Nuclear reactions in the early universe I

Mark Paris – Los Alamos Nat'l Lab
Theoretical Division
ISSAC 2014 UCSD

Acknowledgments

- IGPPS University Collaborative subcontract #257842
  - "Towards a Unitary and Self-Consistent Treatment of Big Bang Nucleosynthesis"
  - Started FY2014
  - $45k of LDRD-DR for first year (before -9%)
- Supporting grant: LANL Institutional Computing
  - Project Name: w14_bigbangnucleosynthesis
  - Project duration: 2 years (commenced April '14)
  - Year1: 1M, Year2: 1M CPU-hours
- LANL Collaborators
  - T-2: Gerry Hale, Anna Hayes & Gerry Jungman

Supporting activities 2013—2014

- Paris – T-2 staff member [Jan. 2012 hire]
  - International conferences (2 invited, 1 contributed), seminars, workshops
  - 4 peer-review publications on light nuclear reactions
  - LANL Institutional Computing 2 year grant
  - **LDRD-ER (FY15): BBN proposal oral review 14 May ’14 (yesterday)**
- Fuller – Director CASS, UCSD
  - Conferences, colloquia, workshops (many)
  - Publications (many)
  - NSF Grant No. PHY- 09-70064 at UCSD
- Grohs – Graduate Program UCSD – ABD
  - 15 Feb 2013-Sterile Neutrinos: Dark Matter, Neff, and BBN Implications-CASS Journal Club-UCSD; 10 Sep 2013-Nucleosynthesis, Neff, and Neutrino Mass Implications from Dark Radiation-NUPAC Seminar-UNM; 13 Jan 2014-Nucleosynthesis, N_{eff}, and Neutrino Mass Implications from Dark Radiation-HEP Seminar-Caltech; 14 Feb 2014-Evidence (to the trained eye) for Sterile Neutrino Dark Matter-CASS Journal Club-UCSD; 28 Mar 2014-Photon Diffusion in the Early Universe-PCGM30 (Pacific Coast Gravity Meeting)-UCSD; 18 Apr 2014-Neutrinos in Cosmology I-CASS Journal Club-UCSD
  - Dissertation targeted Spring 2015

Organization

Nuclear reactions in the early universe

- Lectures (Paris/E. Grohs)
  I. Overview of cosmology/Kinetic theory/Big bang nucleosynthesis (BBN)
  II. Scattering & reaction formalism/Neutrino energy transport
- Workshop sessions (E. Grohs/Paris)
  I. BBN exercises: compute Nuclear Statistical Equilibrium/electron fraction
  II. Compute primordial abundances vs $\Omega_b h^2$: code parallelization
- Lecture notes
  - Will be available online (URL TBA)

IGPPS Los Alamos

Possibly useful references

S. Weinberg, *Gravitation and Cosmology* (John Wiley & Sons, 1972).

S. Detweiler, Classical and Quantum Gravity **22**, S681 (2005), URL `http://stacks.iop.org/0264-9381/22/i=15/a=006`.

L. Landau and E. Lifshitz, *Mechanics*, v. 1 (Elsevier Science, 1982), ISBN 9780080503479, URL `http://books.google.com/books?id=bE-9tUH2J2wC`.

Planck Collaboration, P. A. R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown, F. Atrio-Barandela, J. Aumont, C. Baccigalupi, A. J. Banday, et al., ArXiv e-prints (2013), 1303.5076.

D. Baumann, ArXiv e-prints (2009), 0907.5424.

S. Dodelson, *Modern Cosmology*, Academic Press (Academic Press, 2003), ISBN 9780122191411, URL `http://books.google.com/books?id=3oPRxdXJexcC`.

P. Ade et al., Phys. Rev. Lett. **112**, 241101 (2014).

K. Huang, *Statistical mechanics* (Wiley, 1987), ISBN 9780471815181, URL `http://books.google.com/books?id=M8PvAAAAMAAJ`.

L. Brown, *Quantum Field Theory* (Cambridge University Press, 1994), URL `http://books.google.com/books?id=_qt_nQEACAAJ`.

E. Lifshitz and L. Pitaevskiĭ, *Physical Kinetics*, no. v. 10 in Course of theoretical physics (Butterworth-Heinemann, 1981), ISBN 9780750626354, URL `http://books.google.com/books?id=h7LgAAAAMAAJ`.

M. Peskin and D. Schroeder, *An Introduction to Quantum Field Theory*, Advanced book classics (Addison-Wesley Publishing Company, 1995).

J. Bernstein, *Kinetic Theory in the Expanding Universe* (Cambridge University Press, 1988).

Outline

Lecture I

- Overview
- Cosmological dynamics in GR
- Big bang nucleosynthesis (BBN)
- Boltzmann equation
  - Flat & curved spacetime

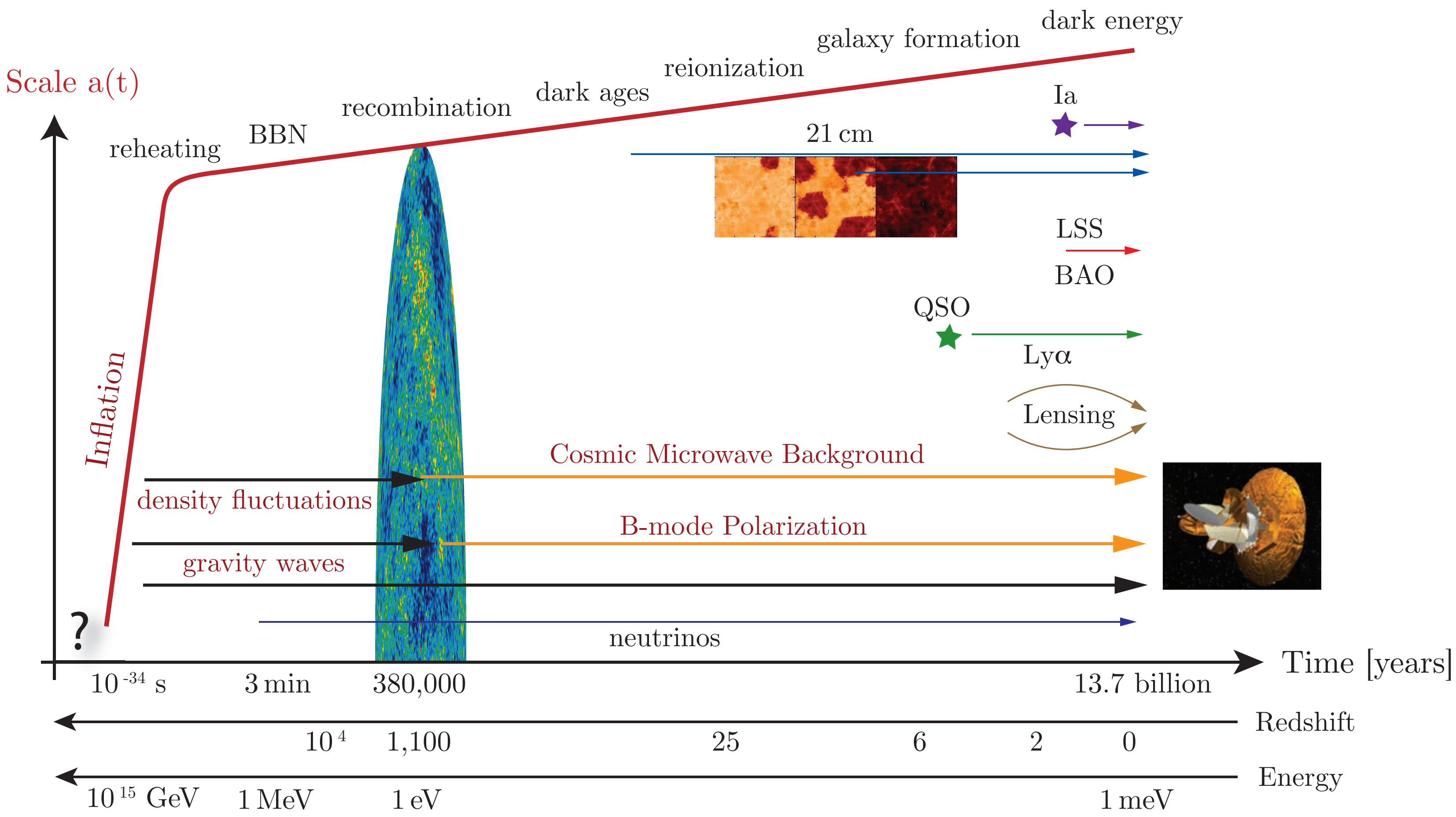
Lecture II

- Unitary reaction network (URN) of light nuclei
- Neutrino energy transport
- Evan Grohs: observations of primordial abundances

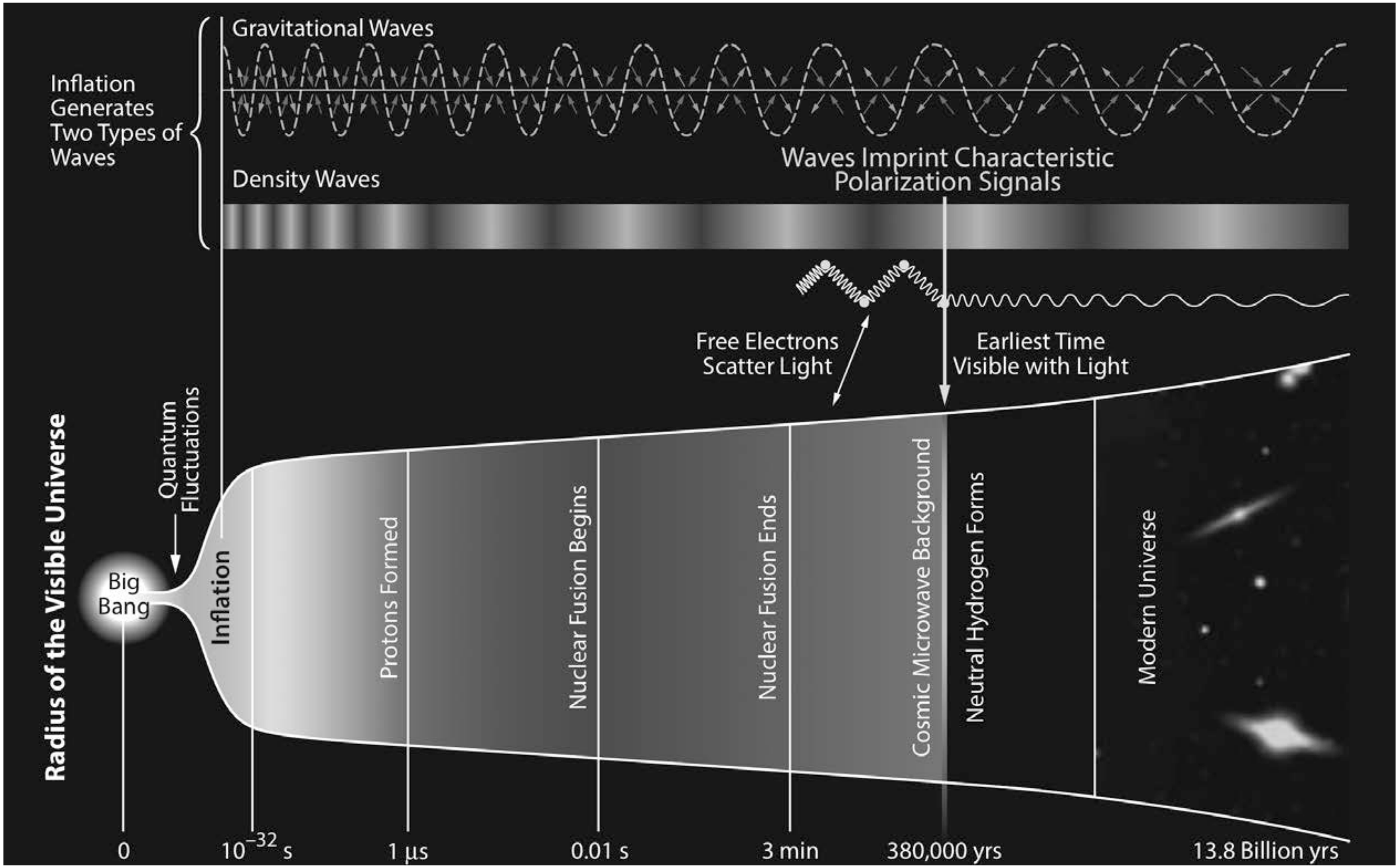
Big Bang nucleosynthesis (BBN) in CSM context

- Cosmological Standard Model – ΛCDM
- Formation of ^{4}He, deuterium (D), ^{3}H, ^{3}He, ^{7}Be/Li, … in the primordial 'fireball'
- Epochs (Hot/dense > cool/rarified)
  - Planck > GUT/Inflation > EWPT > QHT > BBN > RC > GF/LSS
- Time of BBN: ~1sec → ~10^2 sec; T_{BBN}: ~1 MeV → ~10 keV
- Relevant physics: cooling thermonuclear reactor
  - work of expansion cools radiation & matter
  - weak (neutrino) & strong nuclear interactions (& ???)
  - Boltzmann transport, non-equilibrium phenomena
- Comparison to observations
  - stunning successes: CMB, helium, deuterium
  - perplexing anomalies: dark matter/energy, lithium problem

Big Bang nucleosynthesis (BBN) in CSM context



Big Bang nucleosynthesis (BBN) in CSM context



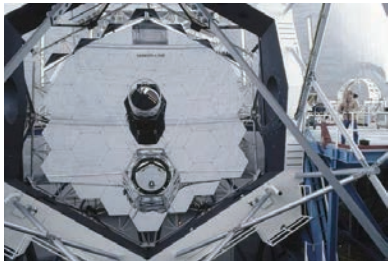
Big Bang nucleosynthesis (BBN) in CSM context

- Cosmological Standard Model – Λ CDM
- Formation of 4He, deuterium (D), 3H, 3He, 7Be/Li, … in the primordial 'fireball'
- Epochs (Hot/dense > cool/rarified)
  - Planck > GUT/Inflation > EWPT > QHT > BBN > RC > GF/LSS
- Time of BBN: ~1sec → ~102 sec; TBBN: ~1 MeV → ~10 keV
- Relevant physics: cooling thermonuclear reactor
  - work of expansion cools radiation & matter
  - weak (neutrino) & strong nuclear interactions (& ???)
  - Boltzmann transport, non-equilibrium phenomena
- Comparison to observations
  - stunning successes: CMB, helium, deuterium
  - perplexing anomalies: dark matter/energy, lithium problem

IGPPS Los Alamos

Observations [more from Evan G. tomorrow]

- Observational astronomy
  - existing 10m-class telescopes: Keck, …
    - Gold-plated: 2% D meas. Pettini & Cooke '13
  - adaptive optics
  - space- & ground-based observatories
  - planned 30^{+}m-class telescopes: ELT, TMT, …
- Cosmic microwave background
  - Planck, WMAP, PolarBear, APT, SPT, CMBPol, …
- Implications
  - test physics beyond SM; lab tests difficult/impossible
  - precision constraints expected to test ***nuclear*** physics
- ***Unprecedented precision for primordial nuclear abundances***



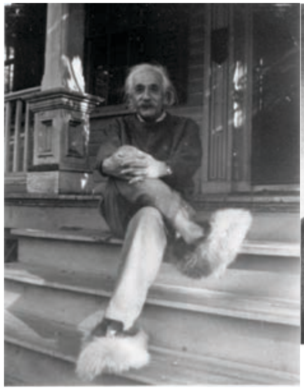
Standard FLRW Cosmology

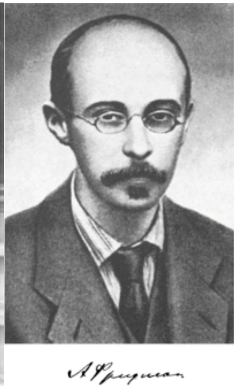
- Robertson, Walker show homogen., isotropic > Friedmann, Lemaître solution to GR unique:

$$G_{00} = 8\pi T_{00}; \quad g_{00} = 1, \quad g_{ij} = -a^2(t), \quad K_{space} \equiv 0; \quad \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho(t)$$

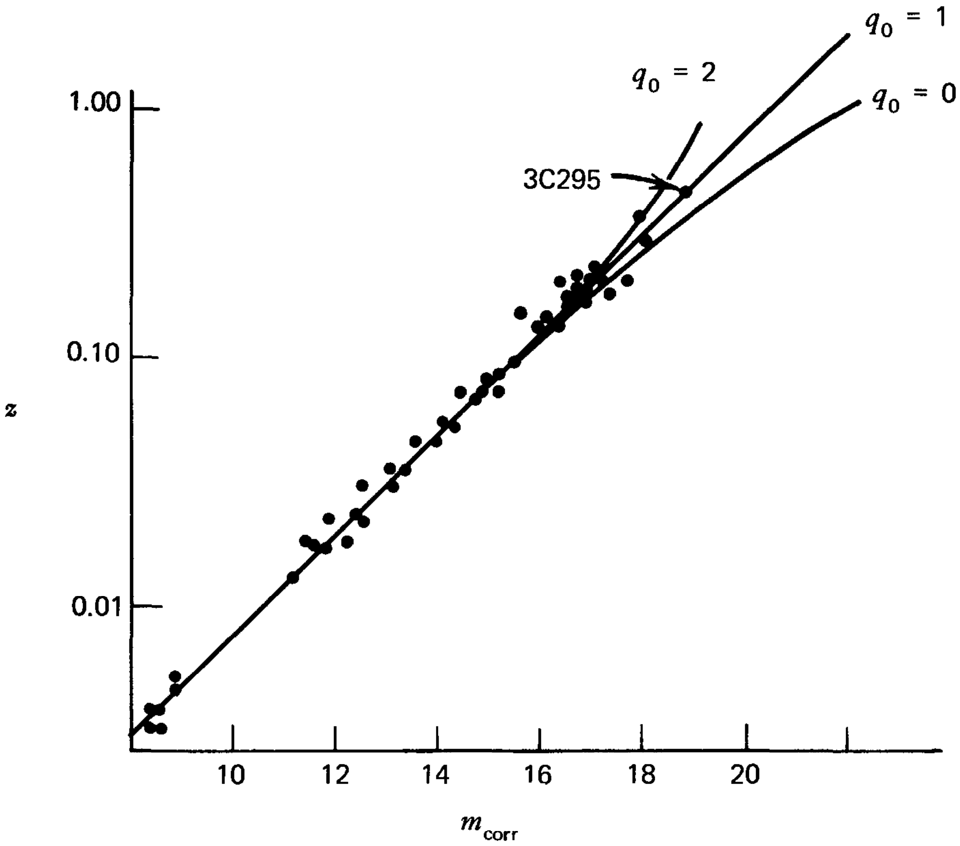
- The 'Old', Big Three observations
  - expansion: Hubble "constant," H_0= 67.1 km/s/Mpc (Planck)
  - CMB: T = 2.73 K
  - BBN: concordance at baryon/photon ratio
- HIF universe ⇨ may *only* tune RHS of Einstein-Friedmann Eqn
  - radiation: photons, neutrinos, dark radiation
  - matter: baryonic, dark
  - ΛCDM model: set of assumptions to confront data
    - Wayne Hu (Uchicago): "alive and well" but issues with growth of density fluctuations

Einstein-Friedmann equations (0)



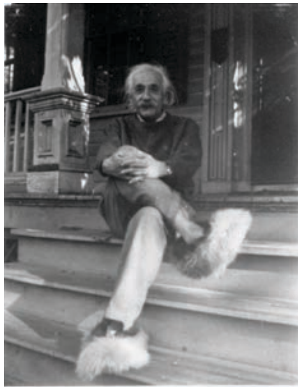


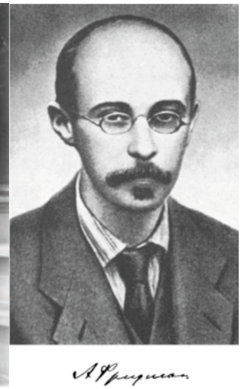
- An enduring legacy…



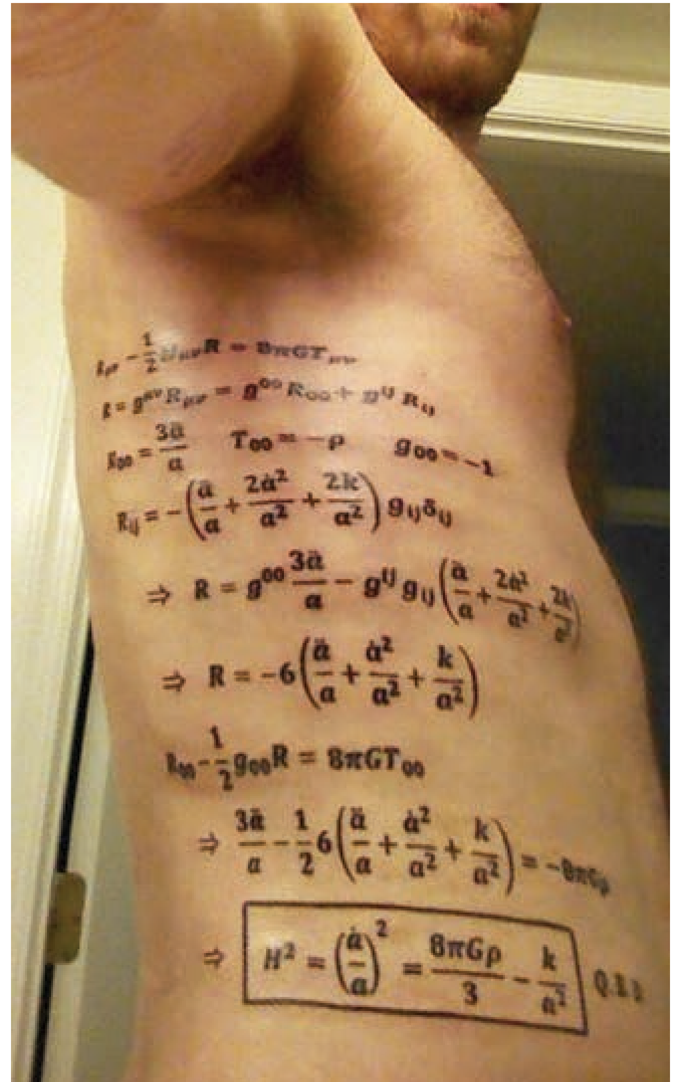
IGPPS Los Alamos

Einstein-Friedmann equations (0)





- An enduring legacy…



Einstein-Friedmann equations (I)

- Universe dynamics from GR ⟺ energy-momentum density

$$\boxed{G_{\mu\nu}} = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = -p g_{\mu\nu} + (p+\rho) u_\mu u_\nu$$

- Einstein/Ricci/Curv Scalar

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R$$

$$R_{\mu\nu} = g^{\alpha\beta} R_{\alpha\mu\beta\nu} = R^{\beta}{}_{\mu\beta\nu} = R_{\alpha\mu}{}^{\alpha}{}_{\nu}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$R^{\mu}{}_{\nu\rho\sigma} = \frac{\partial \Gamma^{\mu}{}_{\nu\sigma}}{\partial x^{\rho}} - \frac{\partial \Gamma^{\mu}{}_{\nu\rho}}{\partial x^{\sigma}} + \Gamma^{\tau}{}_{\nu\sigma}\Gamma^{\mu}{}_{\rho\tau} - \Gamma^{\tau}{}_{\nu\rho}\Gamma^{\mu}{}_{\sigma\tau}$$

- Metric/connection

$$g_{00} = 1, \qquad g_{ij} = a^2(t)\tilde{g}_{ij}$$

$$\begin{aligned}
\Gamma^{0}{}_{00} &= \tfrac{1}{2} g^{0\alpha}(2g_{\alpha 0,0} - g_{00,\alpha}) = 0\\
\Gamma^{0}{}_{i0} &= \tfrac{1}{2} g^{0\alpha}(g_{\alpha i,0} + g_{\alpha 0,i} - g_{i0,\alpha}) = 0\\
\Gamma^{0}{}_{ij} &= \tfrac{1}{2} g^{0\alpha}(g_{\alpha i,j} + g_{\alpha j,i} - g_{ij,\alpha}) = -\tfrac{1}{2} g_{ij,0} = -\dot{a} a \tilde{g}_{ij}\\
\Gamma^{i}{}_{00} &= \tfrac{1}{2} g^{i\alpha}(2g_{\alpha 0,0} - g_{00,\alpha}) = 0\\
\Gamma^{i}{}_{j0} &= \tfrac{1}{2} g^{i\alpha} g_{\alpha j,0} = \tfrac{1}{2}\frac{1}{a^2}\tilde{g}^{ik}\frac{\partial[a^2\tilde{g}_{kj}]}{\partial x^0} = \frac{\dot{a}}{a}\delta^{i}{}_{j} = \Gamma^{i}{}_{0j}\\
\Gamma^{i}{}_{jk} &= \tfrac{1}{2}\tilde{g}^{il}(\tilde{g}_{lj,k} + \tilde{g}_{lk,j} - \tilde{g}_{jk,l}) \equiv \tilde{\Gamma}^{i}{}_{jk}.
\end{aligned}$$

IGPPS Los Alamos

Einstein-Friedmann equations (II)

- Knowing energy density (ρ) and pressure (p)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho,$$

$$-\frac{\ddot{a}}{a} = \frac{4\pi}{3}(\rho + 3p)$$

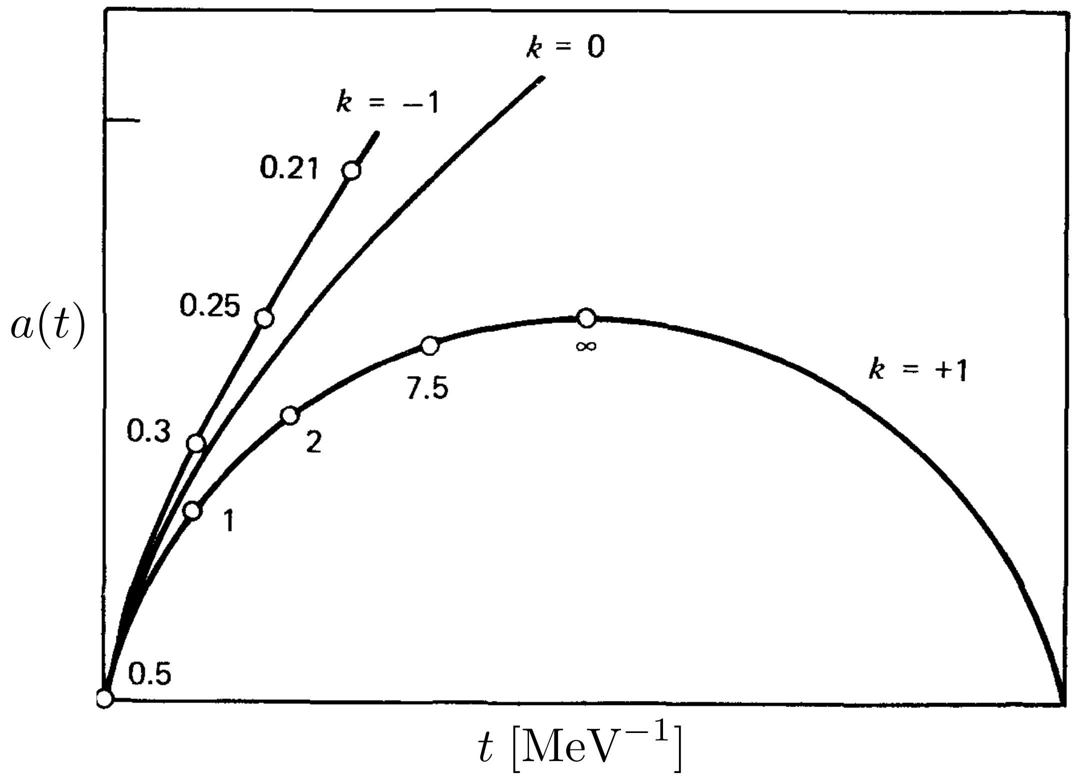
- Covariantly conserved energy-momentum (not indep. eqn.)

$$T^{\mu\nu}{}_{;\nu} = 0 \qquad \dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$$

- Two equations for three unknowns: $a(t), \rho(t), p(t)$
  - Equation of state: $p = w\rho^x$

Einstein-Friedmann equations (III)

- Solution classes



$p = w\rho^x$

- $w > 0 \implies \ddot{a} < 0$ a(t) negative curvature
- $w < 0 \implies \ddot{a} > 0$ a(t) positive curvature; inflation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho,$$

$$-\frac{\ddot{a}}{a} = \frac{4\pi}{3}(\rho + 3p)$$

- Acceleration parameter

$$q(t) = -\frac{\ddot{a}/\dot{a}}{\dot{a}/a}$$

- Hubble constant

$$H_0 = \frac{\dot{a}(t_0)}{a(t_0)} > 0$$

- Redshift

$$1 + z = \frac{a(t_0)}{a(t_1)}, \quad t_0 > t_1$$

- Current critical density

$$\rho_{c,0} = \frac{3}{8\pi}H_0^2 m_{Pl}^2 \approx 5\,\frac{\text{protons}}{\text{m}^3}$$

Einstein-Friedmann equations (IV)

- Maximally symmetric subspace
  - Consequence of homogeneity & isotropy
  - 'Maximal' number L.I. Killing vector fields N(N+1)/2 (dim N)
  - Flows of Killing vector fields generate isometries of manifold
  - Friedman universe has MS spacelike hypersurfaces
- Tensors in MS spaces
  - scalar:

$$\partial_\mu S(x) = 0$$

  - vector:

$$A^i(x) \equiv 0 \qquad\qquad (A^0(x) \neq 0)$$

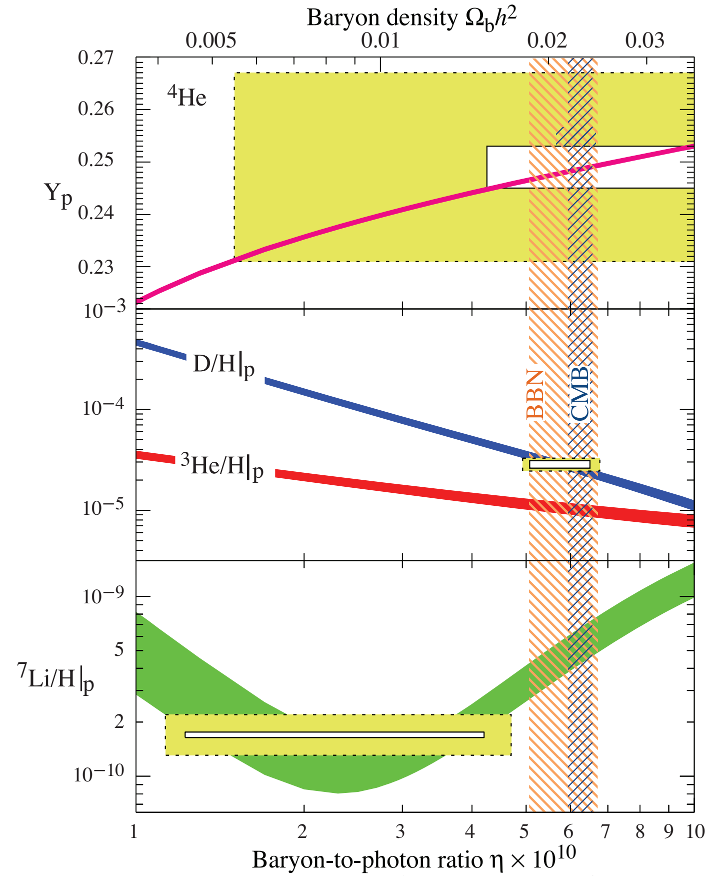
  - rank-2 tensor:

$$B_{ij} = B_{ji} = C g_{ij} \qquad\qquad\qquad C \neq C(x)$$

Standard BBN – ^{7}Li anomaly

- n/p ratio
  - exquisite sensitivity to neutrino *distribution*
  - ~1:5
- Helium
  - exquisite sensitivity to neutrons
  - mass fraction Y_p~1/4 (p:primordial)
- Deuterium
  - ~1:10^5
  - Pettini & Cooke obs. better by fact 5
- Lithium
  - mass A=7
    - 3—5 σ discrepancy > Li anomaly

h=H_0/(100 km/s/Mpc)



Review Particle Properties

Workshop II: generate 'Schramm plot'

The New, 'Big Five' observations

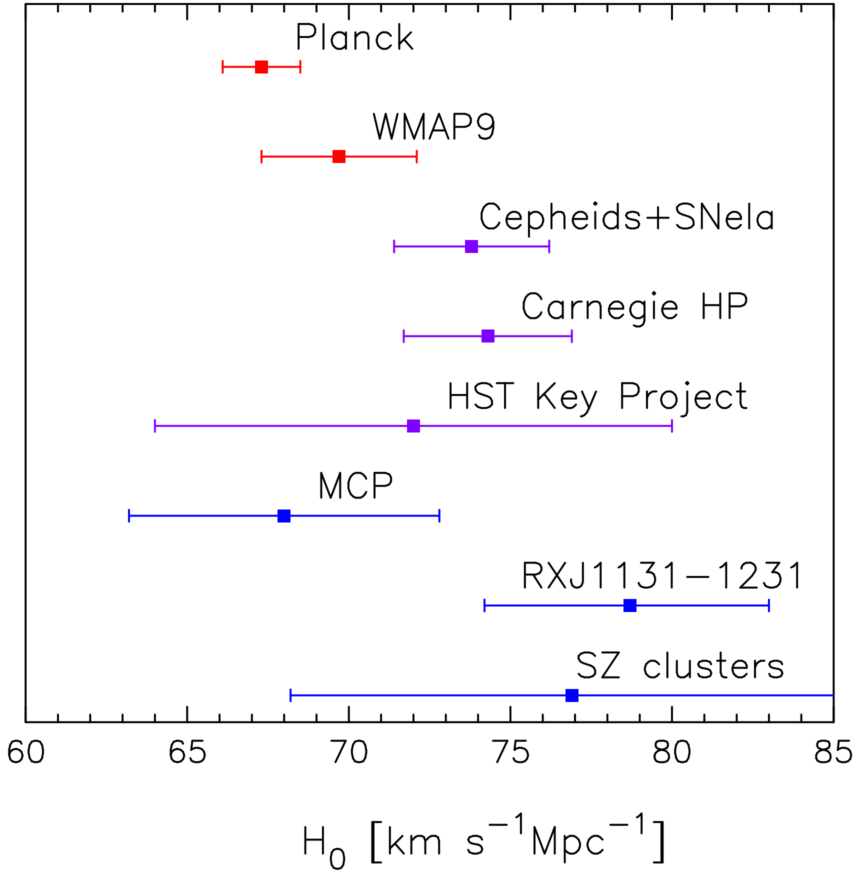
GF: "***VERY EXCITING* situation developing** . . . because of the advent of . . ."

- comprehensive cosmic microwave background (CMB) observations (WMAP, Planck, ACT, SPT, PolarBear, CMBPol,...)
  - N_{eff} : "effective number" of relativistic species; Y_p : ^{4}He mass fraction (relative to proton); η (Ω_b): baryon-to-photon number fraction; Primordial deuterium abundance $(D/H)_p$; Σm_ν
- 10/30-meter class telescopes, adaptive optics, and orbiting observatories
  - e.g., precision determinations of deuterium abundance dark energy/matter content, structure history etc.
- Laboratory neutrino mass/mixing measurements
  - mini/micro-BooNE, EXO, LBNE

GF: "is setting up a nearly over-determined situation where ***new***
Beyond Standard Model **neutrino physics**
likely ***must*** show itself!"

ΛCDM: Possible discrepancies (I)

- Hubble expansion



- Planck XVI (2013)
  - "tension between the CMB-based estimates and the astrophysical measurements of H_0 is intriguing and merits further discussion"
  - "*highly model dependent*"
  - ΛCDM extraction
    - requires assumptions about relativistic energy density (RED)
    - extra RED could explain discrepancy

IGPPS Los Alamos

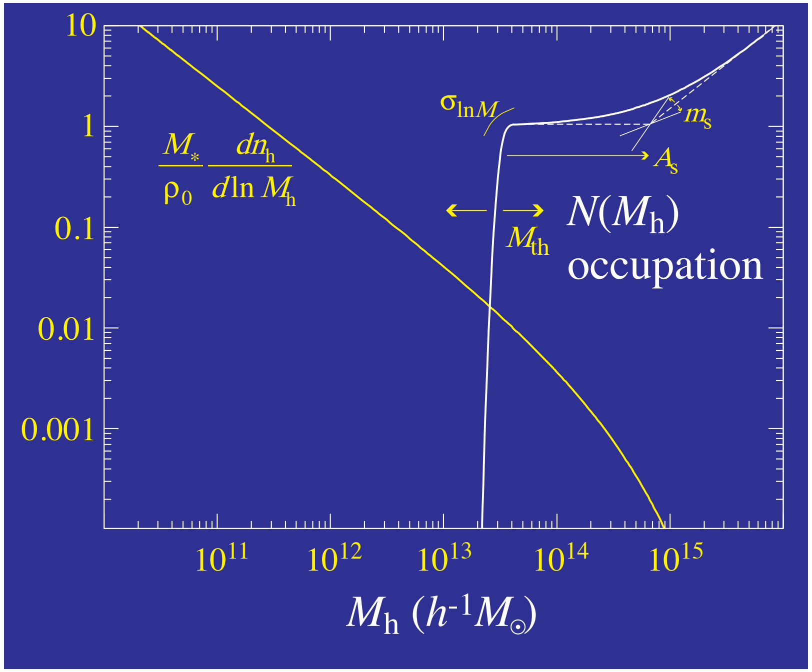
ΛCDM: Possible discrepancies (II)

- Clustering
  - Abundance of rare massive DM halos exponentially sensitive to growth of structure
  - rms fluct. total mass 8 h^{-1} Mpc spheres with variance

$$\sigma_R^2 = \int \frac{dk}{k} \mathcal{P}_m(k) \left[\frac{3 j_1(kR)}{kR} \right]^2$$

  - Discrepancy b/w CMB & lensing
  - extra RED can reconcile CMB-inferred σ_8 with direct observational determinations

- Dark matter & structure formation

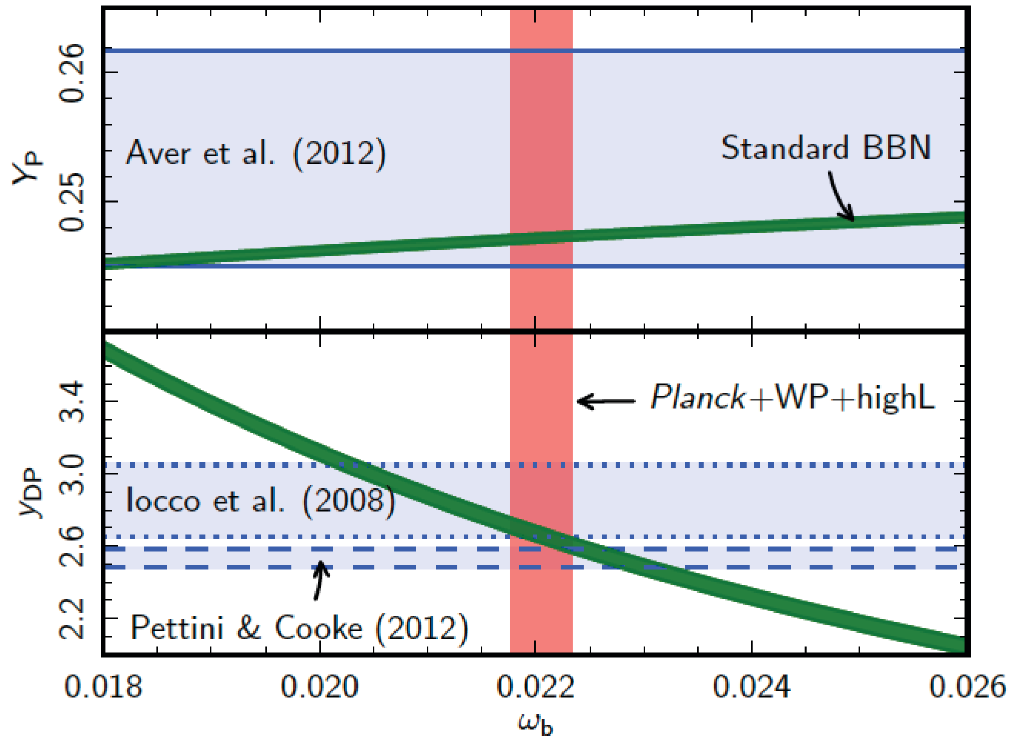


Wayne Hu/2013 October

IGPPS Los Alamos

ΛCDM: Possible discrepancies (III)

- Big Bang nucleosynthesis
  - Lithium anomaly
  - Y_P, Y_{DP} exquisite sensitivity to active neutrino spectrum:
    - Most neutrons → ^{4}He
    - $Y_p \leftarrow n/p \leftarrow f_\nu(p,T)$
  - Thermal effects
    - Hotter later: less neutrons
    - Non-equilibrium ν: less neutrons
  - Probe neutrino sector by studying constraints on various scenarios imposed by precision BBN

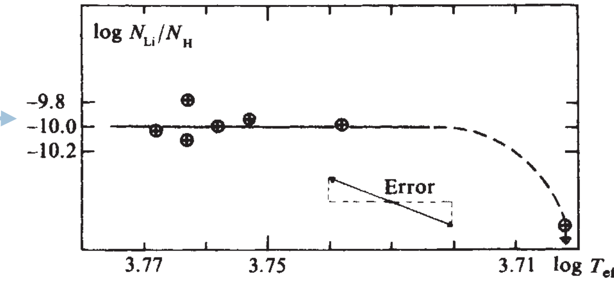


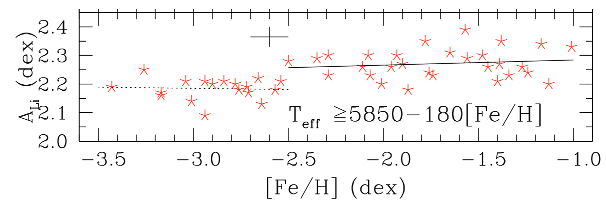
Planck XVI (2013)

IGPPS Los Alamos

Possible solutions to lithium anomaly in BBN

- Astronomical explanation
  - Spite-Spite plateau
  - Robust? Melendez et.al.(2010) ‘broken’





- Nuclear physics
  - resonant destruction of mass 7 nuclides
    - ^{9}B compound system example (below)
  - Unitarity: fundamental, neglected property of QM
- Physics beyond the standard model (BSM)
  - new particles’ effect on thermal history, etc.

Even if the lithium anomaly is not nuclear or BSM in origin, precision cosmology forces better treatment of nuclear and astroparticle physics

Possible refinements to BBN

- Physics beyond the standard model
  - Increasing observational precision requires "sharpening the tool"
    - improve on existing BBN codes from late 60's
    - replace equilibrium thermal history > full neutrino transport
  - BBN can be used to ***test*** BSM & nuclear physics
- Fundamental principle of nuclear physics: Unitarity
  - Existing codes' nuclear reaction networks don't observe unitarity
  - LANL-developed unitary reaction network (URN) for thermonuclear boost & burn
  - Two objectives from nuclear physics perspective
    - Test LANL URN in similar (but different, high-entropy) environment
    - Address fundamental problem in cosmology
- NB: ***without correct URN, req'd. by QM, BSM physics uncertain***

BBN project: introduce a new theoretical tool

- Outline for the rest of talk
  - 1st refinement: neutrino sector
  - 2nd refinement: nuclear physics

BBN project: introduce a new theoretical tool

- Outline for the rest of talk
  - 1st refinement: neutrino sector
  - 2nd refinement: nuclear physics

Unitary, self-consistent primordial nucleosynthesis

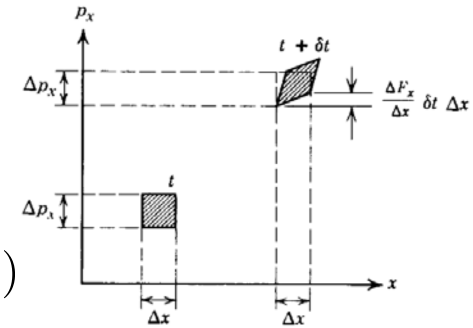
- BBN as a tool for precision cosmology
  - incorporate unitarity into strong & electroweak interactions
  - couple unitary reaction network (URN) to full Boltzmann transport code
    - neutrino energy distribution function evolution/transport code
    - fully coupled to nuclear reaction network
    - calculate light primordial element abundance for non-standard BBN
      - active-sterile neutrino mixing
      - massive particle *out-of-equilibrium* decays→energetic active SM particles
  - New tools/codes for nuc-astro-particle community:
    - test new physics w/BBN
    - existing codes are based on Wagoner's (1969) code
      - we will improve this situation dramatically

Kinetic theory: flat spacetime

- distribution function

$$dN(\mathbf{r},\mathbf{p},t) = d^3r\, dn(\mathbf{r},\mathbf{p},t) = d^3r\,\frac{d^3p}{(2\pi)^3}\, f(\mathbf{r},\mathbf{p},t)$$

$$\frac{d}{dt} f(\mathbf{r},\mathbf{p},t) = \frac{\partial}{\partial t} f(\mathbf{r},\mathbf{p},t) + \frac{\partial \mathbf{r}}{\partial t}\cdot\frac{\partial}{\partial \mathbf{r}} f(\mathbf{r},\mathbf{p},t) + \frac{\partial \mathbf{p}}{\partial t}\cdot\frac{\partial}{\partial \mathbf{p}} f(\mathbf{r},\mathbf{p},t)$$



- Boltzmann equation $\frac{d}{dt} f(\mathbf{r},\mathbf{p},t) = \left(\frac{\partial f}{\partial t}\right)_c$
  - collisionless: $\left(\frac{\partial f}{\partial t}\right)_c = 0$
  - collisional:

$$\left(\frac{\partial f}{\partial t}\right)_c = R_i - R_o,$$

$$= \int \frac{d^3p_2}{2E_2(2\pi)^3}\,\frac{d^3p_1'}{2E_{1'}(2\pi)^3}\,\frac{d^3p_2'}{2E_{2'}(2\pi)^3} \times (2\pi)^4\delta^{(4)}\left(p_1' + p_2' - (p_1 + p_2)\right)|\mathcal{M}_{fi}|^2 \left(F_{1'2'} - F_{12}\right)$$

$$\left(\frac{\partial f}{\partial t}\right)_c = \int \frac{d^3p_2}{(2\pi)^3}\, d\sigma |\mathbf{v}_1 - \mathbf{v}_2| (F_{1'2'} - F_{12})$$

$$d\sigma = \frac{1}{2E_1 2E_2|v_1 - v_2|}\,\frac{d^3p_1'}{2E_{1'}(2\pi)^3}\,\frac{d^3p_2'}{2E_{2'}(2\pi)^3} \times (2\pi)^4\delta^{(4)}\left(p_1' + p_2' - (p_1 + p_2)\right)|\mathcal{M}_{fi}|^2,$$

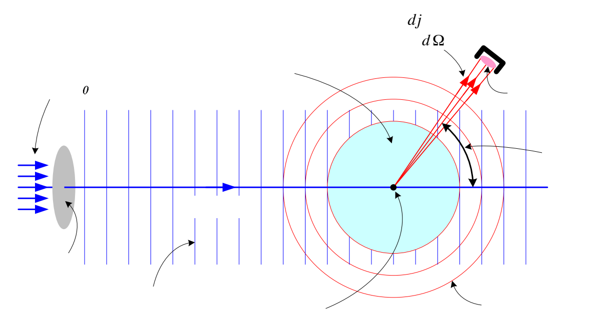
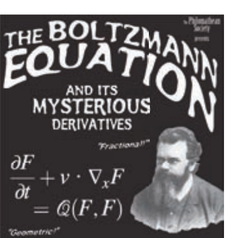


Fig. H. Haberzettl



Kinetic theory: curved spacetime

- Liouville operator

$$\frac{d}{d\lambda} f(x^\mu(\lambda), p^\mu(\lambda)) = \frac{\partial f}{\partial x^\mu}\frac{dx^\mu}{d\lambda} + \frac{\partial f}{\partial p^\mu}\frac{dp^\mu}{d\lambda}$$

$$L_F(f(p,t)) = \left[\frac{\partial}{\partial t} - \frac{\dot{a}}{a} p \frac{\partial}{\partial p}\right] f(p,t) = \frac{1}{E} C(E)$$

$$p^0 = E = (p^2 + m^2)^{1/2}$$

Geodesic equation

$$p^\mu \equiv \frac{dx^\mu}{d\lambda}$$

$$\frac{dp^\mu}{d\lambda} + \Gamma^\mu{}_{\nu\rho} p^\nu p^\rho = 0$$

- Relativistic Boltzmann equation

$$n(t) = \int \frac{d^3p}{(2\pi)^3} f(p,t)$$

$$\dot{n}(t) + 3H(t)n(t) = a^{-3}\frac{d}{dt}(a^3 n(t)) = \int \frac{d^3p}{(2\pi)^3}\frac{C(E)}{E}$$

$$H(t) = \frac{\dot{a}}{a}$$

Hubble expansion

$$\dot{\xi} + 3H\xi = 0$$

$$\dot{n} + 3Hn = \mathcal{C}[n]$$

$$\frac{\partial}{\partial t}\left(\frac{n}{\xi}\right) = \frac{1}{\xi}\mathcal{C}[n]$$

Entropy production

- Boltzmann H-theorem
  - Entropy current

$$S^\mu = -\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p^0}\left[f\log f \mp (1\pm f)\log(1\pm f)\right]$$

$$S^\mu{}_{;\mu} = -\int \frac{d^3p}{(2\pi)^3}\log f C(E) \geq 0$$

$$\text{Equilibrium} \implies S^\mu{}_{;\mu} \equiv 0$$

- Equivalence relations
  - Collision integral is zero; proper entropy is constant; equilibrium distributions
  - Collision integral non-zero; proper entropy generation; non-equilibrium

Equilibrium distributions

- Fermi-Dirac

$$\mathscr{Z}_{FD} = \sum_{N=0}^{1} \left(e^{-\beta(\epsilon-\mu)}\right)^N = 1 + e^{-\beta(\epsilon-\mu)}.$$

$$f_{FD} = \langle N \rangle_{FD} = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$$

- Bose-Einstein

$$\mathscr{Z}_{BE} = \sum_{N=0}^{\infty} \left(e^{-\beta(\epsilon-\mu)}\right)^N = \frac{1}{1 - e^{-\beta(\epsilon-\mu)}},$$

$$f_{BE} = \langle N \rangle_{BE} = \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$$

- Maxwell-Boltzmann

$$f_{BE} = f_{FD} \approx e^{-\beta(\epsilon-\mu)} = f_{MB}$$

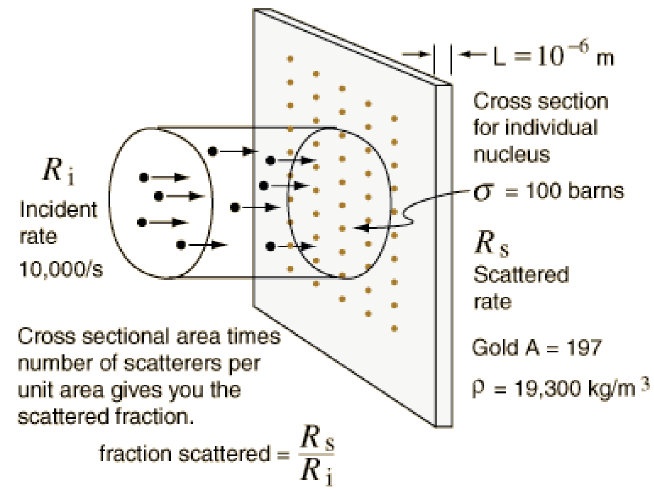
The equilibrium distributions satisfy the condition that the collision integral is zero. But here we derive them from the grand canonical ensemble.

Kinetic regimes

- Equilibrium $\Gamma \gg H(t)$
  - Hubble exp. negligible for kinetics
  - Forward/Reverse rates *detail balance*
  - Reaction rate sufficiently fast to explore much phase space
  - Caveat: FLRW no timelike Killing field
- Kinetic $\Gamma \simeq H(t)$
  - Hubble exp. and reactions compete
  - Non-zero net=F-R rate
  - Boltzmann H-theorem: dS/dt>0 but ~ 0
    - However, assume adiabatic
- Decoupled $\Gamma \ll H(t)$
  - e.g. Relativistic: $T \sim a^{-1}$
  - Free-streaming; distribution frozen

Reaction rate

$$d\Gamma_{34,12} = dn_2 \langle \sigma_{34,12} v_{12,rel} \rangle$$

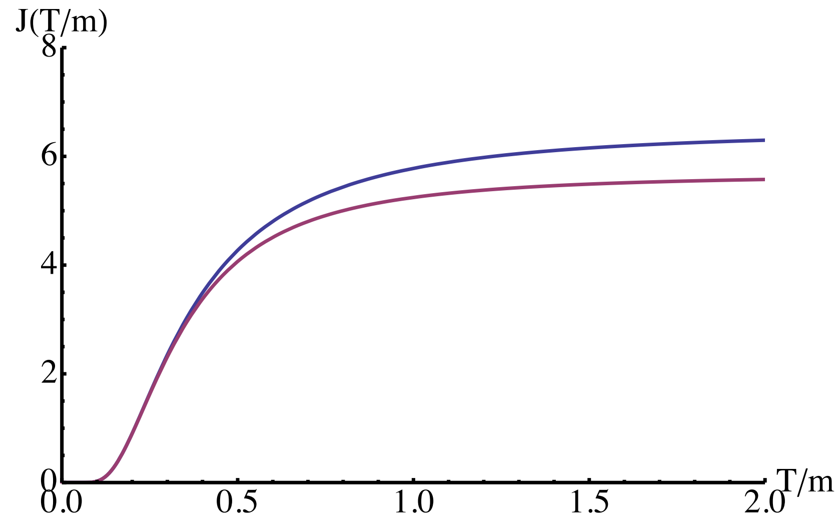


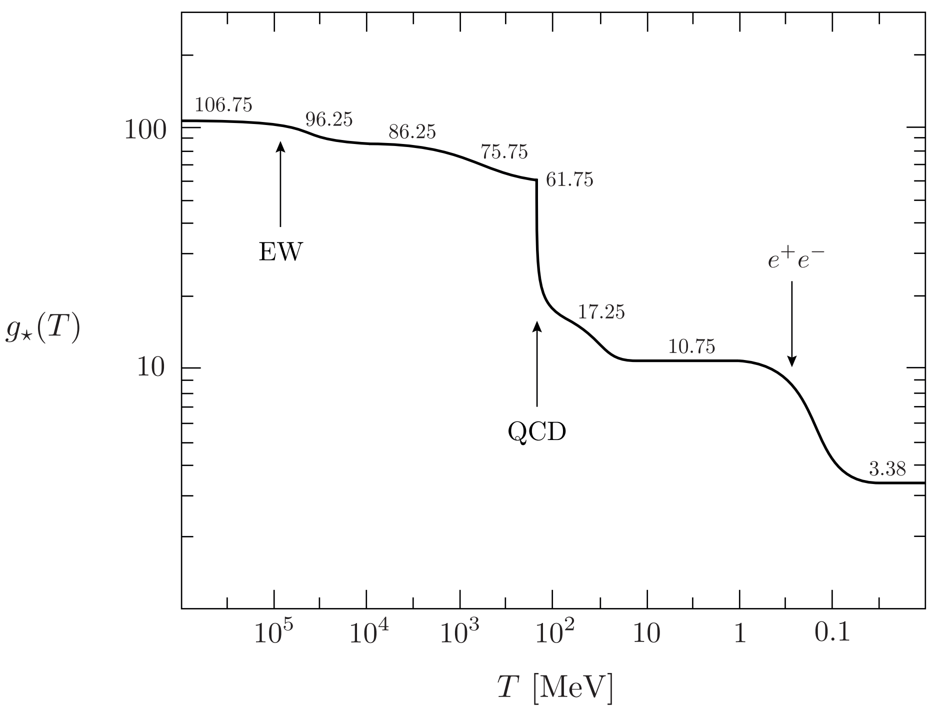
Cosmological transitions (*Caveat Emptor*)

$$\rho(T) = \sum_{i=\gamma,\nu_j,\ell^\pm,\ldots} \int \frac{d^3p}{(2\pi)^3} f_i(p) \sqrt{p^2 + m_i^2}$$

$$= \sum_i g_i \frac{T_i^4}{2\pi^2} J_{\pi_i}(x_i)$$

$$J_{\pi_i}(x_i) = \int_0^\infty d\xi\, \xi^2 \frac{\sqrt{\xi^2 + x_i^2}}{e^{\sqrt{\xi^2+x_i^2}} + \pi_i}, \quad x_i = m_i/T$$





$$\rho(T) = g_\star(T) \frac{\pi^2}{30} T_\gamma^4$$

$$g_\star = \sum_i g_i \left(\frac{T_i}{T_\gamma}\right)^4 \frac{J_{\pi_i}(x_i)}{J_\gamma(0)}$$

NB: $T_i \doteq T_\gamma$

$$J_i(x_i) \to \theta\left(T - \frac{m_i}{6}\right) J(x_i) \to \text{ arbitrary!}$$

Reaction network reduction of Boltzmann eqn

- Reaction network reduction of BEq. (classical, non-degenerate)

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_{\alpha_1}) = \sum_{\alpha_2\beta}\int_{\substack{p_{\beta_1}p_{\beta_2}\\ p_{\alpha_1}p_{\alpha_2}}} (2\pi)^4\delta^{(4)}(p_{\beta_1}+p_{\beta_2}-(p_{\alpha_1}+p_{\alpha_2}))$$

$$\times \left|\mathcal{M}_{\beta\alpha}\right|^2 (f_{\beta_1}f_{\beta_2}-f_{\alpha_1}f_{\alpha_2})$$

$$= -\sum_{\alpha_2\beta} n^{(0)}_{\alpha_1} n^{(0)}_{\alpha_2}\langle\sigma_{\beta\alpha}v_\alpha\rangle\left[\frac{n_{\alpha_1}n_{\alpha_2}}{n^{(0)}_{\alpha_1}n^{(0)}_{\alpha_2}} - \frac{n_{\beta_1}n_{\beta_2}}{n^{(0)}_{\beta_1}n^{(0)}_{\beta_2}}\right]$$

$$\langle\sigma_{\beta\alpha}v_\alpha\rangle \equiv \frac{1}{\mathcal{N}}\int\frac{d^3p_{\beta_1}}{(2\pi)^3}\int\frac{d^3p_{\beta_2}}{(2\pi)^3}|\mathbf{v}_1-\mathbf{v}_2|d\sigma_{\beta\alpha}f_{\alpha_1}f_{\alpha_2}$$

$$\mathcal{N} = \int\frac{d^3p_{\beta_1}}{(2\pi)^3}\int\frac{d^3p_{\beta_2}}{(2\pi)^3}f_{\alpha_1}f_{\alpha_2} \equiv n^{(0)}_{\alpha_1}n^{(0)}_{\alpha_1}$$

$$n_i^{(0)} = g_i\int\frac{d^3p}{(2\pi)^3}e^{-E_i/T} \approx g_i\left(\frac{m_iT}{2\pi}\right)^{3/2}e^{-m_i/T}$$

n-p weak equilibrium [Workshop exercise]

- At high T ~ 10's MeV $X_n \sim X_p \sim 1/2$
- At 10 MeV > T > 1 MeV (ignore nucleons)

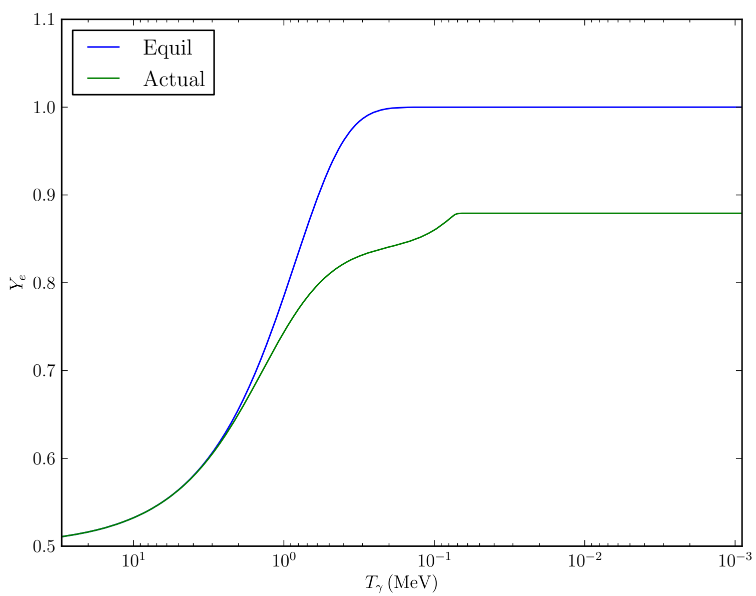
$$n\nu_e \leftrightarrow pe^-, \qquad ne^+ \leftrightarrow p\bar{\nu}_e, \qquad n \leftrightarrow pe^-\bar{\nu}_e.$$

- Equilibrium condition

$$\mu_p = \mu_n \implies \frac{n_n^{(0)}}{n_p^{(0)}} = e^{Q/T}$$

$$Q = m_n - m_p \simeq 1.293\ \text{MeV}$$

Electron Fraction vs. Plasma Temperature ($\Omega_b h^2$ =2.207E-02)



$$\frac{dX_n}{dt} = -\lambda(n \to p)X_n + \lambda(p \to n)(1 - X_n)$$

$$\lambda(i \to j) = n_\ell^{(0)} \langle \sigma_{ji} v_i \rangle$$

$$X_n = \frac{n_n}{n_b} \qquad n_b = n_n + n_p$$

$$X_p \approx X_{e^-}$$

IGPPS Los Alamos

Big bang nucleosynthesis [Workshop exercise]

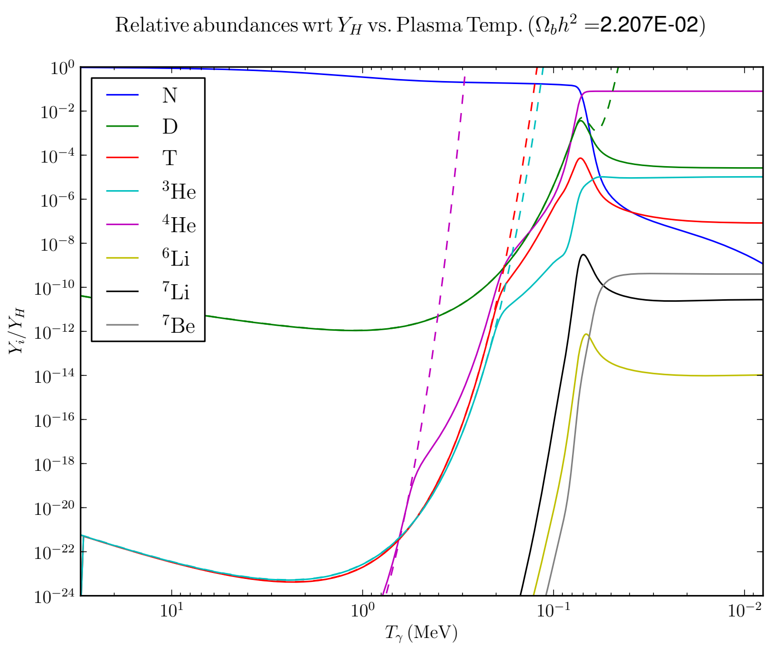
- Full reaction network [*NB: should be unitary*]

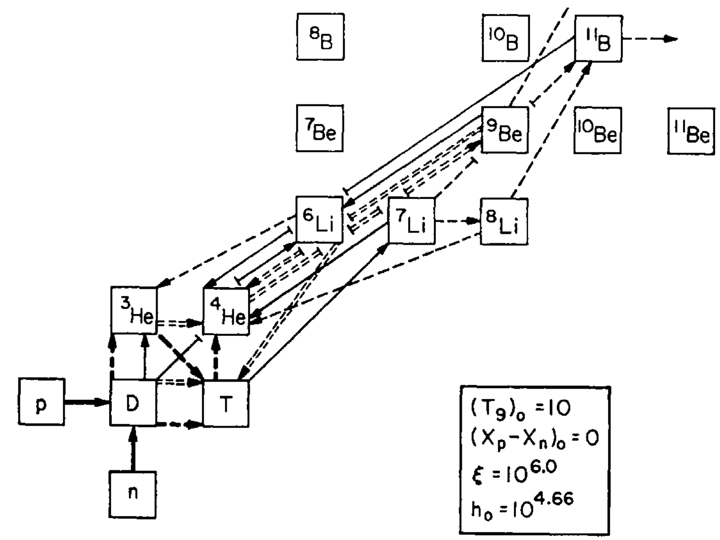
$$Y_i = \frac{n_i}{n_b}$$

$$\frac{dY_{\alpha_1}}{dt} = \sum_{\alpha_2 \beta} \left[-n_b \langle \sigma_{\beta\alpha} \rangle Y_{\alpha_1} Y_{\alpha_2} + n_b \langle \sigma_{\alpha\beta} \rangle Y_{\beta_1} Y_{\beta_2} \right]$$

- Nuclear statistical equilibrium

$$\mu_A = Z\mu_p + N\mu_n$$





End Lecture I