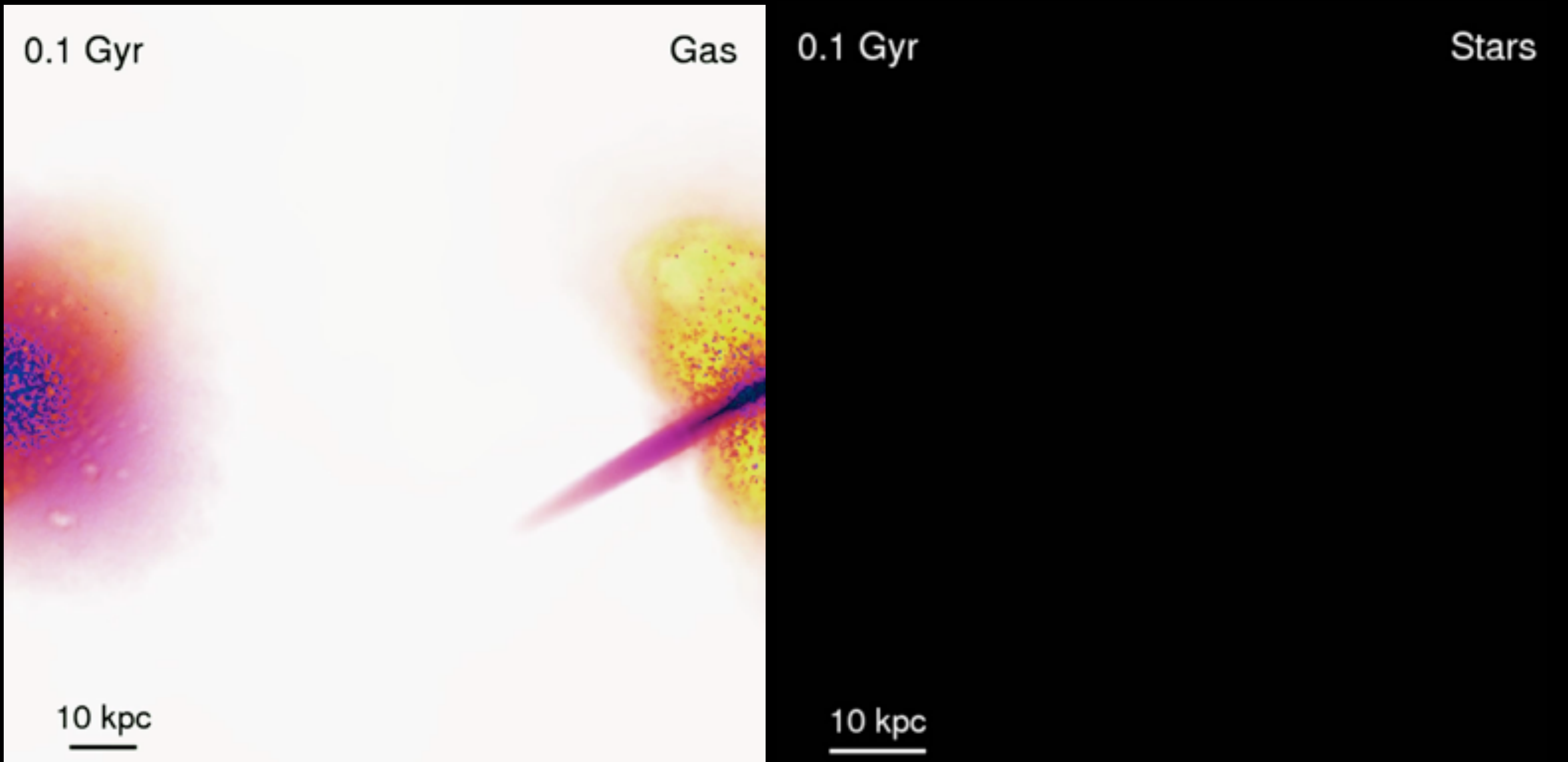
ISM Structure: Order from Chaos

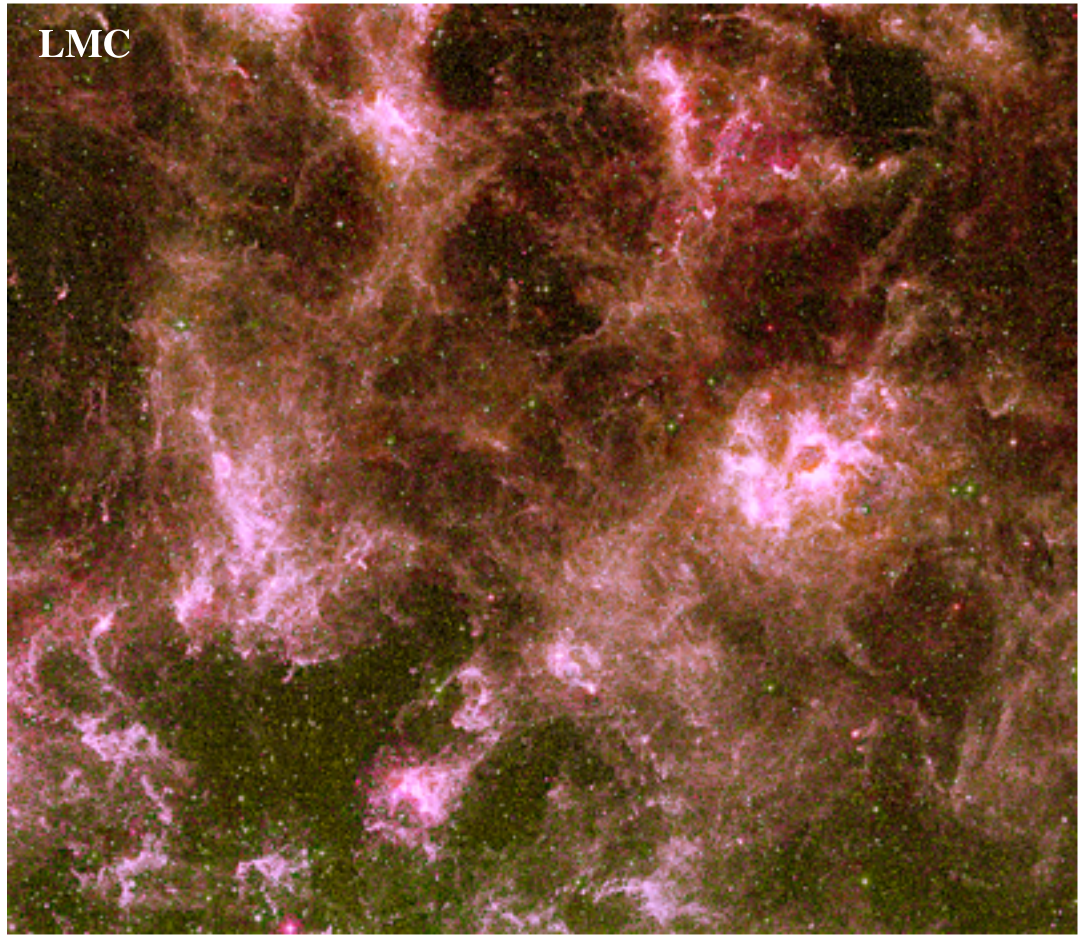


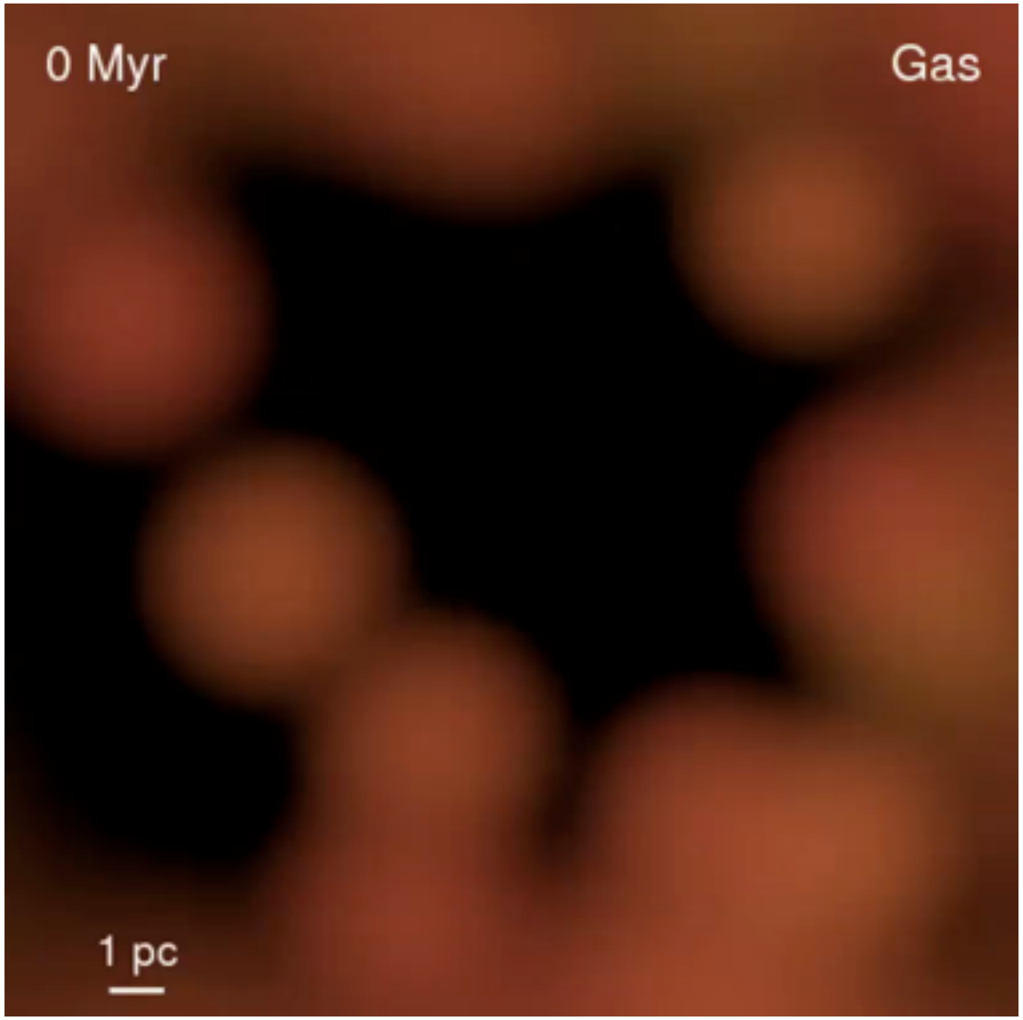
Philip Hopkins

with Eliot Quataert, Norm Murray,
Lars Hernquist, Dusan Keres, Todd Thompson, Desika Narayanan,
Dan Kasen, T. J. Cox, Chris Hayward, Kevin Bundy, & more

The Turbulent ISM

IMPORTANT ON
(ALMOST) ALL SCALES



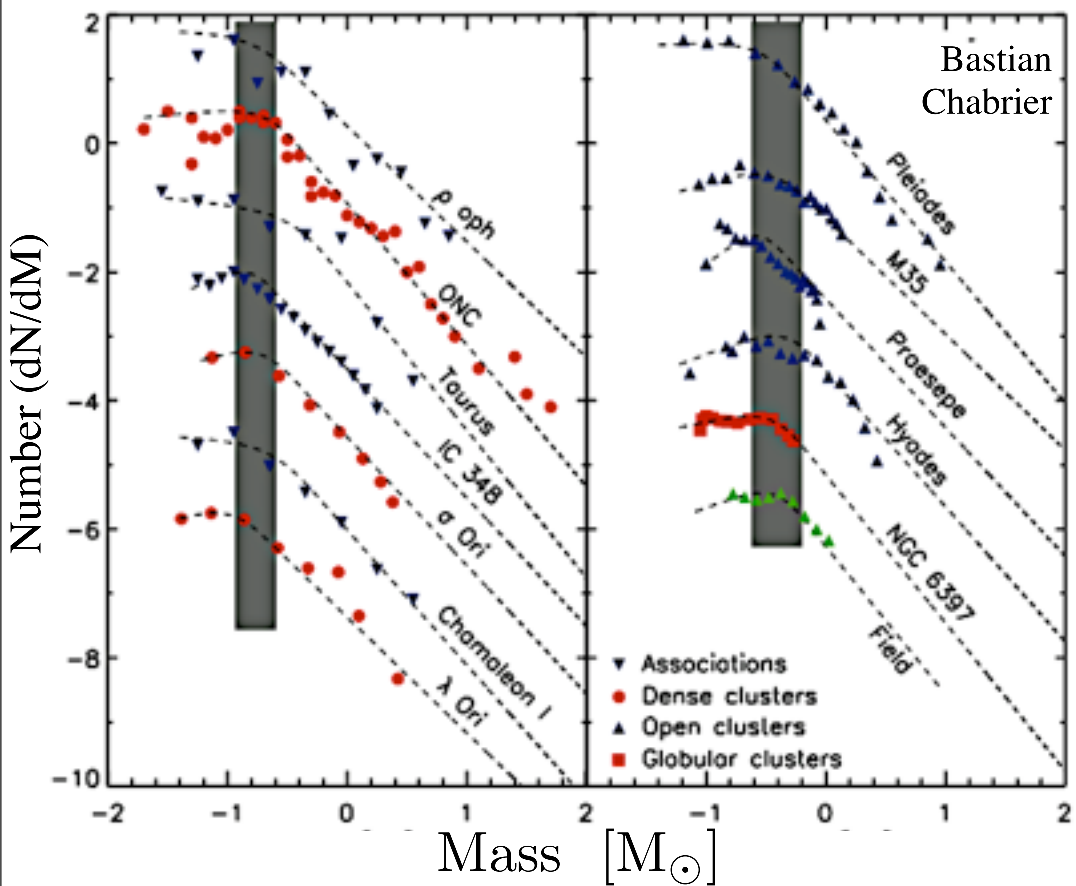


- Gravity
- Turbulence
- Magnetic, Thermal, Cosmic Ray, Radiation Pressure
- Cooling (atomic, molecular, metal-line, free-free)
- Star & BH Formation/Growth
- "Feedback": Massive stars, SNe, BHs,
  external galaxies, etc.

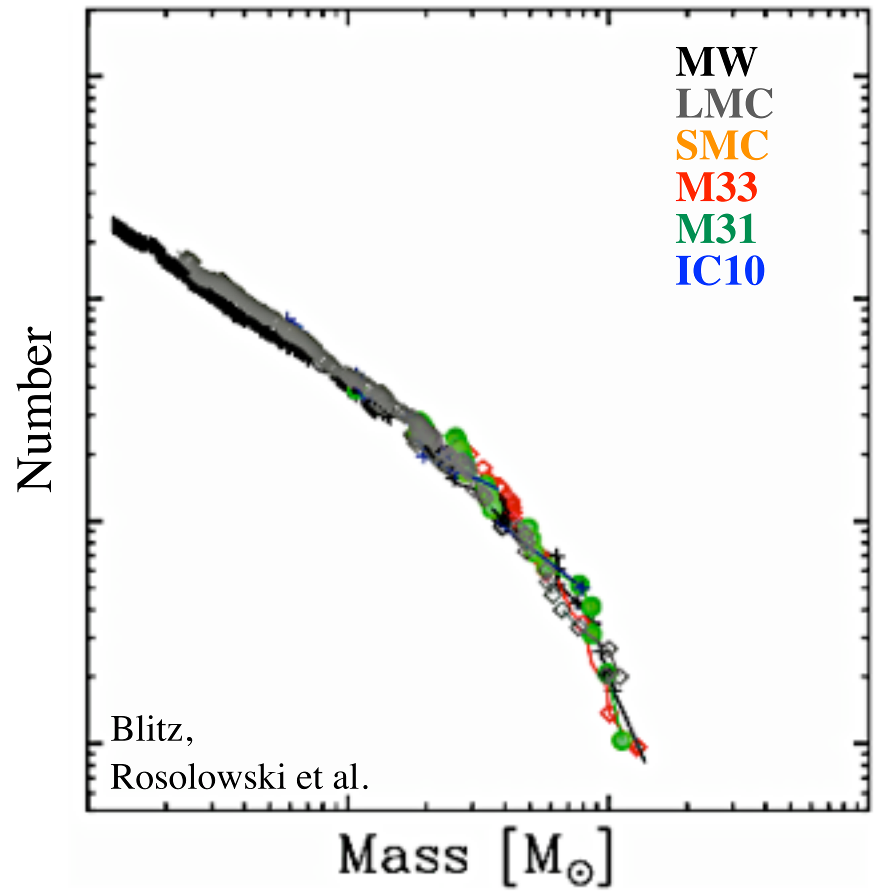
The ISM

YET THERE IS SURPRISING REGULARITY

Stars & Pre-Stellar Gas Cores:



Giant Molecular Clouds:



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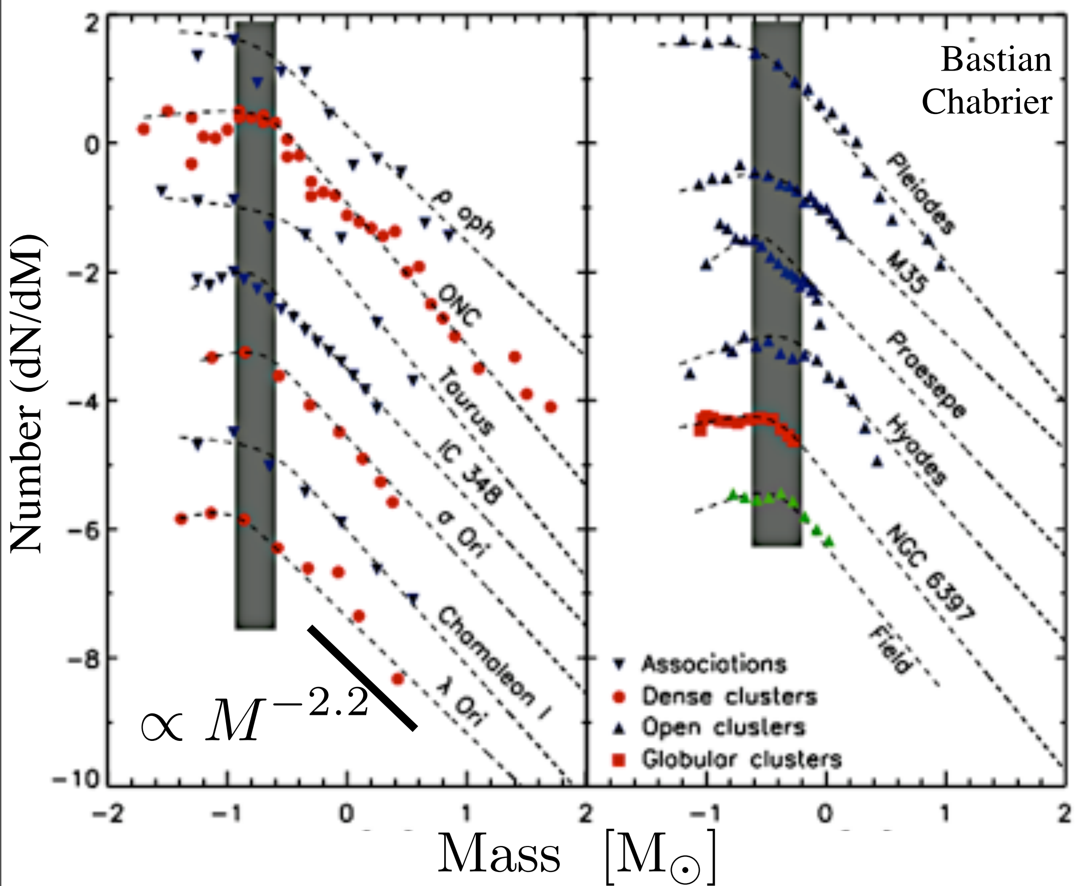

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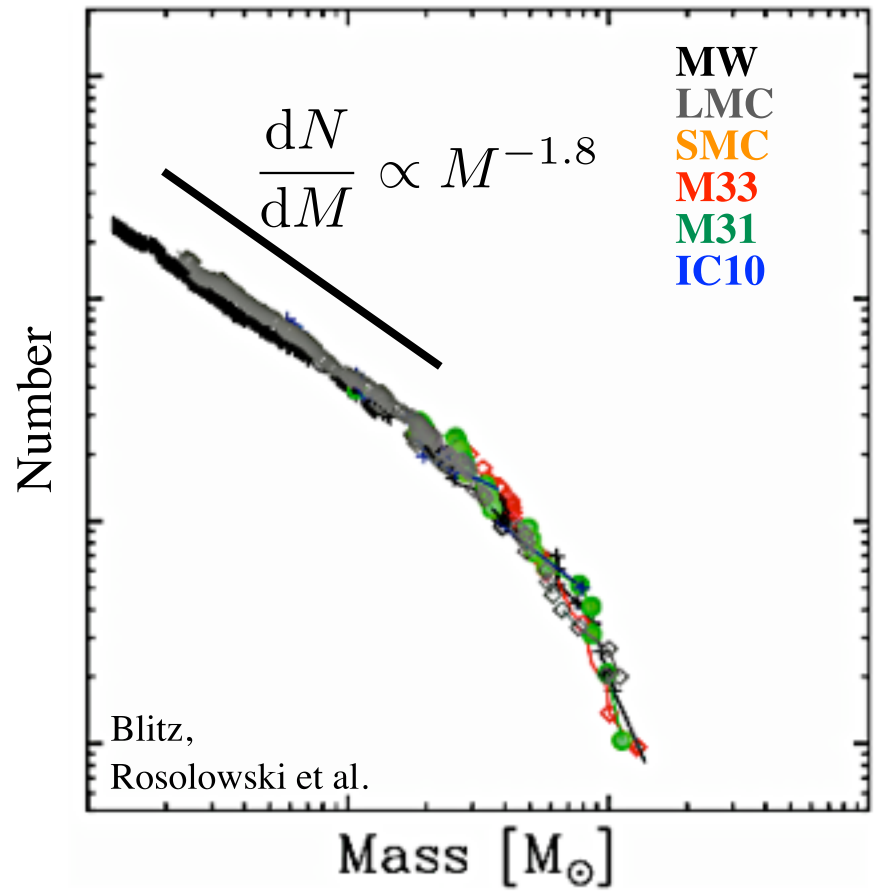

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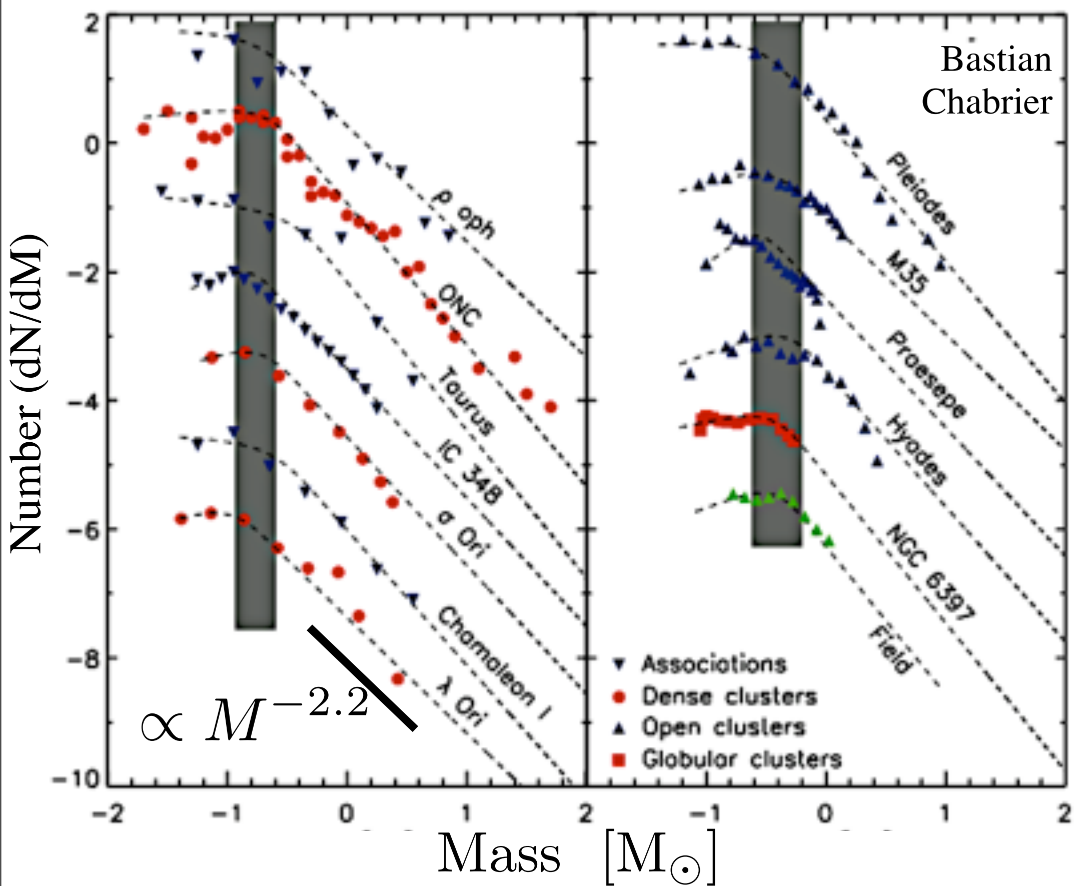

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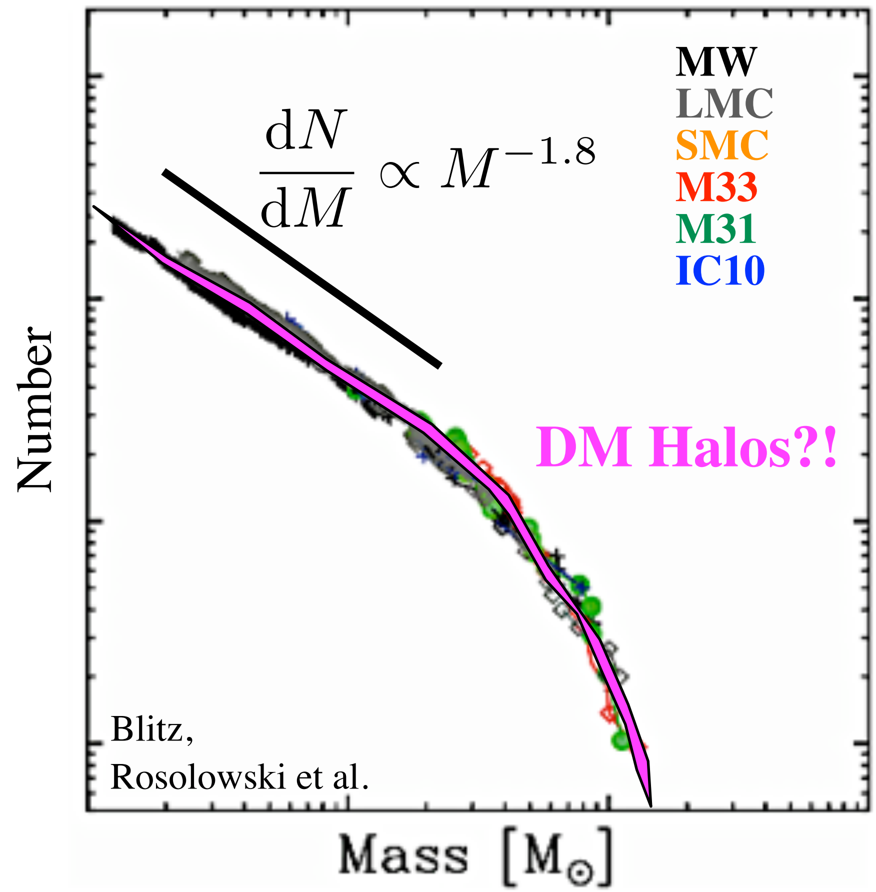
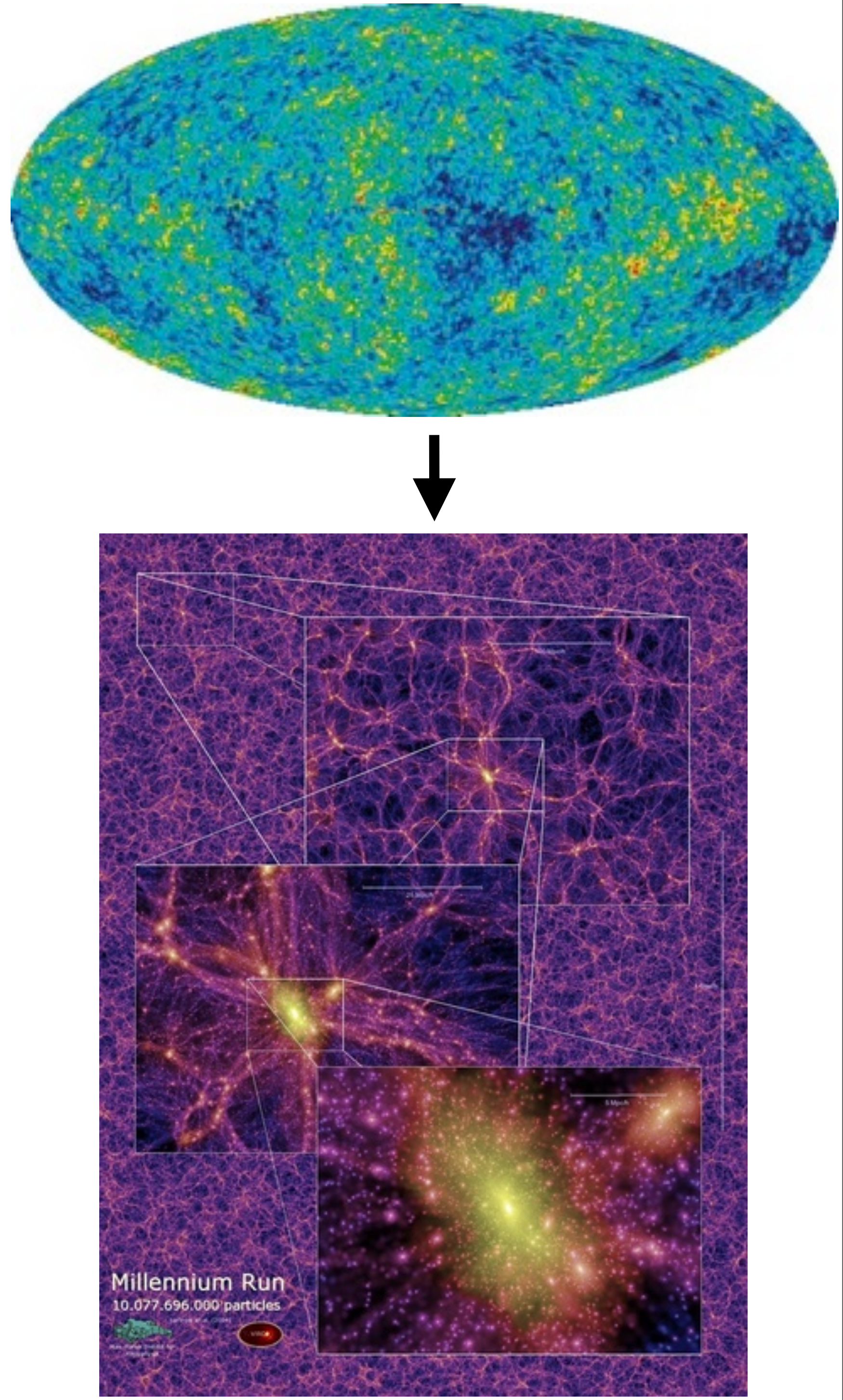


Extended Press-Schechter / Excursion-Set Formalism

- Press & Schechter '74:
  - r Fluctuations a Gaussian random field
  - Know linear power spectrum P(k~1/r): variance ~ k^3 P(k)



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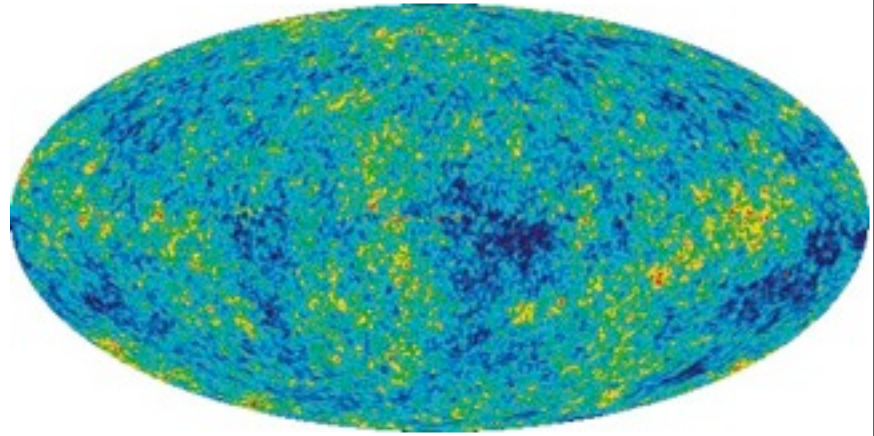

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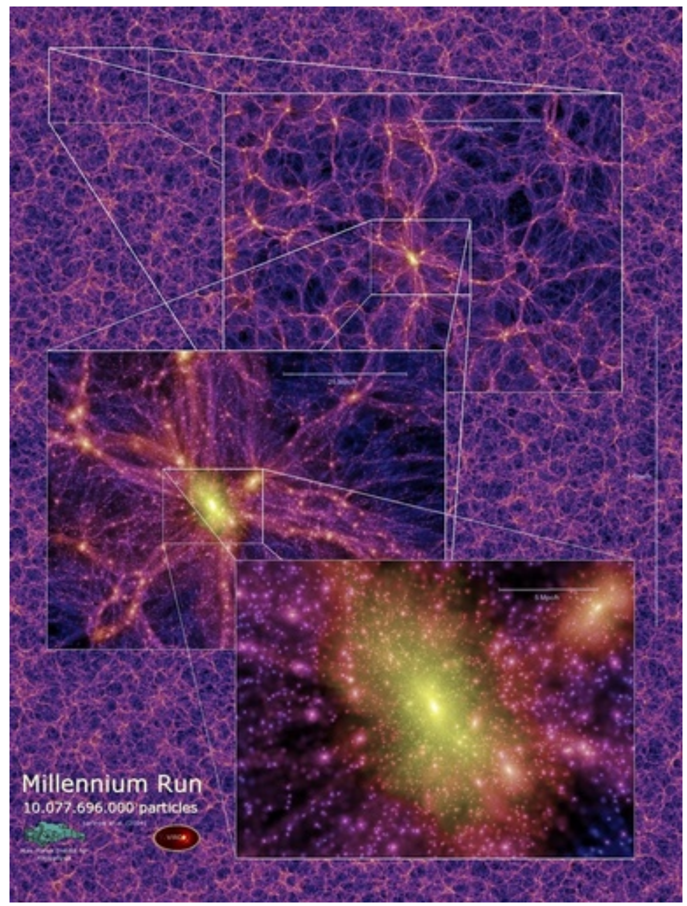

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  - Turnaround & gravitational collapse

$$\bar{\rho}(< R \sim 1/k) > \rho_{\rm crit}$$

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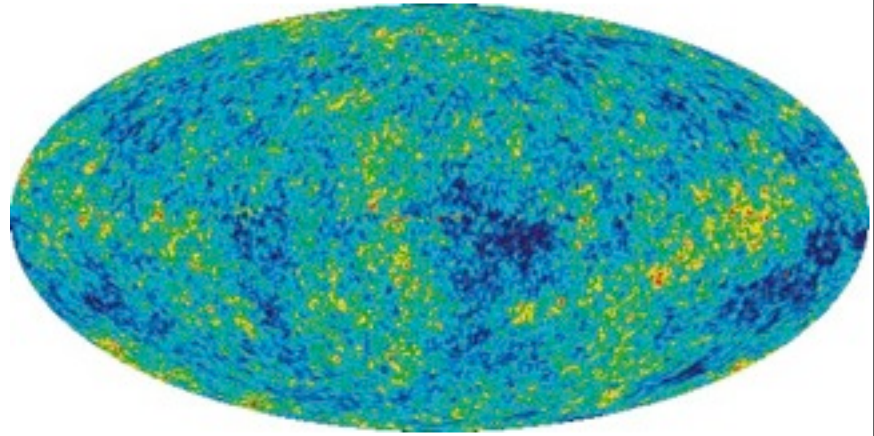

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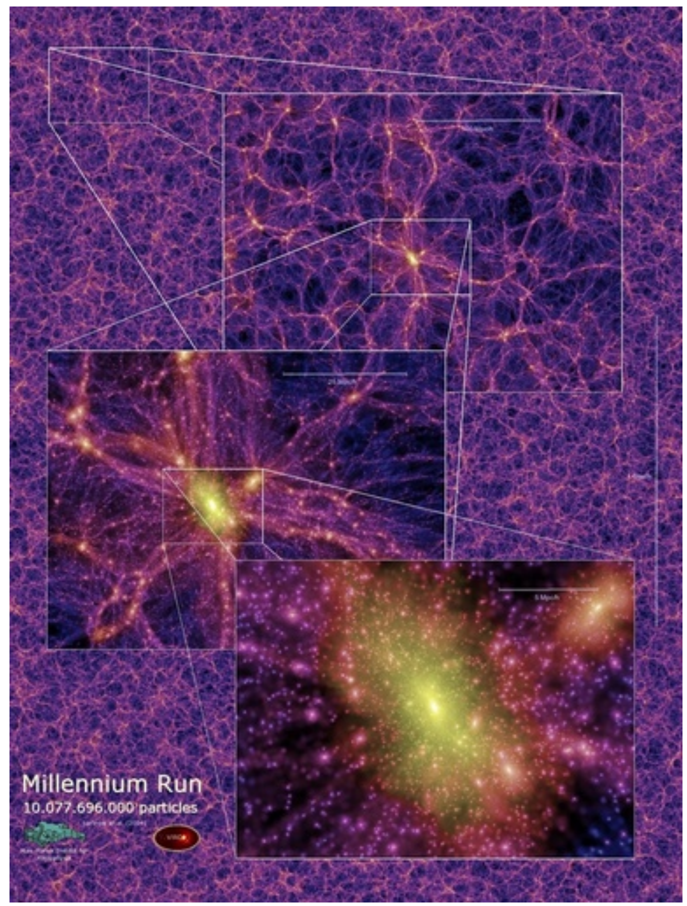

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- Generalize to conditional probabilities,
  N-point statistics, resolve “cloud in cloud” problem
  (e.g. Bond et al. 1991)

Turbulence

BASIC EXPECTATIONS

Velocity: $E(k) \propto k^{-p}$ $\quad (k\,E(k) \sim u_t(k)^2)$

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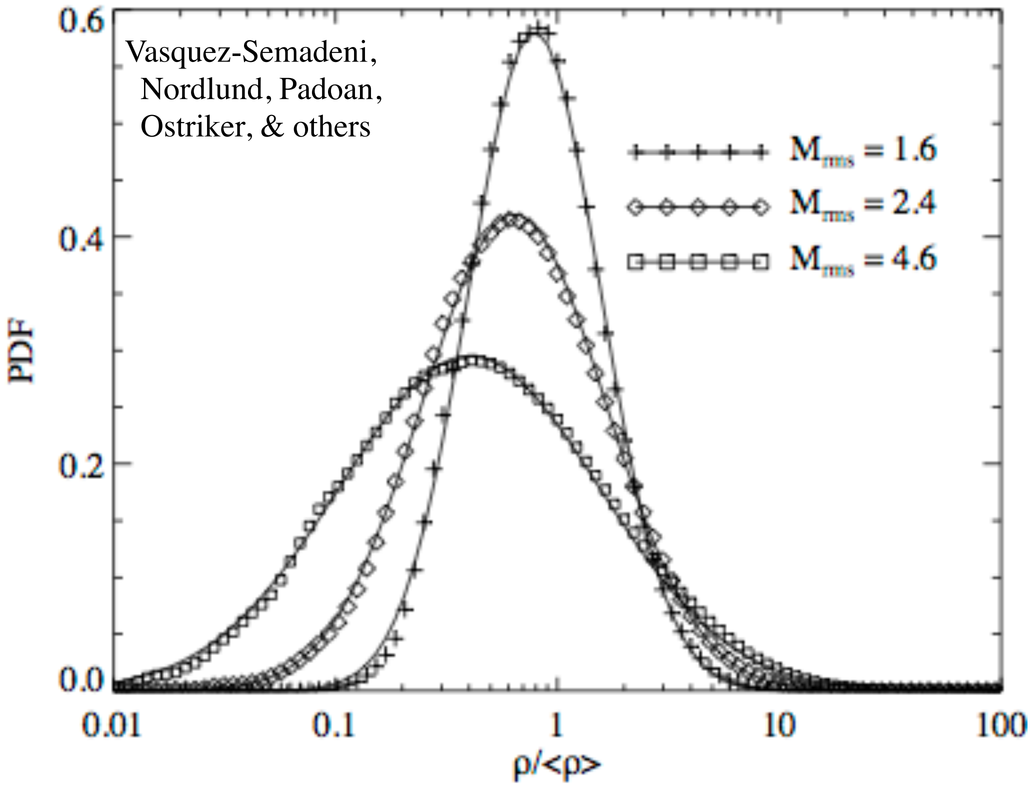

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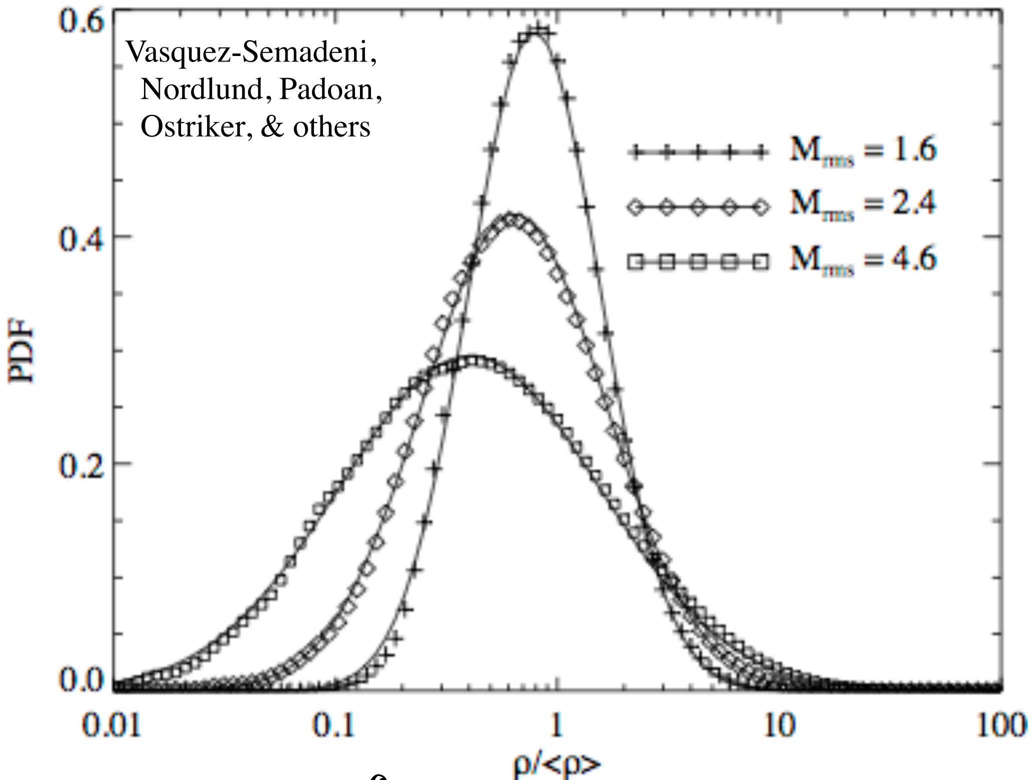

$$S(R) = \int \mathrm{d}\ln k\; S_k\,|W(k,\,R)|^2$$

What Defines a Fluctuation of Interest?

DISPERSION RELATION:

$$\omega^2 = \kappa^2 + c_s^2\, k^2 + u_t(k)^2\, k^2 - \frac{4\pi\, G\, \rho\, |k| h}{1 + |k| h}$$

Chandrasekhar ‘51, Vandervoort ‘70, Toomre ‘77

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Gravity

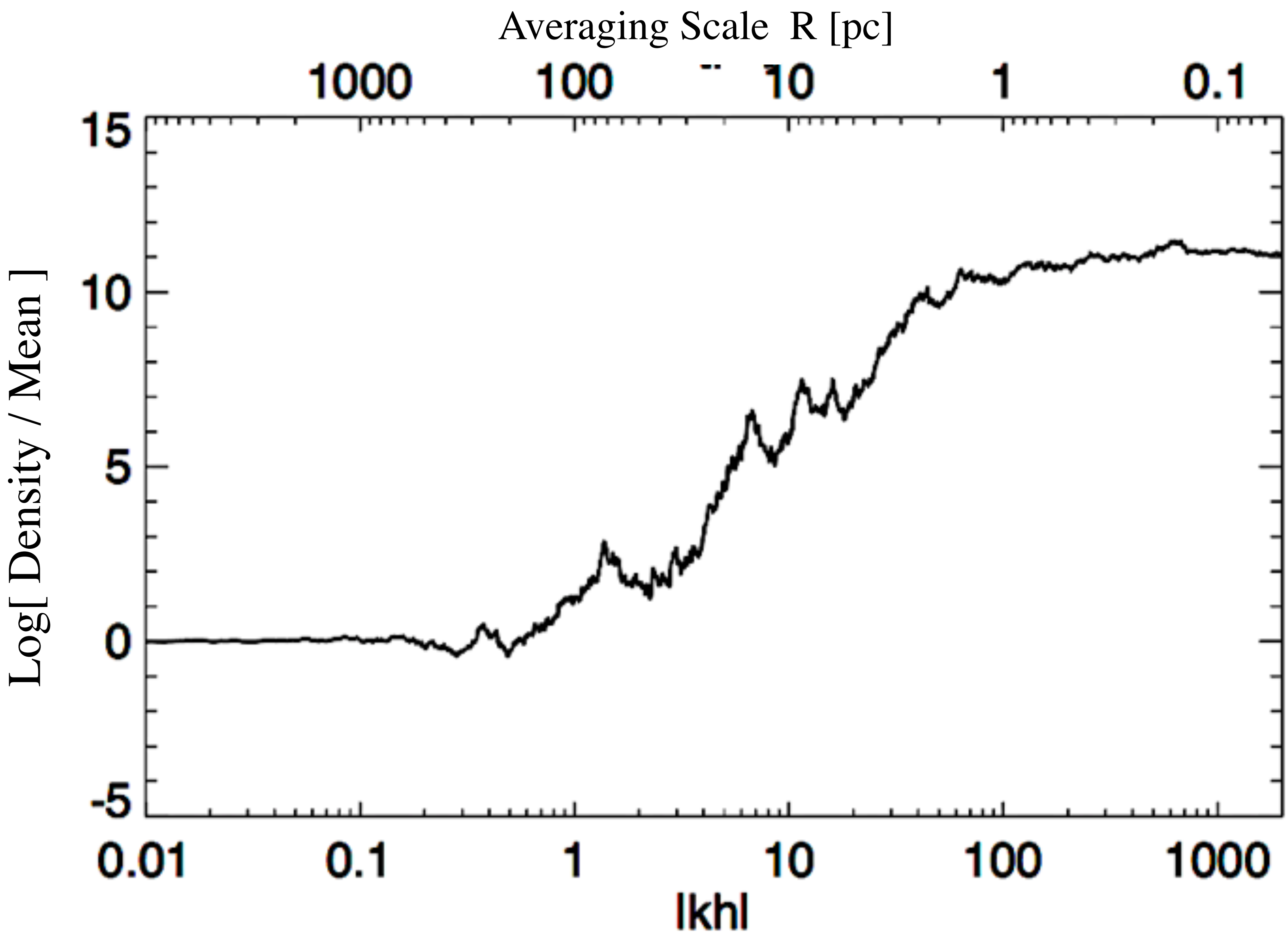
Mode Grows (Collapses) when w<0:

$$\rho > \rho_c(k) = \rho_0\,(1 + |kh|)\left[(\mathcal{M}_h^{-2} + |kh|^{1-p})\, kh + \frac{2}{|kh|}\right]$$

Chandrasekhar ‘51, Vandervoort ‘70, Toomre ‘77

“Counting” Collapsing Objects

EVALUATE DENSITY FIELD vs. “BARRIER”



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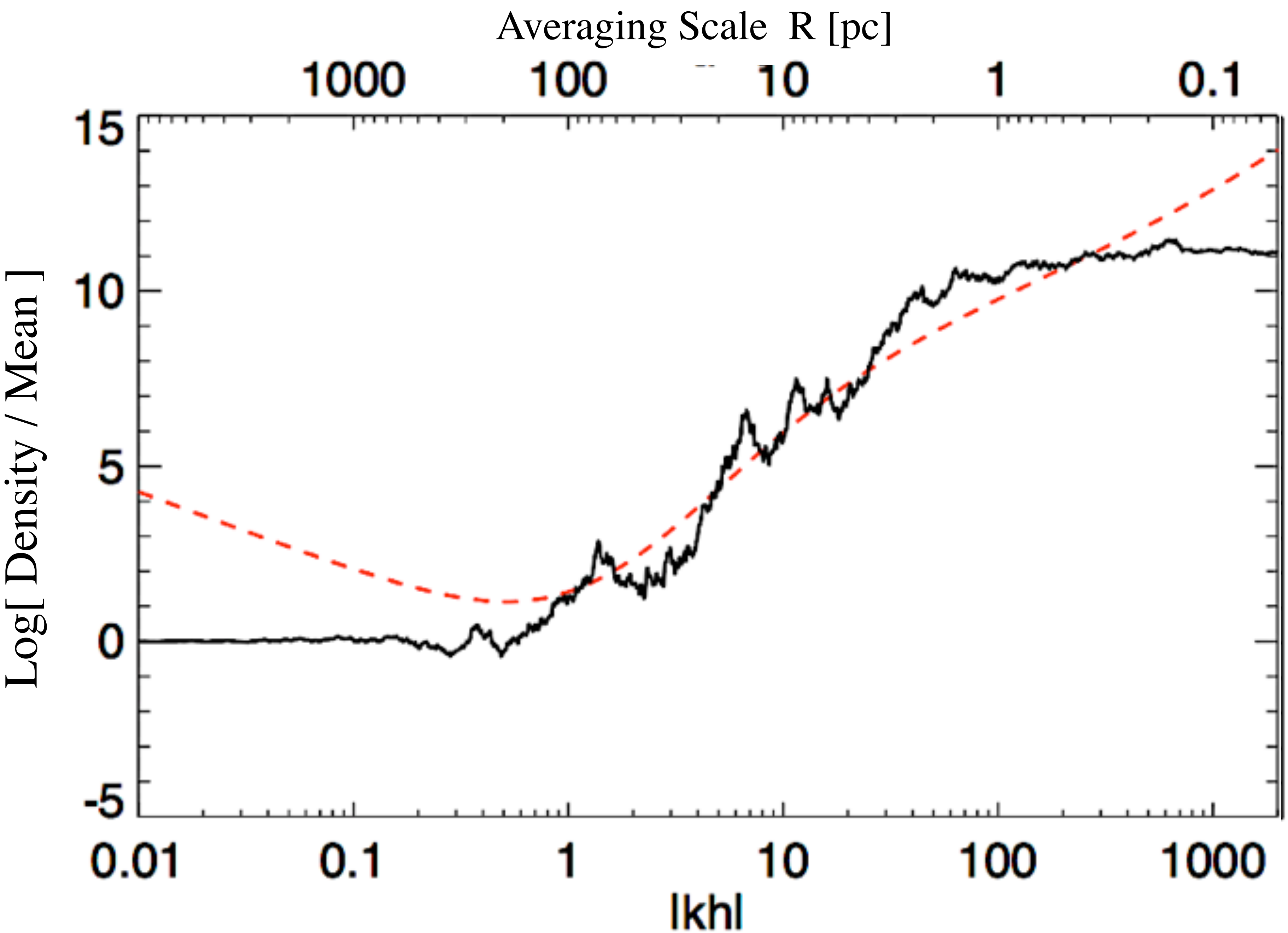

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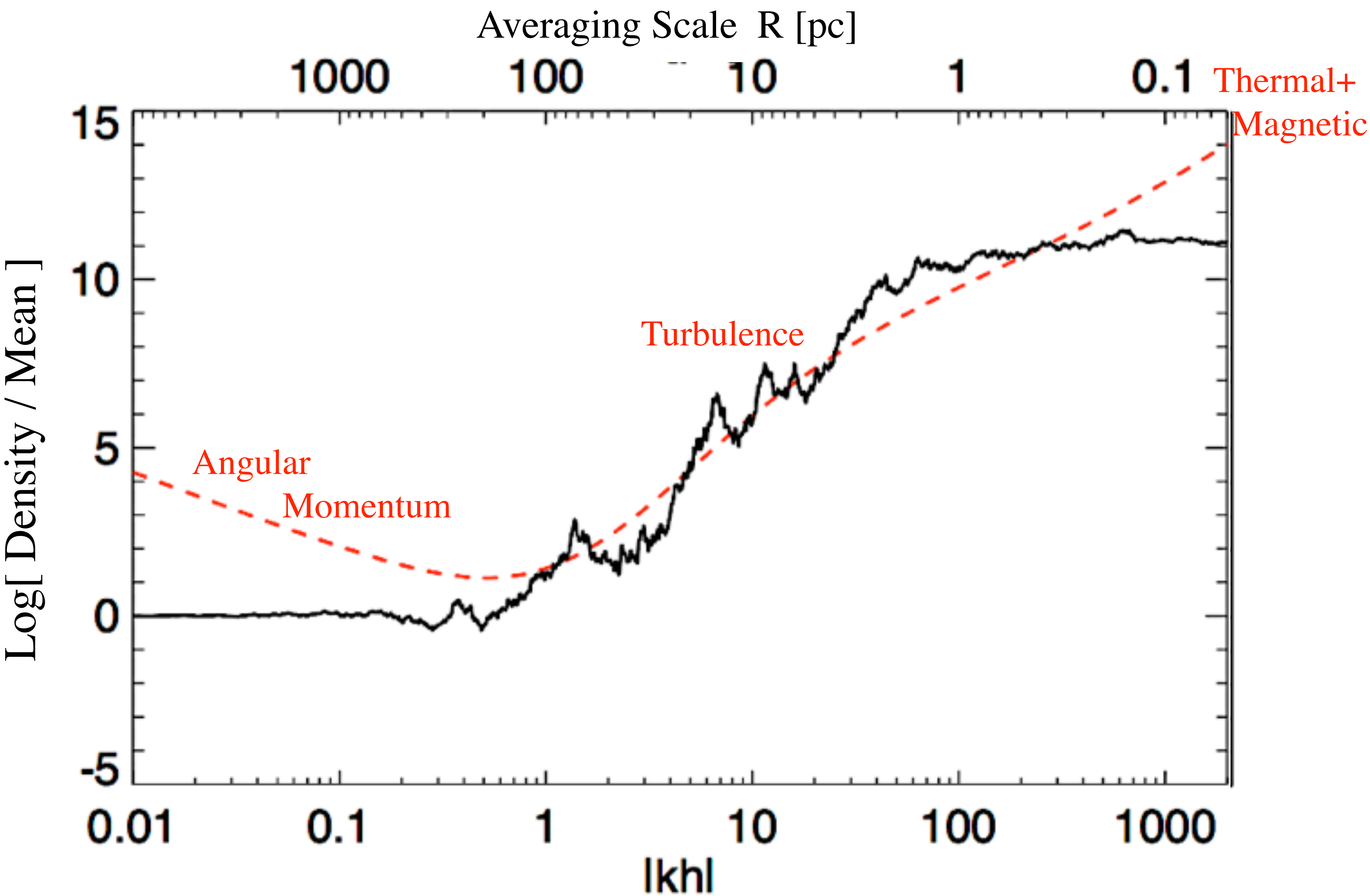

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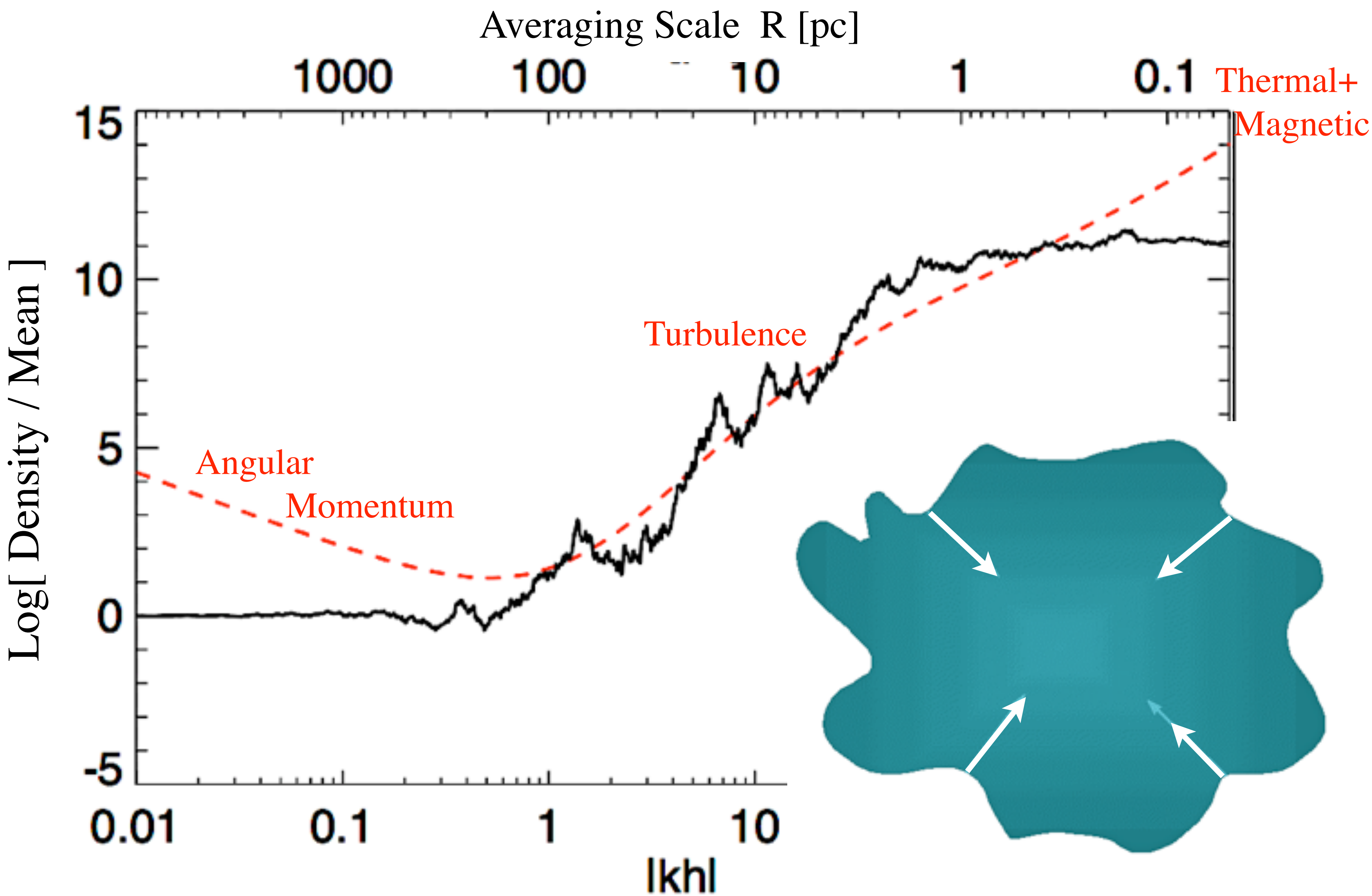

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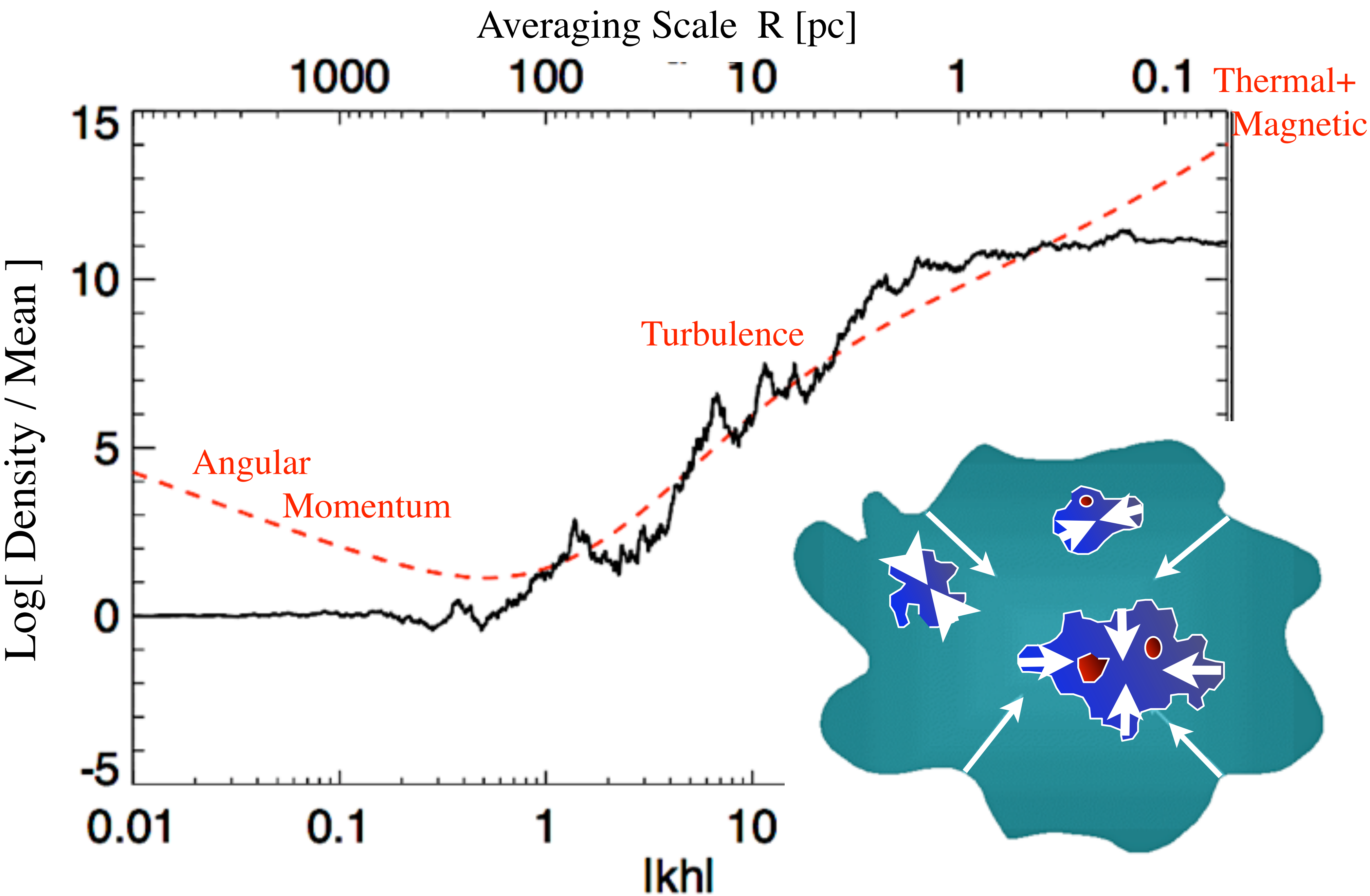

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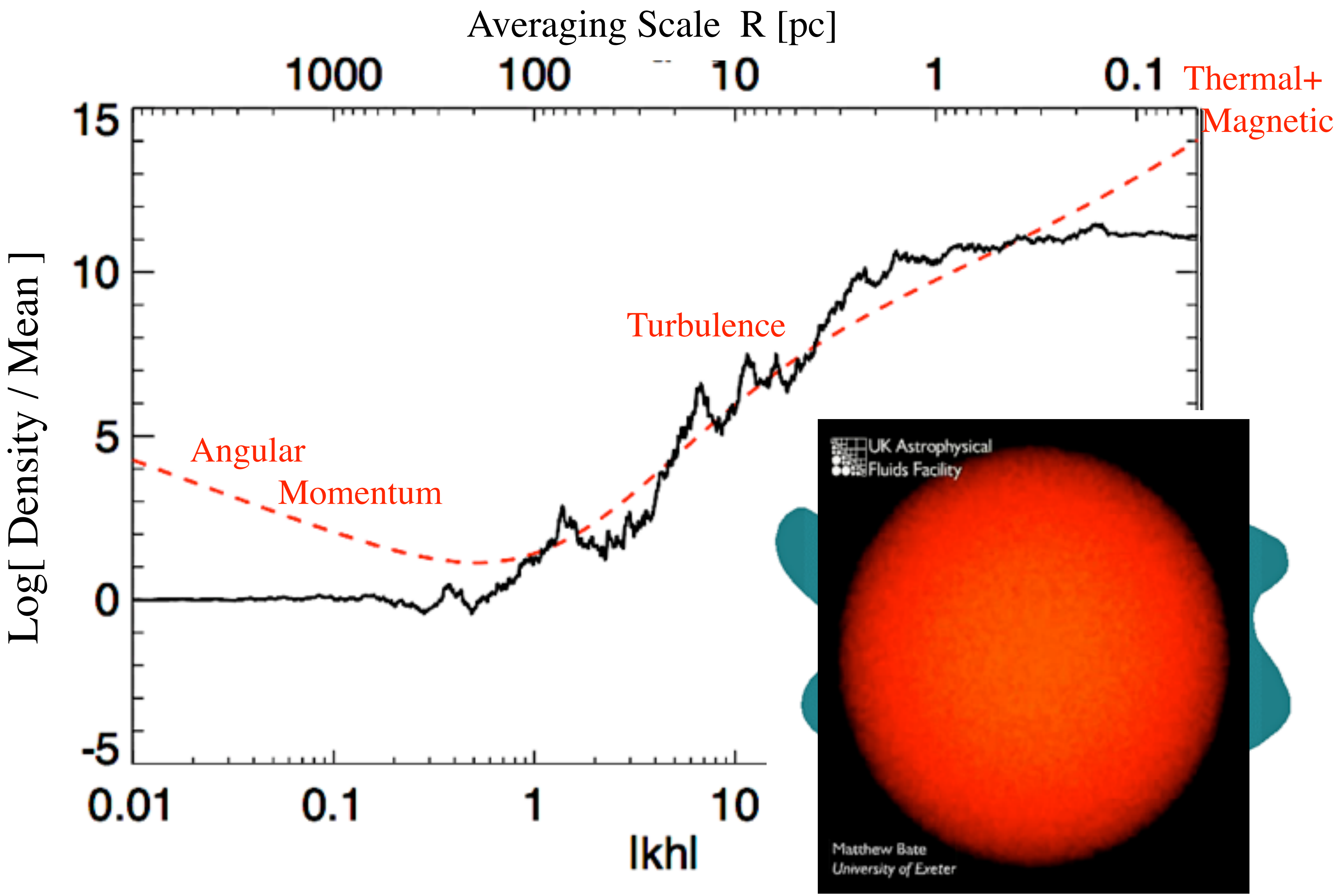

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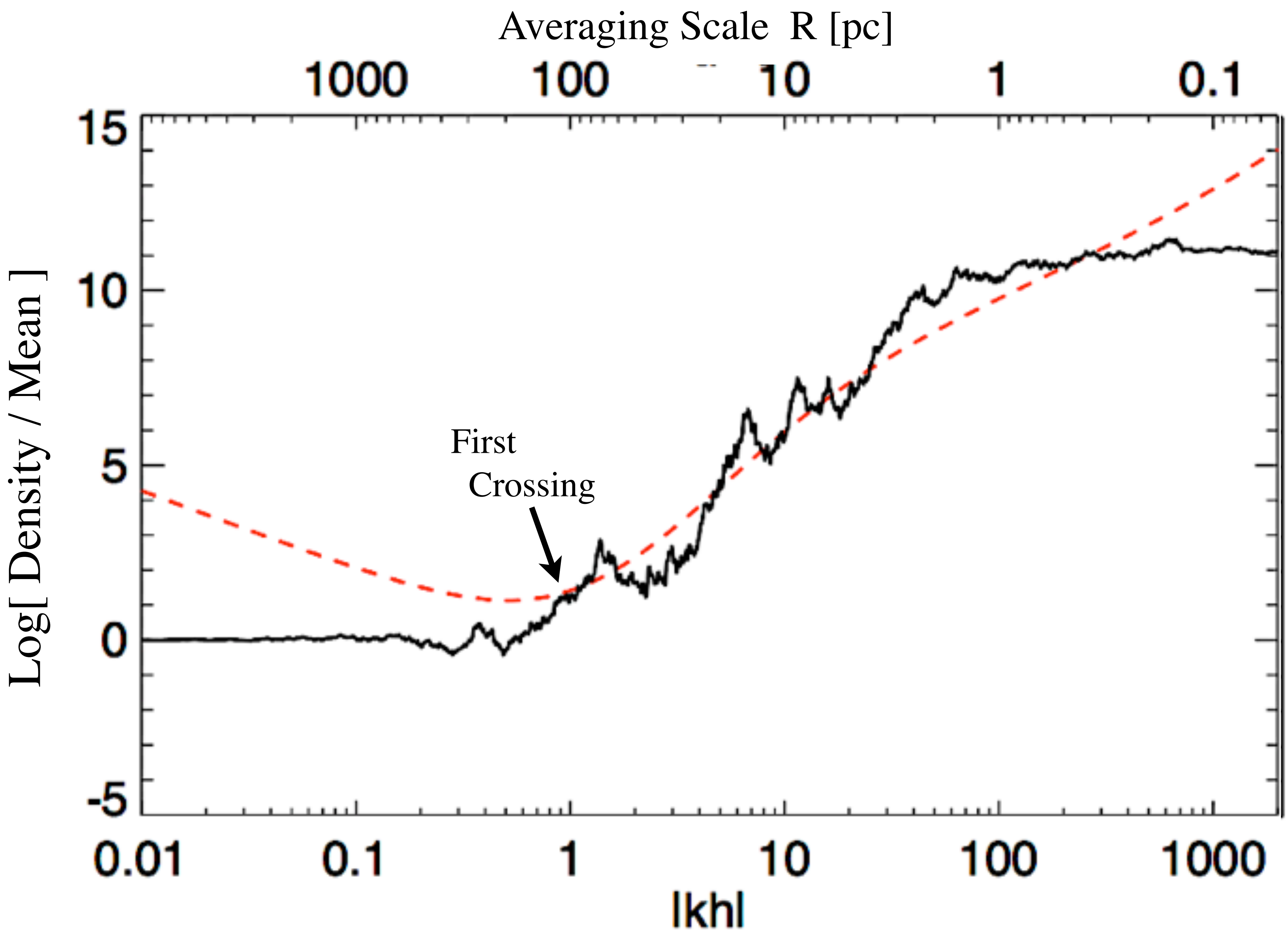

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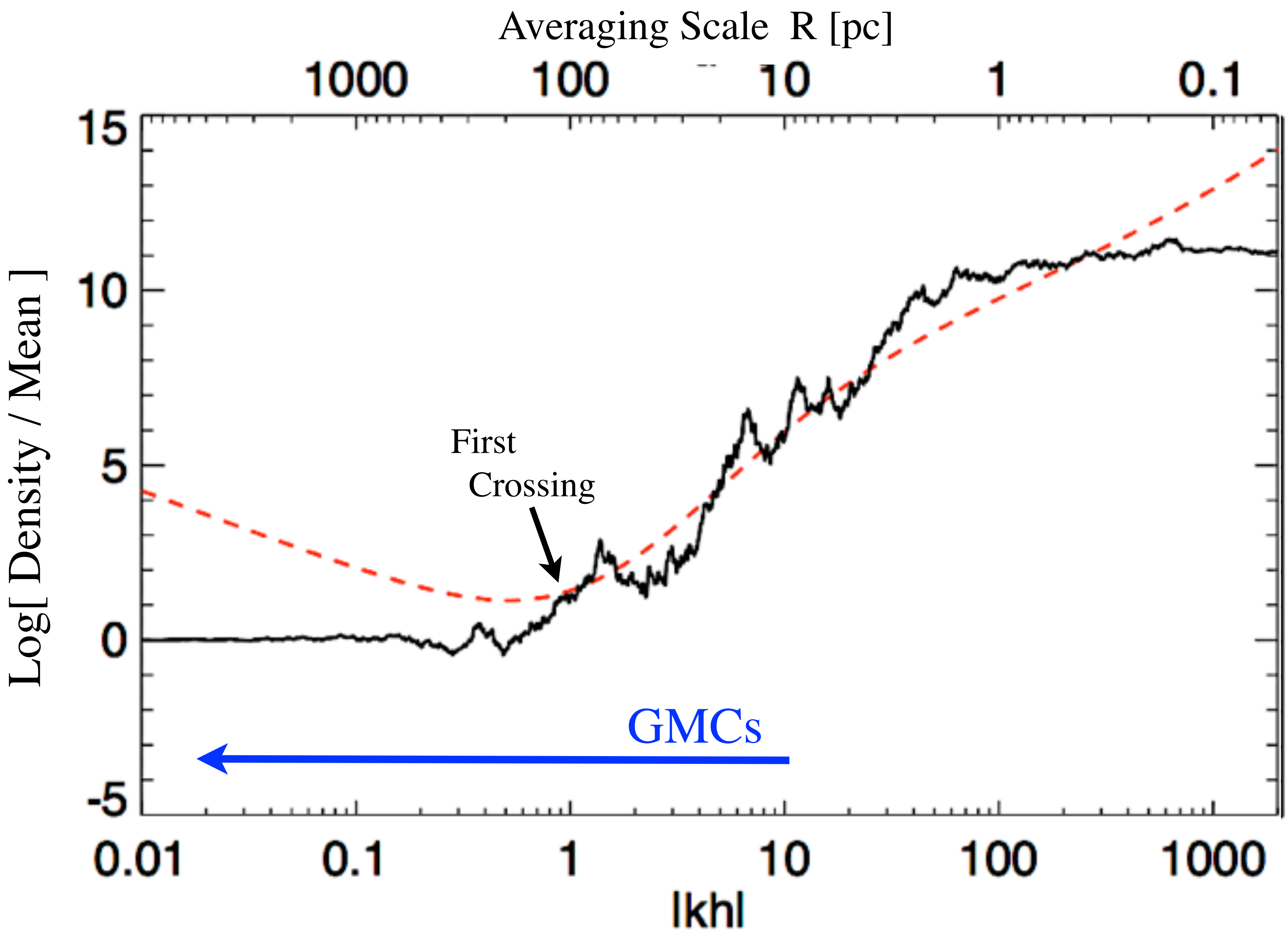

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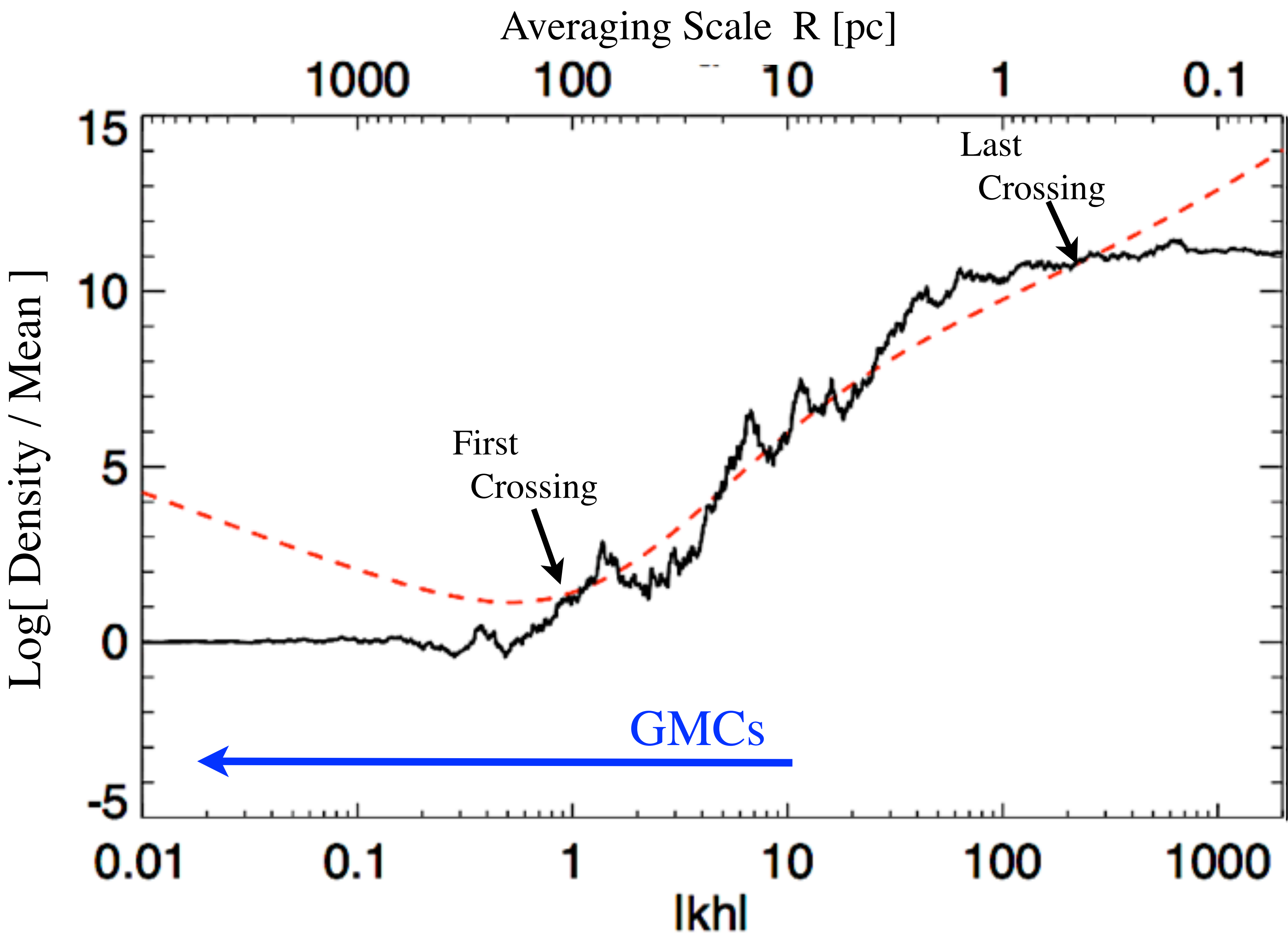

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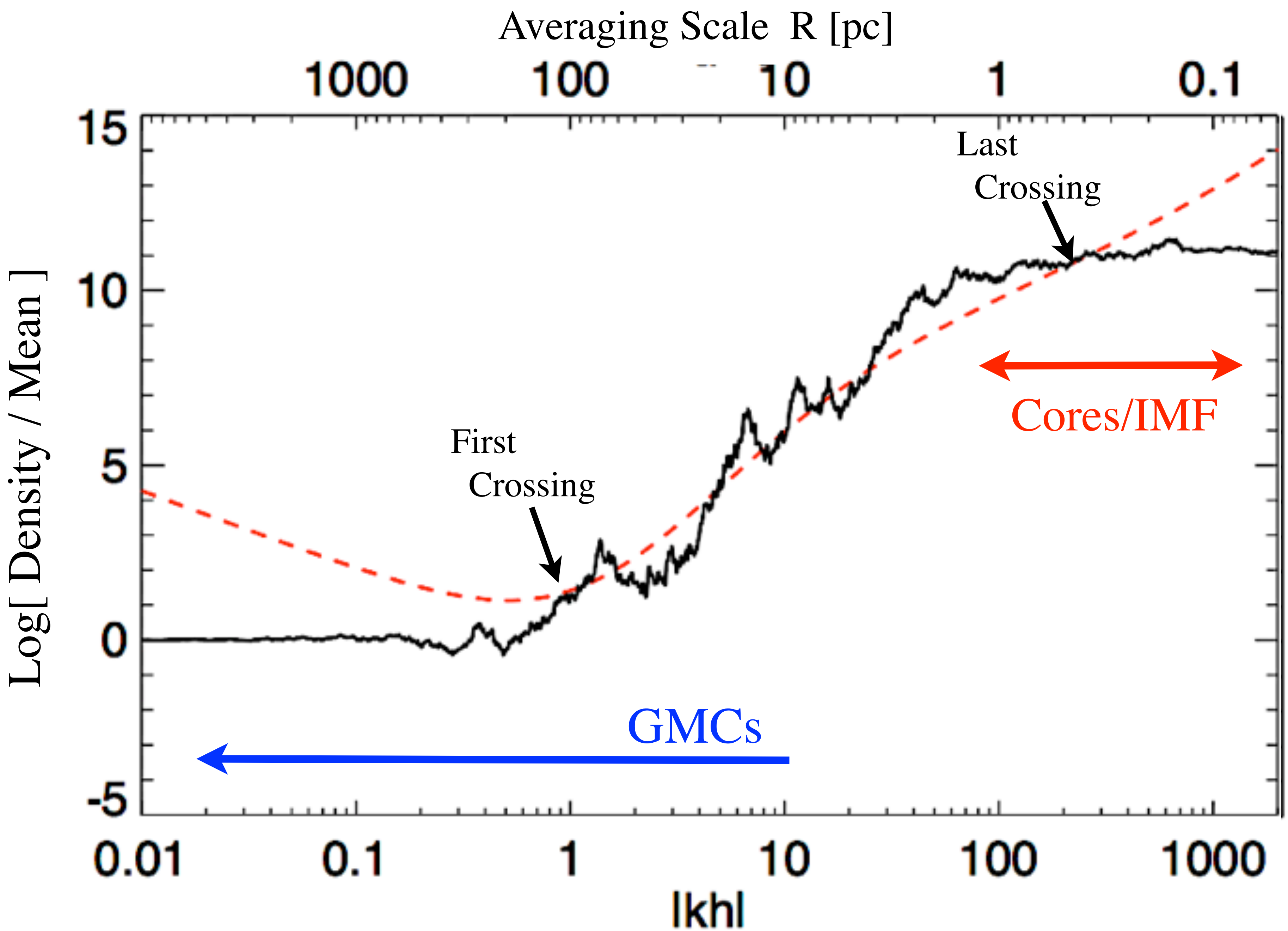
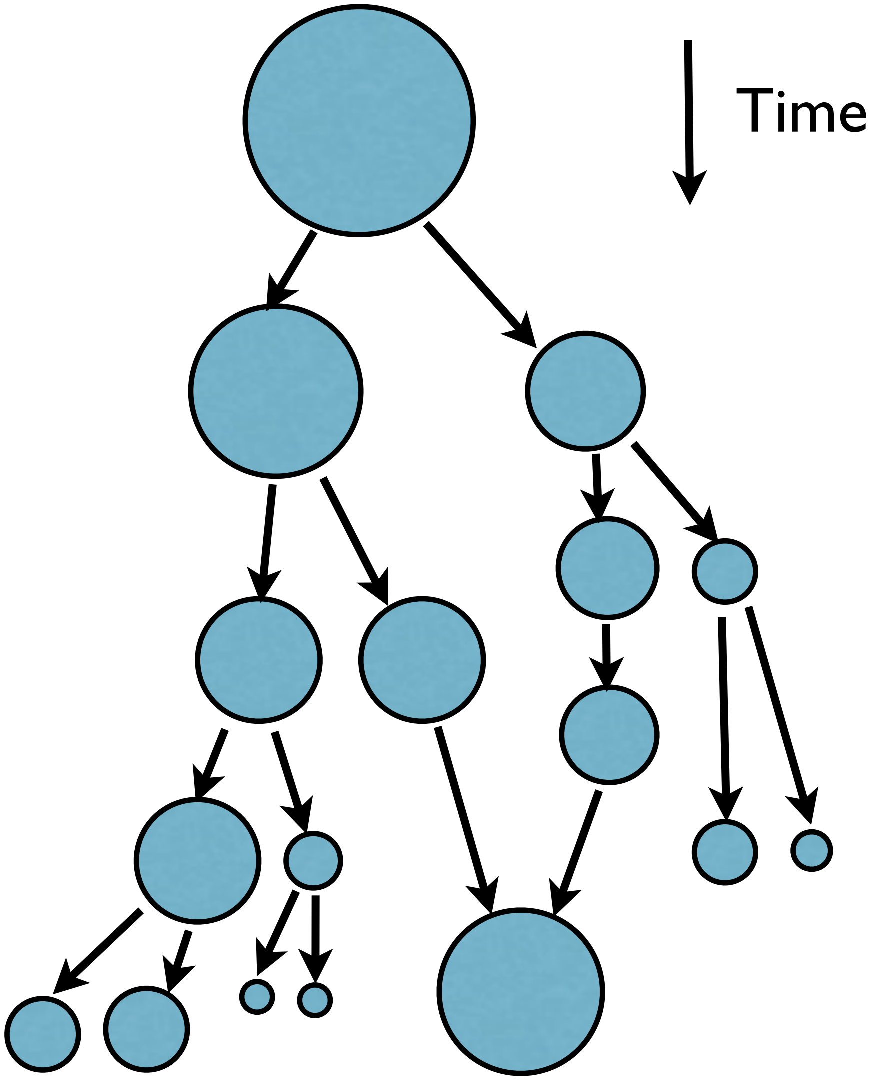


Evolve the Fluctuations in Time

CONSTRUCT "MERGER/FRAGMENTATION" TREES

$$p(\delta\,|\,\tau) = \frac{1}{\sqrt{2\pi\,S\,(1-\exp{[-2\tau]})}}\,\exp{\left[-\frac{(\delta-\delta(t=0)\,\exp{[-\tau]})^2}{2\,S\,(1-\exp{[-2\tau]})}\right]}$$



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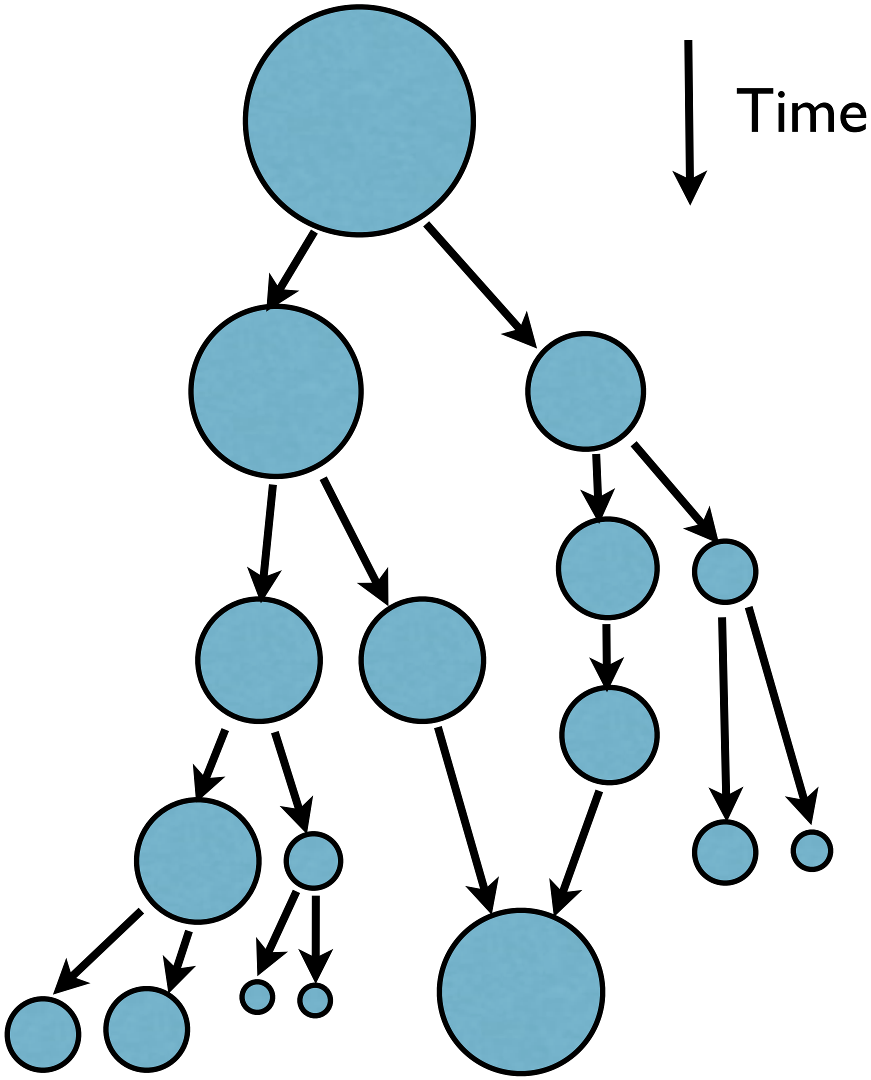
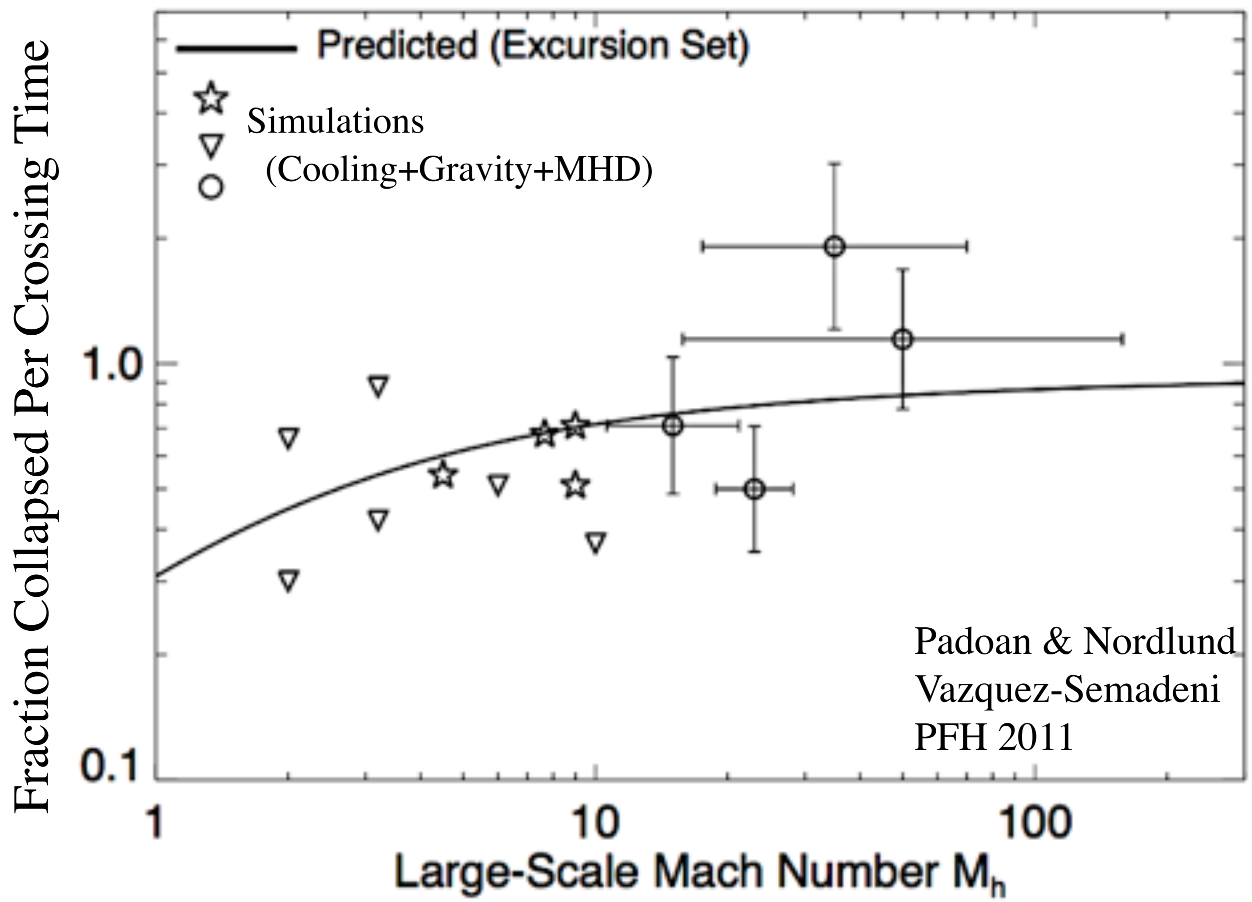
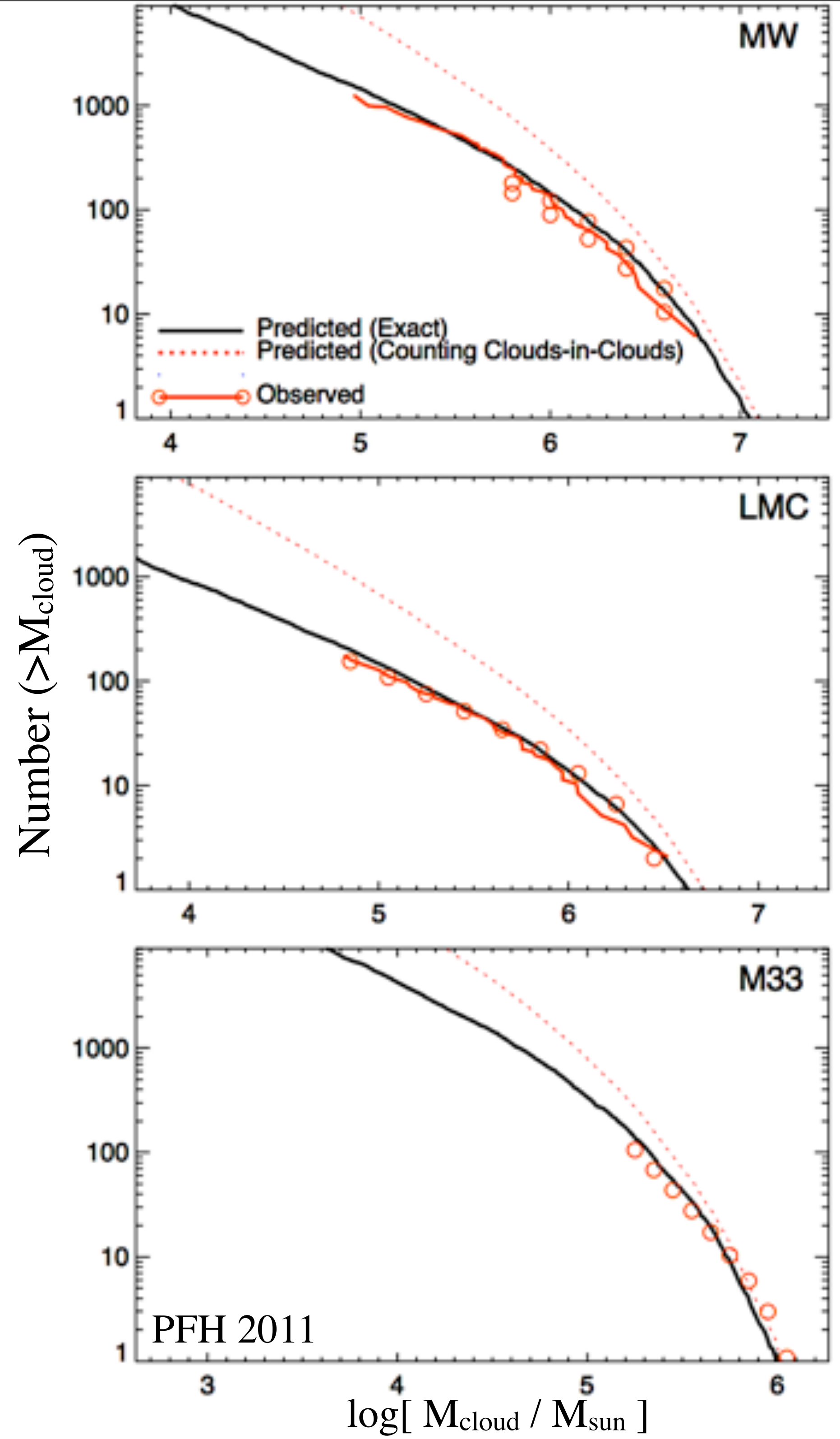




The "First Crossing" Mass Function

VS GIANT MOLECULAR CLOUDS



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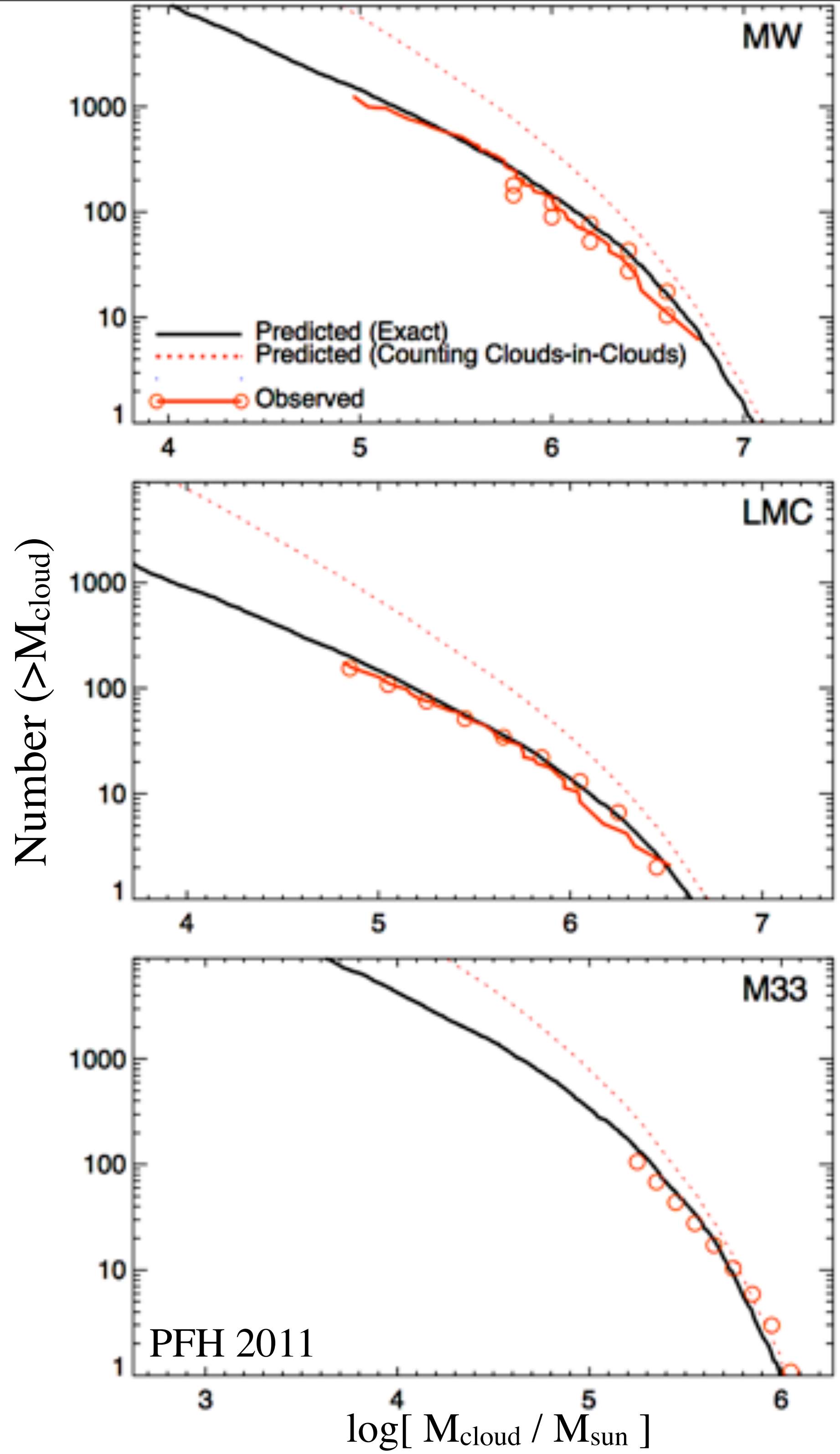

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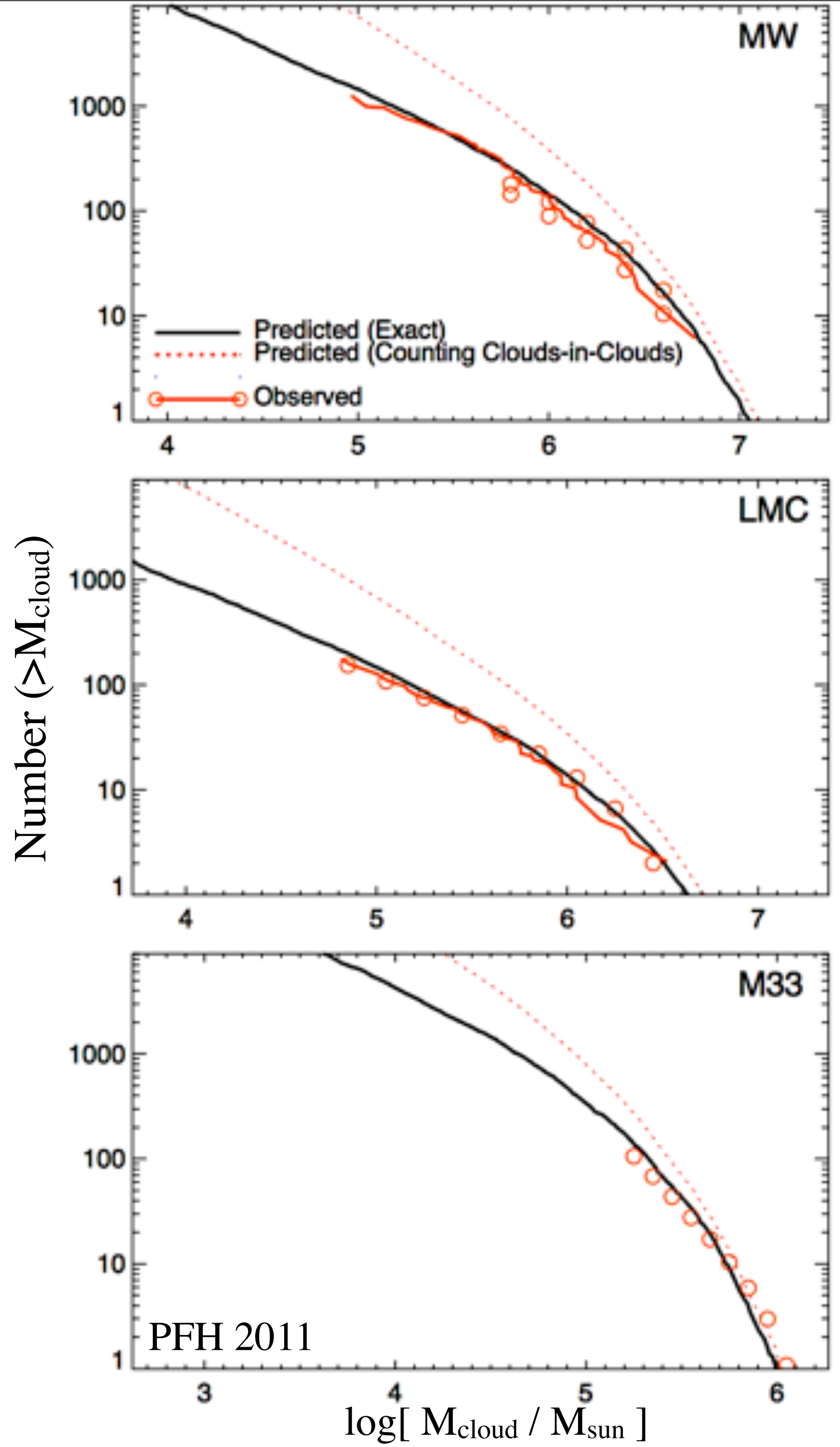

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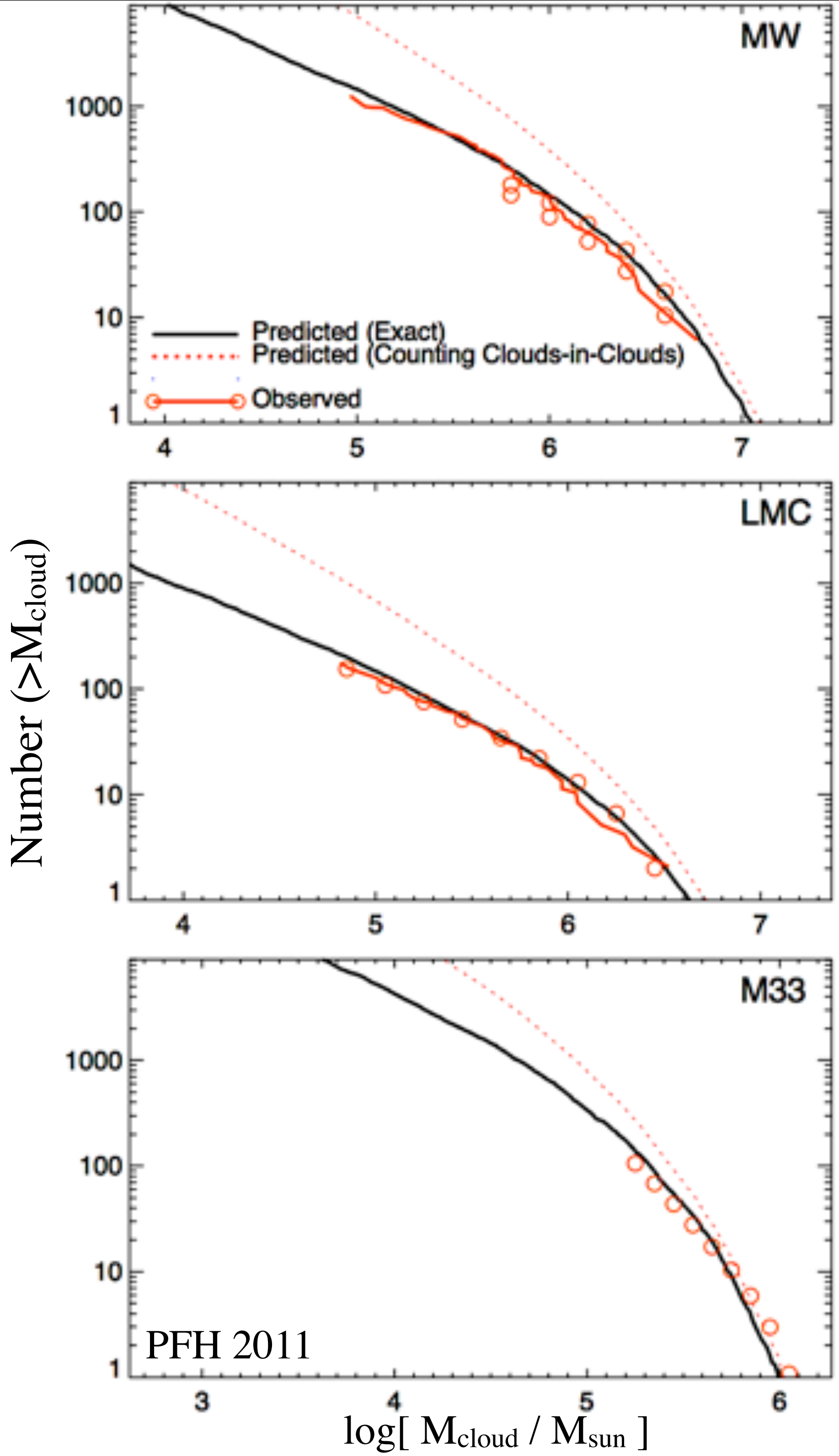
$$\alpha \approx -2 + \frac{(3-p)^2}{2\,S\,p^2}\,\ln\left(\frac{M_J}{M}\right)$$

$$\approx -2 + 0.1\,\log\left(\frac{M_J}{M}\right)$$



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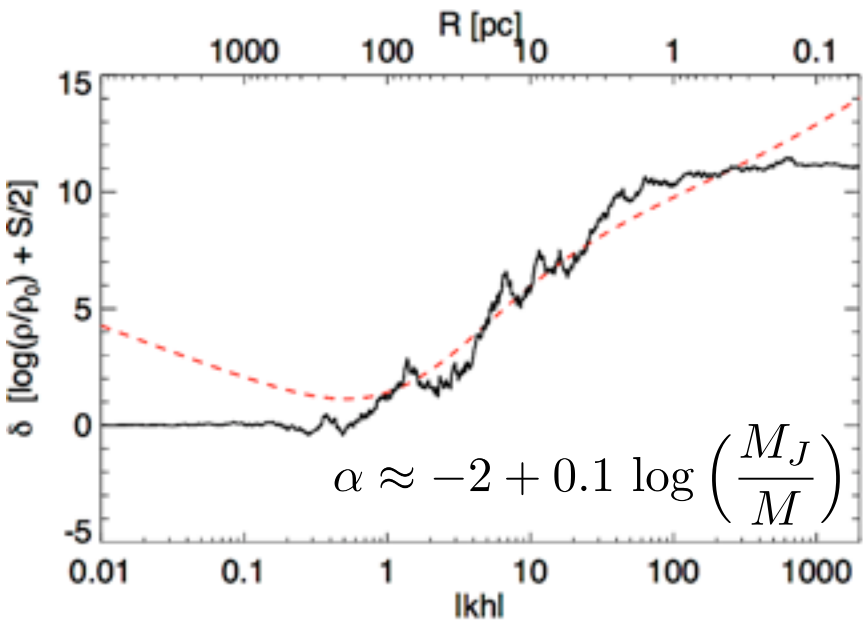

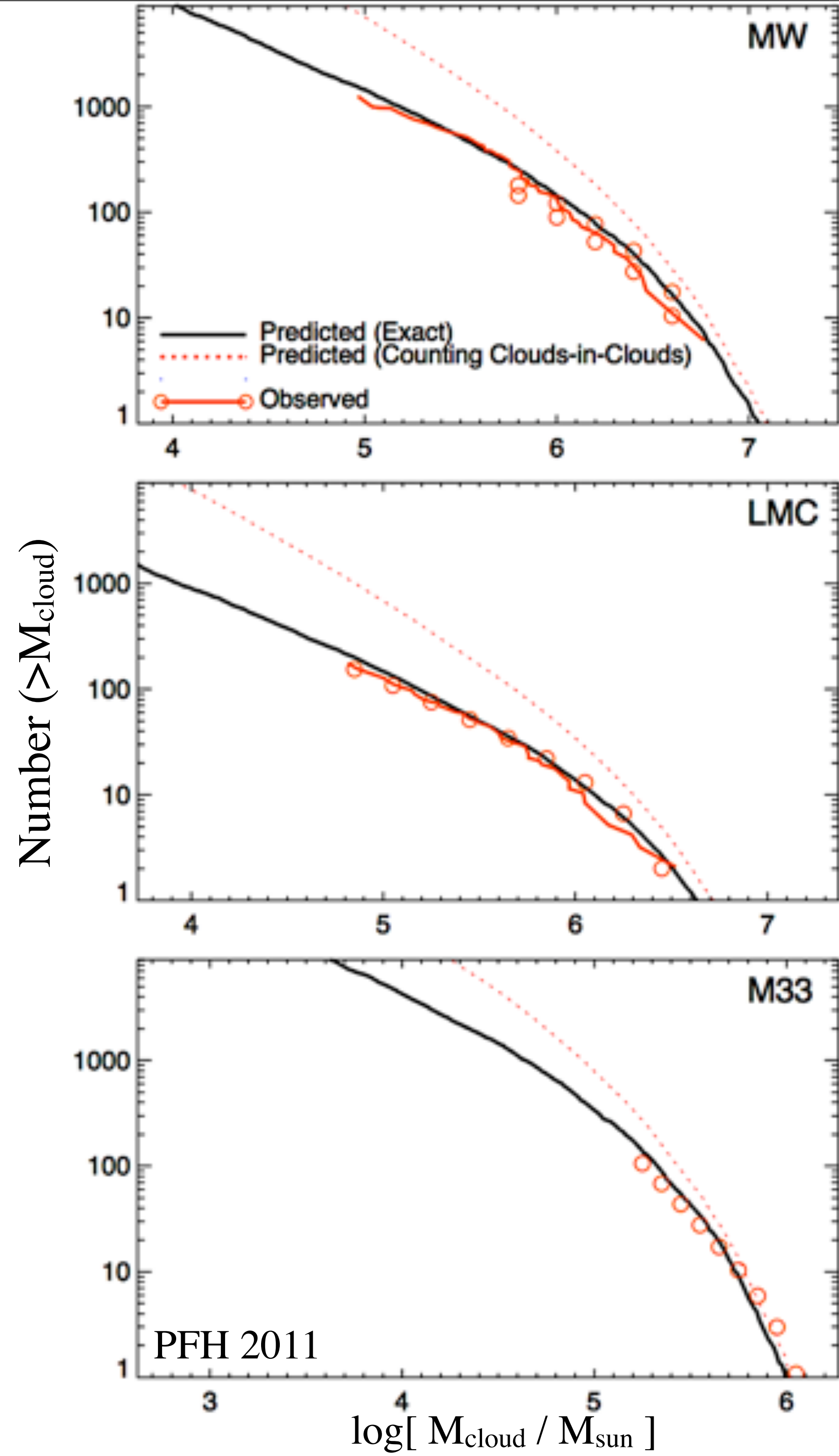

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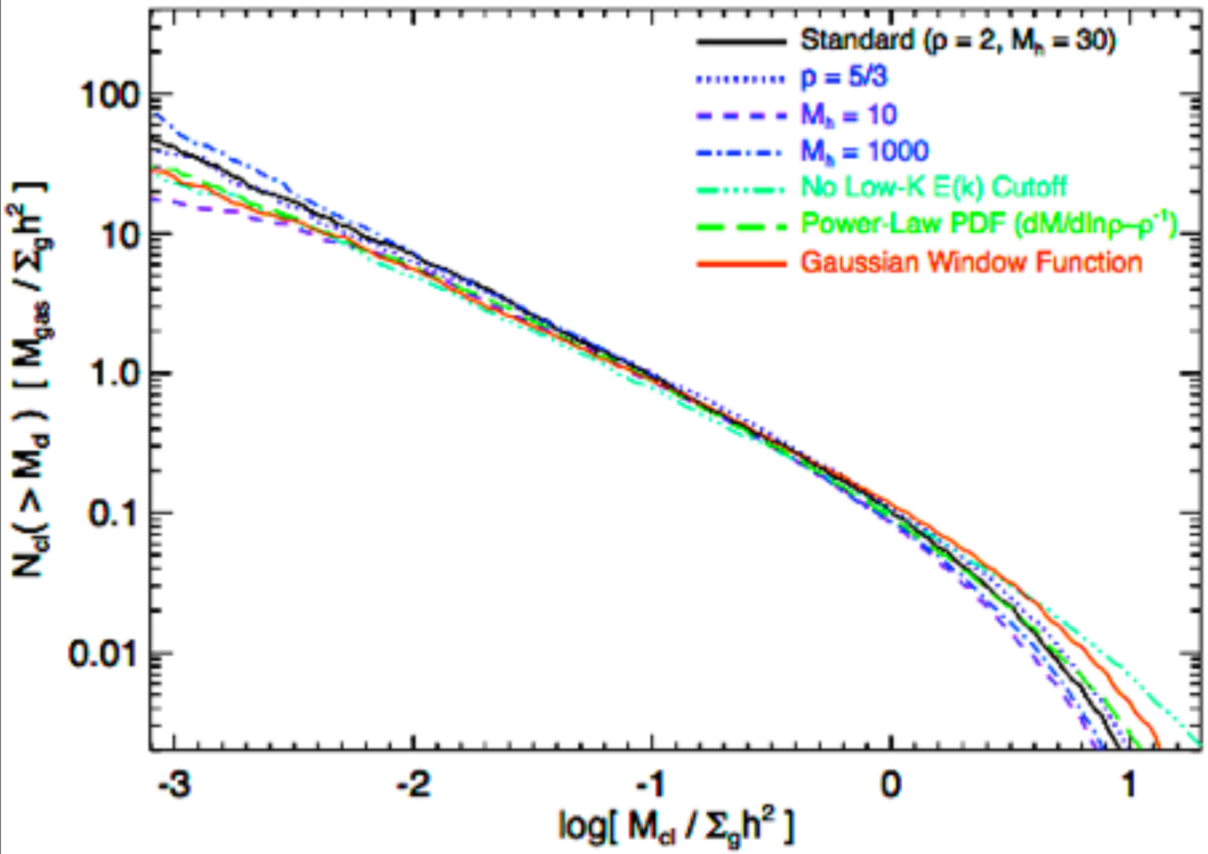
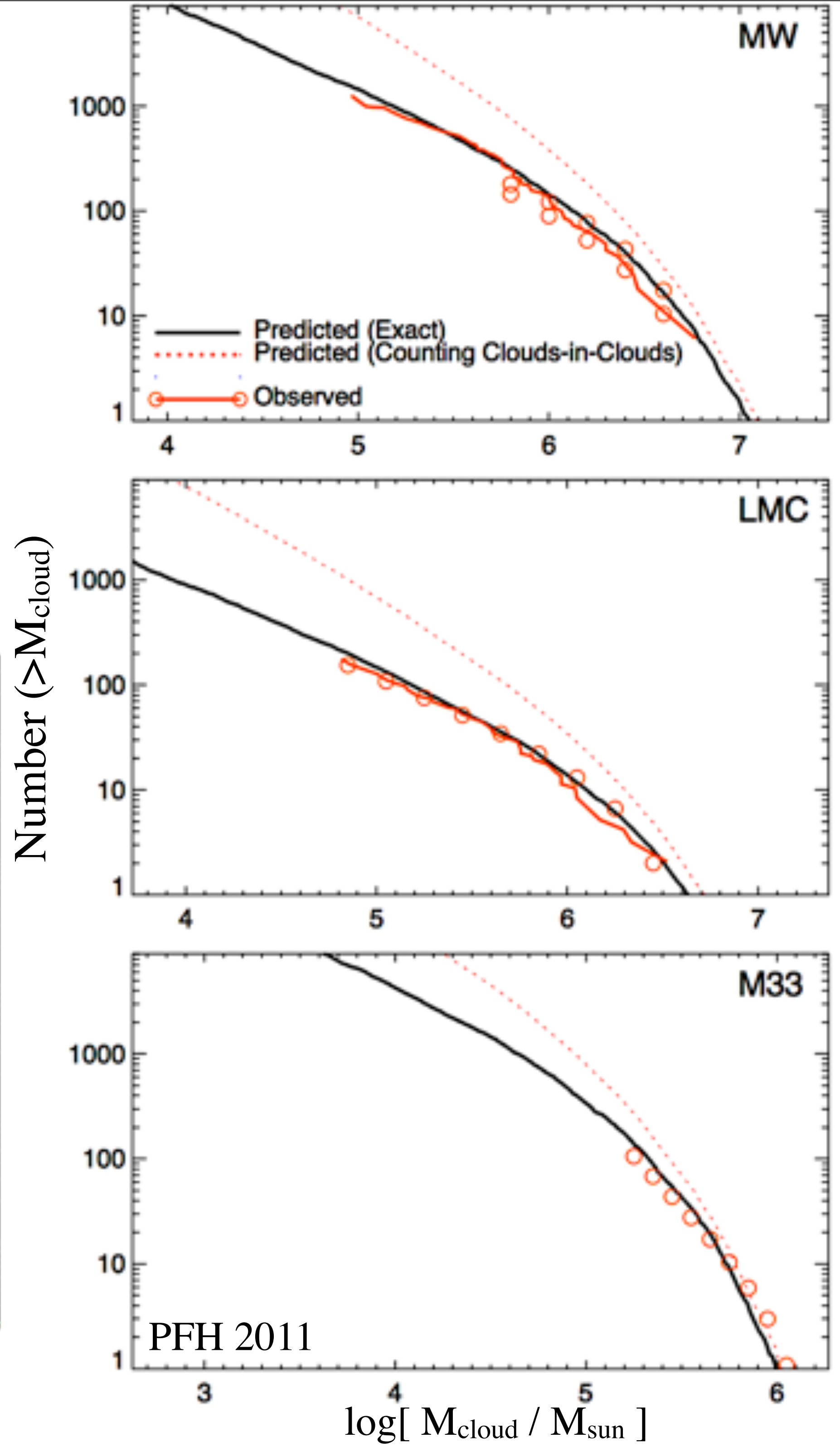
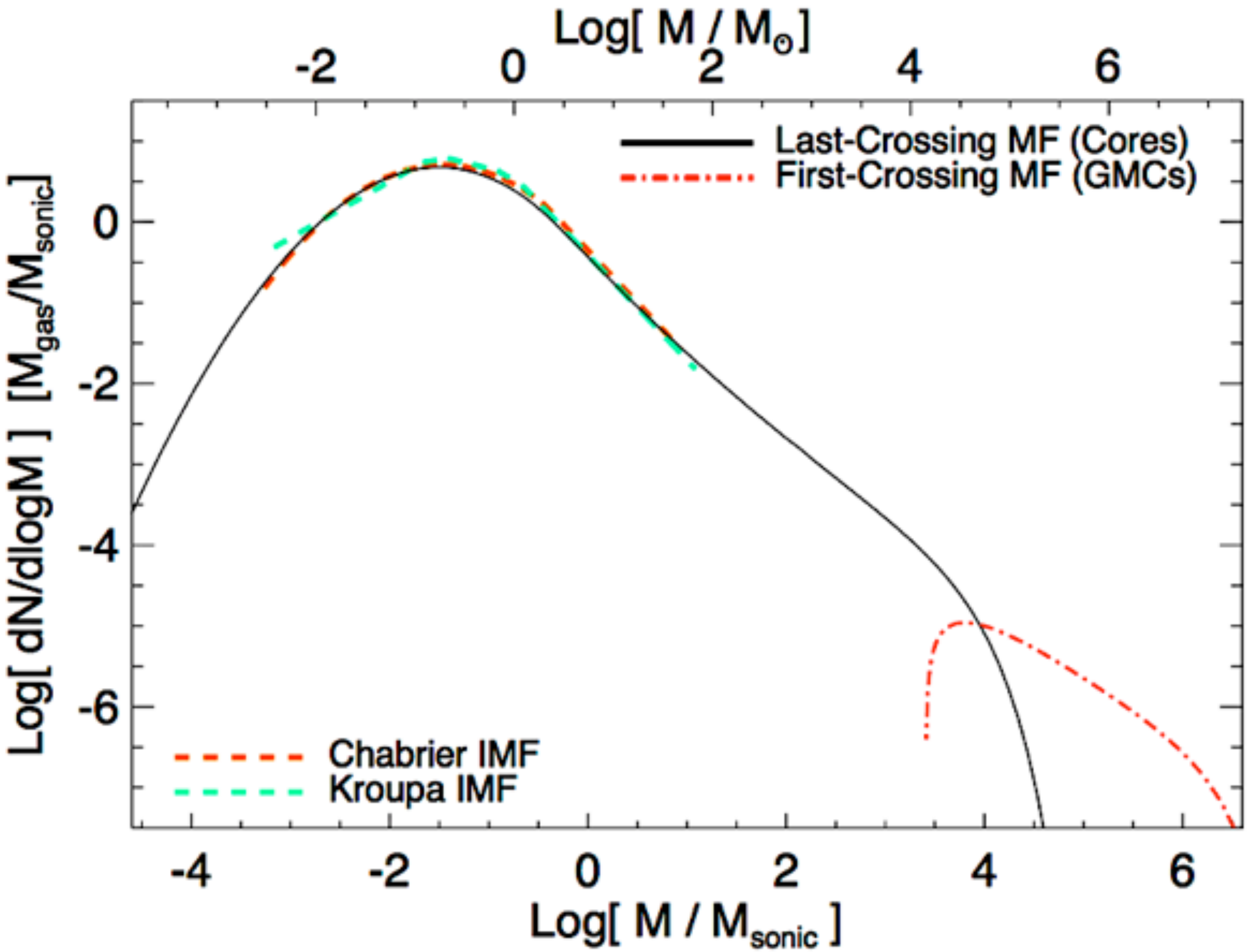




The "Last Crossing" Mass Function

VS PROTOSTELLAR CORES & THE STELLAR IMF



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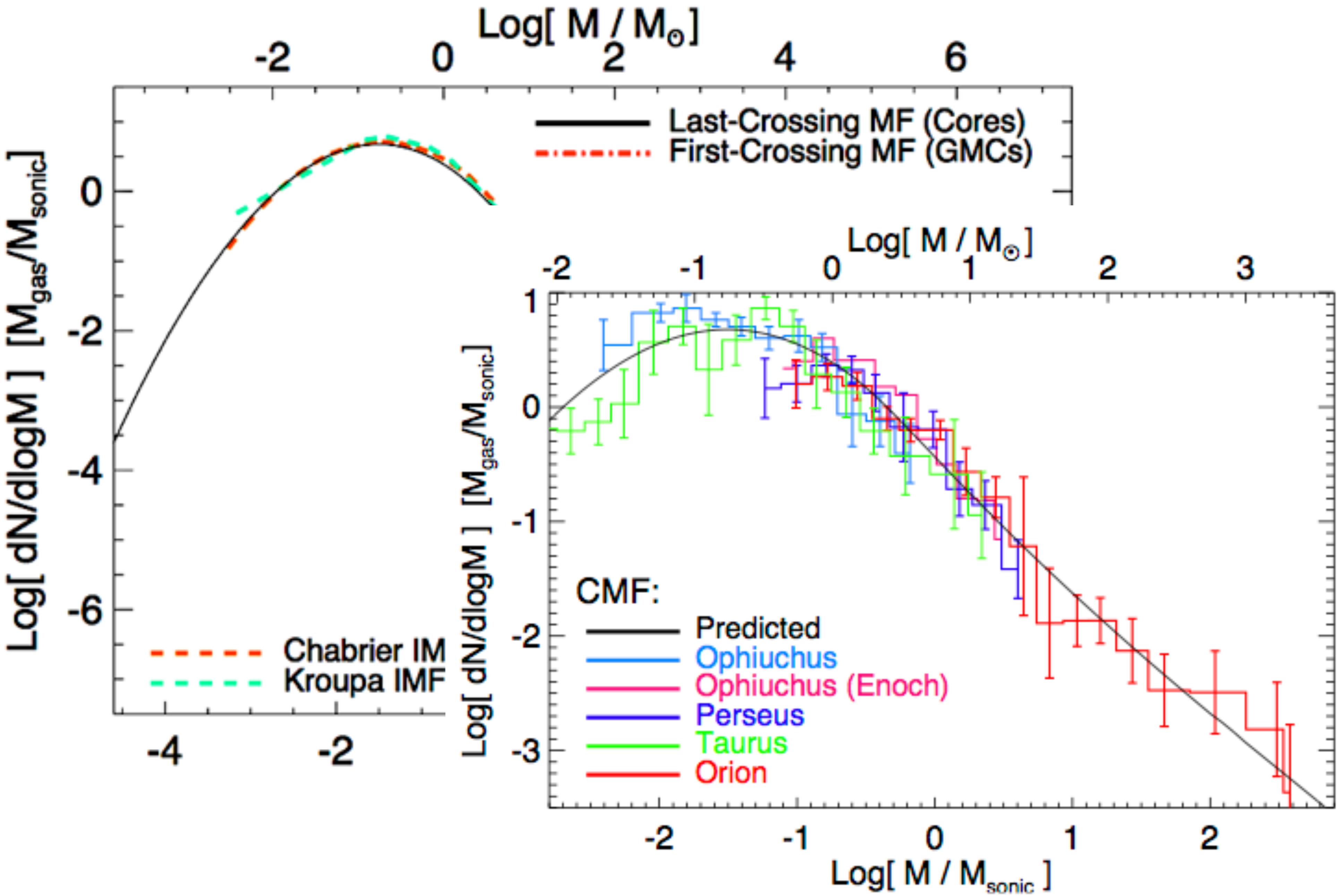
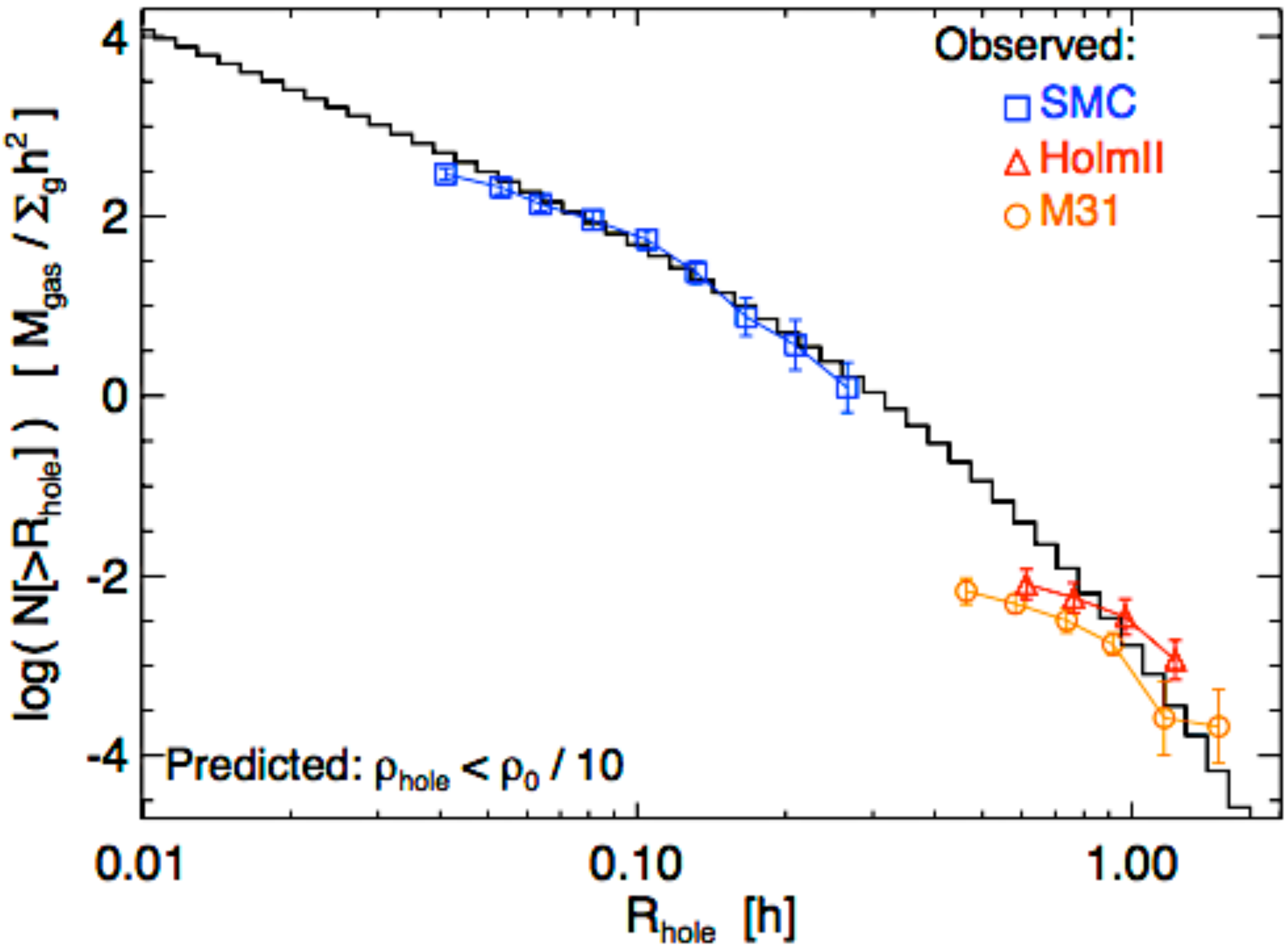


“Void” Abundance

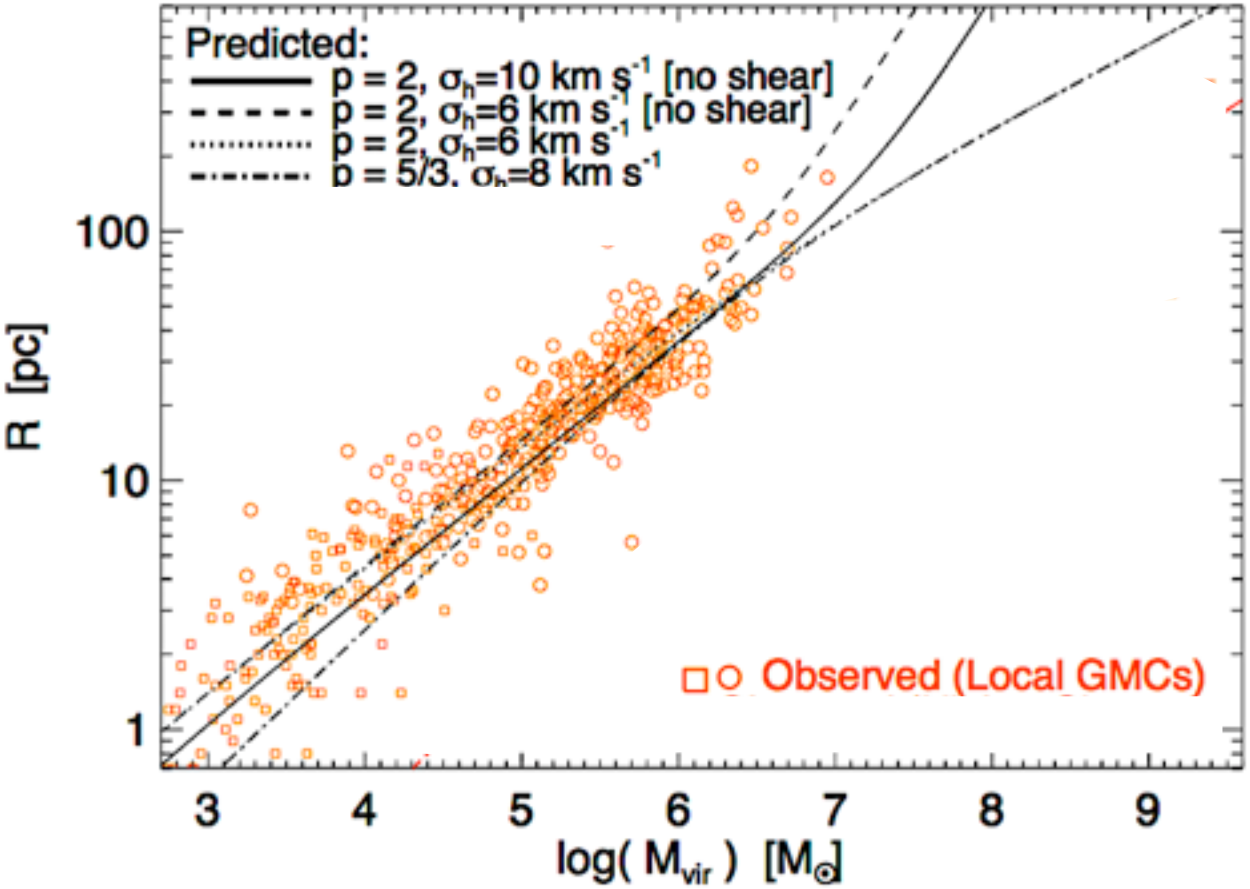
VS HI “HOLES” IN THE ISM



Don’t need SNe to “clear out” voids

Structural Properties of "Clouds"

LARSON'S LAWS EMERGE NATURALLY



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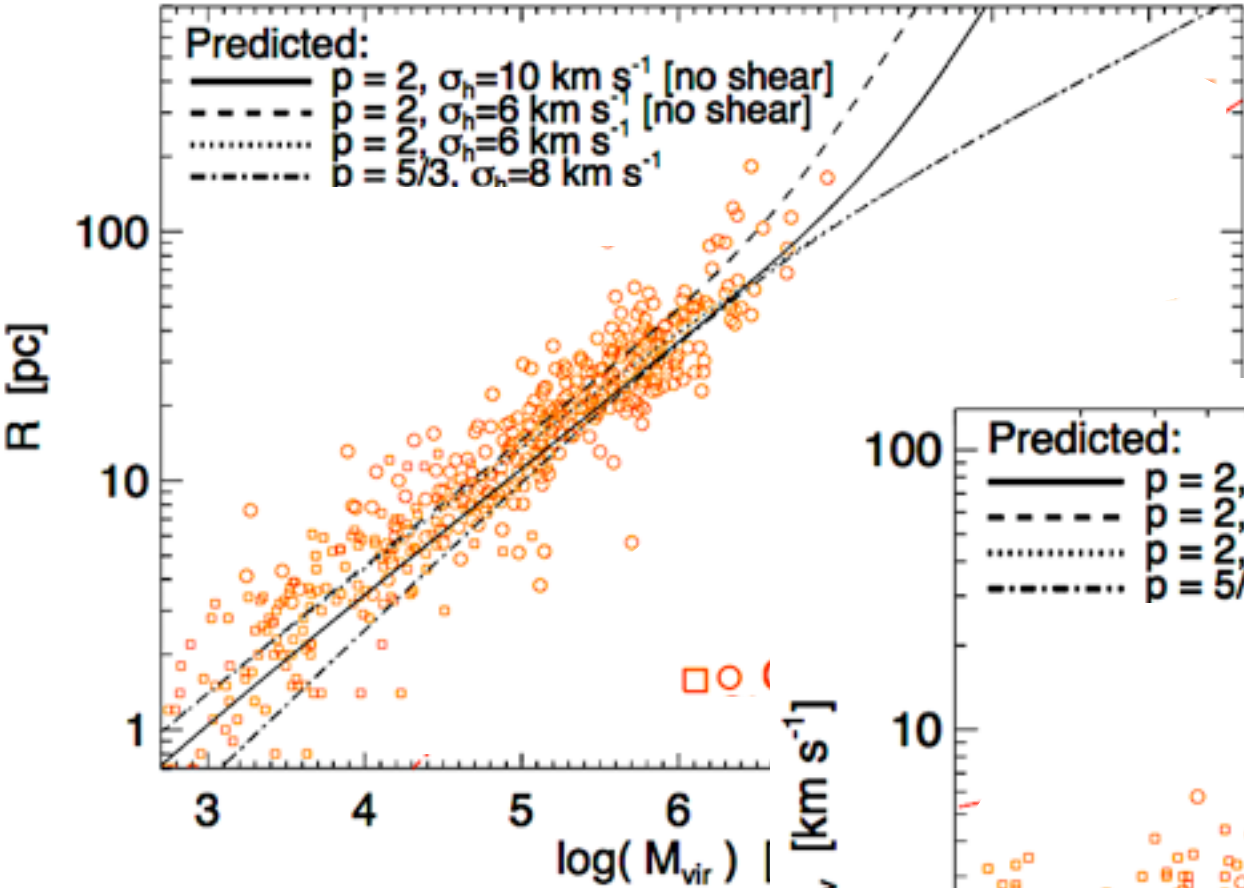
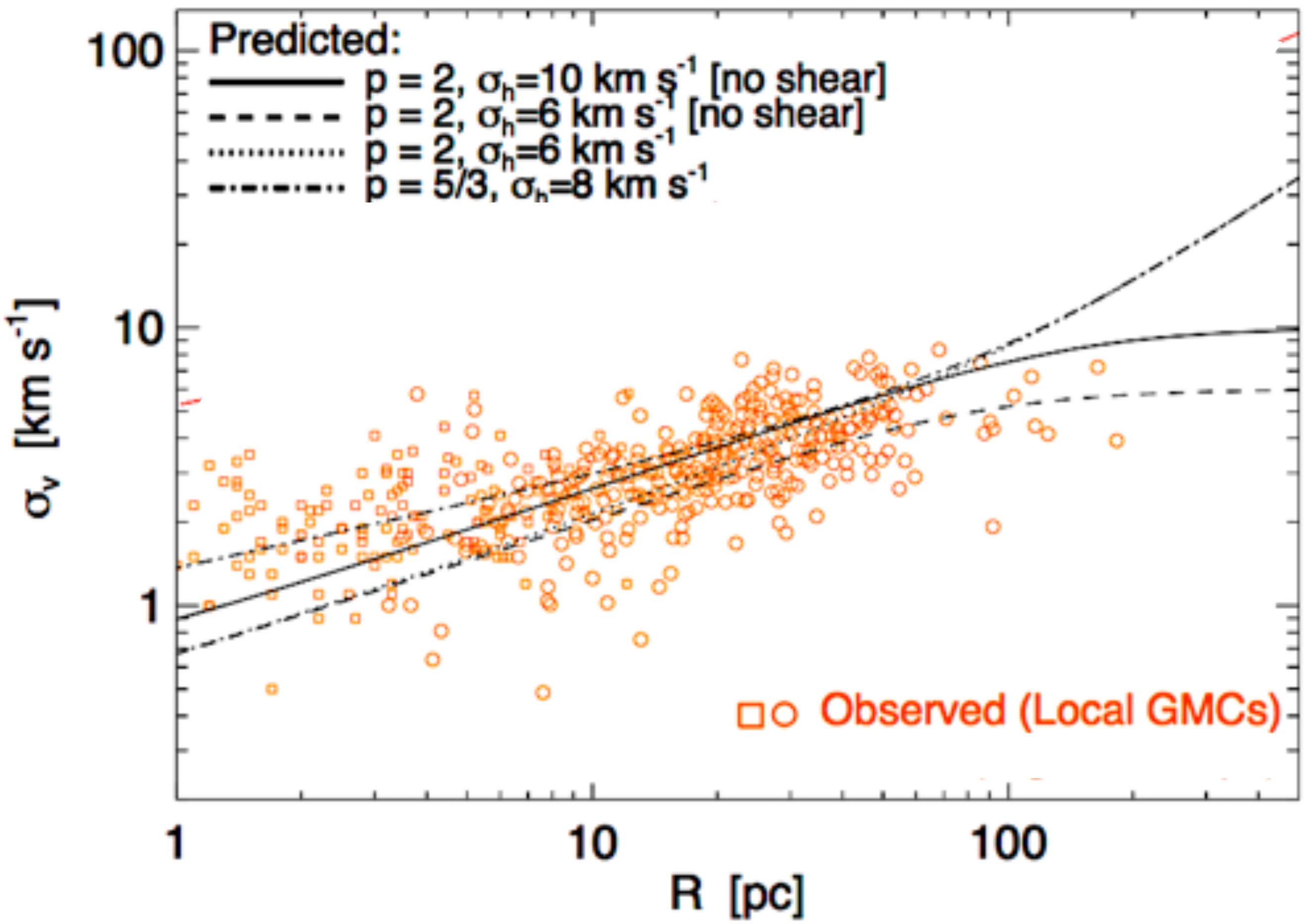
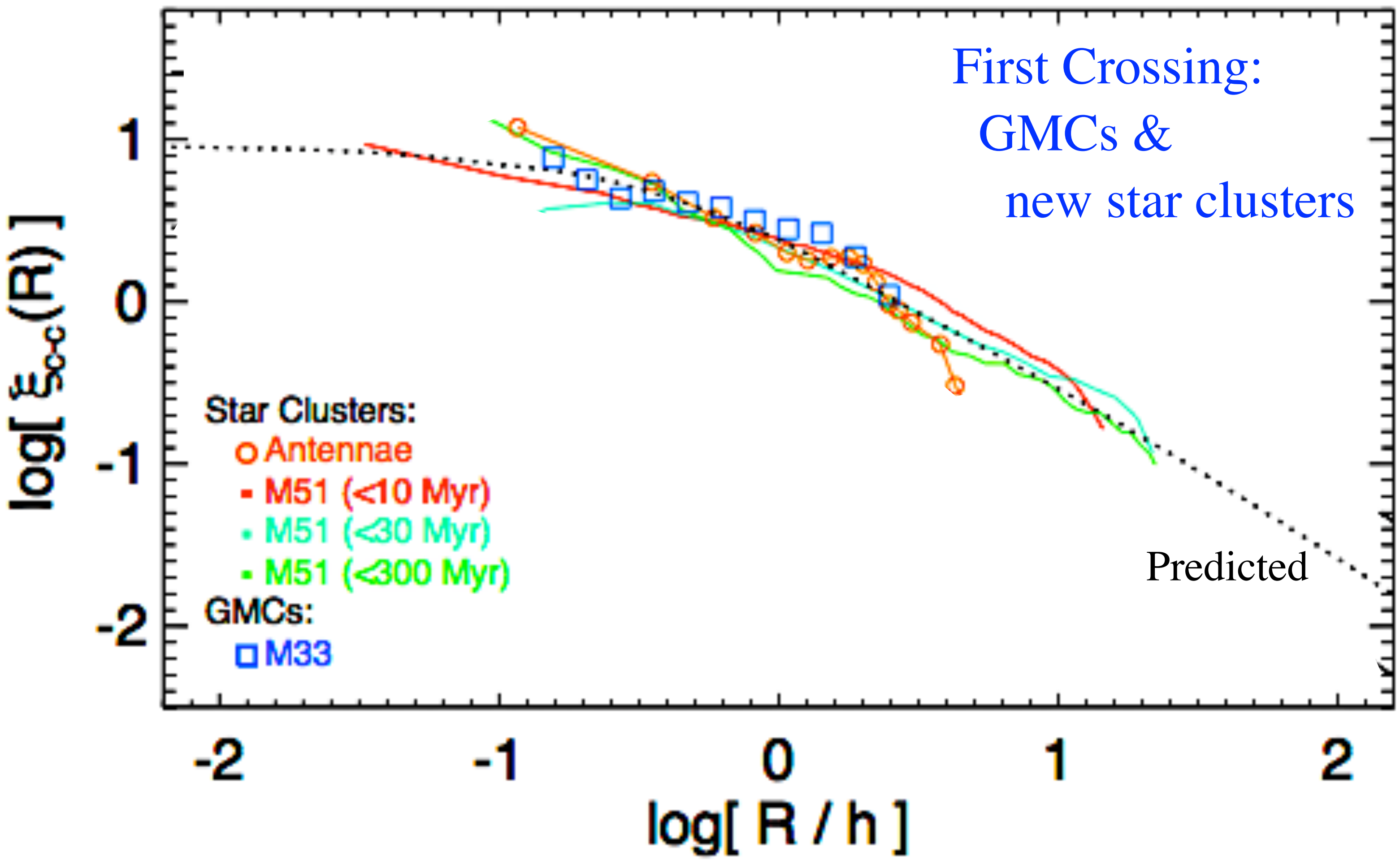




Clustering

PREDICT N-POINT CORRELATION FUNCTIONS

$$1 + \xi(r \,|\, M) \equiv \frac{\langle n[M \,|\, r' < r] \rangle}{\langle n[M] \rangle}$$



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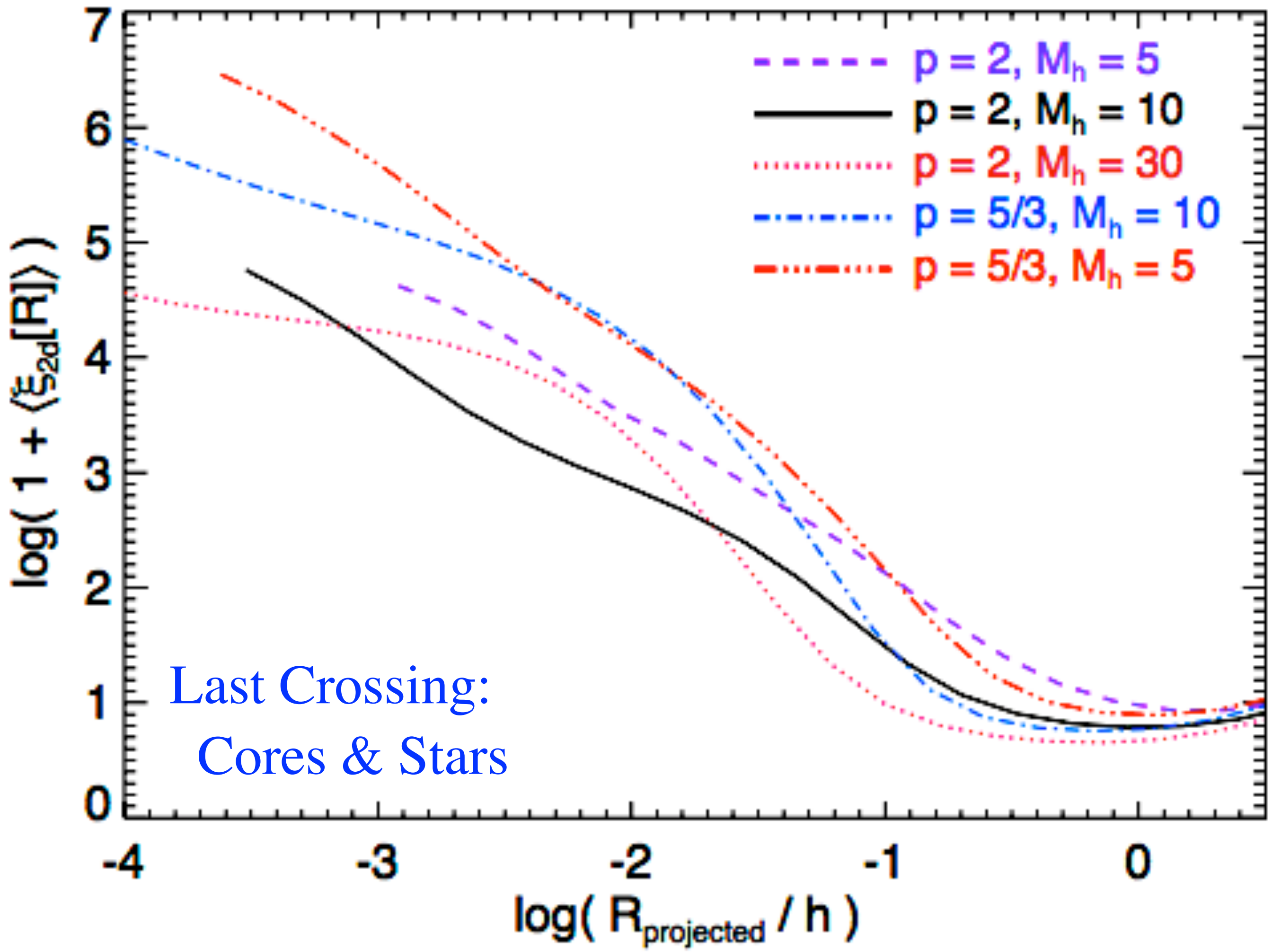
PFH 2012b



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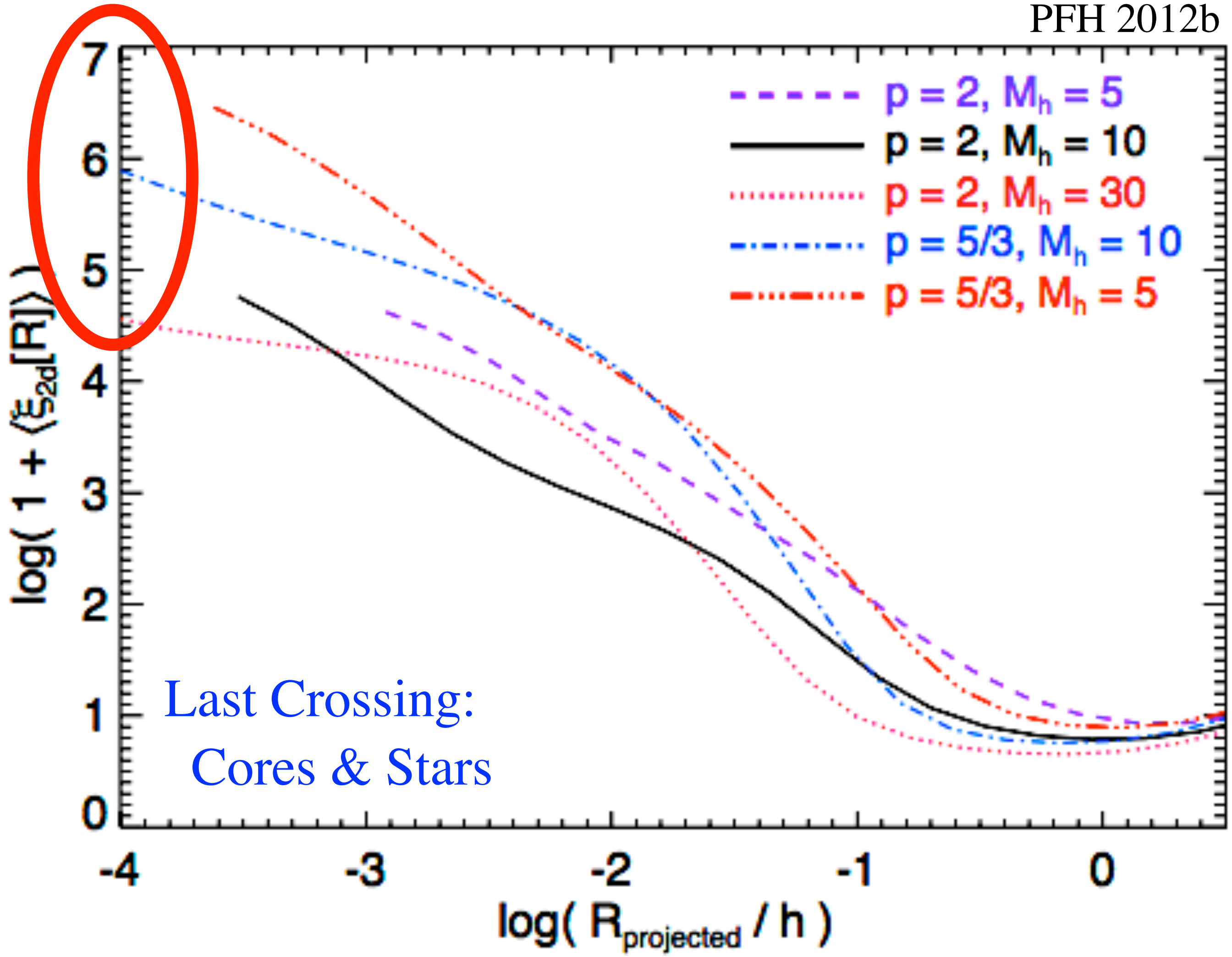

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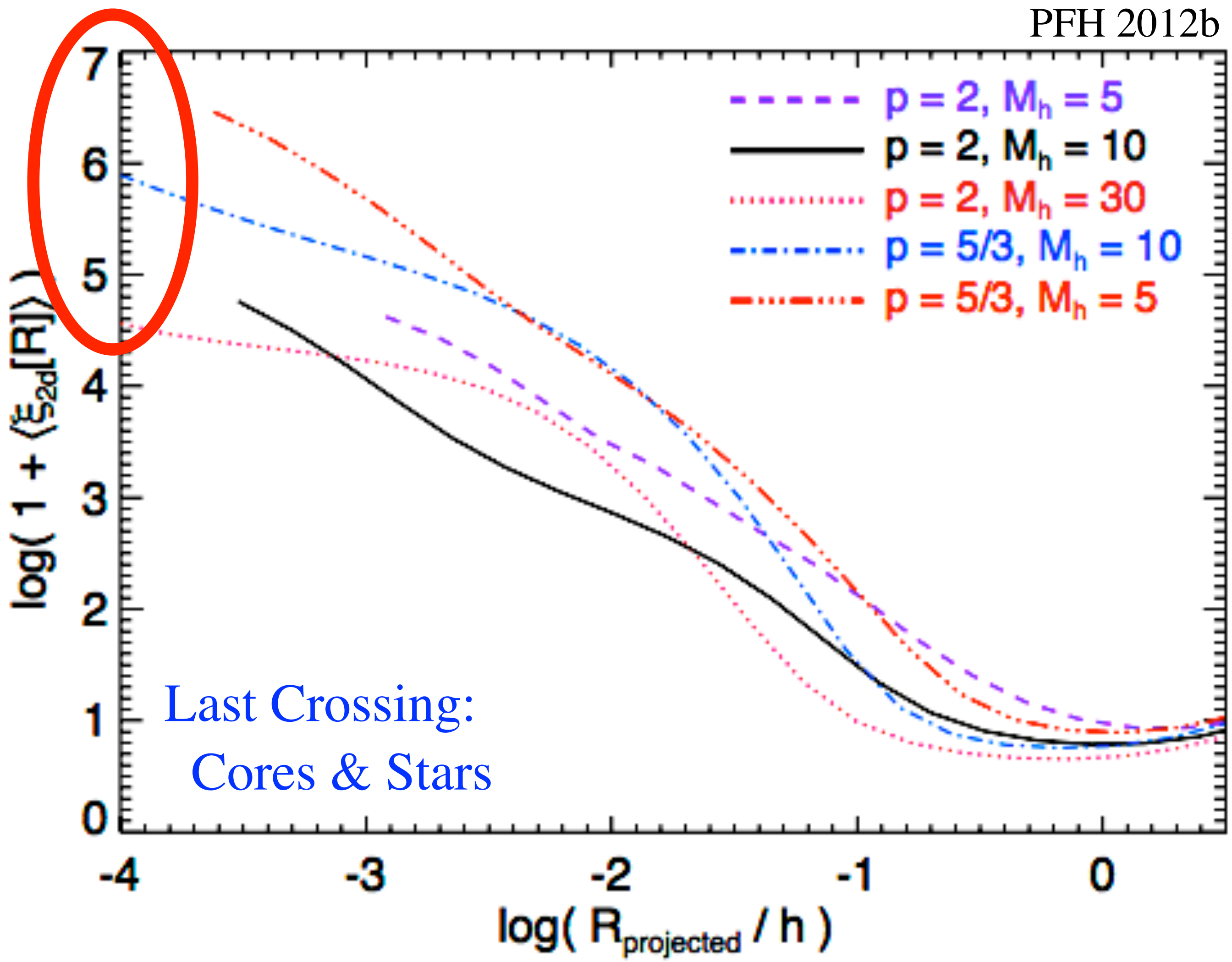
Why is Star Formation Clustered?

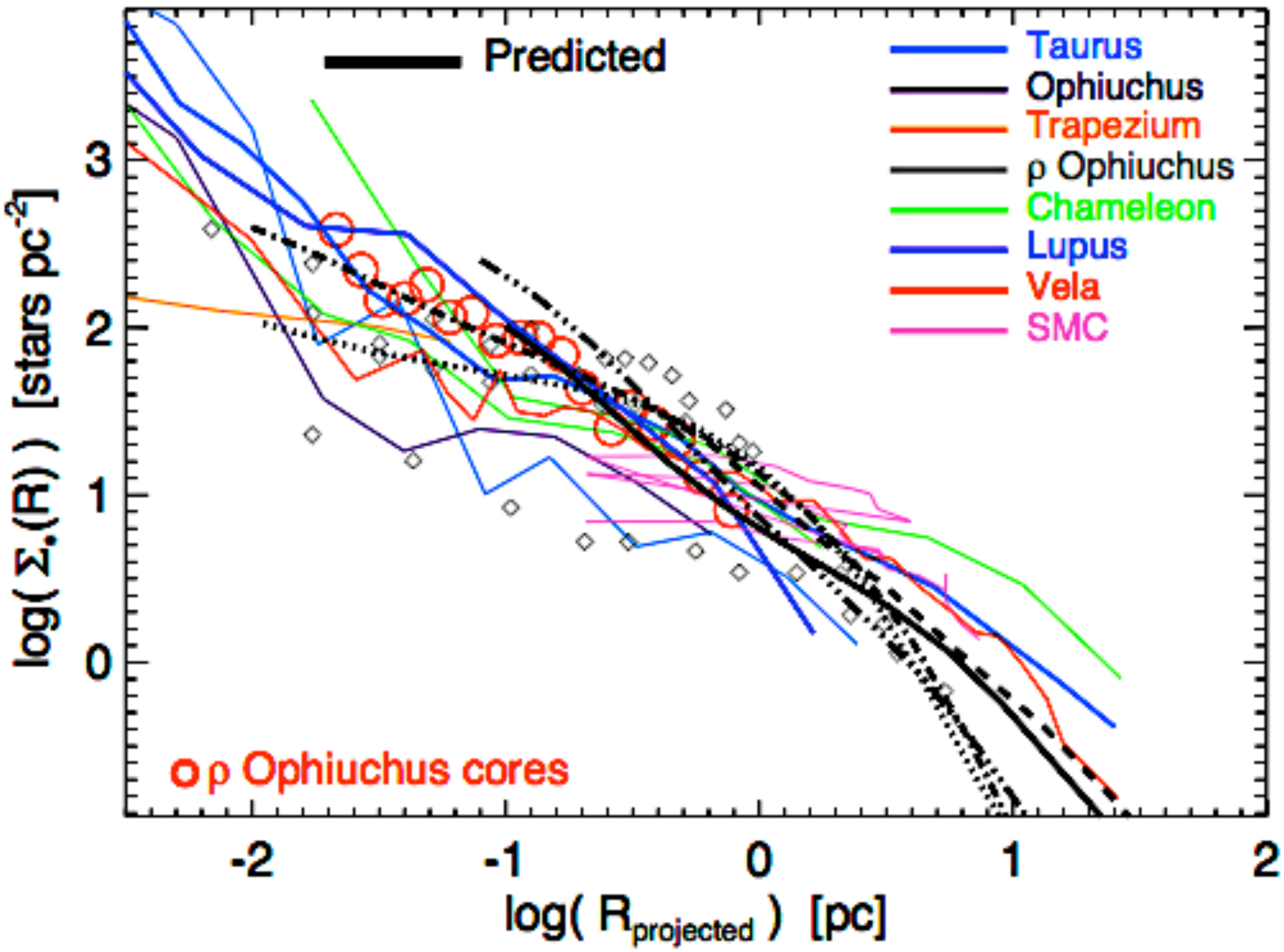
$$S \sim \ln \mathcal{M}(k)^2$$
$$\sim \ln r^{3-p}$$

PFH 2012b

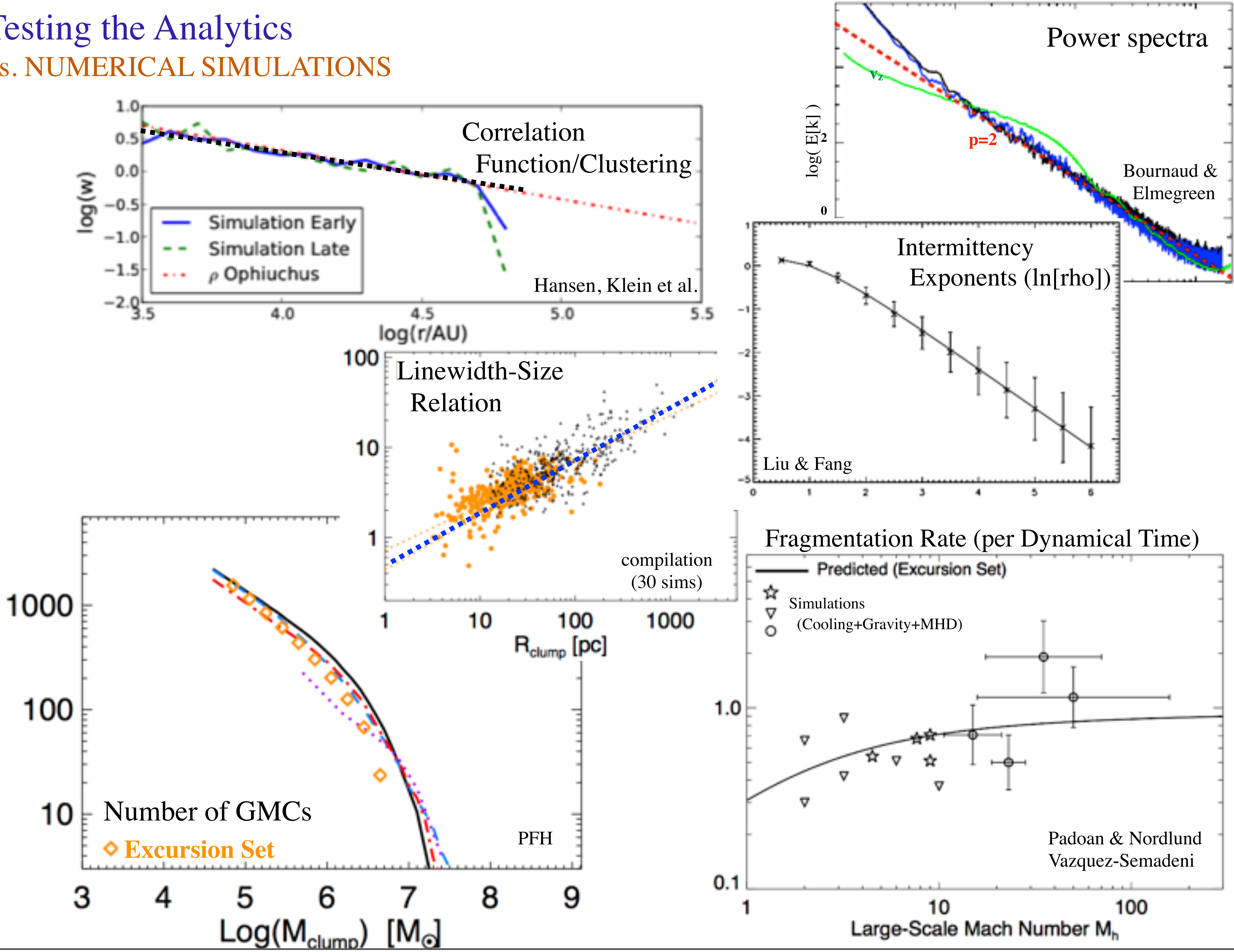


Clustering of Stars: Predicted vs. Observations

PREDICT N-POINT CORRELATION FUNCTIONS



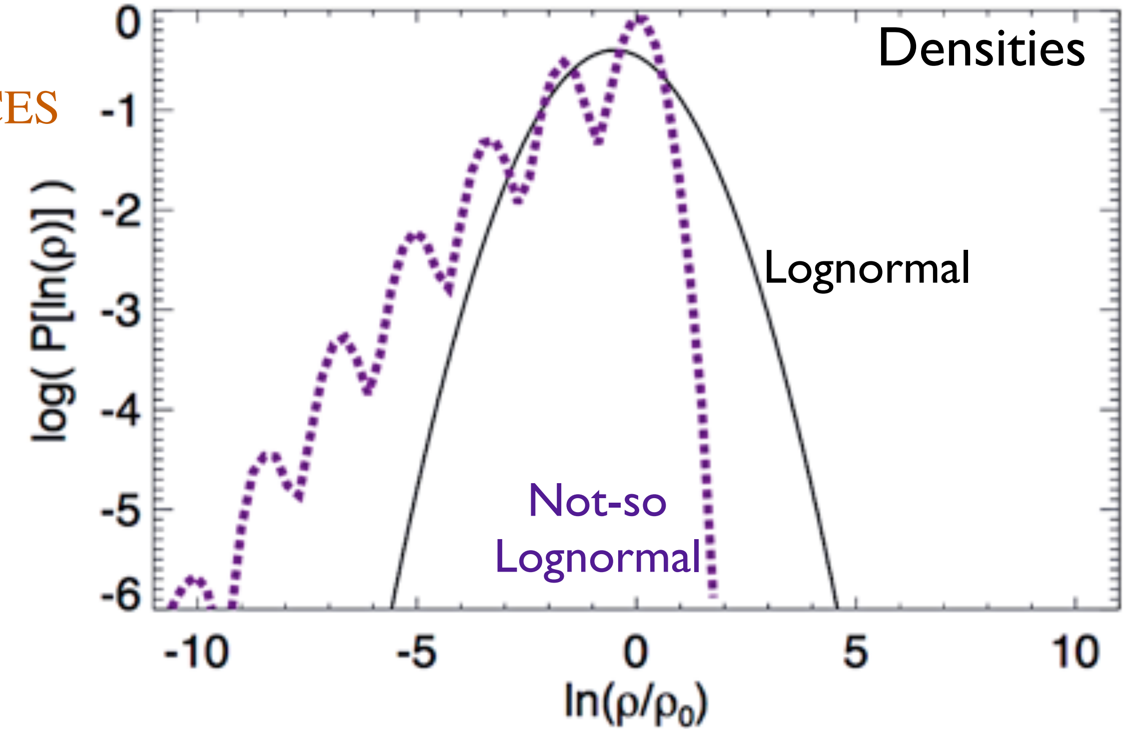
Testing the Analytics
vs. NUMERICAL SIMULATIONS

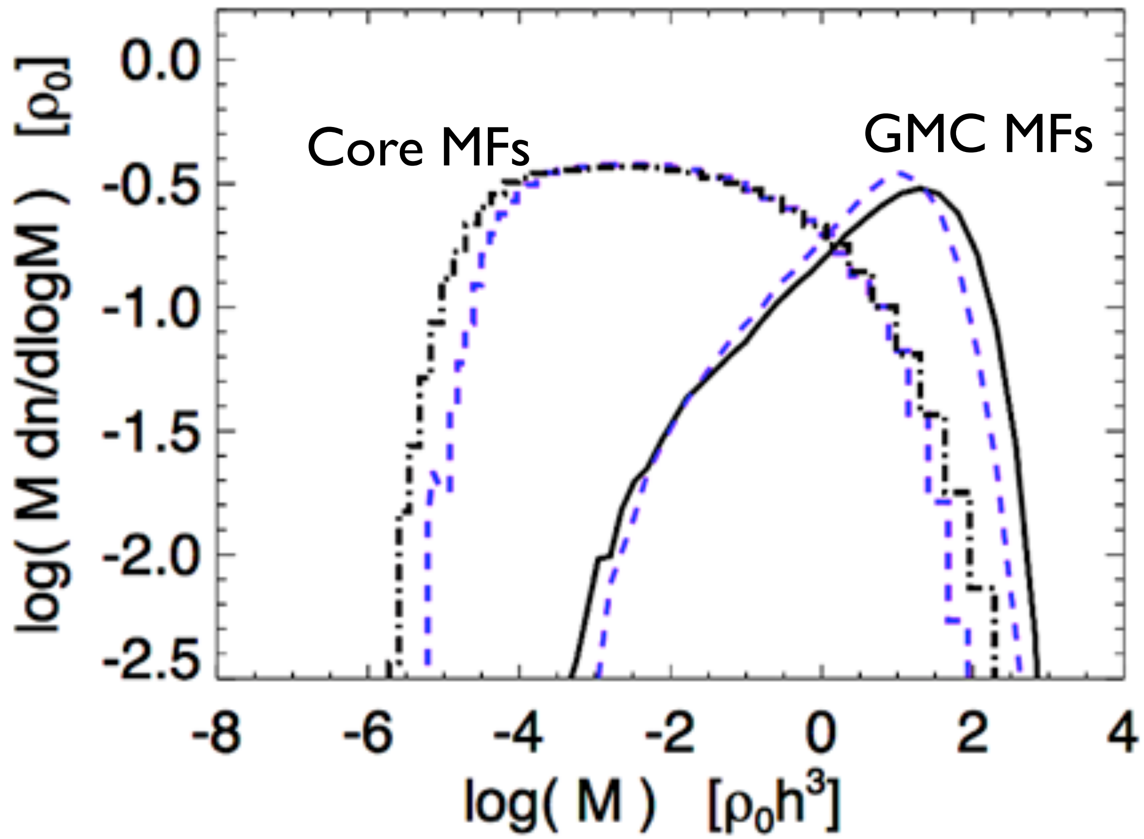


General, Flexible Theory:

EXTREMELY ADAPTABLE TO MOST CHOICES

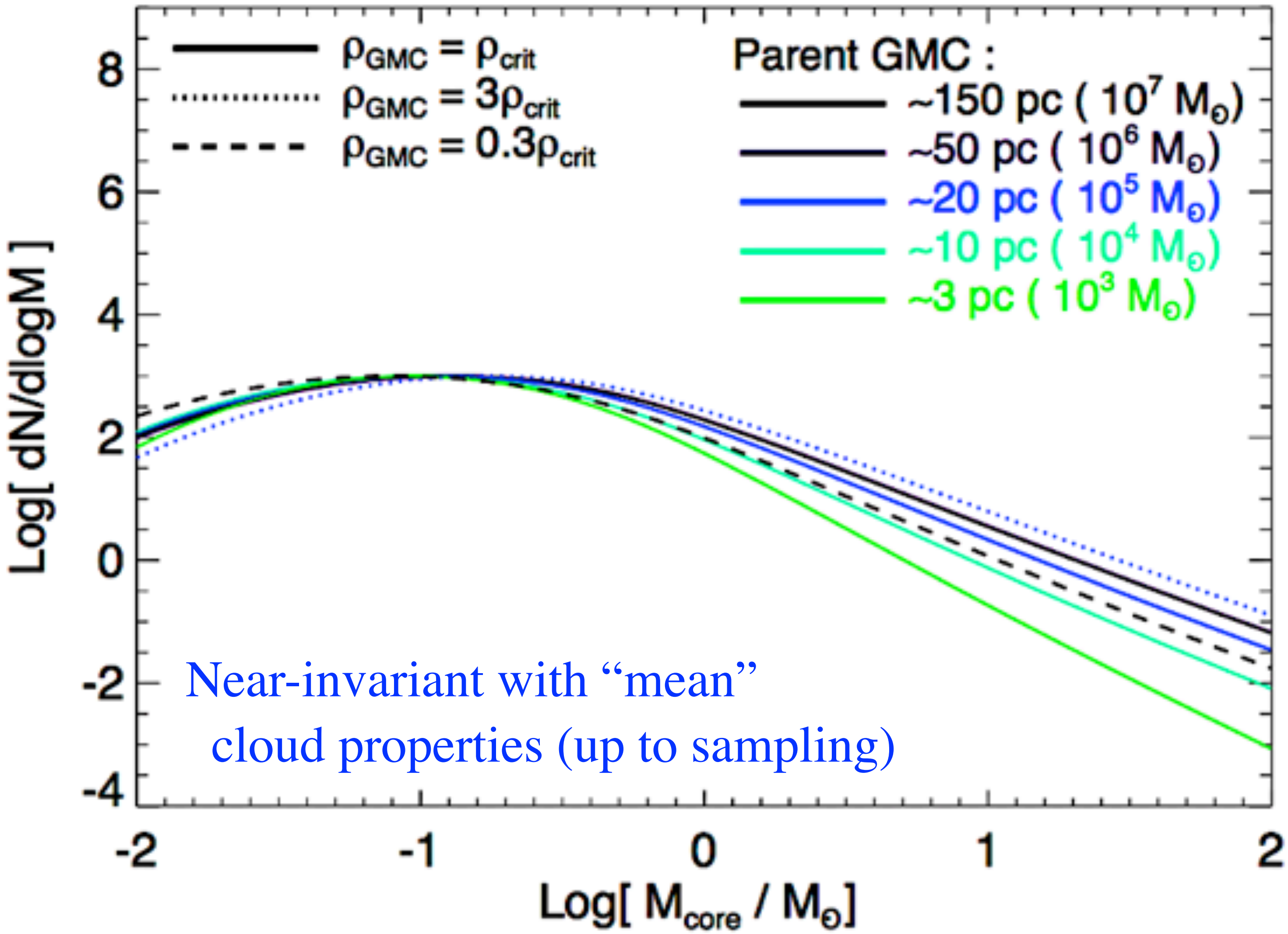
- Complicated, multivariable gas equations of state
- Accretion
- Magnetic Fields
- Time-Dependent Background Evolution/Collapse
- Intermittency
- Correlated, multi-scale driving





Variation in the Core Mass Function

VS "NORMAL" IMF VARIATIONS



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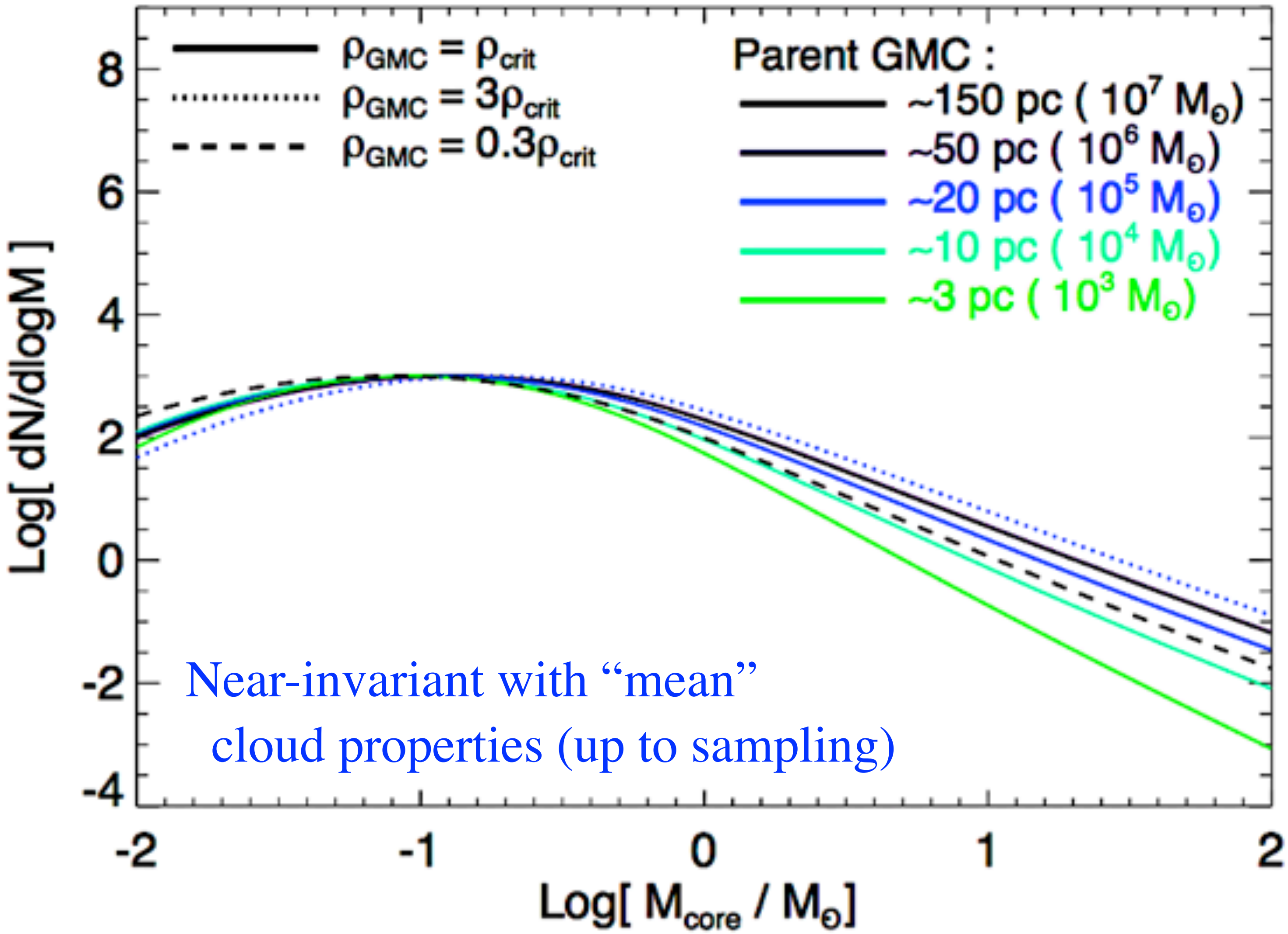

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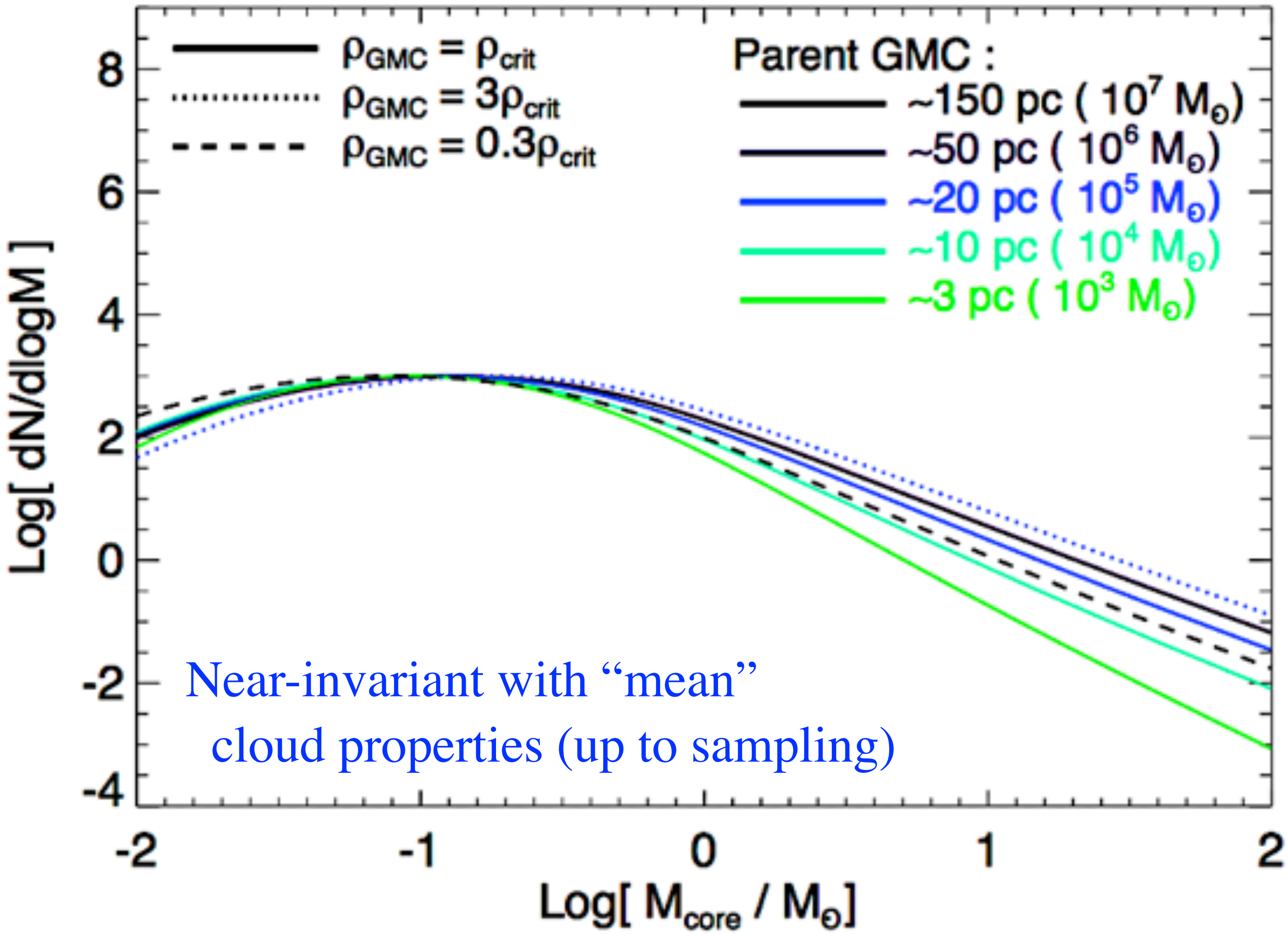

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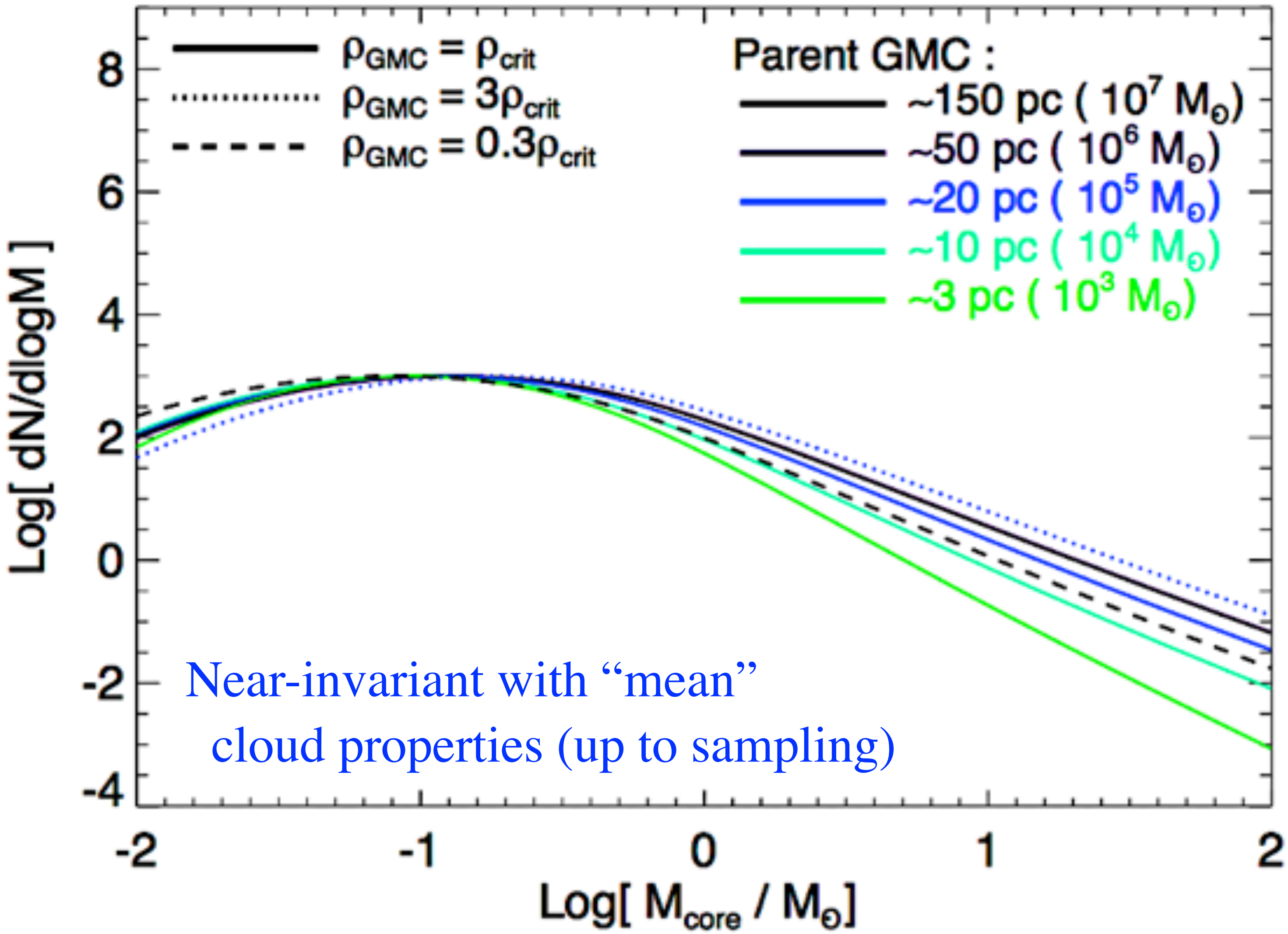

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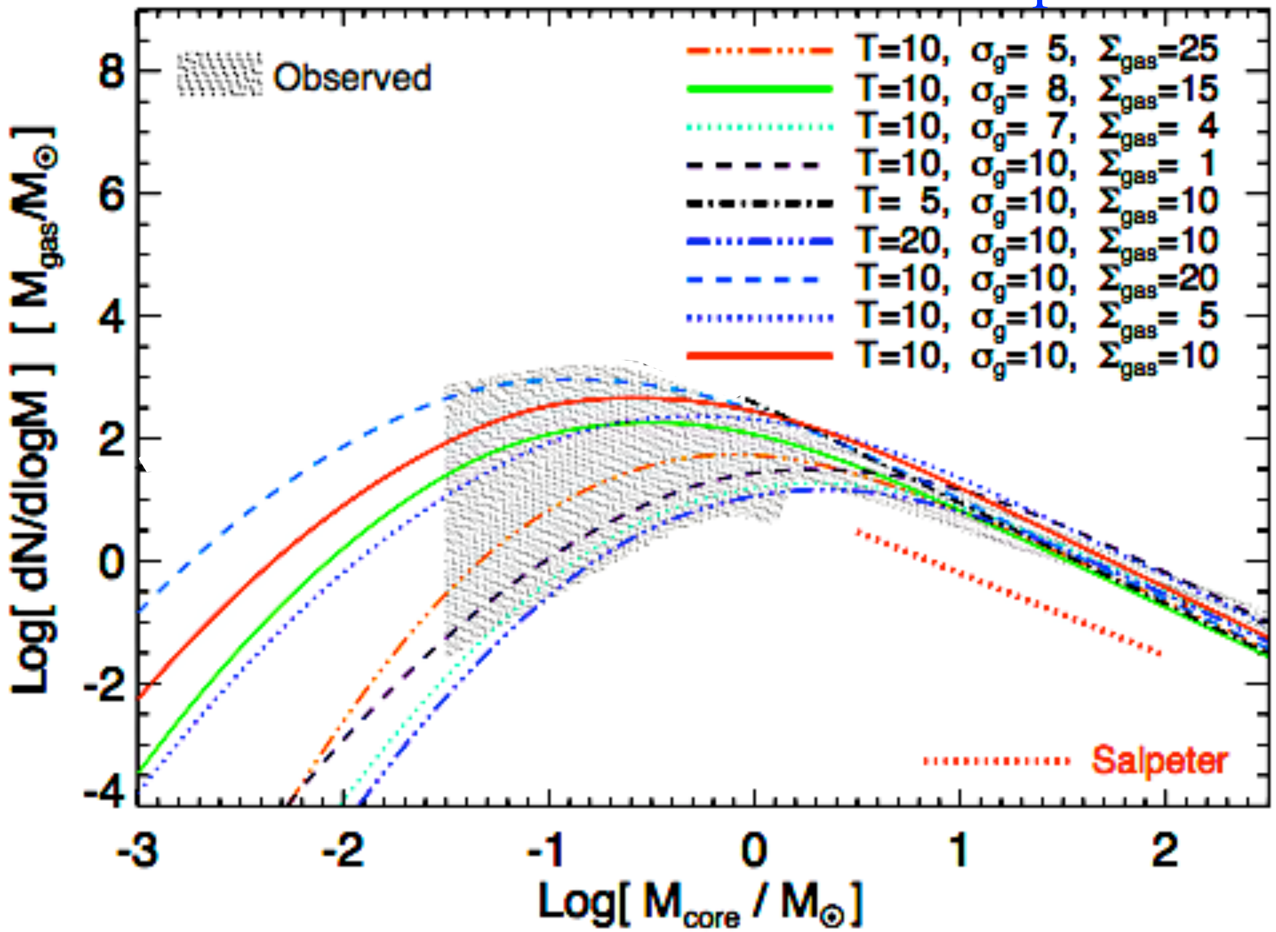
Variation in the Core Mass Function

VS “NORMAL” IMF VARIATIONS

Weak variation with Galactic Properties



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BUT, What About Starbursts?

MW: $T_{\rm cold} \sim 10\,K$

$\sigma_{\rm gas} \sim 10\,{\rm km\,s^{-1}}$

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ULIRG: $T_{\rm cold} \sim 70\,K$

$\sigma_{\rm gas} \sim 80\,{\rm km\,s^{-1}}$

$(Q \sim 1 \text{ for } \Sigma_{\rm gas} \sim 1000\,M_{\odot}\,{\rm pc^{-2}})$

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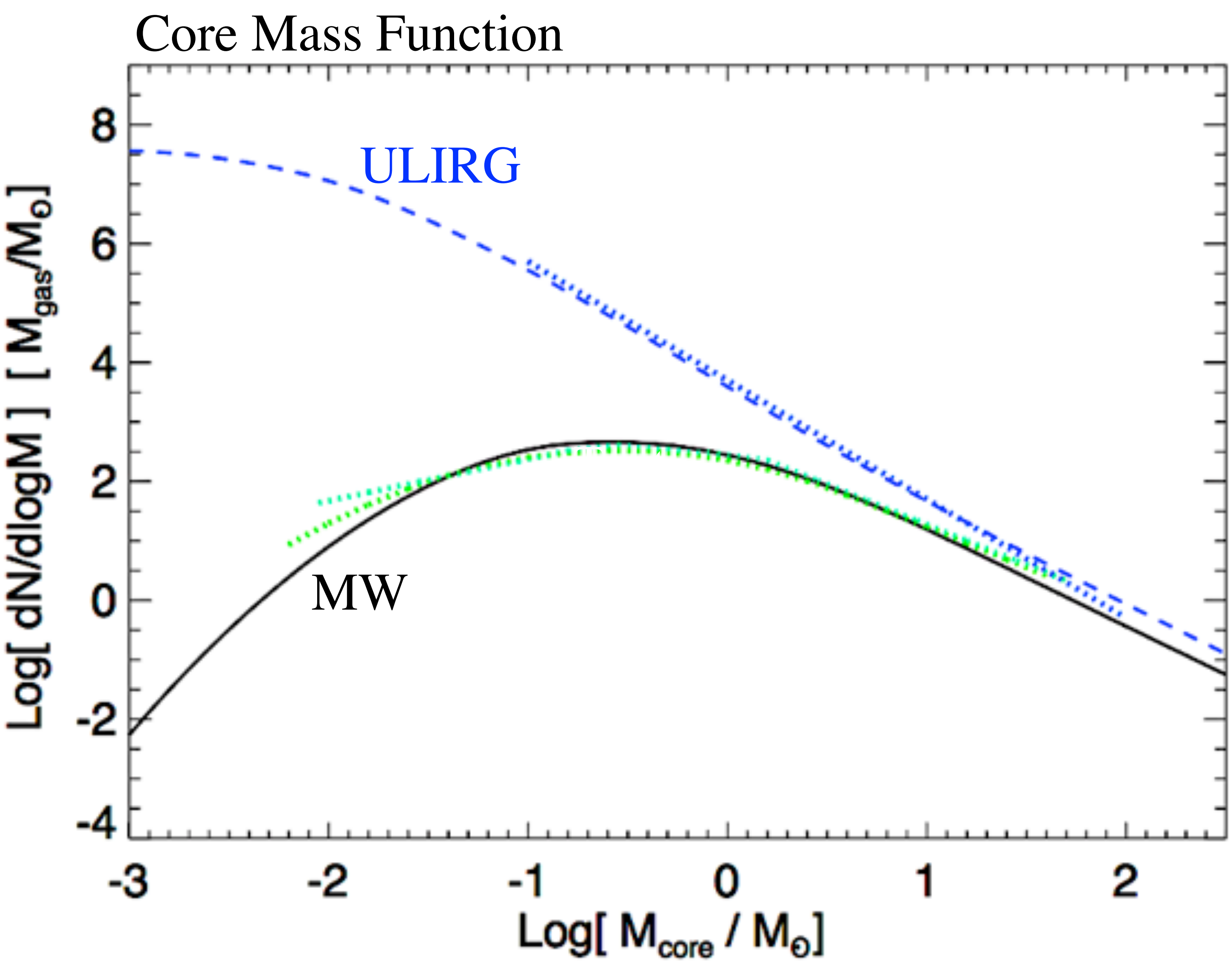

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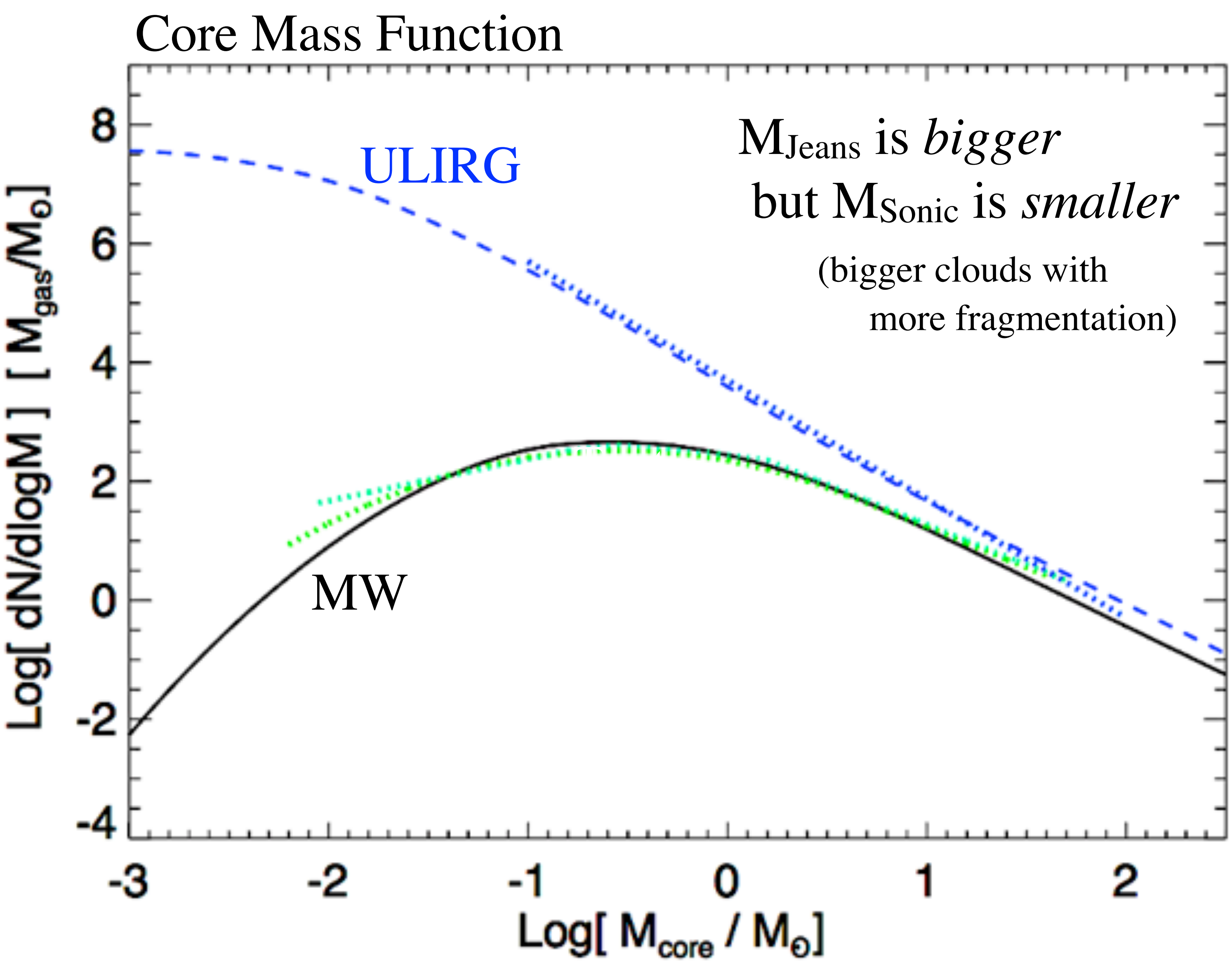
BOTTOM-HEAVY: TURBULENCE WINS!



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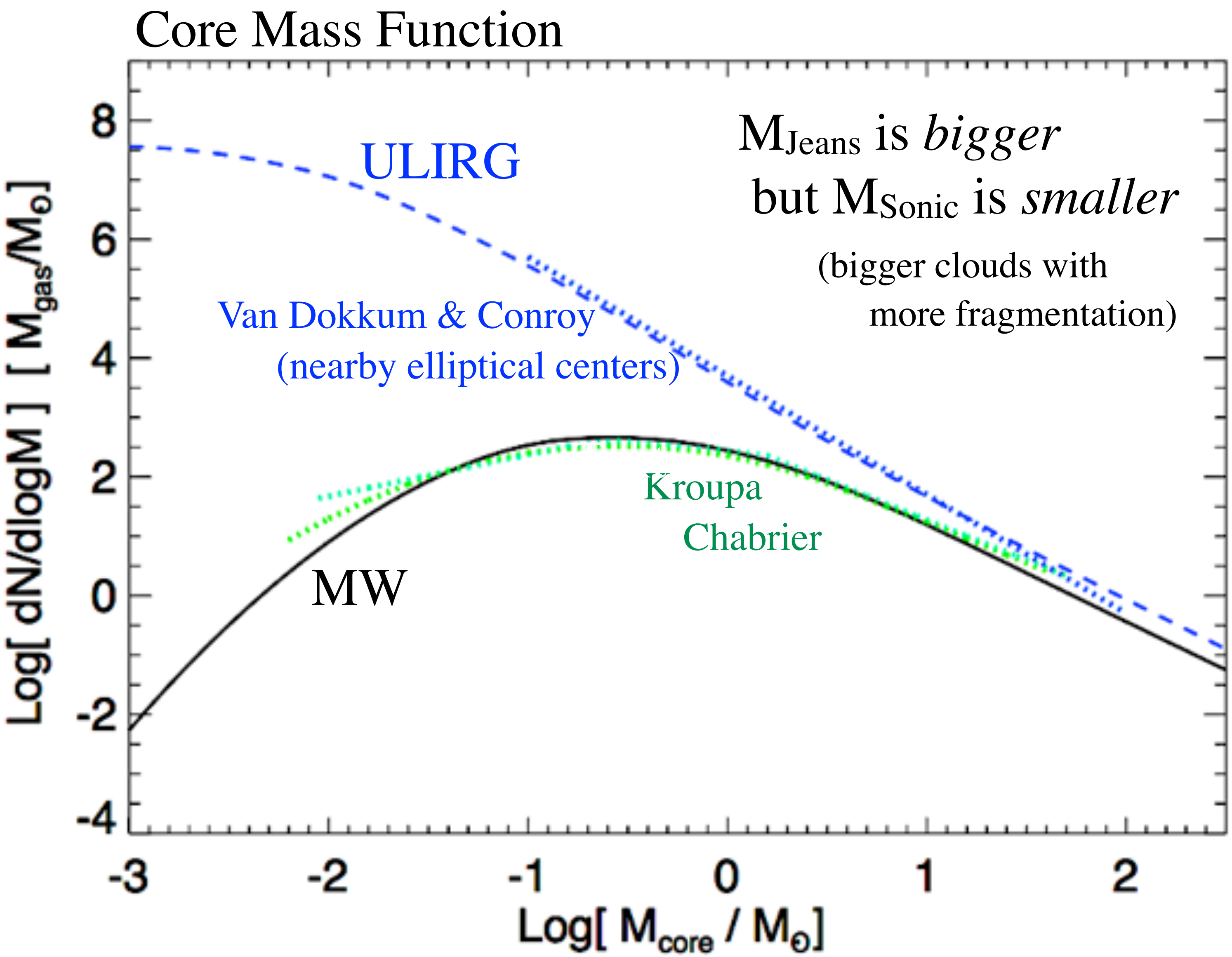

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Open Questions:

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Efficient Cooling: $\dot{P}_{\rm diss} \sim \frac{M_{\rm gas}\, v_{\rm turb}}{t_{\rm crossing}}$

2. Why Doesn't Everything Collapse?

"Top-down" turbulence can't stop collapse once self-gravitating

Fast Cooling: $\dot{M}_{*} \sim \frac{M_{\rm gas}}{t_{\rm freefall}}$

Summary:

*** ISM *statistics* are far more fundamental than we typically assume ***

- **Turbulence + Gravity:** ISM structure follows
  - Lognormal density PDF is ***not*** critical
  - *ANALYTICALLY* understand:
    - **GMC Mass Function & Structure** (“first crossing”)
    - **Core MF** (“last crossing”) & **Linewidth-Size-Mass**
    - **Clustering** of Stars (correlation functions)

- **Feedback Regulates & Sets Efficiencies of Star Formation**
  - K-S Law: ‘enough’ stars to offset dissipation (set by gravity)
    - Independent of small-scale star formation physics (how stars form)