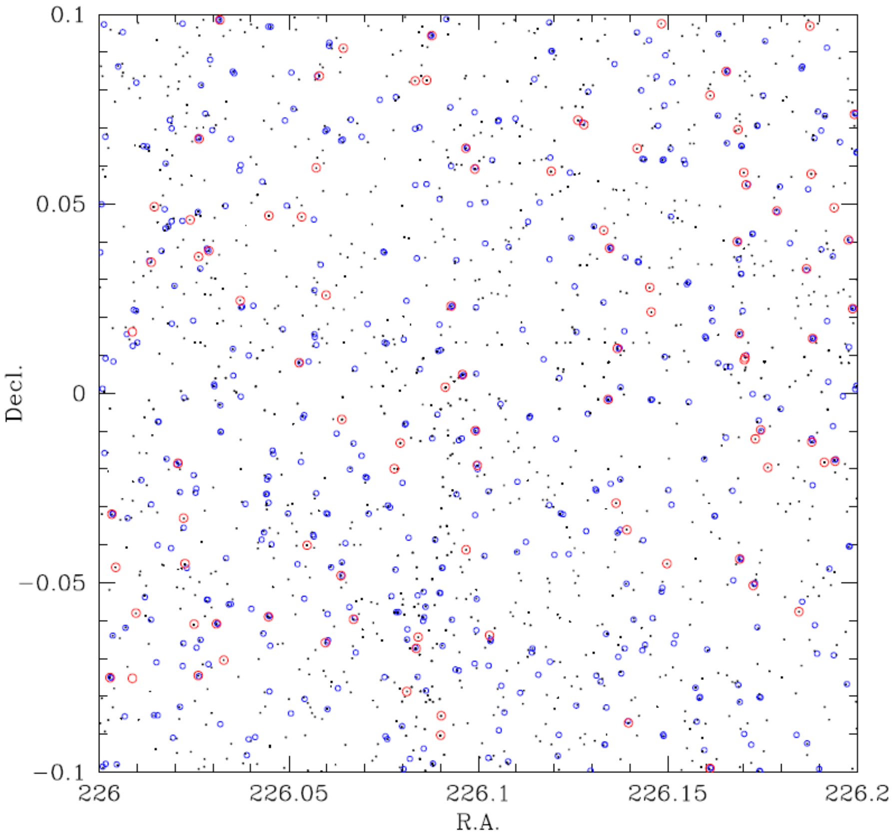
$$P(H|D) = \left[1 + \frac{1 - P(H)}{B\,P(H)}\right]^{-1}$$

- Prior probability of a match
  - Like dice in a bag: $1/N$ and N^{1-n}
  - In general?

From Priors to Posteriors

- Different selections
  - **Nearby** / Distant
  - Red / Blue
- But only 1 number

$$P_0 = \frac{N_\star}{\prod N_i}$$



Self-Consistent Estimates

- Prior has an unknown fudge-factor
  - Educated guess
    $$P(H|D) = \left[1 + \frac{1-P(H)}{B\,P(H)}\right]^{-1}$$
  - Or solve for it:

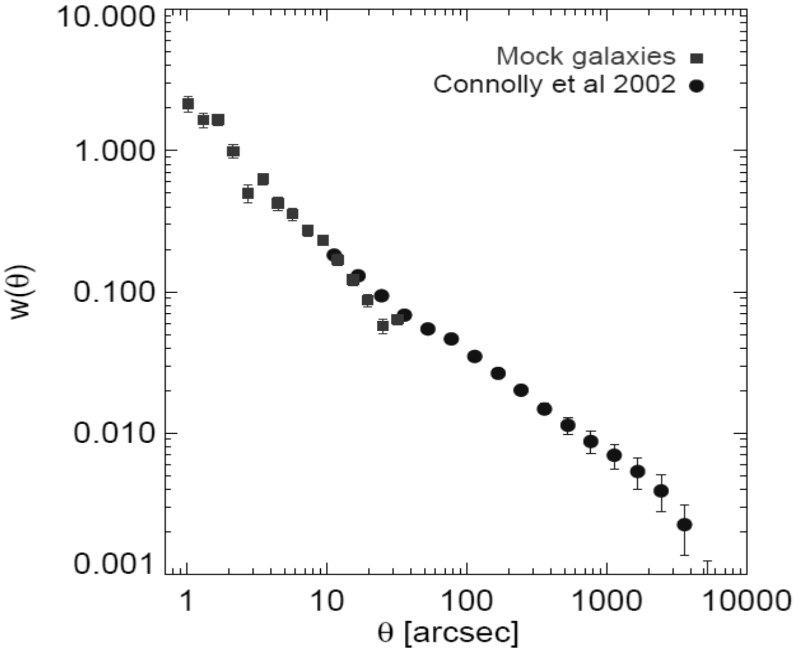
$$\left.\begin{array}{l}\sum P(H) = N_\star \\ \sum P(H|D) = N_\star\end{array}\right\}$$

TB & Szalay (2008)

Simulations

- Mock objects
  - With correct clustering
  - U_{01} values as properties

- Simulated sources
  - Subsets: N_1 N_2
  - Overlap: $N_\star$



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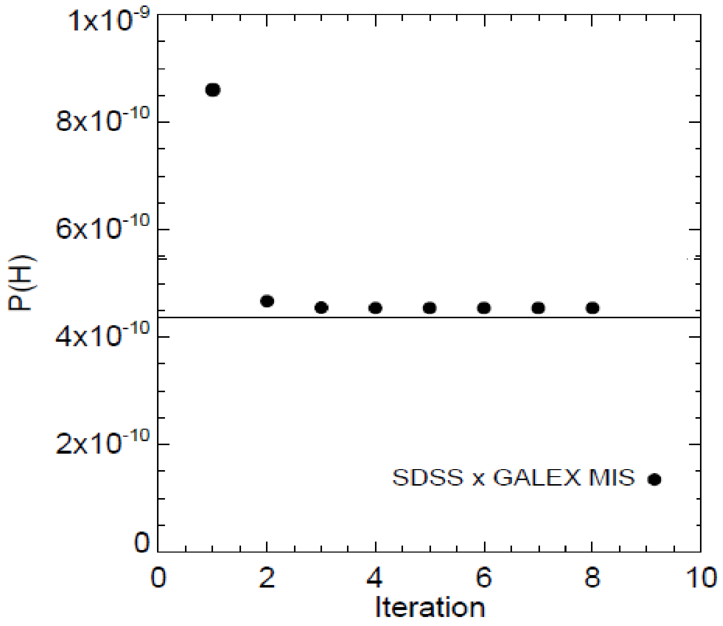
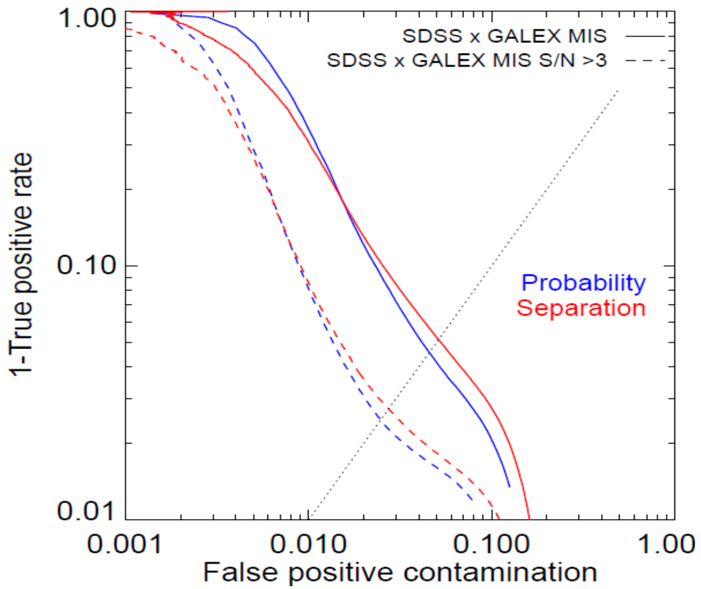


Simulations

- Quality



- Multiple matches

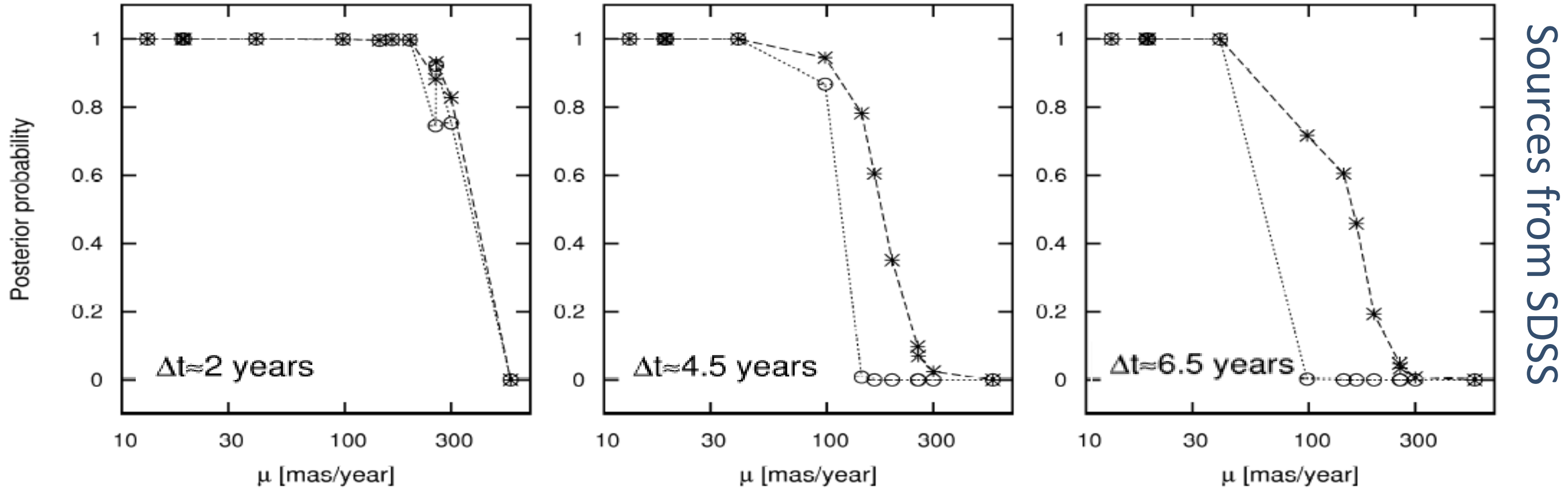
GALEX	SDSS 1	SDSS 2	SDSS Many
1	74.061 (75.870)	21.007 (18.595)	2.577 (2.469)
2	1.146 (2.253)	1.006 (0.697)	0.188 (0.102)
Many	0.006 (0.009)	0.007 (0.004)	0.002 (0.001)

Explained by simple model of point sources!

Heinis, TB, Szalay (2009)

Proper Motion

- Same hypotheses but different parameters
  - Just need μ prior to integrate



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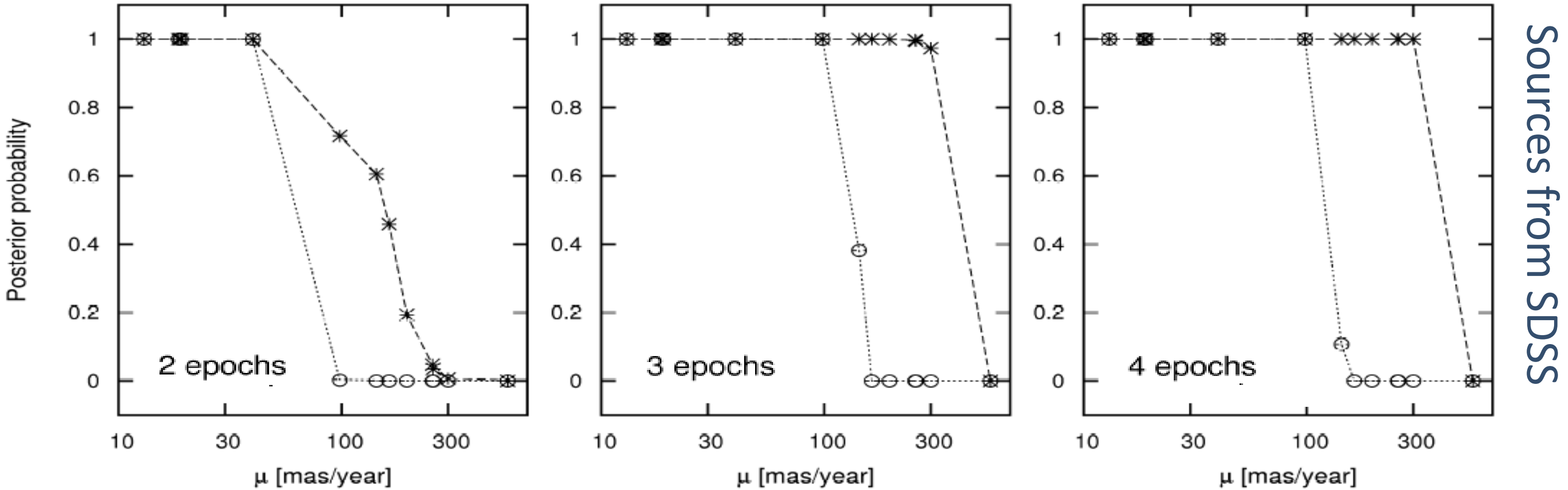
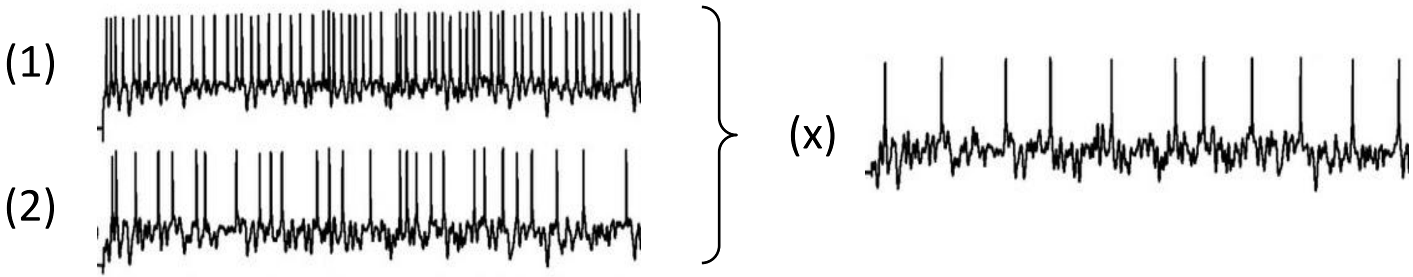
Kerekes, TB+ (2010)

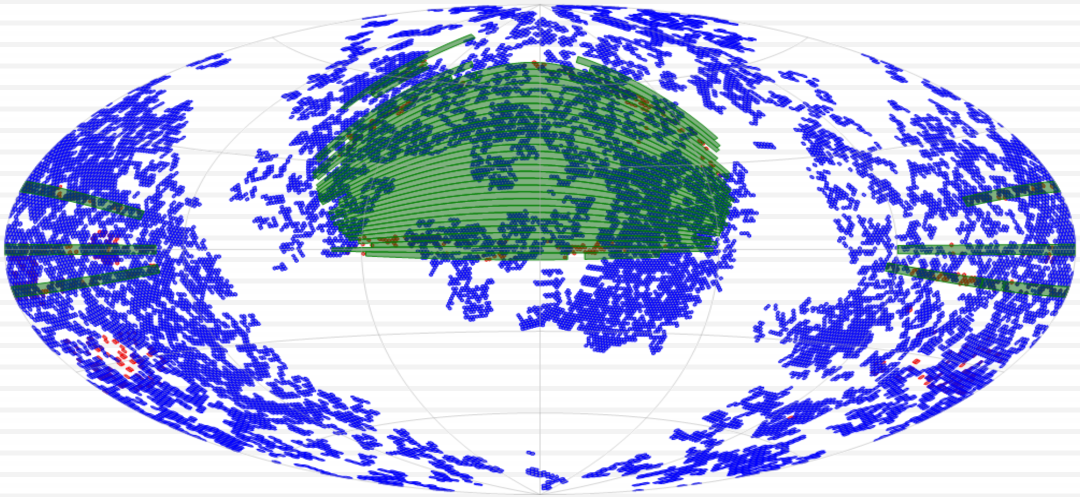


Matching Events

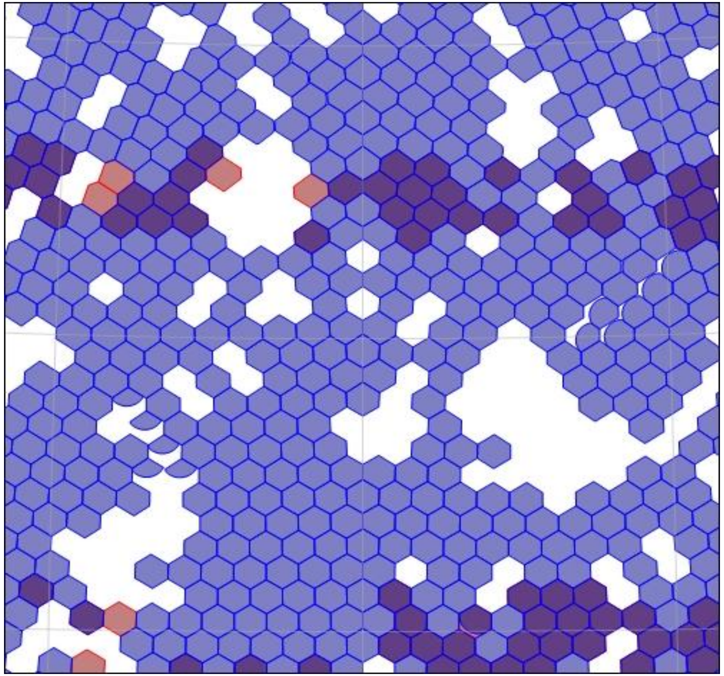
- Streams of events in time and space
  - E.g., thresholded peaks in signal-to-noise



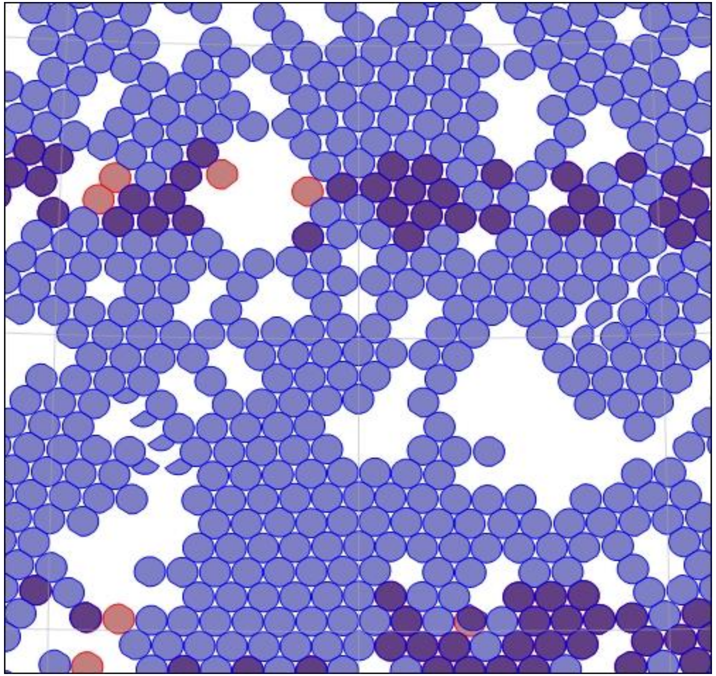
Dropouts from Sky Coverage



Drawing with Equations



$r = 0.6°$



$r = 0.5°$

TB, Szalay & Fekete (2010)

Matching in Practice

Open SkyQuery

- Following our 1st prototype
- Successful
- Not bayesian
- Limitations

SkyQuery – The 3rd Generation

- Dynamic federation of astronomy databases
  - Query the collection as if they were one
- The 3rd gen tool coming this summer
  - Cluster of machines running partitioned jobs
  - Proper probabilistic exec with variable errors

SkyQuery

- Almost pure standard SQL

```sql
SELECT p.ObjID, p.RA, p.Dec,
       s.BestObjID, s.SpecObjID, s.RA, s.Dec
INTO xtest
FROM SDSSDR7:PhotoObjAll AS p
    CROSS JOIN SDSSDR7:SpecObjAll AS s
WHERE
    p.RA BETWEEN 0 AND 5
    AND p.Dec > -9999
    AND s.Dec > -9999
    AND s.RA > -9999
```

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XMATCH BAYESIAN AS x
    MUST p ON Point(p.RA, p.Dec), 0.1, 0.1, 0.1
    MUST s ON Point(s.RA, s.Dec), 0.1, 0.1, 0.1
    HAVING LIMIT 1e3
WHERE
    p.RA BETWEEN 0 AND 5
    AND p.Dec > -9999
    AND s.Dec > -9999
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```

SkyQuery

- Almost pure standard SQL
- Added XMATCH
  - Verifiable
  - Flexible

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    AND s.RA > -9999
```

SkyQuery

username: dobos | account | log out

home | schema browser | query | job history | my db | import | output | help

Check | Submit | Output: | Task name:

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SELECT p.ObjID, p.RA, p.Dec,
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    AND s.RA > -9999
```

HST Crossmatch Catalog β RELEASE AT AAS

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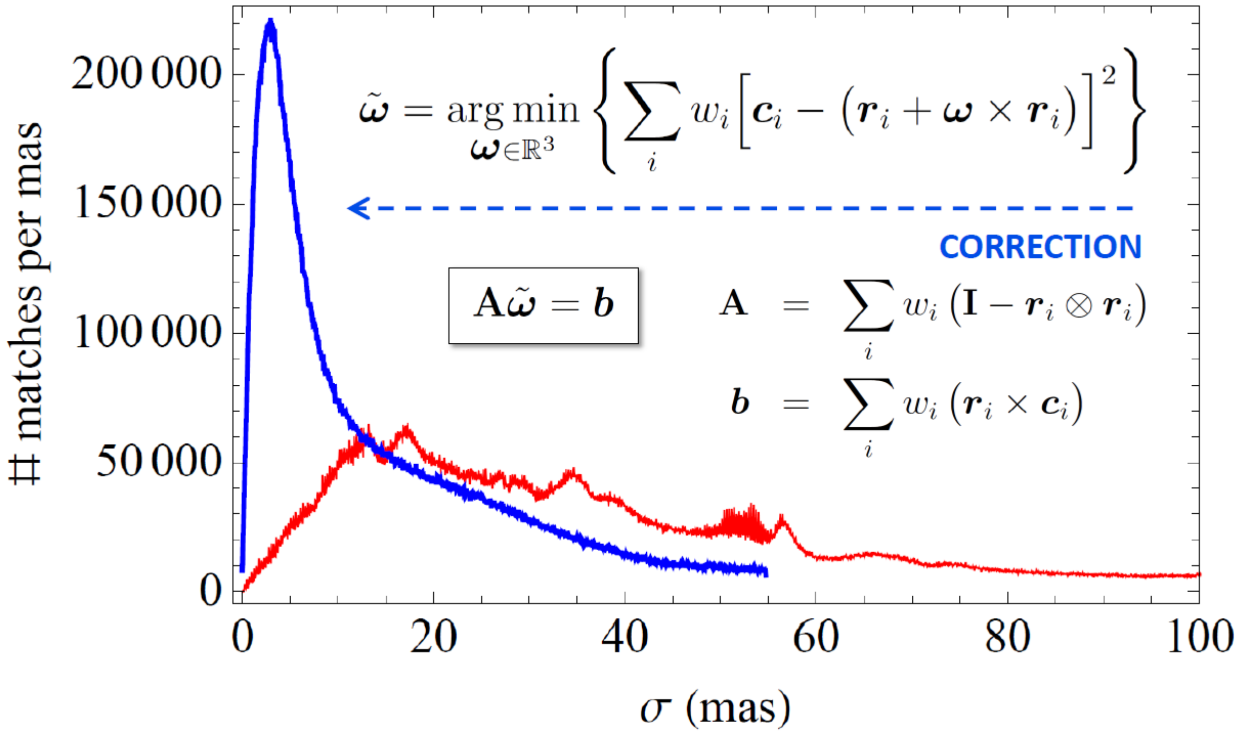
- SQL pipeline
- Astrometric correction
  - Subpixel precision



HST Crosstmatch Catalog β RELEASE AT AAS

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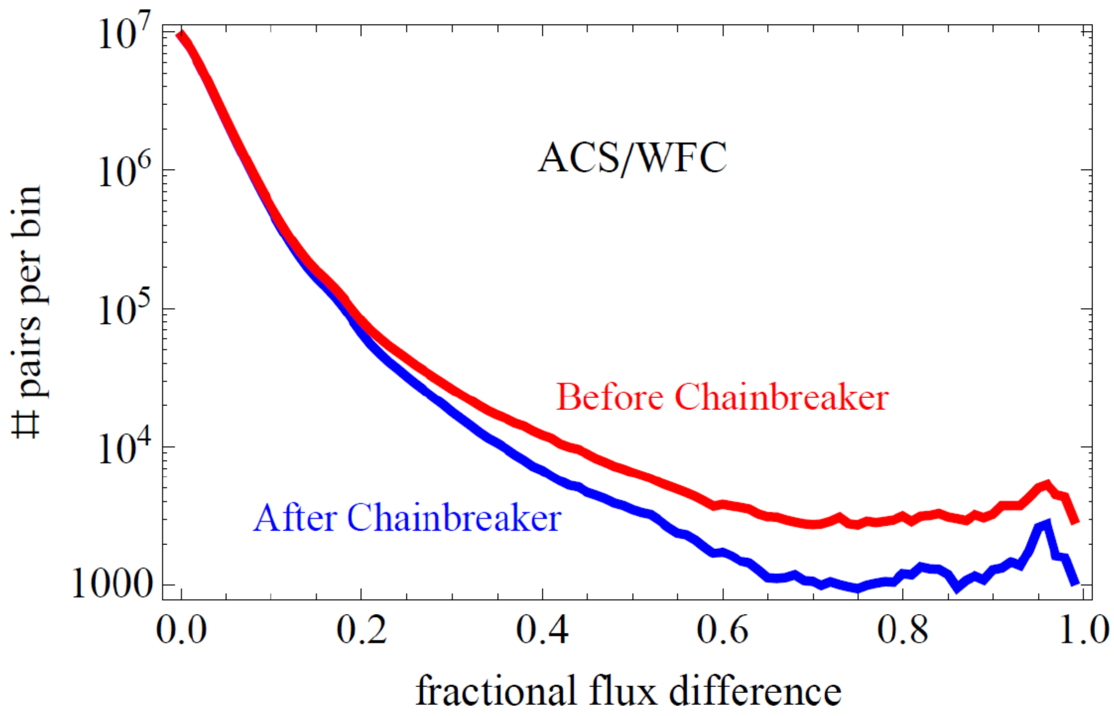
- FoF groups
  - Possible chains
- Bayesian model selection
  - Chainbreaker



HST Crossmatch Catalog β RELEASE AT AAS

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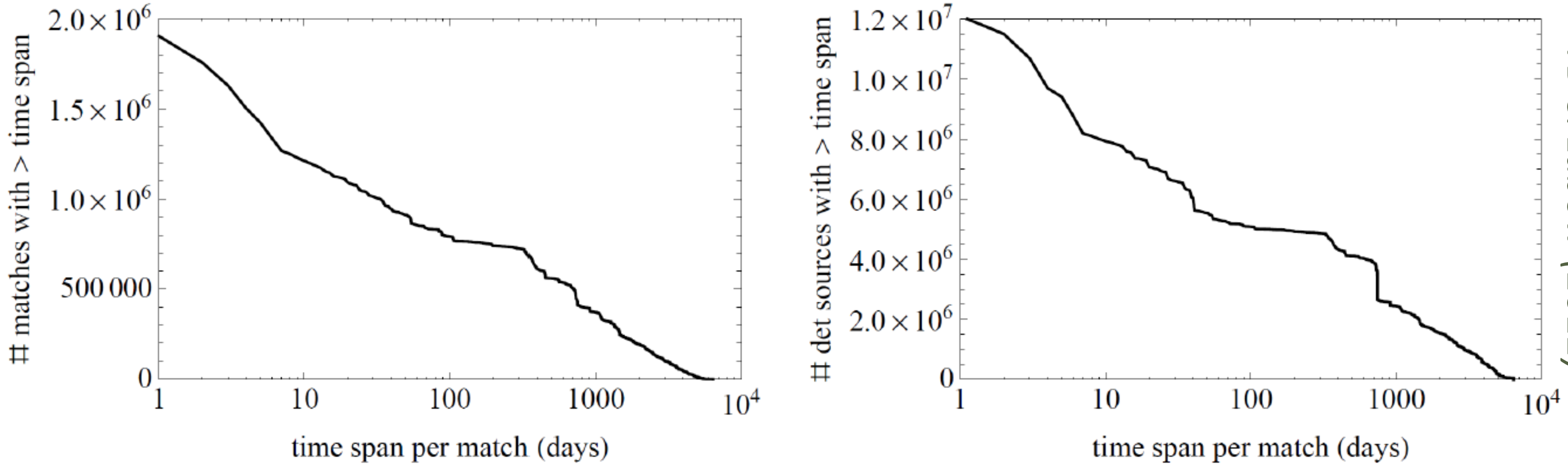
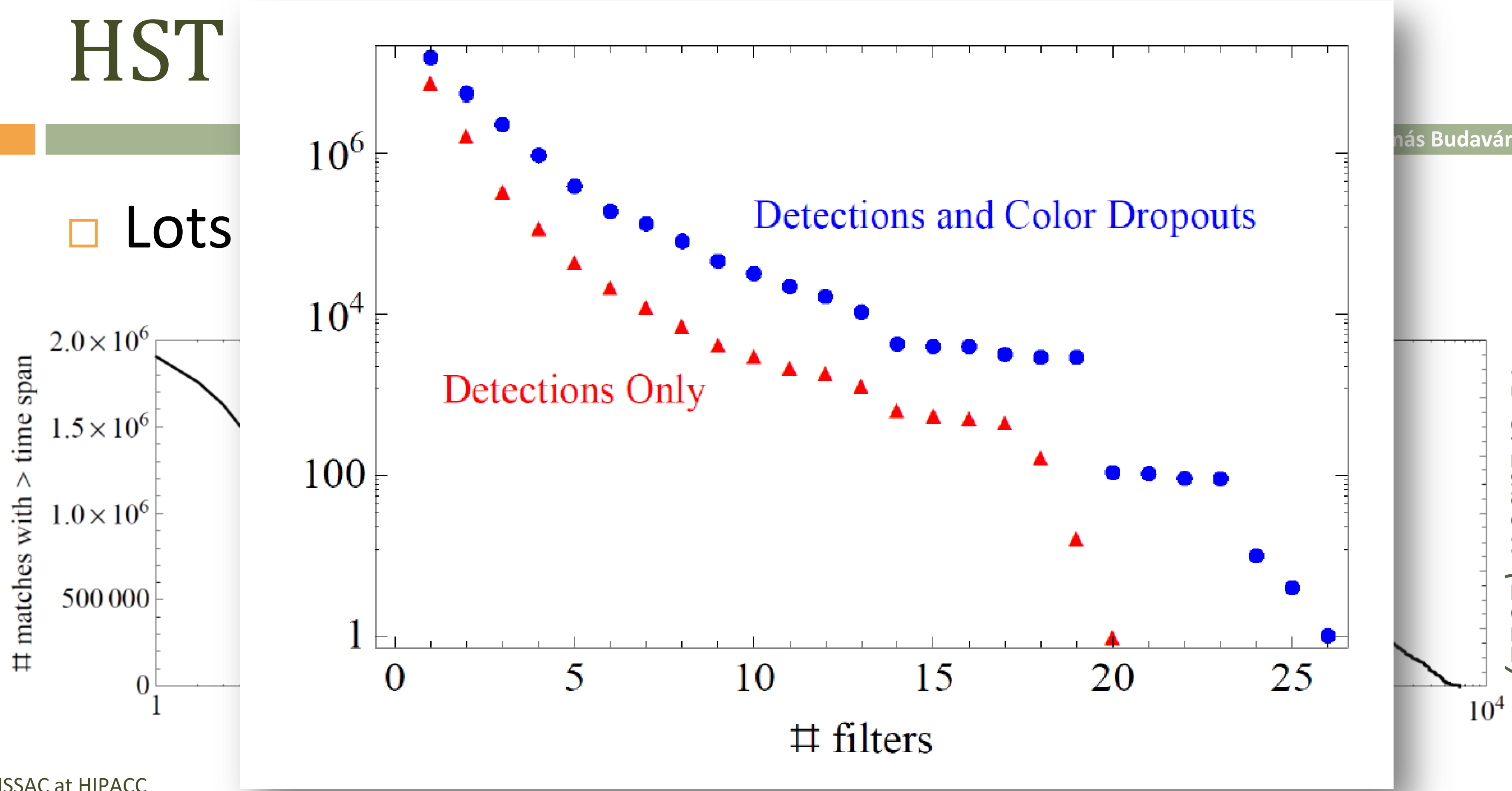
- Lots of matching sources during HST's long life



TB & Lubow (2012)

HST

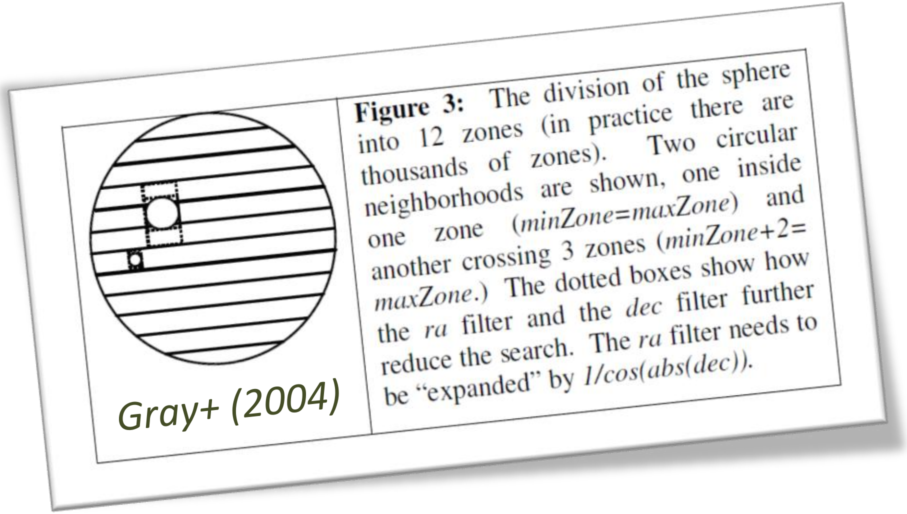
- Lots



TB & Lubow (2012)

Zone Algorithm

- Constant Declination zones
  - Sort by R.A. within
- Fast SQL code
  - SDSS-GALEX in 1 hour
  - CPU limited!



Parallel on GPUs

- Recent Github release
  - Multi-GPU implementation
- Search in 5” – great perf!
  - NVIDIA GTX 480 1.5GB
    - 29M×29M in 11 seconds
  - C2050 Teslas
    - 400M×150M in 3 minutes

```
C:\>CuXmatch.exe dr7.bin 29000000 dr7.bin 29000000 5 5 4
[dbg] n_zones: 129600

[dat] 1
[tmr] Load: 12.776000
[tmr] Copy: 0.452000
[tmr] Sort: 2.605000
[tmr] Lmts: 0.000000
[tmr] Back: 0.499000
[tmr] Splt: 0.921000

[dat] 2
[tmr] Load: 10.296000
[tmr] Copy: 0.453000
[tmr] Sort: 2.823000
[tmr] Lmts: 0.000000
[tmr] Back: 0.499000
[tmr] Splt: 0.905000

[tmr] Cop2: 0.671000
[tmr] Mtch: 10.998000
[tmr] Ftch: 0.265000
[tmr] Main: 47.876000

[res]
587727177914515631 587727177914515631
587727177914515580 587727177914515580
587727177914515797 587727177914515797
587727177914581686 587727177914581686
...
```