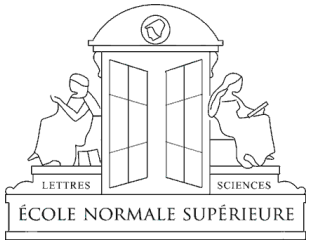

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AMR and GRAVITY in RAMSES

Patrick Hennebelle

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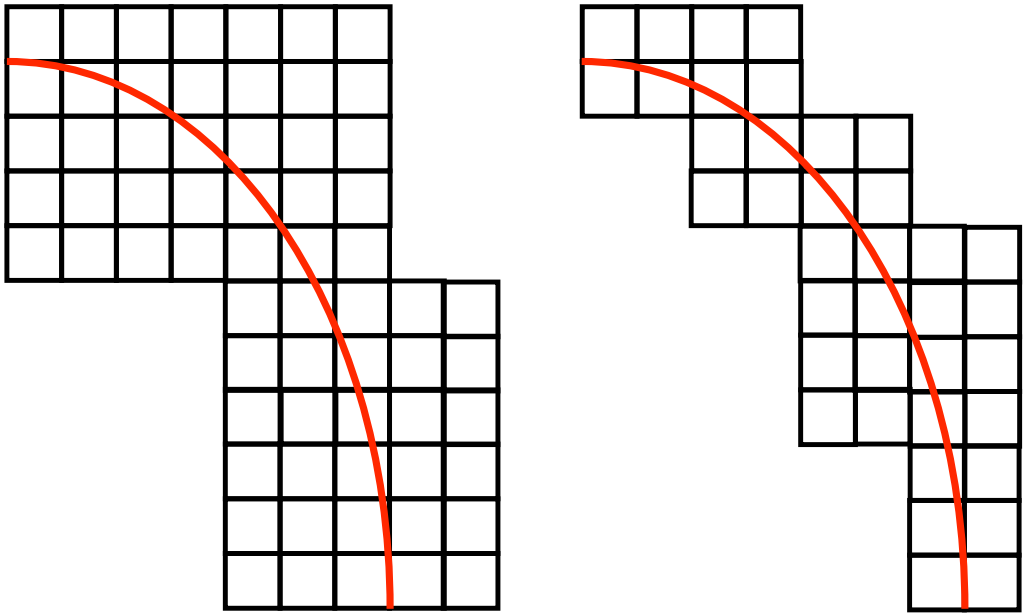
Patch-based versus tree-based



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Graded Octree structure

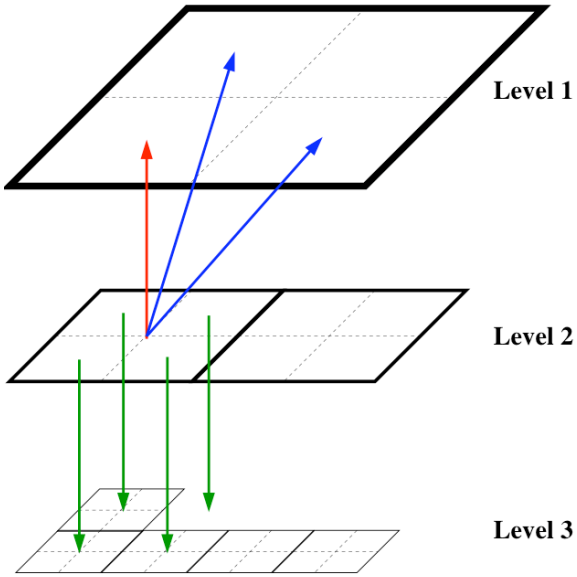
Fully Threaded Tree (Khokhlov 98).

Cartesian mesh refined on a *cell by cell basis.*

`octs`: small grid of 8 cells

Pointers (arrays of index)

- 1 parent cell
- 6 neighboring parent cells
- 8 children octs
- 2 linked list indices



Cell-centered variables are updated level by level using linked lists.

Cost = 2 integer per cell.

Optimize mesh adaptation to complex flow geometries, but CPU overhead compared to unigrid can be as large as 50%.

2 type of cell: “leaf” or active cell
“split” or inactive cell

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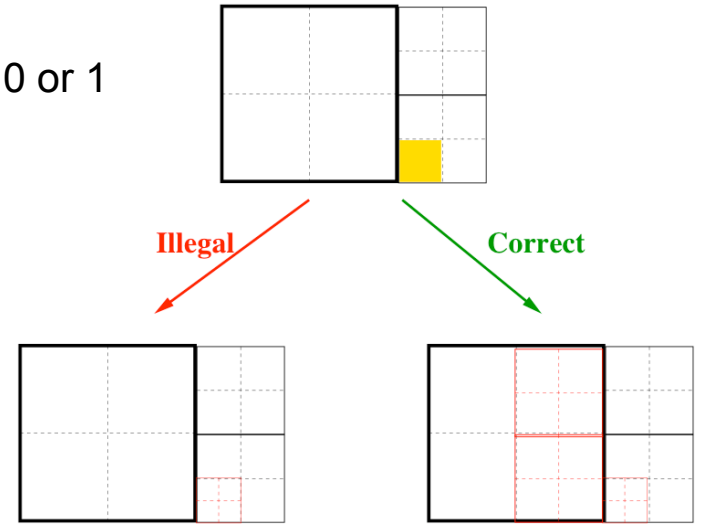
Refinement rules for graded octree

Compute the refinement map: flag = 0 or 1



Step 1: mesh consistency

if a split cell contains at least one split or marked cell, then mark the cell with flag = 1 and mark its 26 neighbors

Step 2: physical criteria

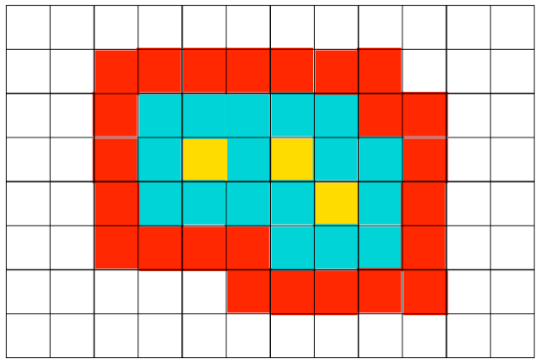
quasi-Lagrangian evolution, Jeans mass

geometrical constraints (zoom)

Truncation errors, density gradients…

Step 3: mesh smoothing

apply a dilatation operator (mathematical morphology) to regions marked for refinement → convex hull



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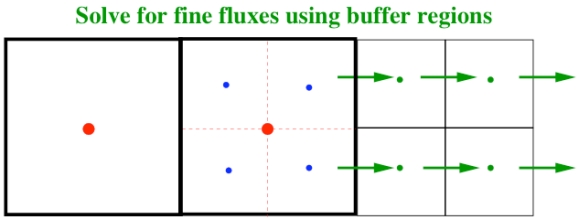
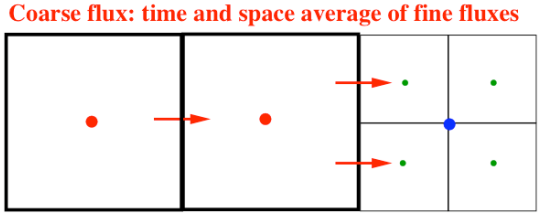
Godunov schemes and AMR

Berger & Oliger (84), Berger & Collela (89)



Prolongation (interpolation) to finer levels

- fill buffer cells (boundary conditions)
- create new cells (refinements)



Restriction (averaging) to coarser levels

- destroy old cells (de-refinements)

Flux correction at level boundary

$$(\mathbf{F}^{n+1/2,\ell}_{i+1/2,j}) = \frac{(\mathbf{F}^{n+1/2,\ell+1}_{i+1/2,j-1/4}) + (\mathbf{F}^{n+1/2,\ell+1}_{i+1/2,j+1/4})}{2}$$

Careful choice of interpolation variables (conservative or not ?)

Several interpolation strategies (with $R^T P = I$) :

- straight injection
- tri-linear, tri-parabolic reconstruction

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Godunov schemes and AMR

Buffer cells provide boundary conditions for the underlying numerical scheme. The number of required buffer cells depends on the kernel of the chosen numerical method. *The kernel is the ensemble of cells on the grid on which the solution depends*.

- First Order Godunov: 1 cell in each direction

$$u_i^{n+1} = u_i^n(1-C) + u_{i-1}^n C$$

- Second order MUSCL: 2 cells in each direction

$$u_i^{n+1} = u_i^n(1-C) + u_{i-1}^n C - \frac{C}{2}(1-C)\left(\Delta u_i - \Delta u_{i-1}\right) = 0$$

- Runge-Kutta or PPM: 3 cells in each direction

Simple octree AMR requires 2 cells maximum. For higher-order schemes (WENO), we need to have a different data structure (patch-based AMR or augmented octree AMR).

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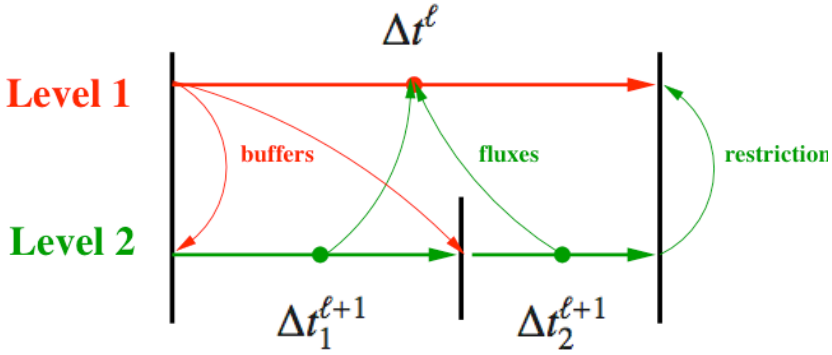
Adaptive Time Stepping



Time integration: single time step or recursive sub-cycling

- froze coarse level during fine level solves (one order of accuracy down !)
- average fluxes in time at coarse fine boundaries

$$(\mathbf{F}_{i+1/2,j}^{n+1/2,\ell}) = \frac{1}{\Delta t_1^{\ell+1} + \Delta t_2^{\ell+1}} \left(\Delta t_1^{\ell+1} \frac{(\mathbf{F}_{i+1/2,j-1/4}^{n+1/4,\ell+1}) + (\mathbf{F}_{i+1/2,j+1/4}^{n+1/4,\ell+1})}{2} + \Delta t_2^{\ell+1} \frac{(\mathbf{F}_{i+1/2,j-1/4}^{n+3/4,\ell+1}) + (\mathbf{F}_{i+1/2,j+1/4}^{n+3/4,\ell+1})}{2} \right)$$

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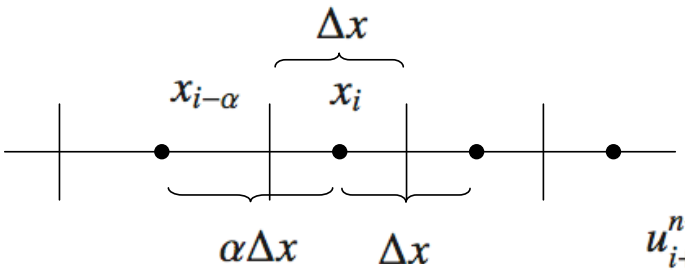
The AMR catastrophe



Assume a and C>0.

$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+a\frac{u_{i+1/2}^{n+1/2}-u_{i-1/2}^{n+1/2}}{\Delta x}=0$$

$$u_{i+1/2}^{n+1/2}=u_i^n+(1-C)\frac{\Delta x}{2}\left(\frac{\partial u}{\partial x}\right)_i$$

$$u_{i-1/2}^{n+1/2}=u_{i-\alpha}^n+(2\alpha-1-C)\frac{\Delta x}{2}\left(\frac{\partial u}{\partial x}\right)_{i-1}$$

First order scheme: $$\left(\frac{\partial u}{\partial t}\right)+\alpha a\left(\frac{\partial u}{\partial x}\right)=a\frac{\Delta x}{2}(\alpha^2-C)\left(\frac{\partial^2 u}{\partial x^2}\right)+O(\Delta t^2,\Delta x^2)$$

Second order scheme: $$\left(\frac{\partial u}{\partial t}\right)+a\left(\frac{\partial u}{\partial x}\right)=a\frac{\Delta x}{2}(\alpha-C)(1-\alpha)\left(\frac{\partial^2 u}{\partial x^2}\right)+O(\Delta t^2,\Delta x^2)$$

At level boundary, we loose one order of accuracy in the modified equation.

First order scheme: the AMR extension is *not consistent* at level boundary.

Second order scheme: for α=1.5, AMR is *unstable* at level boundary.

Solutions: 1- refine gradients, 2- enforce first order, 3- add artificial diffusion

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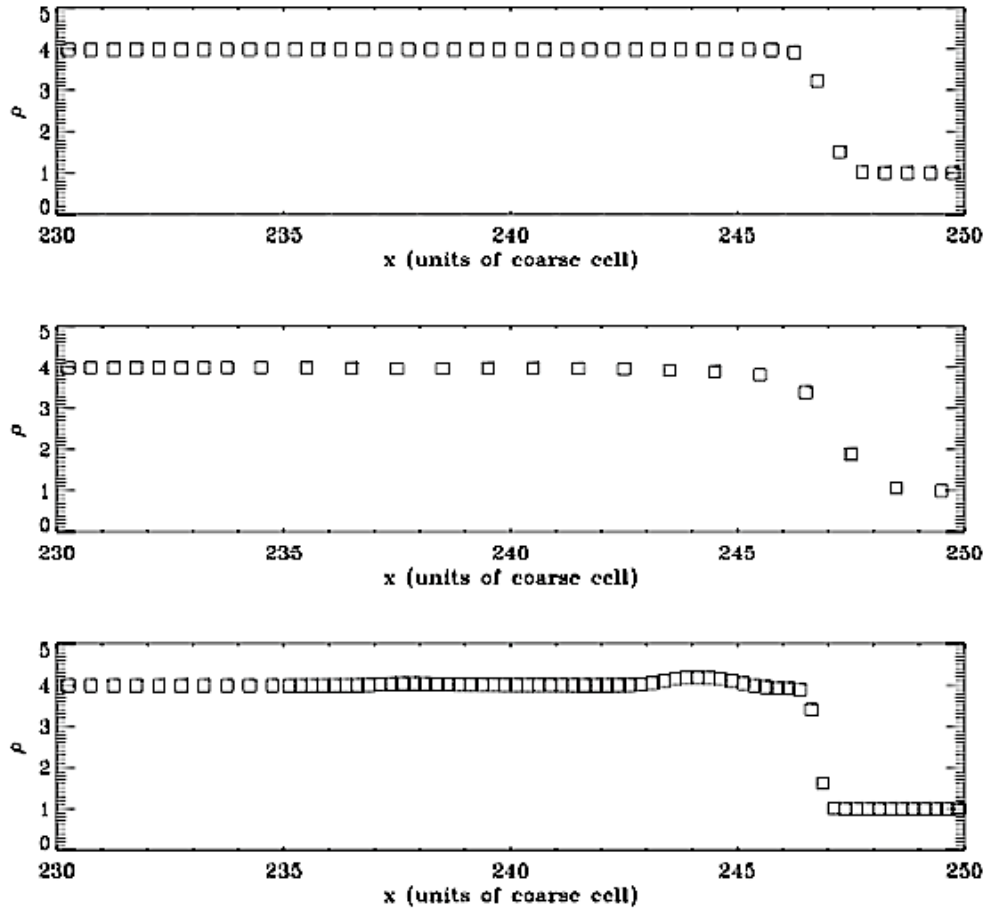
Shock wave propagating through level boundary



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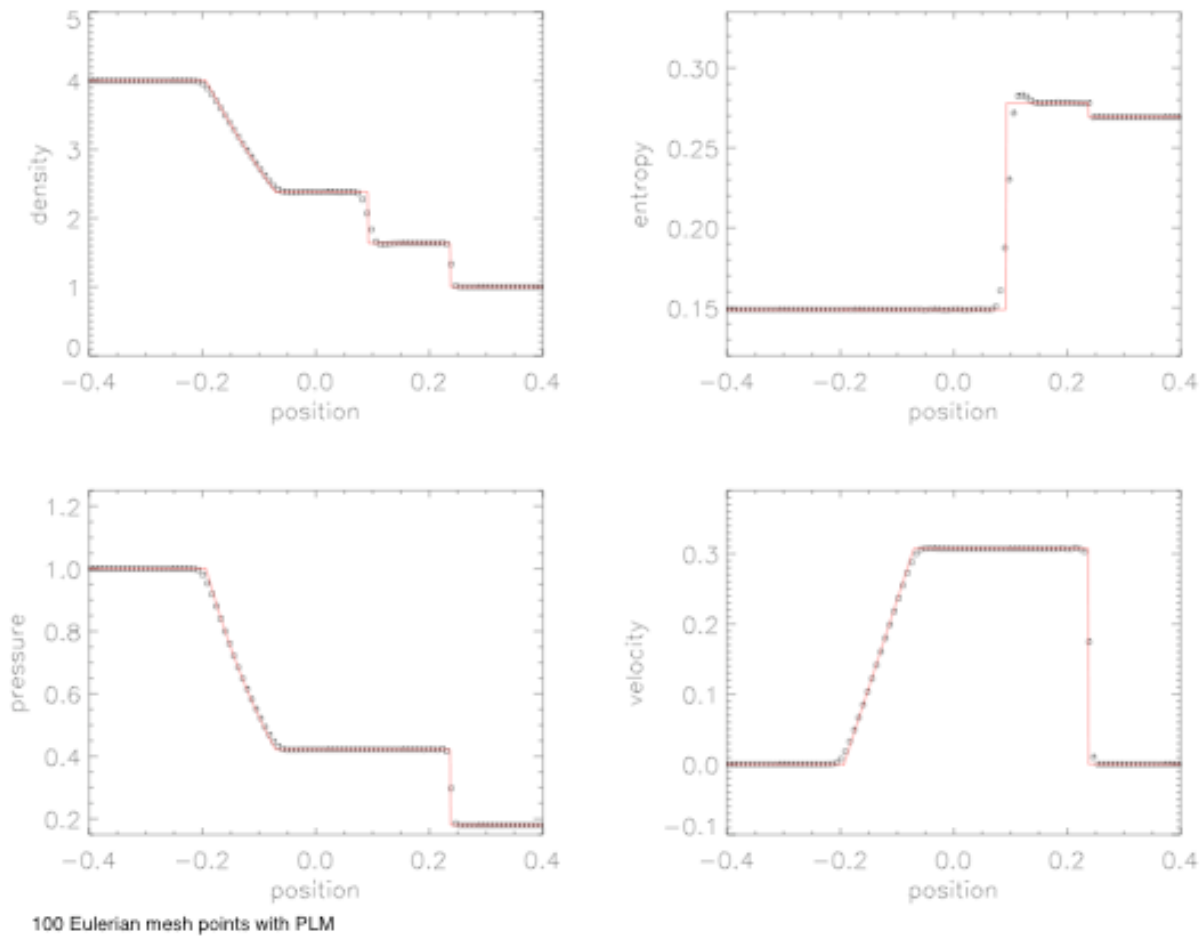
Shock-tube test with regular grid



100 Eulerian mesh points with PLM

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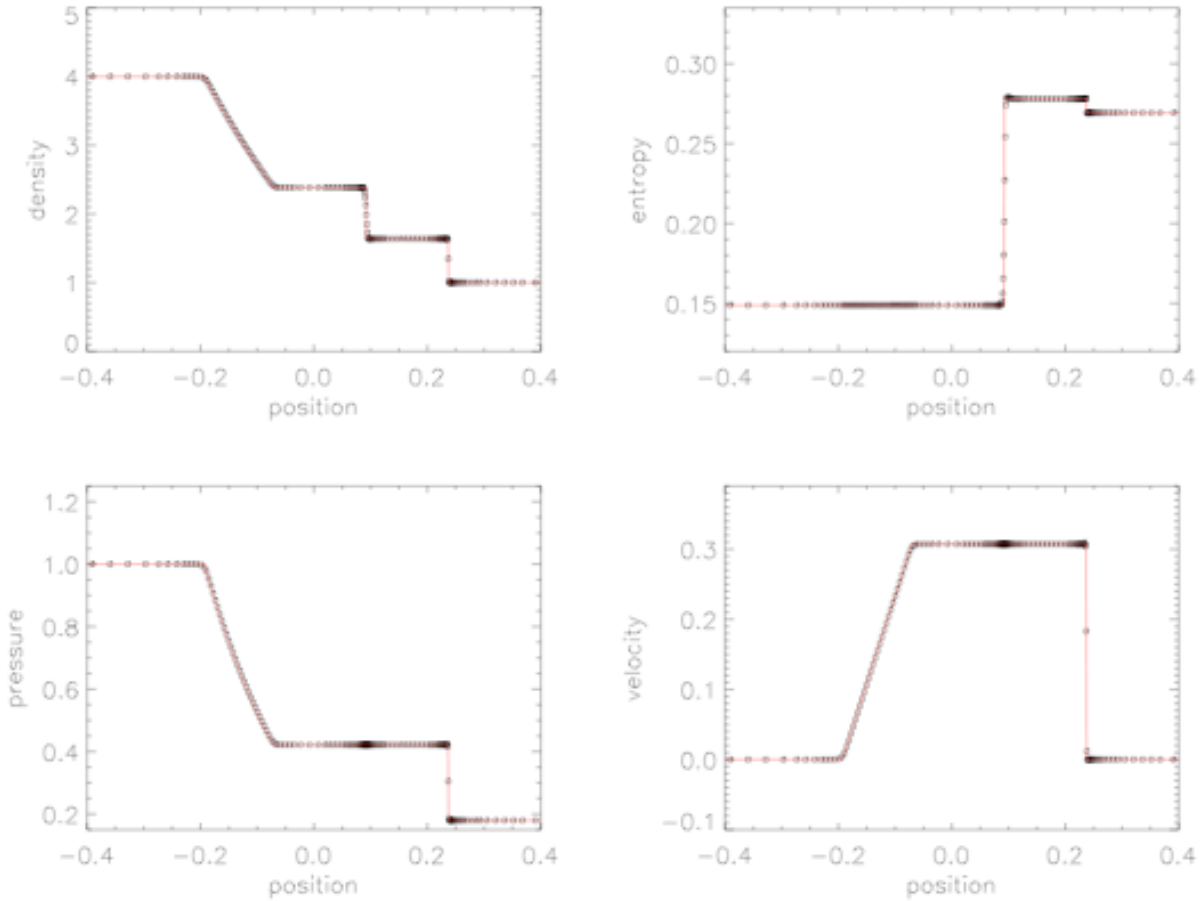
Shock-tube test with AMR



120 AMR mesh points with RAMSES

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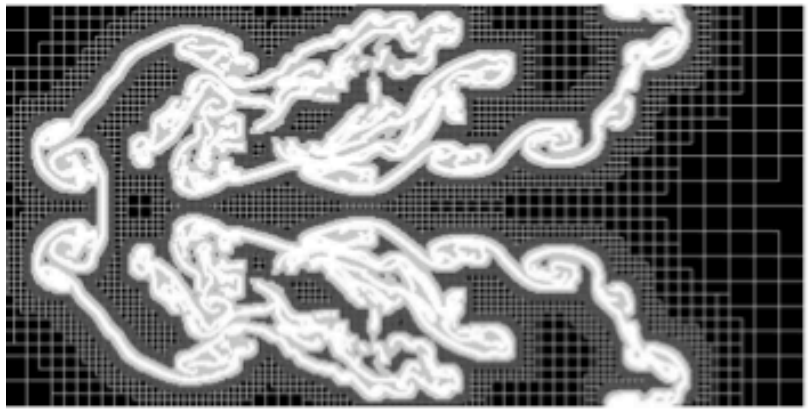
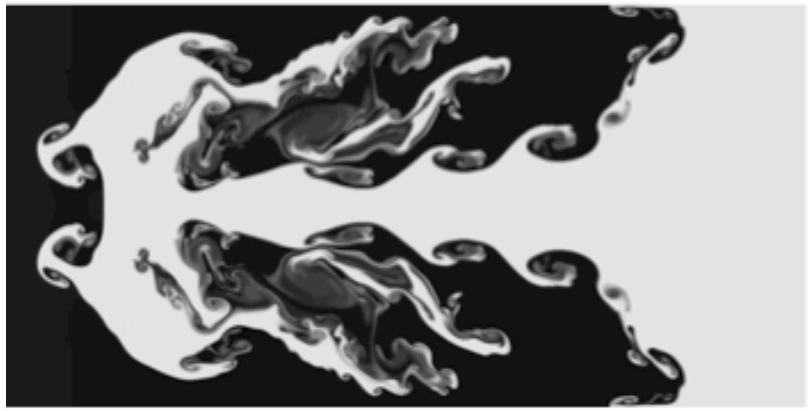
Complex geometry with AMR





Maximum numerical dissipation occurs at the 2 fluids interface.

The optimal refinement strategy is based on density gradients.

The number of required cells is directly related to the *fractal exponent* n of the 2D surface.

$$N_{cell} \propto (\Delta x)^{-n}$$

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Cosmology with AMR

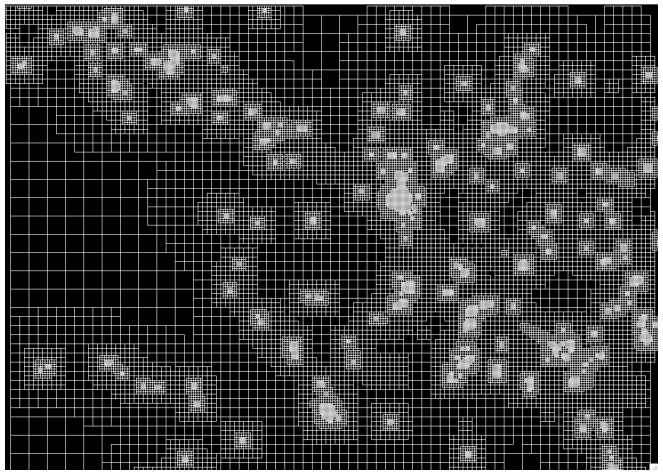
Particle-Mesh on AMR grids:
Cloud size equal to the local mesh spacing

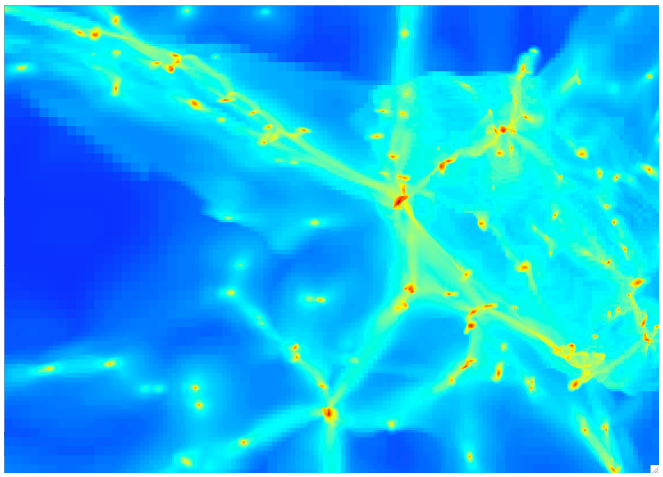
Poisson solver on the AMR grid
Multigrid or Conjugate Gradient
Interpolation to get Dirichlet boundary conditions (one way interface)

Quasi-Lagrangian mesh evolution:
roughly constant number of particles per cell

$$n = \frac{\rho_{DM}}{m_{DM}} + \frac{\rho_{gas}}{m_{gas}} + \frac{\rho_*}{m_*}$$

Trigger new refinement when n > 10-40 particles. The fractal dimension is close to 1.5 at large scale (filaments) and is less than 1 at small scales (clumps).





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RAMSES: a parallel graded octree AMR

- Tree-based AMR (octree structure) : the cartesian mesh is recursively refined *on a cell by cell basis*.
- Full connectivity : each "oct" have direct access to neighboring parent cells and to children "octs". (memory overhead : 2 integers per cell).

→ Optimize the mesh adaptivity to complex geometries, but CPU overhead can be as large as 50%.

N body module :	Particle-Mesh method on AMR grids (similar to the ART code).
	Poisson equation solved using Conjugate Gradient and Multigrid.
Hydro module :	*Unsplit* second order Godunov method : Riemann solver with
	piecewise linear reconstruction (option : MUSCL or PLMDE).
Time integration :	Single time step or W cycle (fine levels subcycling)
Other	Cooling & UV heating, Zoom simulation technology
	MPI based parallel implementation → *Space Filling Curves*

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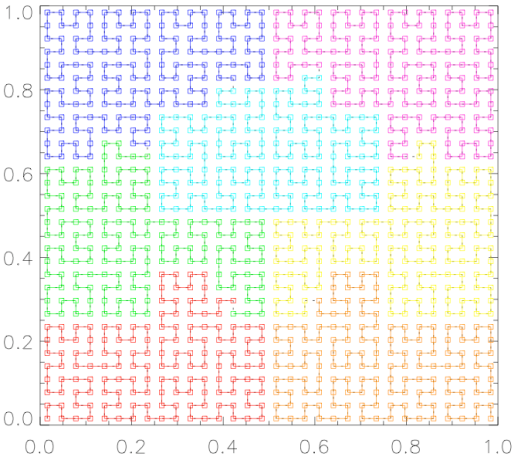
Domain decomposition for parallel computing



Parallel computing using the MPI library with a domain decomposition based on the *Peano-Hilbert curve*.

Algorithm inspired by gravity solvers (tree codes).

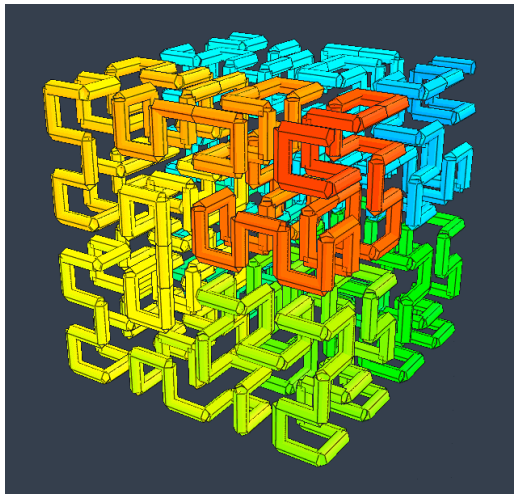
Use locally essential tree.

Tested and operational up to 6144 core.

Scaling depends on problem size and complexity.

Salmon, J.K. and Warren, M.S., "Parallel out-of-core methods for N-body simulation", In Eighth SIAM Conference on Parallel Processing for Scientific Computing, SIAM,1997.

Peter MacNeice, Kevin M. Olsonb, Clark Mobarryc, Rosalinda de Fainchteind and Charles Packer, « PARAMESH: A parallel adaptive mesh refinement community toolkit, », 2000, Computer Physics Communications, 126, 3.



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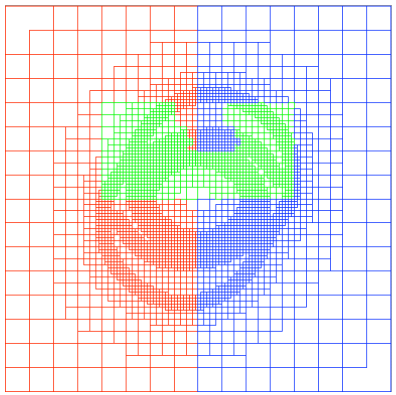
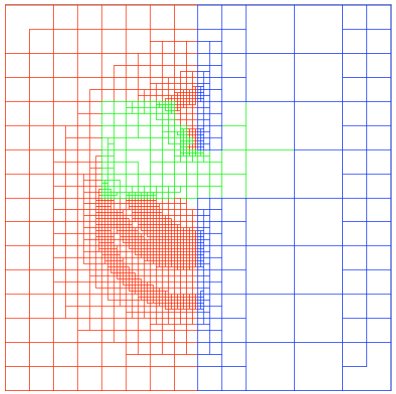
Locally essential trees

Salmon 90,
Warren 92,
Dubinsky 96

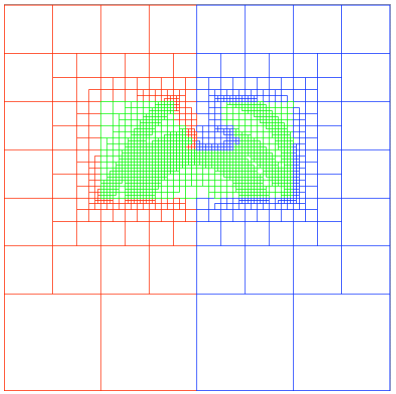


Domain decomposition over 3 processors

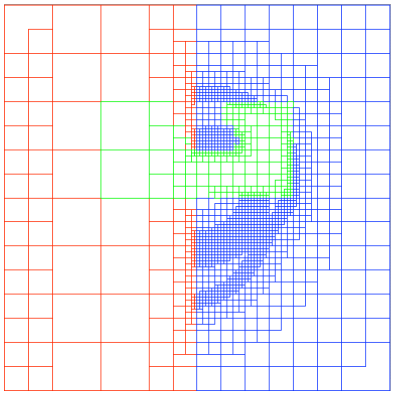
Each processor octree is surrounded by ghost cells (local copy of distant processor octrees) so that the resulting local octree contains all the necessary information.



Locally essential tree in processor #1



Locally essential tree in processor #2



Locally essential tree in processor #3

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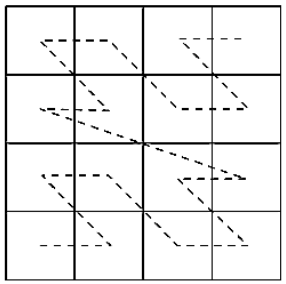
Dynamic partitioning in RAMSES

Several cell ordering methods:

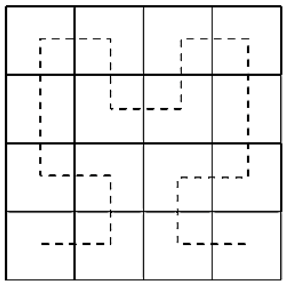
1- column, row or plane major

2- Hilbert or Morton

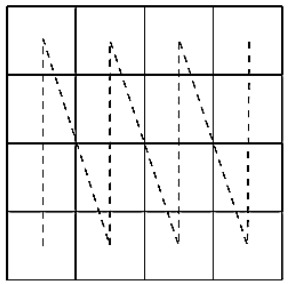
3- User defined (angular, column+Hilbert…)



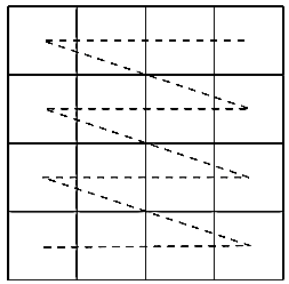
Morton



Hilbert



Column major



Row major

Dynamic partioning is performed every N steps by sorting each cell along chosen ordering and redistributing the mesh across processors. Usually, a good choice is N=10 (10% overhead).

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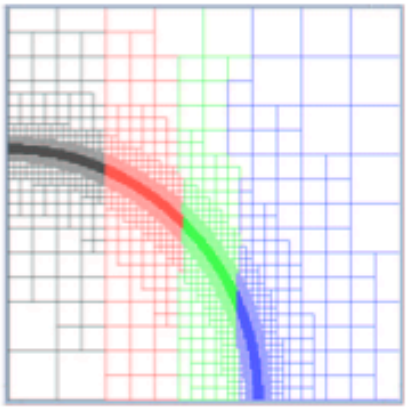
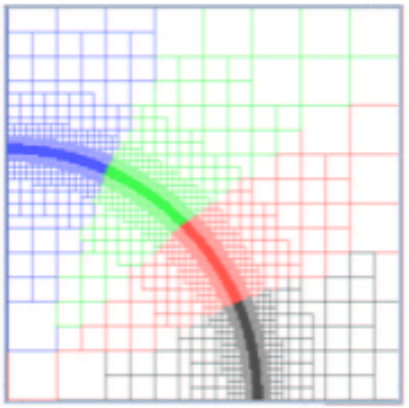
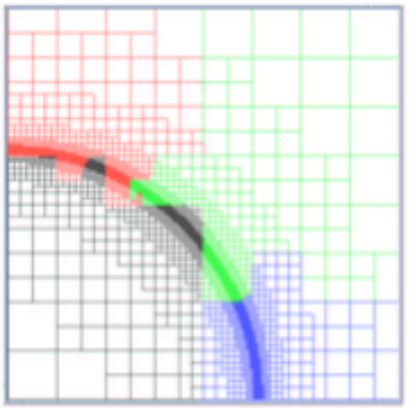
Are there any optimal load balancing strategy ?



Column major



Angular



Hilbert

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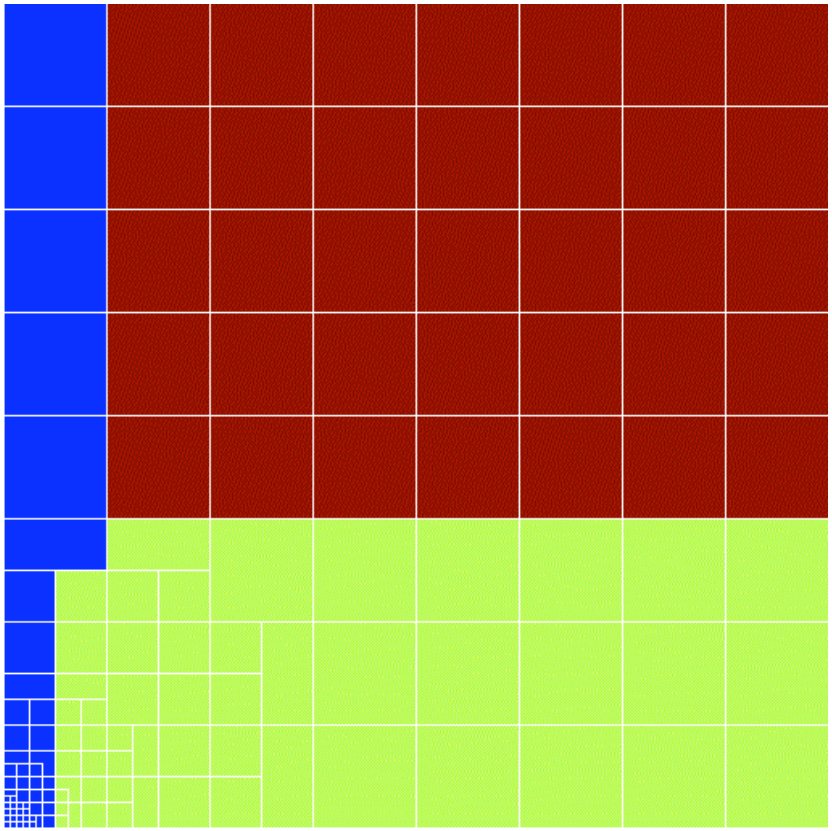
Are there any optimal load balancing strategy ?



Recursive bisection

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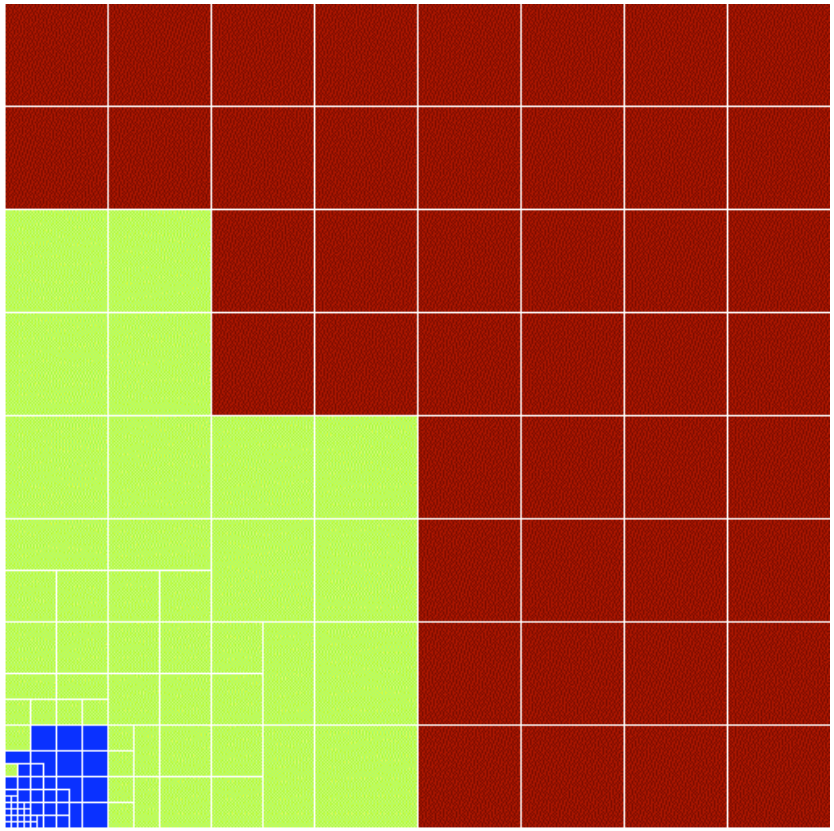
Are there any optimal load balancing strategy ?



Hilbert ordering

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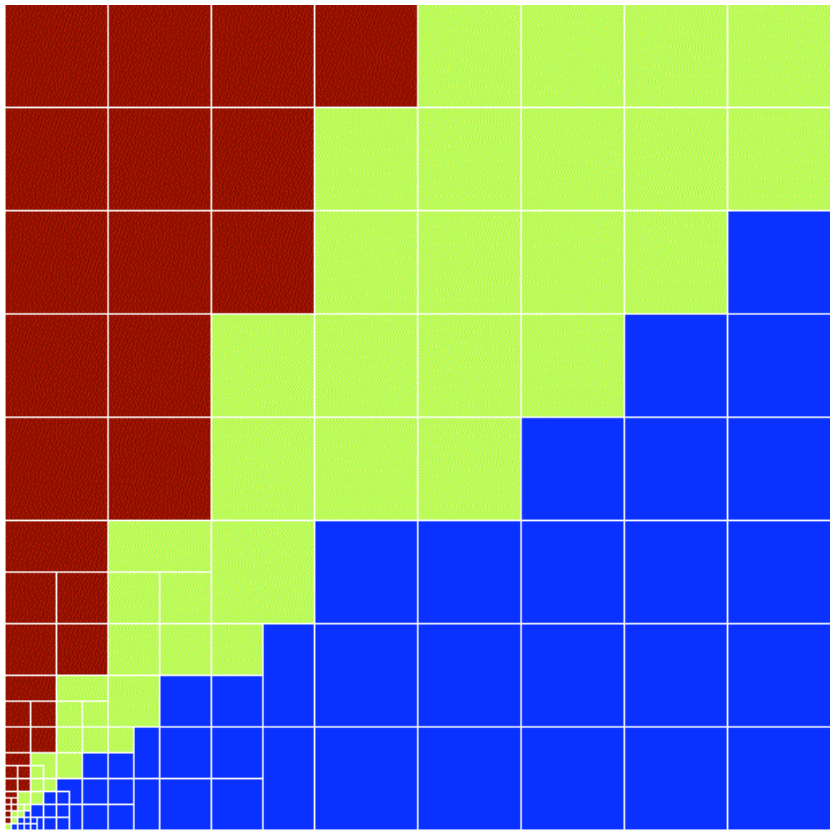
Are there any optimal load balancing strategy ?



Angular ordering

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Gravity in RAMSES

$$\Delta\Phi = 4\pi G\rho$$

$$\Rightarrow \Phi_{i+1} - 2\Phi_i + \Phi_{i-1} = 4\pi G dx^2 \rho_i \quad \text{(in 1D)}$$

$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & . & . & 0 \\ 0 & . & . & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} \Phi_1 \\ . \\ . \\ \Phi_n \end{pmatrix} = 4\pi G dx^2 \begin{pmatrix} \rho_1 \\ . \\ . \\ \rho_n \end{pmatrix}$$

Because it is linear it can usually be solved efficiently in a grid using Fourier transform. However, when the mesh structure is complex, this is not a good strategy.

Gravity in RAMSES is used on the grid using iterative methods.

Two methods have been developed:

-conjugate gradient
-multi-grid (relaxion method)

The conjugate gradient method

Generic method to solve the system Ax=b when A is symmetrical.

Since A, is symmetrical, there is a basis of normal eigenvectors.

$$p_i^T A p_j = 0, i \neq j$$

$$x = \sum_{i=1}^{n} \alpha_i p_i$$

$$\Rightarrow Ax = \sum_{i=1}^{n} \alpha_i A p_i$$

$$p_k^T Ax = \sum_{i=1}^{n} \alpha_i p_k^T A p_i = \alpha_k p_k^T A p_k$$

$$\Rightarrow \alpha_k = \frac{p_k^T Ax}{p_k^T A p_k} = \frac{p_k^T b}{p_k^T A p_k}$$

If the eigenvectors of A are known, x is easily found.
However, this is quite a difficult task to get them all since A is a very big matrix.

The idea of the method is to find a good approximation by considering only a few of them.
This leads to an iterative procedure.

The conjugate gradient method

The idea is then to try minimizing the quantity: $f(x) = \frac{1}{2} x^T Ax - x^T b$

Indeed the gradient of f is Ax-b. Thus a minimum of f corresponds to a solution.

Let us start with some guess x_0. We can calculate the residual:

$r_0 = b - Ax_0$ which is the gradient of f in x_0. More generally if we have an approximation x_k, the residual is simply $r_k = b - Ax_k$

If we were trying to use to minimize the system directly along the gradient, the algorithm would be very inefficient. Instead one can construct a new direction that will be orthogonal. It can be shown that the vectors

$$p_k = r_k - \sum_{i<k} \frac{p_i^T A r_k}{p_i^T A p_i} p_i$$

are all orthogonal to each other and thus provide an efficient set of direction for minimisation.

The new value of x is minimised along p_k. It is going to be efficient because the direction along which it is done is perpendicular to all the previous directions along which the minimisation has been done.

$$x_{k+1} = x_k + \alpha_k p_k$$

The optimal choice turns out to be:

$$\alpha_k = \frac{p_k^T b}{p_k^T A p_k}$$

This provides an efficient minimisation algorithm not too difficult to implement. In particular, there is no need to store large matrix.

The gradient conjugate method can be used to solve second order terms in other equations (e.g. heat and radiative diffusion).