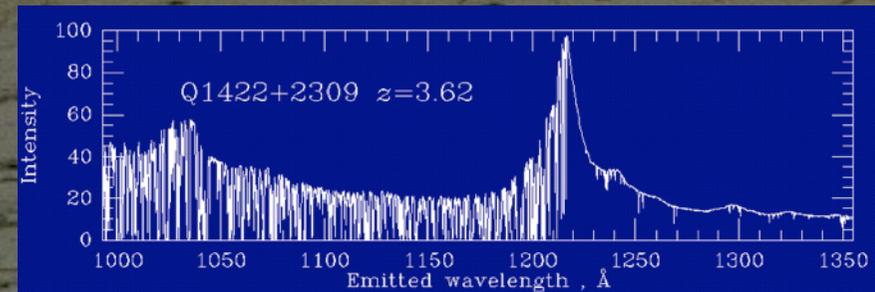
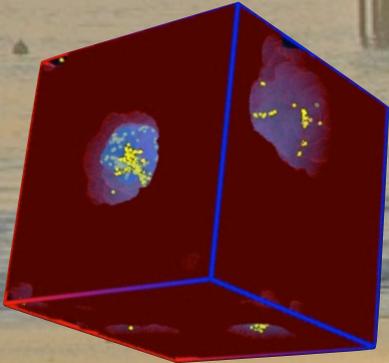


Radiative Transfer in a Clumpy Universe: the UVB



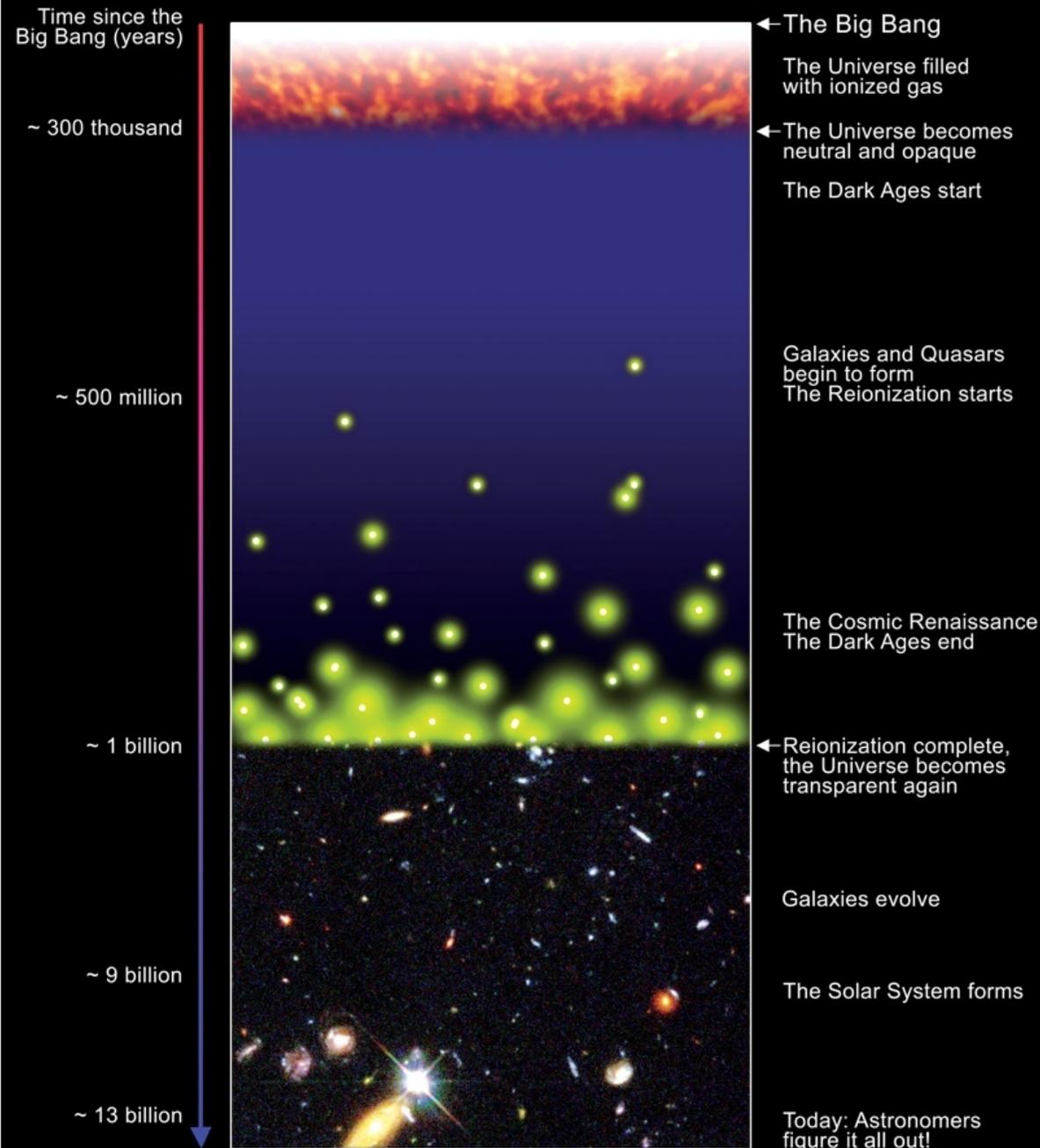
Piero Madau
UC Santa Cruz

The cosmic UVB originates from the integrated emission of star-forming galaxies and QSOs. It determines the thermal and ionization state of the IGM, the repository of most of the baryons in the Universe at high redshift.

It is a crucial yet most uncertain input parameters for cosmological simulations of LSS and galaxy formation, for interpreting QSO absorption-line data and derive information on the distribution of primordial gas -- **traced by HI, HeI, HeII transitions** -- and of the nucleosynthetic products of star formation -- **CIII, CIV, SiIII, SiIV, OVI, etc.**

What is the Reionization Era?

A Schematic Outline of the Cosmic History



S.G. Djorgovski et al. & Digital Media Center, Caltech

Outline

Hydrogen recombination

The dark ages

Cosmic structure formation

The reionization equation

Quasars or galaxies?

The Gunn-Peterson trough

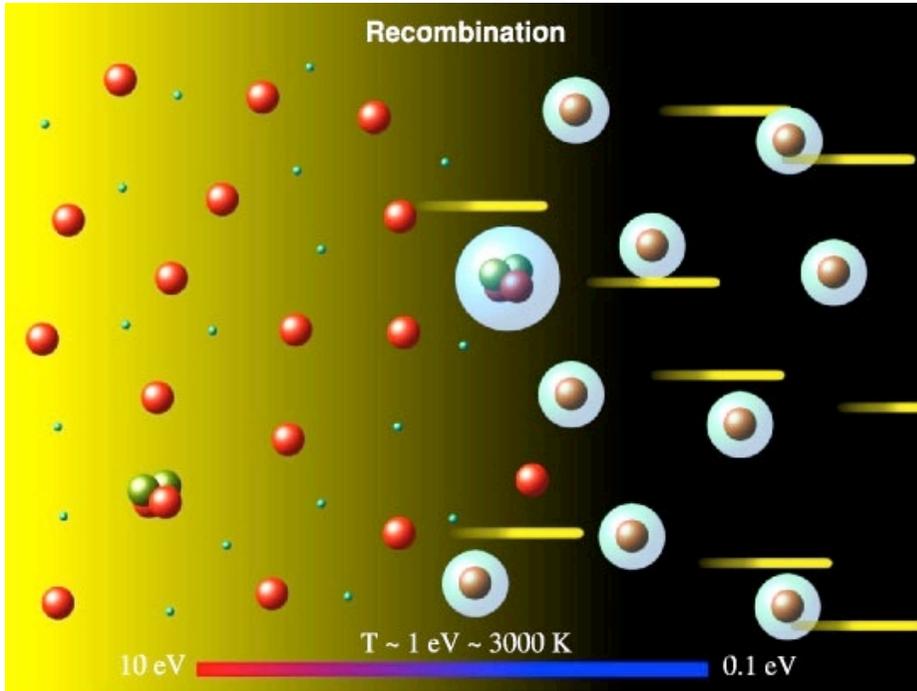
Quasar absorbers along the LOS

Effective optical depth of the Universe

Cosmological radiative transfer

CUBA (the code)

Hydrogen recombination



$e^- + p \rightarrow H + \gamma$ @ $z \approx 1100$
marks the end of the plasma era

equilibrium (Saha) eq. for $x \equiv n_e/n_H$

$$\frac{x^2}{1-x} = \underbrace{2.5 \times 10^6}_{\text{big coefficient}} \eta^{-1} \left(\frac{I}{k_B T} \right)^{3/2} \exp\left(-\frac{I}{k_B T} \right)$$

$$I = 13.6 \text{ eV}$$

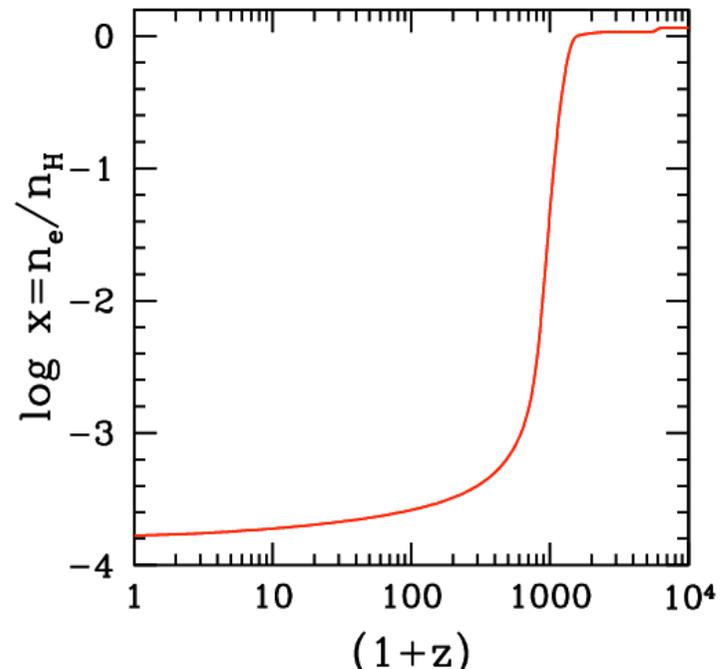
$$\eta^{-1} \equiv n_\gamma/n_b$$

Two factors delay recombination:

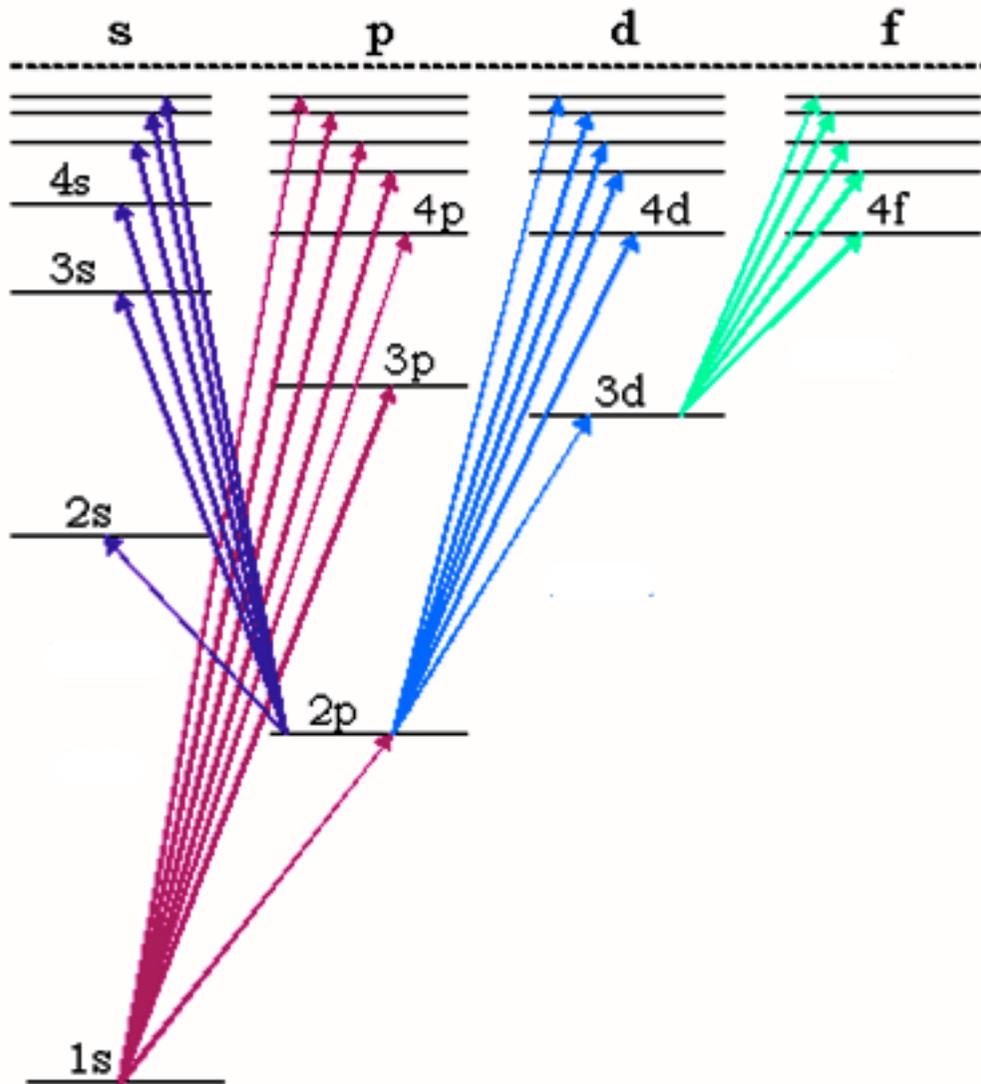
(1) $\eta^{-1} = 1.5 \times 10^9 \gg 1$

(2) inability to maintain equilibrium

as $H^{-1} \ll t_{\text{rec}} \equiv x/\dot{x}$



Hydrogen recombination (a digression)



Zeldovich, Kurt, Sunyaev 1968
Peebles 1968

When an e^- is captured to the ground state of HI, it produces a photon that immediately ionizes another atom, leaving no net change.

When it is captured to an excited state, the allowed decay to the ground state produces a resonant Lyman series photon, which has a large capture cross-section \rightarrow puts another atom in a high energy state that is easily photoionized again, thereby annulling the effect.

Two main routes to the production of atomic hydrogen:

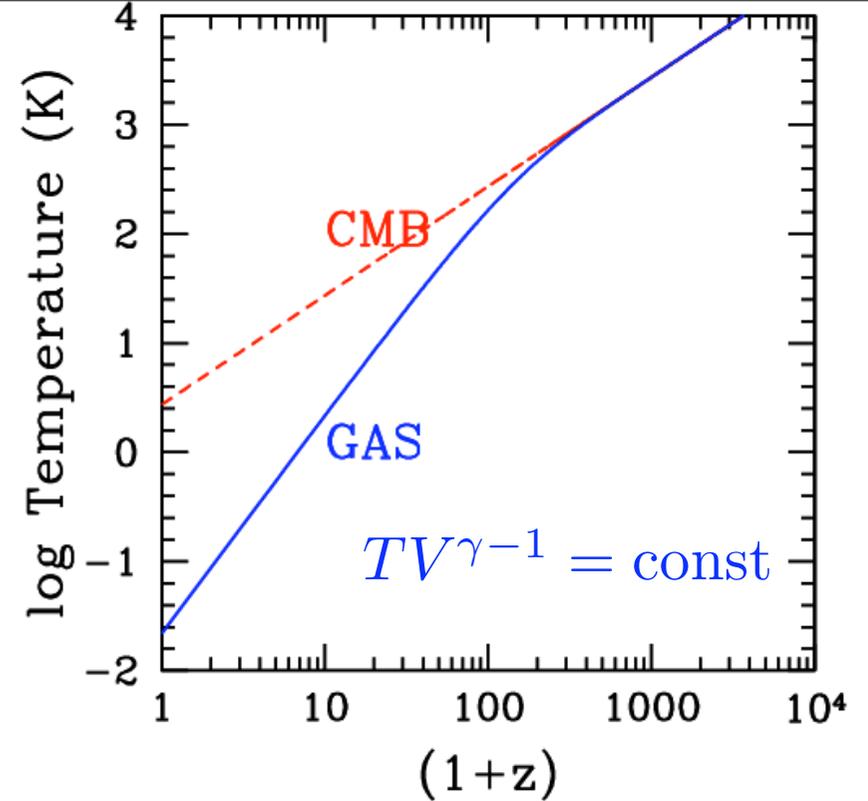
- 1) two-photon decay from the 2s level to 1s.
- 2) loss of Ly α resonance photons by the cosmological redshift.

The “dark ages” ...

Timescale for relaxation of matter temperature is

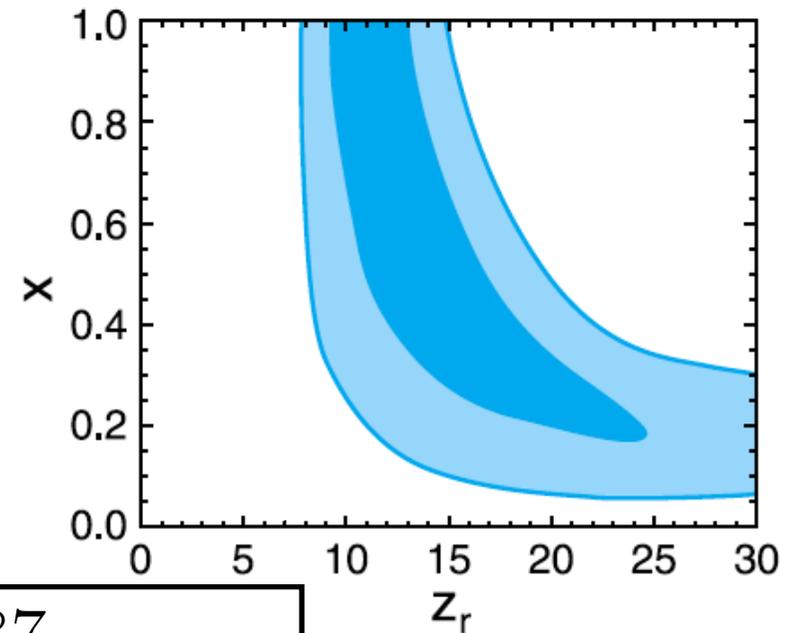
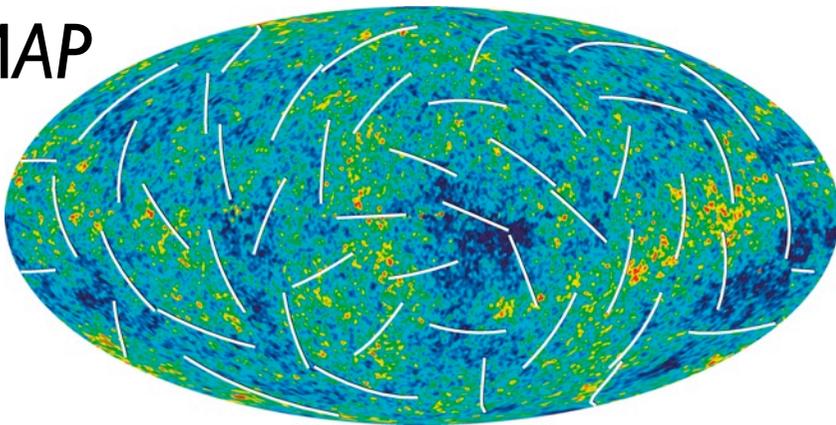
$$t_{\text{comp}} = \frac{3m_e c}{4\sigma_T a_B T^4} \frac{1+x}{2x}$$

$$\rightarrow z_{\text{th}} \simeq 580(\Omega_b h^2)^{2/5} \simeq 130.$$



..and their end

WMAP



$$\tau_T(z) = \int_0^z n_e(z) \sigma_T |cdt/dz| dz = 0.087$$

→ Universe becomes semi-opaque after reionization

The “dark ages” ...

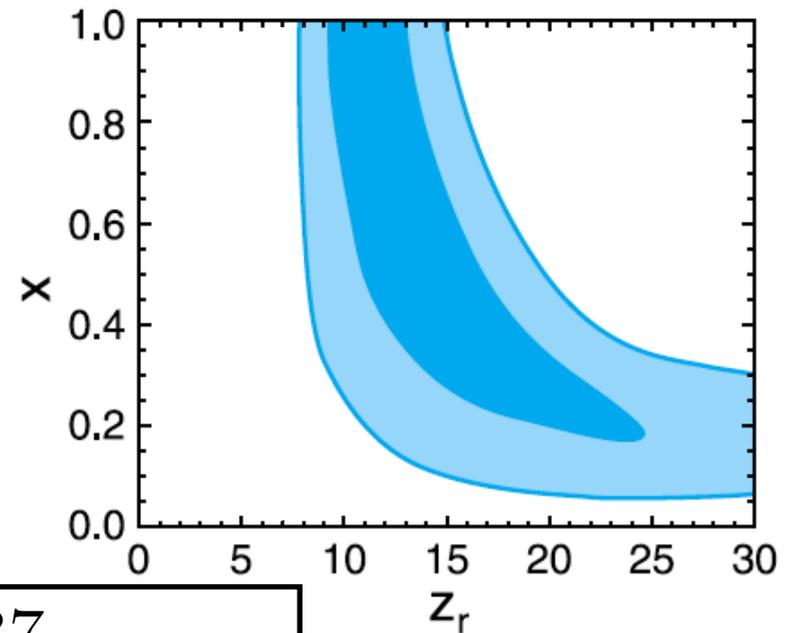
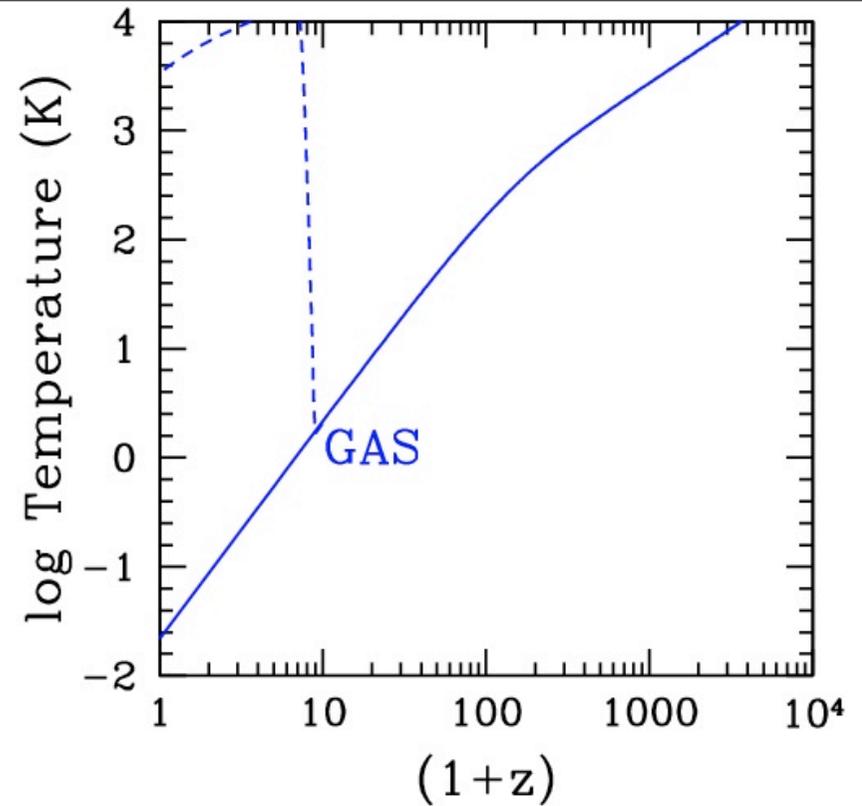
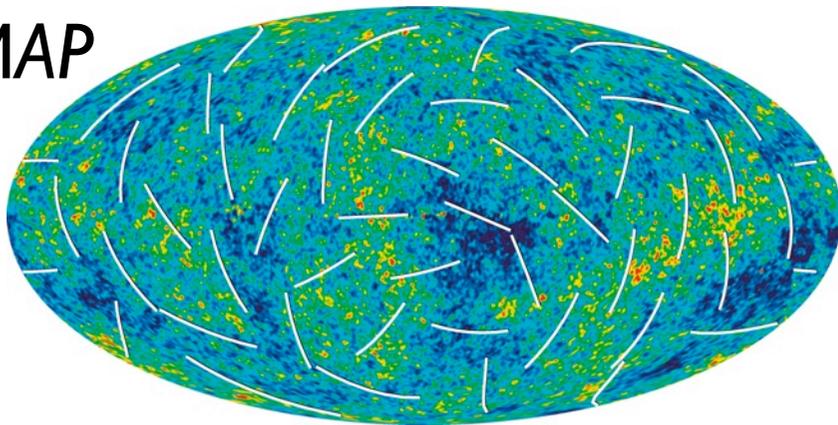
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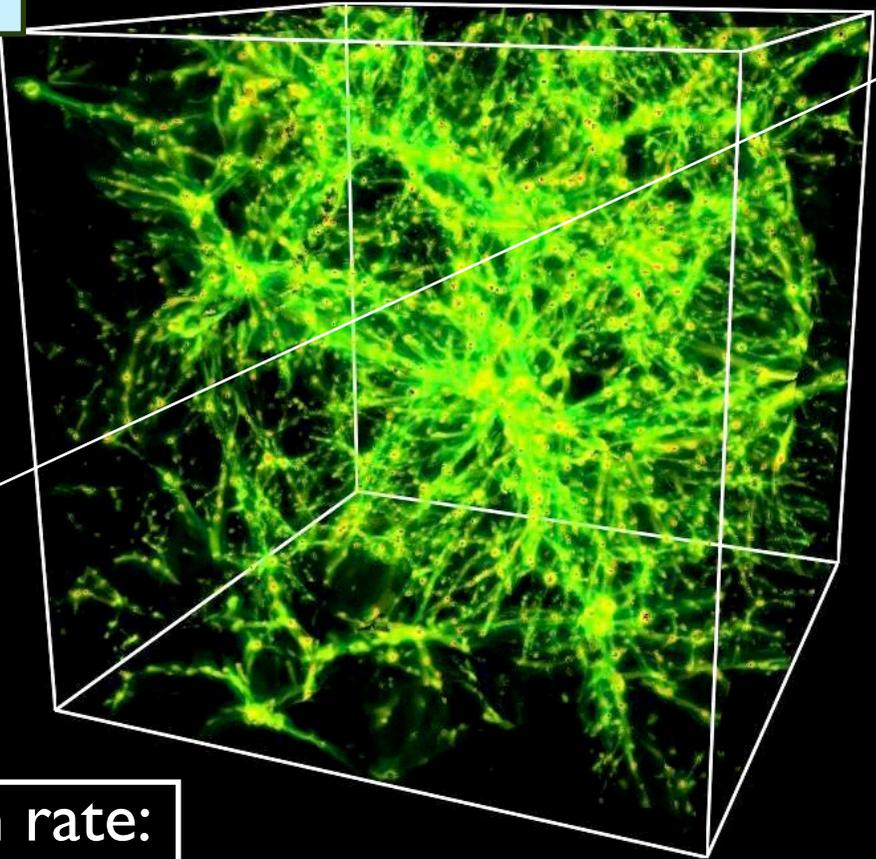
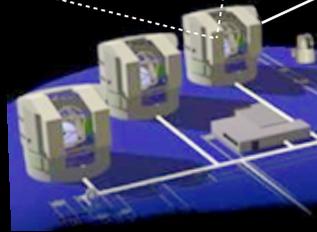
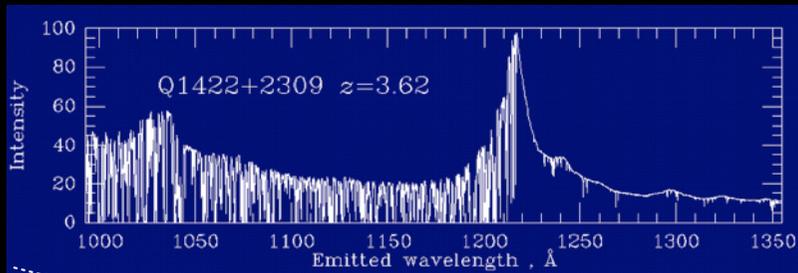


$$\tau_T(z) = \int_0^z n_e(z) \sigma_T |cdt/dz| dz = 0.087$$

→ Universe becomes semi-opaque after reionization

Cosmic structure formation: I

QSO



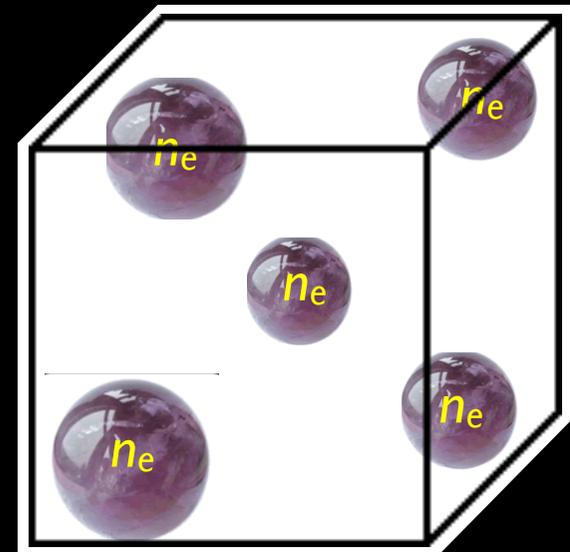
Clumpiness boosts H recombination rate:

$$\langle n_e n_p \rangle \alpha_B(T) = C \langle n_p \rangle^2 \alpha_B(T)$$
$$t_{\text{rec}} = (n_p \alpha_B C)^{-1}$$

Example: ionized gas of density n_e filling uniformly a fraction f of the available volume, rest is empty space.

Then

$$\langle n_e^2 \rangle = f n_e^2; \quad \langle n_e \rangle = f n_e$$
$$\rightarrow \langle n_e^2 \rangle = \langle n_e \rangle^2 / f \rightarrow C = 1/f$$

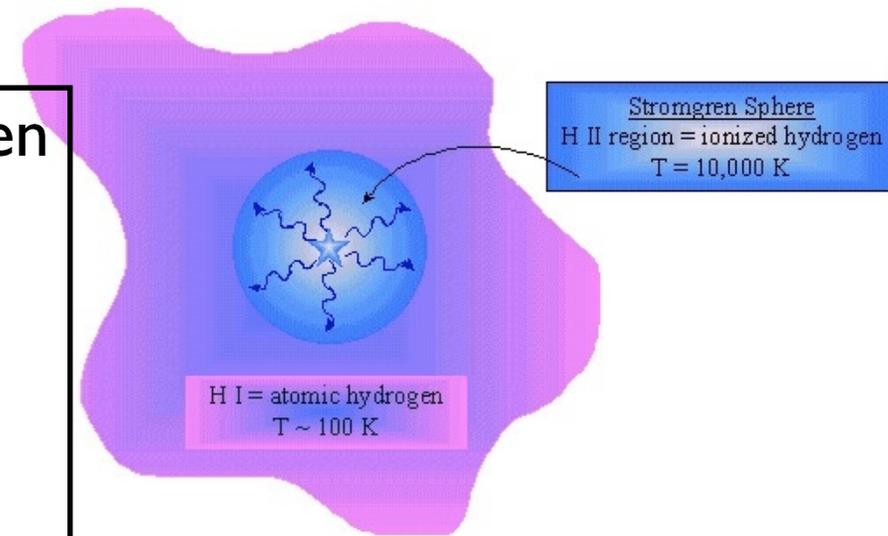


Cosmic structure formation: II

HII region in homogeneous ISM (Stromgren analysis):

$$n_H \frac{dV}{dt} = \dot{N}_\gamma - V \alpha_B n_H^2$$

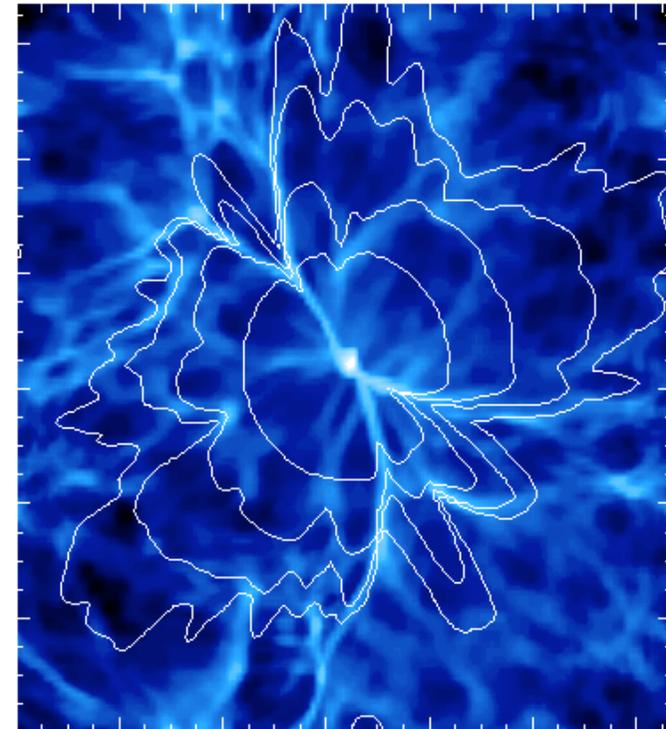
$$\rightarrow V = \frac{\dot{N}_\gamma t_{\text{rec}}}{n_H} (1 - e^{-t/t_{\text{rec}}})$$



HII region in expanding clumpy IGM (Shapiro & Giroux 1987):

$$n_H(t) \left(\frac{dV}{dt} - 3HV \right) = \dot{N}_\gamma - V \alpha_B \langle n_H(t) \rangle^2 C$$

V=proper volume



The reionization equation

Reionization @ milliFLOP speed (PM, Haardt, & Rees 1999)

$Q_I(t)$ = volume filling factor of HII regions at t

$$Q_I(t) = \int_0^t \frac{\dot{n}_\gamma(t')}{\langle n_H(t') \rangle} dt' - \int_0^t \frac{Q_I(t')}{t_{\text{rec}}} dt'$$

source

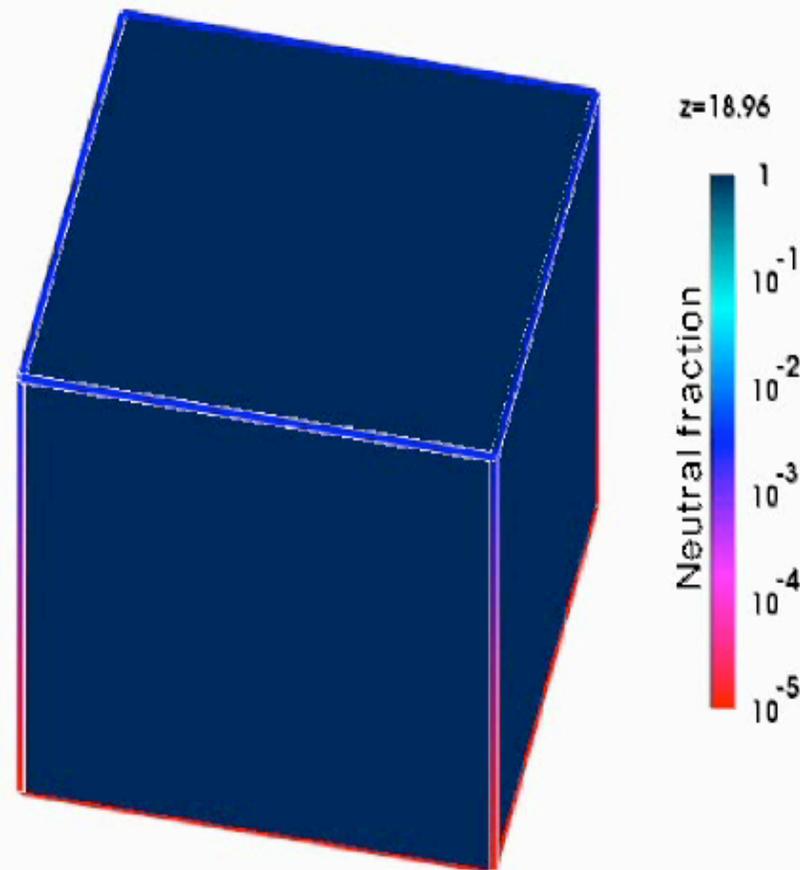
sink

(no redshifting, ionizing photons absorbed locally). Differentiating:

$$\frac{dQ_I}{dt} = \frac{\dot{n}_\gamma}{\langle n_H \rangle} - \frac{Q_I}{t_{\text{rec}}}$$

simple diff. eq. statistically describes transition from a neutral Universe to a fully ionized one!

Contrary to the static case, cosmological HII regions will always percolate in an expanding universe with constant comoving ionizing emissivity...



The reionization equation

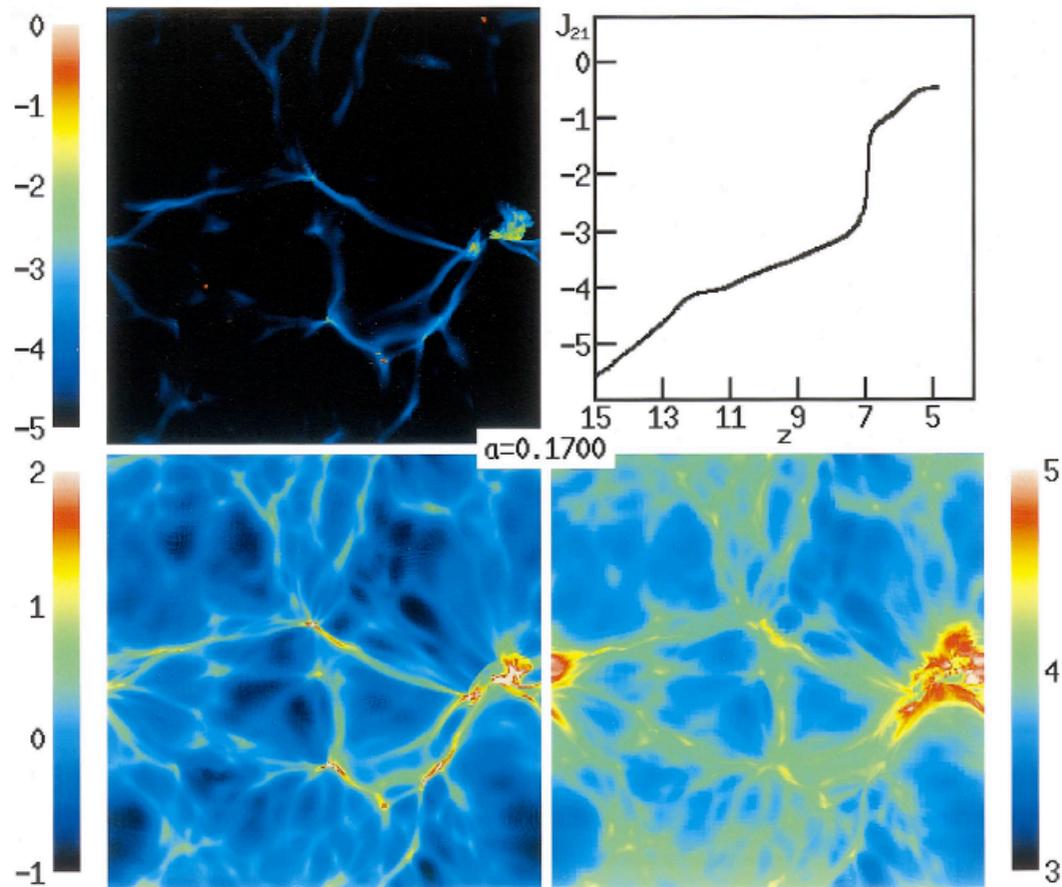
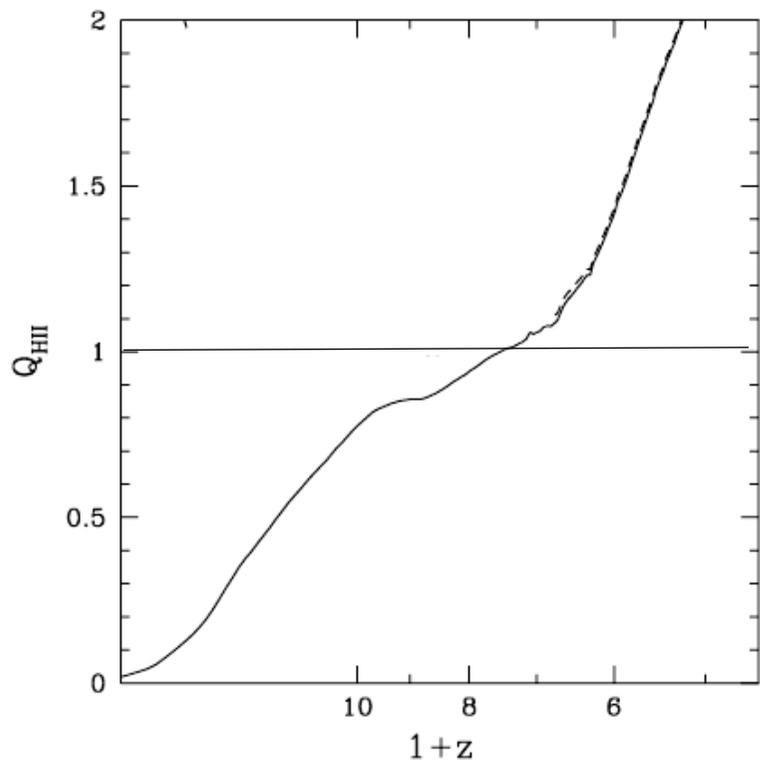
$$t_{\text{rec}} \ll t \rightarrow Q_I \approx \frac{\dot{n}_\gamma}{\langle n_H \rangle} t_{\text{rec}}$$

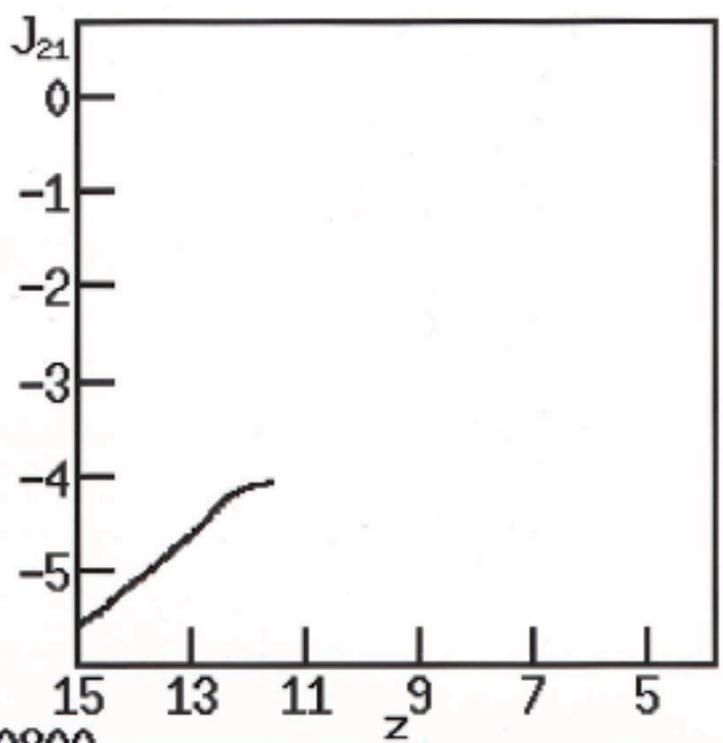
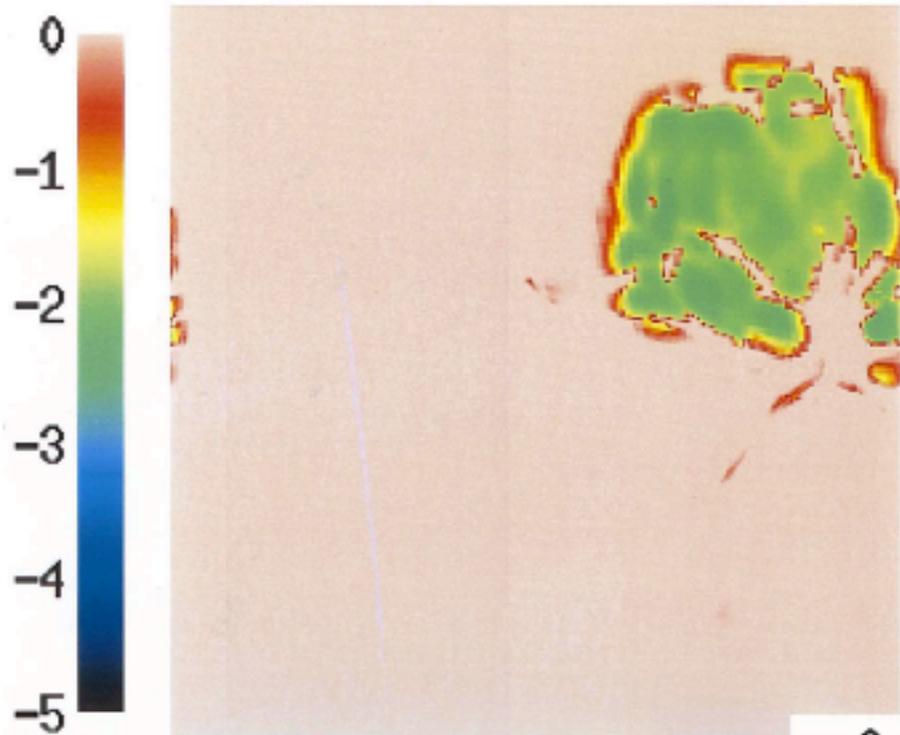
The universe is completely reionized when $Q_I=1$, i.e. when

$$\dot{n}_\gamma t_{\text{rec}} = \langle n_H \rangle$$

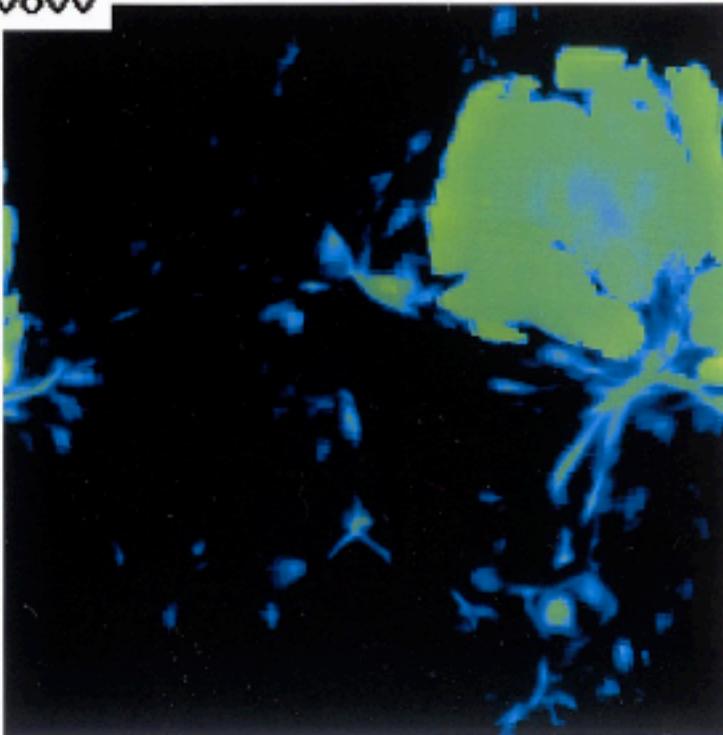
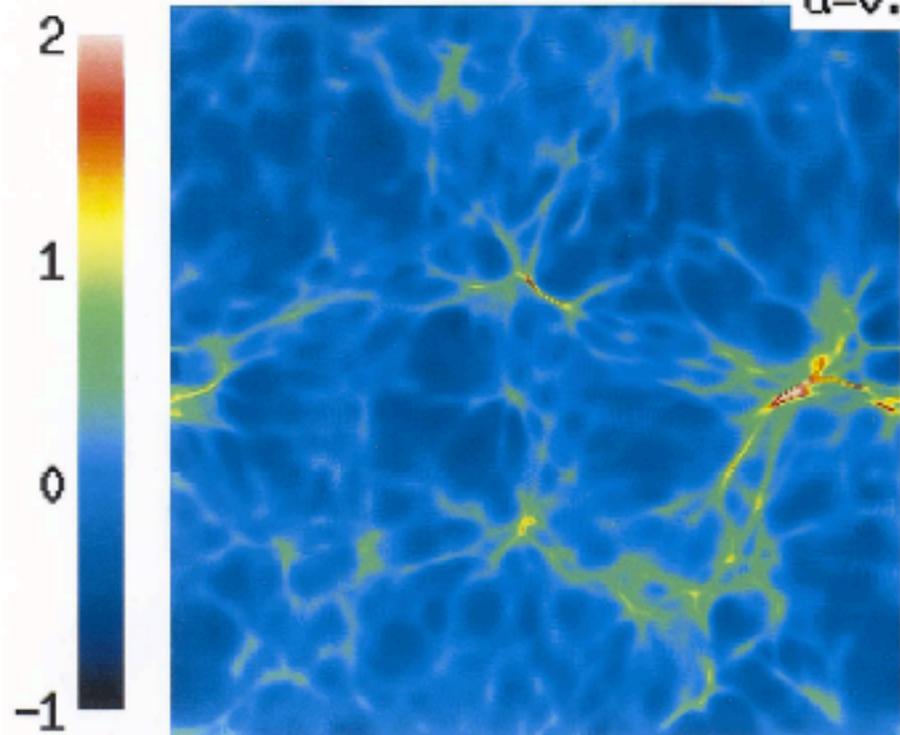
Because of hydrogen recombinations, only a fraction t_{rec}/t of the photons emitted above 13.6 eV is actually used to ionize new IGM material.

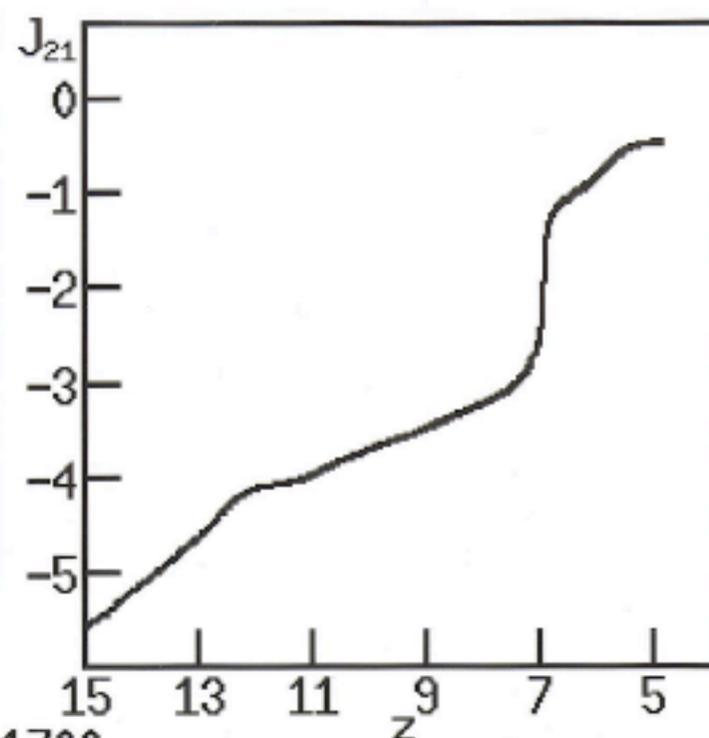
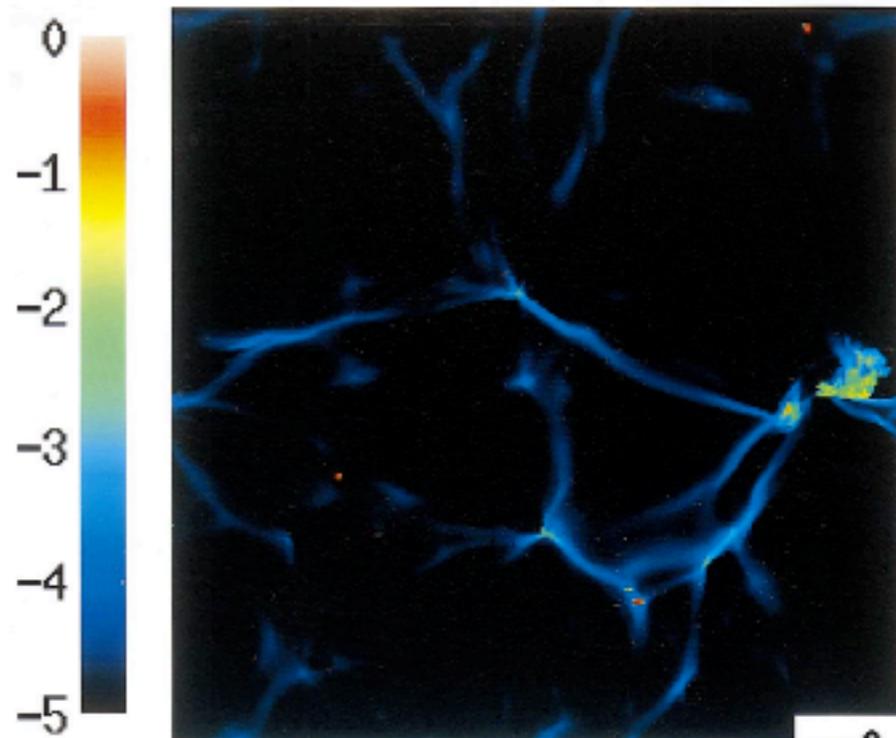
Numerical simulation of stellar reionization (Gnedin 2000)



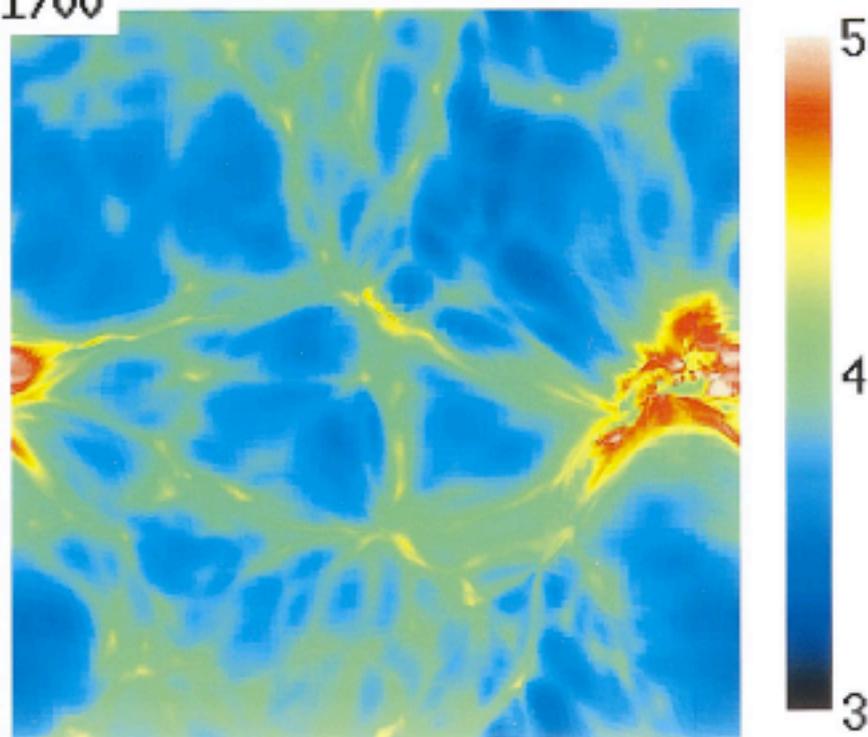
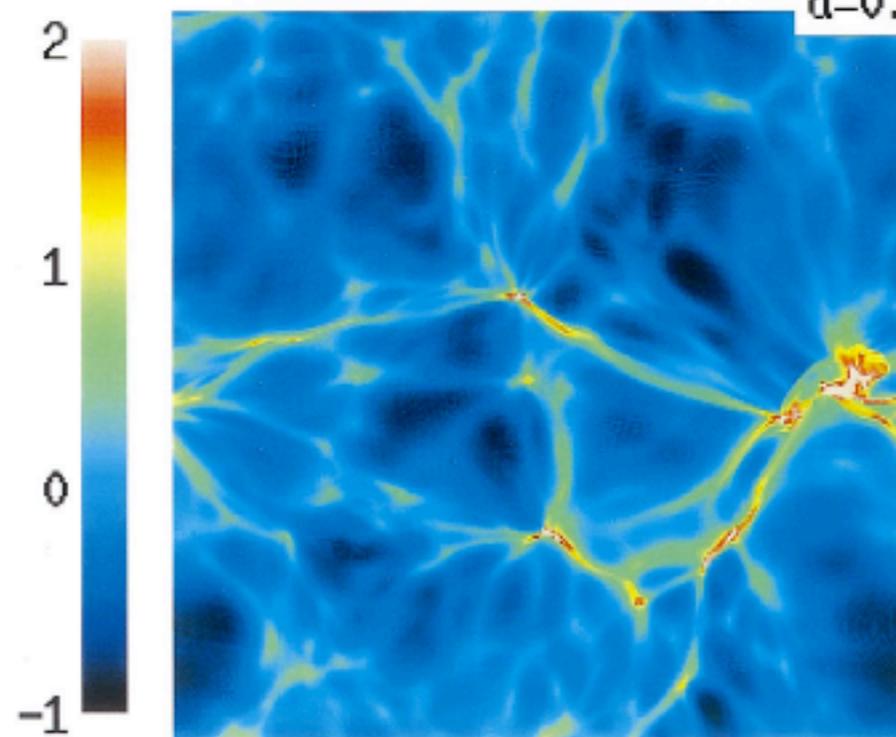


$\alpha=0.0800$

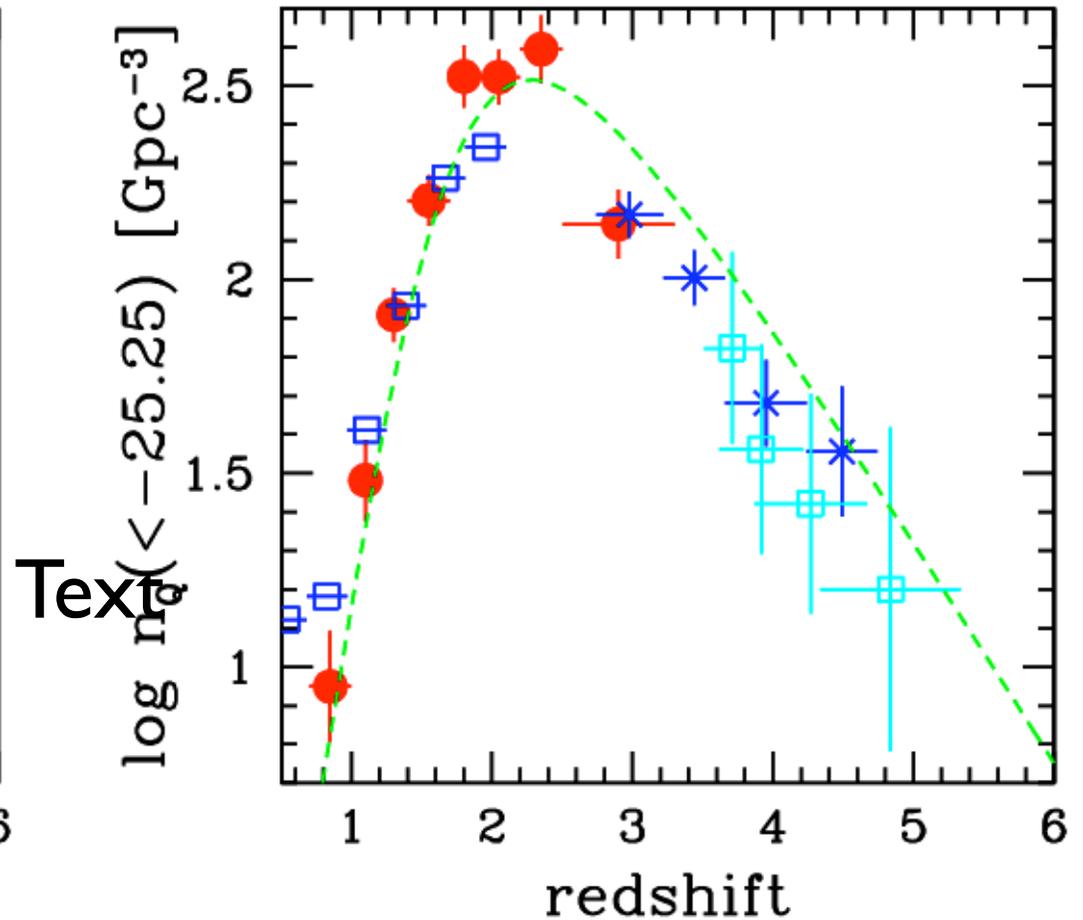
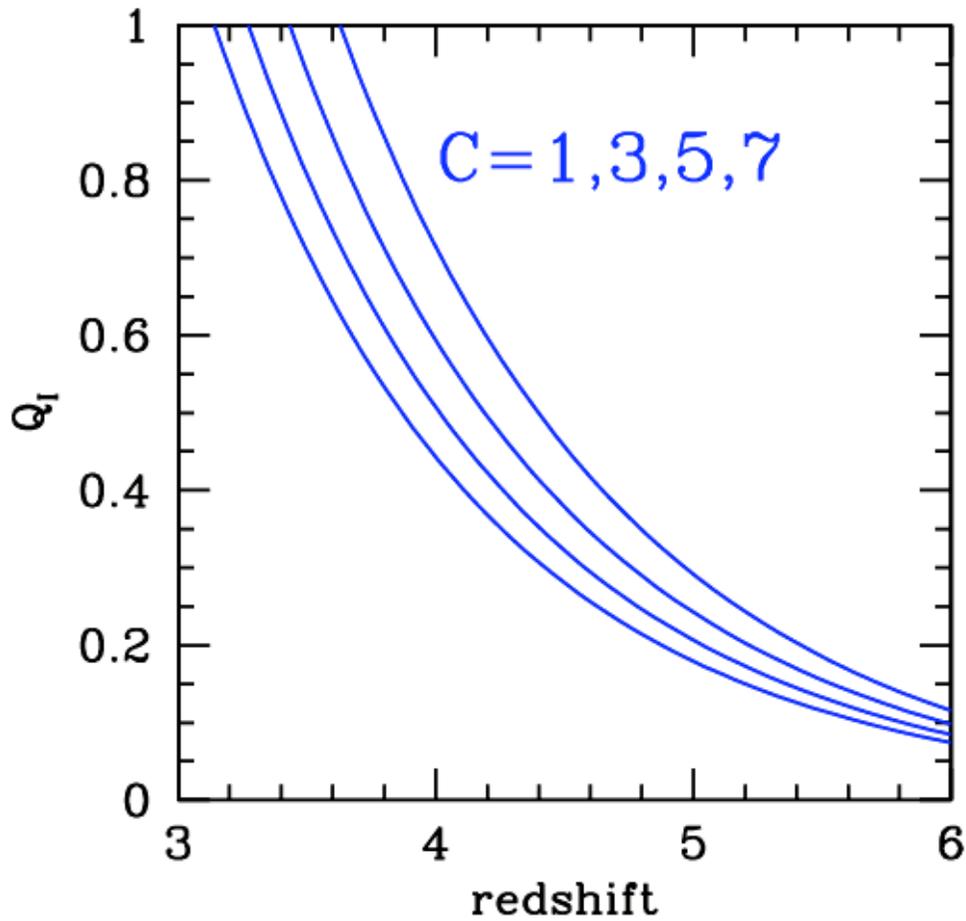




$\alpha=0.1700$

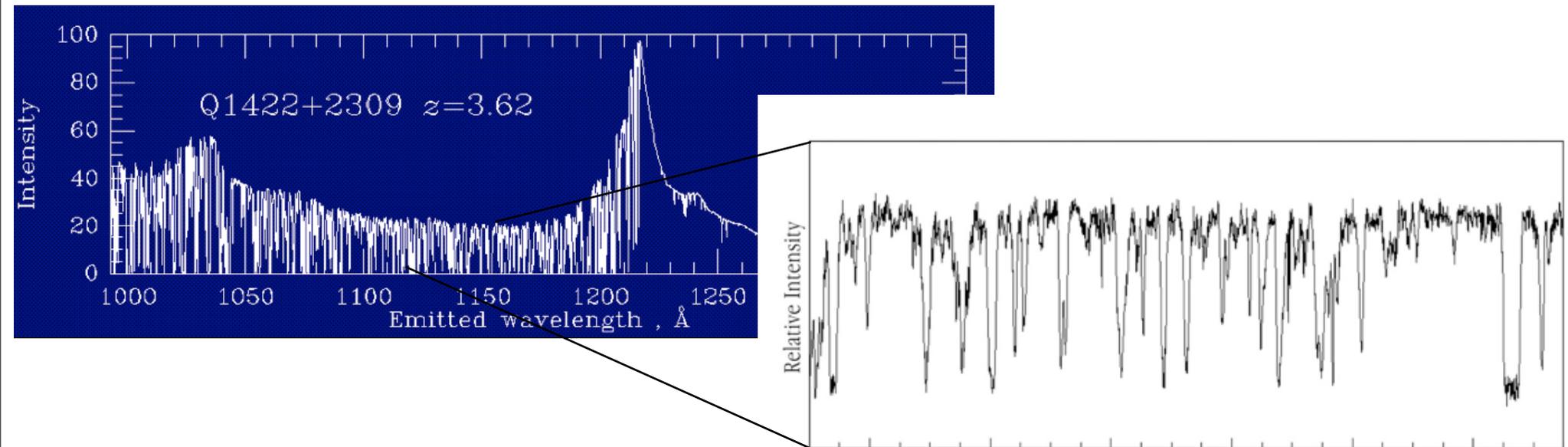


Quasars or galaxies?



(Bright) QSOs are not responsible for the reionization of cosmic hydrogen!

The Gunn-Peterson trough



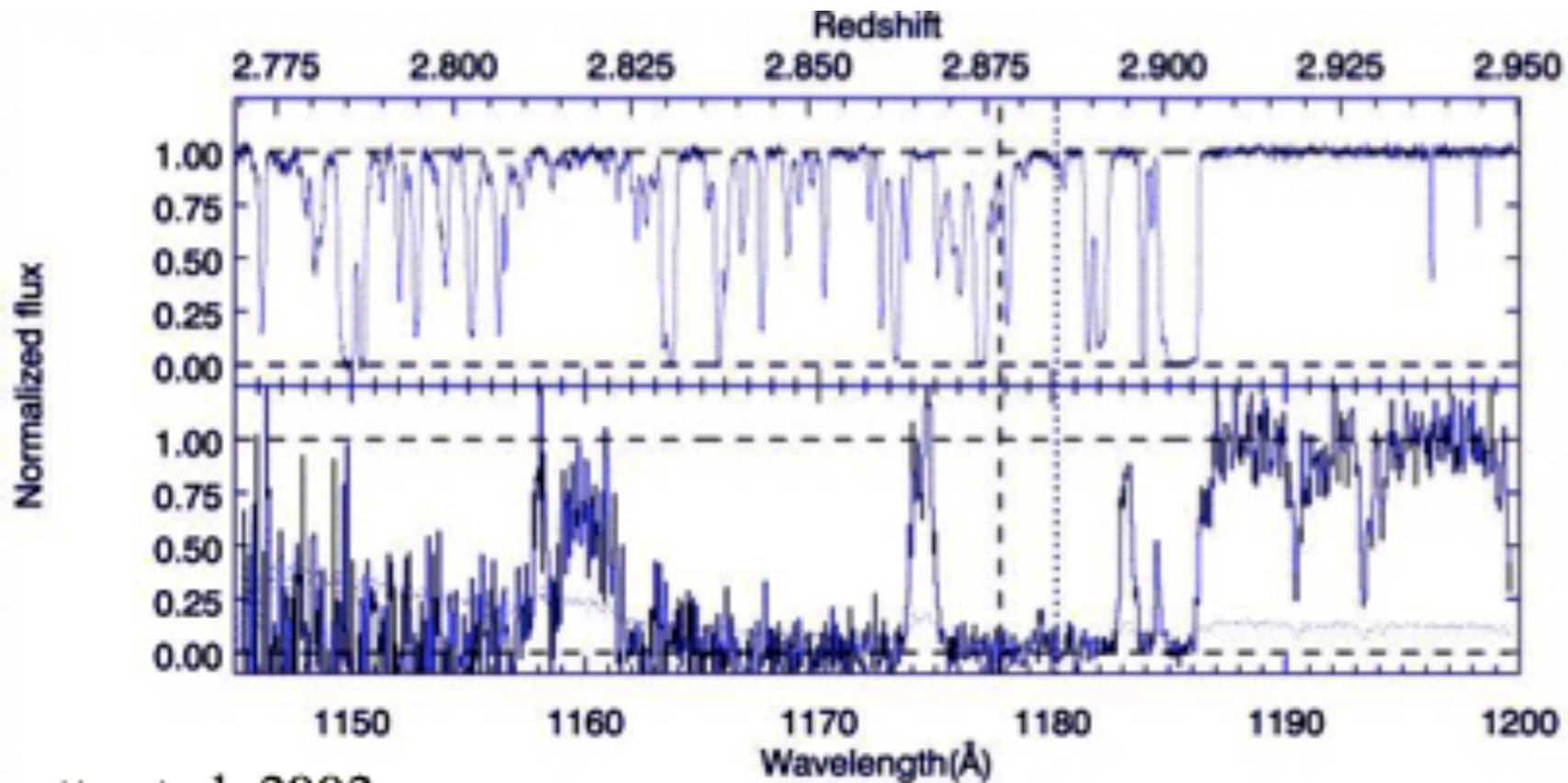
radiation emitted at ν_e and z_e becomes resonant (Ly α) at $(1+z) = (1+z_e) \nu_\alpha/\nu_e \rightarrow$ scattered off the Ios with cross-section:

$$\sigma(\nu) = \frac{\pi e^2}{m_e c} f \phi(\nu)$$

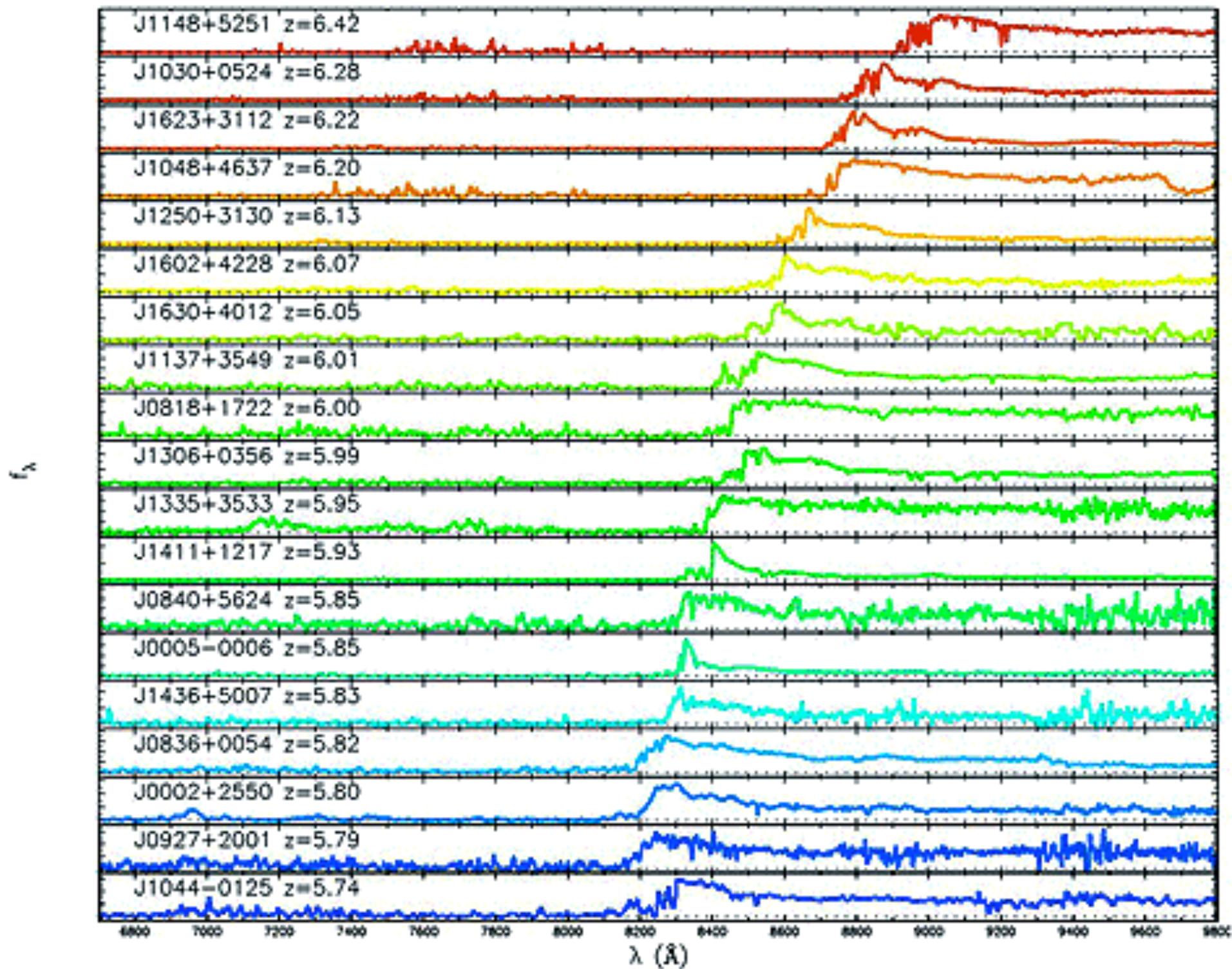
total optical depth for resonant scattering (Gunn-Peterson)

$$\tau_{\text{GP}}(z_e) = \int_0^{z_e} \sigma n_{\text{HI}}(z) |cdt/dz| dz = \left(\frac{\pi e^2 f}{m_e \nu_\alpha} \right) \frac{n_{\text{HI}}}{H}.$$

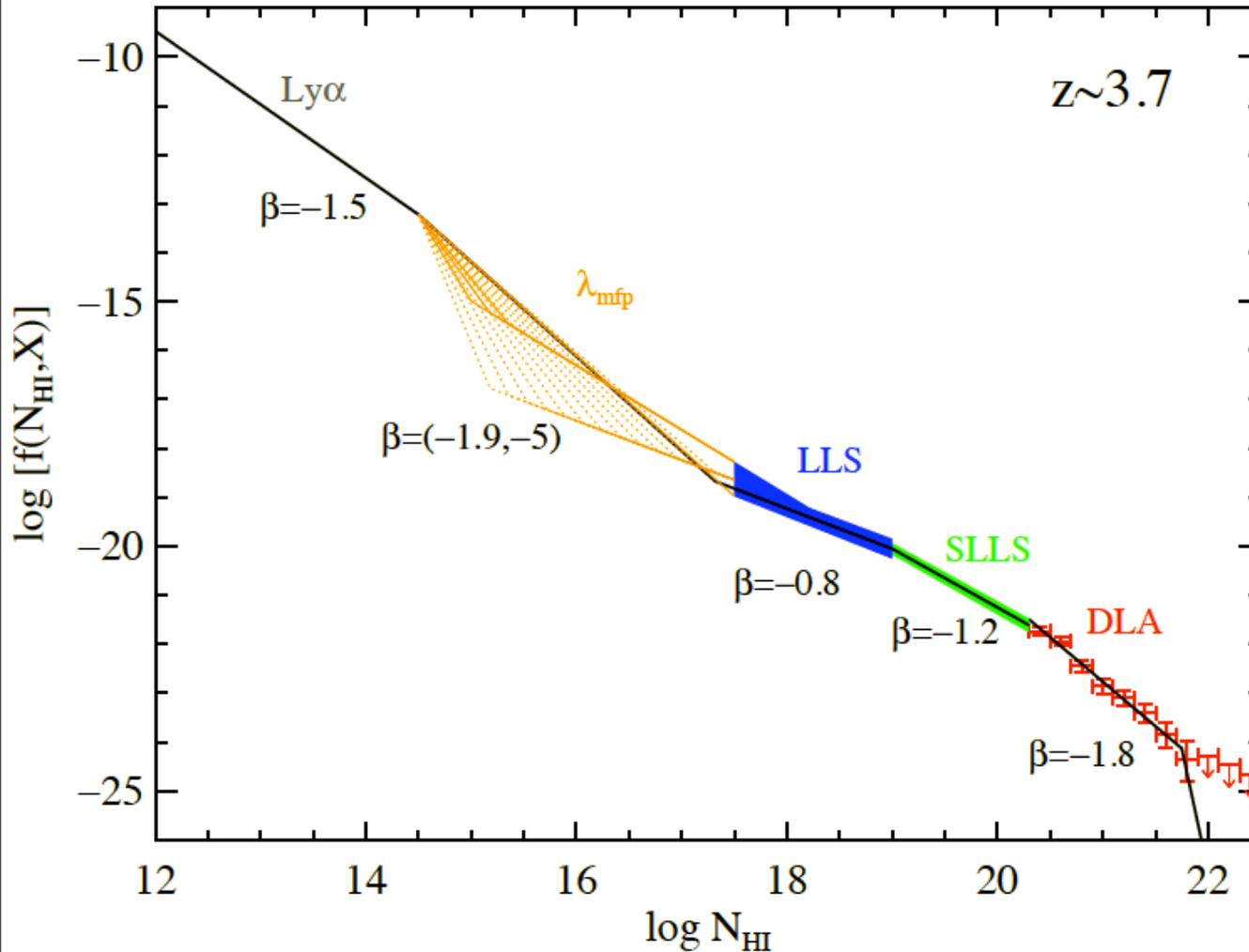
Hydrogen is highly ionized at $z < 5.7$



Smette et al. 2002



Quasar absorbers along the LOS



$f(N_{\text{HI}}, z)$ defined by the probability dP that a los intersects an absorber with column density N_{HI} in the range dN_{HI} , at redshift z in the range dz ,

$$dP = f(N_{\text{HI}}, z) dN_{\text{HI}} dz$$

We can now quantify the degree of attenuation of UV radiation in a clumpy Universe by introducing the concept of an effective continuum optical depth τ_{eff} along the line-of-sight to redshift z : $\langle e^{-\tau} \rangle = e^{-\tau_{\text{eff}}}$ where the average is taken over all lines-of-sight.

Effective optical depth

Assume random distribution of absorbers in column density and redshift space, then:

$$\tau_{\text{eff}}(\nu_o, z_o, z) = \int_{z_o}^z dz' \int_0^\infty f(N_{\text{HI}}, z) (1 - e^{-\tau})$$

$$\tau = N_{\text{HI}}\sigma_{\text{HI}}(\nu) \quad [\dots + N_{\text{HeI}}\sigma_{\text{HeI}} + N_{\text{HeII}}\sigma_{\text{HeII}}]$$

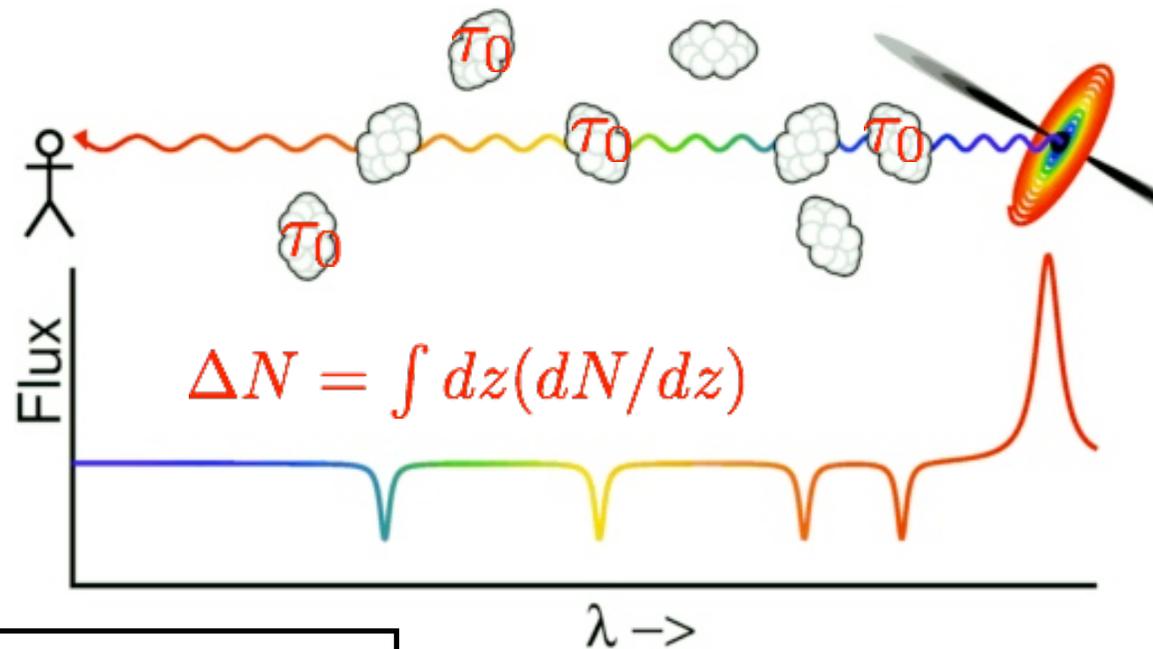
$$\nu = \nu_o(1 + z)/(1 + z_o); \quad \sigma_i = \text{photoionization cross section}$$

→ Poissonian probability of encountering a total optical depth $k\tau_0$ is:

$$p(k\tau_0) = e^{-\Delta N} \Delta N^k / (k!) \tau_0^k$$

$$\langle e^{-\tau} \rangle = e^{-k\tau_0} p(k\tau_0) = \exp[-\Delta N(1 - e^{-\tau_0})]$$

$$\langle \tau \rangle = \Delta N \tau_0 > \tau_{\text{eff}} = \Delta N(1 - e^{-\tau_0})$$



Cosmological radiative transfer

The equation of cosmological radiative transfer describes the time evolution of the space- and angle-averaged monochromatic intensity J_ν :

$$\left(\frac{\partial}{\partial t} - \nu H \frac{\partial}{\partial \nu} \right) J_\nu + 3H J_\nu = -c\kappa_\nu J_\nu + \frac{c}{4\pi} \epsilon_\nu$$

$$* J_{\nu_o}(z_o) = \frac{c}{4\pi} \int_{z_o}^{\infty} |dt/dz| dz \frac{(1+z_o)^3}{(1+z)^3} \epsilon_\nu(z) e^{-\tau_{\text{eff}}}$$

$$\tau_{\text{eff}}(\nu_o, z_o, z) = \int_{z_o}^z dz' \int_0^\infty f(N_{\text{HI}}, z) (1 - e^{-\tau})$$

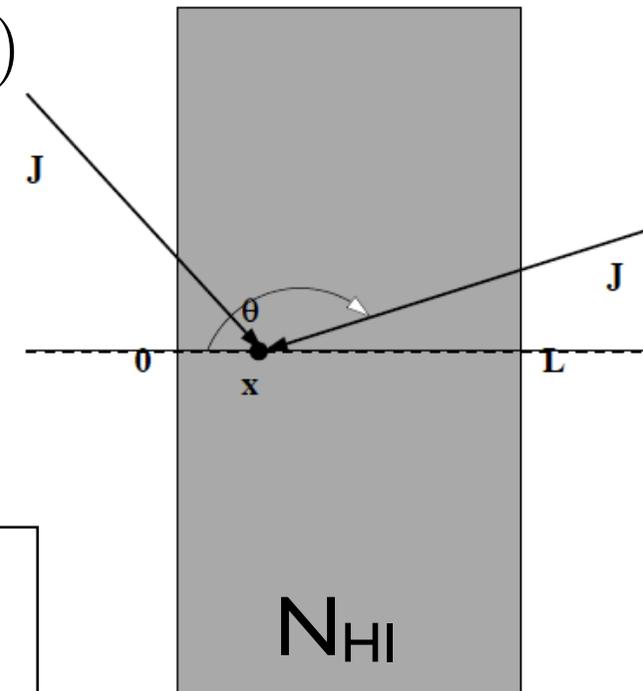
$$\tau = N_{\text{HI}} \sigma_{\text{HI}}(\nu) \left[\dots + N_{\text{HeI}} \sigma_{\text{HeI}} + N_{\text{HeII}} \sigma_{\text{HeII}} \right]$$

$$\nu = \nu_o (1+z)/(1+z_o)$$

σ_i = photoionization cross section

observed

must be modeled



☛ equation * must be solved by iteration since $\tau = \tau(J)$

Two important effects must be included:

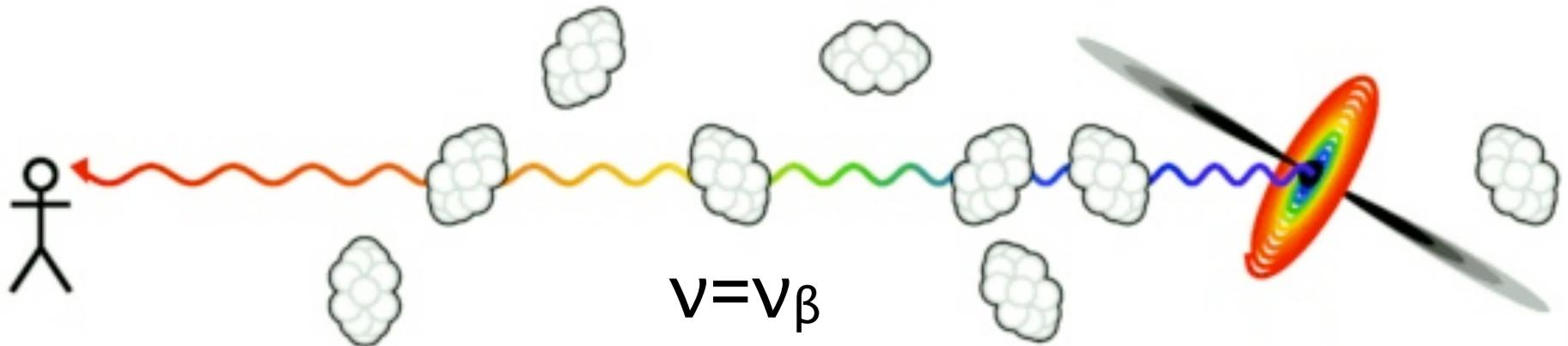
1) absorbers are not only sinks but also sources of ionizing radiation. In particular, Hell reprocesses soft X-rays He-ionizing photons into UV H-ionizing ones.

$$\epsilon(\nu, z) = \epsilon_{\text{QSO}} + \epsilon_{\text{Gal}} + \epsilon_{\text{rec}}$$
$$\epsilon(z) = (1 + z)^3 \int dL L \phi(L, z)$$

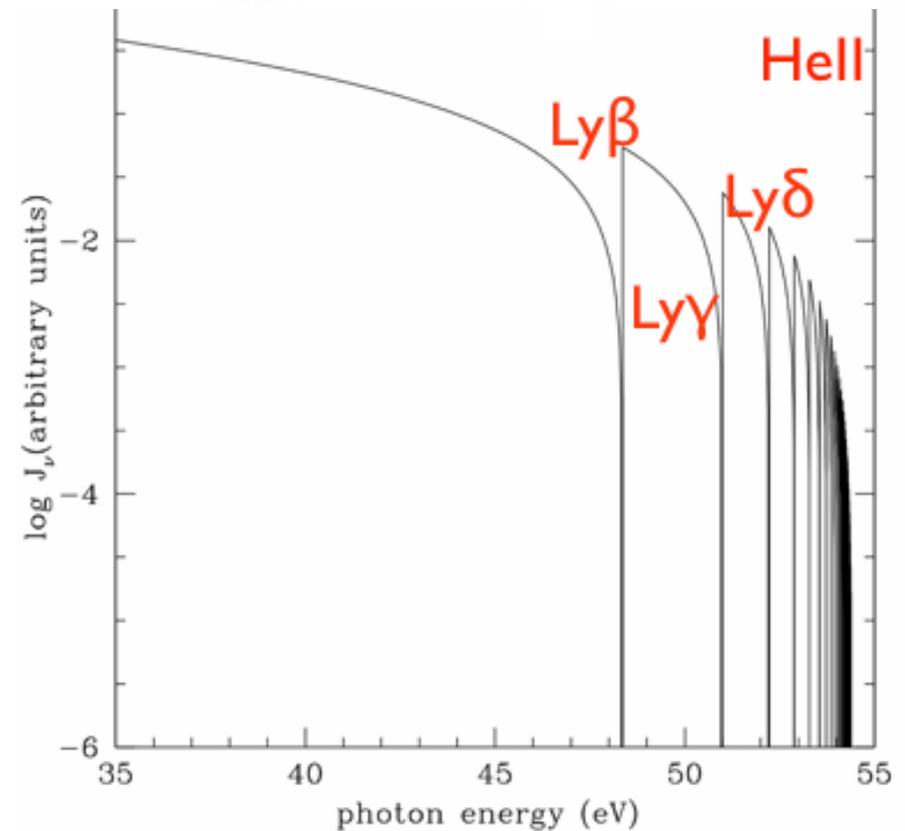
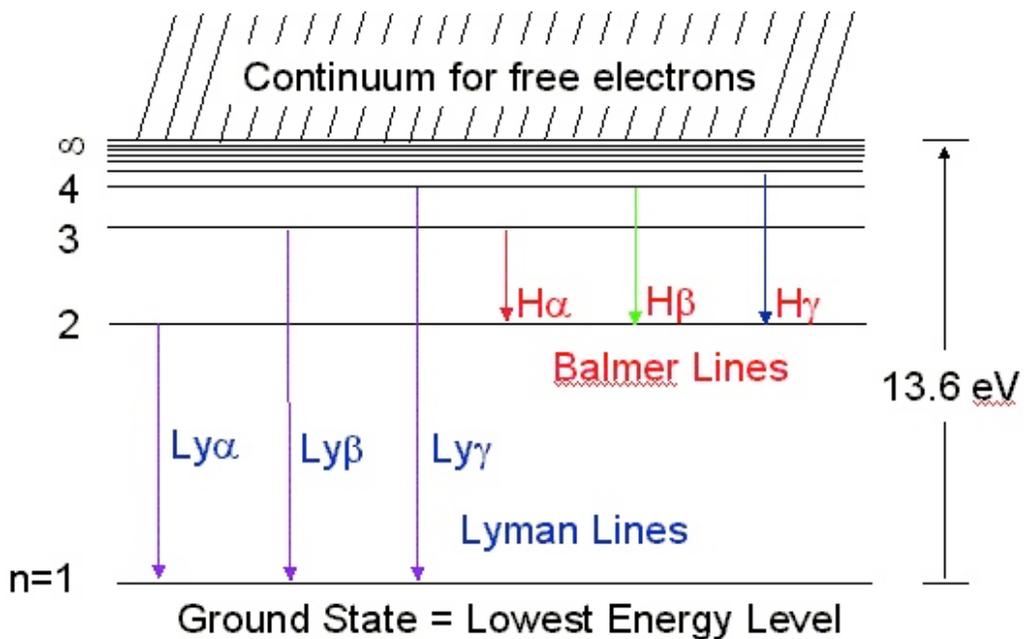
Ionizing recombination radiation includes:

- recombinations to ground state of H I, He I, He II
- He II Balmer and 2-photon continuum
- He II Lyman-alpha

2) besides photoelectric absorption, resonant absorption by H and He Lyman series will produce a sawtooth modulation of the spectrum.



$$v = v_{\alpha}$$



J-solution flow chart

ABSORBERS

HI distribution

SOURCES

QSO/GAL LF

SED

J-solution flow chart

ABSORBERS

HI distribution

SOURCES

QSO/GAL LF

SED

cosmological
radiative transfer
→ J

J-solution flow chart

ABSORBERS

HI distribution

local radiative
transfer \rightarrow H/He
ionization state

SOURCES

QSO/GAL LF

SED

cosmological
radiative transfer
 \rightarrow J

J-solution flow chart

ABSORBERS

HI distribution

local radiative
transfer \rightarrow H/He
ionization state

$\tau_{eff}, \epsilon_{rec}$

SOURCES

QSO/GAL LF

SED

cosmological
radiative transfer
 $\rightarrow J$

J-solution flow chart

ABSORBERS

HI distribution

local radiative
transfer \rightarrow H/He
ionization state

$\tau_{eff}, \epsilon_{rec}$

SOURCES

QSO/GAL LF

SED

cosmological
radiative transfer
 $\rightarrow J$



J-solution flow chart

ABSORBERS

HI distribution

local radiative
transfer \rightarrow H/He
ionization state

$\tau_{eff}, \epsilon_{rec}$

SOURCES

QSO/GAL LF

SED

cosmological
radiative transfer
 $\rightarrow J$

UVB



J-solution flow chart

ABSORBERS

HI distribution

local radiative
transfer \rightarrow H/He
ionization state

$\tau_{\text{eff}}, \epsilon_{\text{rec}}$

SOURCES

QSO/GAL LF

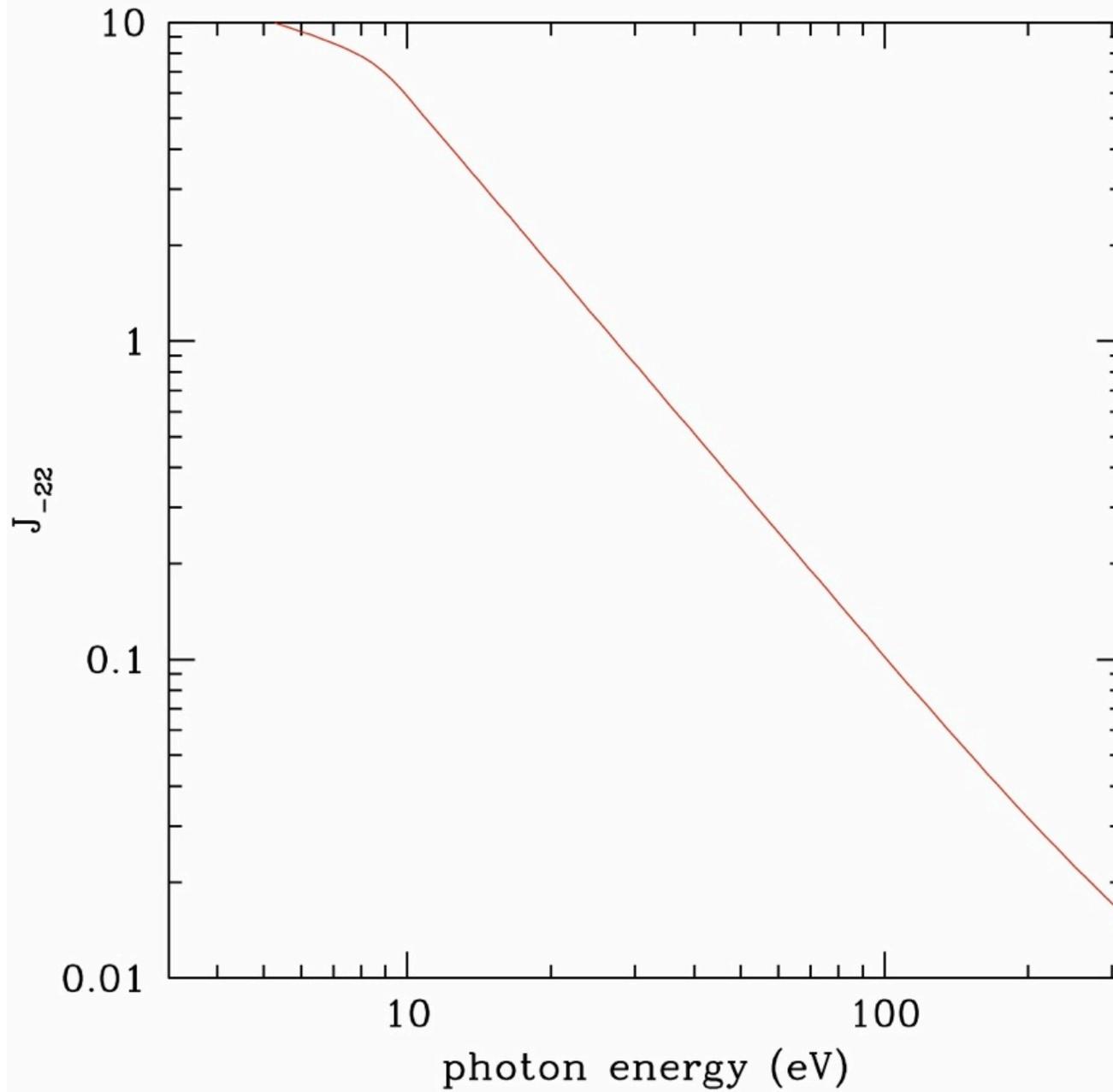
SED

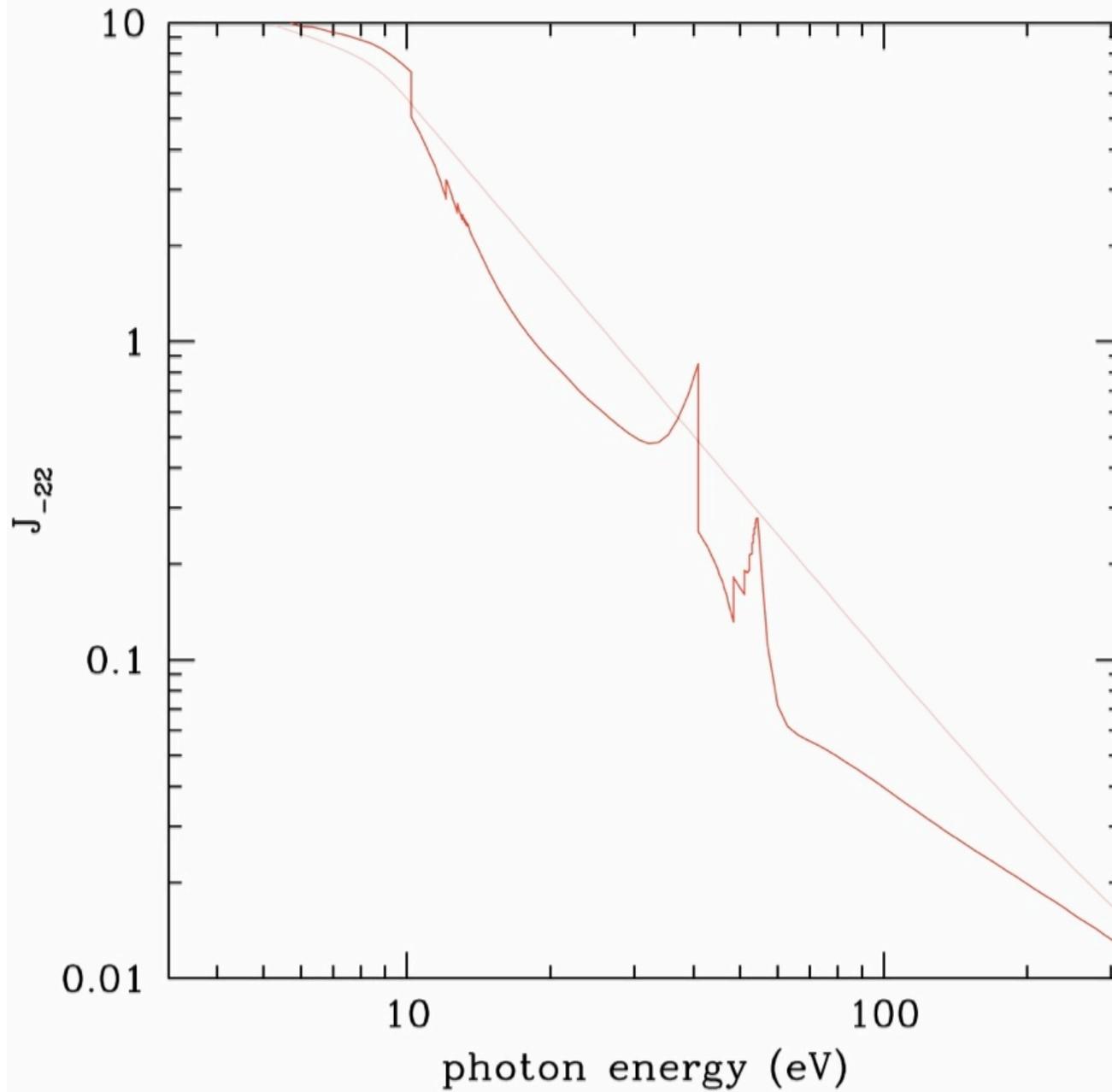
CUBA

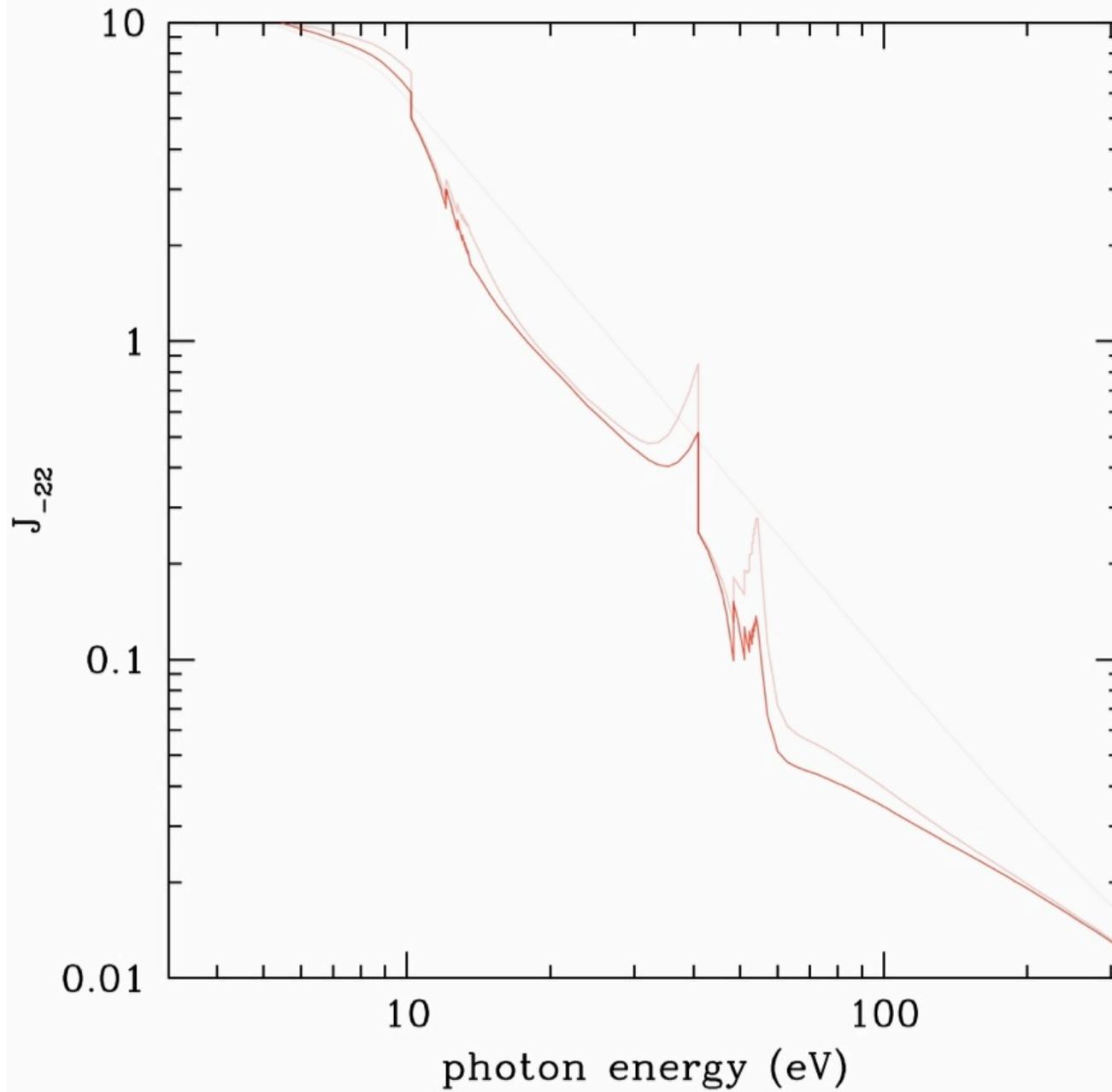


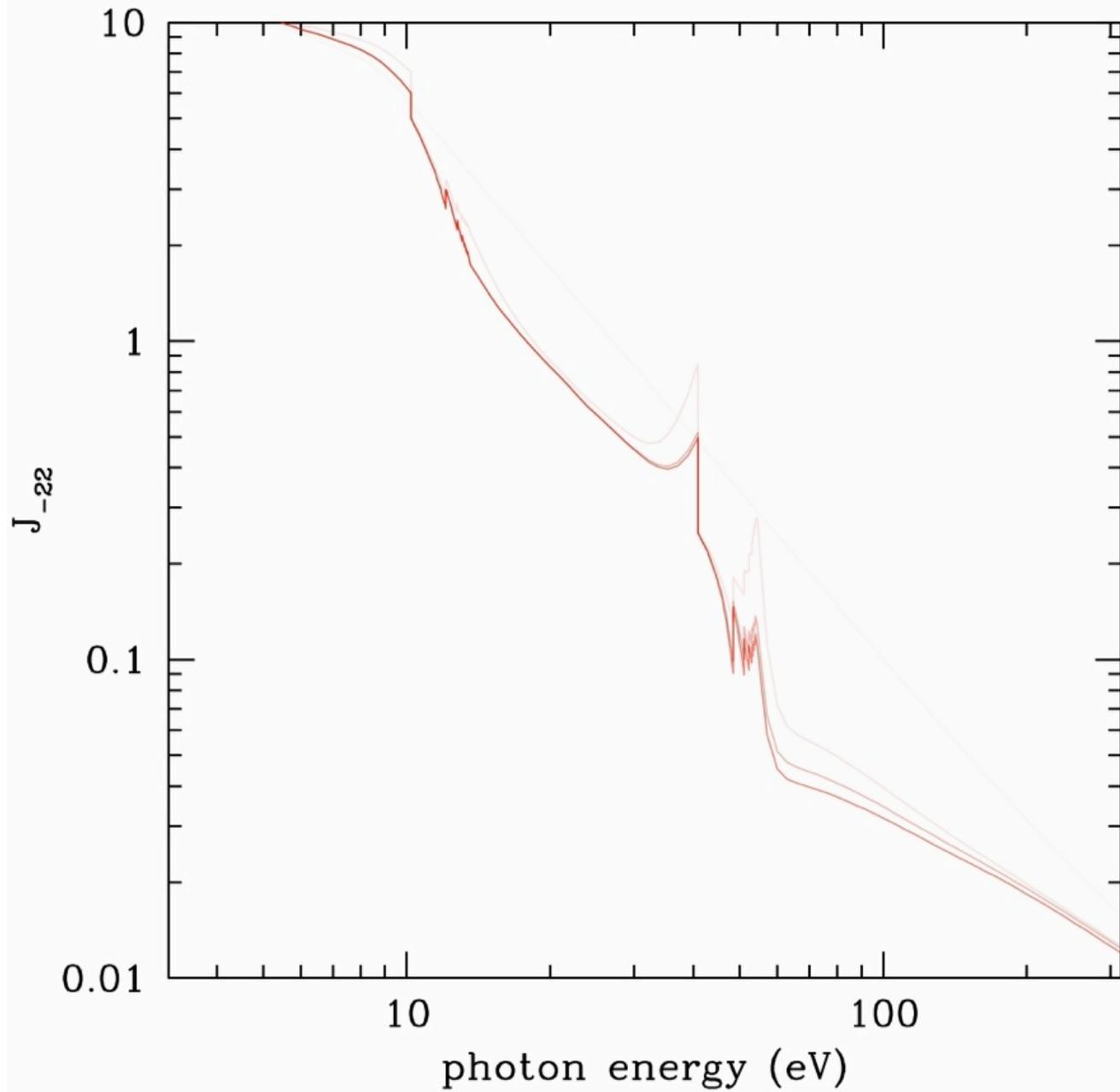
cosmological
radiative transfer
 $\rightarrow J$

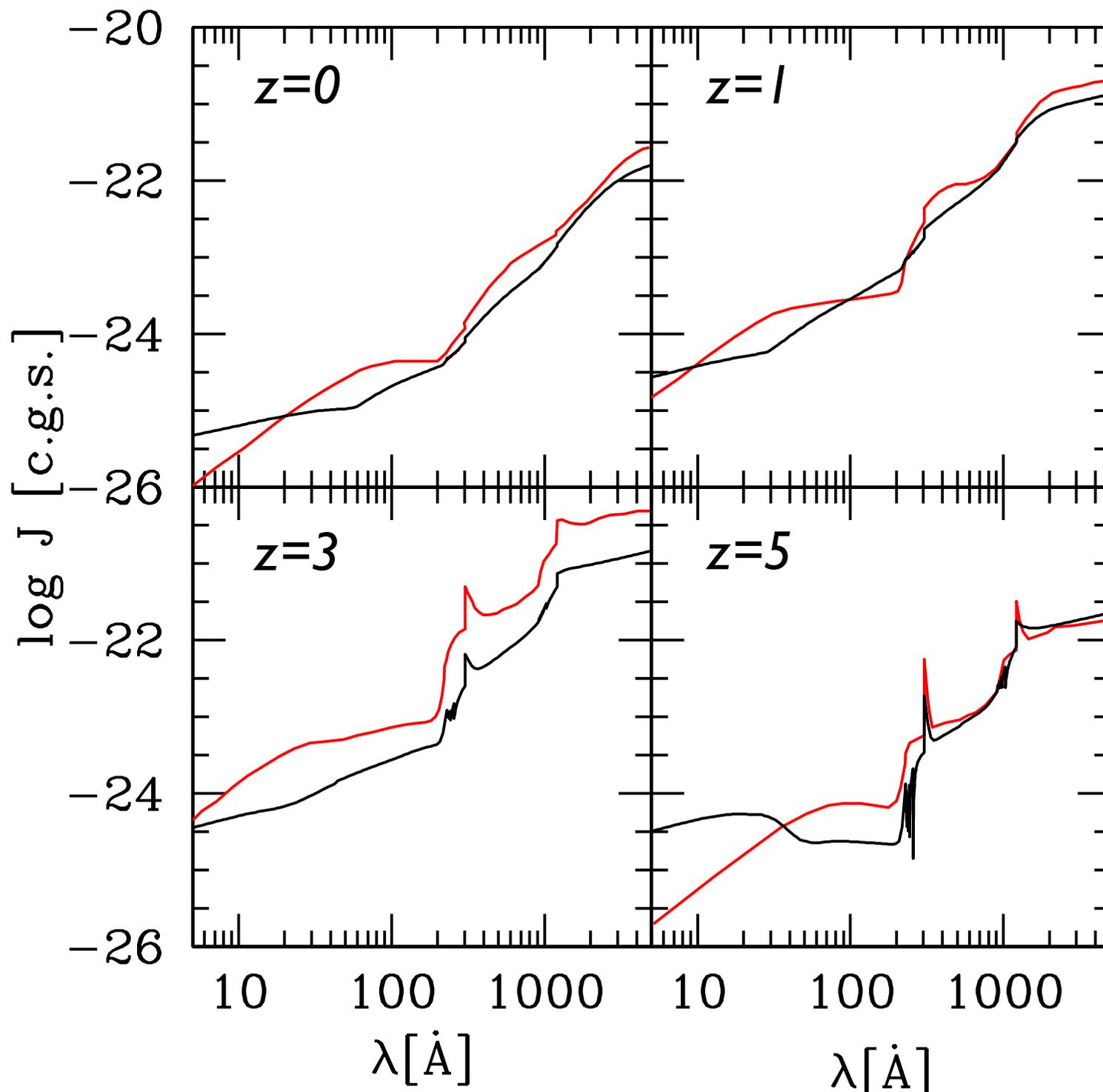
UVB

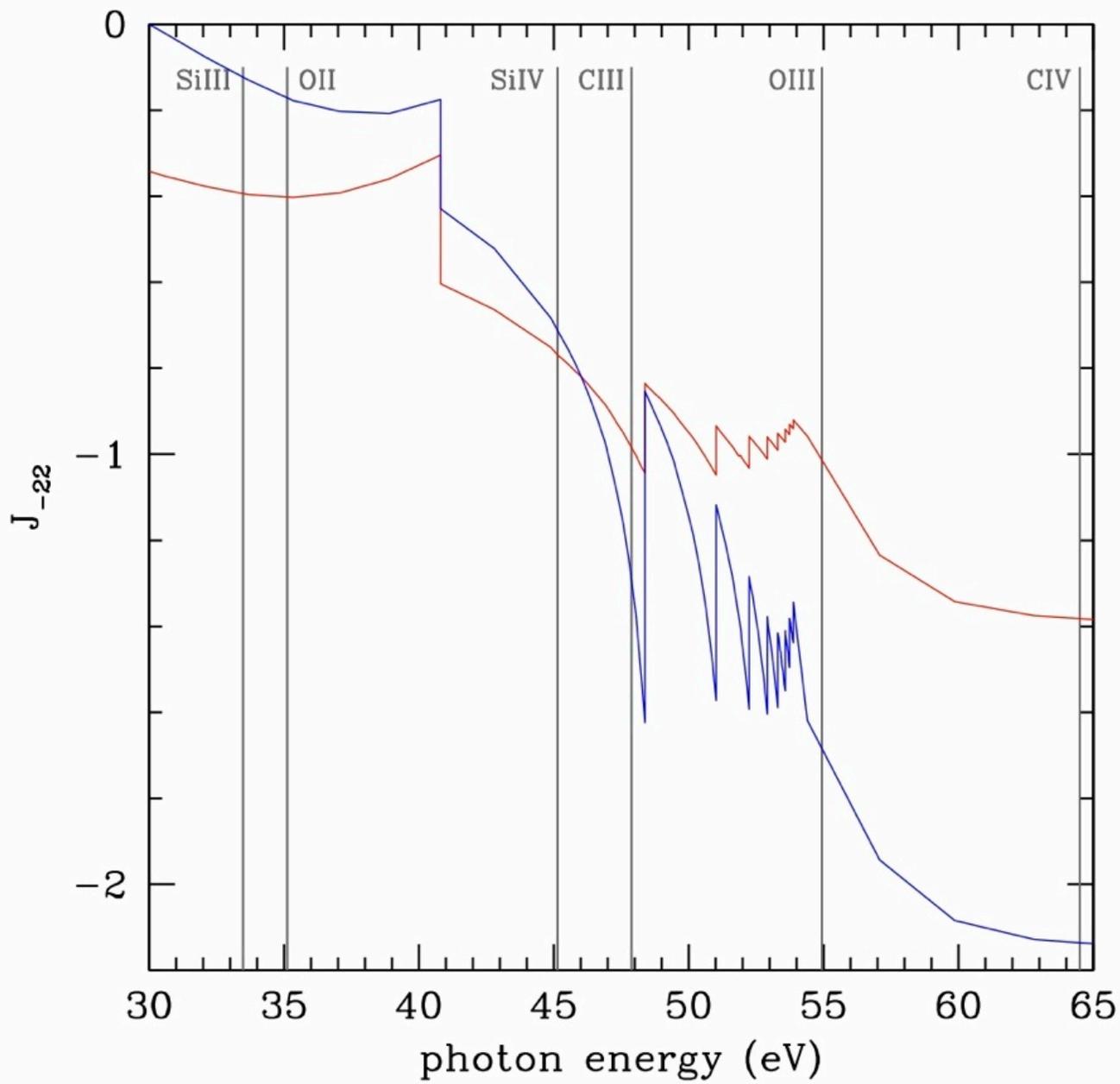












Haardt & PM 1996, ApJ, 461, 20

PM, Haardt, & Rees 1999, ApJ, 514, 648

PM et al, 2004 ApJ, 604, 484

PM & Haardt 2009, ApJ, 693, L100

Gilmore et al. 2009, MNRAS, 399, 1694

Haardt & PM 2020, in preparation

<http://pism.ucolick.org/CUBA>

THE END