

Computational Neutrino Flavor (and **Spin**) Astrophysics



HiPACC –
Computational Astrophysics 2014-2012:
Approaching the Exascale

Lawrence Berkeley National Laboratory
Berkeley, CA, March 21, 2014

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SDSC PRESENT

NEUTRINO AND NUCLEAR ASTROPHYSICS

THE 2014 INTERNATIONAL SCHOOL ON ASTROCOMPUTING

JULY 21 - AUGUST 1, 2014

SAN DIEGO SUPERCOMPUTER CENTER
UNIVERSITY OF CALIFORNIA, SAN DIEGO

[HTTP://HIPACC.UCSC.EDU/ISSAC2014.HTML](http://hipacc.ucsc.edu/issac2014.html)

THE INTERPLAY OF FRONTIER RESEARCH IN NEUTRINO PHYSICS, NUCLEOSYNTHESIS, ABUNDANCE OBSERVATIONS, AND HIGH-PERFORMANCE COMPUTING LIES AT THE HEART OF EFFORTS TO UNDERSTAND CORE COLLAPSE SUPERNOVAE, COMPACT OBJECT MERGERS, AND THE MASS ASSEMBLY HISTORY OF GALAXIES. NEW OBSERVATIONS ARE DRIVING EXCITING NEW DEVELOPMENTS IN THESE FIELDS. THIS SCHOOL WILL PROVIDE THE BACKGROUND FOR ADDRESSING THESE ISSUES, INCLUDING USE OF SEVERAL OF THE RELEVANT COMPUTER CODES. THE SCHOOL WILL BE HOSTED AT THE SDSC, WHOSE DATA-INTENSIVE COMPUTING FACILITIES, INCLUDING THE NEW GORDON SUPERCOMPUTER WITH A THIRD OF A PETABYTE OF FLASH STORAGE, ARE AMONG THE BEST IN THE WORLD. ALL STUDENTS AT ISSAC 2014 WILL HAVE ACCOUNTS ON GORDON, AND WILL PARTICIPATE IN HANDS-ON CODE SESSIONS IN THE AFTERNOONS WITH LECTURES IN THE MORNINGS.

DIRECTOR: GEORGE FULLER (UCSD)

SPEAKERS WILL INCLUDE:

MAIN LECTURERS

BAHA BALANTEKIN (UNIVERSITY OF WISCONSIN)
JOE CARLSON (LOS ALAMOS NATIONAL LAB)
HUALYU DUAN (UNIVERSITY OF NEW MEXICO)
ALEX FRIEDLAND (LOS ALAMOS NATIONAL LAB)
DAN KASEN (UC BERKELEY/LAWRENCE BERKELEY LAB)
EVAN KIRBY (UC IRVINE)
TONY MEZZACAPPA (OAK RIDGE NATIONAL LAB)
CHRISTIAN OTT (CALTECH)
YONG-ZHONG QIAN (UNIVERSITY OF MINNESOTA)

ADDITIONAL LECTURERS

JOHN CHERRY (LOS ALAMOS NATIONAL LAB)
VINCENZO CIRIGLIANO (LOS ALAMOS NATIONAL LAB)
GEORGE FULLER (UC SAN DIEGO)
MARK PARIS (LOS ALAMOS NATIONAL LAB)
JOEL PRIMACK (UC SANTA CRUZ)

HOUSING: STUDENTS WILL BE STAYING AT CONFERENCE HOUSING NEAR SDSC ON THE UCSD CAMPUS

THE REGISTRATION FEE FOR ISSAC 2014 WILL BE \$300; PAYMENT WILL BE REQUIRED AT THE TIME OF ACCEPTANCE. UC-HIPACC WILL COVER LODGING FOR ALL STUDENTS, AND SOME FINANCIAL ASSISTANCE MAY BE AVAILABLE FOR TRAVEL EXPENSES.

APPLY BY APRIL 14, 2014, AT THE WEBSITE [HTTP://HIPACC.UCSC.EDU/ISSAC2014.HTML](http://hipacc.ucsc.edu/issac2014.html)



Why are the neutron star merger and core collapse supernova environments so sensitive to neutrino flavor/spin physics ?

In a nutshell:

Core collapse supernovae are cold, highly electron lepton number degenerate systems.

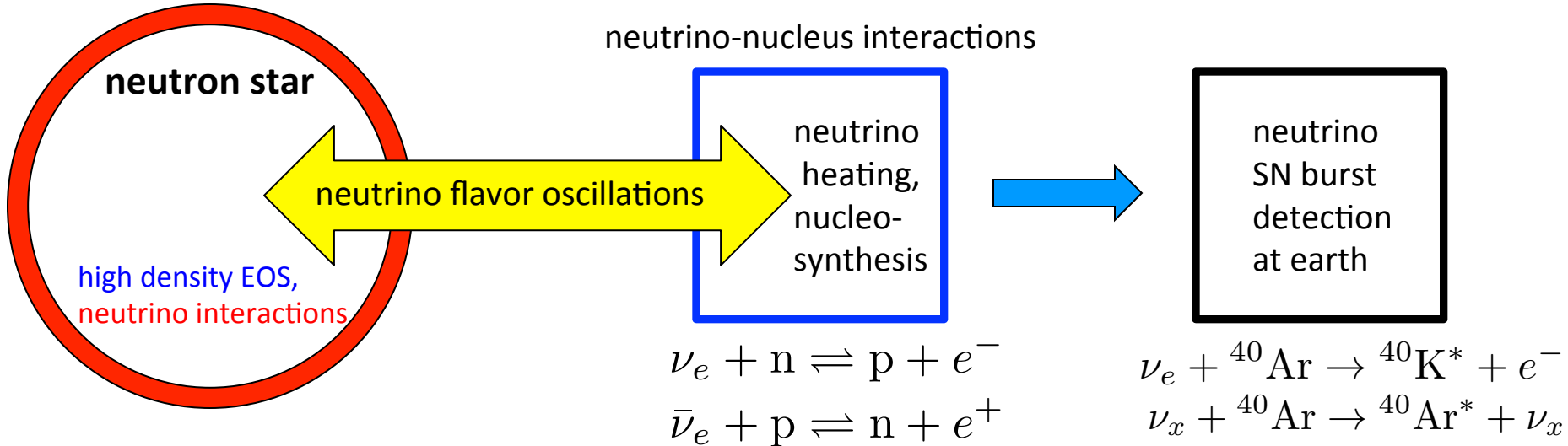
They are *exquisitely sensitive* to lepton number violating processes.

Macroscopic effects in SN physics or signal from:

flavor oscillations: very sensitive to neutrino mass hierarchy;

spin coherence: sensitive to Majorana/Dirac nature of neutrinos
& absolute neutrino masses

Calculating neutrino flavor transformation in the core collapse supernova/merger environment is a vexing problem, but one whose solution may lie at the heart of many aspects of the physics of stellar collapse, nucleosynthesis, and the n signal.



We need the fluxes and energy spectra of each flavor/type of neutrino at all epochs and at all radii.

The state of collapse/merger modeling

simulations of core collapse supernovae are very sophisticated:
multi-dimensional radiation hydrodynamics;
Boltzmann neutrino transport, and *detailed microphysics/EOS* . . .

Our understanding of the effects of nonzero neutrino mass (flavor oscillations; spin flip), though numerically sophisticated, is *crude*, and difficult to incorporate into the SN simulations, probably *even at the exascale*.

There are *unsettled issues* in the story of supernova neutrinos.

One thing we can say for certain is that the current collapse/merger simulations **leave out** many features of neutrino mass and flavor/spin physics *that we know are there*

We must follow *quantum mechanical phases* and *high frequency complex amplitudes* if we want to compute neutrino mass/flux/spin physics.

This is an essential difference between neutrino flux/spin astrophysical simulations and conventional radiation hydrodynamics treatments.

The main take-away message from the experiments:

Neutrino energy (mass) eigenstates $|\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$

are not coincident with the weak interaction (flavor) eigenstates

$|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle$

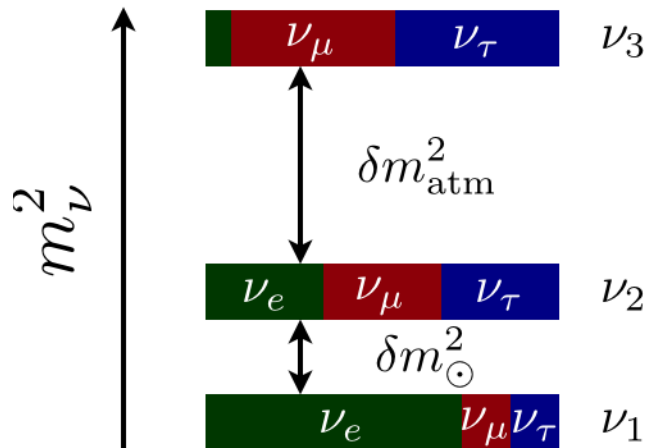
Neutrino Mass: what we know and don't know

We know the *mass-squared* differences: $\left\{ \begin{array}{l} \delta m_{\odot}^2 \approx 7.6 \times 10^{-5} \text{ eV}^2 \\ \delta m_{\text{atm}}^2 \approx 2.4 \times 10^{-3} \text{ eV}^2 \end{array} \right.$

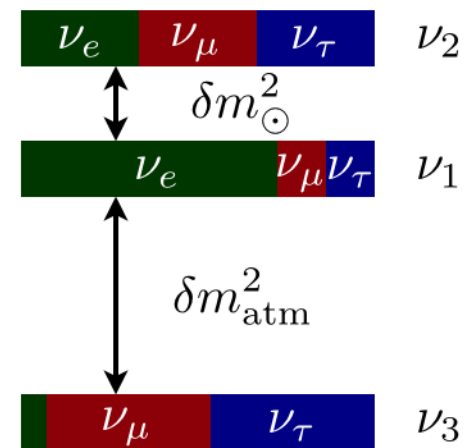
e.g., $\delta m_{21}^2 \equiv m_2^2 - m_1^2$

We *do not* know the *absolute masses* or the *mass hierarchy*:

normal mass hierarchy



inverted mass hierarchy



P-Maki-Nakagawa-Sakata matrix

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U_m \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

$$U_m = U_{23} U_{13} U_{12} M$$

$$U_{23} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix}$$

$$U_{13} \equiv \begin{pmatrix} \cos \theta_{13} & 0 & e^{i\delta} \sin \theta_{13} \\ 0 & 1 & 0 \\ -e^{-i\delta} \sin \theta_{13} & 0 & \cos \theta_{13} \end{pmatrix}$$

$$U_{12} \equiv \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4 parameters

$\theta_{12}, \theta_{23}, \theta_{13}, \delta$

$$\theta_{12} \approx 0.59^{+0.02}_{-0.015}$$

$$\theta_{23} \approx 0.785^{+0.124}_{-0.124} \approx \frac{\pi}{4}$$

$$\theta_{13} \approx 0.154^{+0.065}_{-0.065}$$

$\delta = CP$ violating phase = ?

in medium it's a different story . . .

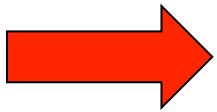
neutrinos can scatter on *any* particles that carry weak charge, including *other neutrinos*, and this generates potentials that can make the neutrinos change flavors

like photons acquire an index of refraction when traveling through glass

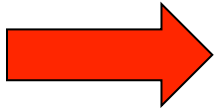
But, unlike for photons . . .

Potentials that govern how a neutrino changes its flavor depend on the flavor states of neutrino: **NONLINEAR**

How Quantum Mechanical Systems Evolve – The Rules



when you make a measurement you have to get an eigenvalue and system is “**collapsed**” into the corresponding eigenstate



two ways system can evolve in time:

Schroedinger-like evolution **SMOOTH/Continuous**

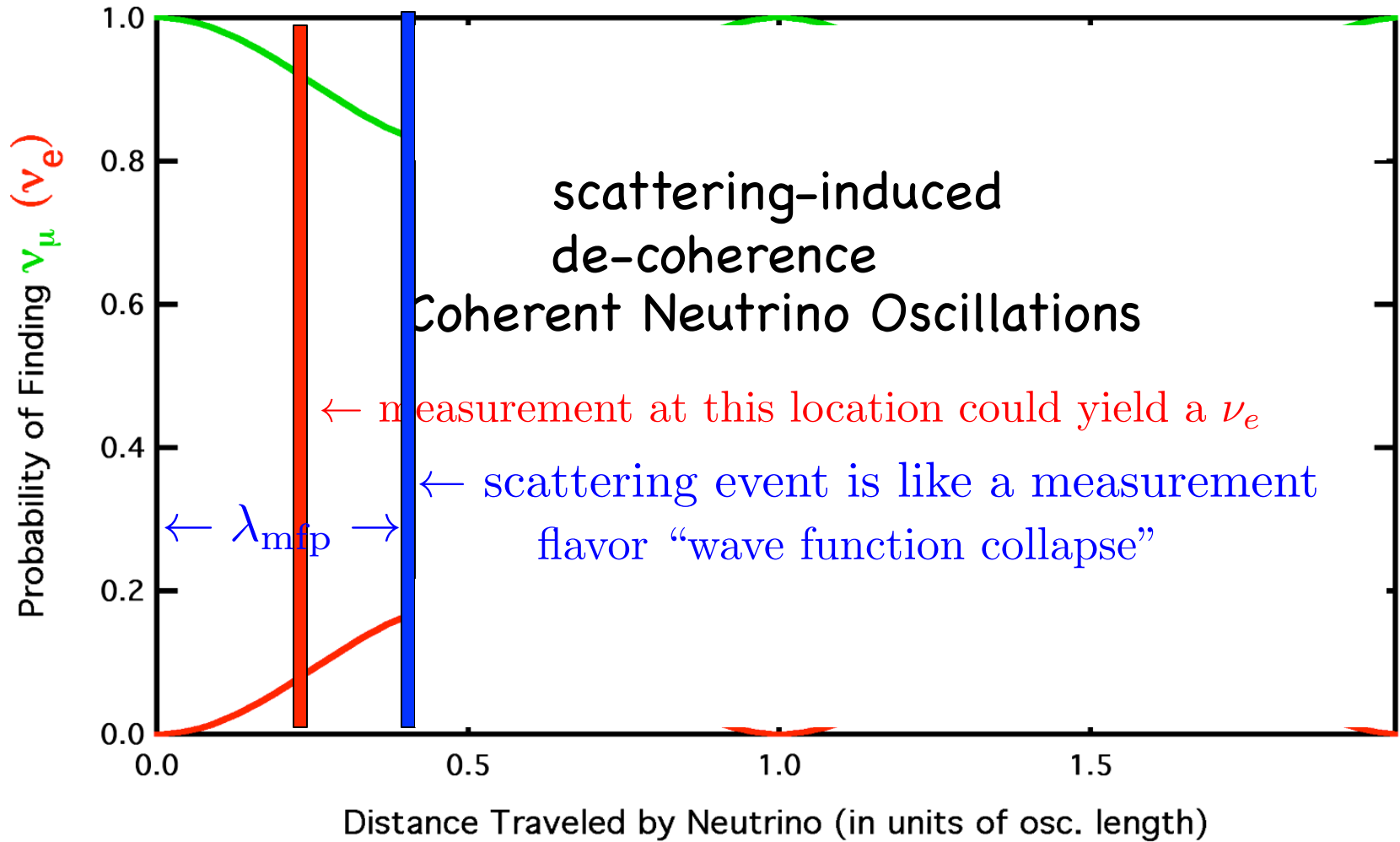
state reduction (“wave function collapse”) because of a “measurement”
ABRUPT (scattering can be like a “measurement”)

**The neutrino flavor problem in SN/Mergers
forces us to seek a unified treatment
of these time evolution modes**

Simple Example: two-by-two vacuum neutrino oscillations

$$|\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle$$



$$|\Psi(t=0)\rangle = |\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle$$

$$|\Psi(t)\rangle = -\sin \theta e^{-iE_1 t} |\nu_1\rangle + \cos \theta e^{-iE_2 t} |\nu_2\rangle$$

Quantum Kinetic Equations

$$i D \hat{f} - [\hat{\mathcal{H}}, \hat{f}] - \hat{U} [\hat{\phi}] = \text{collision terms} (\hat{f}, \hat{f})$$

where \hat{f} and \hat{f} are 3×3 Hermitian density operators for neutrinos and antineutrinos, respectively, and $\hat{\phi}$ is a 3×3 complex matrix encoding spin coherence.

and where $\hat{\mathcal{H}}$ & \hat{U} give neutrino interactions with matter and other neutrinos

separation of scales ??

Schroedinger-like:

$$i \frac{\partial |\Psi\rangle}{\partial t} = \hat{H} |\Psi\rangle \text{ with } |\Psi\rangle = (\psi_e, \psi_\mu, \psi_\tau)$$

@ “low” density where
neutrinos propagate coherently

Boltzmann equation

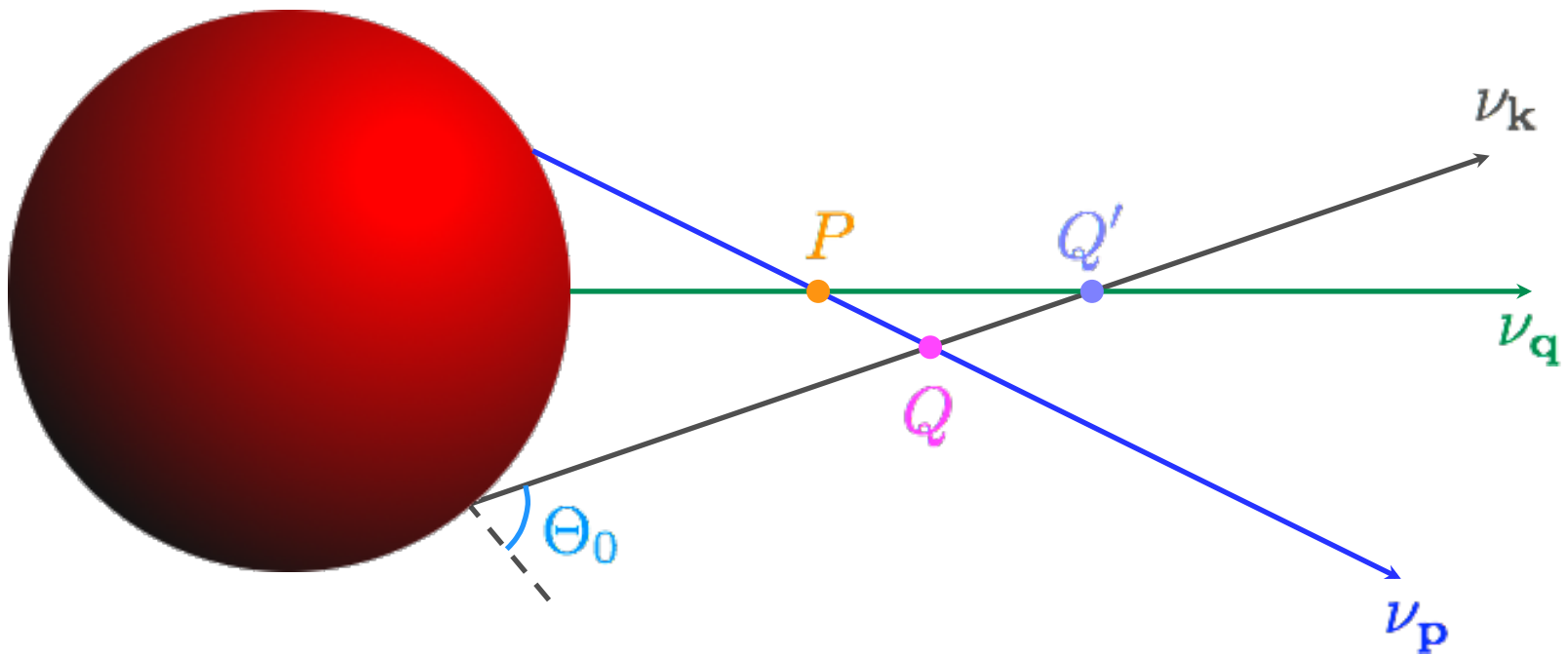
@ “high” density where
inelastic scattering dominates

The advent of supercomputers has allowed us to follow neutrino flavor transformation in core collapse supernovae, including the first self-consistent treatment of **nonlinearity** stemming from neutrino-neutrino forward scattering.

The results are startling. Despite the small measured neutrino mass-squared differences, **collective** neutrino flavor transformation can take place deep in the supernova envelope

Pushing the frontier of high performance computing with a unique new kind of transport problem

- Anisotropic, nonlinear quantum coupling of all neutrino flavor evolution histories

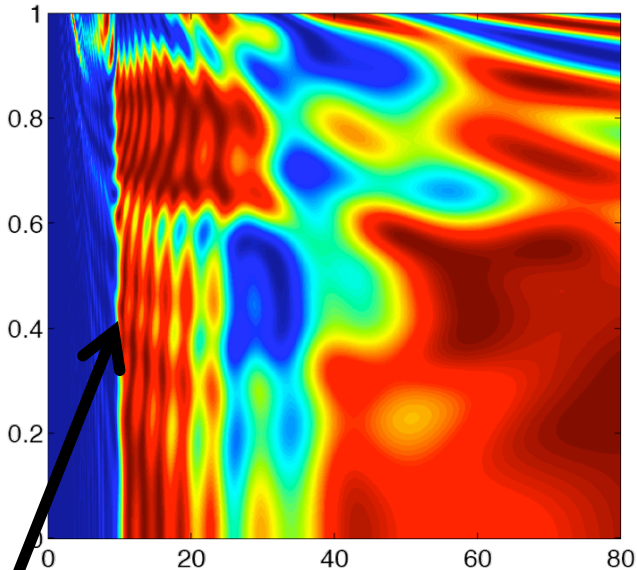


Must solve many *millions* of coupled, nonlinear partial differential equations!!

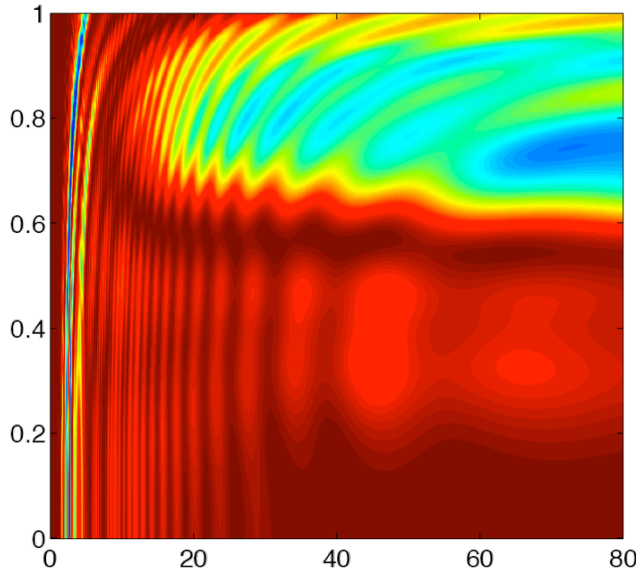
cosine of θ trajectory angle wrt. normal to n.s. surface

$\cos \vartheta_0$

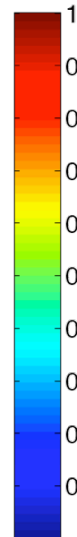
$$P_{\nu_\alpha \rightarrow \nu_\alpha}$$



$$P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha}$$

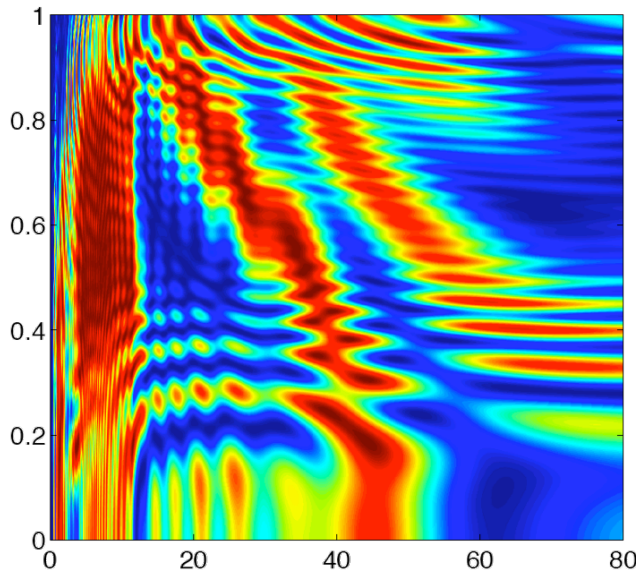
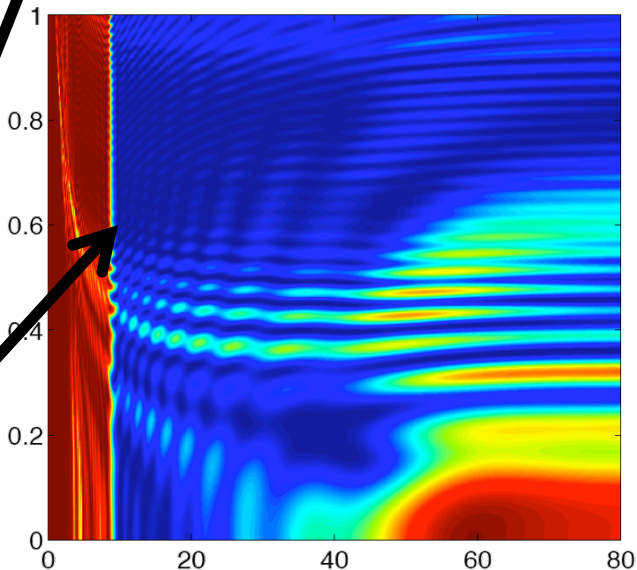


survival probability



normal mass hierarchy
 $\delta m^2 > 0$

inverted mass hierarchy
 $\delta m^2 < 0$



E_ν (MeV)

Spectral Swap

consequences of neutrino mass and quantum coherence in supernovae

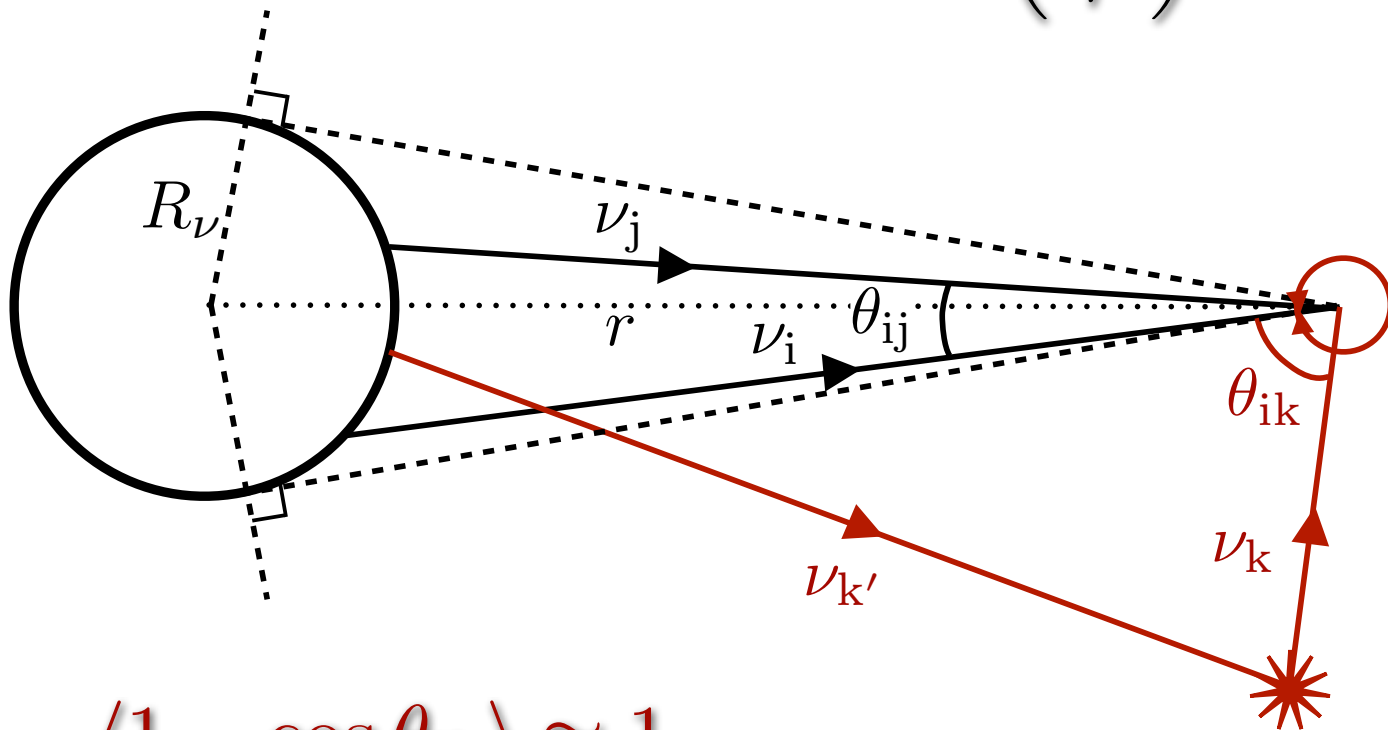
H. Duan, G. M. Fuller, J. Carlson, Y.-Z. Qian, Phys. Rev. Lett. 97, 241101 (2006) astro-ph/0606616

Toward Quantum Kinetics

- (a) Effects of a small amount of direction-changing scattering on the neutrino flavor transformation? – The “Halo”
- (b) *Spin Coherence*: neutrino-antineutrino inter-conversion

The Neutrino Halo

$$r \gg R_\nu \Rightarrow \langle 1 - \cos \theta_{ij} \rangle \propto \left(\frac{R_\nu}{r} \right)^2$$

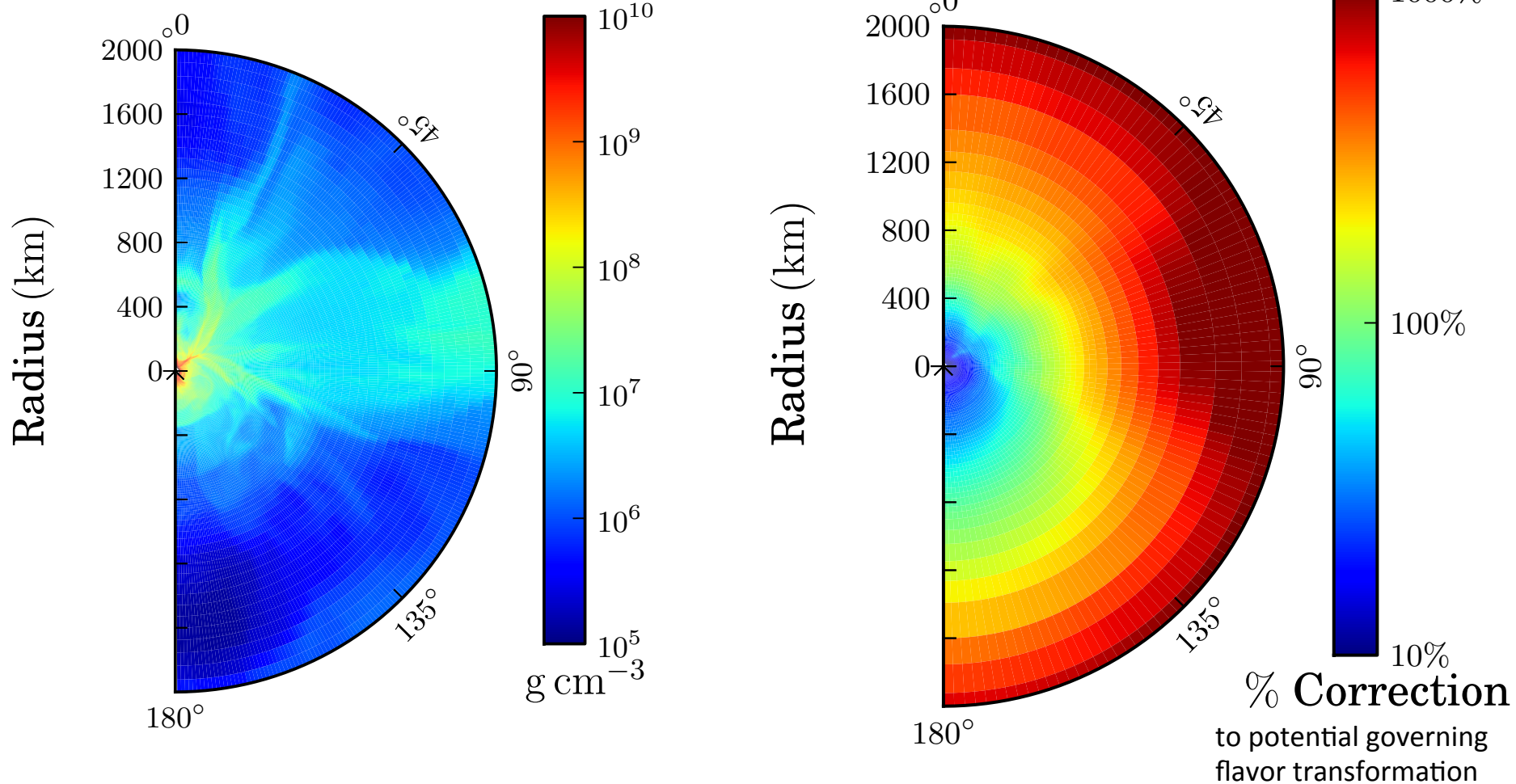


$$\langle 1 - \cos \theta_{ik} \rangle \approx 1$$

$\sim 10^{-3}$ of all ν 's

How large is the Halo effect for free nucleons?

$$\sigma_{\text{coherent}} \propto A^2 \Rightarrow \mathcal{H}_{\text{halo}} \propto \langle A \rangle$$



the **Halo** of scattered neutrinos converts the coherent neutrino flavor evolution problem from an *initial value problem* into a *boundary value problem*

(quantum flavor information *coming down* from outer regions of star)

and moreover couples in nuclear composition in a completely new way

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neutrino flavor evolution problem
from an *initial value problem* into
a *boundary value problem*

(quantum flavor information *coming down* from outer regions of star)

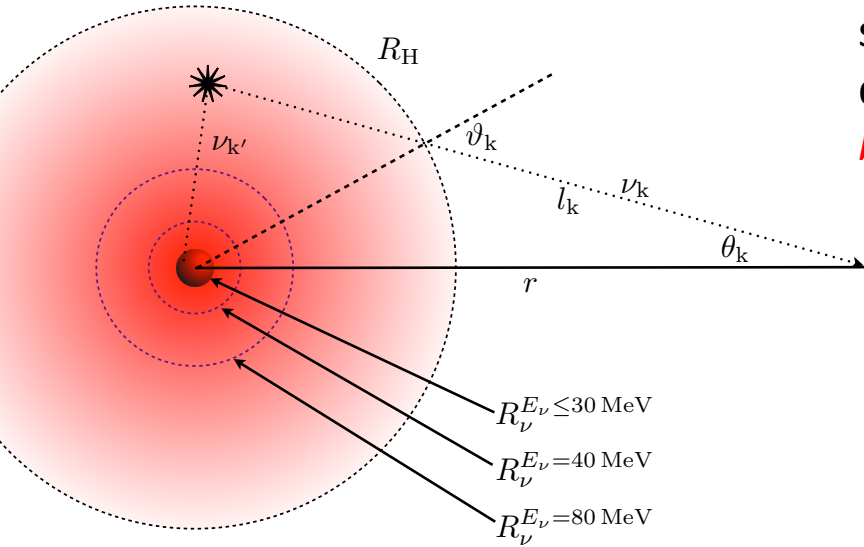
**and moreover couples in nuclear composition
in a completely new way**

stability analyses suggest little effect from Halo during shock re-heating/accretion phase

(S. Sarikas, I. Tamborra, G. Raffelt, L. Hudepohl, H.T. Janka PRD **85**, 113007 (2012) 1204.0971;

A. Mirizzi & P.D. Serpico, PRD **86**, 085010 (2012) 1208.0157) – **But these studies leave out much of the halo
and do not capture the composition/inhomogeneous effects**

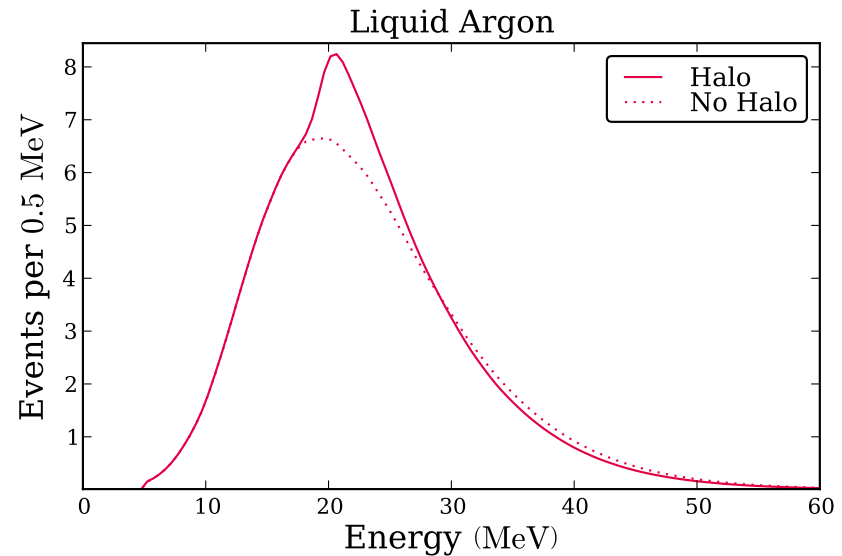
O-Ne-Mg Core Collapse – *very centrally-condensed*, so we *can* model the Halo with our initial value code: quantum mechanical information all coming from *below* region of collective oscillations!



Dispersion/de-coherence in Halo causes neutrino trajectory-dependent swap energy, which could have consequences for a detected neutrino signal

With Halo fewer high energy ν_e 's are transformed

\Rightarrow more ν_e -induced events in detector



Quantum Kinetic Equations

A. Vlasenko, G.M.F., V. Cirigliano (2013), arXiv:1309.2628

$$i\mathcal{D}[\mathcal{F}] - [\mathcal{H}, \mathcal{F}] - (\Delta\mathcal{H}\mathcal{F}_\phi - \mathcal{F}_\phi\Delta\mathcal{H}^\dagger) = i\mathcal{C}[\mathcal{F}]$$

$$i\mathcal{D}[\mathcal{F}] - [\mathcal{H}, \mathcal{F}] \approx i\mathcal{C}[\mathcal{F}] \quad \text{a 6X6 matrix formulation}$$

$$\mathcal{F} = \begin{bmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{bmatrix} \quad \begin{array}{l} f(x,p) \text{ and } \bar{f}(x,p) \text{ are neutrino/antineutrino} \\ \text{density operators, so they are } 3 \times 3 \text{ matrices} \Rightarrow \end{array} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Here ϕ is a **Coherent limit** quantity

encoding neutrino spin helicity low densities \neq potentials small, $\Sigma \ll m$,

collision term smaller still, so drop Σ^2 and Boltzmann limit

Collision terms \Rightarrow $\mathcal{C} = \begin{bmatrix} C & C_\phi \\ C_\phi^\dagger & C^T \end{bmatrix}$ Then ϕ decouples and we have

neutrino-antineutrino transformation

$$i \frac{p^\mu}{E} \partial_\mu f - [H, f] = 0 \quad \text{with } H = \Sigma^\kappa \frac{m^\dagger m}{2E} \quad \text{collision terms mix different energy \& flavor states, averaging out the off-diagonal terms, so } f \text{ diagonal}$$

and the Hamiltonian is

$$i \frac{p^\mu}{E} \partial_\mu \bar{f} - [\bar{H}, \bar{f}] = 0 \quad \text{with } \bar{H} = \Sigma^\kappa \frac{m^\dagger m}{2E}$$

with $H_{\nu\bar{\nu}} = \frac{1}{|\vec{p}|} \left(\Sigma^+ m^* + m^* \Sigma^{+T} \right) \Rightarrow$ no coherence $[f, H] = 0 \Rightarrow$ particles must have mass

This is the Schrödinger Equation for the wave functions ψ given earlier, but must be careful because of nonlinearity with components Σ to neutrino momentum

orthogonal to neutrino trajectory p^μ

$$\Rightarrow \frac{p^\mu}{E} \partial_\mu f_\alpha = \Pi_\alpha^+ (f_\alpha) - \Pi_\alpha^- f_\alpha \quad \text{with } \alpha = e, \mu, \tau$$

Π functions \Rightarrow usual Boltzmann gain-loss terms (depend on matter and ν densities)

Neutrino-Antineutrino inter-conversion

interesting analogy to Majorana neutrino spin precession in a *real magnetic field*

A. de Gouvea & S. Shalgar [arXiv:1301.5637](#) showed that *standard model neutrino transition magnetic moment* ($\sim 10^{-24}$ Bohr magnetons) could engender *collective* neutrino-antineutrino oscillations – require $\sim 10^{12}$ Gauss fields

similar process with **QKE spin coherence**, but no magnetic field required
--- sensitive to Majorana/Dirac nature of neutrinos, absolute mass

neutrino-antineutrino conversion

potentially very important for nucleosynthesis
because the relative mix of neutrinos and antineutrinos
determines neutron-to-proton ratio