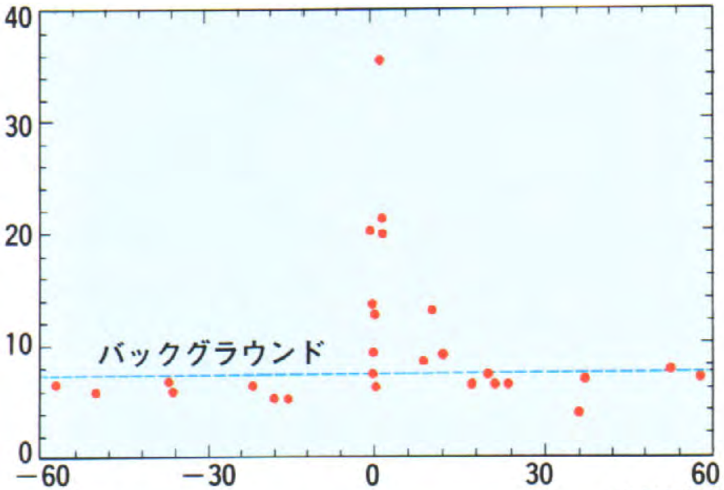
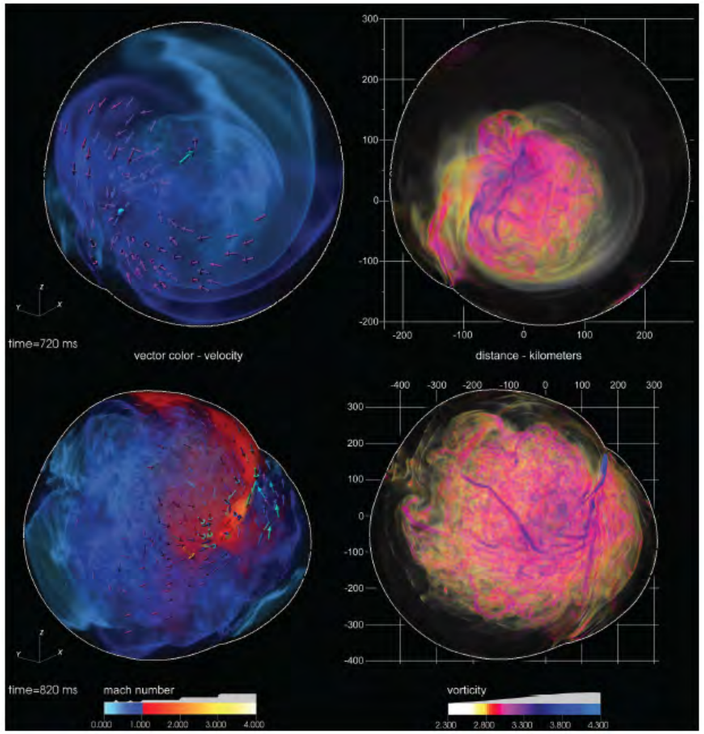
Neutrinos from core-collapse supernovae

- $M_{prog} \geq 8 M_{Sun}$
- $\Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
- 99% of the energy is carried away by neutrinos and antineutrinos with

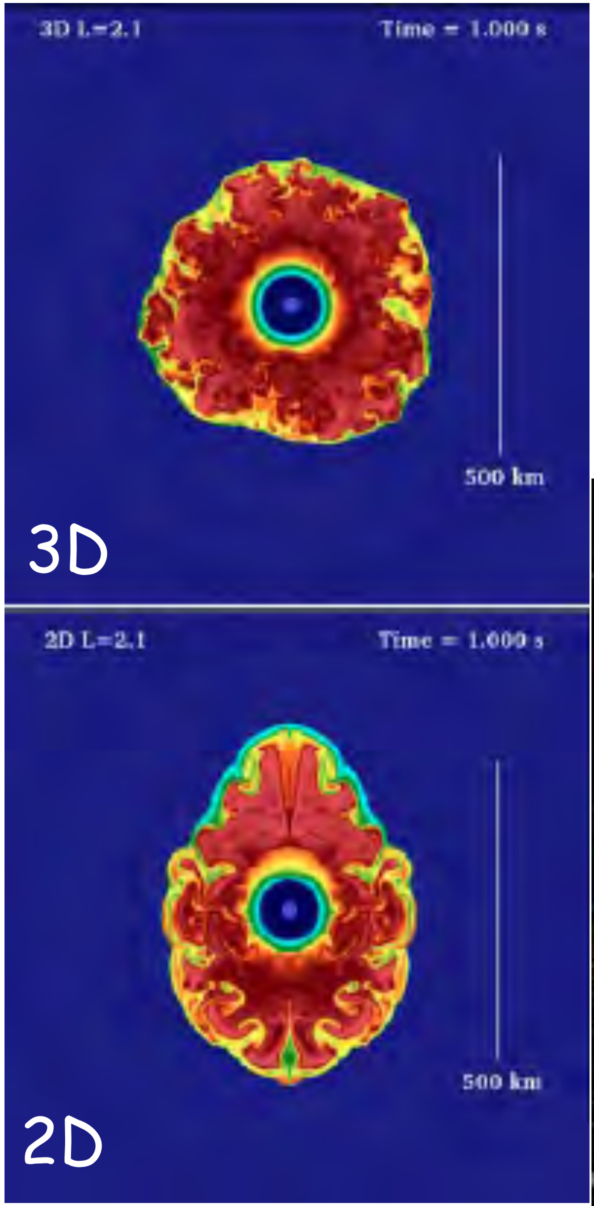
  $10 \leq E_{\nu} \leq 30$ MeV
- ~ 10^{58} Neutrinos!



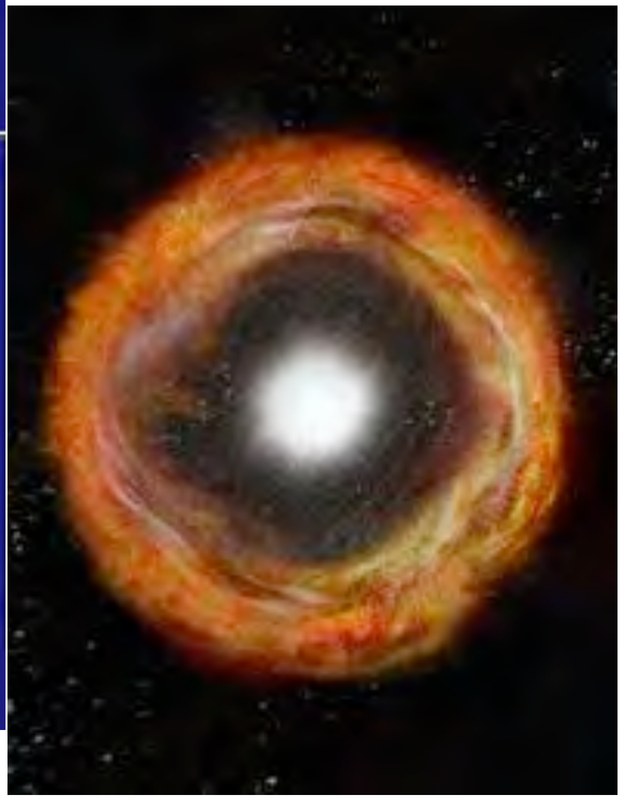
ORNL/UT

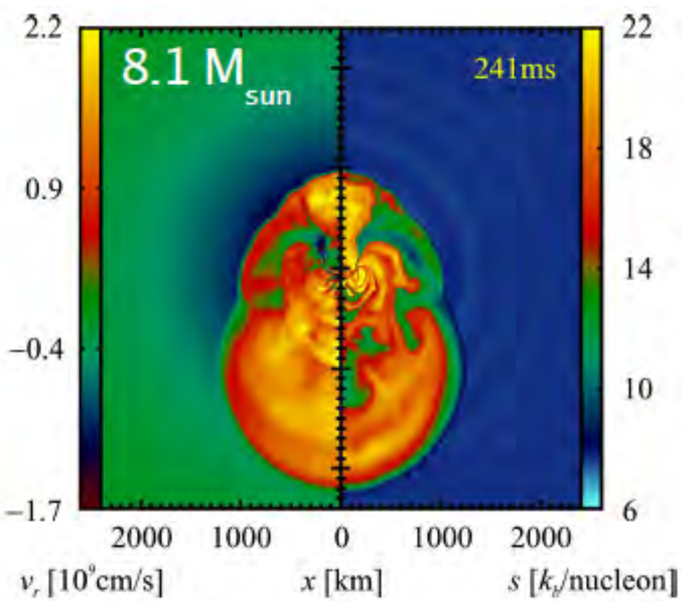


Development of 2D and 3D models for core-collapse supernovae: Complex interplay between turbulence, neutrino physics and thermonuclear reactions.



Princeton

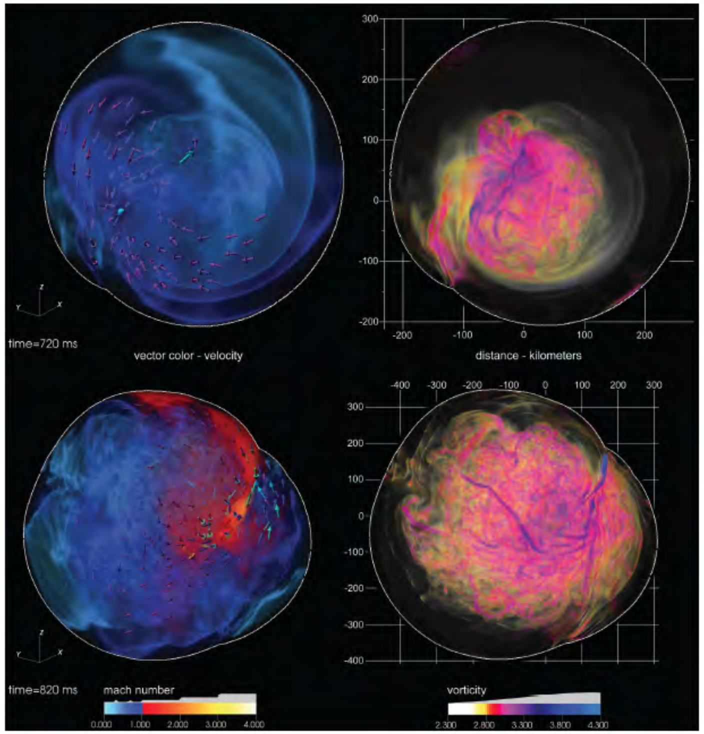


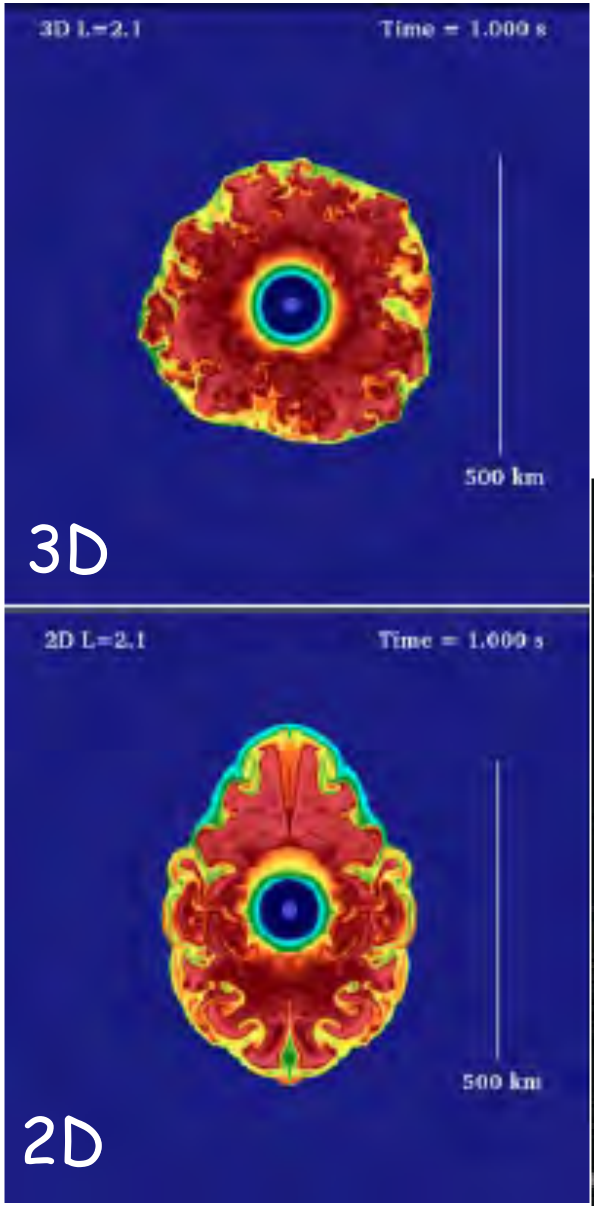


Munich

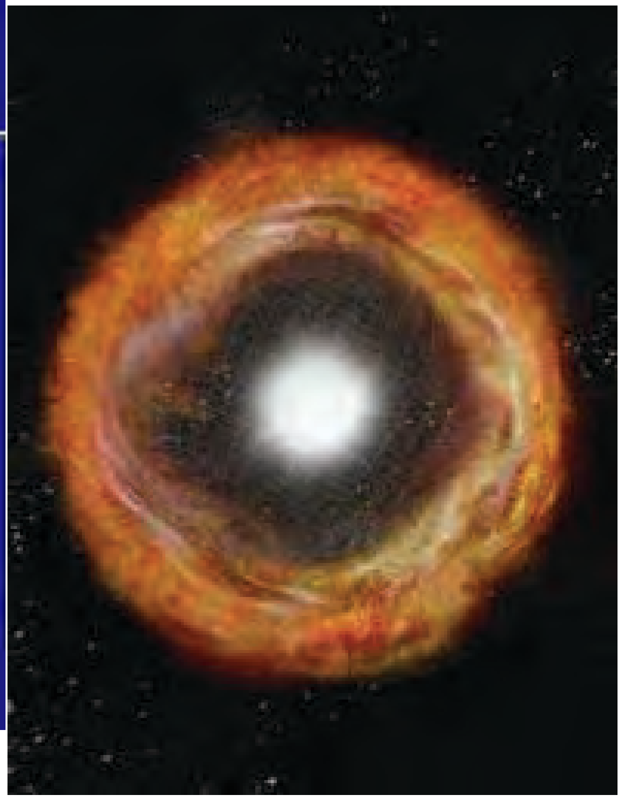
Development of 2D and 3D models for core-collapse supernovae: Complex interplay between turbulence, neutrino physics and thermonuclear reactions.

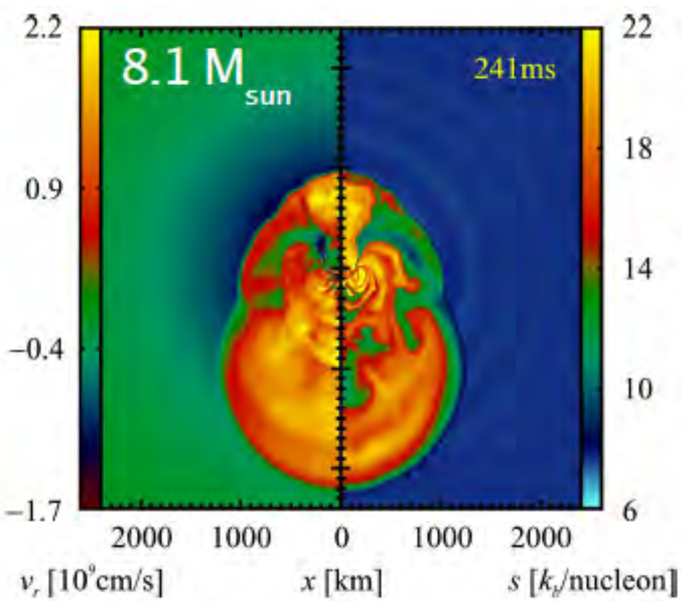
ORNL/UT





Princeton

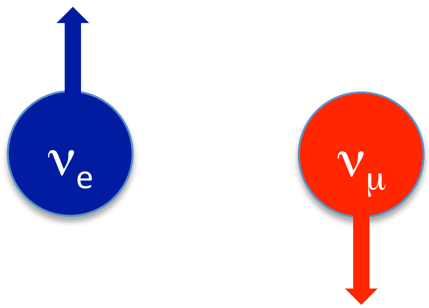




Munich

If we want to catch a supernova with neutrinos we'd better know what neutrinos do inside a supernova.

Neutrino flavor isospin



$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

Free neutrinos (only mixing)

$$\hat{H} = \frac{m_1^2}{2E} a_1^\dagger a_1 + \frac{m_2^2}{2E} a_2^\dagger a_2 + (\cdot\cdot)\hat{1}$$

$$= \frac{\delta m^2}{4E}\cos 2\theta\left(a_\mu^\dagger a_\mu - a_e^\dagger a_e\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(a_e^\dagger a_\mu + a_\mu^\dagger a_e\right) + (\cdot\cdot)'\,\hat{1}$$

Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(a_\mu^\dagger a_\mu - a_e^\dagger a_e\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(a_e^\dagger a_\mu + a_\mu^\dagger a_e\right) + (\cdot\cdot)''\,\hat{1}$$

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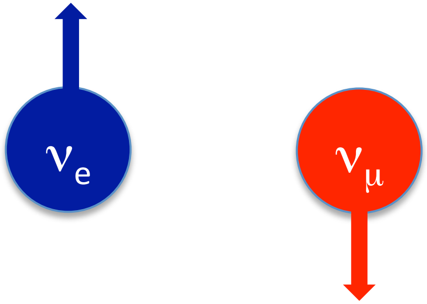

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$$a_e = \cos\theta\, a_1 + \sin\theta\, a_2$$

$$a_\mu = -\sin\theta\, a_1 + \cos\theta\, a_2$$

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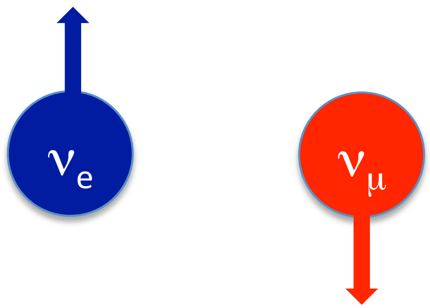

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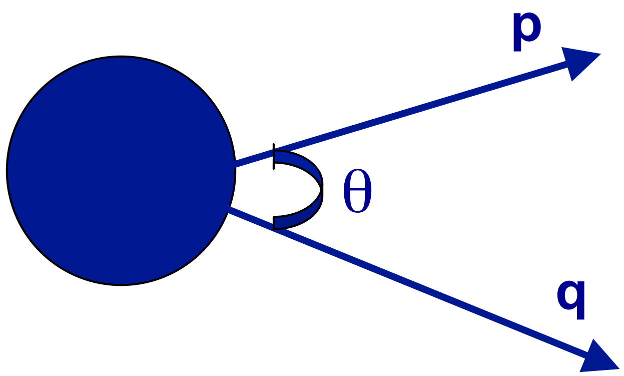
Neutrino-Neutrino Interactions

Smirnov, Fuller and Qian, Pantaleone, McKellar, Friedland, Lunardini, Duan, Raffelt, Balantekin, Kajino, Pehlivan …

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\, dq \left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q$$



This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

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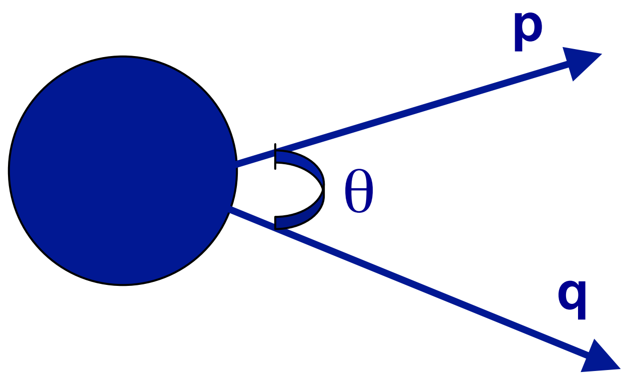


This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

$$\hat{H} = \int dp\left(\frac{\delta m^2}{2E}\vec{\mathbf{B}}\cdot\vec{\mathbf{J}}_p - \sqrt{2}G_F N_e \mathbf{J}_p^0\right) + \frac{\sqrt{2}G_F}{V}\int dp\, dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q$$

$$\vec{\mathbf{B}} = \left(\sin 2\theta,\ 0, -\cos 2\theta\right)$$

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral "swaps" or "splits").

Including antineutrinos

$$H = H_\nu + H_{\bar{\nu}} + H_{\nu\nu} + H_{\bar{\nu}\bar{\nu}} + H_{\nu\bar{\nu}}$$

Requires introduction of a second set of SU(2) algebras!

Including three flavors

Requires introduction of SU(3) algebras.

Both extensions are straightforward, but tedious!
Balantekin and Pehlivan, J. Phys. G **34**, 1783 (2007).

Many neutrino system

This is the only many-body system driven by the weak interactions:

Table: Many-body systems

Nuclei	Strong	at most ∼250 particles
Condensed matter	E&M	at most N_A particles
ν**'s in SN**	Weak	$\sim 10^{58}$particles

Astrophysical extremes allow us to test physics that cannot be tested elsewhere!

Path Integral for the Evolution Operator

$$i\frac{\partial U}{\partial t} = (H_\nu + H_{\nu\nu})\,U$$

Use SU(2) coherent states to write the evolution operator as a path integral:

$$|z(t)\rangle = \exp\left(\int dp z(p,t) J_+(p)\right)|\phi\rangle$$

$$|\phi\rangle = \prod_p a_e^\dagger(p)|0\rangle$$

$$\langle z'(t_f)|U|z(t_i)\rangle = \int \mathcal{D}[z,z^*]\exp\left(iS[z,z^*]\right)$$

Stationary Phase Approximation

$$\langle z'(t_f)|U|z(t_i)\rangle = \int \mathcal{D}[z, z^*] \exp\left(iS[z, z^*]\right)$$

$$S(z, z^*) = \int_{t_i}^{t_f} dt \frac{\langle z(t)|i\frac{\partial}{\partial t} - H(t)|z(t)\rangle}{\langle z(t)|z(t)\rangle} + \log\langle z'(t_f)|z(t_f)\rangle$$

$$H = H_\nu + H_{\nu\nu}$$

$$\left(\frac{d}{dt}\frac{\partial}{\partial \dot{z}} - \frac{\partial}{\partial z}\right) L(z, z^*) = 0 \qquad \left(\frac{d}{dt}\frac{\partial}{\partial \dot{z}^*} - \frac{\partial}{\partial z^*}\right) L(z, z^*) = 0$$

Mean-field evolution equations

$$\Delta = \frac{\delta m^2}{2p}, \qquad A = \sqrt{2} G_F N_e$$

$$D = \sqrt{2} G_F \int dq (1 - \cos\theta_{pq}) \left[(|\psi_e(q,t)|^2 - |\psi_x(q,t)|^2) \right]$$

$$D_{ex} = 2\sqrt{2} G_F \int dq (1 - \cos\theta_{pq}) \left(\psi_e(q,t) \psi_x^*(q,t) \right)$$

$$i \frac{\partial}{\partial t} \begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix} = \frac{1}{2} \begin{pmatrix} A + D - \Delta \cos 2\theta & D_{e\mu} + \Delta \sin 2\theta \\ D_{\mu e} + \Delta \sin 2\theta & -A - D + \Delta \cos 2\theta \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix}$$

The duality between $H_{\nu\nu}$ and BCS Hamiltonians

The ν-ν Hamiltonian

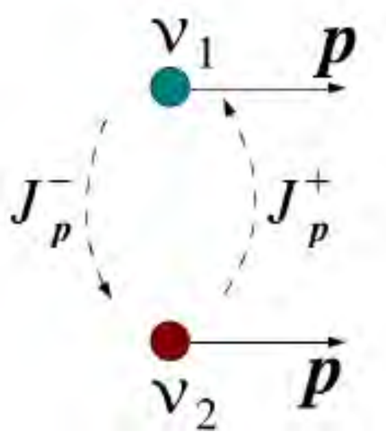
$$\hat{H} = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \vec{J} \cdot \vec{J}$$

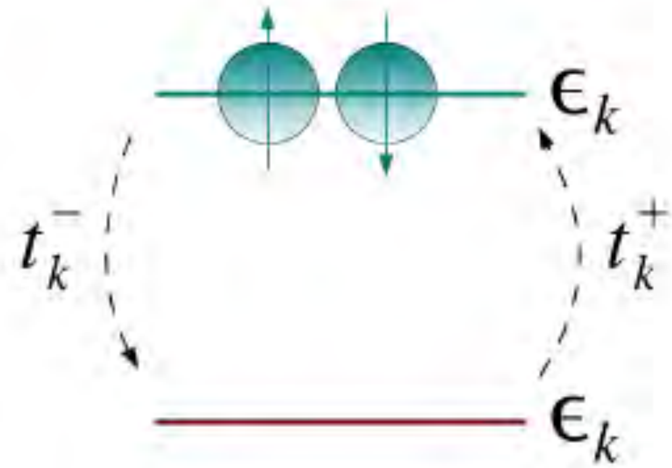
$\Longleftrightarrow$

The BCS Hamiltonian

$$\hat{H}_{\mathrm{BCS}} = \sum_k 2\epsilon_k \hat{t}_k^0 - |G| \hat{T}^+ \hat{T}$$

Same symmetries leading to Analogous (dual) dynamics!
Pehlivan, Balantekin, Kajino, and Yoshida, Phys.Rev. D **84**, 065008 (2011)



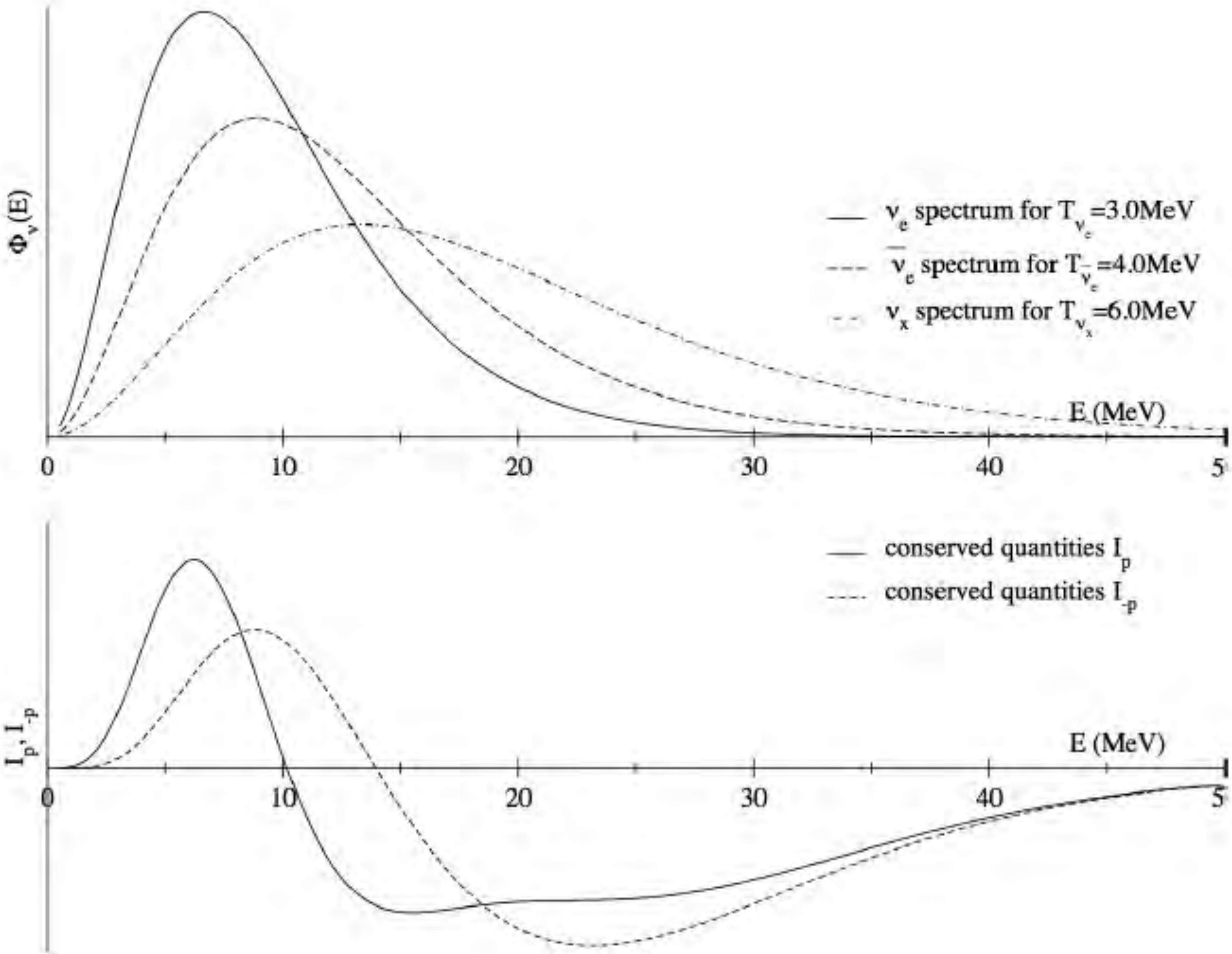


This symmetry naturally leads to splits in the neutrino energy spectra and was used to find conserved quantities in the single-angle case.

Conserved quantities of the collective motion

$$h_p = \vec{\mathbf{B}} \cdot \vec{\mathbf{J}}_p + \frac{4\sqrt{2}G_F}{\delta m^2 V} \sum_{p \neq q} qp \frac{\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q}{q-p}$$

- There is a second set of conserved quantities for antineutrinos.
- Note the presence of volume. In fact h_p/V are the conserved quantities for the neutrino densities.
- For three flavors a similar expression is written in terms of SU(3) operators.



$\Phi_\nu(E)$
ν_e spectrum for T_{ν_e}=3.0MeV
$\bar{\nu}_e$ spectrum for $T_{\bar{\nu}_e}$=4.0MeV
ν_x spectrum for T_{ν_x}=6.0MeV
E (MeV)
0
10
20
30
40
5
conserved quantities I_p
conserved quantities I_{-p}
I_p, I_{-p}
E (MeV)
0
10
20
30
40
5

The ν-ν Hamiltonian

$$\hat{H} = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \vec{J} \cdot \vec{J}$$

$\Longleftrightarrow$

The BCS Hamiltonian

$$\hat{H}_{\mathrm{BCS}} = \sum_k 2\epsilon_k \hat{t}^0_k - |G| \hat{T}^+ \hat{T}$$

Recall how we treat the BCS Hamiltonian. We diagonalize it in a quasiparticle basis. However that basis does not preserve particle number. We enforce the particle number conservation by introducing a Lagrange multiplier. This Lagrange multiplier turns out to be the chemical potential.

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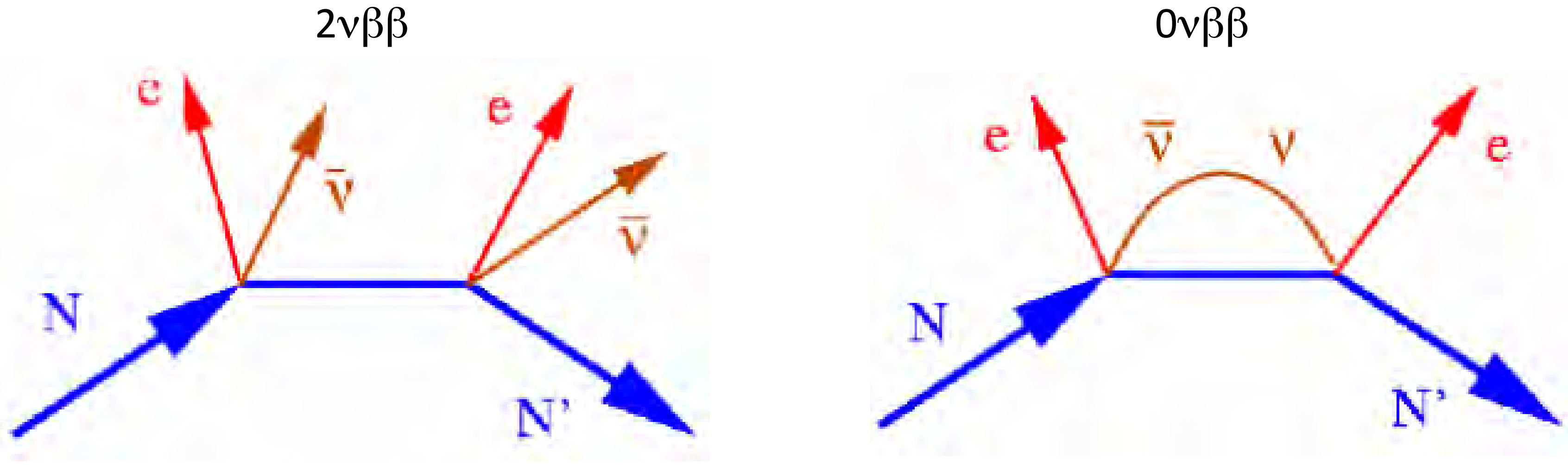
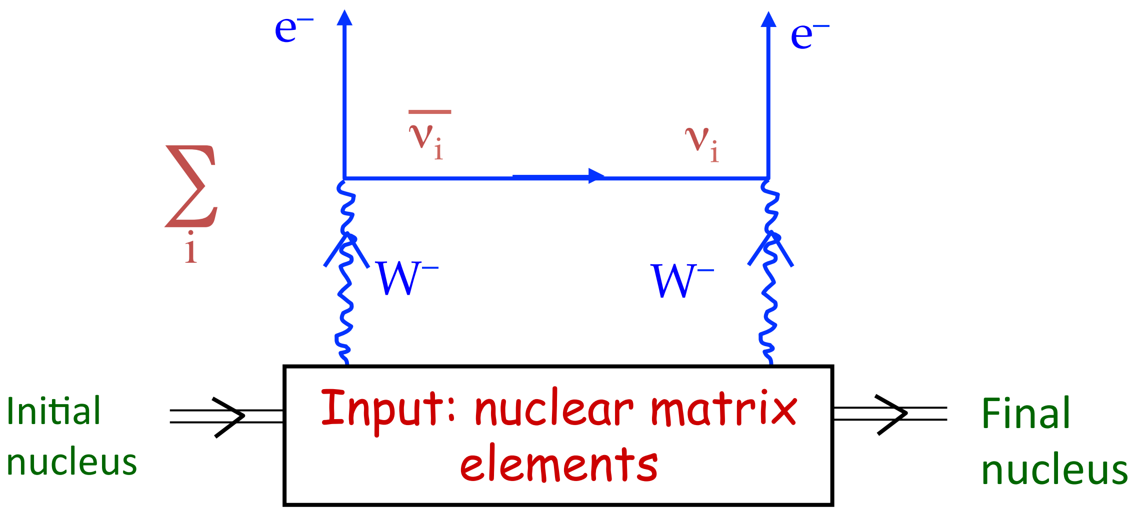
Recall how we treat the BCS Hamiltonian. We diagonalize it in a quasiparticle basis. However that basis does not preserve particle number. We enforce the particle number conservation by introducing a Lagrange multiplier. This Lagrange multiplier turns out to be the chemical potential.
In the many neutrino case we can do the same. The Lagrange multiplier we have to introduce to preserve the total neutrino number shows up the the final neutrino energy spectra as a "split". This is the origin of the spectral splits (or swaps) numerically observed in many calculations.

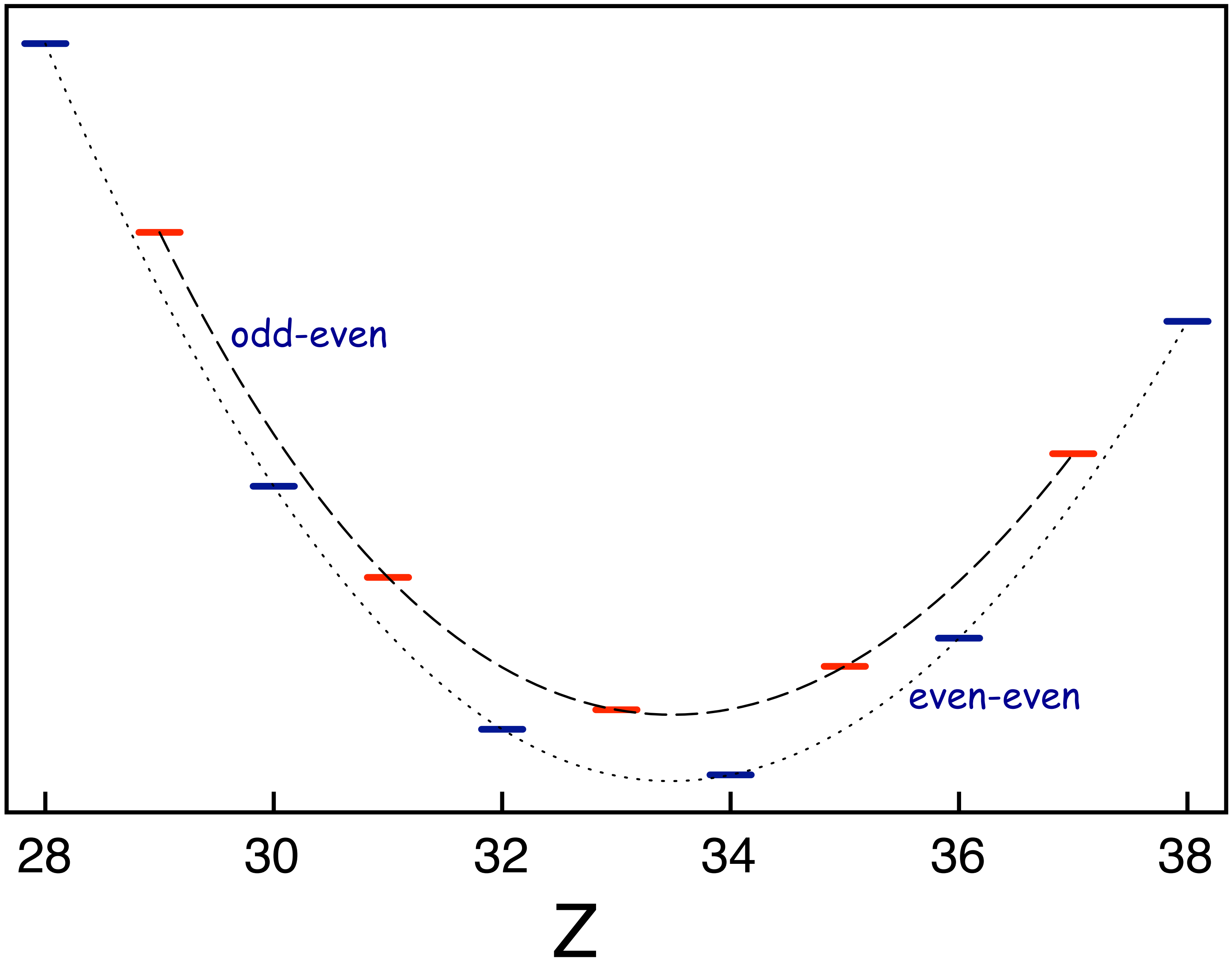
Maria Goeppert *Mayer* was *awarded* the 1963 *Nobel* for the nuclear shell model, the San Diego Union headline read “*San Diego Housewife Wins Nobel Prize*”.

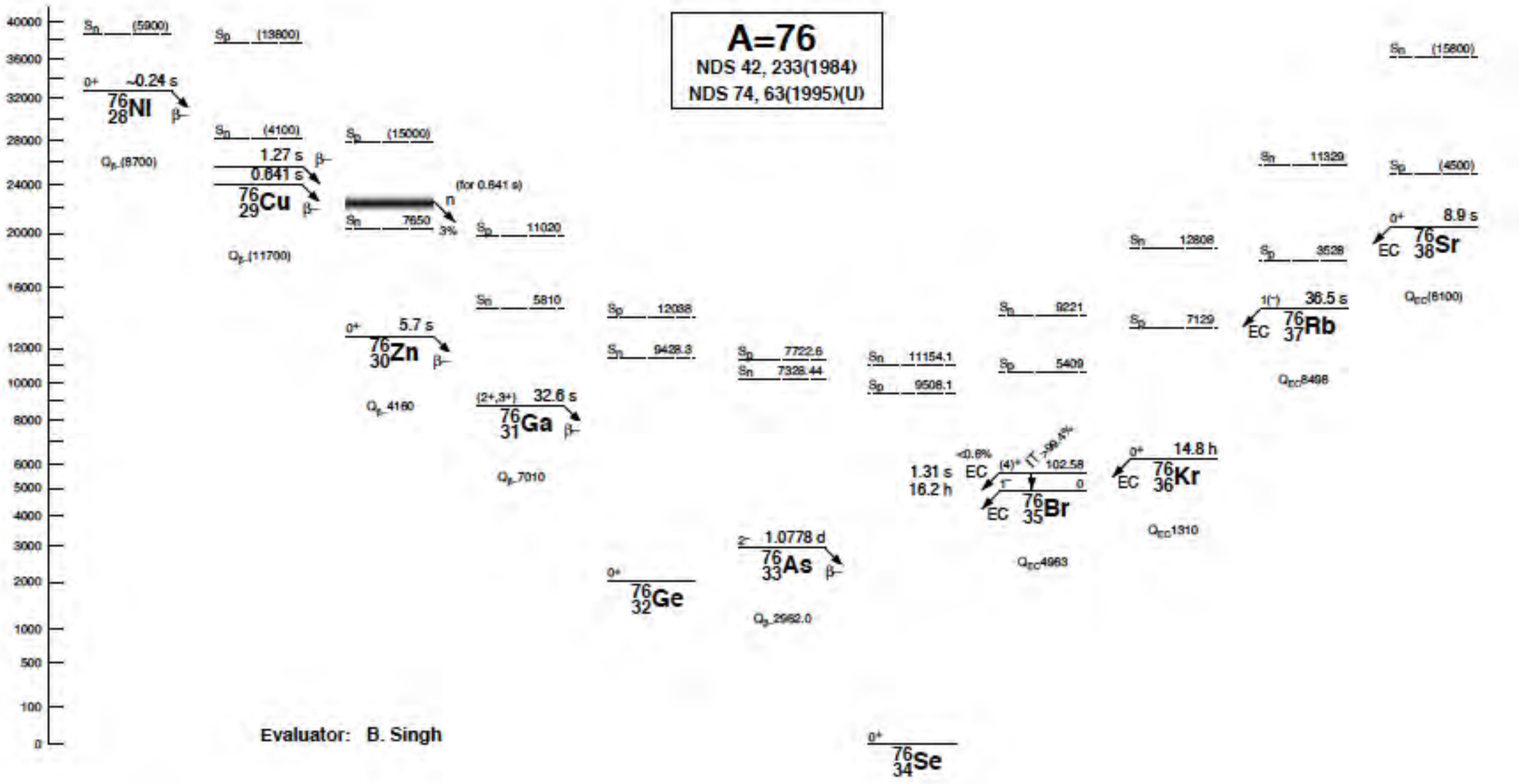


Majorana nature of the neutrinos permit neutrinoless double beta decay:

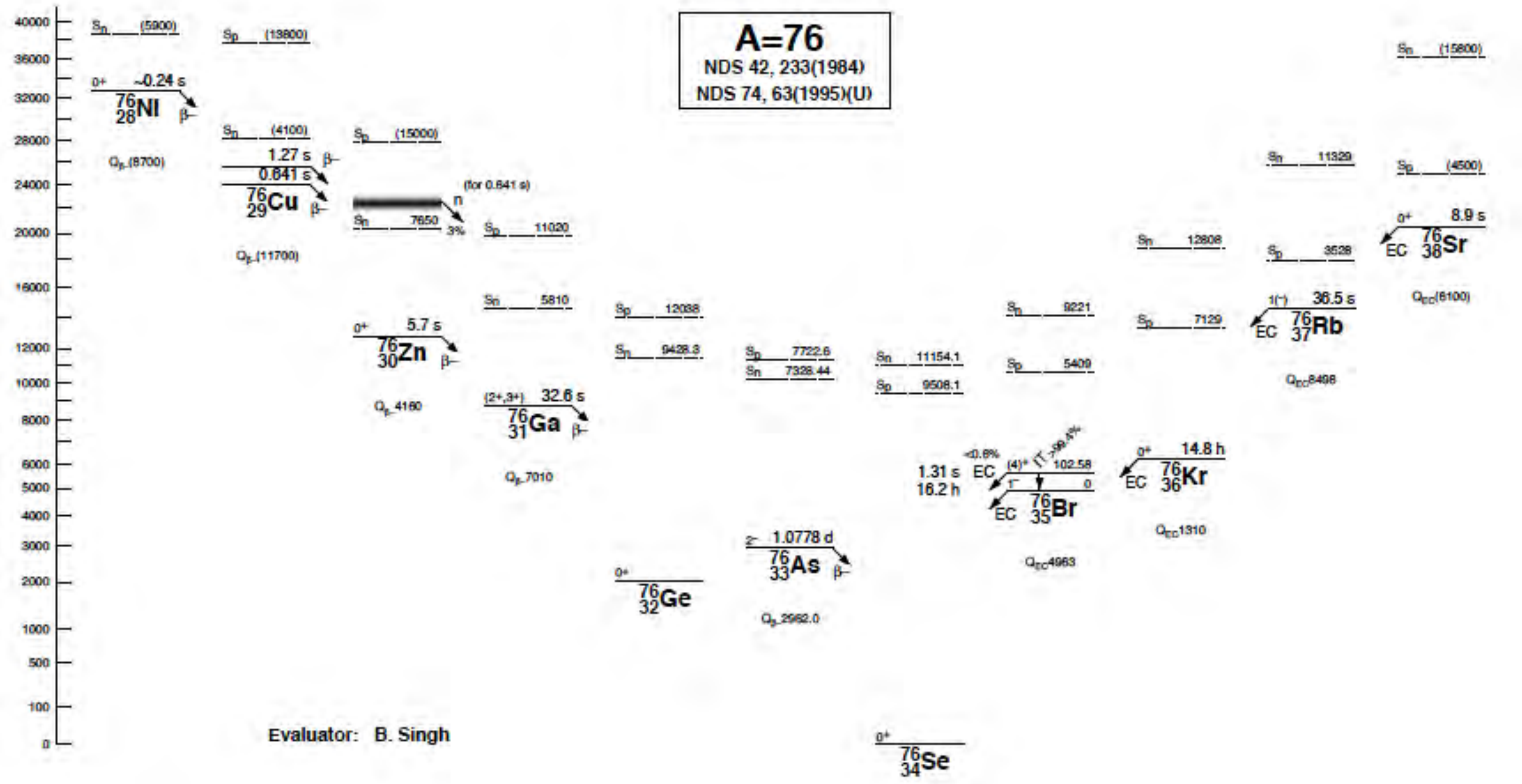


Pairing gives rise to double beta decay:





A=76
NDS 42, 233(1984)
NDS 74, 63(1995)(U)
Sn (5900)
Sp (13800)
0+ ~0.24 s
76 28 Ni β−
Qβ-(8700)
Sn (4100)
1.27 s β−
0.641 s
76 29 Cu β−
Qβ-(11700)
Sp (15000)
(for 0.641 s)
n
Sn 7650
3%
0+ 5.7 s
76 30 Zn β−
Qβ-4160
Sp 11020
Sn 5810
(2+,3+) 32.6 s
76 31 Ga β−
Qβ-7010
Sp 12038
Sn 9428.3
0+
76 32 Ge
Sp 7722.6
Sn 7328.44
2− 1.0778 d
76 33 As β−
Qβ-2962.0
Sn 11154.1
Sp 9506.1
0+
76 34 Se
Sn 9221
Sp 5409
1.31 s
16.2 h
EC
<0.6%
(4)+ IT >99.4%
102.58
1− 0
EC 76 35 Br
QEC4963
Sn 12808
Sp 7129
0+ 14.8 h
EC 76 36 Kr
QEC1310
Sn 11329
Sp 3528
1(−) 36.5 s
EC 76 37 Rb
QEC8496
Sn (15800)
Sp (4500)
0+ 8.9 s
EC 76 38 Sr
QEC(6100)
Evaluator: B. Singh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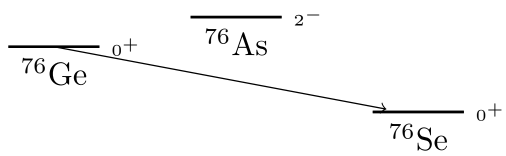
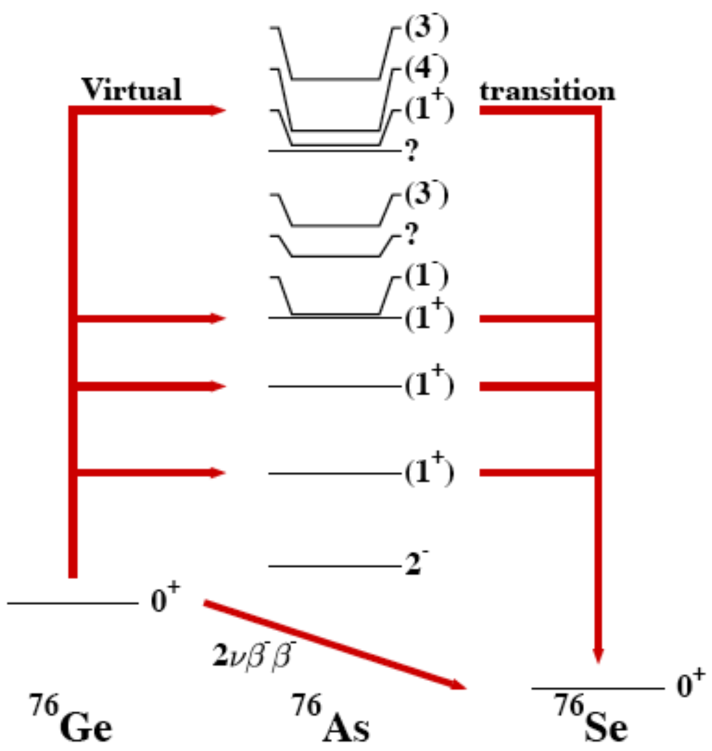
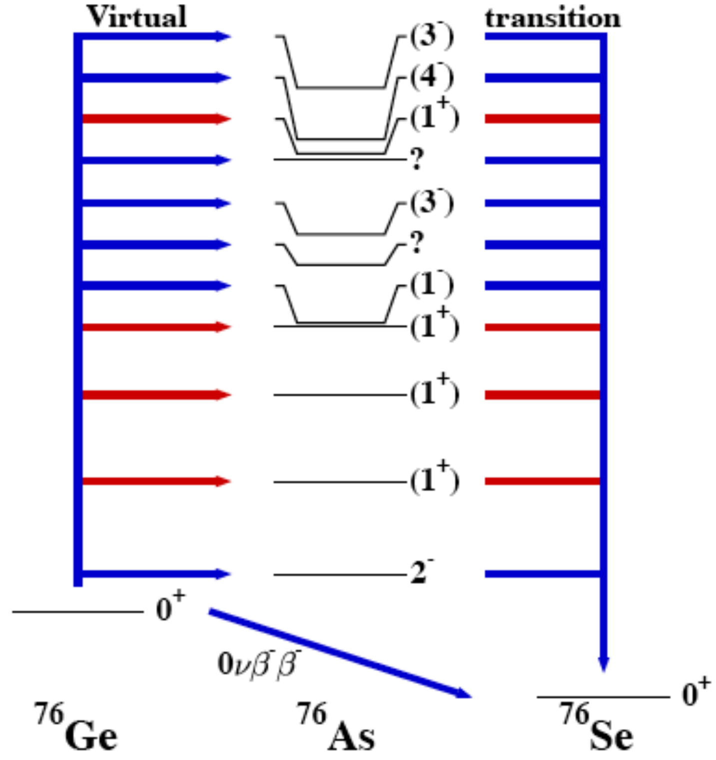
76 28 Ni
0+ ~0.24 s
76 29 Cu
1.27 s
0.641 s
76 30 Zn
0+ 5.7 s
76 31 Ga
(2+,3+) 32.6 s
76 32 Ge
0+
76 33 As
2- 1.0778 d
76 34 Se
0+
76 35 Br
1.31 s
16.2 h
76 36 Kr
0+ 14.8 h
76 37 Rb
1(-) 36.5 s
76 38 Sr
0+ 8.9 s
(for 0.641 s)
3%


76Ge 0+
76As 2−
76Se 0+

Why are matrix elements of 0νββ and 2νββ different?





2νββ

0νββ

Only intermediate 1+ states contribute (single-state dominance approximation?)

All intermediate states contribute (closure approximation?)

Both approximations could be problematic!

Nuclear matrix elements for double beta decay

$$M^{2\nu} = \sum_n \frac{< f \| \vec{\sigma}\tau_+ \| n > \cdot < n \| \vec{\sigma}\tau_+ \| i >}{E_n - E_i + E_0}$$

Two-neutrino ββ decay

$$M^{0\nu} = M_{GT}^{0\nu} - \frac{M_F^{0\nu}}{g_A^2} + M_T^{0\nu}$$

$$M_{GT}^{0\nu} \approx < f | \sum_{j,k} \frac{1}{r_{jk}} \vec{\sigma}(j) \cdot \vec{\sigma}(k)\, \tau_+(j) \tau_+(k) | f >$$

Neutrinoless ββ decay

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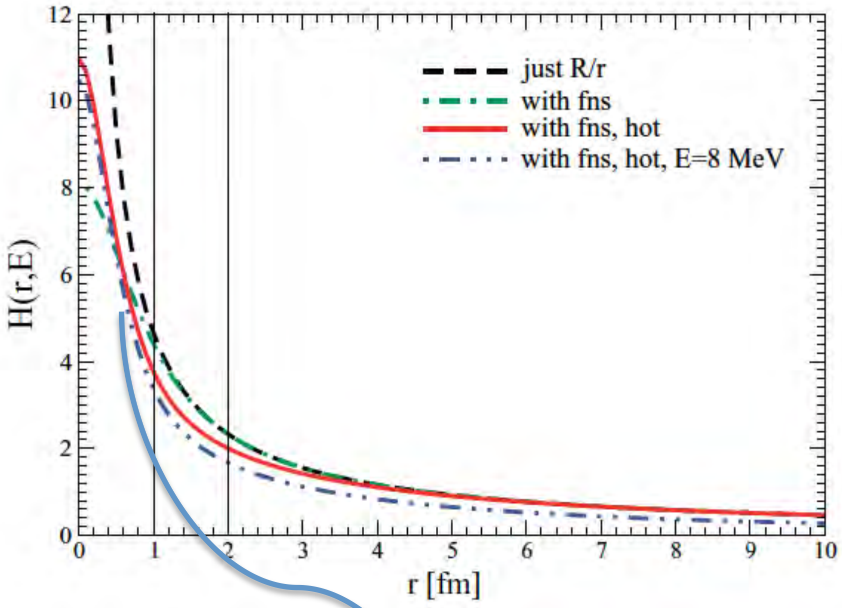
Two-neutrino ββ decay



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$$M_{GT}^{0\nu} \approx < f | \sum_{j,k} \frac{1}{r_{jk}} \vec{\sigma}(j) \cdot \vec{\sigma}(k)\, \tau_+(j) \tau_+(k) | f >$$

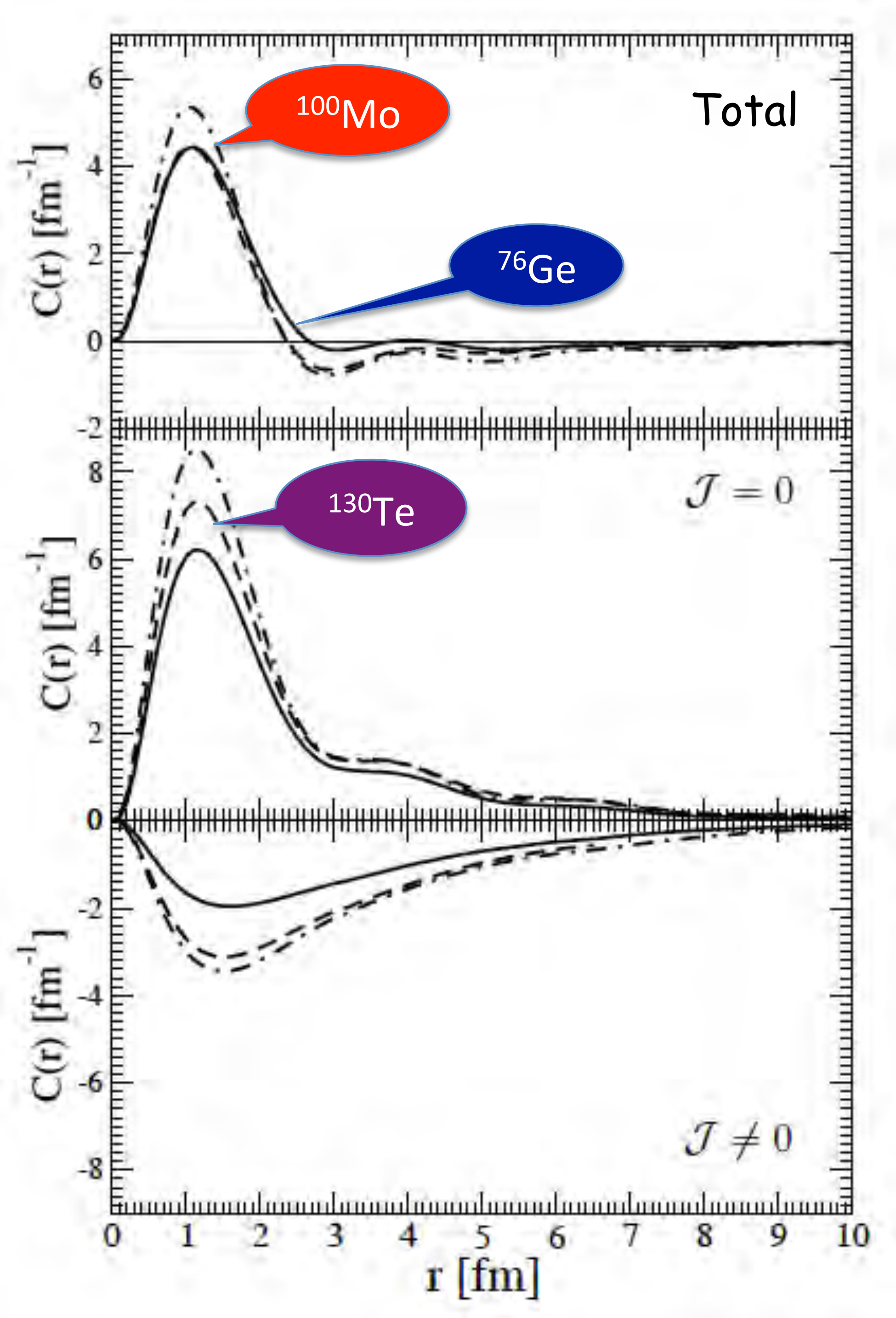
Neutrinoless ββ decay

Nuclear matrix elements

$$M_{GT}^{0\nu} = \int_0^\infty C_{GT}^{0\nu}(r)\,dr$$

Momentum of virtual neutrino, q ~ 1/r
r ~ 2 fm
q ~ 100 MeV

P. Vogel, J. Phys G **39**, 124002 (2012)



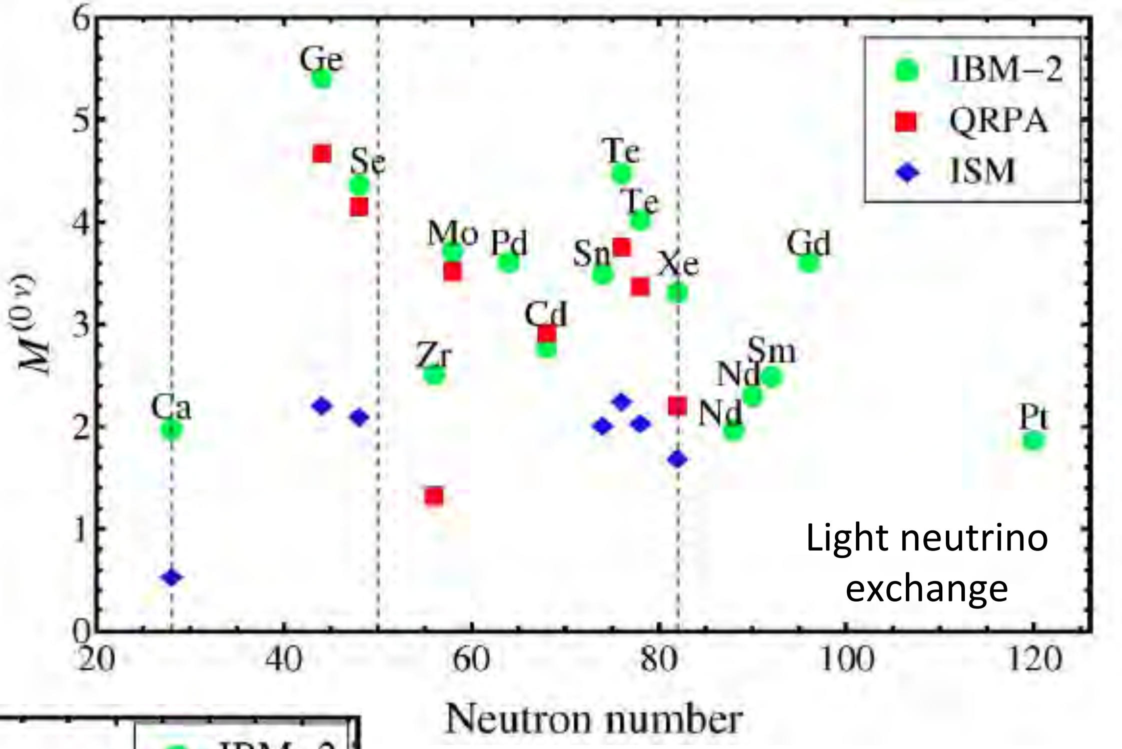
0ν double beta decay

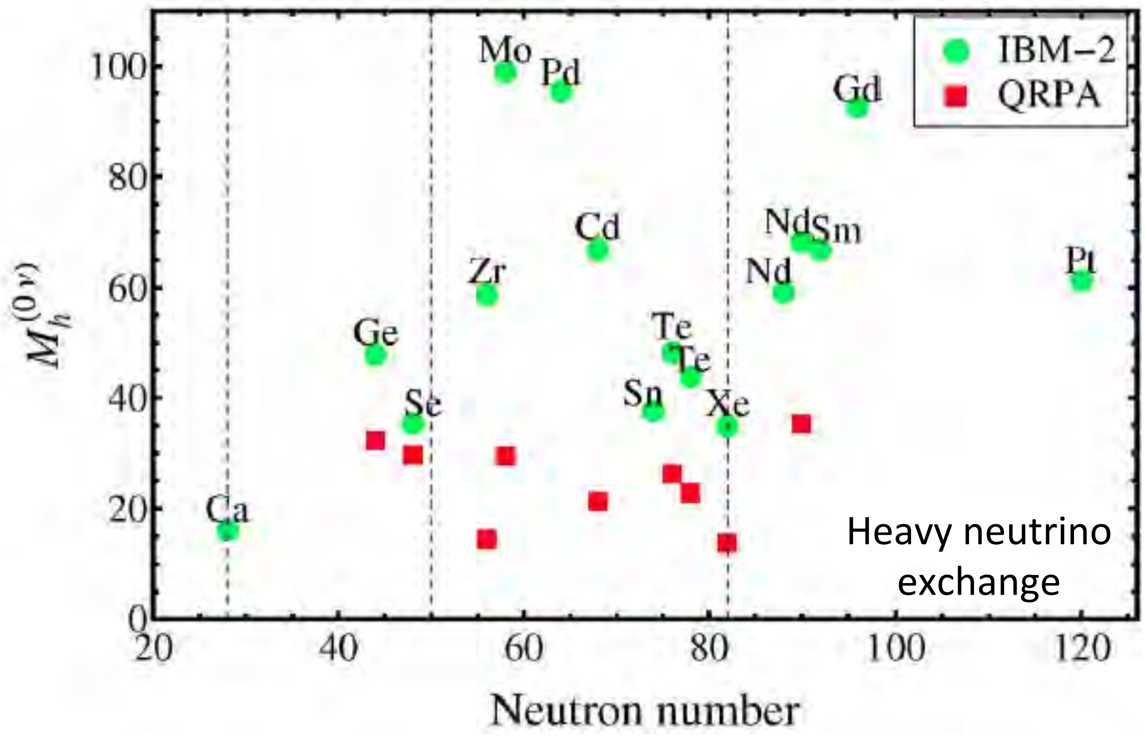
$$(1/T_{1/2}) = G(E,Z)\, M^2 \langle m_{\beta\beta}\rangle^2$$

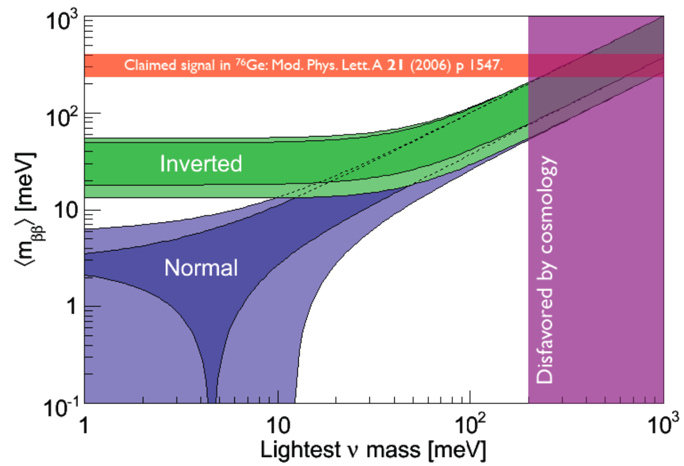
$G(E,Z)$: phase space

M : nuclear matrix element

$$\langle m_{\beta\beta}\rangle = |\Sigma_j\, |U_{ej}|^2\, m_j\, e^{i\delta(j)}|$$



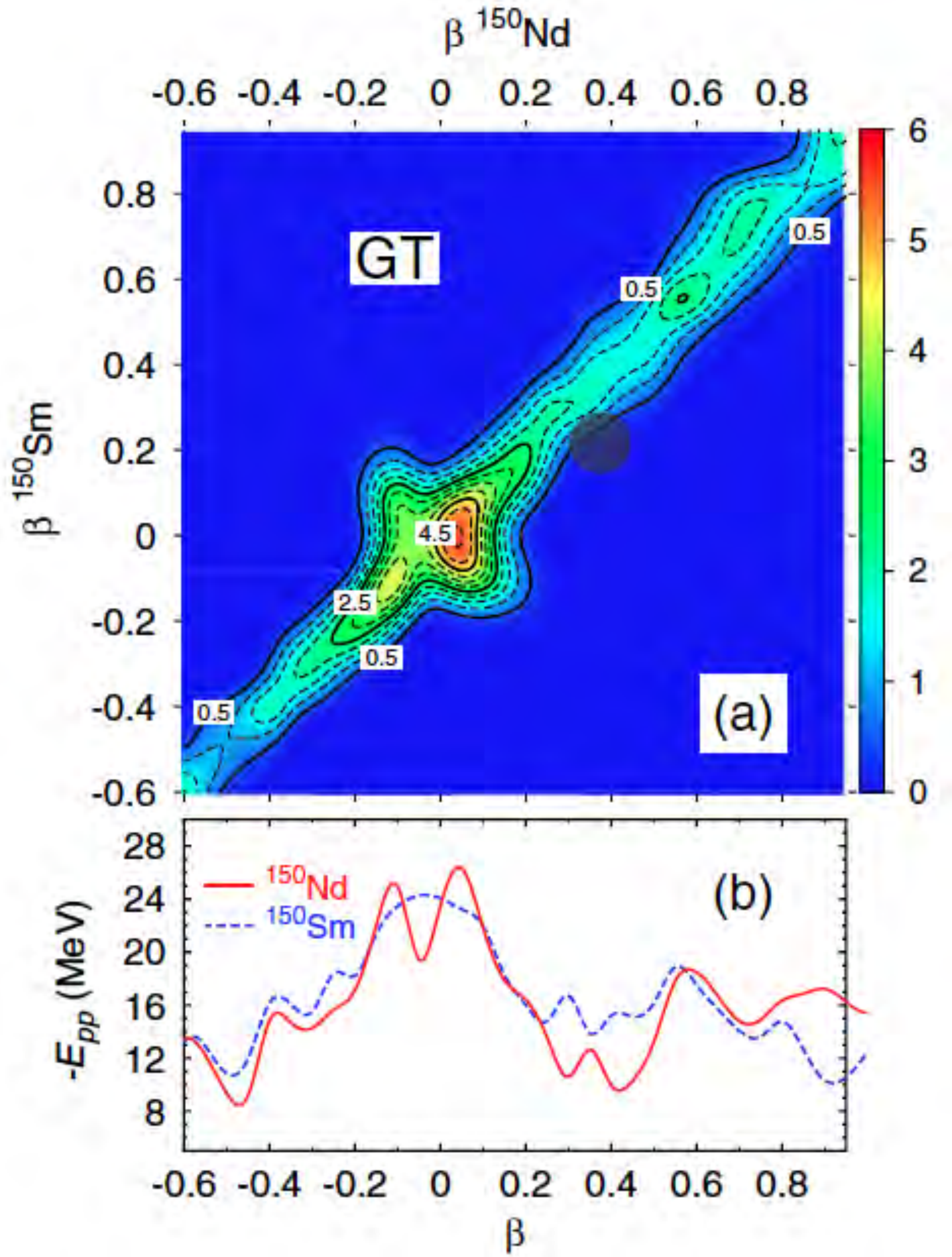




In neutrinoless double beta decay, the overlap between initial and final states should be not too small!

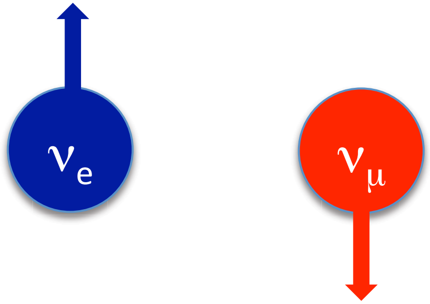
Example:

$^{150}Nd \rightarrow ^{150}Sm+ee$



Rodriguez & Martinez-Pinedo,
PRL **105,** 252503 (2010)

Neutrino flavor isospin



$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

Free neutrinos (only mixing)

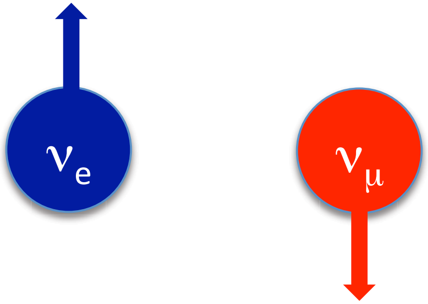
$$\hat{H} = \frac{m_1^2}{2E} a_1^\dagger a_1 + \frac{m_2^2}{2E} a_2^\dagger a_2 + (\cdot\cdot)\hat{1}$$

$$= \frac{\delta m^2}{4E} \cos 2\theta \left(a_\mu^\dagger a_\mu - a_e^\dagger a_e\right) + \frac{\delta m^2}{4E} \sin 2\theta \left(a_e^\dagger a_\mu + a_\mu^\dagger a_e\right) + (\cdot\cdot)'\,\hat{1}$$

Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E} \cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(a_\mu^\dagger a_\mu - a_e^\dagger a_e\right) + \frac{\delta m^2}{4E} \sin 2\theta \left(a_e^\dagger a_\mu + a_\mu^\dagger a_e\right) + (\cdot\cdot)''\,\hat{1}$$

Neutrino flavor isospin



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$$a_e = \cos\theta\, a_1 + \sin\theta\, a_2$$
$$a_\mu = -\sin\theta\, a_1 + \cos\theta\, a_2$$

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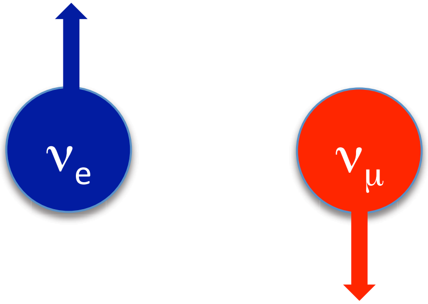
Neutrino flavor isospin



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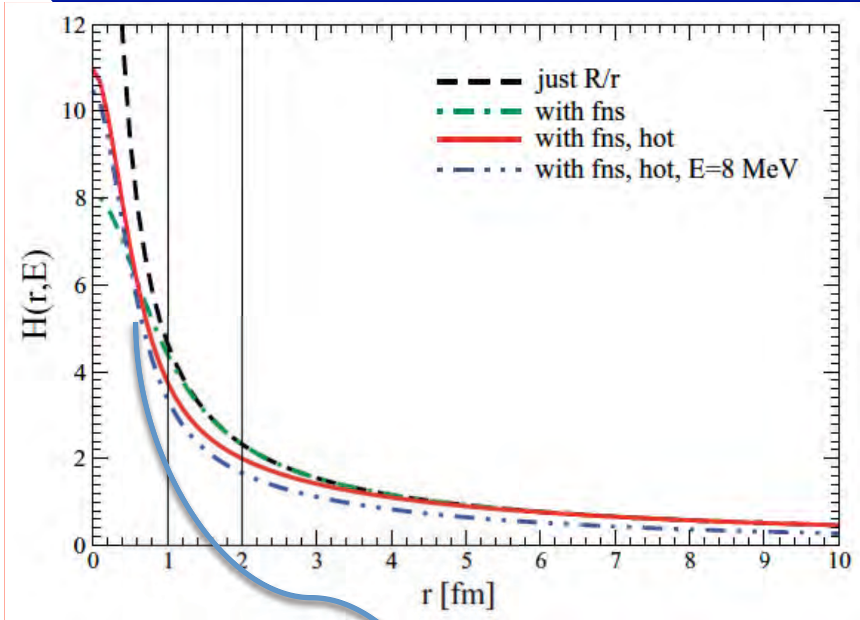
Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)''\,\hat{1}$$

Nuclear matrix elements for double beta decay

$$M^{2\nu} = \sum_n \frac{< f \| \vec{\sigma}\tau_+ \| n > \cdot < n \| \vec{\sigma}\tau_+ \| i >}{E_n - E_i + E_0}$$

Two-neutrino ββ decay



$$M^{0\nu} = M_{GT}^{0\nu} - \frac{M_F^{0\nu}}{g_A^2} + M_T^{0\nu}$$

$$M_{GT}^{0\nu} \approx < f | \sum_{j,k} \frac{1}{r_{jk}} \vec{\sigma}(j) \cdot \vec{\sigma}(k)\ \tau_+(j)\tau_+(k) | f >$$

Neutrinoless ββ decay

Nuclear matrix elements

$$M_{GT}^{0\nu} = \int_0^\infty C_{GT}^{0\nu}(r)\,dr$$

Momentum of virtual neutrino, q ~ 1/r
r ~ 2 fm
q ~ 100 MeV

P. Vogel, J. Phys G **39**, 124002 (2012)