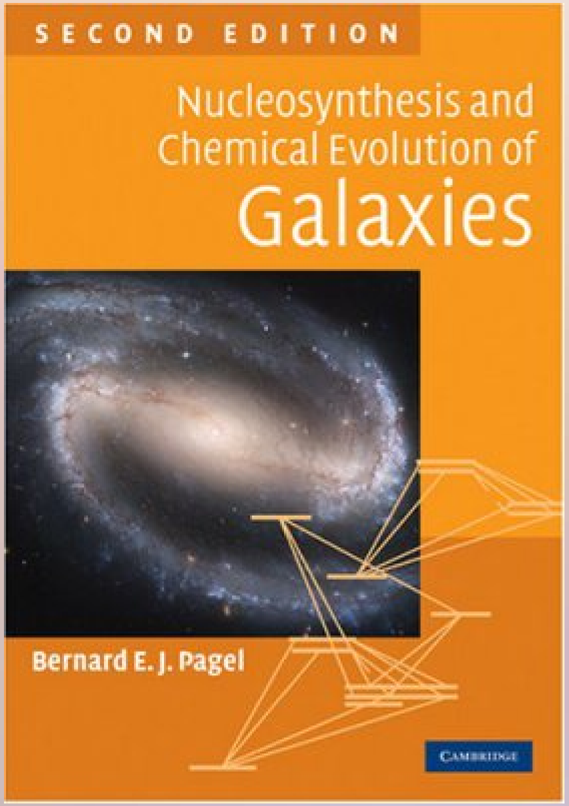
Analytic Models of Chemical Evolution

- Pagel: *Nucleosynthesis and Chemical Evolution of Galaxies*, chs. 7-8
- Equations of GCE
- Specific models
  - closed box
  - leaky box
  - inflow model



Helpful references

- Beatrice Tinsley 1980, Fundamentals of Cosmic Physics, 5, 287
  - “Evolution of the Stars and Gas in Galaxies”
- Andy McWilliam 1997, ARA&A, 35, 503
  - “Chemical Evolution of the Galaxy”
- Francesca Matteucci 2012, Springer
  - *Chemical Evolution of Galaxies*
- Kirby et al. 2011, ApJ, 727, 79
  - “Alpha Element Distributions in Milky Way Satellite Galaxies”

Initial mass function

$$\phi(m) \propto dN/dm \qquad [M_\odot^{-1}]$$

$$\int_{m_L}^{m^U} m\,\phi(m)\,dm = 1$$

m^U: 100 $M_\odot$

m_L: 0.08 $M_\odot$

$$\phi(m) = 0.17 m^{-2.35}$$

(Salpeter IMF)

$$\begin{aligned}
\phi(m) &= 13\,m^{-0.3} && \text{if } 0.01 \le m < 0.08 \\
\phi(m) &= 0.29\,m^{-1.8} && \text{if } 0.08 \le m < 0.5 \\
\phi(m) &= 0.15\,m^{-2.7} && \text{if } 0.5 \le m < 1.0 \\
\phi(m) &= 0.15\,m^{-2.3} && \text{if } m \ge 1.0
\end{aligned}$$

(Kroupa 2001 IMF)

Stellar lifetimes

$Z = 0.02 = Z_\odot$:

$$\tau = 950m^{-2.8} + 36m^{-0.65} + 1.5\ \mathrm{Myr}$$

$Z = 0.001 = 10^{-1.3}\, Z_\odot$:

$$\tau = 650m^{-2.7} + 10m^{-0.90} + 1.7\ \mathrm{Myr}$$

Basic GCE equations

galaxy mass

$$M = g + s \qquad [M_\odot]$$

infall onto galaxy

galactic wind (ejection)

$$dM/dt = F - E$$

stellar ejecta

$$dg/dt = F - E + e - \psi$$

SFR

$$ds/dt = \psi - e$$

$$e(t) = \int_{m_{\tau=t}}^{m_U} (m - m_{\mathrm{rem}})\, \psi(t - \tau(m))\, \phi(m)\, dm \qquad [M_\odot\ \mathrm{yr}^{-1}]$$

mass in stellar remnants

$$\frac{d}{dt}(gZ) = e_Z - Z\psi + Z_F F - Z_E E \qquad [M_\odot\ \mathrm{yr}^{-1}]$$

ISM abundance
M(element)/Σ*M*

stellar ejecta in metal *Z*

ejecta in metals as a
fraction of initial
stellar mass

$$e_Z(t) = \int_{m_{\tau=t}}^{m_U} \left[(m - m_{\mathrm{rem}})\, Z(t - \tau(m)) + m q_Z(m)\right]\, \psi(t - \tau(m))\, \phi(m)\, dm$$

Basic GCE equations

return fraction

$$R = \int_{m_\tau}^{m_U} (m - m_{\rm rem})\, \phi(m)\, dm \qquad [\ ]$$

$$\alpha = 1 - R$$

$$\begin{aligned} m_{\rm rem} &= 0.11m + 0.45 && \text{if } m \leq 6.8 \\ &= 1.5 && \text{if } m > 6.8 \end{aligned}$$

yield of element *i*

$$p_i = \alpha^{-1} \int_{m_\tau}^{m_U} m\, q_i(m)\, \phi(m)\, dm \qquad [M_\odot]$$

fraction of initial stellar mass (*m*) in element *i*

Instantaneous recycling approximation

all stars born by time t,
not accounting for mass loss

$$S(t) = \int_0^t \psi(t')\, dt' \qquad [M_\odot]$$

accounting for mass loss

$$s(t) = \alpha S(t)$$

$$\frac{dg}{dt} = F - E - \frac{ds}{dt}$$

$$\left(\frac{ds}{dt} = \alpha \frac{dS}{dt} = \alpha\psi\right)$$

$$\frac{dg}{ds} = \frac{F - E}{\alpha\psi} - 1$$

Instantaneous recycling approximation

$$\begin{aligned} e_Z(t) &= \int_{m_{\tau=t}}^{m_U} \left[(m - m_{\text{rem}})\, Z(t - \cancel{\tau(m)}) + m q_Z(m)\right] \psi(t - \cancel{\tau(m)})\, \phi(m)\, dm \\ &= \int_{m_t}^{m_U} \left[(m - m_{\text{rem}})\, Z(t) + m q_Z(m)\right] \psi(t)\, \phi(m)\, dm \\ &= \psi\, (RZ + \alpha p) \end{aligned}$$

$$\tau(m) = 0$$

$$\frac{d}{dt}(gZ) = e_Z - Z\psi + Z_F F - Z_E E$$

(from slide 5)

$$\frac{d}{dS}(gZ) = \alpha p + RZ - Z + Z_F \frac{F}{\psi} - Z_E \frac{E}{\psi}$$

$$\frac{d}{ds}(gZ) = p - Z\left(1 + \frac{E}{\alpha\psi}\right) + Z_F \frac{F}{\alpha\psi}$$

<u>homogeneous wind</u>

$$Z_E = Z$$

Instantaneous recycling approximation

$$= Z\frac{dg}{ds} + g\frac{dZ}{ds}$$

$$\frac{dg}{ds} = \frac{F-E}{\alpha\psi} - 1$$ (from slide 7)

$$g\frac{dZ}{ds} = p + (Z_F - Z)\frac{F}{\alpha\psi}$$

$$\frac{d}{ds}(gZ) = p - Z\left(1 + \frac{E}{\alpha\psi}\right) + Z_F\frac{F}{\alpha\psi}$$

Instantaneous recycling approximation

$$M(t) = s(t) + g(t) = M_0 - M_{\rm ej} + M_{\rm accr}$$

initial galaxy mass

mass in galactic wind

accreted mass

$$\frac{d}{ds}(gZ) = p - Z\left(1 + \frac{E}{\alpha\psi}\right) + Z_F\frac{F}{\alpha\psi}$$

$$s + g = M_0 - M_{\rm ej} + M_{\rm accr}$$

$$1 + \frac{dg}{ds} = -\frac{dM_{\rm ej}}{ds} + \frac{dM_{\rm accr}}{ds}$$

$$g\frac{dZ}{ds} = p + (Z_F - Z)\frac{F}{\alpha\psi}$$

$$1 + \frac{dg}{ds} = \frac{F - E}{\alpha\psi}$$

(from slide 7)

$$\frac{d}{ds}(gZ) = p - Z\left(1 + \frac{dM_{\rm ej}}{ds}\right) + Z_F\frac{dM_{\rm accr}}{ds}$$

$$g\frac{dZ}{ds} = p + (Z_F - Z)\frac{dM_{\rm accr}}{ds}$$

Closed box

Posit that:

$$g(t) + s(t) = M = \text{constant}$$

$$Z = Z(t)$$

not a function of position; instantaneous mixing

$$g(0) = M$$

$$Z(0) = S(0) = 0$$

Closed box: IRA

Closed box + IRA

$$dg = -ds$$

$$g\frac{dZ}{ds} = p + (Z_F - Z)\cancel{\frac{dM_{accr}}{ds}}$$

(from slide 10)

$$g\frac{dZ}{ds} = -g\frac{dZ}{dg} = p$$

$$\begin{aligned} Z &= -p\int_0^t \frac{dg}{g} \\ &= -p[\ln g(t) - \ln g(0)] \end{aligned}$$

current metallicity of *gas*

$$\boxed{\begin{aligned} Z &= p\ln\frac{M}{g} \\ &= p\ln\mu^{-1} \end{aligned}}$$

gas fraction

$$\mu \equiv \frac{g}{M}$$

No dependence on SFR (ψ)!

Closed box + IRA

$$z \equiv Z/p$$

$$\begin{aligned} < z > &= \frac{1}{s}\int_0^s z(s')\,ds' \\ &= \frac{1}{s}\int_0^s \ln\frac{s'+g}{g}\,ds' \\ &= \frac{1}{1-\mu}\int_1^\mu \ln\mu'\,d\mu' \\ &= \frac{(\mu\ln\mu-\mu)+1}{1-\mu} \\ &= 1+\frac{\mu\ln\mu}{1-\mu} \end{aligned}$$

average metallicity of *stars*

$$\begin{aligned} \mu &\equiv g/M \\ &= \frac{M-s}{M} \\ &= 1-\frac{s}{M} \end{aligned}$$

$$\int \ln x\,dx = x\ln x - x$$

Closed box + IRA

$$\begin{aligned} z &= -\ln \mu \\ &= -\ln \frac{g}{M} \end{aligned}$$

$$\begin{aligned} \frac{g}{M} &= 1 - \frac{s}{M} \\ &= e^{-z} \end{aligned}$$

$$\frac{s(z)}{M} = 1 - e^{-z}$$

$$z\frac{ds}{dz} \propto \frac{ds}{d\log z} \propto z\, e^{-z}$$

MDF

No dependence
on SFR (ψ)!

Closed box + IRA

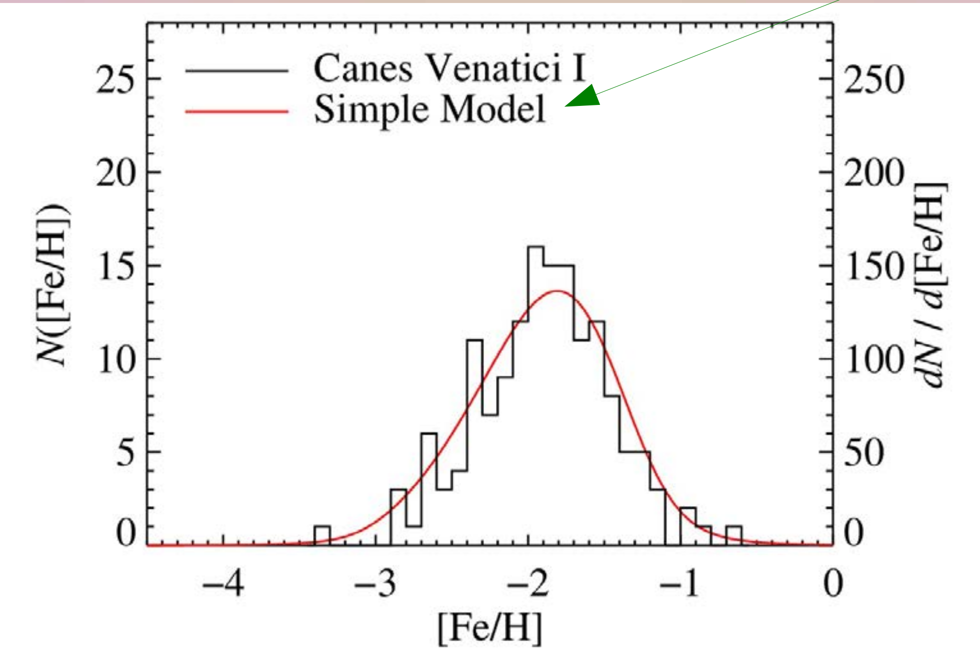
Closed box + IRA

$$< Z > = p\left[1 + \frac{\mu \ln \mu}{1-\mu}\right]$$

$$< Z > \leq p \qquad \text{always}$$

$$\lim_{\mu \to 0} < Z > = p$$

Simple Model = Closed Box



Leaky box + IRA

$$\frac{dg}{ds} = -\frac{E}{\alpha\psi} - 1 \equiv -\eta - 1$$

constant

outflow tracks SFR

$$\begin{aligned} g\frac{dz}{ds} &= 1 \\ g\frac{dz}{dg} &= -\frac{1}{1+\eta} \end{aligned}$$

(from slide 13)

$$\frac{g}{M_0} = e^{-(1+\eta)z}$$

$$\frac{ds}{d\log z} \propto z e^{-(1+\eta)z}$$

$$\boxed{p \rightarrow \frac{p}{1+\eta}}$$

G dwarf problem

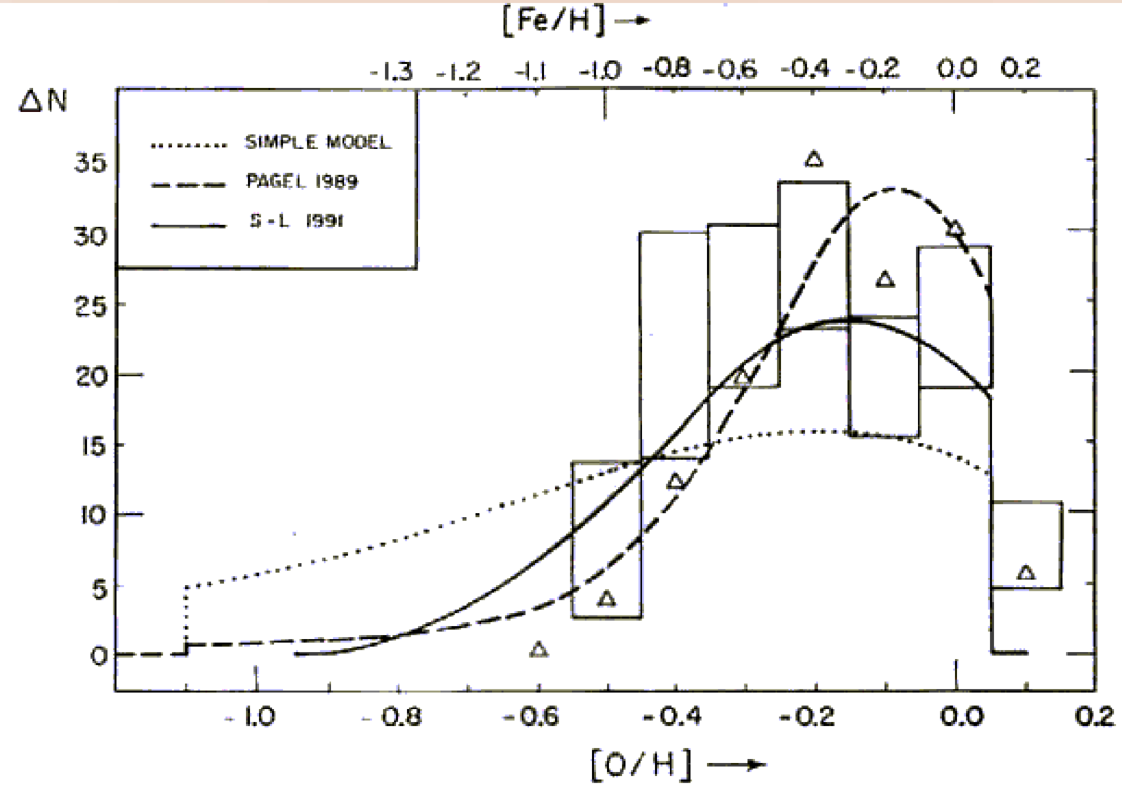


Fig. 8.19. Distribution function of oxygen abundances of 132 G-dwarfs in the solar cylinder, binned in intervals of 0.1 in [O/H]. Triangles show the data points after Pagel (1989ab), based on a reanalysis of those discussed by Pagel & Patchett (1975), and boxes show lower and upper limits based on a new discussion of the dependence of the scale height on age and metallicity by Sommer-Larsen (1991a). The dotted curve shows predictions of an instantaneous Simple model with an initial enrichment [O/H] = −1.1 from the halo. The other model curves are discussed below. After B.E.J. Pagel, 'Abundances in Galaxies', in H. Oberhummer (ed.), *Nuclei in the Cosmos*, p. 98, Fig. 9. ©Springer-Verlag Berlin Heidelberg 1991.

Extreme inflow model

gas inflow keeps gas mass constant

inflowing gas is metal-free

no outflow

$$g = \text{constant}$$
$$Z_F = 0$$
$$E = 0$$

$$dg/dt = F - E - ds/dt = 0$$
$$F = ds/dt = \alpha\psi$$

(from slide 7)

$$\begin{aligned} g\frac{dZ}{ds} &= p + (Z_F - Z)\frac{F}{\alpha\psi} \\ \frac{dZ}{ds} + \frac{Z}{g} &= \frac{p}{g} \\ Z &= Z_0 e^{-s/g} + p(1 - e^{-s/g}) \end{aligned}$$

(from slide 9)

$$\boxed{\lim_{s/g \to \infty} Z = p}$$

"Best accretion model" (Lynden-Bell 1975)

<u>Posit that</u>:

$$E = Z_F = 0$$
$$g(s) = (1 - s/M)(1 + s - s/M)$$

- simplest quadratic possible
- has a maximum in *g*
- smooth decay in *g* as star formation ebbs

$$\text{if } \; M = 1, \; g(s) = 1 - s \longleftarrow$$ closed box

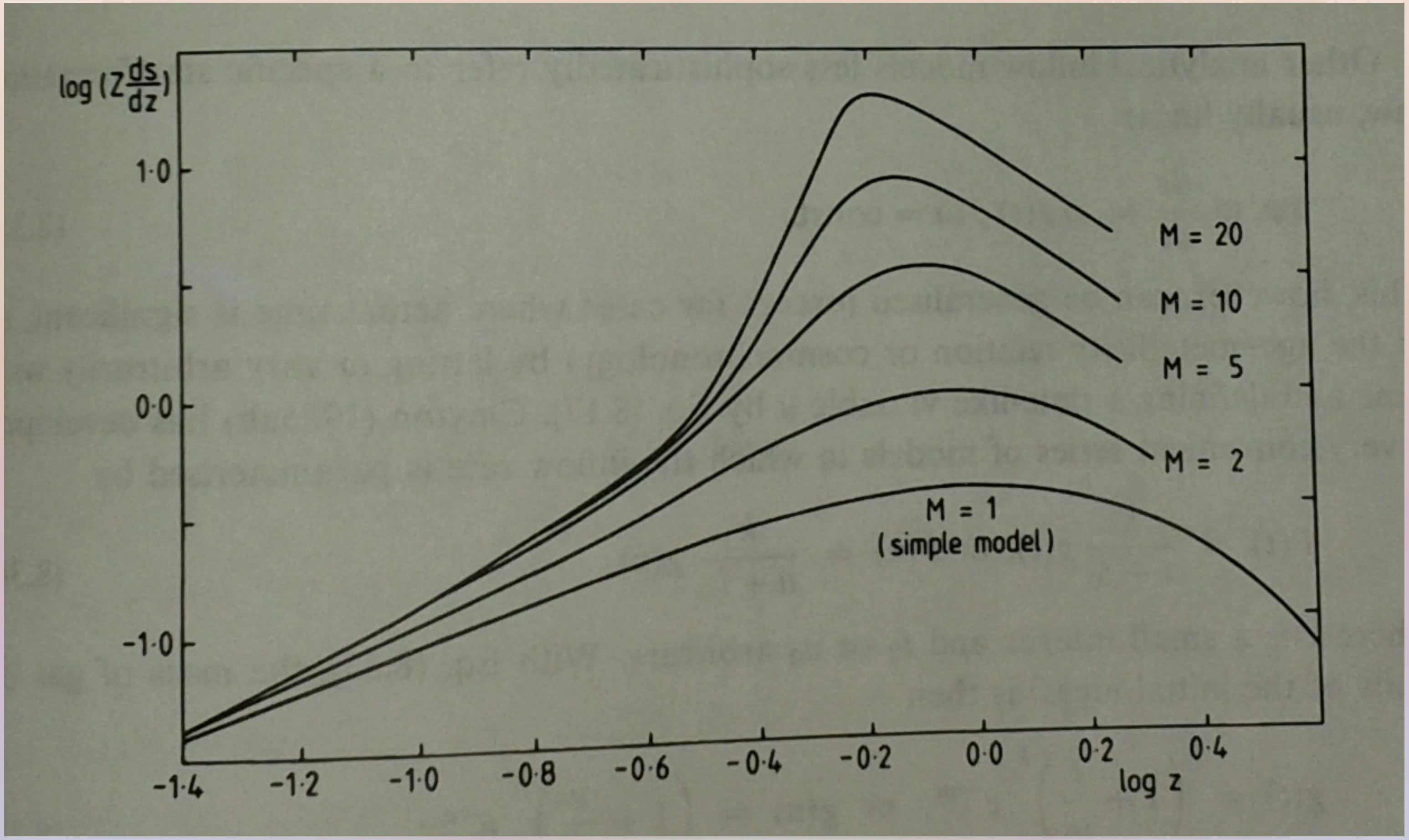
$$\frac{d}{ds}(gZ) = p - Z$$

(from slide 8)

$$z(s) = \left(\frac{M}{1 + z - s/M}\right)^2 \left[\ln \frac{1}{1 - s/M} - \frac{s}{M}\left(1 - \frac{1}{M}\right)\right]$$

$$\frac{ds}{d\ln z} = \frac{z[1 + s(1 - 1/M)]}{(1 - s/M)^{-1} - 2z(1 - 1/M)}$$

“Best accretion model” (Lynden-Bell 1975)



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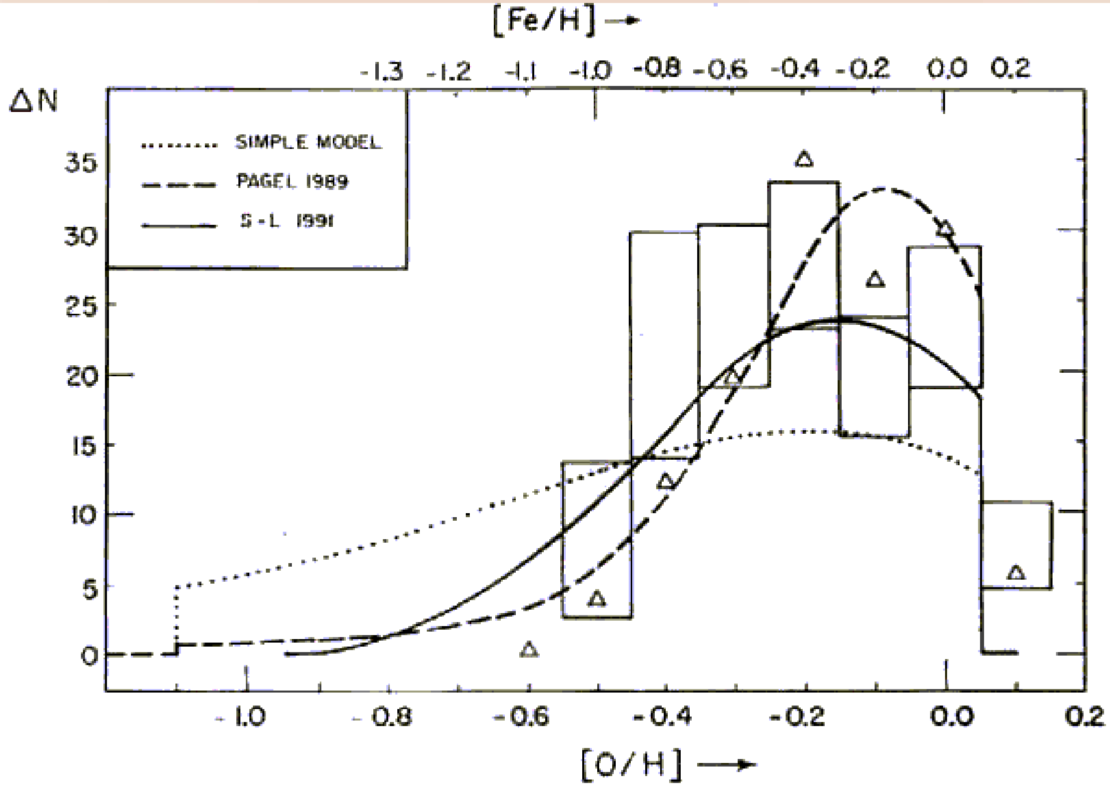
G dwarf problem



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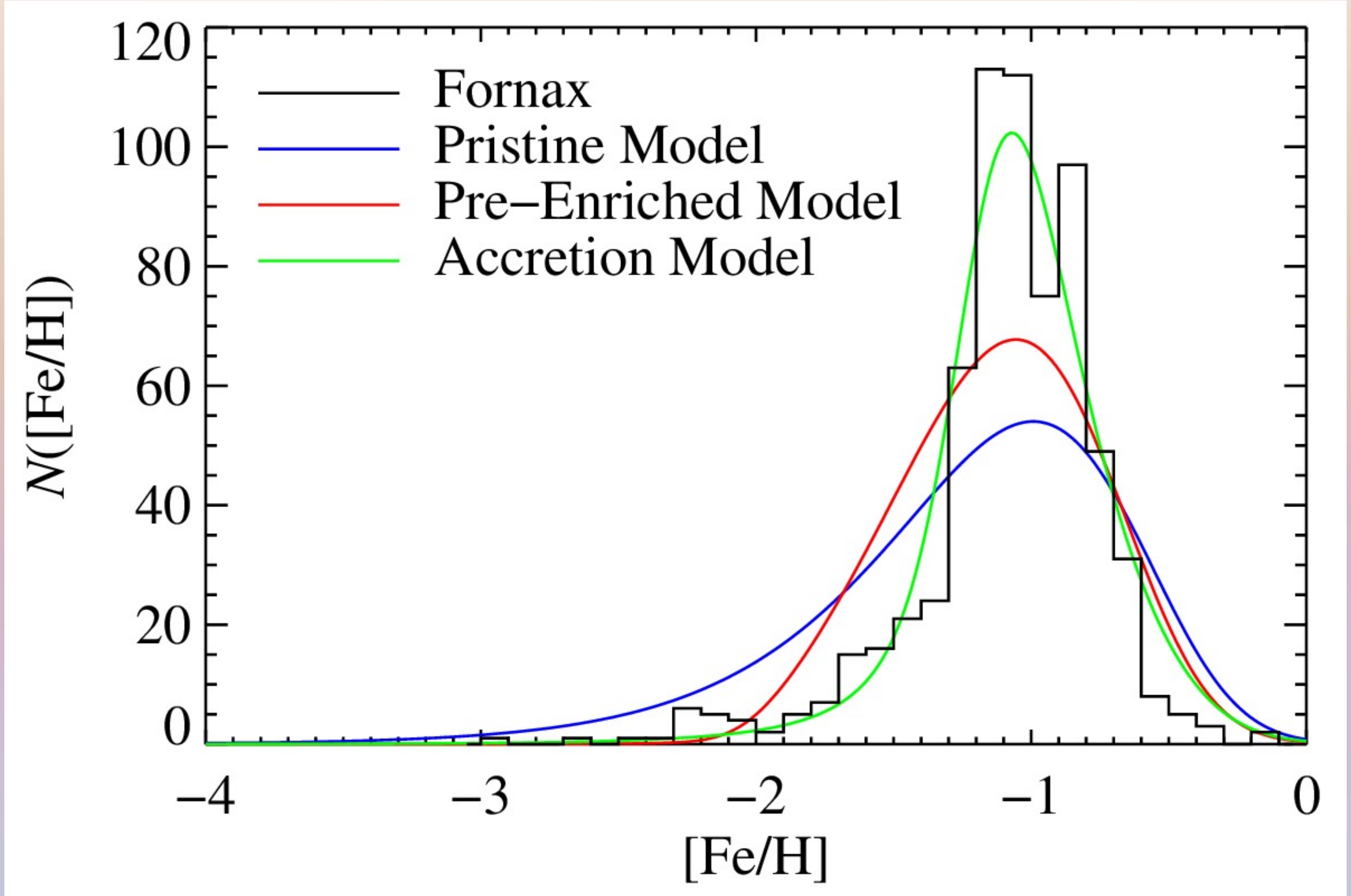
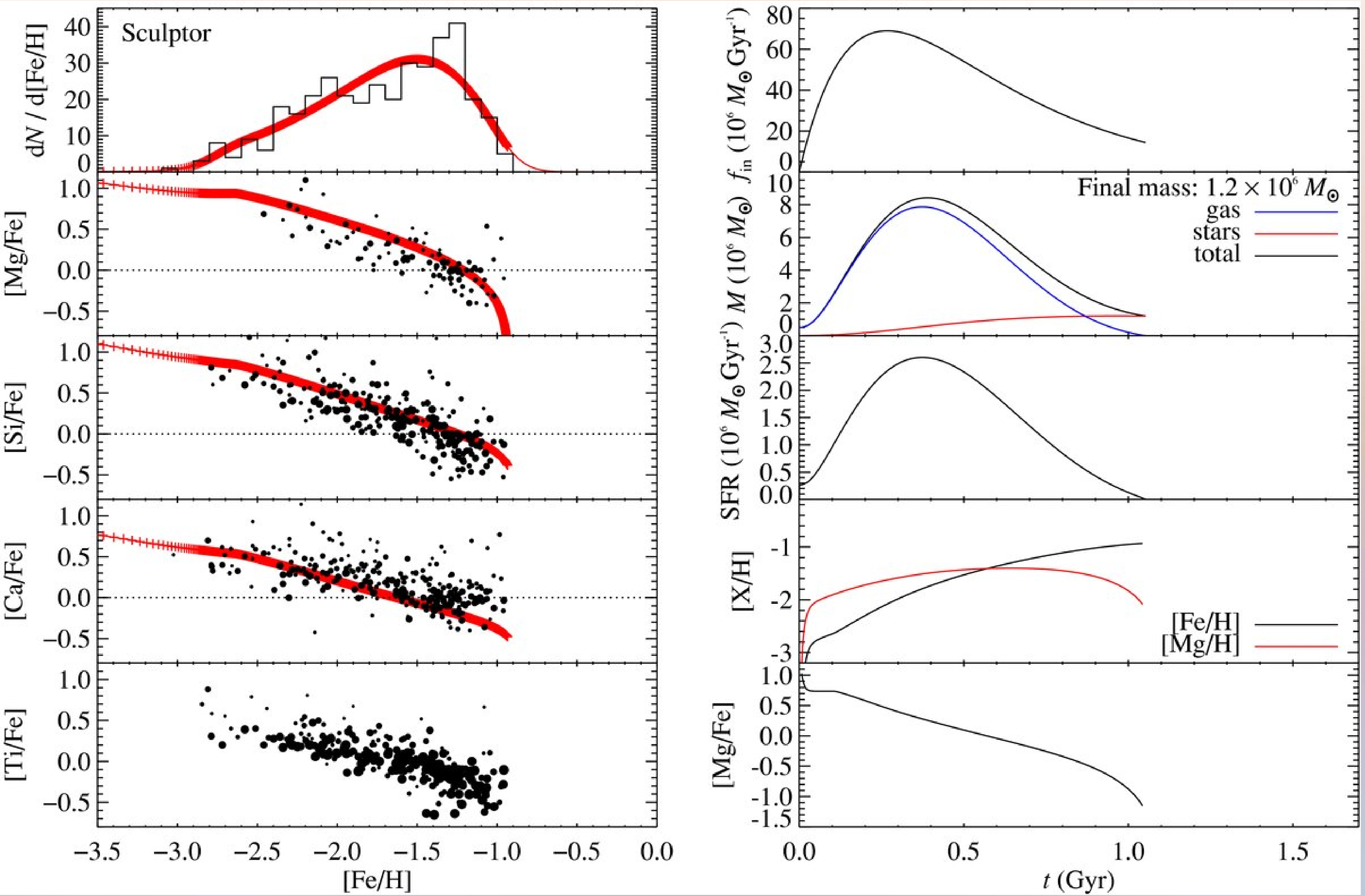
Fornax dwarf spheroidal galaxy



Numerical models



Kirby et al. 2011, ApJ, 727, 79