Collective Neutrino Oscillations

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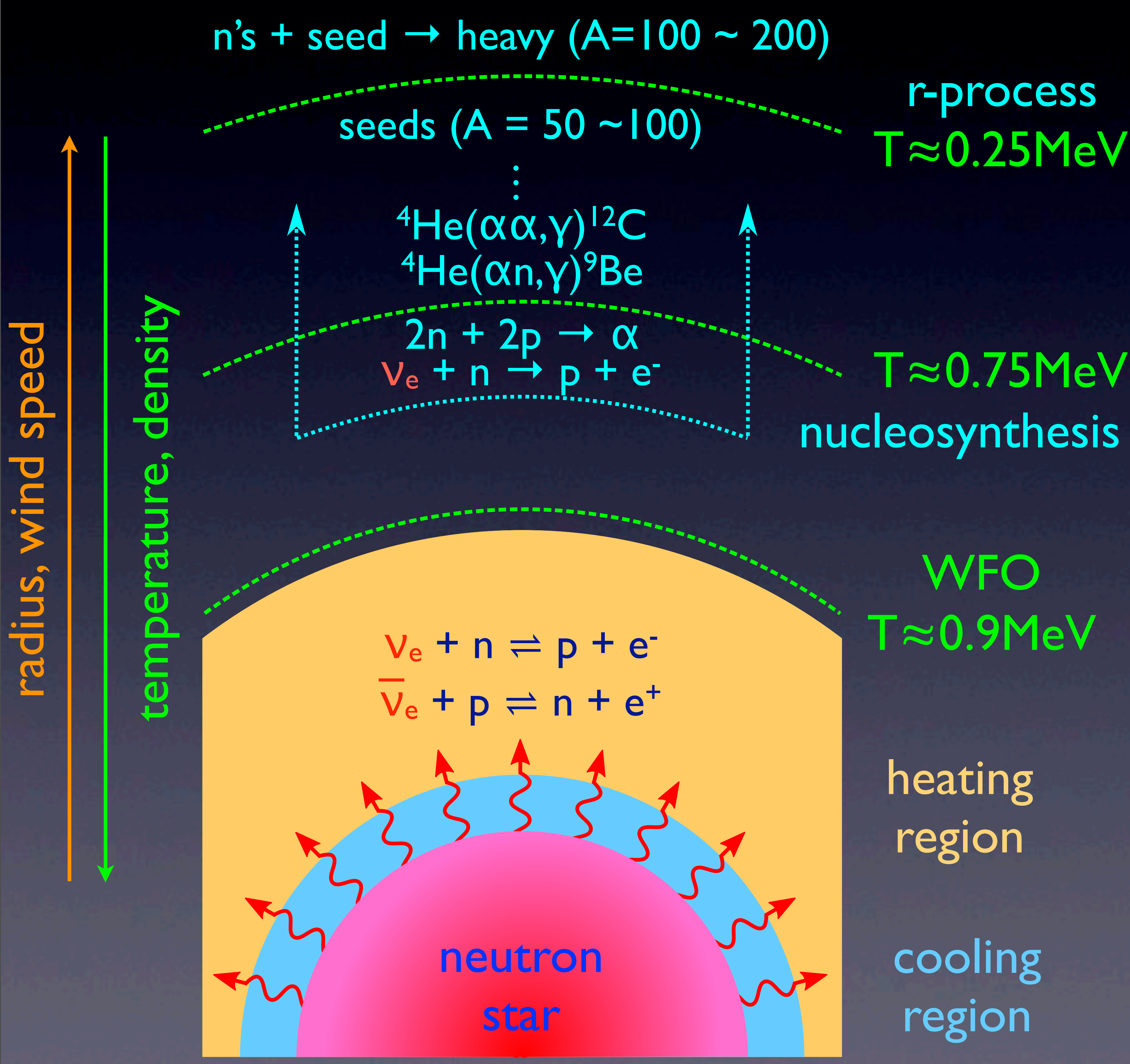
Neutrino & Nuclear Astrophysics

Outline

- ✦ Introduction & overview
- ✦ Understandings & insights
- ✦ New developments & challenges

Neutrinos in Supernovae

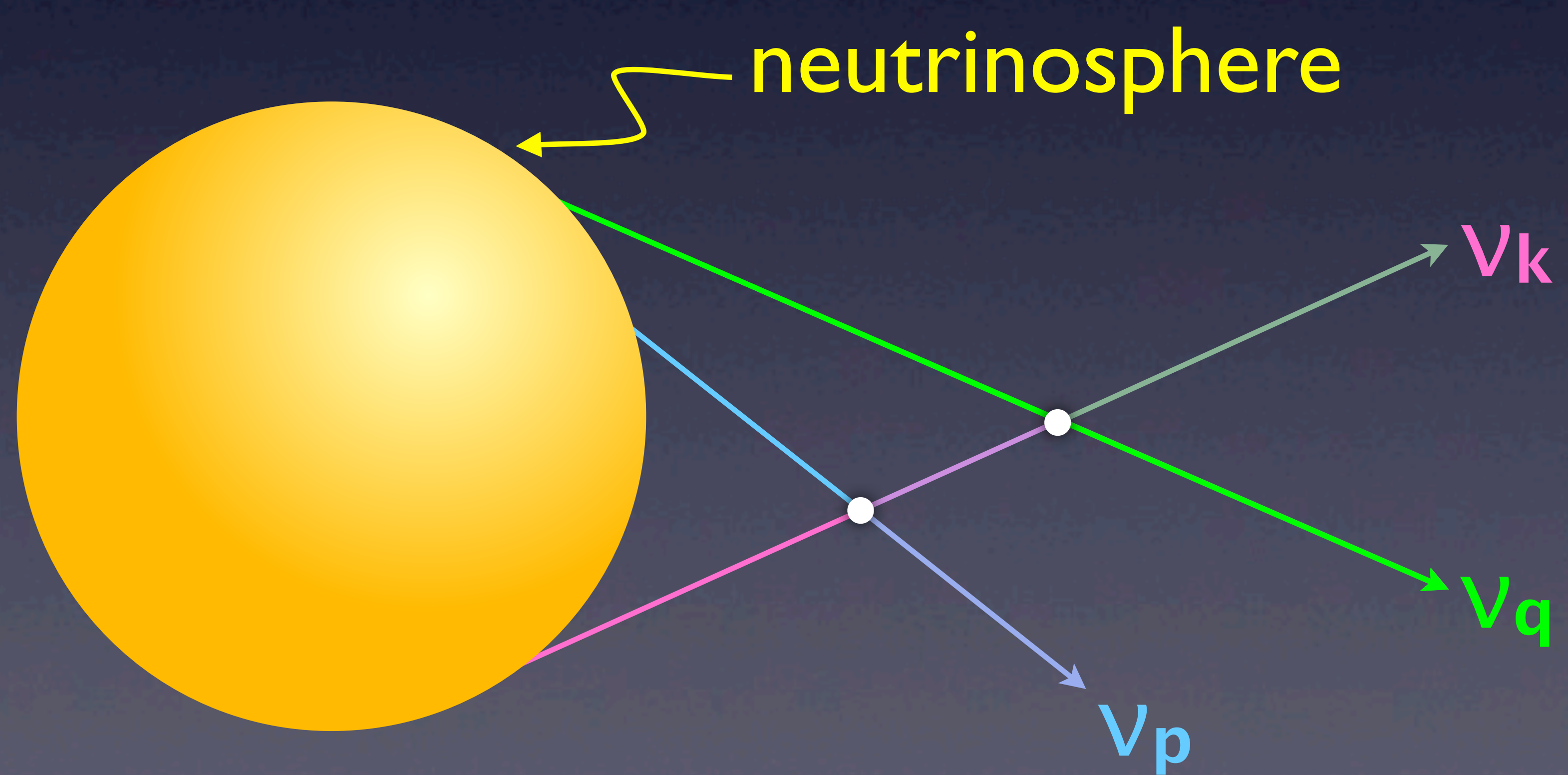


- $\sim 10^{53}$ ergs, 10^{58} neutrinos in ~10 seconds
- All neutrino species, 10~30 MeV
- Dominate energetics
- Influence nucleosynthesis
- Probe into SNe

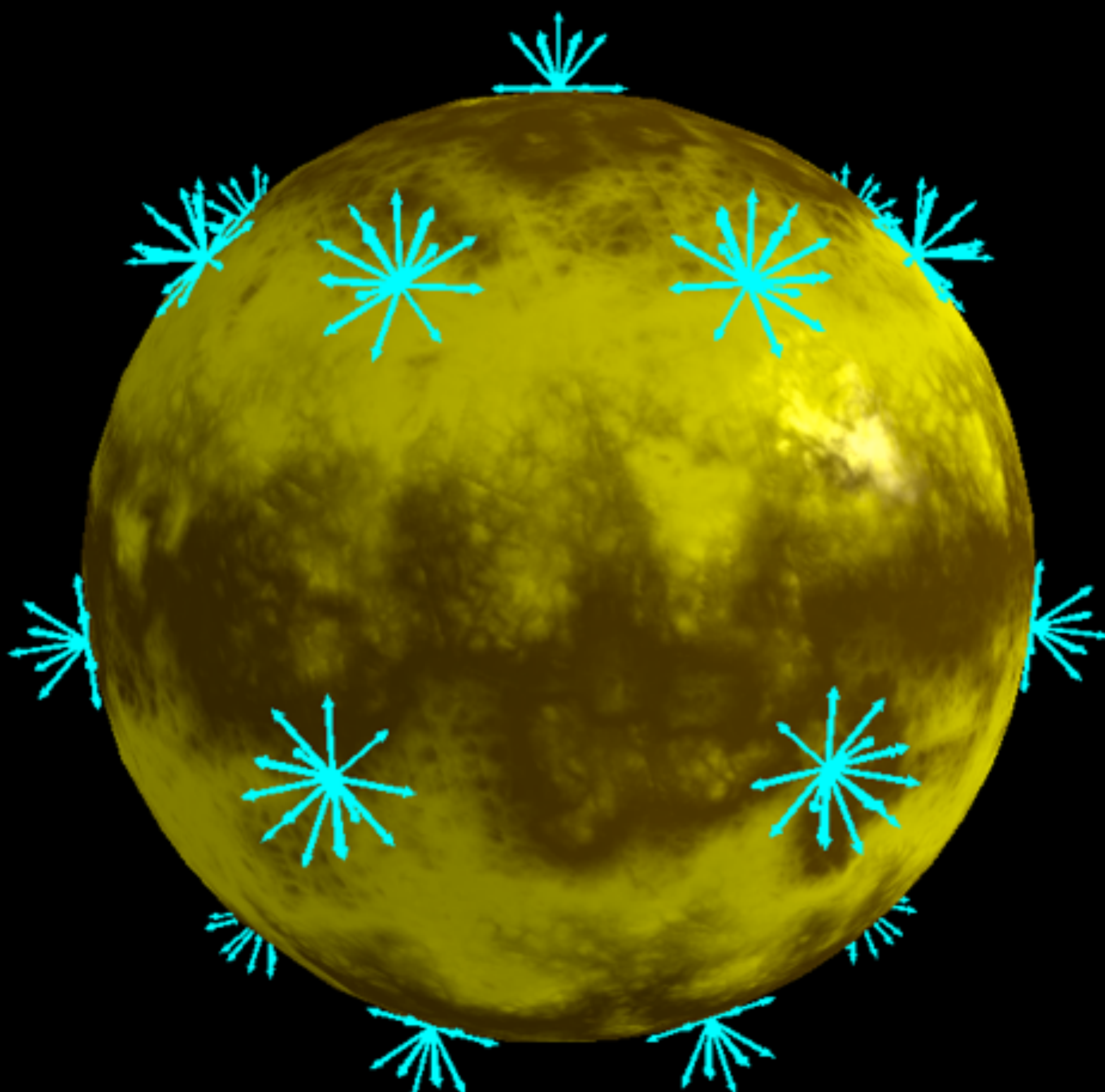
ν oscillations in SN

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$



(3+3)D



$$\psi(r, E, \vartheta, \varphi, \Theta, \Phi)$$

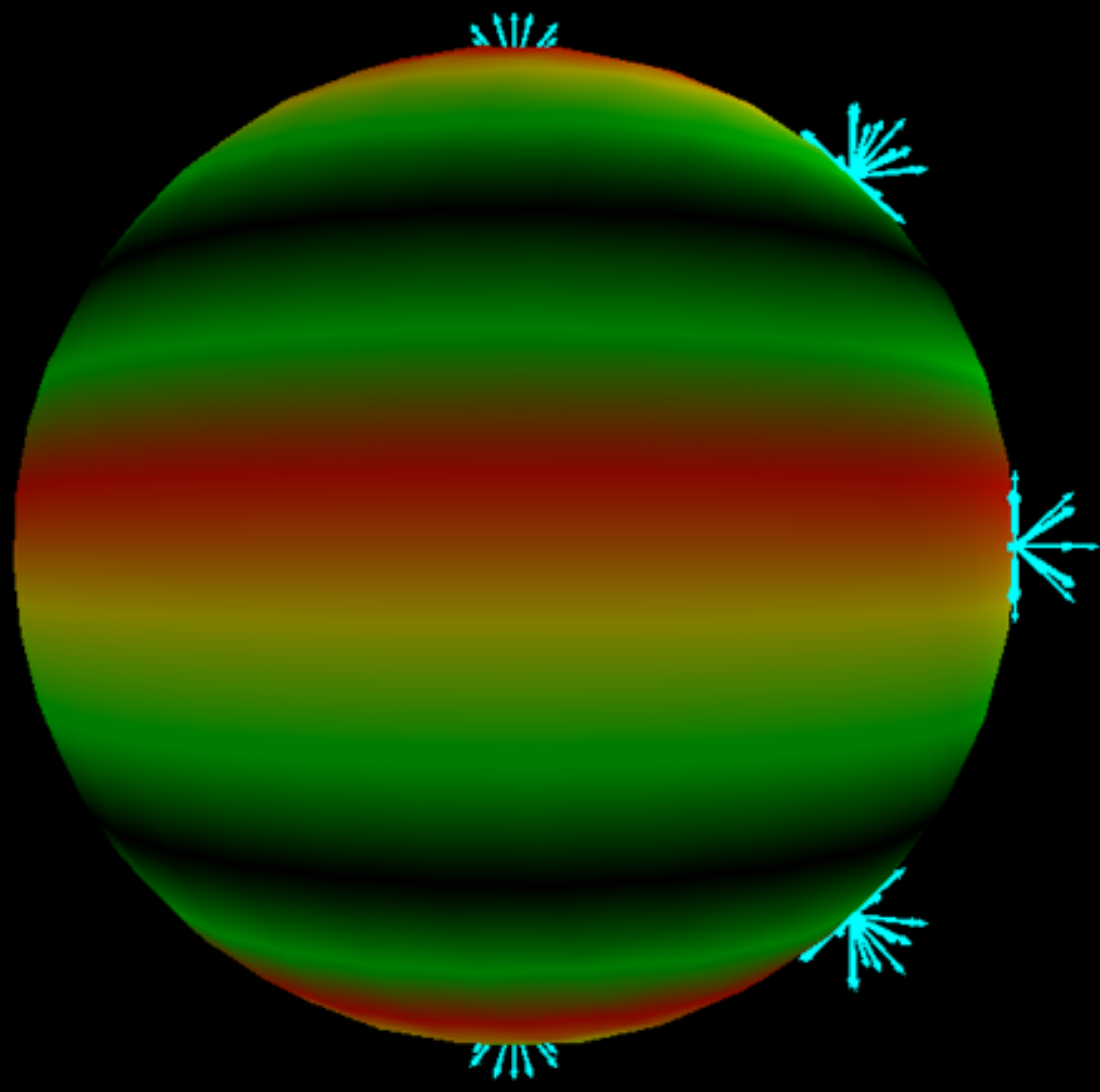
emission direction → ϑ, φ

energy → E

emission points → Θ, Φ

Coherent forward scattering only outside neutrino sphere.

(2+3)D



propagation direction

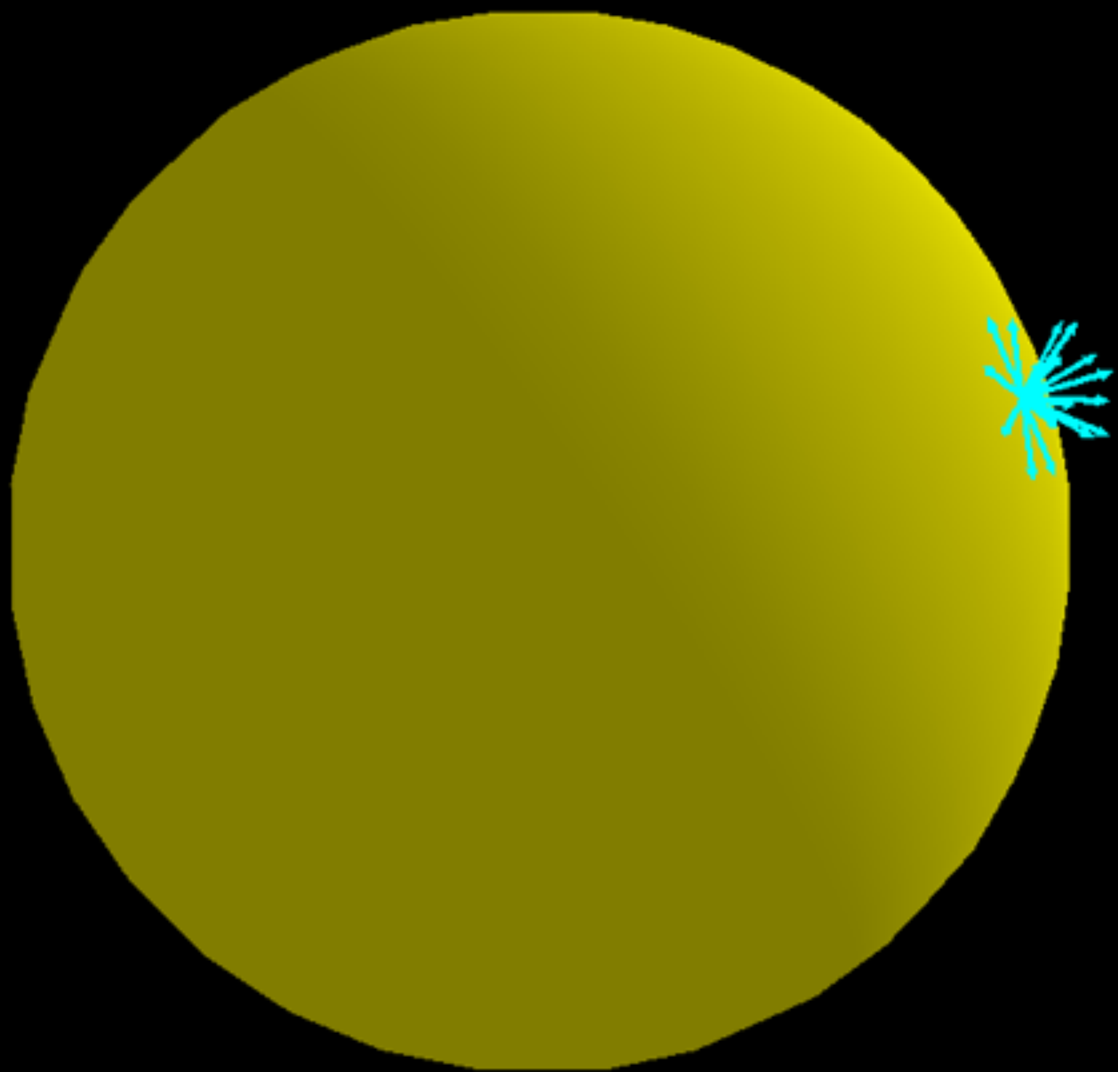
$$\psi(r, E, \vartheta, \varphi, \Theta)$$

energy

emission points

previous assumptions + Axial symmetry around the Z axis.

(1+3)D



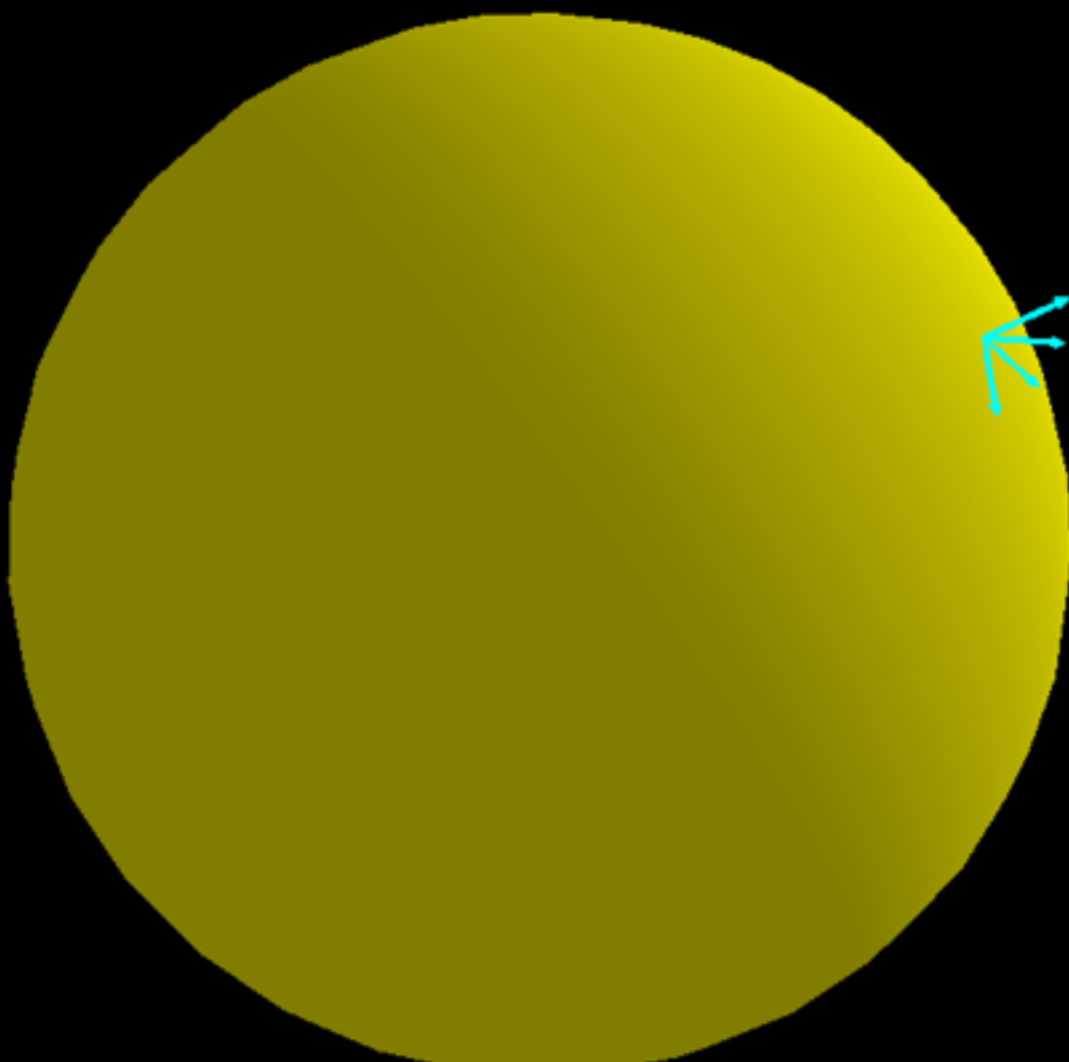
propagation direction

$$\psi(r, E, \vartheta, \varphi)$$

energy

previous assumptions +
Spherical symmetry about the center (Consistency?)

(1+2)D
Multi-Angle/Bulb Model



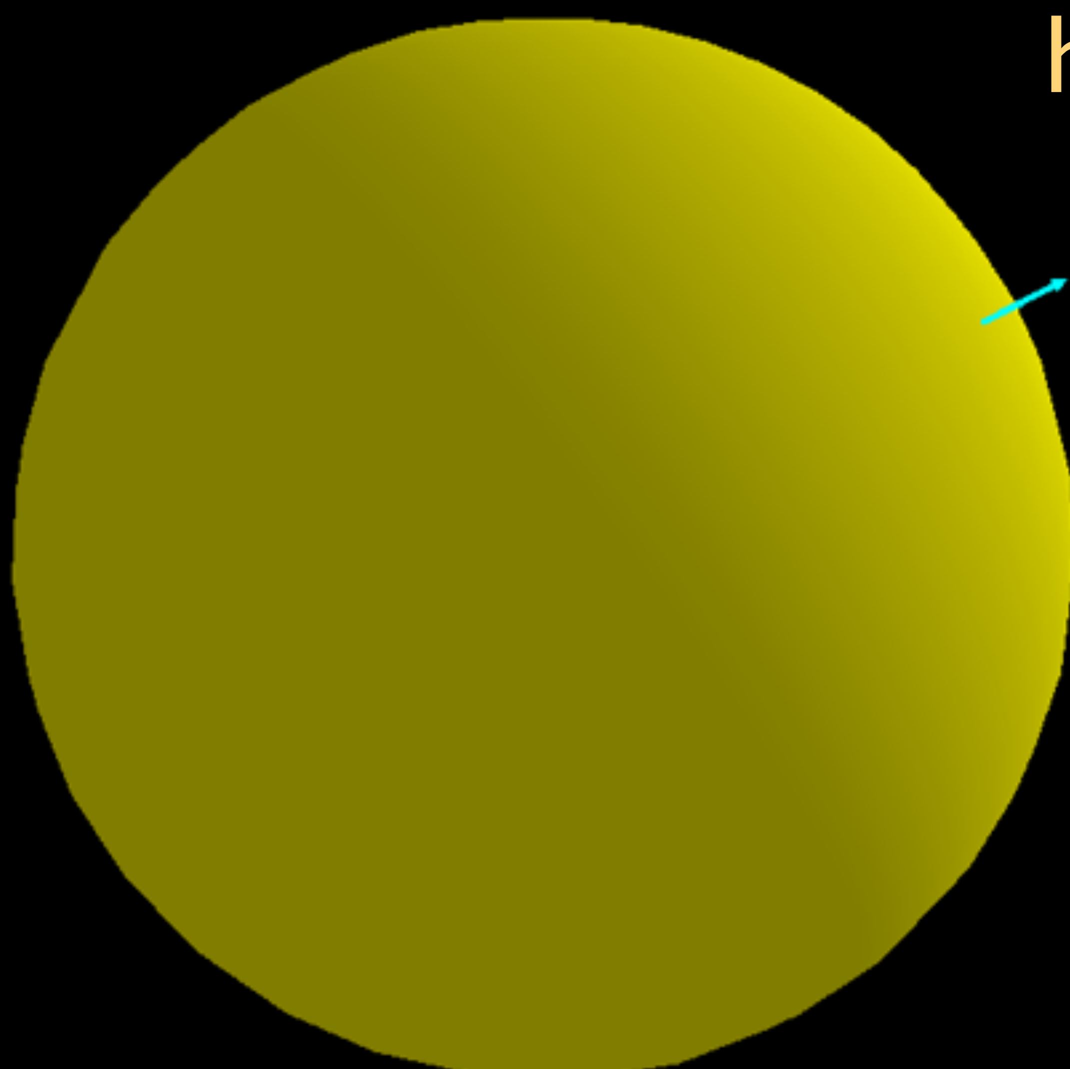
propagation direction

$$\psi(r, E, \vartheta)$$

energy

previous + assumptions + Azimuthal symmetry around any radial direction

(1+1)D
Single-Angle



Equivalent to an expanding homogeneous neutrino gas

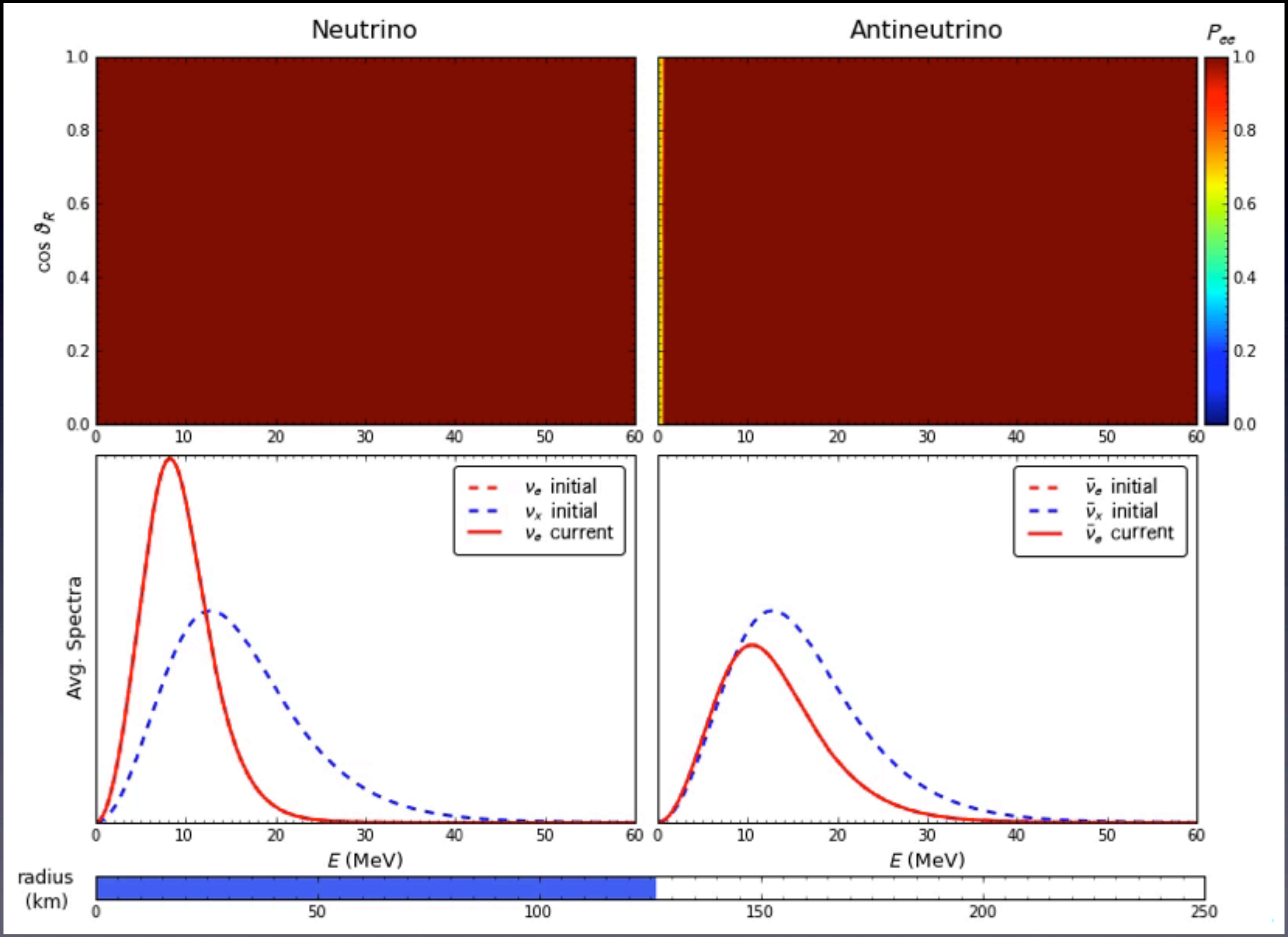
$\psi(r, E)$

↑ energy

previous assumptions +
Trajectory independent neutrino flavor evolution

$$\langle L_{\nu_e}\rangle = 4.1\,\mathrm{foe},\ \langle L_{\bar\nu_e}\rangle = 4.3\,\mathrm{foe},\ \langle L_{\nu_x,\bar\nu_x}\rangle = 7.9\,\mathrm{foe}$$

$$\langle E_{\nu_e}\rangle = 9.4\,\mathrm{MeV},\ \langle E_{\bar\nu_e}\rangle = 13.0\,\mathrm{MeV},\ \langle E_{\nu_x,\bar\nu_x}\rangle = 15.8\mathrm{MeV}$$



YES!! You can make discoveries through numerical calculations.

But are you sure whether the numerical calculations are correct?

Back-of-the-envelope Estimates

Neutrino Self-Coupling

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$

mass squared matrix → M^2

neutrino energy → $2E$

electron density → n_e

ν–ν forward scattering (self-coupling) → $\mathrm{H}_{\nu\nu}$

$$\mathrm{H}_{\nu\nu} = \sqrt{2}G_\mathrm{F}\int \mathrm{d}\mathbf{p}'(1-\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\rho_{\mathbf{p}'} - \bar{\rho}^*_{\mathbf{p}'})$$

3 Energy Scales

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$

- vacuum term: $\Lambda_{vac}=\Delta m^2/2E$
- matter (electron) density: $\Lambda_{mat}=\sqrt{2}G_F n_e$
- neutrino density: $\Lambda_\nu=\sqrt{2}G_F(n_\nu-n_{\bar{\nu}})$

Three Flavor MSW

- $\Lambda_{vac} \approx \Lambda_{mat}$: MSW resonance
- $\Lambda_{vac}(\Delta m^2_{\odot}, 10\text{MeV}) \sim (49\text{ km})^{-1}$
- $\Lambda_{vac}(\Delta m^2_{atm}, 10\text{MeV}) \sim (1.6\text{ km})^{-1}$
- $\Lambda_{mat}(10^{12}\text{ g/cc}) \sim (0.005\text{ mm})^{-1}$
- Both mass splittings are effective

$\Lambda_{vac} = \Delta m^2/2E$ $\quad\quad$ $\Lambda_{mat} = \sqrt{2} G_F n_e$

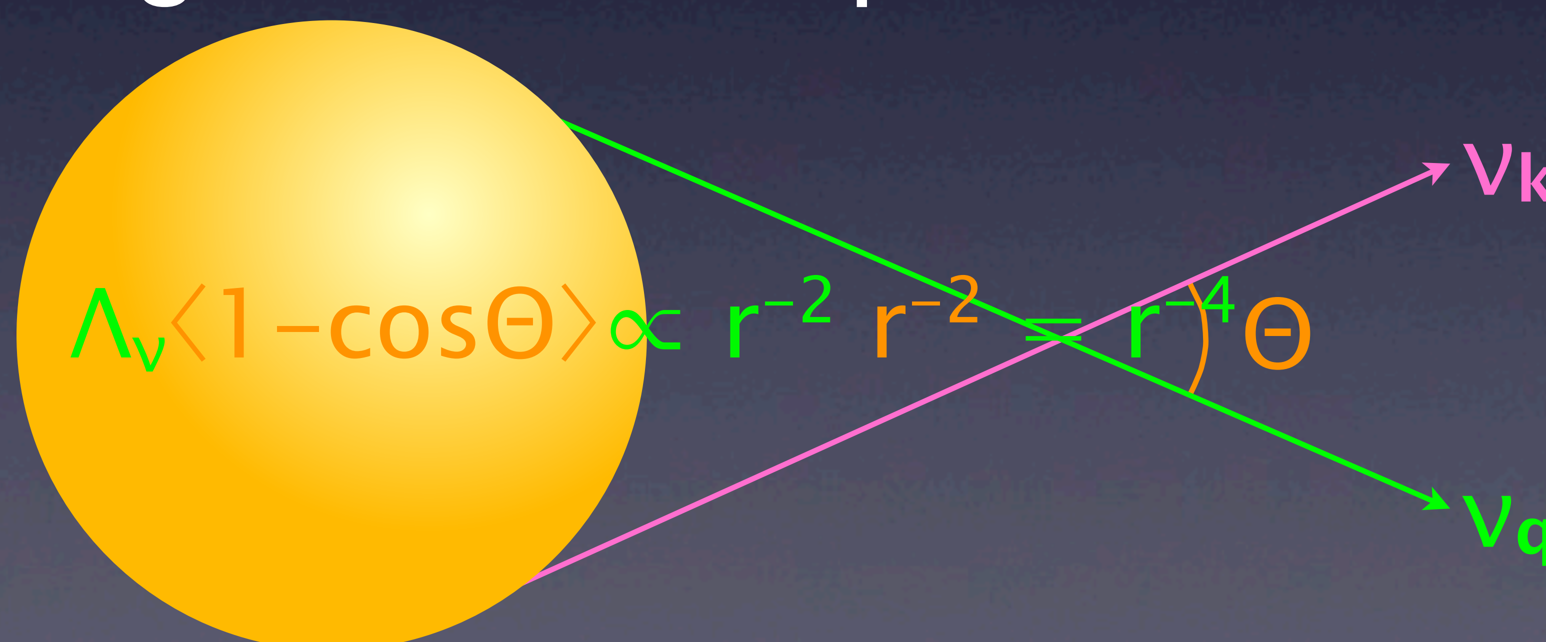
Bumpy Matter Profile

- $\Lambda_{vac} \approx \Lambda_{mat}$: MSW resonance
- $d(\ln \Lambda_{mat})/dr \gtrsim \Lambda_{vac}$: non-adiabatic
- Track propagation of shockwaves
  (Schirato & Fuller, 2002; and others)
- Flavor depolarization at turbulence (cannot distinguish spectra of different flavor)
  (Friedland & Gruzinov, 2006; and others)

$\Lambda_{vac} = \Delta m^2 / 2E$ $\Lambda_{mat} = \sqrt{2} G_F n_e$

Collective Oscillations

- Λ_{vac} depends on neutrino energy $\Rightarrow$ $\Delta\Lambda_{vac}$: dispersion in energies
- $\Delta\Lambda_{vac}$ (~ Λ_{vac}) $\lesssim$ $\Lambda_\nu\langle 1\text{-}\cos\Theta\rangle$: neutrinos with different energies oscillate in phase



$\Lambda_{vac}=\Delta m^2/2E$ $\Lambda_\nu=\sqrt{2}G_F(n_\nu-n_{\bar{\nu}})$

Self-Suppression

- $\Delta\Lambda_{vac}$ ($\sim \Lambda_{vac}$) $\ll \Lambda_{\nu}\langle 1\text{-}\cos\Theta\rangle$: synchronization; no significant oscillations unless experiencing MSW resonance (Pastor et al 2001, 2002)
- Criterion for significant collective oscillations: $\Delta\Lambda_{vac}$ ($\sim \Lambda_{vac}$) $\sim \Lambda_{\nu}\langle 1\text{-}\cos\Theta\rangle$ (HD, Fuller & Qian, 2005)

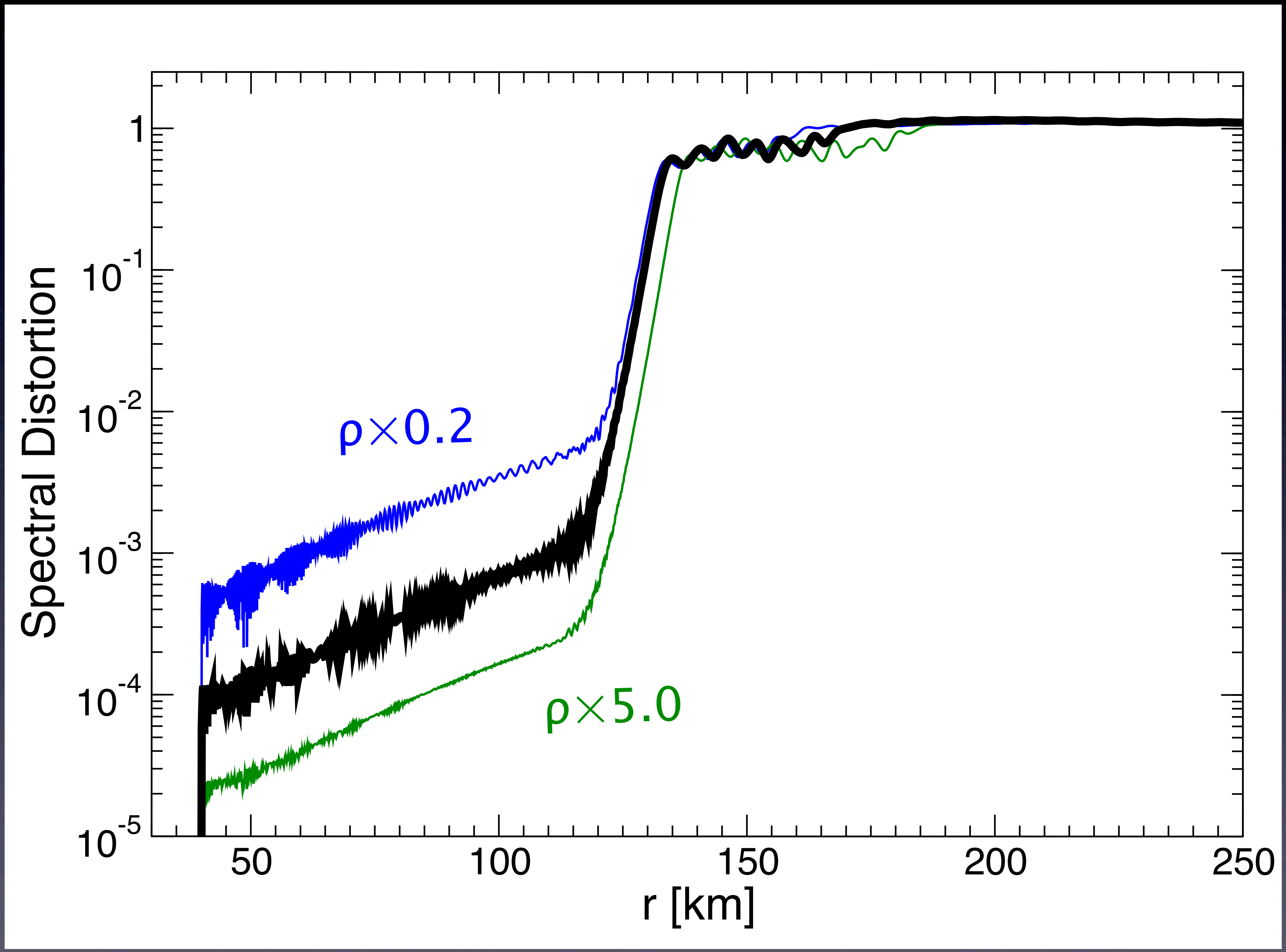
$\Lambda_{vac}=\Delta m^2/2E$ $\Lambda_{\nu}=\sqrt{2}G_F(n_{\nu}-n_{\bar{\nu}})$

Matter Suppression

- $\Lambda_{mat} \gtrsim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$: suppression of collective oscillations?
- No. Uniform matter distribution does not suppress collective oscillations in the homogeneous and isotropic neutrino gas.
(HD, Fuller & Qian, 2005)

$\Lambda_{mat} = \sqrt{2} G_F n_e$ $\Lambda_\nu = \sqrt{2} GF (n_\nu - n_{\bar{\nu}})$

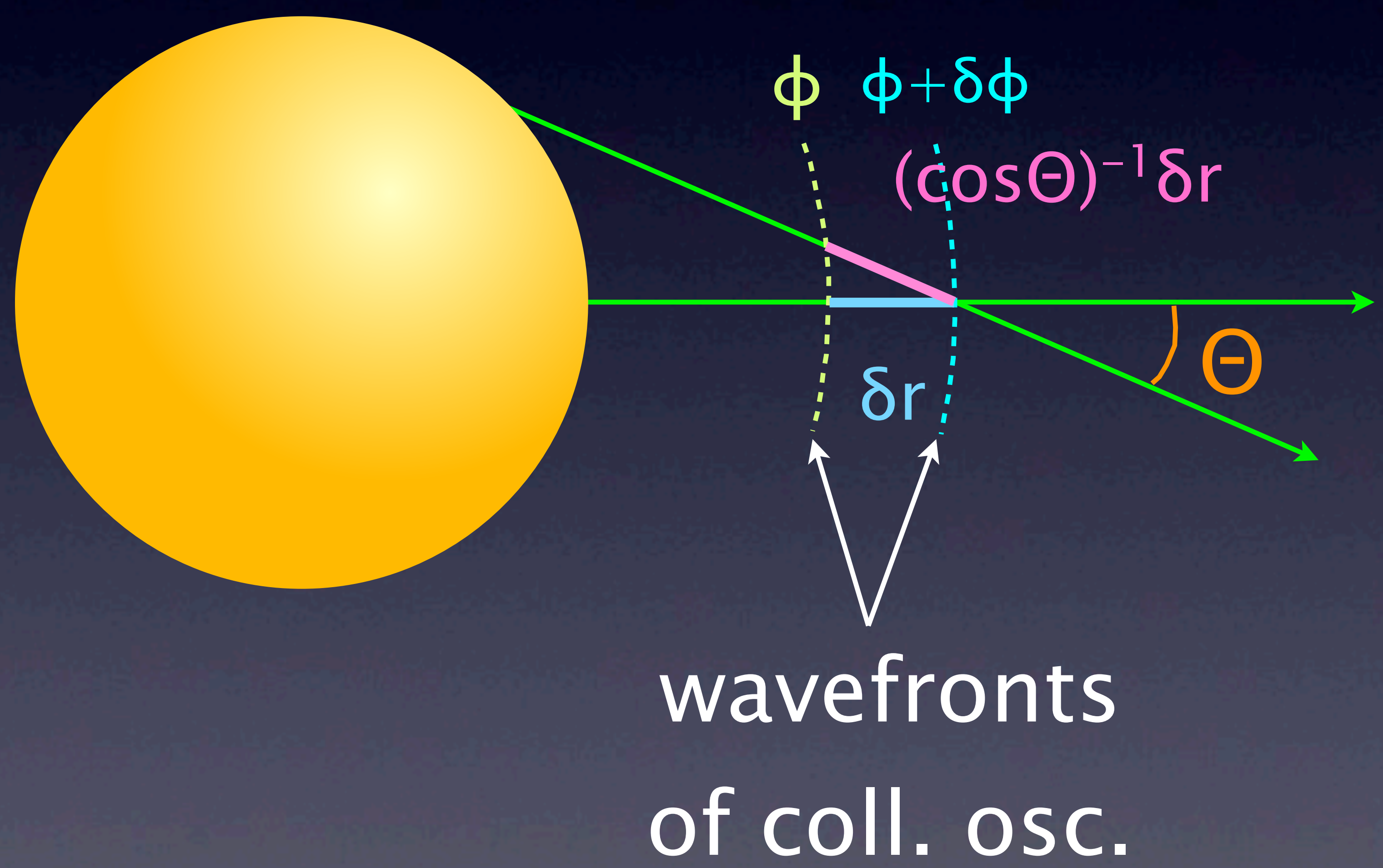
Matter Suppression



HD & Friedland (2010)

Matter Suppression

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

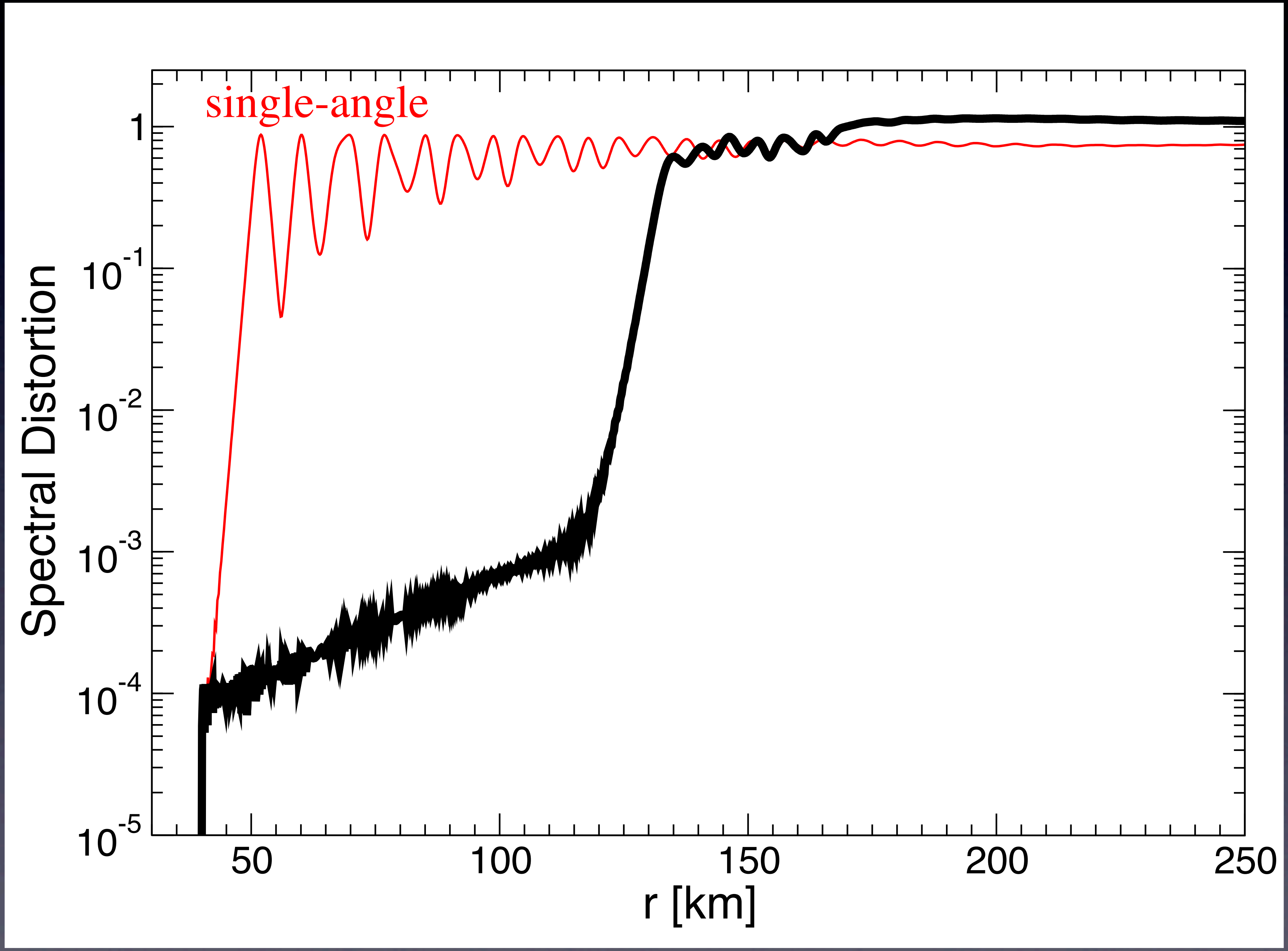


Matter Suppression

- Dispersion in $\Lambda_{mat} d\lambda/dr$: $\Delta\Lambda_{mat}$
- Criterion for significant collective oscillations: $\Delta\Lambda_{mat} \sim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$
- $\Delta\Lambda_{mat} \gtrsim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle \Rightarrow$ suppression of collective oscillations (Esteban-Pretel et al, 2008)
- $\Delta\Lambda_{mat} \propto r^{-2} \rho(r) \Rightarrow$ suppression only at early-time and/or very close to NS

$\Lambda_{vac} = \Delta m^2/2E$ $\Lambda_{mat} = \sqrt{2} G_F n_e$ $\Lambda_\nu = \sqrt{2} GF(n_\nu - n_{\bar{\nu}})$

Multiangle Suppression



HD & Friedland (2010)

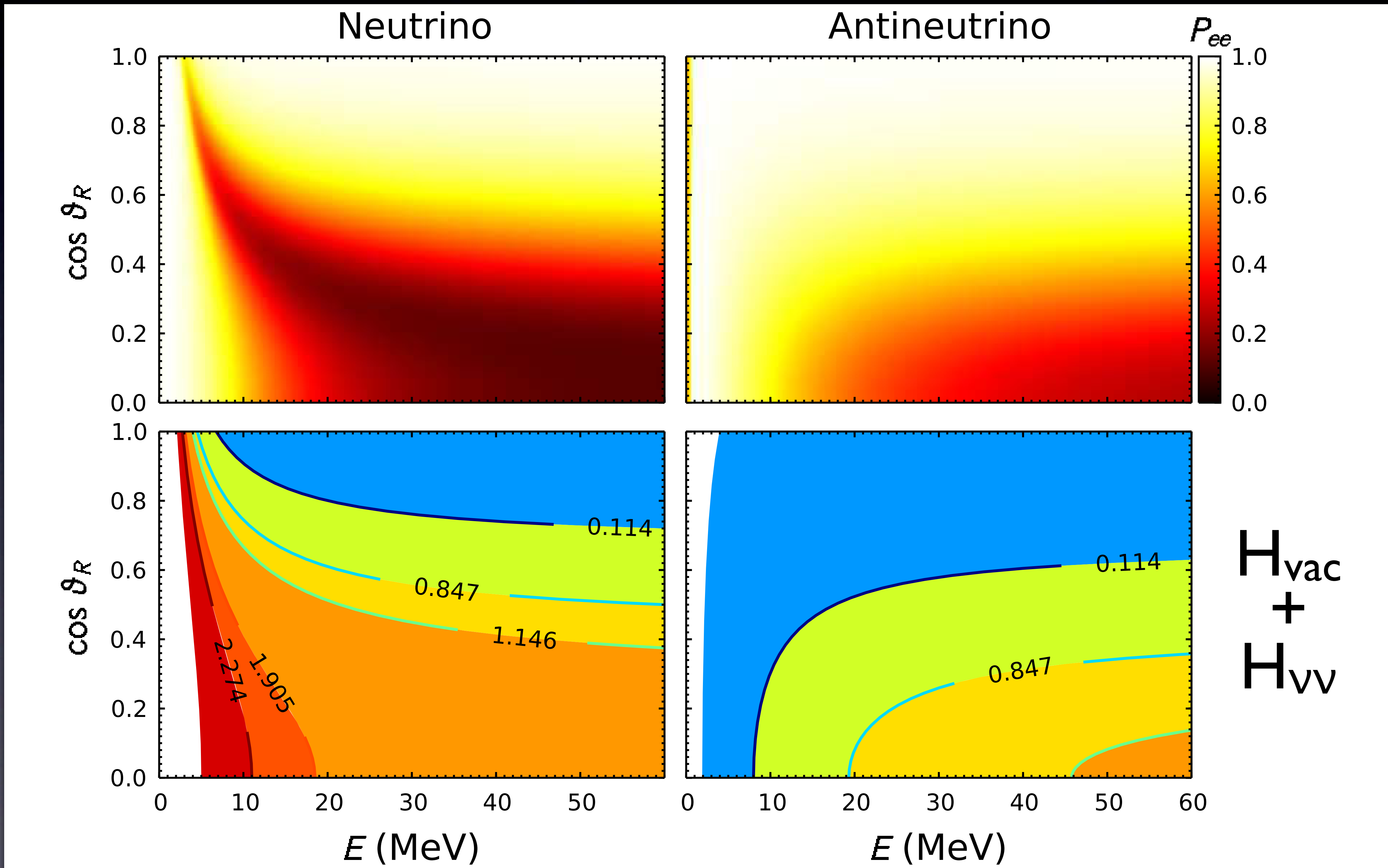
Multiangle Suppression

- $\Lambda_\nu\langle 1\text{-}\cos\Theta\rangle$ depends on neutrino emission angle ⇒ dispersion in angles $\Delta\Lambda_\nu$ (~$\Lambda_\nu\langle 1\text{-}\cos\Theta\rangle$)
- Criterion for significant collective oscillations:
  $\Delta\Lambda_{vac} \sim \Delta\Lambda_\nu \sim \Lambda_\nu\langle 1\text{-}\cos\Theta\rangle$ (HD & Friedland, 2010)

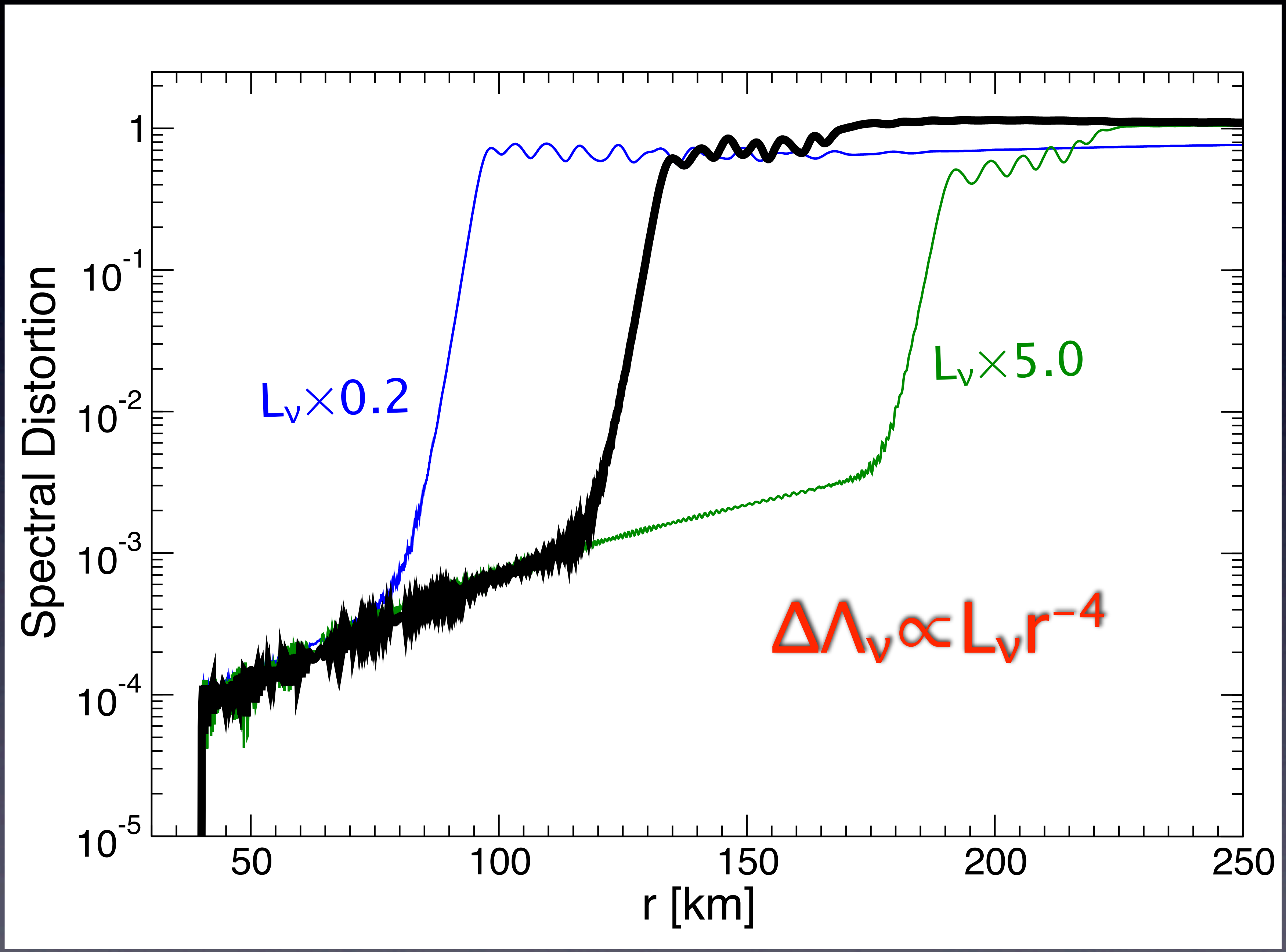
$$\mathrm{H}_{\nu\nu} = \sqrt{2}G_{\mathrm{F}} \int \mathrm{d}\mathbf{p}'(1 - \hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\rho_{\mathbf{p}'} - \bar{\rho}^*_{\mathbf{p}'})$$

$\Lambda_{vac}=\Delta m^2/2E$ $\Lambda_\nu=\sqrt{2}G_F(n_\nu-n_{\bar{\nu}})$

Multiangle Suppression



Multiangle Suppression



HD & Friedland (2010)

Tools & Toy Models

Vacuum Oscillations

neutrinos are generated/detected in flavor states

neutrino mass eigenstates ≠ neutrino flavor states

$$|\nu_1\rangle = \cos\theta_{\rm v}|\nu_e\rangle + \sin\theta_{\rm v}|\nu_\mu\rangle \quad \text{with mass } m_1$$

$$|\nu_2\rangle = -\sin\theta_{\rm v}|\nu_e\rangle + \cos\theta_{\rm v}|\nu_\mu\rangle \quad \text{with mass } m_2$$

vacuum mixing angle

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-\omega\cos 2\theta_{\rm v} & \omega\sin 2\theta_{\rm v}\\ \omega\sin 2\theta_{\rm v} & \omega\cos 2\theta_{\rm v}\end{bmatrix}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}$$

vac. osc. freq. $\omega = \dfrac{\delta m^2}{2E_\nu}$

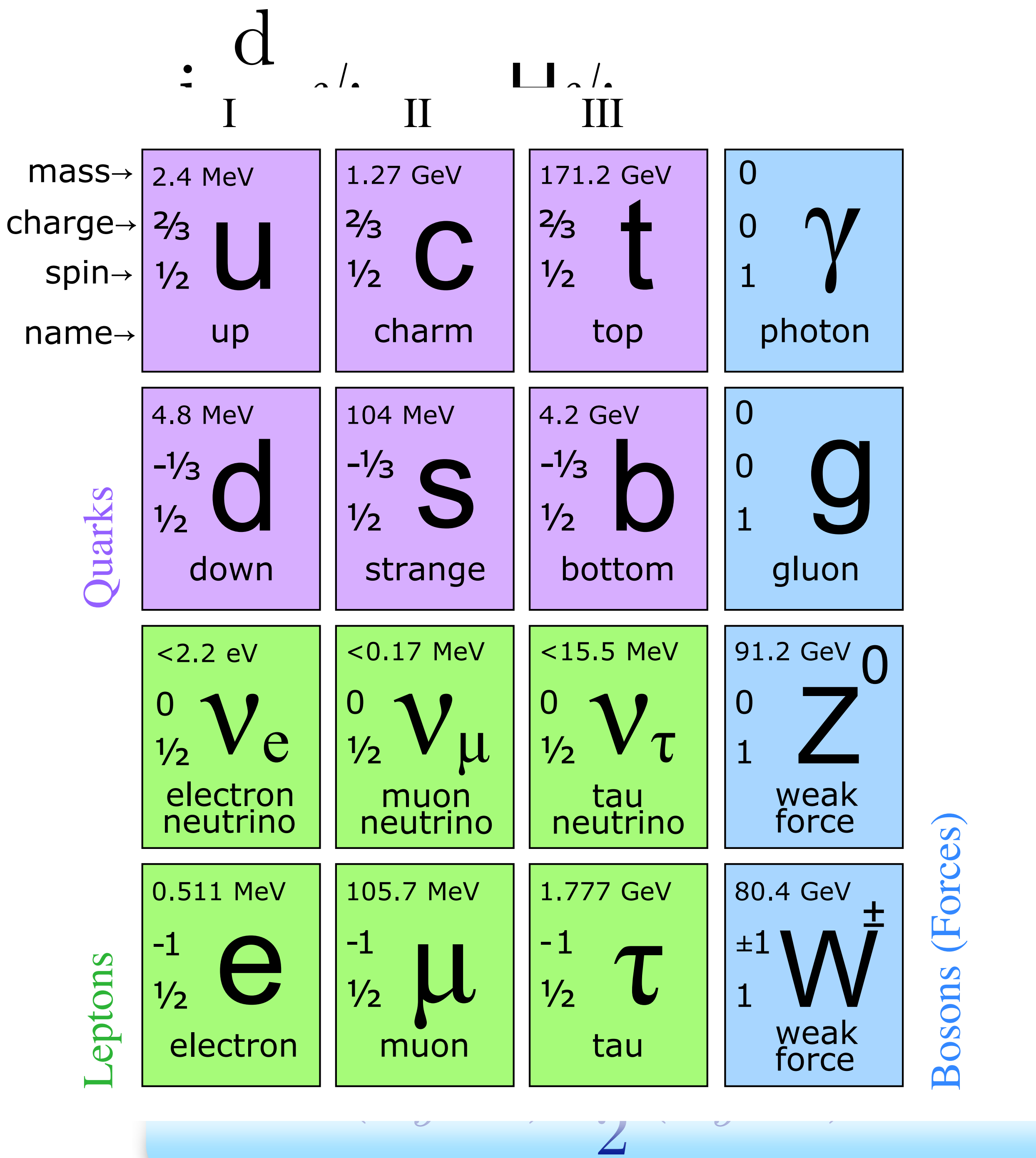
$$\delta m^2 = m_2^2 - m_1^2$$

Neutrino Flavor Isospin

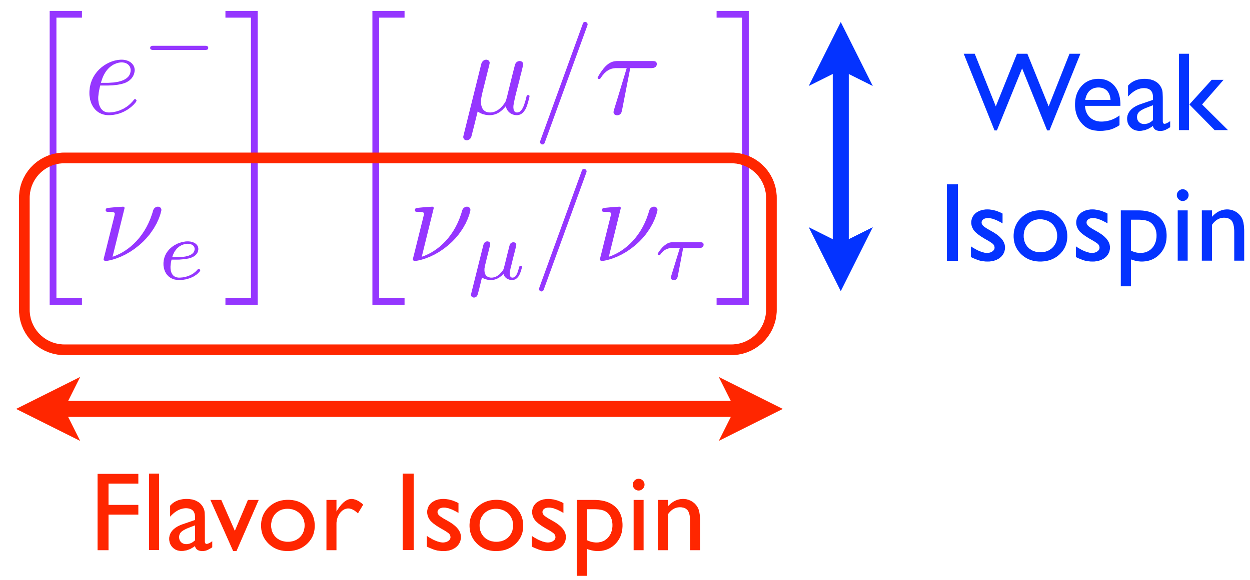
Two-component system ⟷ spin-1/2

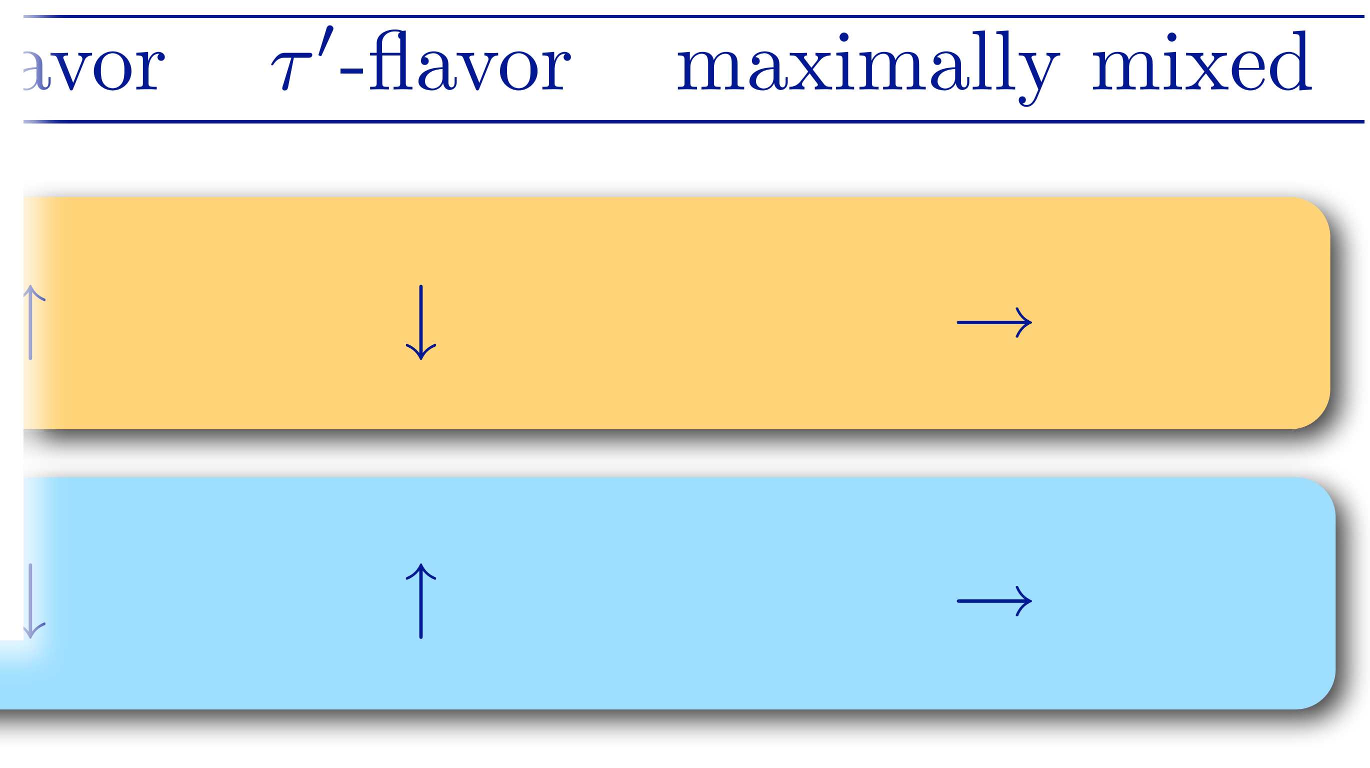
2×2 Hermitian matrix $\mathsf{H} = H_0 \mathbb{1} + \mathbf{H} \cdot \boldsymbol{\sigma}$

Neutrino Flavor Isospin



Wikimedia: Standard Model of Elementary Particles





Vacuum Oscillations Again

$$\vec{H} = \omega \vec{H}_{\rm vac}$$

$$\vec{H}_{\rm vac} \equiv -\hat{e}^{\rm f}_x \sin 2\theta_{\rm v} + \hat{e}^{\rm f}_z \cos 2\theta_{\rm v}$$

$$\omega \equiv \pm \frac{\delta m^2}{2E}$$