The r-Process: Status & Challenges

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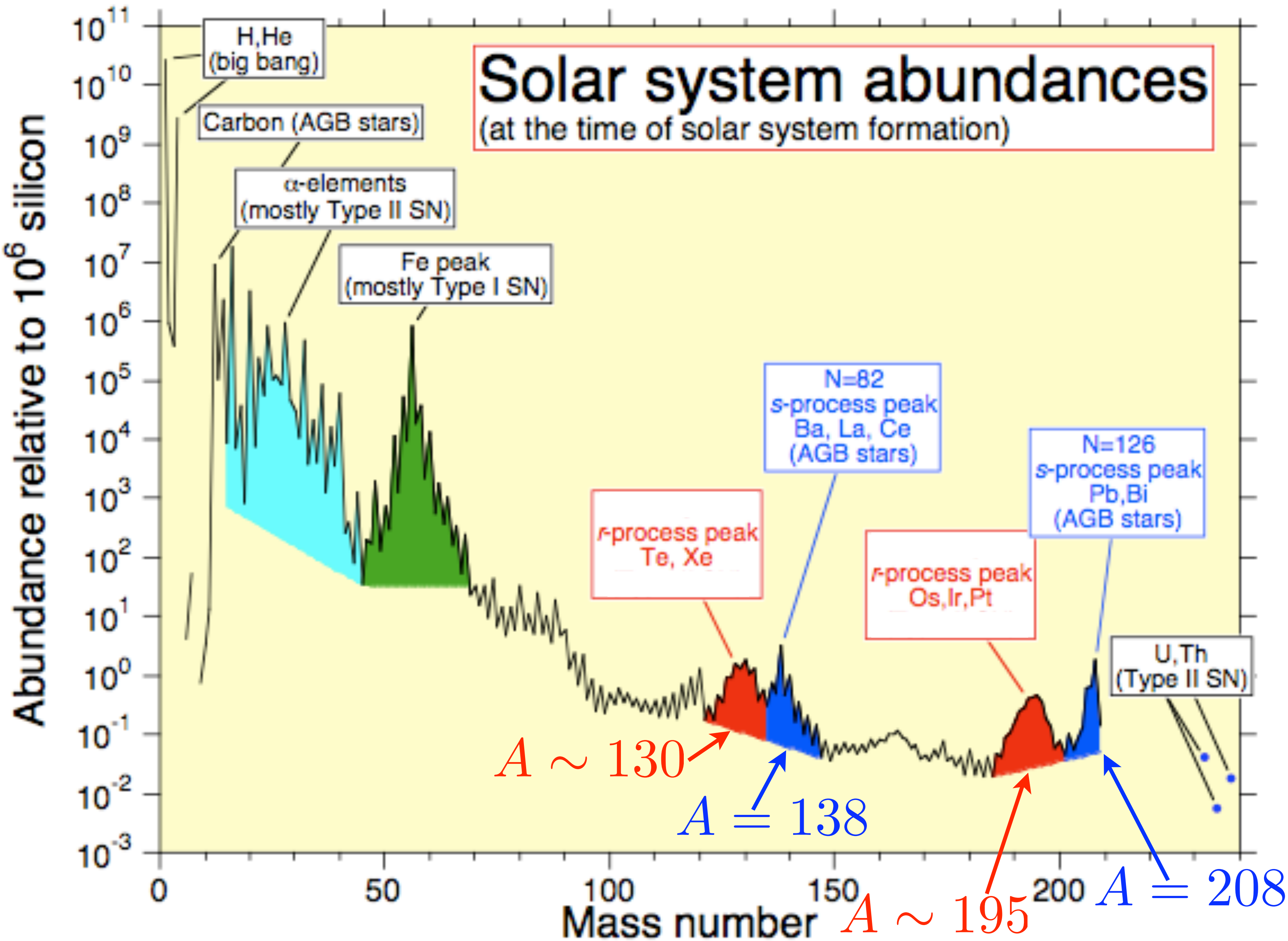
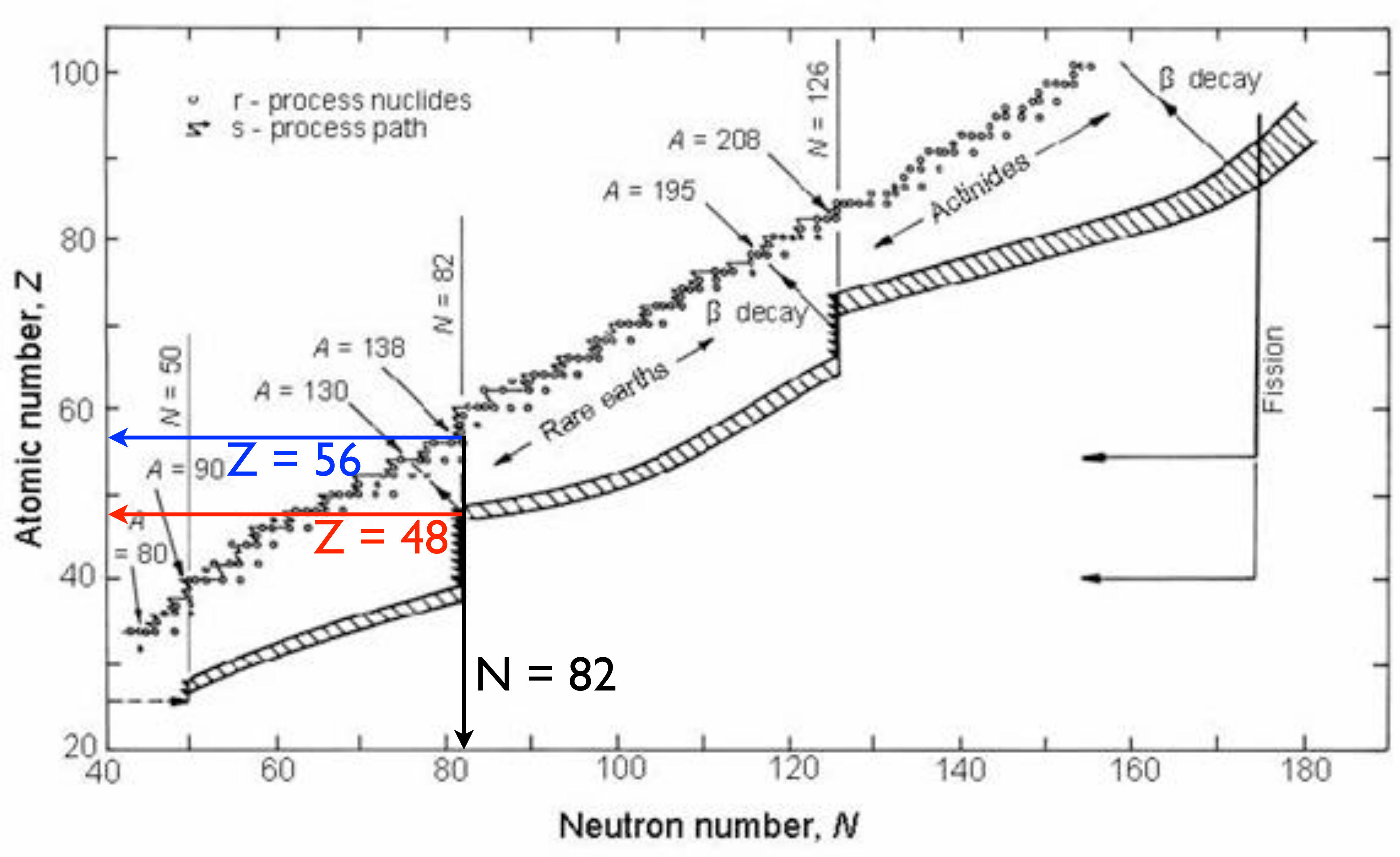
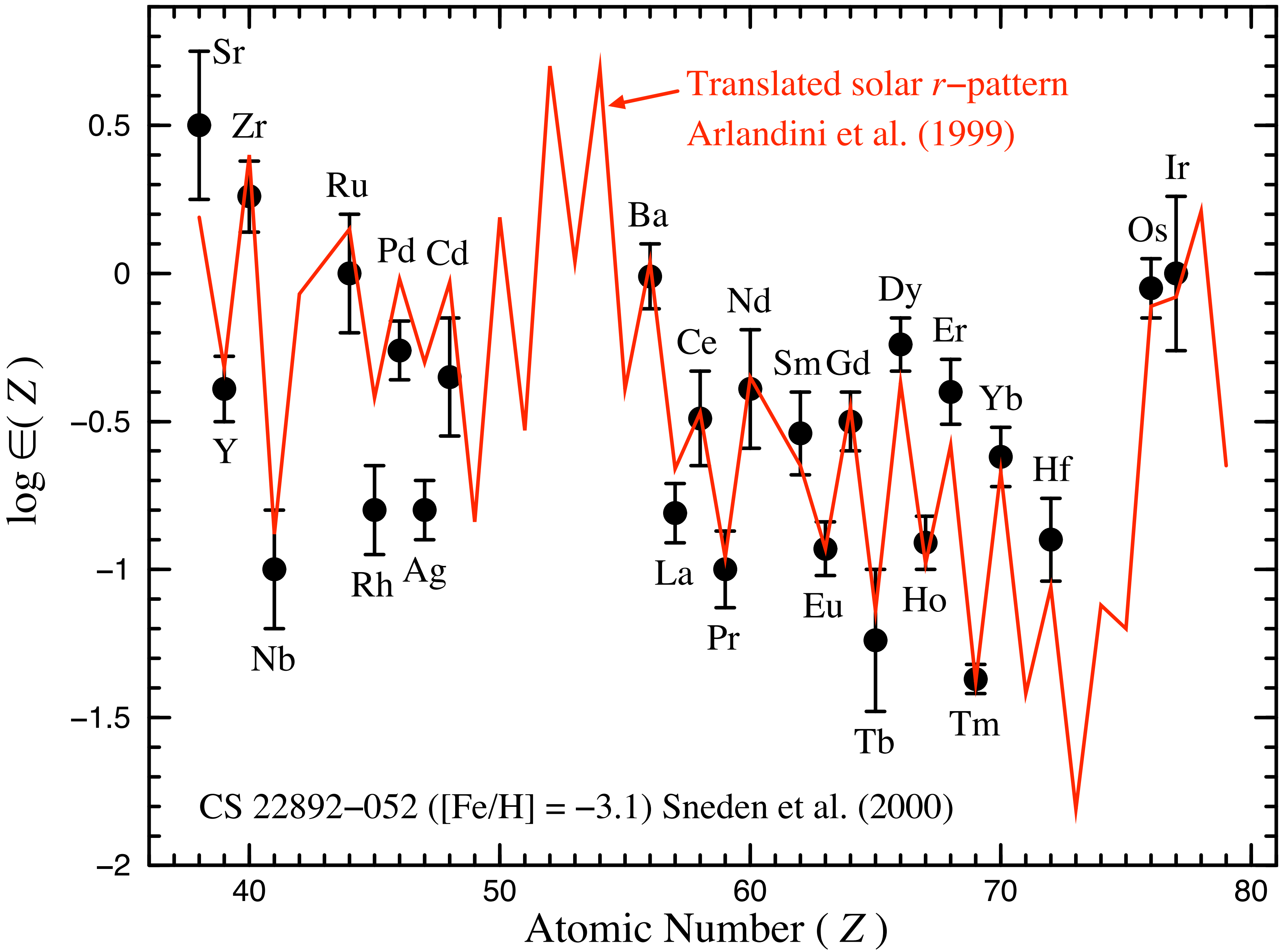
Cosmic Abundances



slow (s) and rapid (r) neutron capture processes





Translated solar r−pattern
Arlandini et al. (1999)
CS 22892−052 ([Fe/H] = −3.1) Sneden et al. (2000)
Sr
Zr
Y
Nb
Ru
Rh
Pd
Ag
Cd
Ba
La
Ce
Pr
Nd
Sm
Eu
Gd
Tb
Dy
Ho
Er
Tm
Yb
Hf
Os
Ir
log ∈(Z)
Atomic Number (Z)
0.5
0
−0.5
−1
−1.5
−2
40
50
60
70
80

Basics of Big Bang Nucleosynthesis

initial state ($T > 1$ MeV): $n,\ p$

$$X_n + X_p = 1 \Rightarrow \text{need } n/p$$

rate of change in abundance:

$$\frac{dY_i}{dt} = P(t) - D(t)Y_i\,,\ Y_i = \frac{X_i}{A_i},\ n_i = \rho_b N_A Y_i$$

$$\left.\begin{array}{l} P(t):\ \text{production rate} \\ D(t):\ \text{destruction rate} \end{array}\right\} \text{both depend on } T(t) \text{ and } \rho_b(t)$$

$T(t)$ specified by dynamics of expansion

$\rho_b(t)$ specified by conservation of entropy per baryon

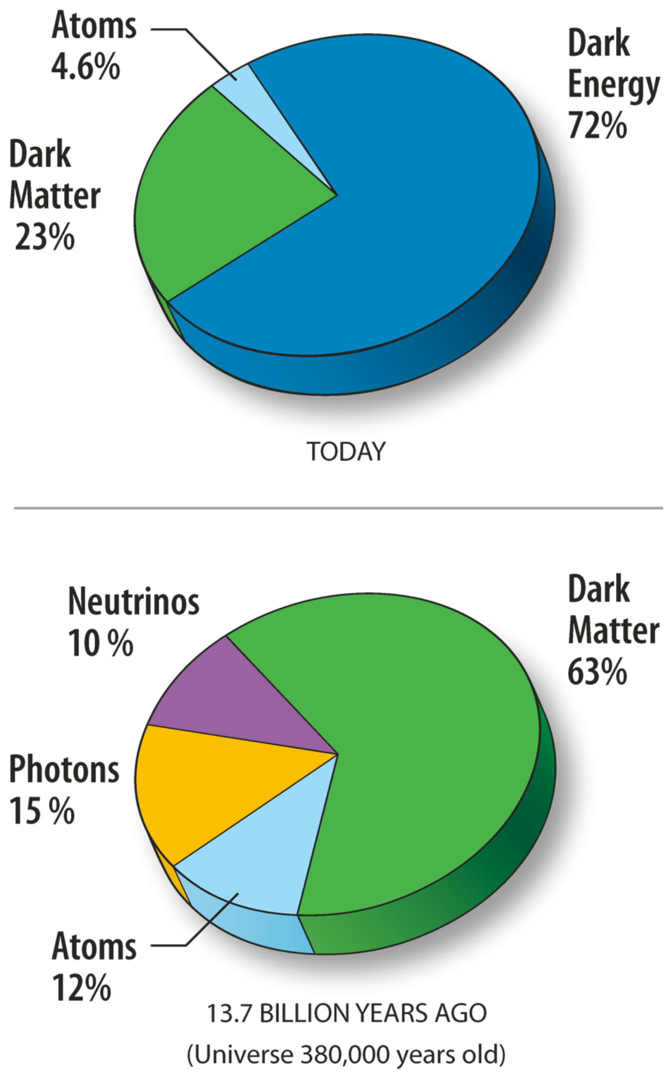
$$s \propto g^*_{\text{eff}}(t)\frac{T^3}{\rho_b} \propto g^*_{\text{eff}}(t)\frac{n_\gamma}{n_b} = \text{const.}$$

$$\text{baryon-to-photon ratio: } \eta = \frac{n_{b,0}}{n_{\gamma,0}} \Rightarrow s \approx \frac{3.6}{\eta}$$

expansion of the early universe

mass conservation $\Rightarrow \rho_b(t) + \rho_{\rm dm}(t) = \rho_m(t) = \rho_{m,0}\left[\frac{R_0}{R(t)}\right]^3$

photon number conservation: $n_\gamma(t)R(t)^3 = n_{\gamma,0}R_0^3$



$$n_\gamma \propto T_\gamma^3 \Rightarrow T_\gamma(t) = T_{\gamma,0}\frac{R_0}{R(t)}$$

$$\rho_\gamma \propto T_\gamma^4 \Rightarrow \rho_\gamma = \rho_{\gamma,0}\left[\frac{R_0}{R(t)}\right]^4$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho + \frac{\Lambda}{3} - \frac{Kc^2}{R^2}$$

$$\Rightarrow \left(\frac{\dot{R}}{R}\right)^2 \approx \frac{8\pi}{3}G\rho_{\rm rel}$$

entropy conservation $\Rightarrow$ evolution of $\rho_{\rm rel}$ at $100 > T > 1$ MeV

$$TS = E + PV - \mu N \Rightarrow S = \frac{E + PV - \mu N}{T}$$

fully relativistic: $S_{\rm rel} = \dfrac{\rho_{\rm rel} V + (\rho_{\rm rel}/3)V}{T} \propto g_{\rm eff} T(t)^3 R(t)^3$

$$g_{\rm eff} = \text{const.} \Rightarrow T(t) \propto R(t)^{-1},\ \dot{T}/T = -\dot{R}/R$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{\dot{T}}{T}\right)^2 = \frac{8\pi}{3} G \rho_{\rm rel} = \left(\frac{8\pi}{3} G\right) g_{\rm eff} \frac{\pi^2}{15} T^4$$

$$T \to \infty \text{ as } t \to 0 \Rightarrow \frac{\dot{T}}{T} = -\sqrt{\frac{8\pi^3}{45} g_{\rm eff} G T^4}$$

$$t \approx \frac{1}{2} \sqrt{\frac{45}{8\pi^3}} \frac{1}{\sqrt{g_{\rm eff} G}} \frac{1}{T^2} = \frac{1.71}{\sqrt{g_{\rm eff}}} \left(\frac{\rm MeV}{T}\right)^2 \ \text{s}$$

$$N_\nu = 3 \Rightarrow g_{\rm eff} = \frac{43}{8},\ t \approx 0.74 \left(\frac{\rm MeV}{T}\right)^2 \ \text{s}$$

BBN and Neutrinos

freeze-out of n/p: $\nu_e + n \rightleftharpoons p + e^-, \;\; \bar{\nu}_e + p \rightleftharpoons n + e^+$

$$\sigma_{\nu_e n} \approx \frac{G_F^2}{\pi} \cos^2\theta_C (f^2 + 3g^2)(E_{\nu_e} + \Delta)^2$$

$$\sigma_{\bar{\nu}_e p} \approx \frac{G_F^2}{\pi} \cos^2\theta_C (f^2 + 3g^2)(E_{\bar{\nu}_e} - \Delta)^2$$

$$\cos^2\theta_C = 0.95, \;\; f = 1, \;\; g = 1.26, \;\; \Delta = M_n - M_p = 1.293 \text{ MeV}$$

$$\text{rate per nucleon: } \lambda_{\nu N} \approx \frac{4\pi}{(2\pi)^3} \int_0^\infty \frac{\sigma_{\nu N} E_\nu^2}{\exp(E_\nu/T) + 1} dE_\nu \approx 0.4 \left(\frac{T}{\text{MeV}}\right)^5 \text{s}^{-1}$$

$$\int_{t_{\text{FO}}}^\infty \lambda_{\nu N} dt \sim \int_0^{T_{\text{FO}}} 0.4 \left(\frac{T}{\text{MeV}}\right)^5 \times 2 \times 0.74 \left(\frac{\text{MeV}}{T}\right)^3 dT \sim 0.2 \left(\frac{T_{\text{FO}}}{\text{MeV}}\right)^3 \sim 1 \Rightarrow T_{\text{FO}} \sim 1.7 \text{ MeV}$$

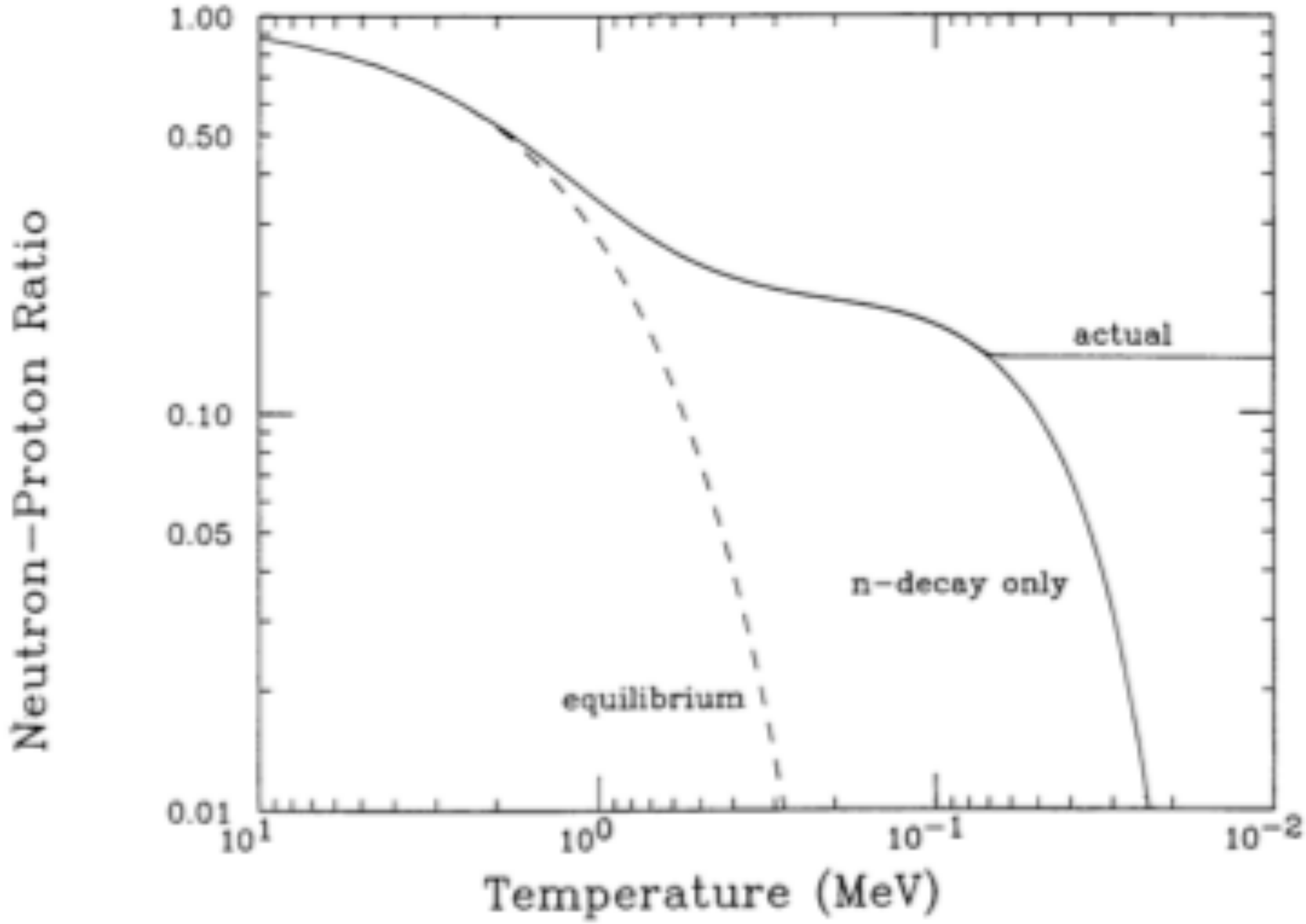
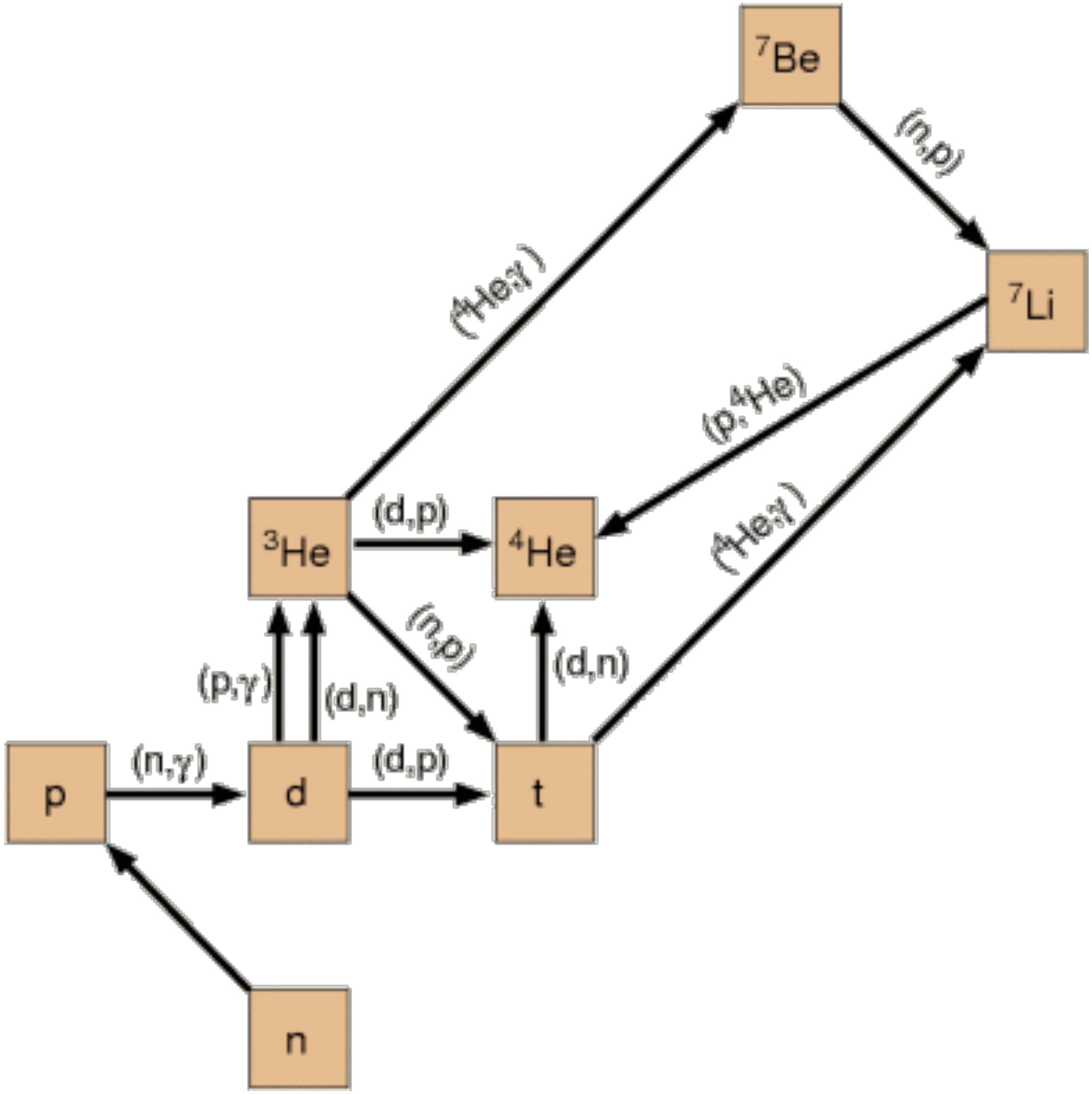
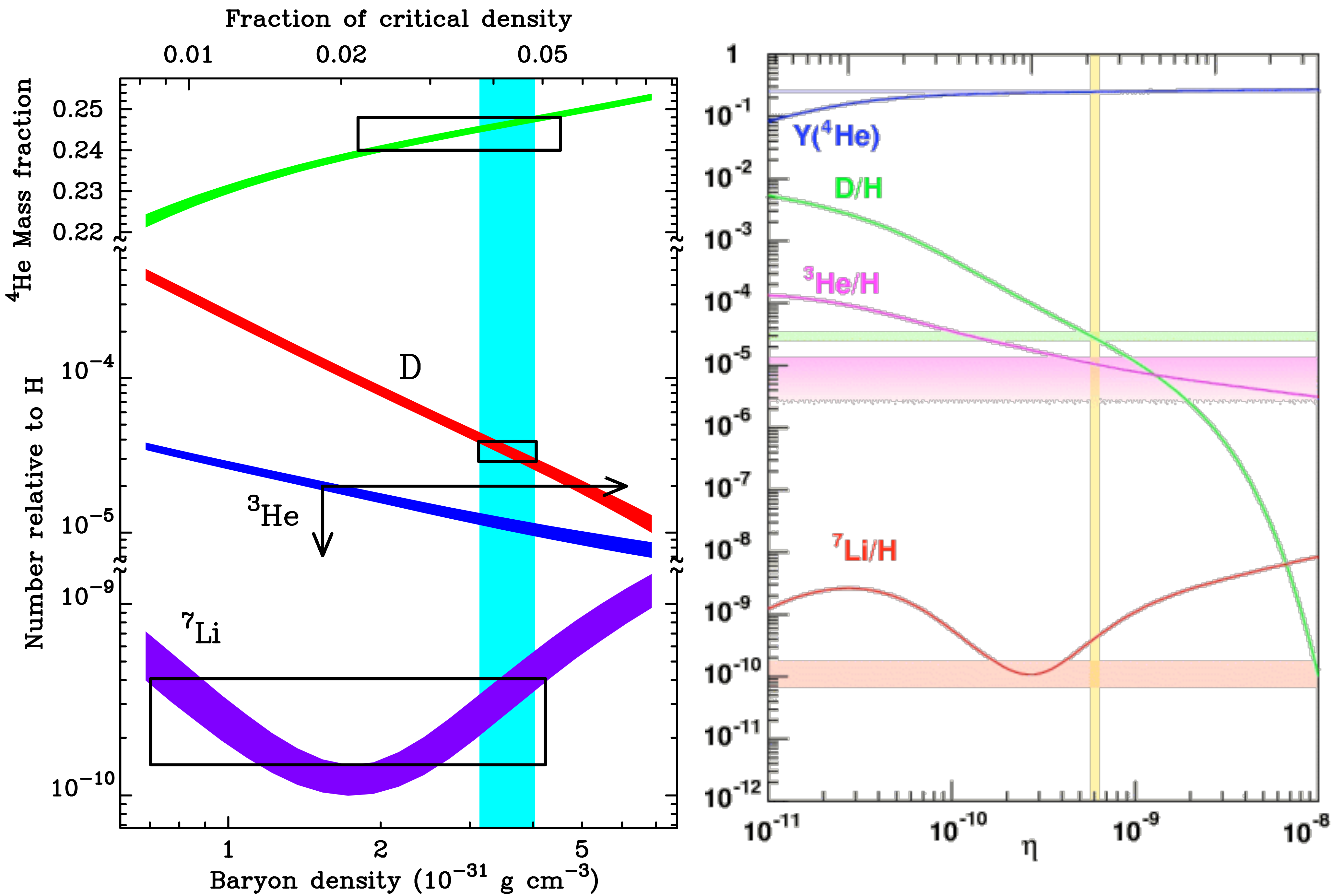


FIG. 1.—Evolution of the neutron-proton ratio with temperature. The NSE ratio is given by the dashed curve. If neutron decay is the only reaction (all other reactions are shut off), the n/p ratio follows the solid curve. The actual final value of the ratio is shown by the straight horizontal line.

$$\frac{n}{p} = \left(\frac{n}{p}\right)_{\rm FO} \exp\left(-\frac{t - t_{\rm FO}}{\tau_n}\right) \sim \exp\left(-\frac{\Delta}{T_{\rm FO}} - \frac{t - t_{\rm FO}}{\tau_n}\right)$$



⁷Be
(n,p)
⁷Li
(⁴He,γ)
(p,⁴He)
³He
(d,p)
⁴He
(⁴He,γ)
(n,p)
(p,γ)
(d,n)
(d,n)
p
(n,γ)
d
(d,p)
t
n



Fraction of critical density
0.01
0.02
0.05
4He Mass fraction
0.25
0.24
0.23
0.22
Number relative to H
10−4
10−5
10−9
10−10
D
3He
7Li
1
2
5
Baryon density (10−31 g cm−3)
1
10-1
10-2
10-3
10-4
10-5
10-6
10-7
10-8
10-9
10-10
10-11
10-12
Y(4He)
D/H
3He/H
7Li/H
10-11
10-10
10-9
10-8
η

Nuclear Statistical Equilibrium (NSE)

$$Zp + (A-Z)n \rightleftharpoons (Z,A) + \gamma \Rightarrow Z\mu_p + (A-Z)\mu_n = \mu(Z,A)$$

considering excited states of nuclei:

$$\begin{aligned} n(Z,A) &= \sum_i \frac{2J_i+1}{(2\pi)^3} \int_0^\infty \frac{4\pi p^2 dp}{\exp\{[(p^2/2M) + M + E_i - \mu]/T\}} \\ &= G(Z,A) \left(\frac{MT}{2\pi}\right)^{3/2} \exp\left[\frac{\mu(Z,A) - M(Z,A)}{T}\right] \end{aligned}$$

nuclear partition function: $G(Z,A) = \sum_i (2J_i + 1) \exp\left(-\frac{E_i}{T}\right)$

$$\begin{aligned} \Rightarrow X(Z,A) &= X_p^Z X_n^{A-Z} \frac{G(Z,A)}{2^A} A^{5/2} \\ &\times \left(\frac{\rho_b}{M_N}\right)^{A-1} \left(\frac{2\pi}{M_N T}\right)^{3(A-1)/2} \exp\left[\frac{B(Z,A)}{T}\right] \end{aligned}$$

In NSE, no rates are needed to calculate abundances:

$$1 = X_n + X_p + \sum_{(Z,A)} X(Z,A)$$

$$Y_e = X_p + \sum_{(Z,A)} \frac{Z}{A} X(Z,A)$$

$$X(Z,A) = X_p^Z X_n^{A-Z} \frac{G(Z,A)}{2^A} A^{5/2}$$

$$\times \left(\frac{\rho_b}{M_N}\right)^{A-1} \left(\frac{2\pi}{M_N T}\right)^{3(A-1)/2} \exp\left[\frac{B(Z,A)}{T}\right]$$

$$\left[\eta \left(\frac{T}{M_N}\right)^{3/2}\right]^{A-1} \exp\left[\frac{B(Z,A)}{T}\right] \sim 1$$

$\Rightarrow X(Z,A)$ dominates NSE abundances

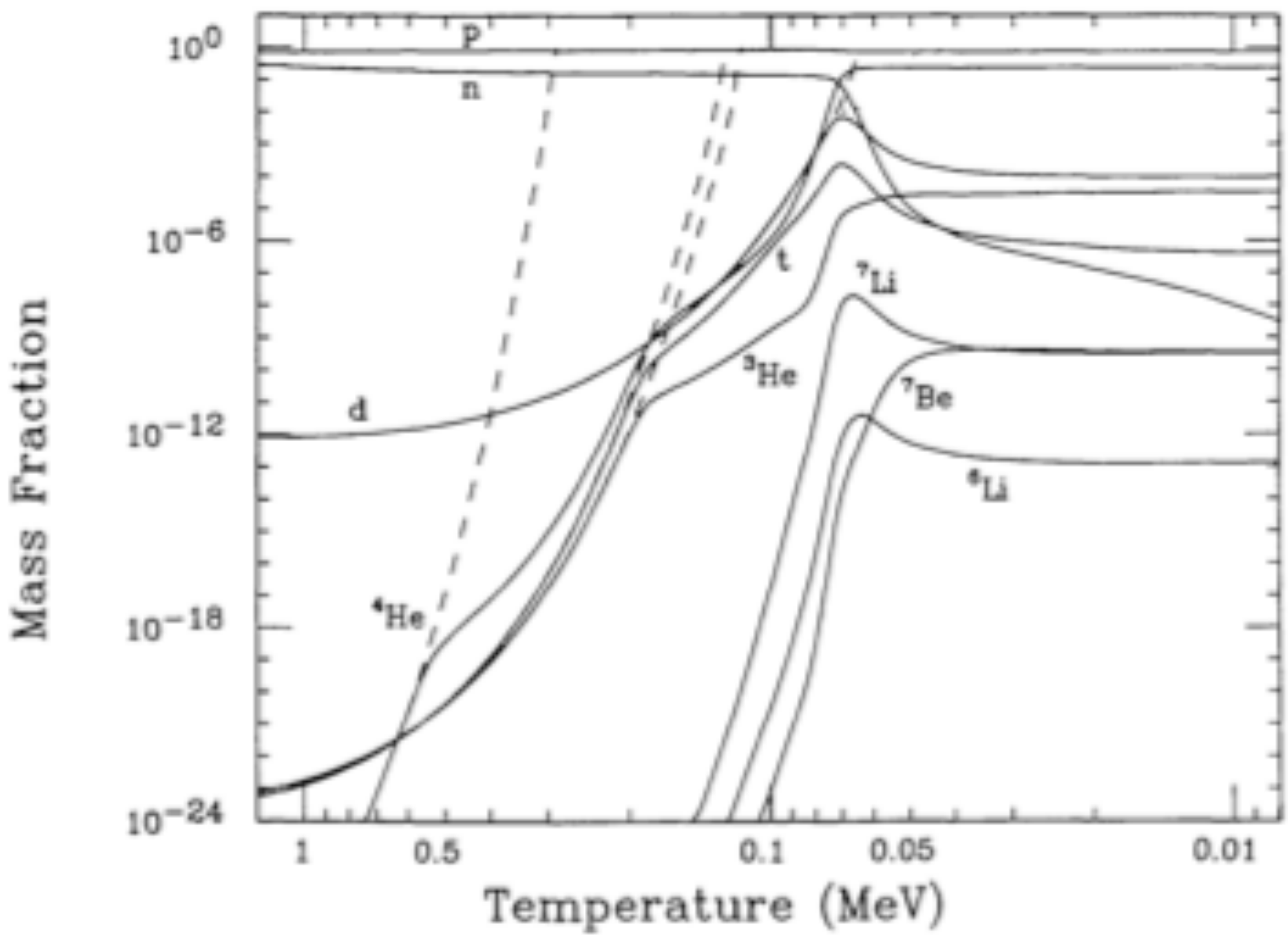


FIG. 2.—Evolution of light-element abundances with temperature, for a baryon-to-photon ratio $\eta_{10} = 3.16$. The dashed curves give the NSE curves of ^{4}He, t, ^{3}He, and d, respectively. The dotted curve is explained in the text.

$$p(n,\gamma)d$$

$$d(d,n)^3\mathrm{He}$$

$$d(d,p)t$$

$$^3\mathrm{He}(d,p)^4\mathrm{He}$$

$$t(d,n)^4\mathrm{He}$$

$$T_{\rm NSE} \sim \frac{B(Z,A)/(A-1)}{\ln \eta^{-1} + (3/2)\ln(M_N/T)}$$

Expansion from high temperature & density

- nuclear statistical equilibrium (NSE)

all strong & electromagnetic reactions in equilibrium

$$(A-Z)n + Zp \rightleftharpoons (Z,A) + \gamma$$

- quasi-statistical equilibrium (QSE)

clusters of nuclei form & reactions involving n, p, & light nuclei in equilibrium within each cluster

$$(n,\gamma), (p,\gamma), (n,p), (\alpha,\gamma), (\alpha,n), (\alpha,p)$$

- hot r-process

QSE within each isotopic chain only

$$(n,\gamma) \rightleftharpoons (\gamma,n) \text{ equilibrium}$$