Self-Interacting Dark Matter



Miguel Rocha - UC Irvine

Santa Cruz Galaxy Workshop 08/14/2013

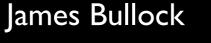




In Collaboration With

Annika Peter











Manoj Kaplinghat Shea Garrison-Kimmel Jose Onorbe

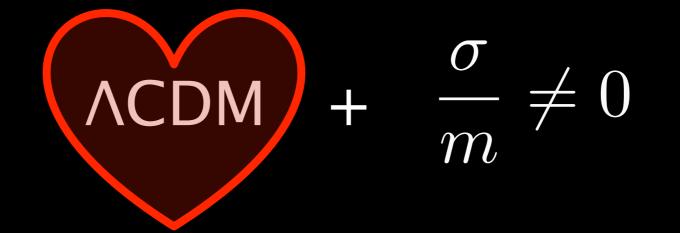




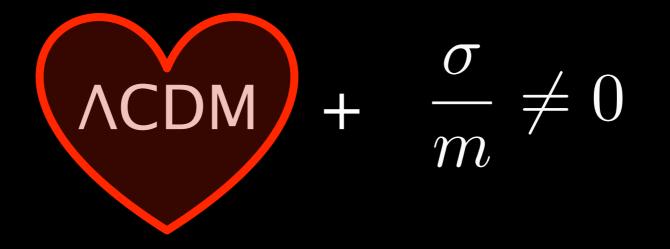
Oliver Elbert



The SIDM model



The SIDM model



Interesting phenomenology if

$$\frac{\sigma}{m} = 0.1 - 100 \text{ cm}^2/\text{g}$$
Spergerl & Steinhardt 2000

The SIDM model

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Interesting phenomenology if

 $\sigma/m = 1 \text{ cm}^2/g = 2 \text{ barn/Gev} = \text{neutron-proton scattering}$

Spergerl & Steinhardt 2000

For most astronomers it is exotic

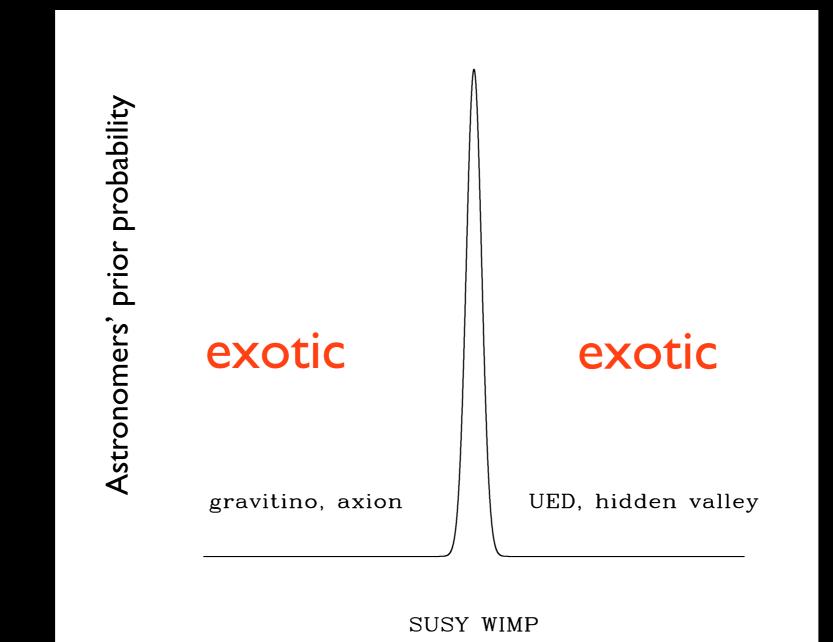


Image courtesy of Annika Petter

For physicist it is a generic consequence of many models beyond the Standard Model Just add a very force of the standard Consequence of $m_{\phi} = O(\text{Mev})$

Х | ф | X

If your prejudice is that new physics can only be at O(TeV), then this large cross-sections will seem crazy

For physicist it is a generic consequence of many models beyond the Standard Model

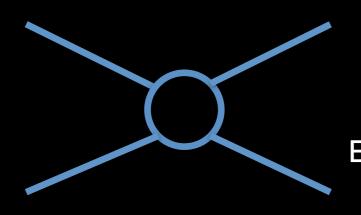
Examples:

Asymmetric DM - Nussinov (1985);Kaplan (1992);Kaplan,Luty, Zurek (2009); Shelton, Zurek (2011); Buckley, Randall (2011); Morrissey, Sigurdson, Tulin (2010); Buckley (2011); Lin, Hai-Bo Yu, Zurek (2011).

Hidden Charge DM - Feng, Tu, Hai-Bo Yu (2008); Ackerman, Buckley, Carroll, Kamionkowski (2008); Feng, Kaplinghat, Tu, HBY (2009).

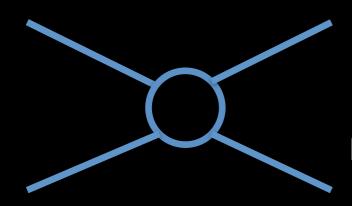
Atomic DM – Foot (2003); Kaplan, Krnjaic, Rehermann, Wells (2009); Feng, Kaplinghat, Tu, Hai-Bo Yu (2009); Cline, Liu, Wei Xue (2012); Francis-Yan Cyr-Racine, Kris Sigurdson (2013).

Double Disk DM - Fan, Katz, Randall, Reece (2013); McCullough, Randall (2013)



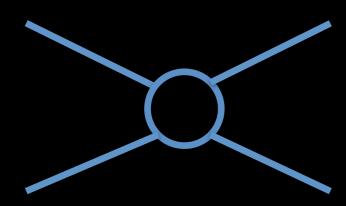
Spergerl & Steinhardt 2000

 $\rightarrow \Gamma \sim \rho \left(\frac{\sigma}{m}\right) v_{rel}$ Elastic - Velocity Independent - Isotropic



Elastic - Velocity Independent - Isotropic $\Gamma \sim \rho\left(\frac{\sigma}{m}\right) v_{rel}$

$\Gamma/H_0 \gtrsim 1$ (central regions of DM halos) Where

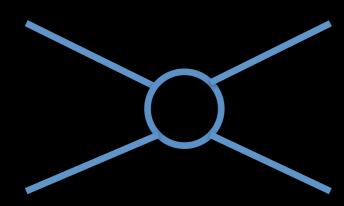


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The energy transfer results in isothermal low density cores

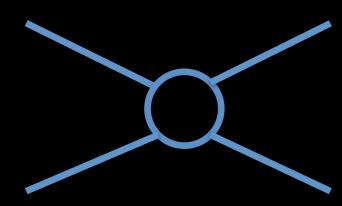


Spergerl & Steinhardt 2000

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$\Gamma/H_0 \gtrsim 1$ (central regions of DM halos) Where

- The energy transfer results in isothermal low density cores
- The isotropic scattering produces near-spherical cores

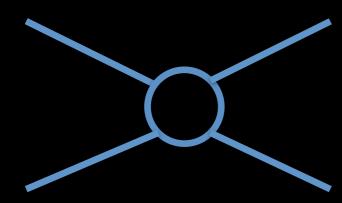


Spergerl & Steinhardt 2000

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Where $\Gamma/H_0 \gtrsim 1$ (central regions of DM halos)

- The energy transfer results in isothermal low density cores
- The isotropic scattering produces near-spherical cores
- The hot dense medium results in substructure evaporation



Spergerl & Steinhardt 2000 $\Gamma\sim\rho\left(\frac{\sigma}{m}\right)v_{rel}$ Elastic - Velocity Independent - Isotropic

Where $\Gamma/H_0 \gtrsim 1$ (central regions of DM halos)

- The energy transfer results in isothermal low density cores
- The isotropic scattering produces near-spherical cores
- The hot dense medium results in substructure evaporation
- In merging systems the drag that the DM experiences would be different to that of the collisionless galaxies, resulting in an offset between the surface mass centroids and the galaxy centroids + lower M/L ratios

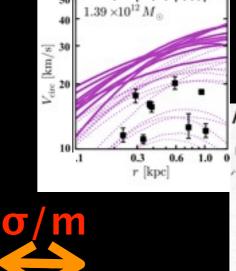
Astrophysical Constraints

Predictions

Core sizes & densities
Shapes

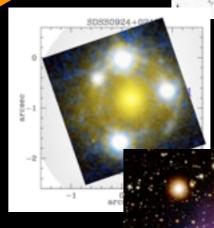
•Substructure

• Merging clusters: offsets & M/L ratios Most reliable predictions from cosmological simulations

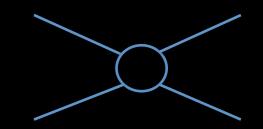


Observations

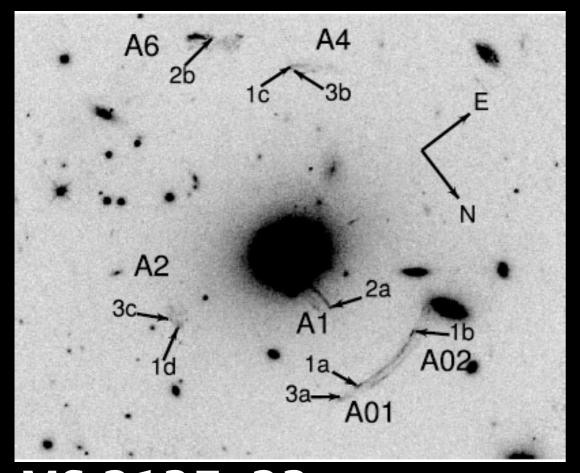
σ/m



The TKO of SIDM



Miralda-Escude (2002)



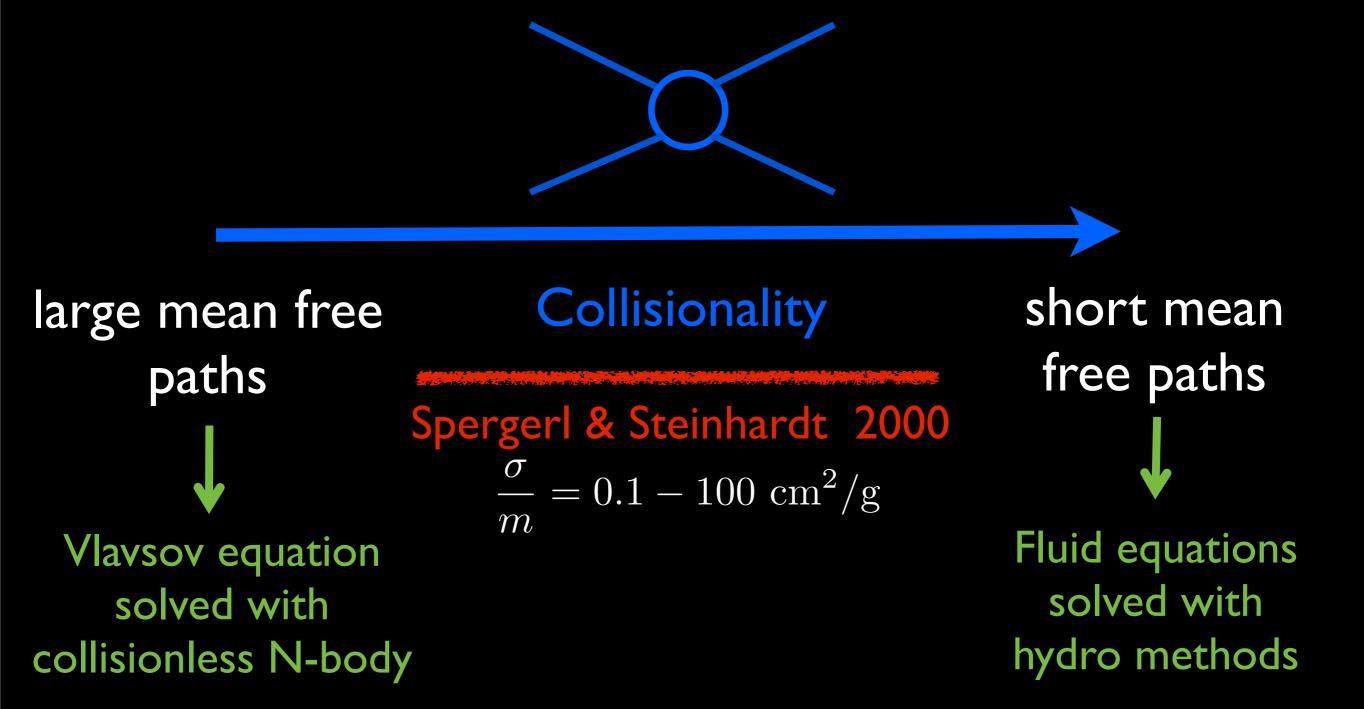
Requires a non-circularly-symmetric surface density at r > 70 kpc.

Assume $\epsilon = 0$ if $\Gamma/H_0 \gtrsim 1$ $\Rightarrow \sigma/m < 0.02 \text{ cm}^2/\text{g}.$

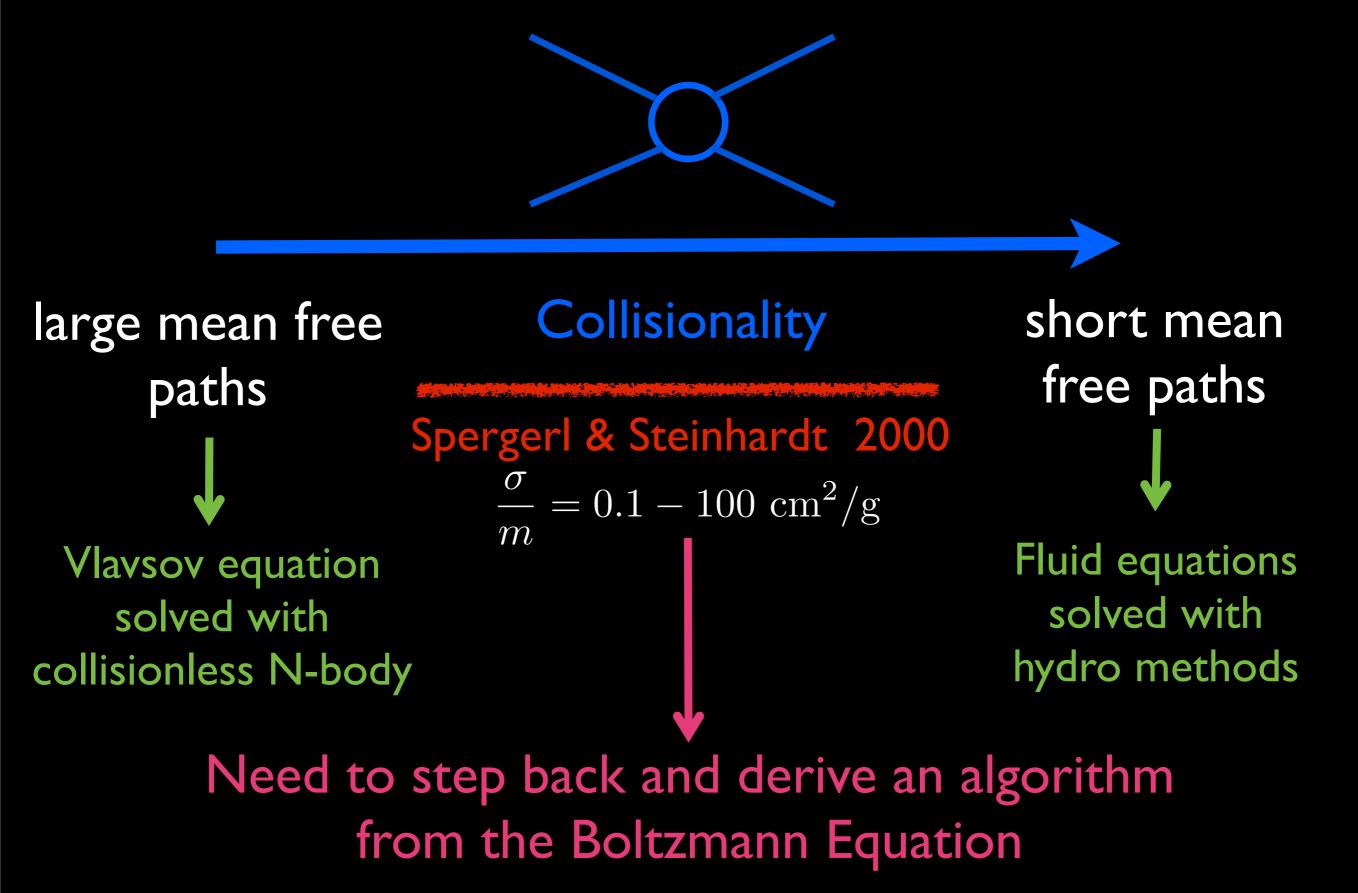
MS 2137-23 Sand et al. 2008

Tightest constraint by far (by > 10x)!

Simulating DM Self-Interactions



Simulating DM Self-Interactions



Lower central phase-space density in SIDM halos

٨CDM

200 Kpc/h

 $\frac{\text{ASIDM}}{\sigma/m = 1 \text{ cm}^2 \text{ /g}}$

Results from cosmological simulations - Summary

Core Sizes & Central Densities

See also Vogelsberger et al. 2012 & Zavala et al. 2013

σ/m < 1

 $\sigma/m = 0.1-0.5$ in better agreement with observations

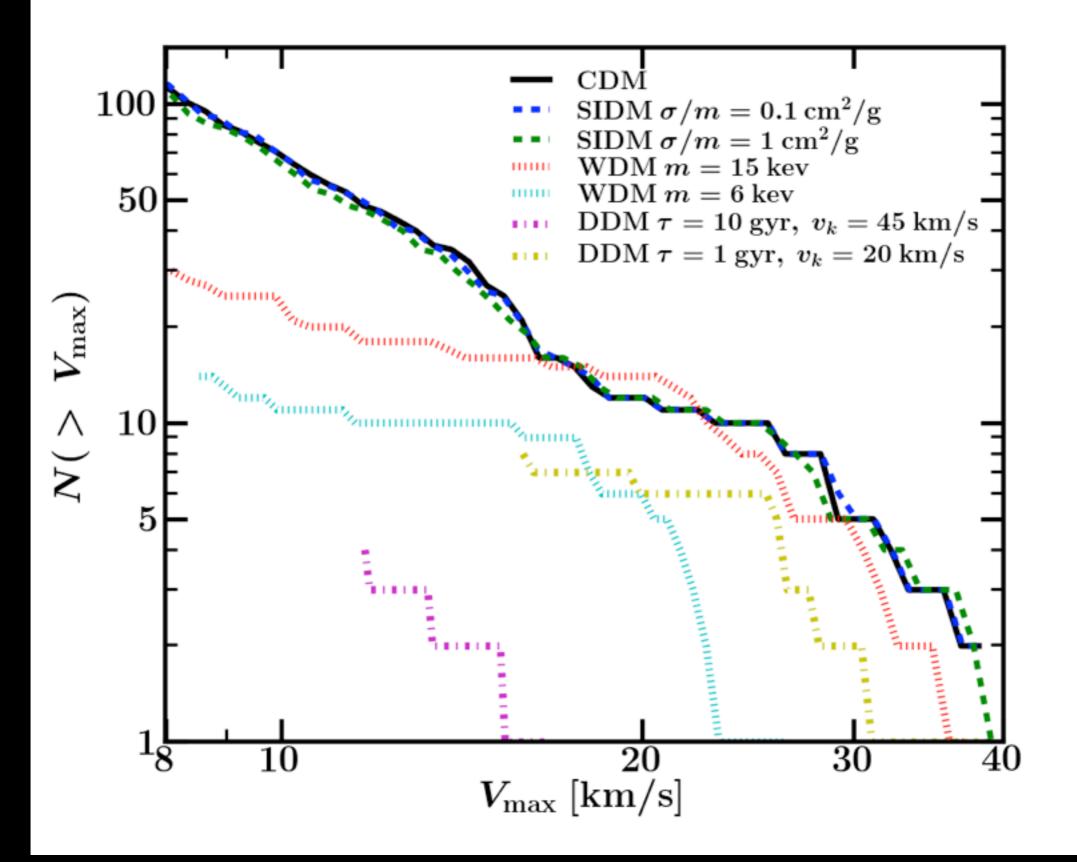
Shapes

σ/m < Ι

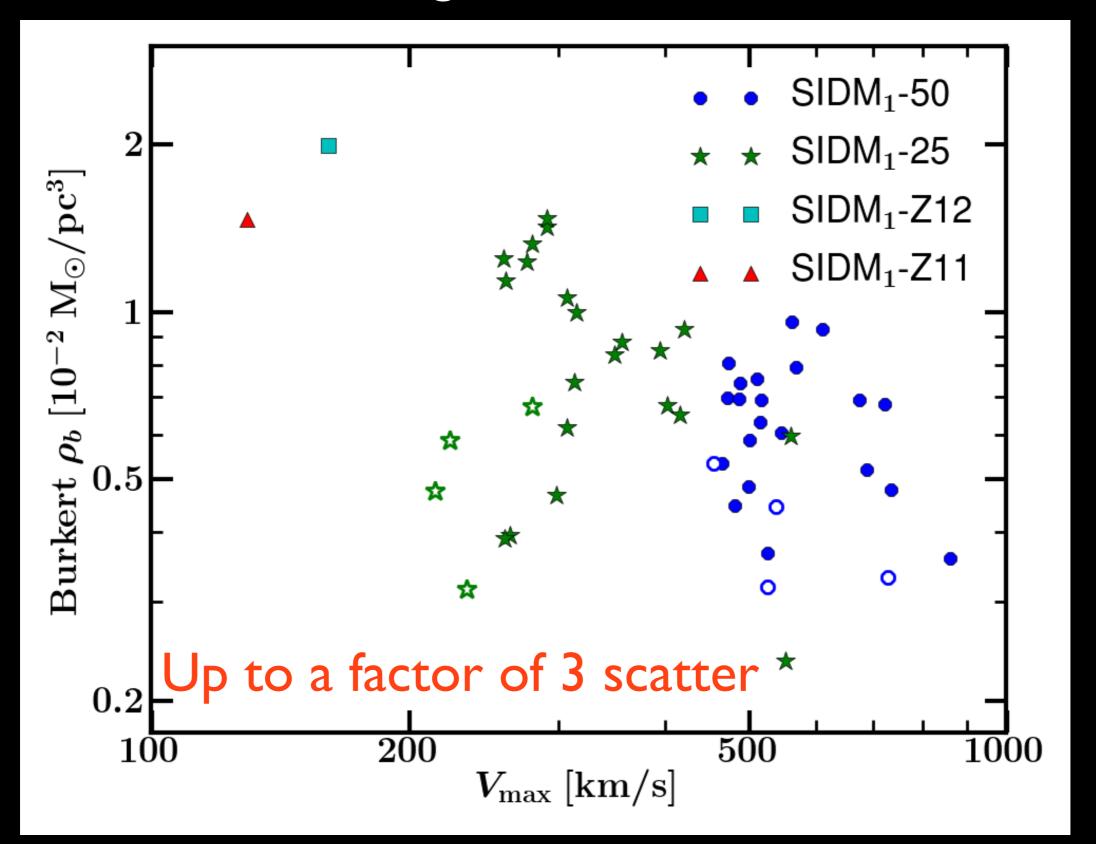
Subhalo count within I Rvir of the host are identical to CDM For r < Rvir/2 we only see differences of ~10-15%

Substructure

Results from cosmological simulations - Substructure

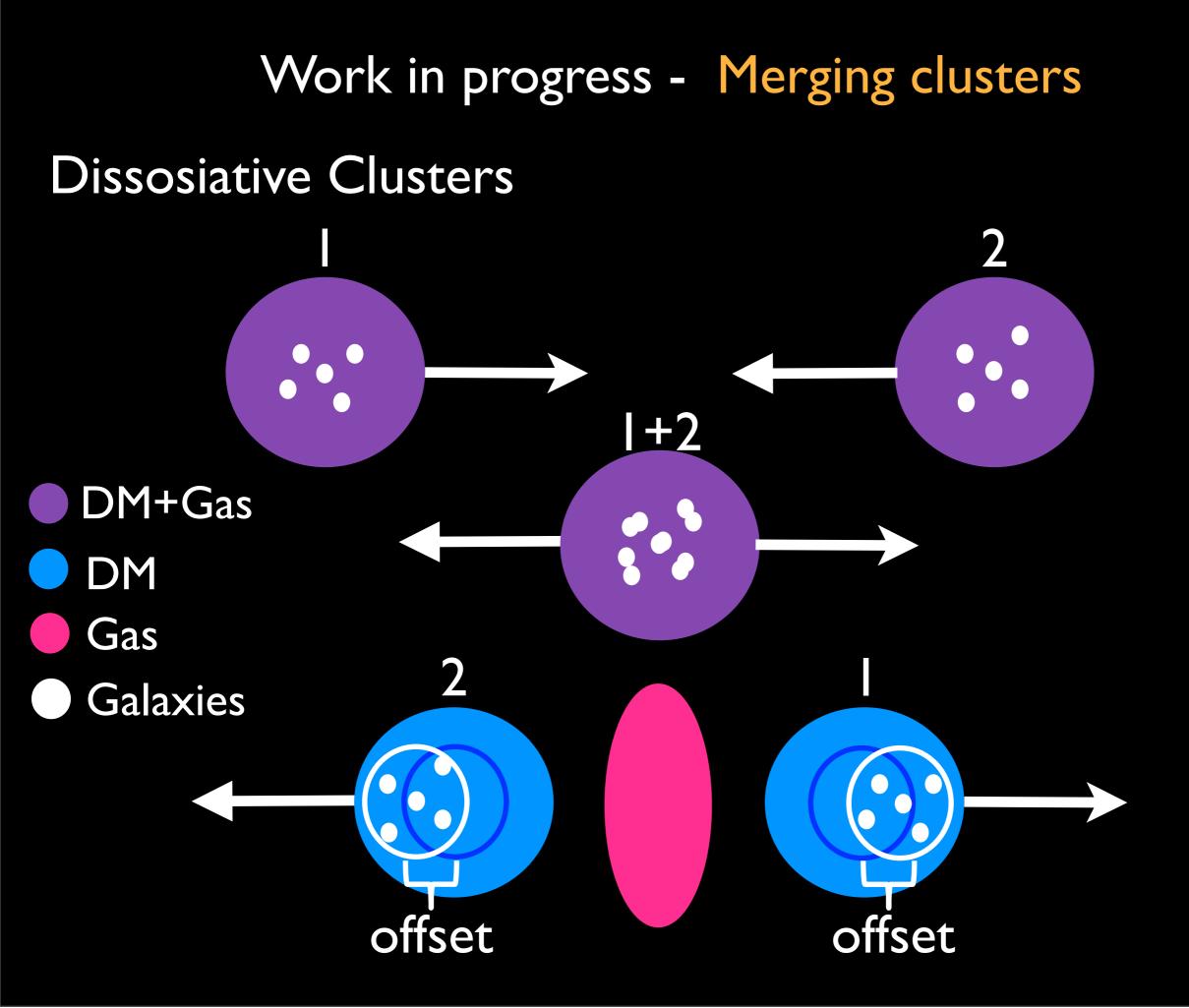


Results from cosmological simulations - Halo densities



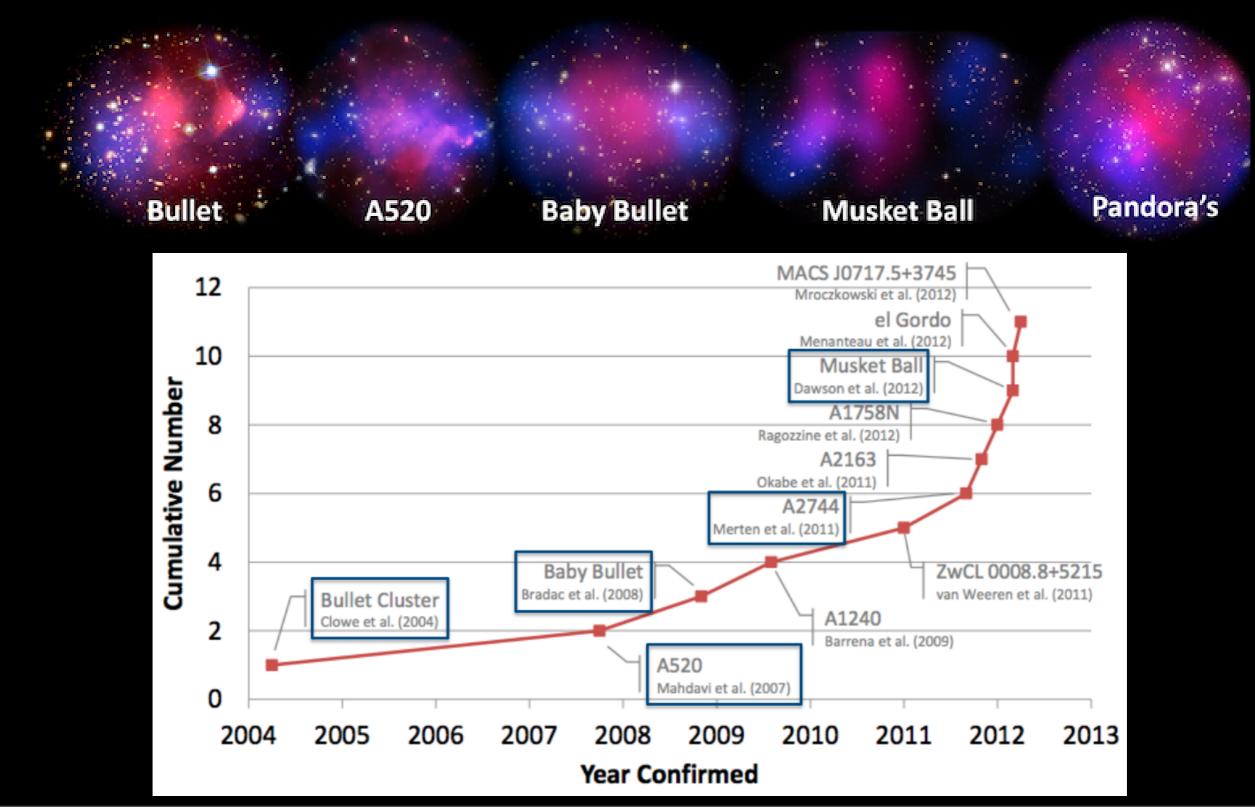
 $\sigma/m = 1 \text{ cm}^2/g$

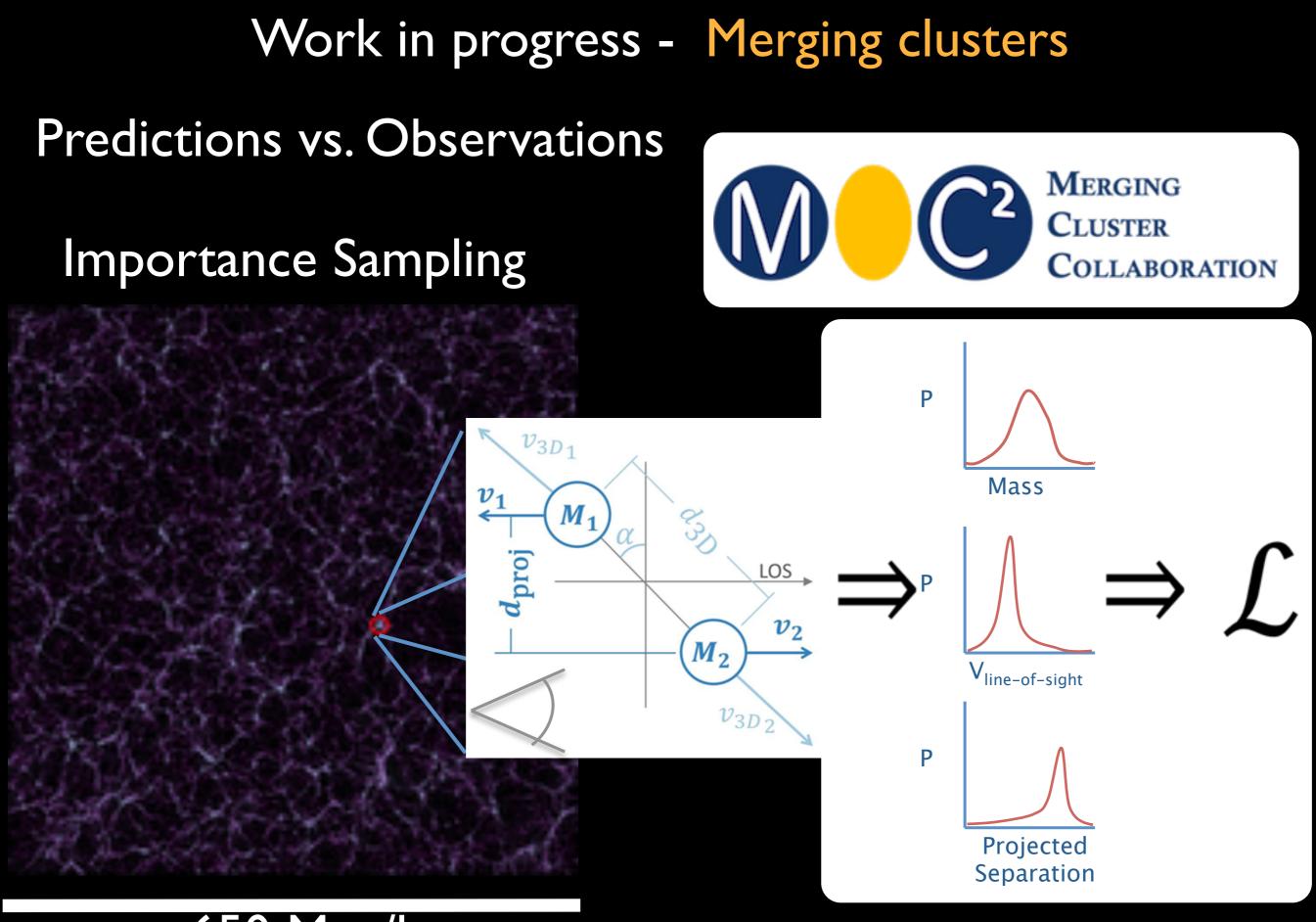
Rocha et al. 2013



Work in progress - Merging clusters

Observations



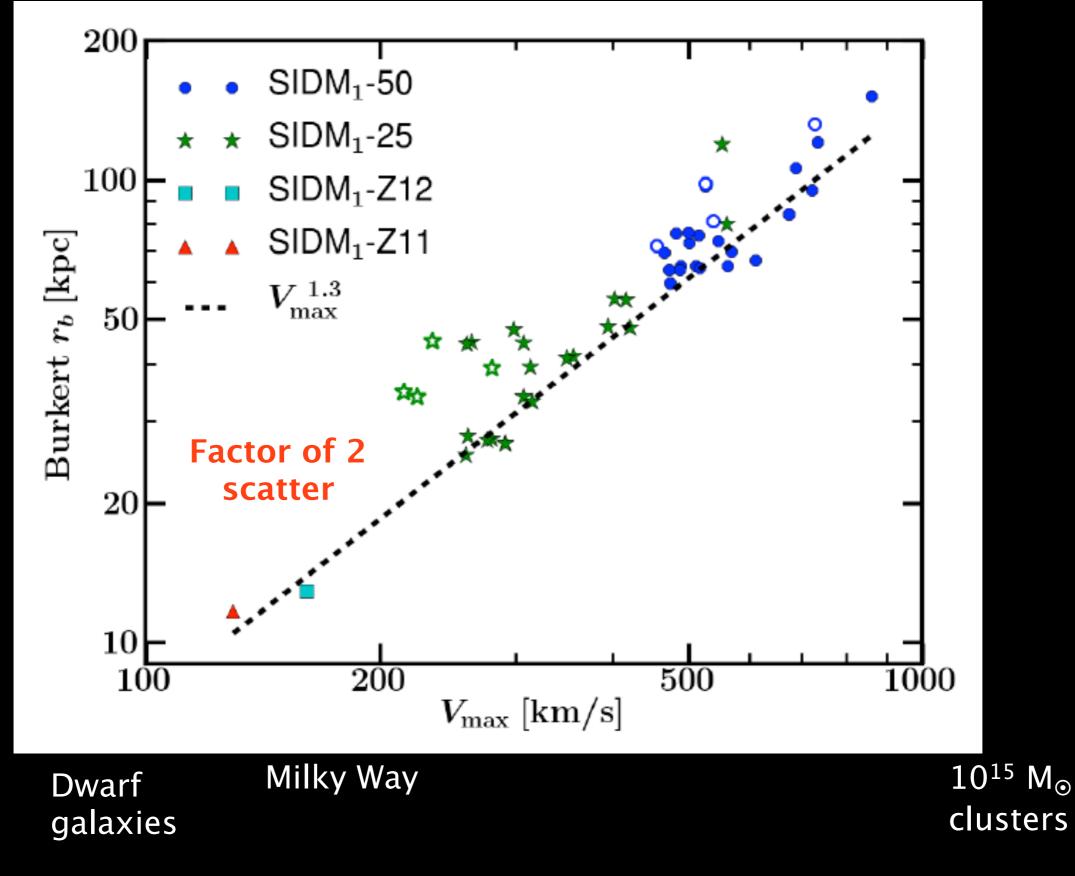


650 Mpc/h

Conclusions

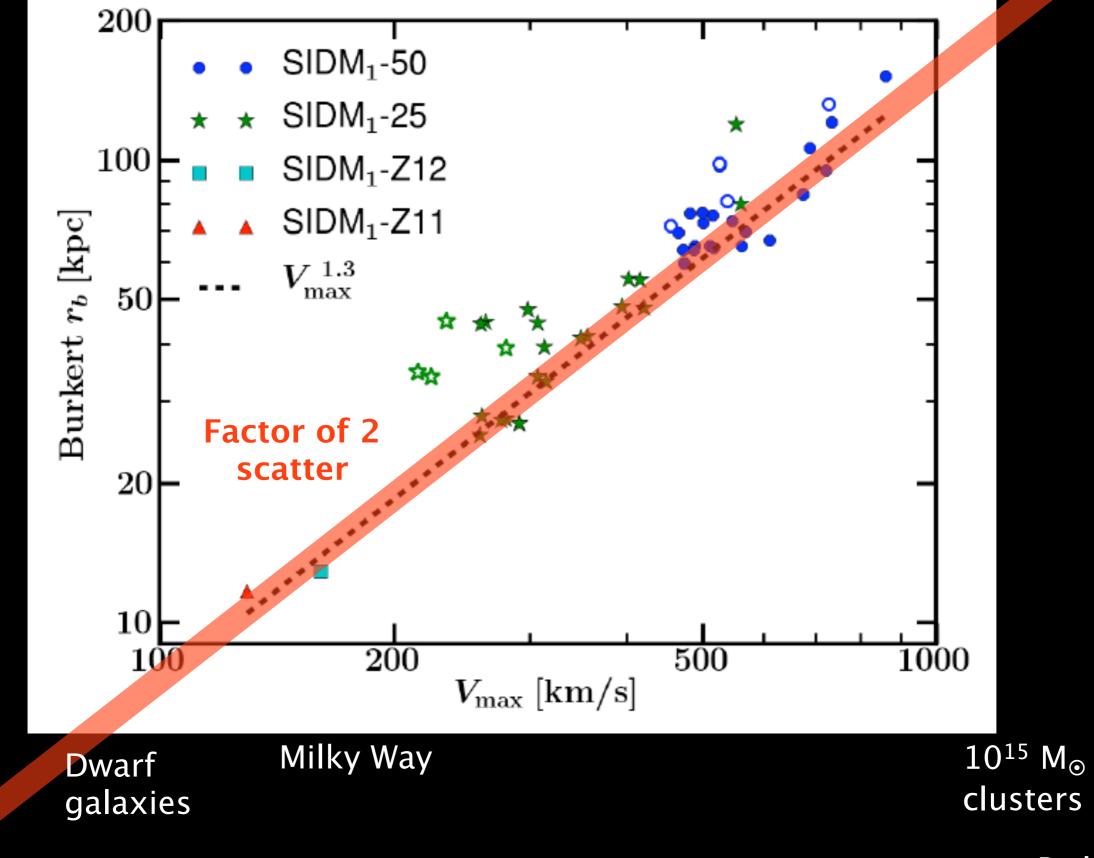
- SIDM with $\sigma/m < 1 \text{ cm}^2/\text{g}$ is alive!
- For constant cross-sections 0.1 < σ/m < 0.5 cm²/g is the interesting regime, able to solve the cusp/core problem and TBTF while still consistent with cluster observations.
- However, we still need to make predictions for SIDM + Baryons.
- Merging clusters are a promising way to probe σ/m
 > 0.1 cm²/g. MCC will either yield a measurement or rule out the astrophysical interesting cross sections.

Thank You



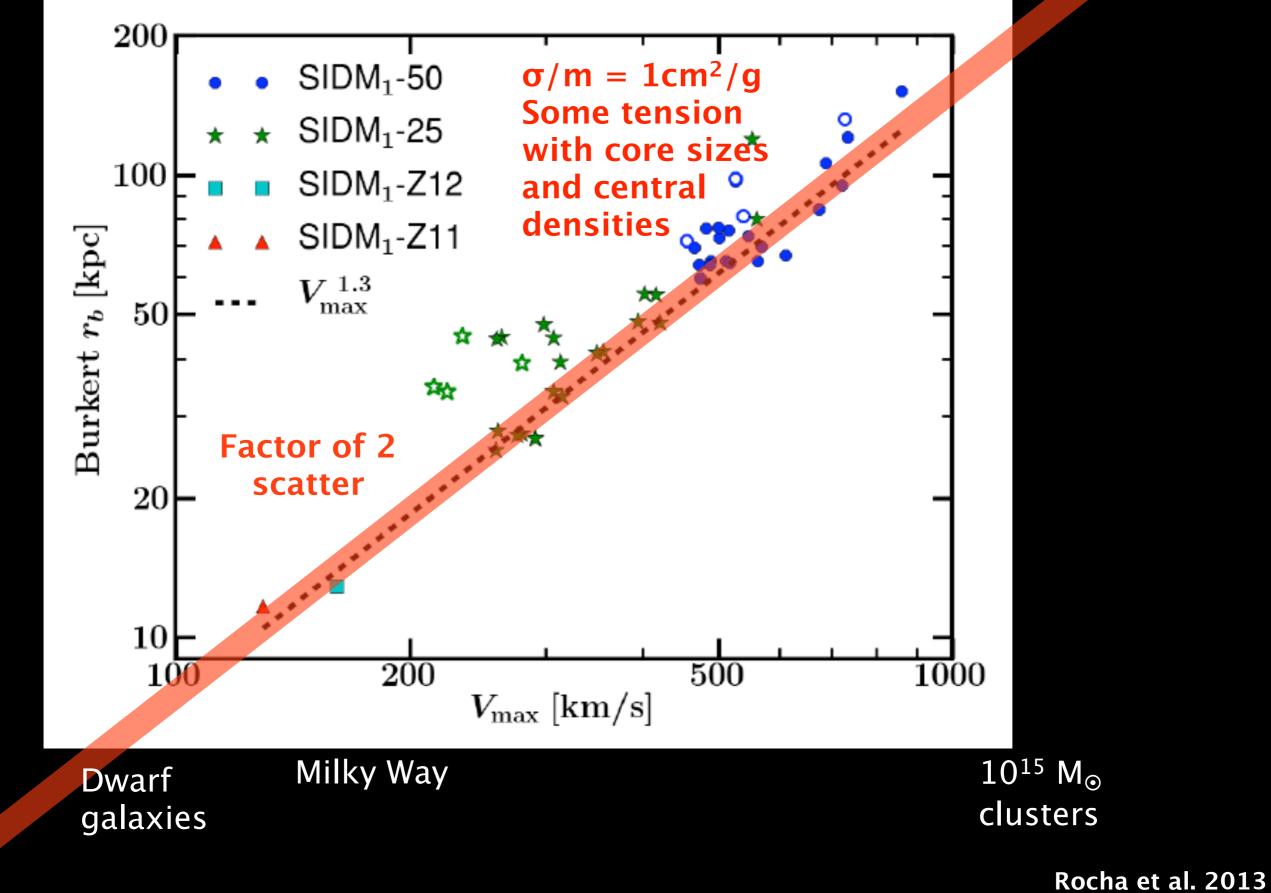
Core Sizes

Rocha et al. 2013

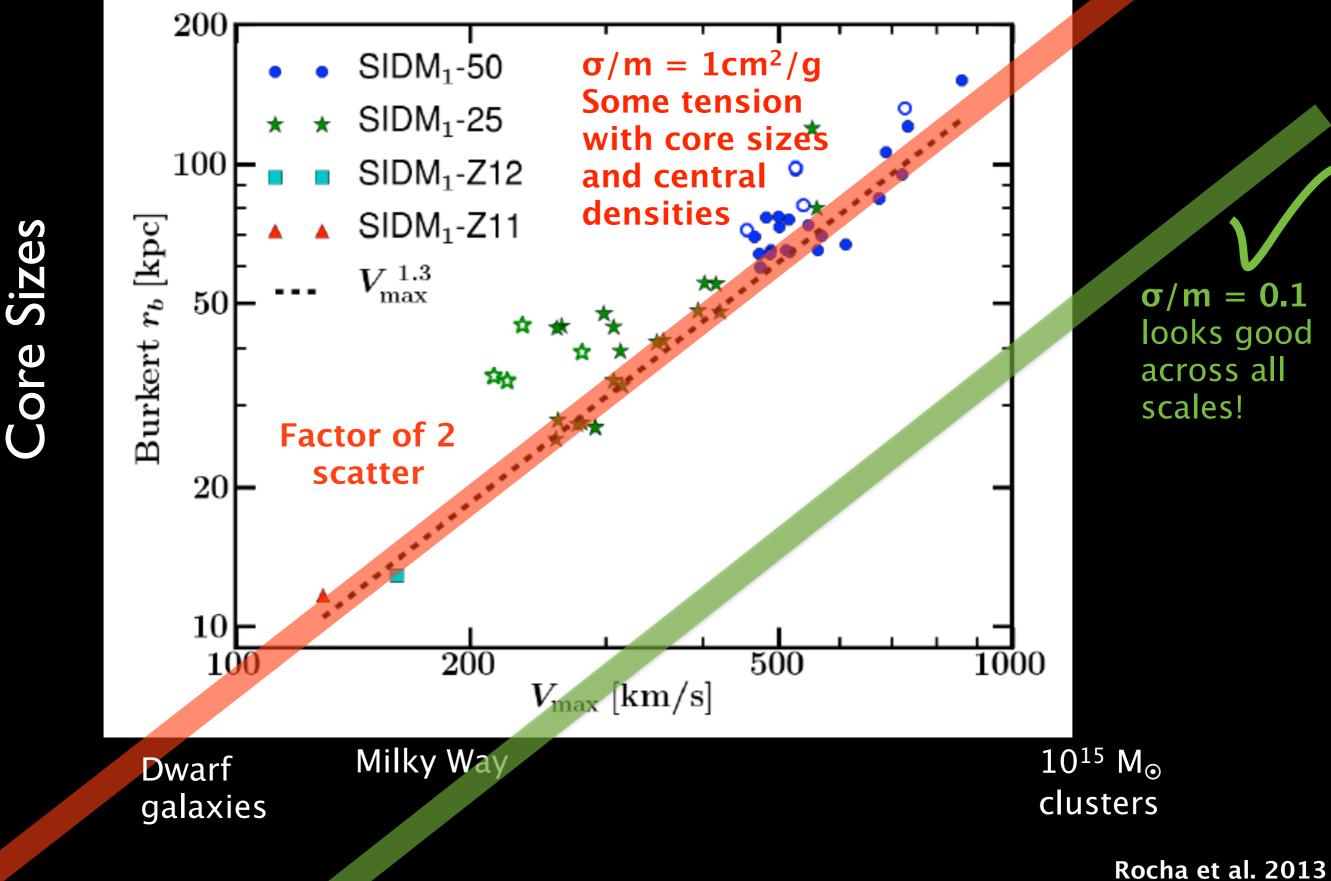


Core Sizes

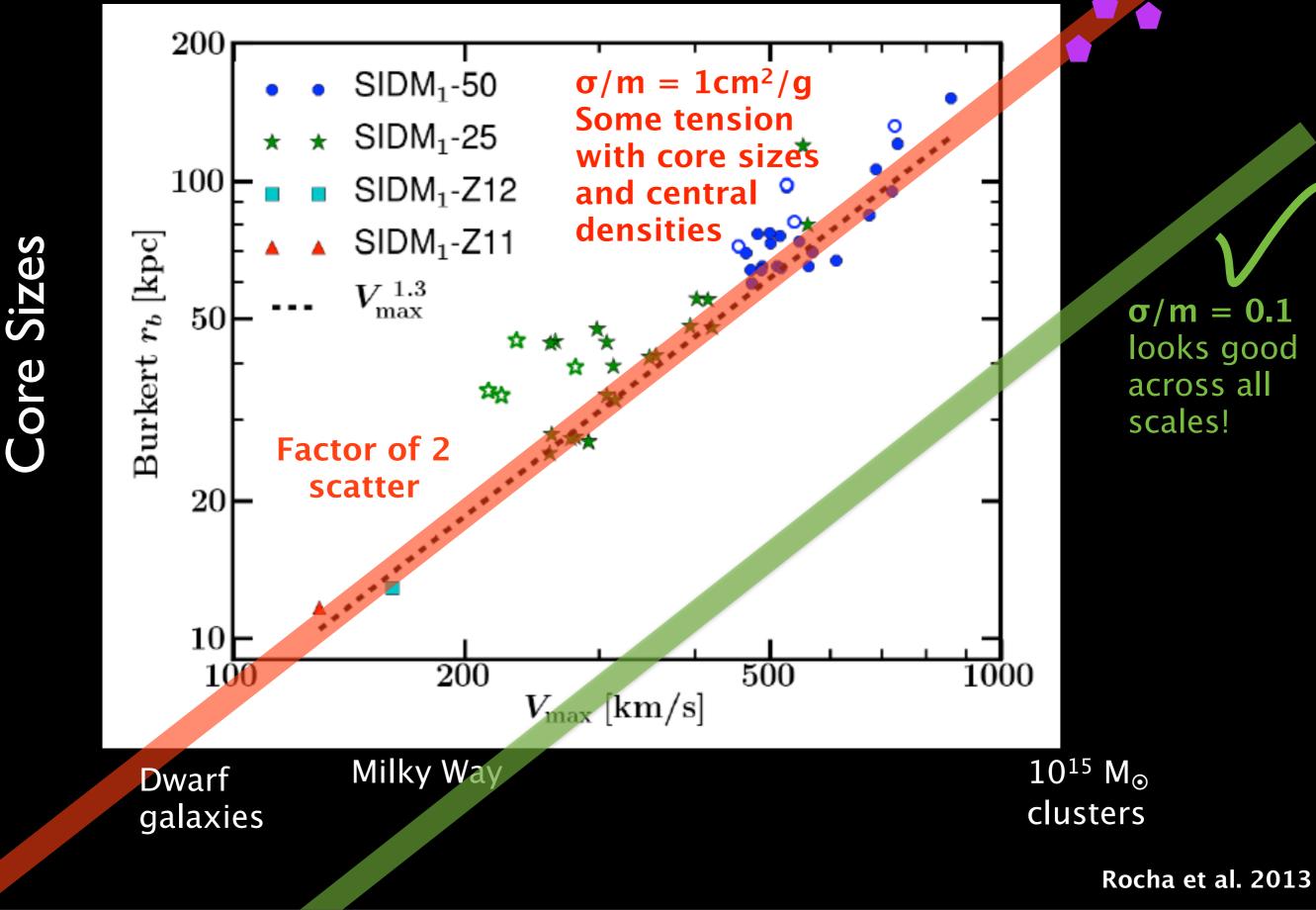
Rocha et al. 2013

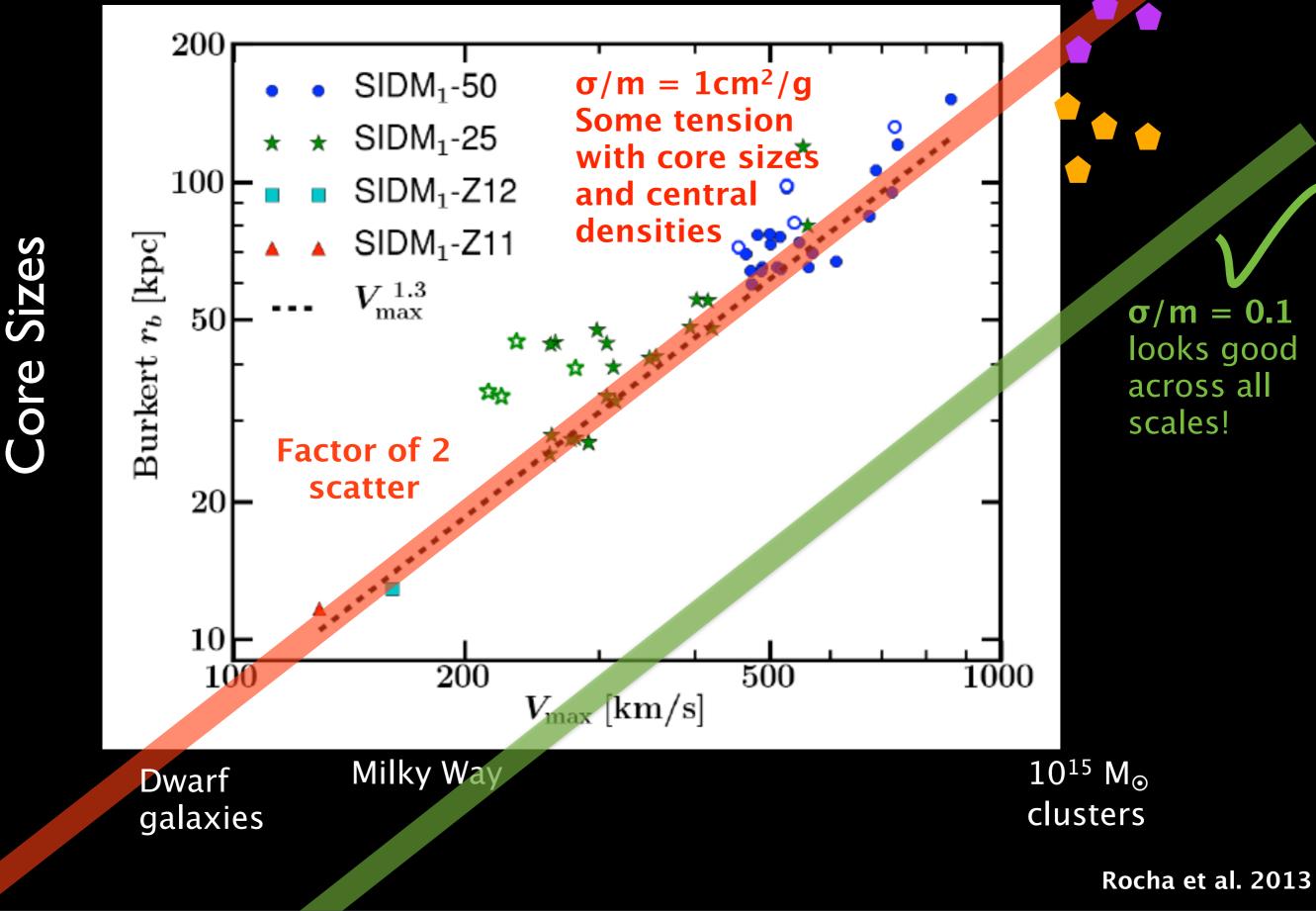


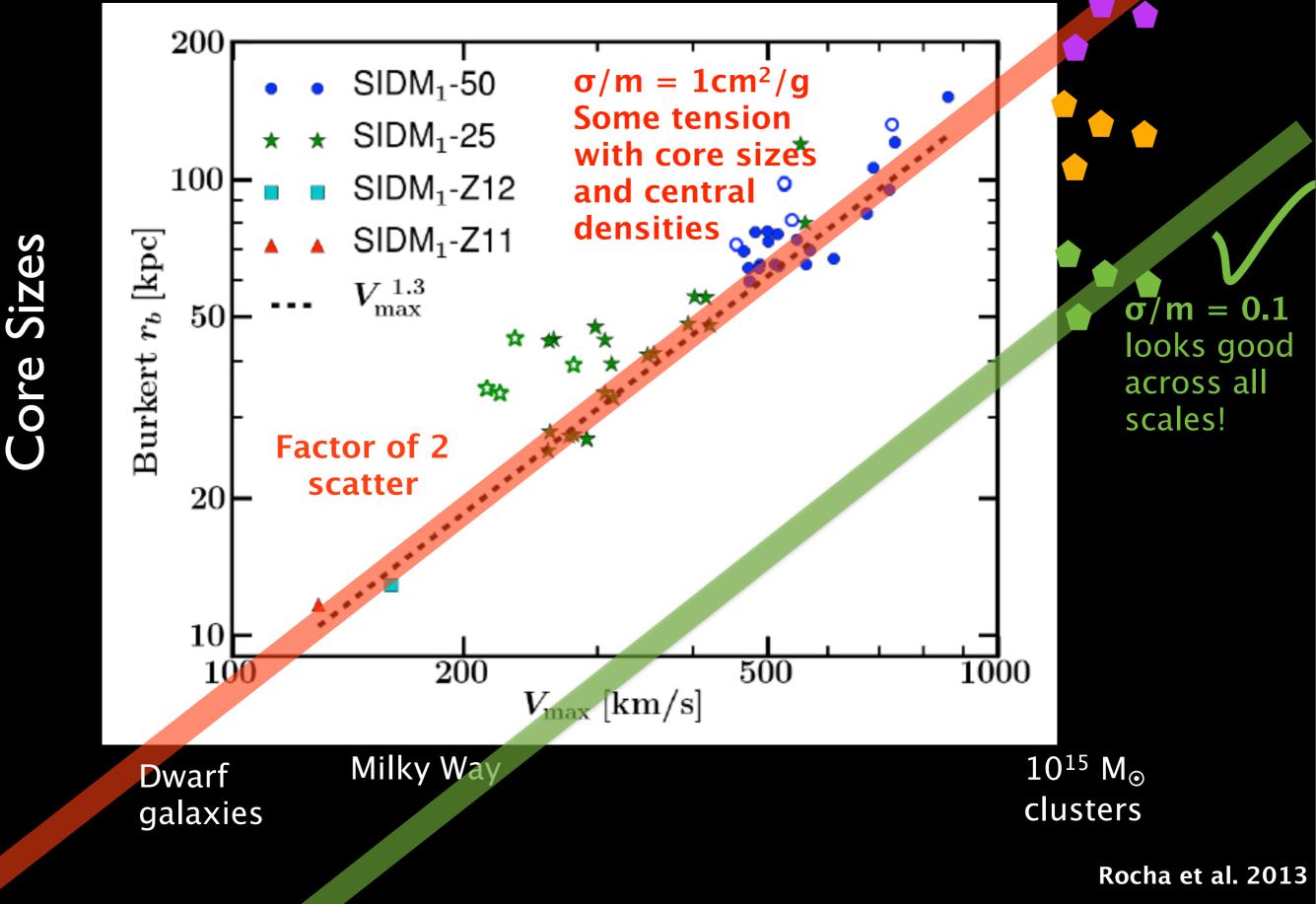
Core Sizes

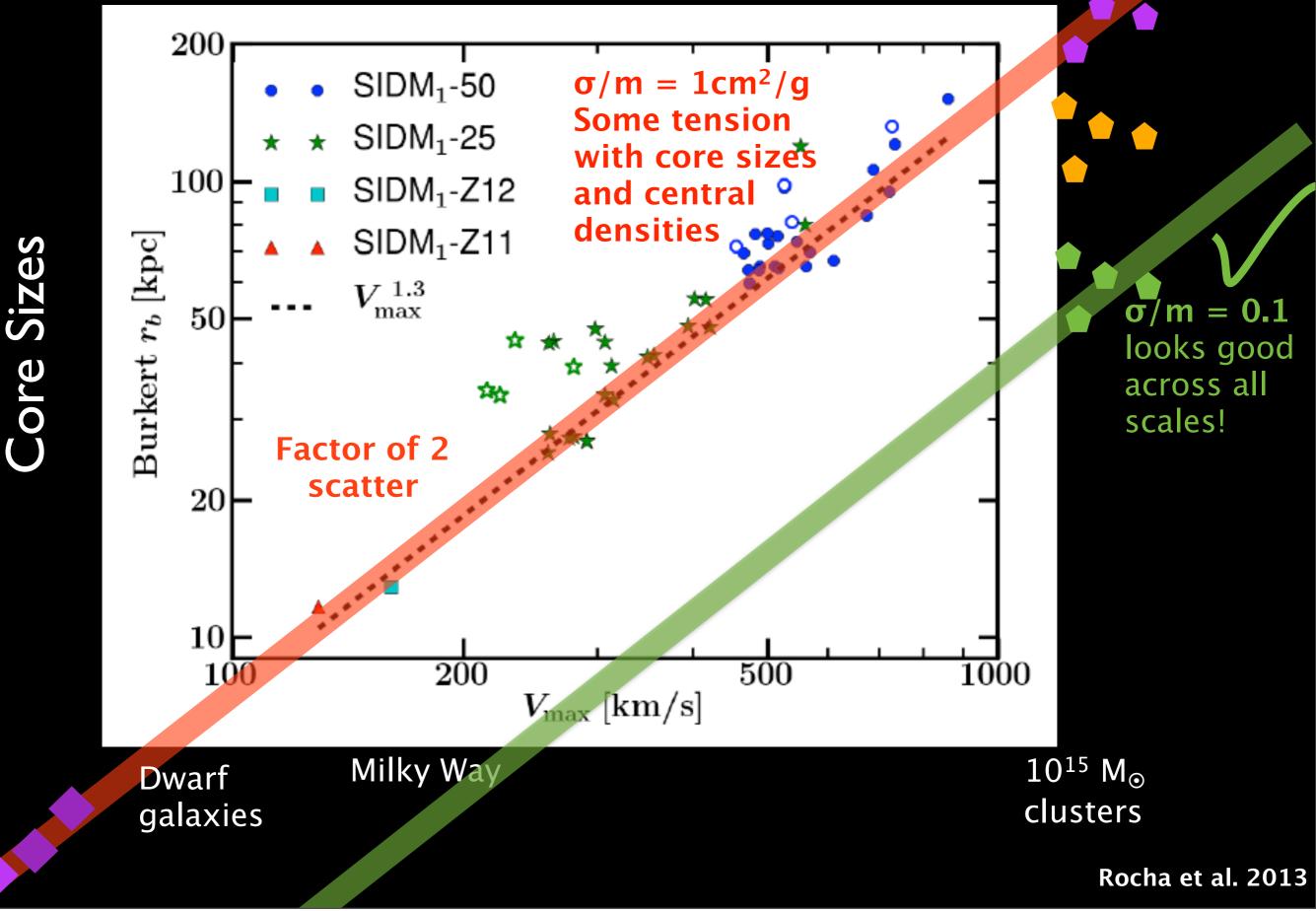


Thursday, August 15, 13

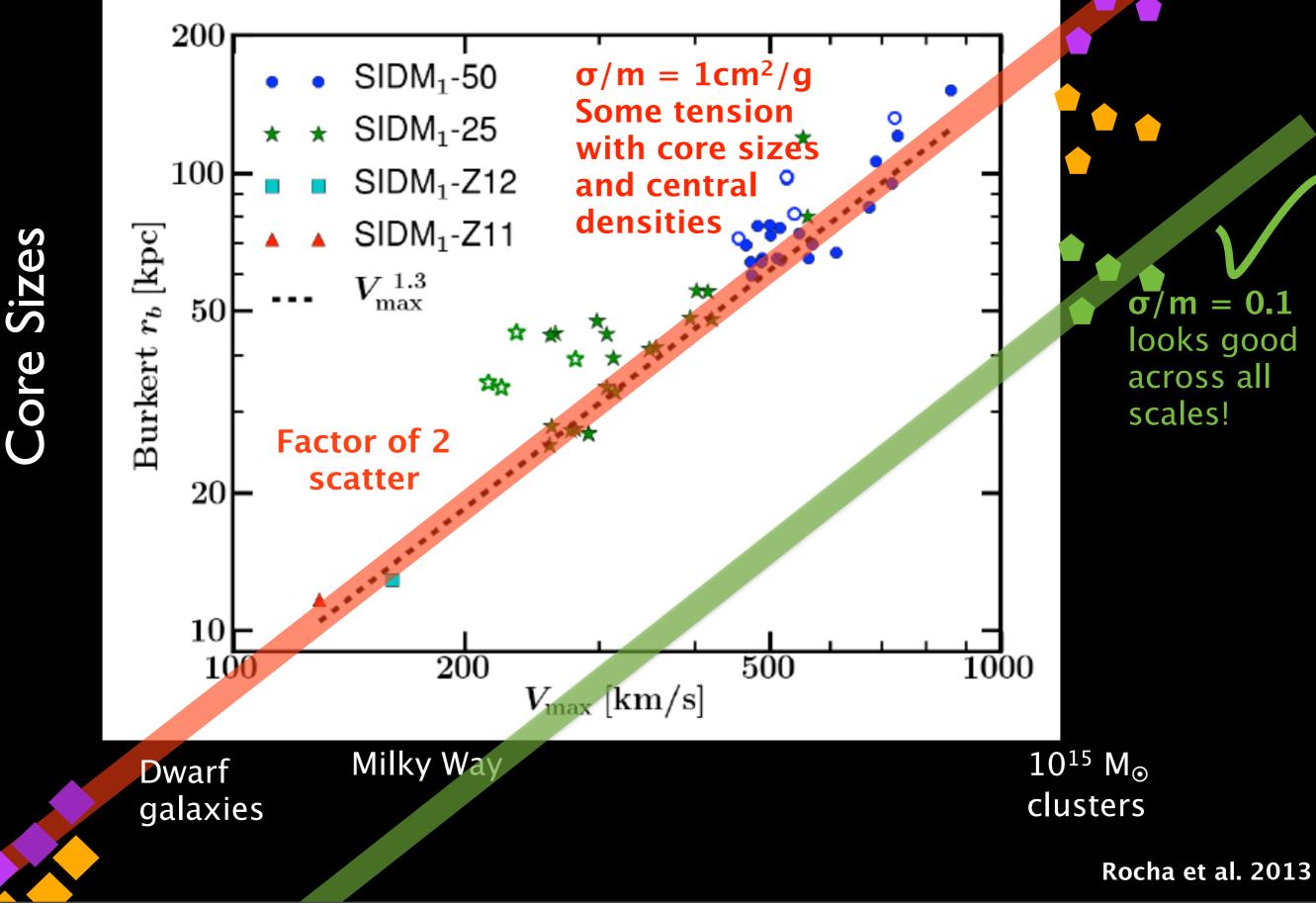




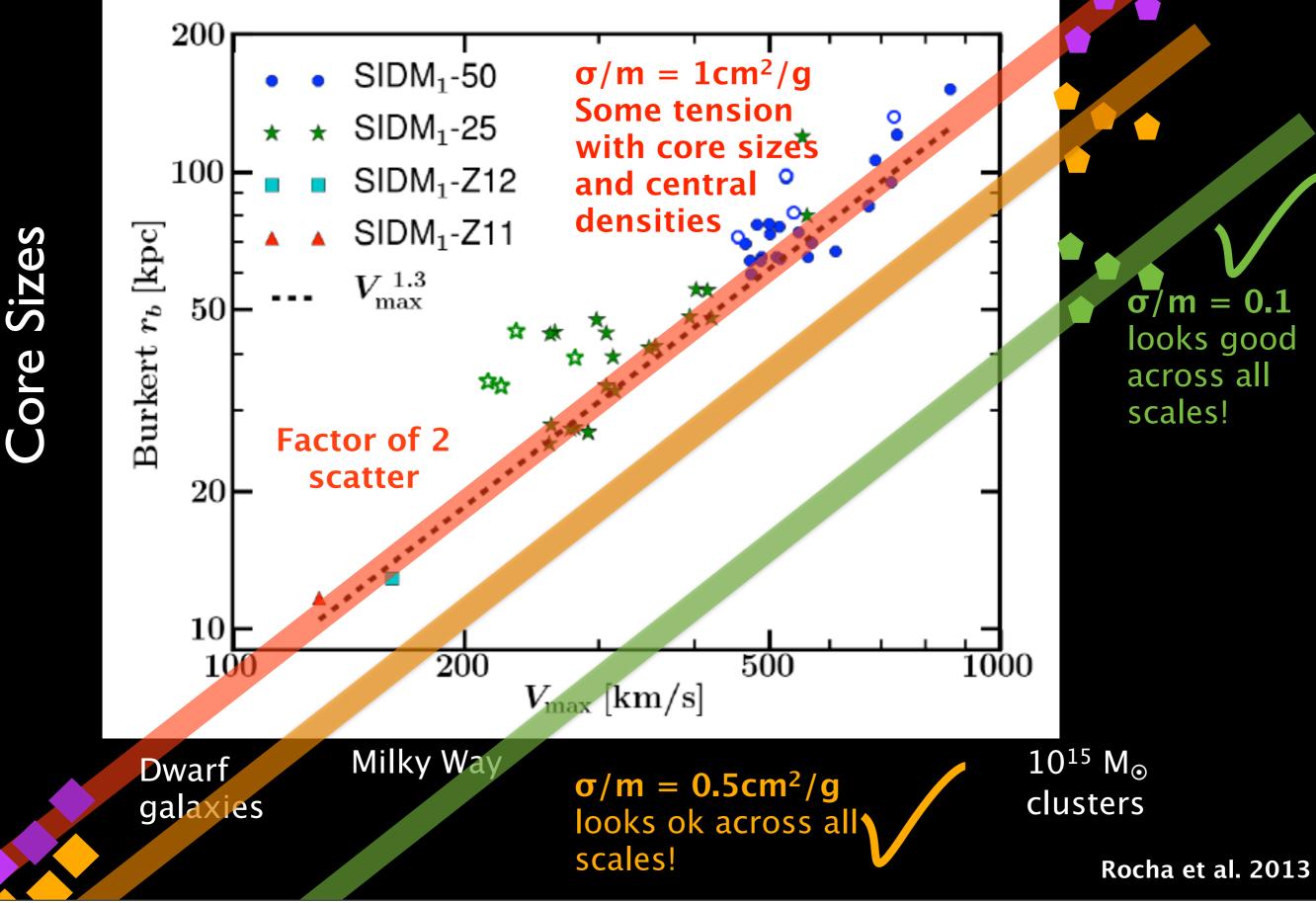


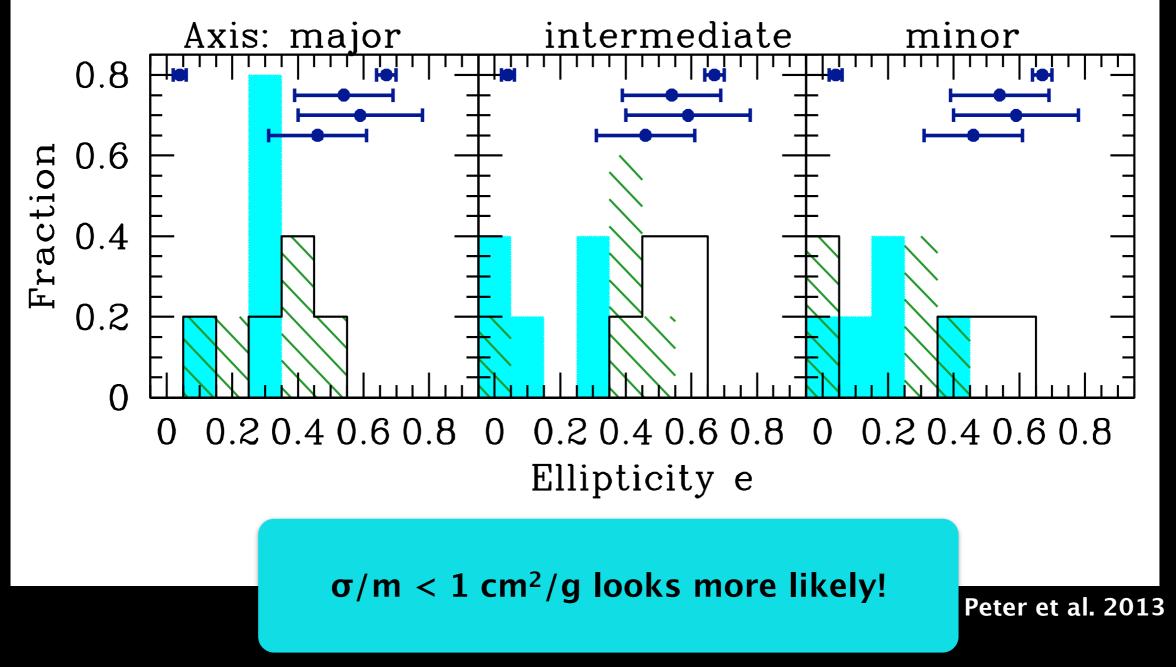


Results from cosomological simulations - Core sizes



Results from cosomological simulations - Core sizes





This is more than an order of magnitude less stringent than Miralda-Escude (2002), the reason is that:

Halos get spherical only within the cores

If inner parts have flattened density, outer parts have even greater weight.

	Observed	$\sigma/m=1 \text{ cm}^2/g$	$\sigma/m=0.1 \text{ cm}^2/\text{g}$
Clusters 700-1000 km/s	0.06-0.025 [Msun/pc ³] Arabadjis et al. 2002, Sand et al. 2004, 2008, Saha et al 2006, Saha & Read 2009 Newman et al. 2009,2011	~0.005-0.004 [Msun/pc ³]	~0.04 [Msun/pc ³]
Low-Mass Spirals 50-130 km/s	0.5-0.01 [Msun/pC ³] de Blok et al. 2001, Simon et al. 2005, Sanchez-Salcedo 2005, Kuzio de Naray et al. 2008,2010, Oh et al . 2011, Salucci et al. 2012	~0.02-0.01 [Msun/pc ³]	~0.2-0.1 [Msun/pc ³]
MW dSphs 20-50 km/s	~0.1 [Msun/pc ³] Strigari et al. 2008, Wolf et al. 2010, Walker & Penarrubia 2011, Amorisco & Evans 2012, Wolf & Bullock 2012	~0.04-0.02 [Msun/pc ³]	~0.5-0.2 [Msun/pc ³]

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Comparison to observed core sizes

 $\sigma/m=1 \text{ cm}^2/\text{g}$

0.9-3 kpc

 $\sigma/m=0.1 \text{ cm}^2/\text{g}$

0.2-0.6 kpc

Observed

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SS n/s	0.5-8 kpc de Blok et al. 2001, Simon et al. 2005, Sanchez-Salcedo 2005, Kuzio de Naray et al. 2008.2010. Oh et	3-10 kpc	0.6-2.5 kpc

Walker & Penarrubia 2011

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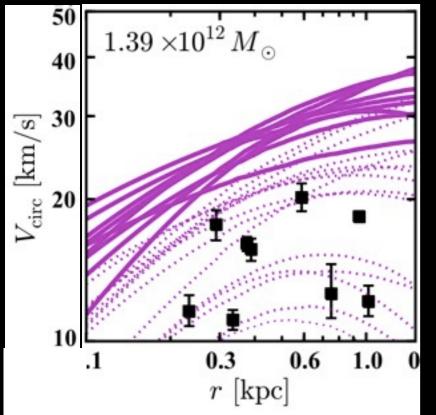
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Results from cosmological simulations - Halo densities, shapes & substructure Observations

Results from cosmological simulations - Halo densities, shapes & substructure

Observations

Milky Way dwarfs

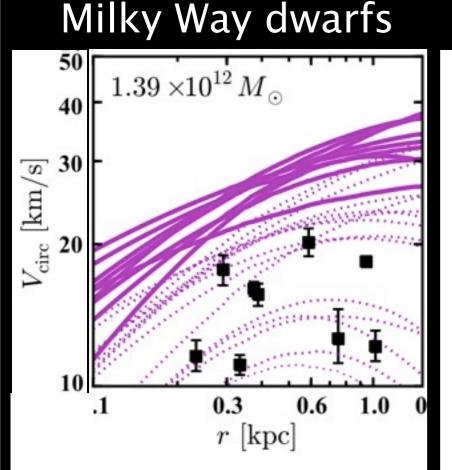


"Too big to fail" (Boylan-Kolchin+ 2011)

Need less DM in ~100 pc in 10^9-10^{10} M $_{\odot}$ halos Cores in ~0.5 kpc observed Walker&Penarrubia 2011

Results from cosmological simulations - Halo densities, shapes & substructure

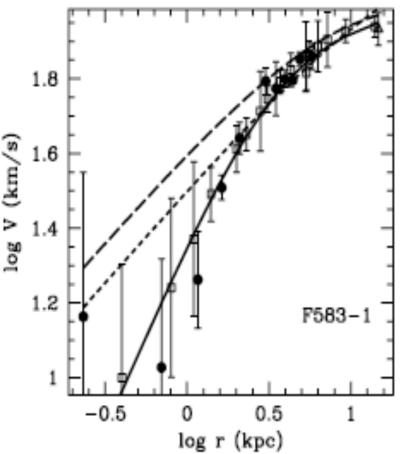
Observations



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Low-mass Spirals



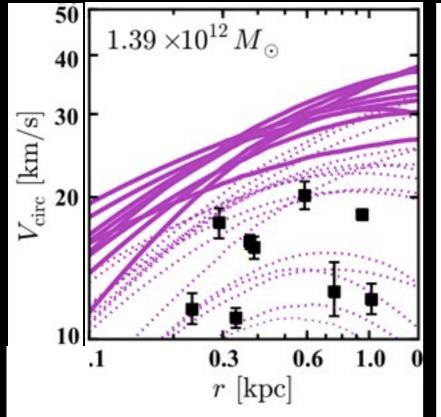
Dwarf core problem (Kuzio de Naray+ 2008)

Need cores in ~0.5-5 kpc in $10^{11}~M_{\odot}$ halos

Results from cosmological simulations - Halo consities, shapes & substructure

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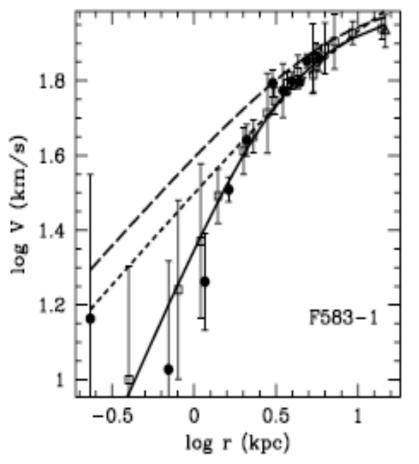
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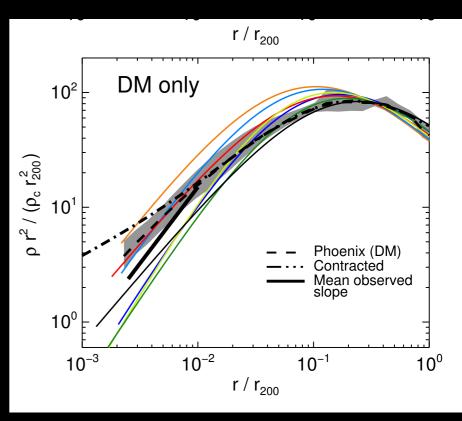
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Dwarf core problem (Kuzio de Naray+ 2008)

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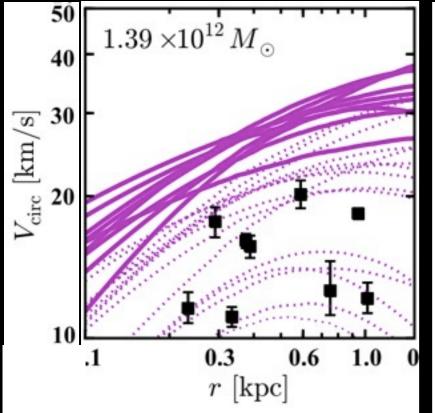
Galaxy cluster densities $\rho \sim r^{-\beta}$ Drew Newman (Newman+ 2012a,b)

Allow cores of ~30 kpc in $10^{15}~M_{\odot}$ halos

Results from cosmological simulations - Halo consities, shapes & substructure

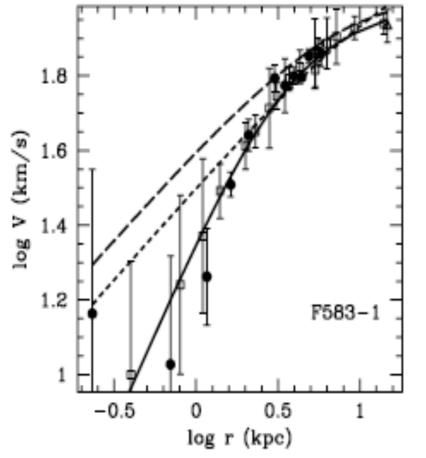
Observations

Milky Way dwarfs



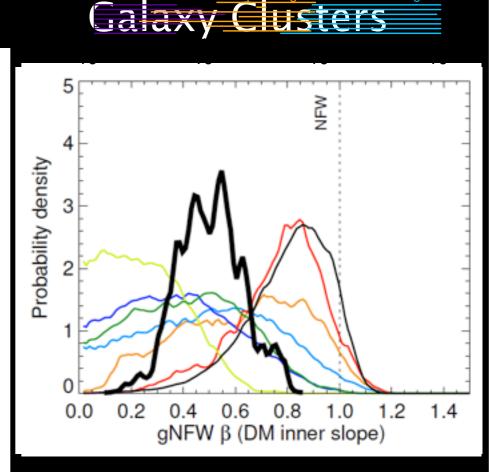
"Too big to fail" (Boylan-Kolchin+ 2011)

Need less DM in ~100 pc in 10^9-10^{10} M $_{\odot}$ halos Cores in ~0.5 kpc observed Walker&Penarrubia 2011 Low-mass Spirals



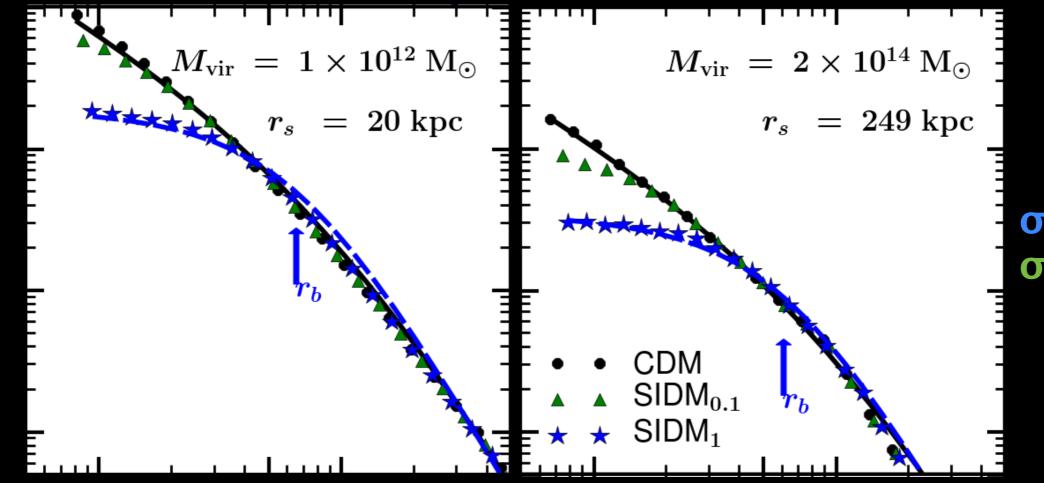
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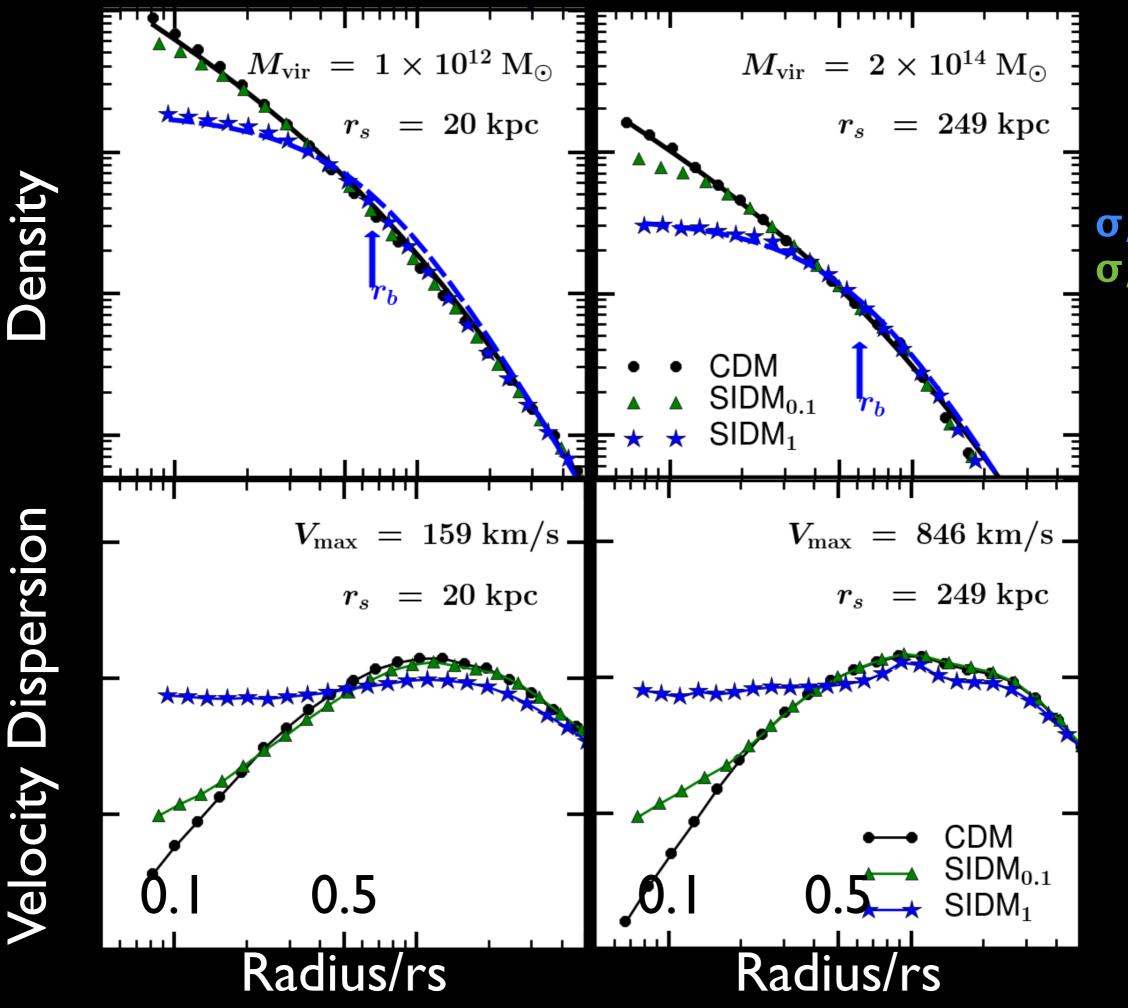
 $\frac{\sigma/m}{\sigma/m} = 1$

Radius/rs

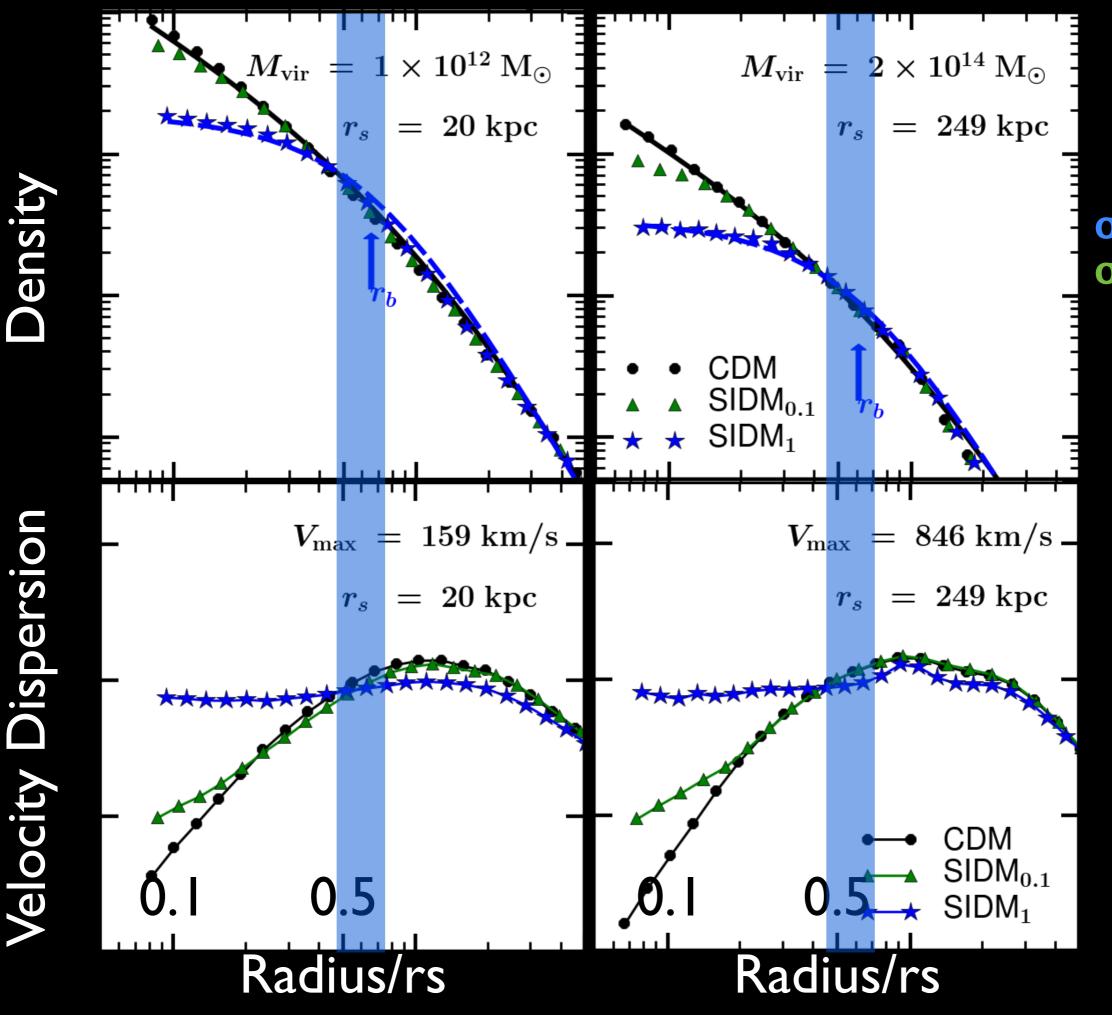
Radius/rs

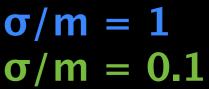
Thursday, August 15, 13

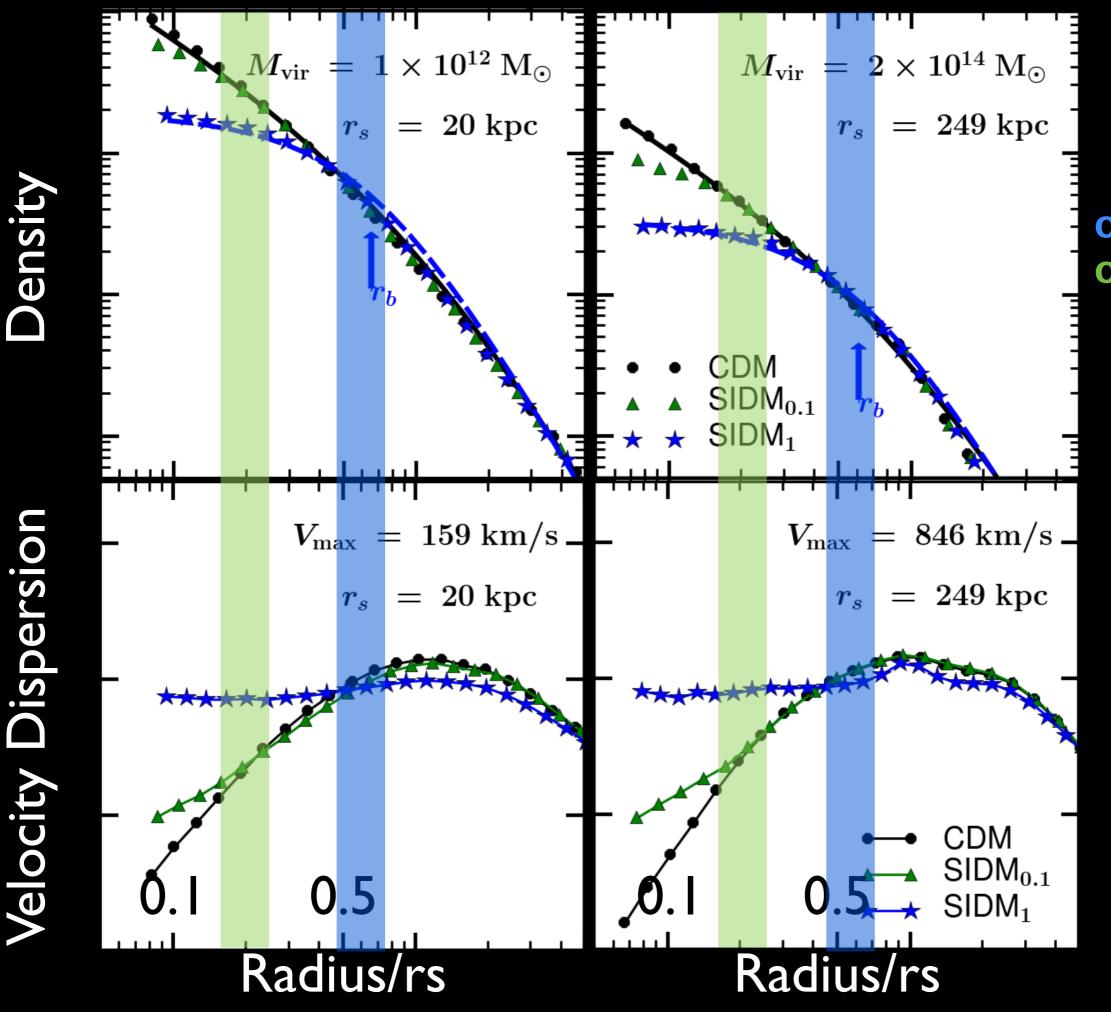
Density



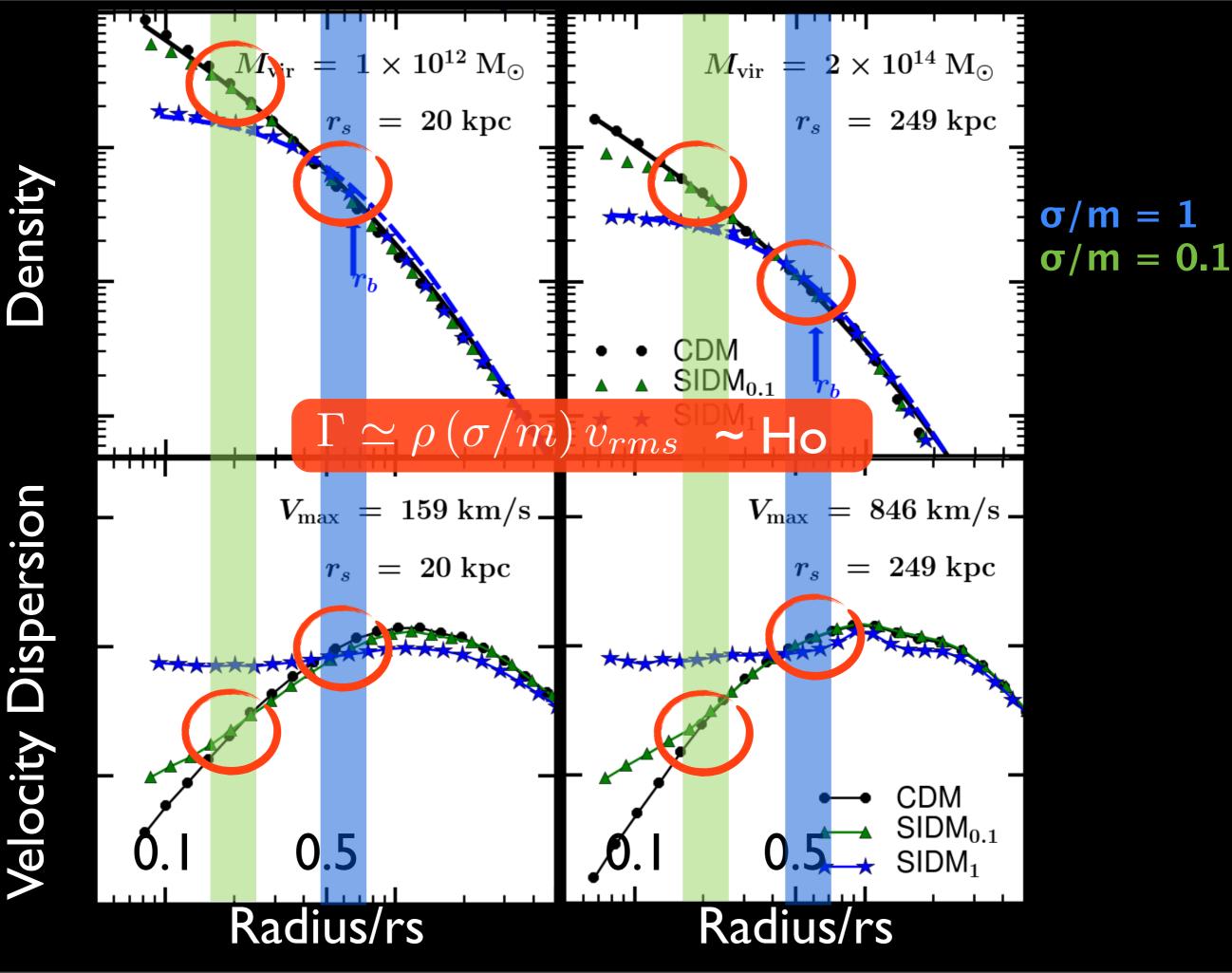
 $\frac{\sigma/m}{\sigma/m} = 0.1$

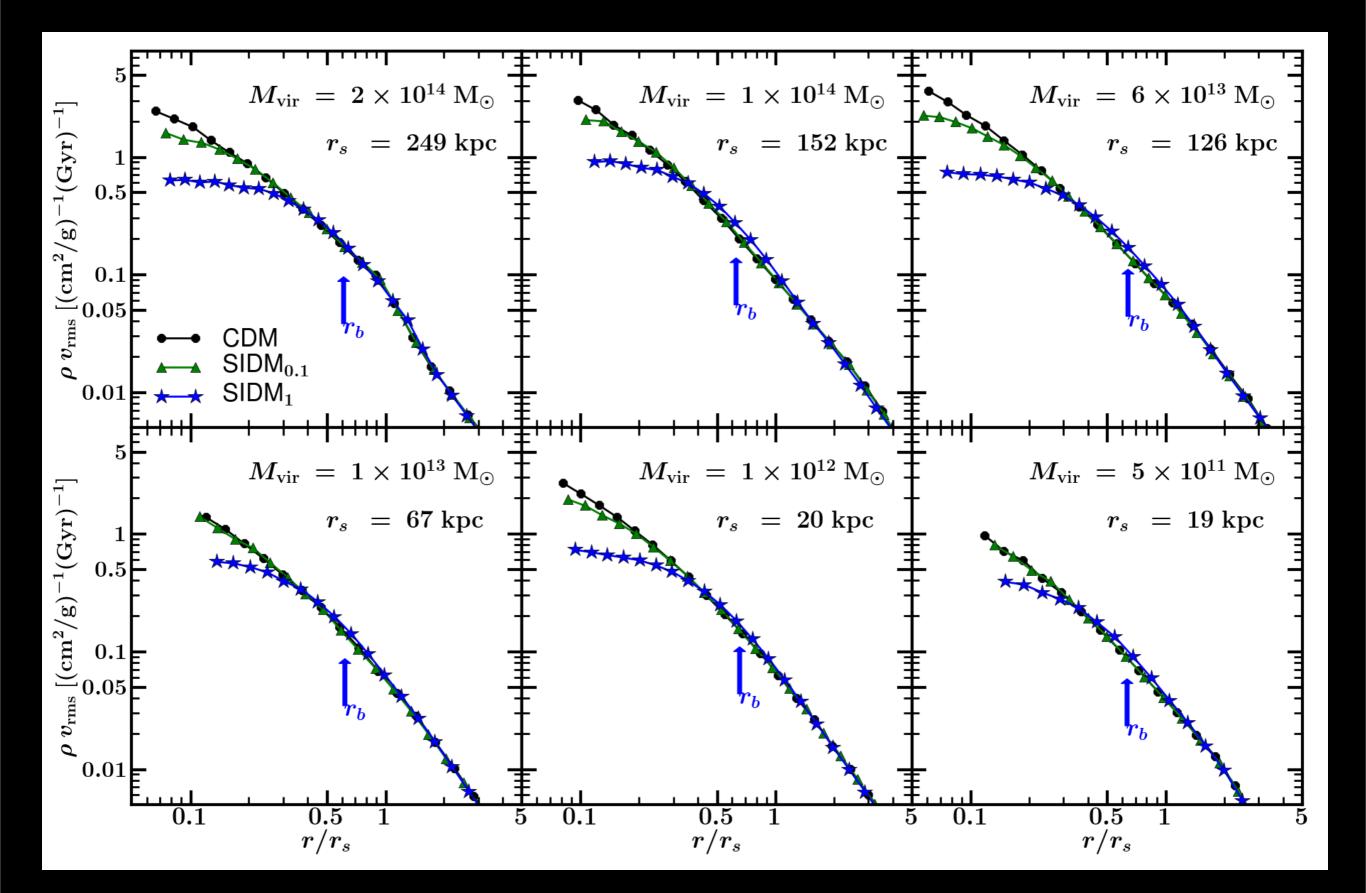


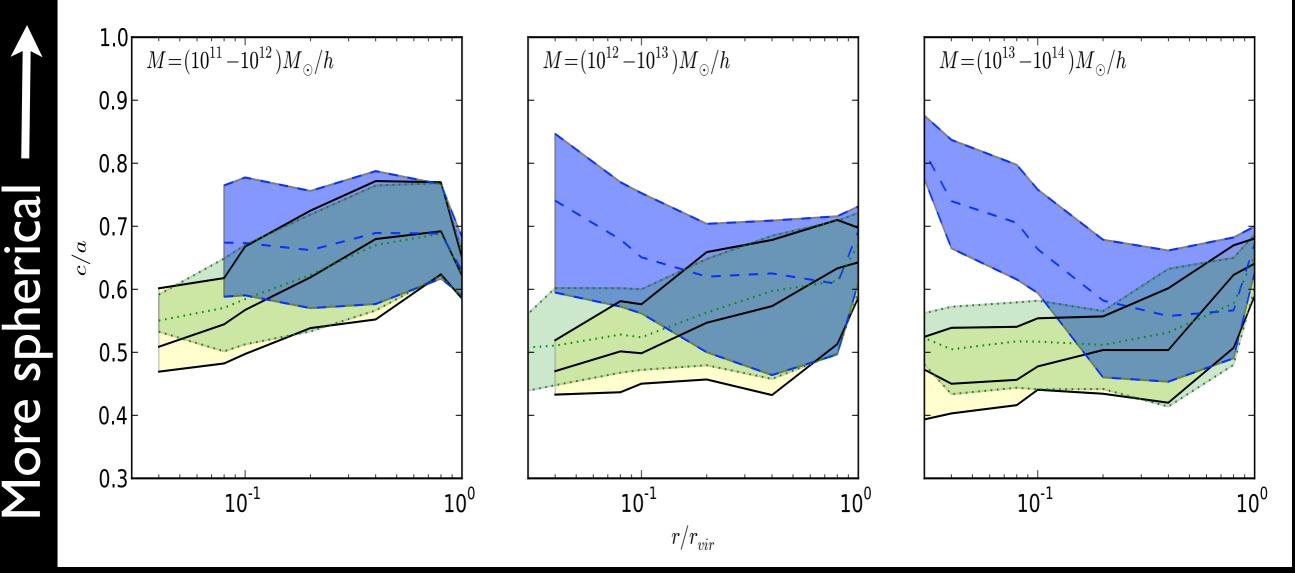




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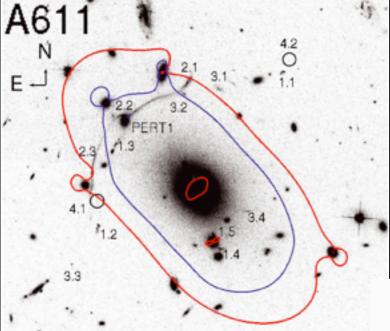




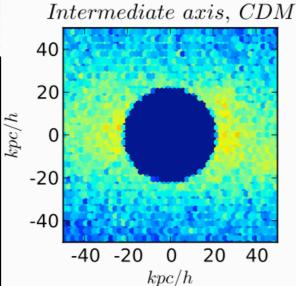
Radius/rvir

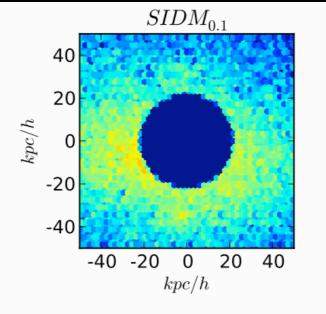
 $\sigma/m = 1 \text{ cm}^2/g$ $\sigma/m = 0.1 \text{ cm}^2/g$ collisionless

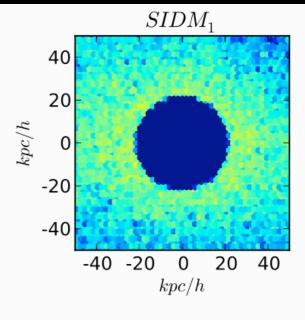
Rocha et al. 2013 Peter et al. 2013

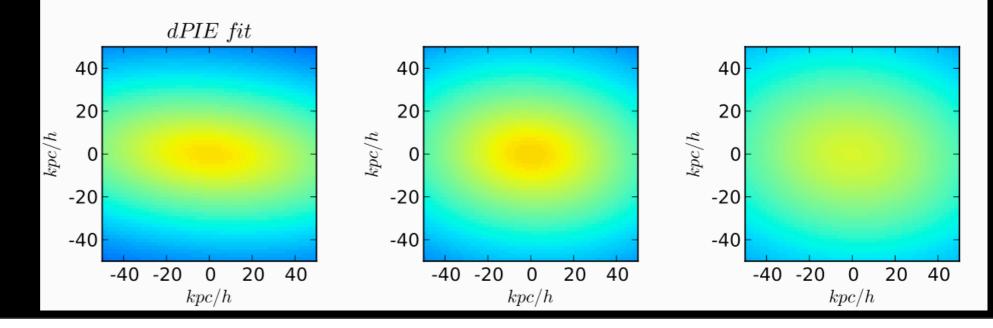


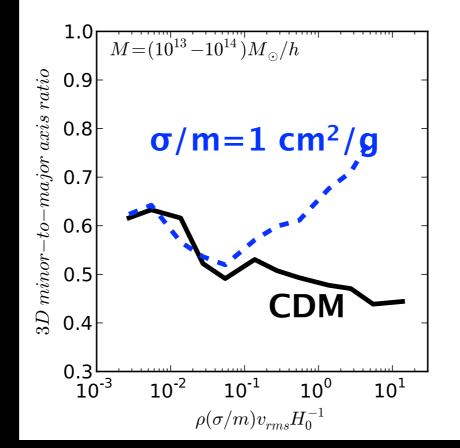
From LoCuSS sample Richard+ 2010 We see surface density (or gravitational potentials) in projection.



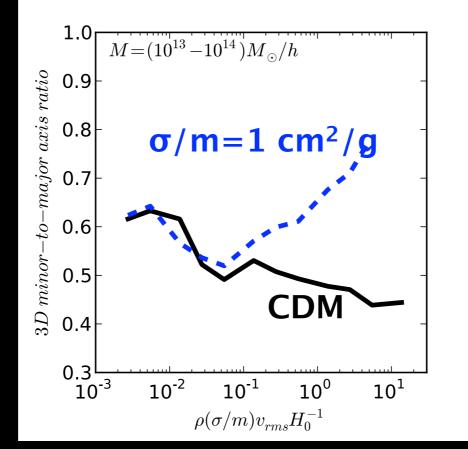


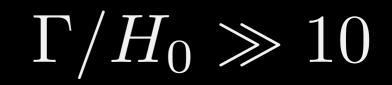




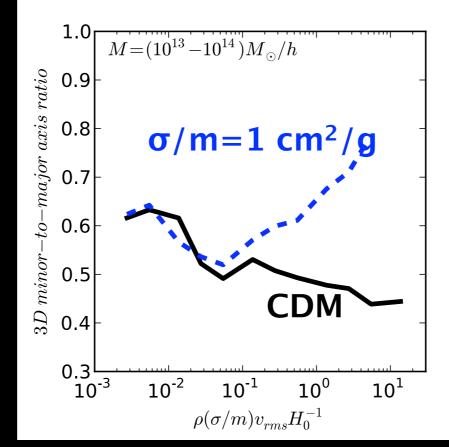


- We see surface density (or gravitational potentials) in projection.
- If inner parts have flattened density, outer parts have even greater weight.



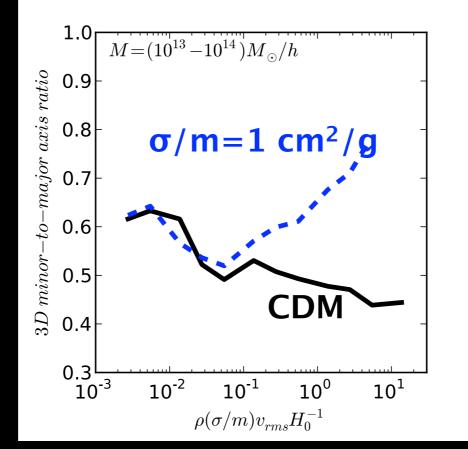


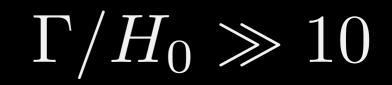
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 $\Gamma/H_0 \gg 10$

• We see surface density (or gravitational potentials) in projection.



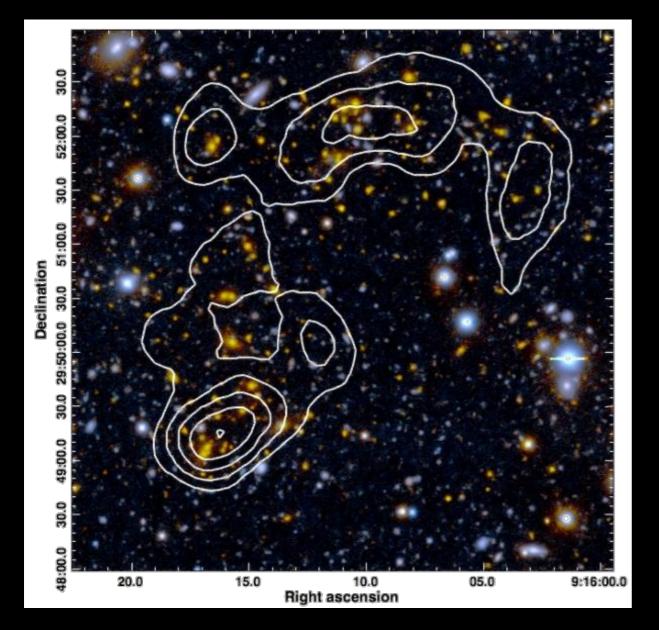


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Work in progress - Merging clusters

Observations

The Musket Ball

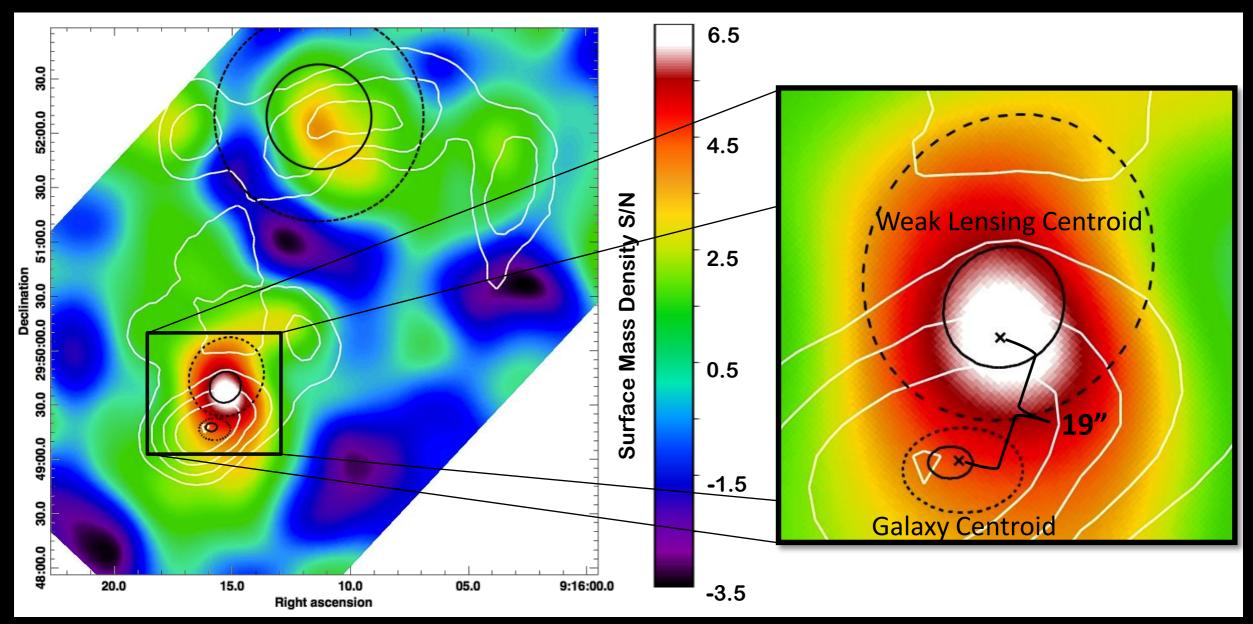


Dawson et al. 2012

Work in progress - Merging clusters

Observations

The Musket Ball

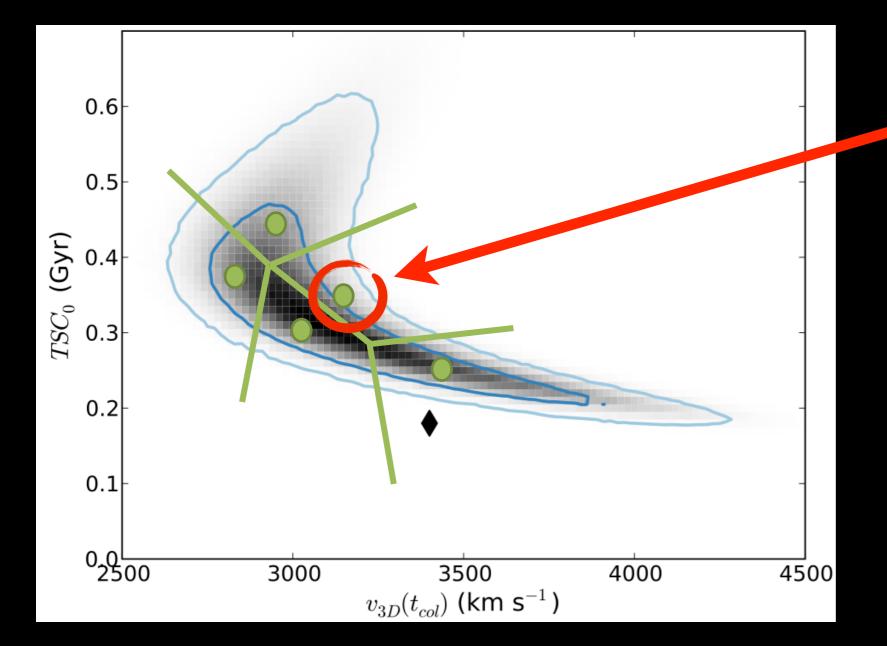


Dawson et al. 2012

Work in progress - Merging clusters

Predictions vs. Observations





Zoom in simulations with hi-res and SIDM

MCC will either yield a measure or rule out the astrophysically interesting SIDM cross sections!!



phase-space evolution given by the Boltzmann Eq. with a hard-sphere collision operator

$$\frac{Df(\mathbf{x}, \mathbf{v}, t)}{Dt} = \Gamma[f, \sigma]$$

= $\int d^3 \mathbf{v}_1 \int d\Omega \frac{d\sigma}{d\Omega} |\mathbf{v} - \mathbf{v}_1| \left[f(\mathbf{x}, \mathbf{v}', t) f(\mathbf{x}, \mathbf{v}'_1, t) - f(\mathbf{x}, \mathbf{v}, t) f(\mathbf{x}, \mathbf{v}_1, t) \right]$



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$$\hat{f}(\mathbf{x}, \mathbf{v}, t) = \sum_{i} (M_{i}/m) W(|\mathbf{x} - \mathbf{x}_{i}|; h_{i}) \delta^{3}(\mathbf{v} - \mathbf{v}_{i})$$

Consistent Pair-Wise Probability

 h_{si}

$$\Gamma(i|j) = (\sigma/m)m_{\rm p}|\mathbf{v}_i - \mathbf{v}_j|g_{ji}$$

$$g_{ji} = \int_0^{h_{\rm si}} d^3 \mathbf{x}' W(|\mathbf{x}'|, h_{\rm si}) W(|\delta \mathbf{x}_{ji} + \mathbf{x}'|, h_{\rm si})$$

$$P(i|j) = \Gamma(i|j) \,\delta t$$

P(i|j) = P(j|i)

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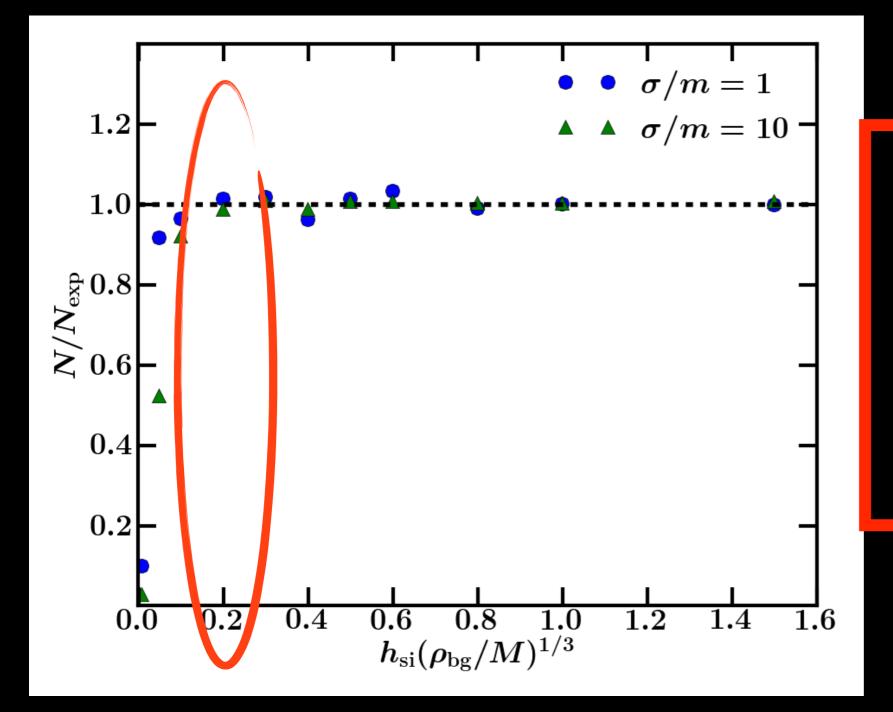
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$$P(i|j) = \Gamma(i|j) \, \delta t$$

$$f(i|j) = P(i|j) = P(j|i)$$

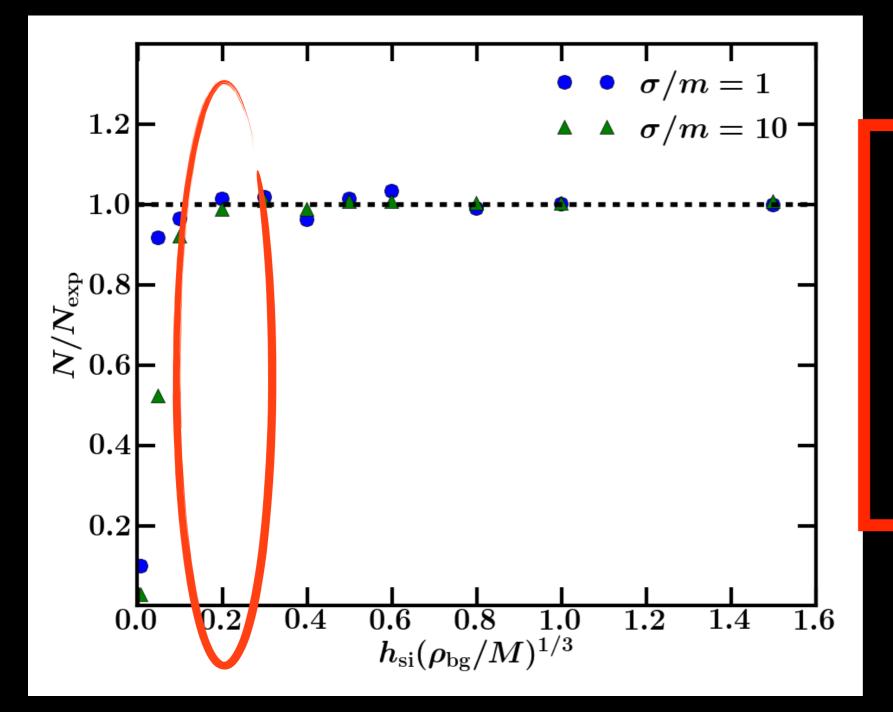
$$f(i|j) = P(j|i)$$

Wind Tunnel Test



Interaction rate converges to the expected value when h_{si} > 0.2* (the interparticle separation)

Wind Tunnel Test



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Wind Tunnel Test

