Modeling Type Ia Supernovae

Alan Calder

A. Jackson, B. Krueger (Stony Brook),

D. Townsley (Alabama) E. Brown (MSU), F. Timmes (ASU),
D. Chamulak (ANL)

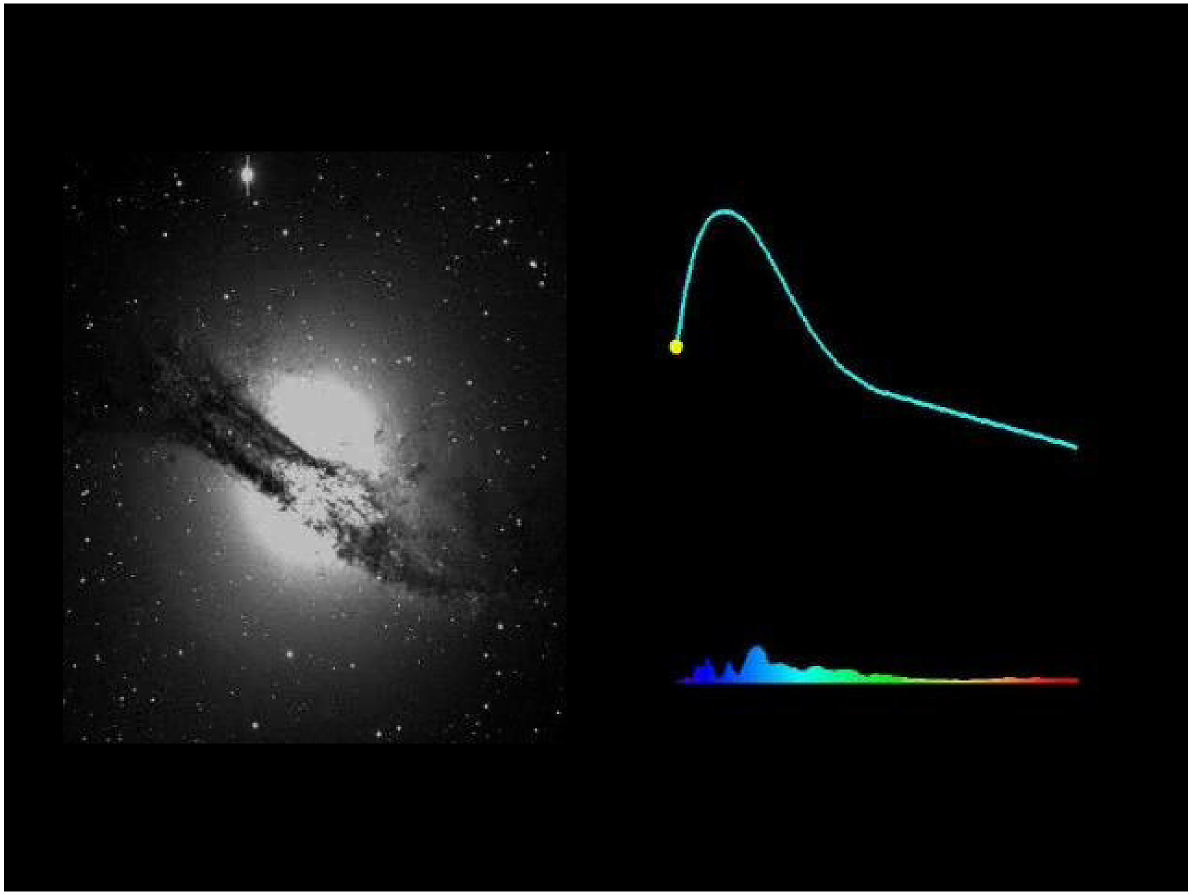
HIPACC Summer School
July 21, 2011

Outline

- Modeling type Ia (thermonuclear) supernovae
  - Introduction to the problem
  - Requisite physics
  - Deflagrations
  - Detonations
  - Role of Rayleigh-Taylor instability
  - Nuclear energetics, turbulence/flame interaction

- Research into SN Ia
  - Deflagration to detonation paradigm (DDT)
  - Research into the systematic effects
    - Effect of metallicity on DDT models.
    - Effect of changing DDT density (influenced by metallicity)
    - Effect of changing central density (influenced by accretion history)

SNe Ia Astronomical Appearance



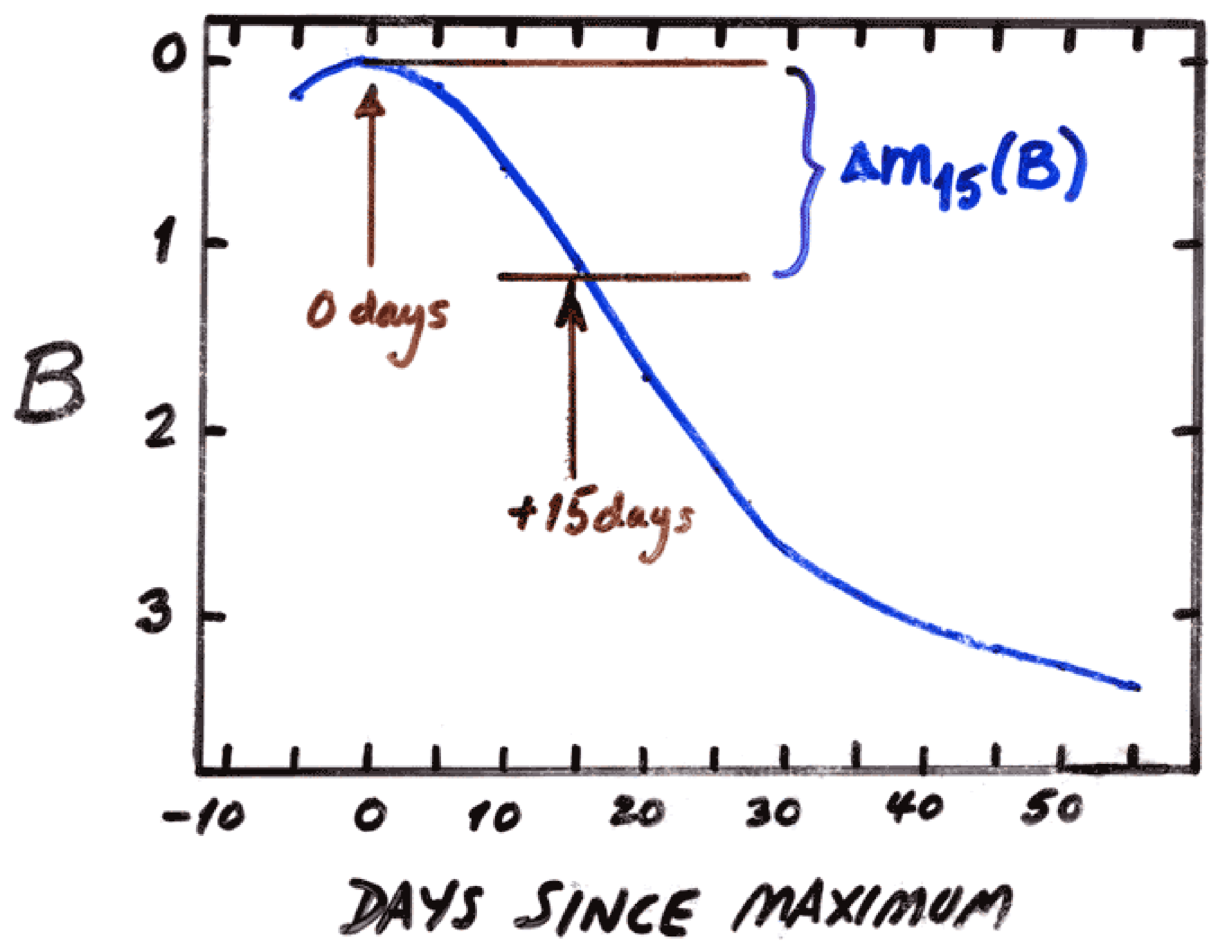
Observations: light curve, the observed intensity of light, and spectrum.

Light curve rises in days, falls off in weeks.

P. Nugent (LBNL)

Phillips Relation (1993)

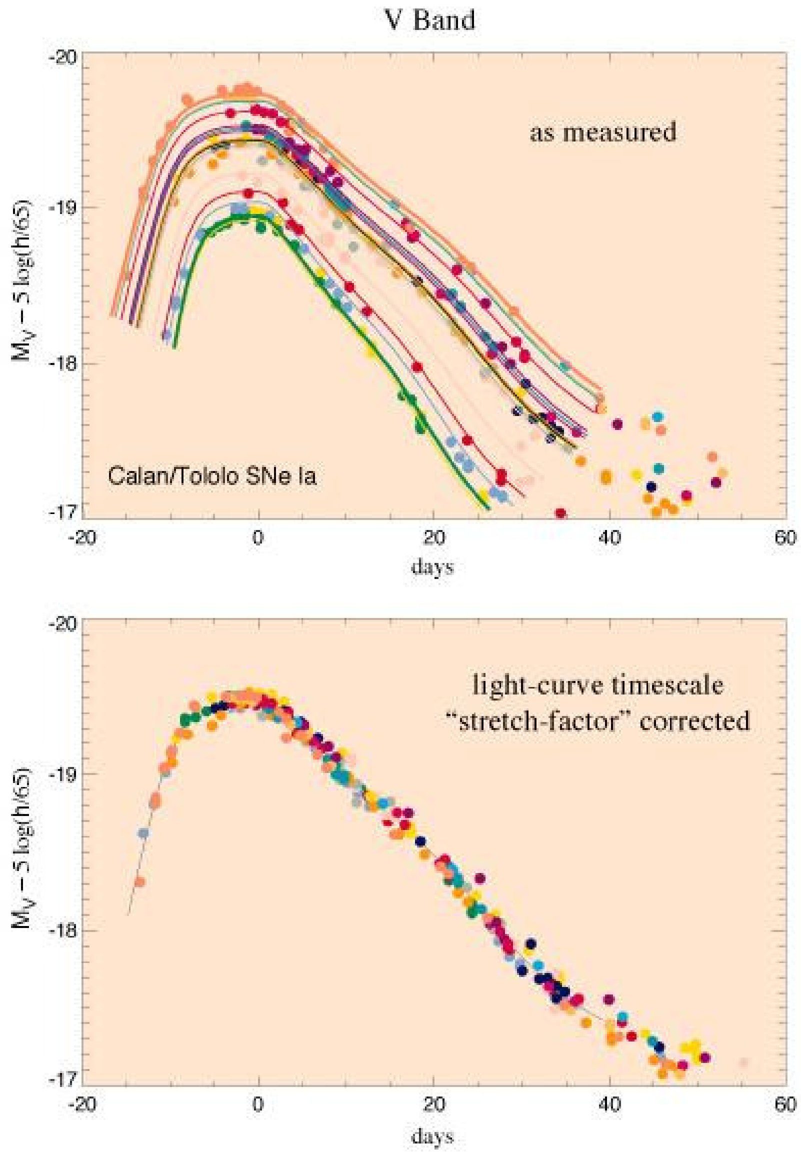
- Mark Phillips considered the change in B-band magnitude with time.
- Found fainter Ia's fade faster.



From Wikipedia

Phillips Relation (1993)

- Mark Phillips considered the change in B-band magnitude with time.
- Found fainter Ia's fade faster.
- Brighter = broader leads to a one-parameter stretch factor (from templates)
- "Standardizable candle"
- Key property: the radioactive decay of ^{56}Ni powers the light curve.



Kim et al.

Three Models Under Investigation

- Single-degenerate: accretion onto a white dwarf
  - Models can explain many aspects of problem- velocities, distribution of nuclides in remnant, etc.
  - Models not completely robust.
- Merging white dwarf pairs
  - Gilfanov & Bogdán (Nature 463 924, 2010) claimed that observed X-ray fluxes of early-type galaxies are too low to be consistent with the prediction of the SD scenario.
  - Hachisu, Kato, and Nomoto (ApJ 724 L212, 2010) argue the Super Soft X-ray Source (SSS) phase is shorter and thus a lower flux is to be expected
  - Models are somewhat preliminary, but getting there.
- Sub-Chandrasekhar (double detonation) model
  - Accreted He shell detonates, triggering a detonation in the core
  - Models may explain some events.

Peculiar Sne Ia

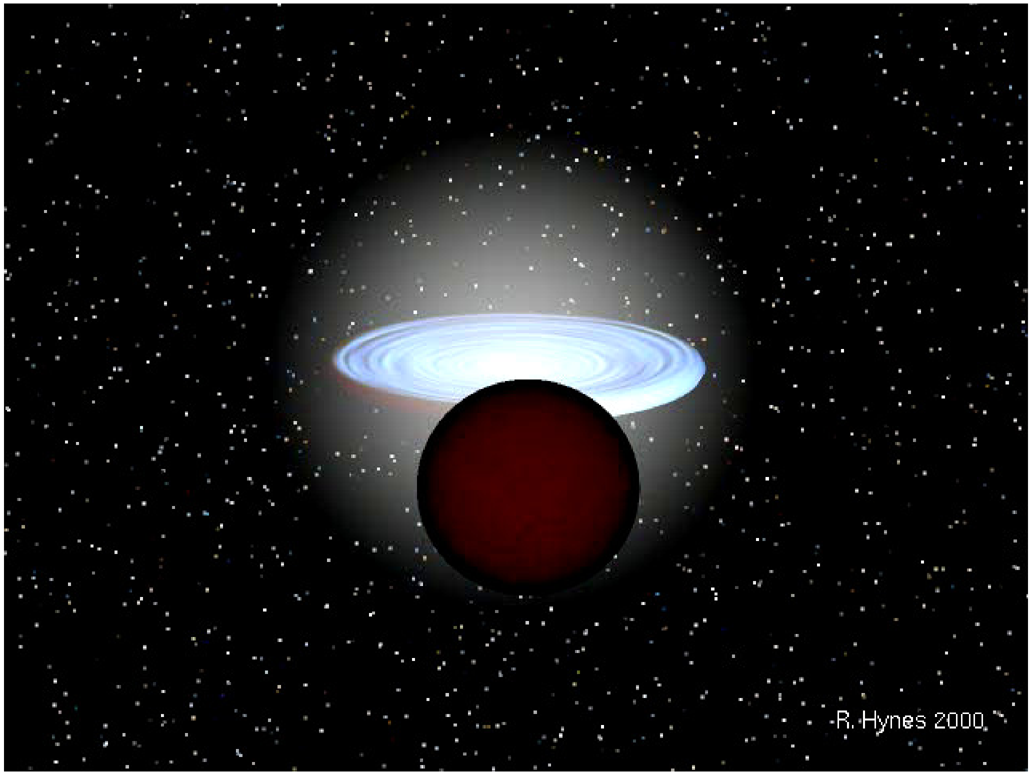
- SN 1991t : Fe and Ni at high velocities. How to get core elements to surface?

- SNLS-03D3bb (SN 2003fg): high luminosity and low kinetic energy. Too bright for normal SN Ia?

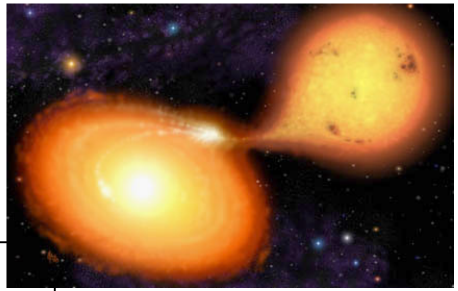
- SN 2007if: bright SN Ia that implied more mass consumed than in a single white dwarf.

Favored Scenario

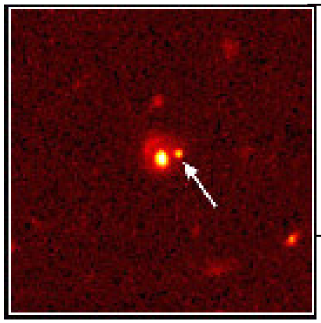
- Mass accretes from a companion onto a white dwarf that then ignites thermonuclear burning.
- Nature of that burning has been the fundamental problem for 30+ years.
  - Is it a deflagration (subsonic flame)?
  - Is it a detonation (supersonic flame)?
  - Will all of star burn? Burn to what?
- Can models reproduce observed nuclear abundances and light curves?



Modeling SN Ia's in SD Scenario



Mark A. Garlick



P. Garnavich/CfA

Light curve

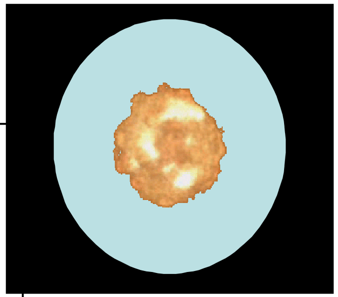
- free expansion of envelope
- multi-group (non-LTE) radiation transport

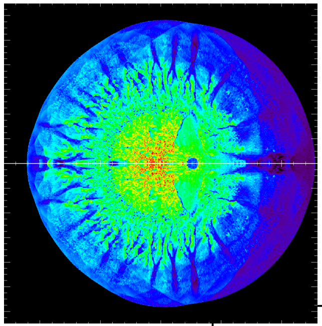
Accretion

- stellar evolution code with accretion/binary evolution code

>10^8 yr

~ seconds





Smoldering

- subsonic convection in core of white dwarf
- low Mach number flow solver
- conductive heat transport

~ 1000 yr

ignition

Flame/Explosion

- initial deflagration
- DDT or expansion/recollapse
- FLASH (compressible module) with subgrid model for flame.

Physics of Type Ia Supernovae

- Studying SN Ia requires large-scale (~1000s of processors for days) fluid dynamics simulations for any hope of progress!
  - Realistic progenitor model
  - Multi-physics:
    - Reactive Euler equations with self-gravity (multi-dimensional!)
    - Equation of state for degenerate matter
    - Flame model (width/radius < 10^{-9})
    - Nuclear Energetics: ${}^{12}C+{}^{12}C$; burn to Nuclear Statistical Quasi-equilibrium (Si group); burn to Nuclear Statistical Equilibrium (Fe group).
    - Emission of ν's result in energy loss, ΔY_e (neutronization)
    - Turbulence-flame interaction.
- Realistic models should include:
  - Rotation
  - Magnetic fields

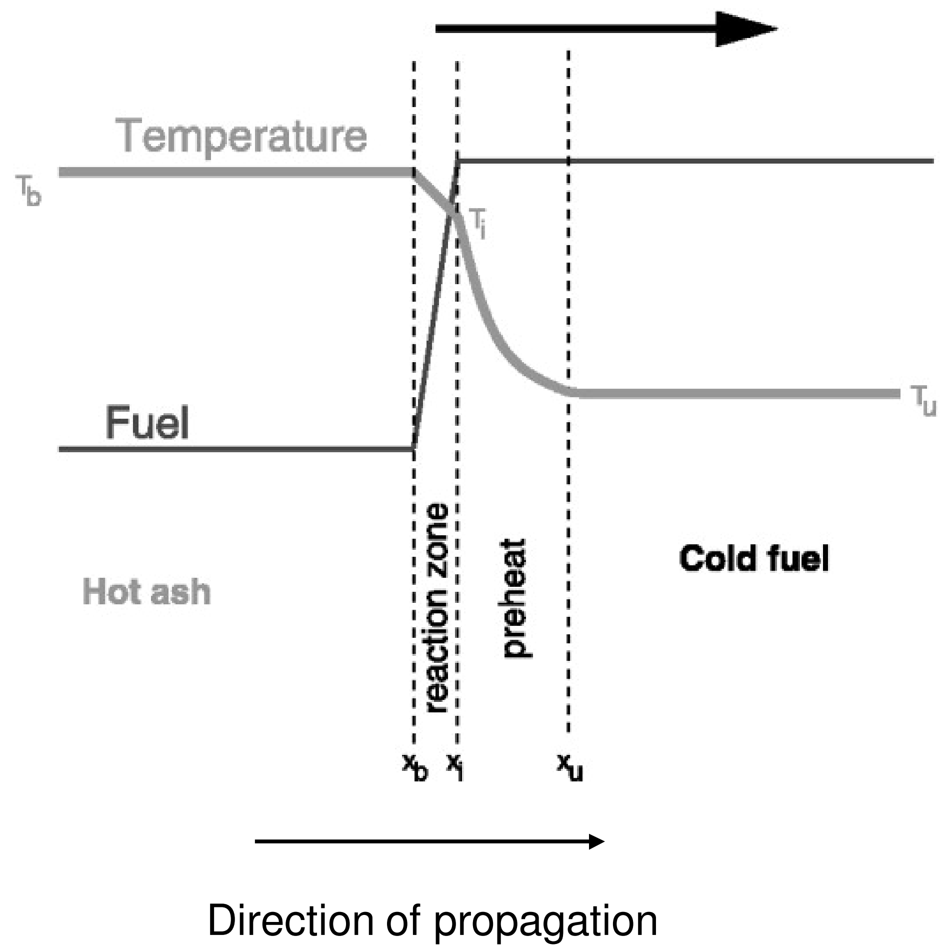
Types of Combustion Waves

- Detonation: Rapid combustion of a material that propagates as a shock wave at supersonic speeds.
- Deflagration: Combustion of a material that propagates as a burn wave at subsonic speeds
- Lewis number: ratio of thermal diffusivity to mass diffusivity.
- Astrophysical flames propagate by heat conduction, hence large Lewis numbers. Terrestrial flame have Lewis number of order unity.

Astrophysical Flames

An astrophysical flame (deflagration) propagates via the conduction (transport) of heat that pre-heats the fuel, initiating the reactions.

The schematic shows a simple, one-reaction case of a deflagration.

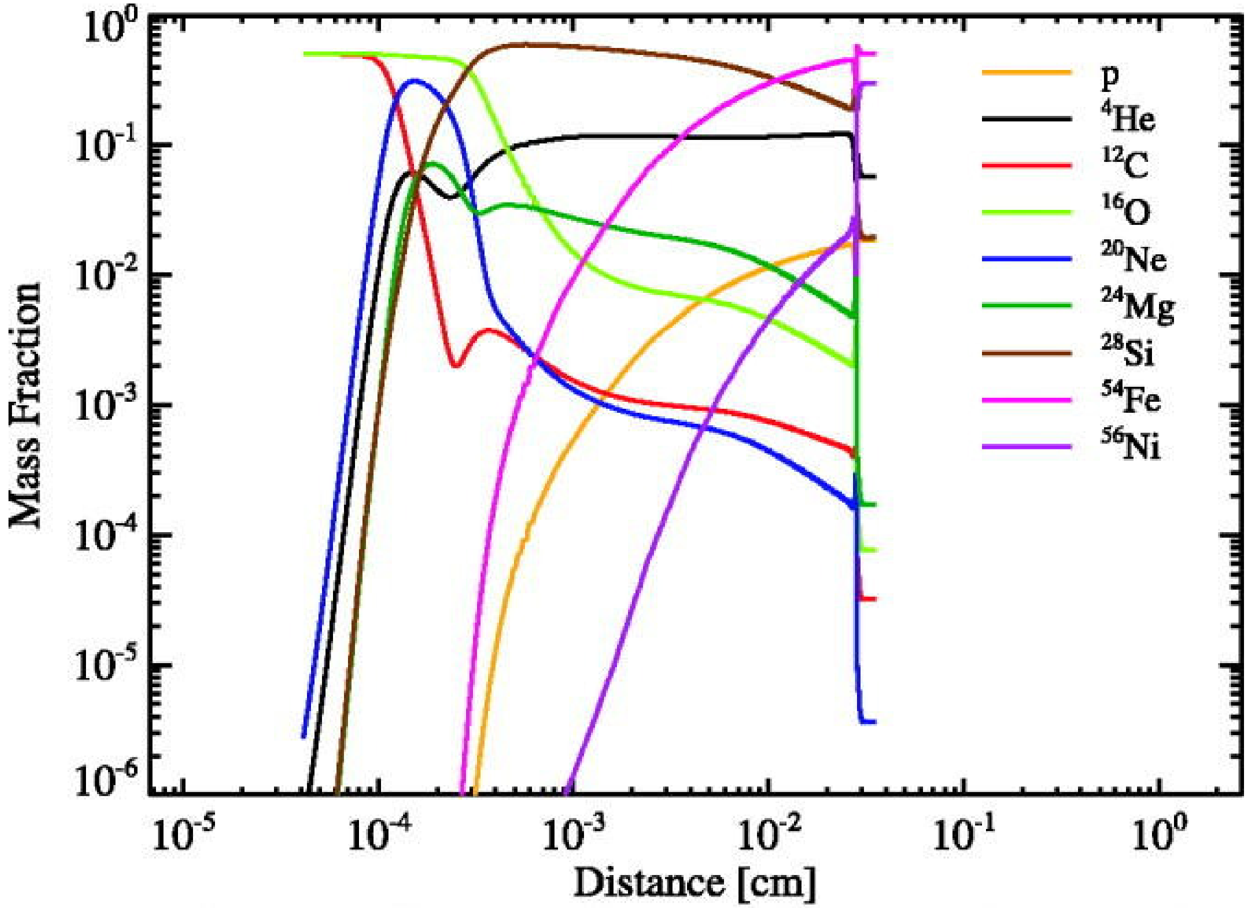


Dursi et al. ApJ 595, 955 (2003)

DNS of Nuclear Flames

$\rho = 10^9$ g/cm^3

C/O Fuel



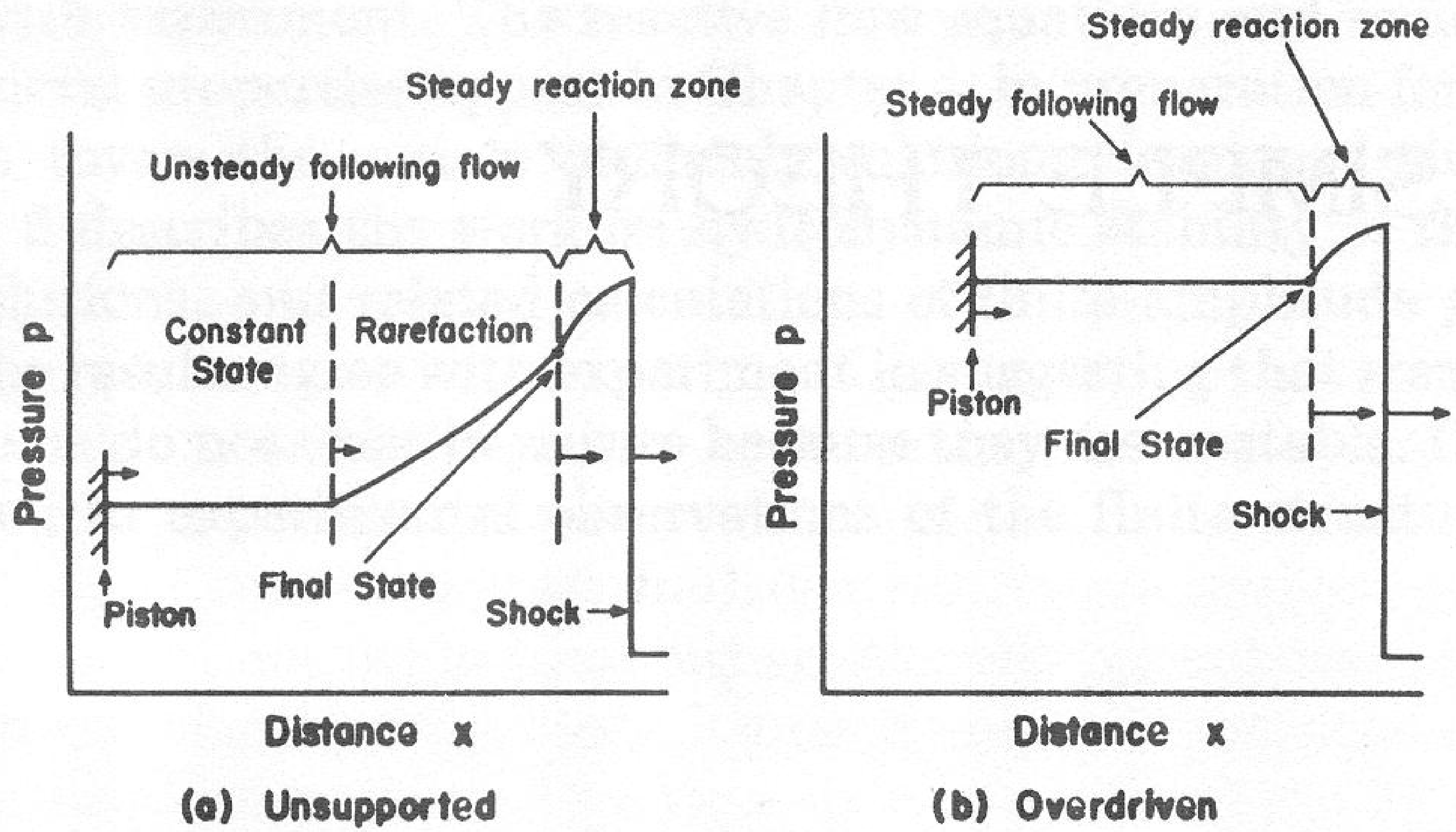
Ash (→ NSE)

aprox19 network

← Flame propagation

Calder et al. ApJ 656, 313 (2007)

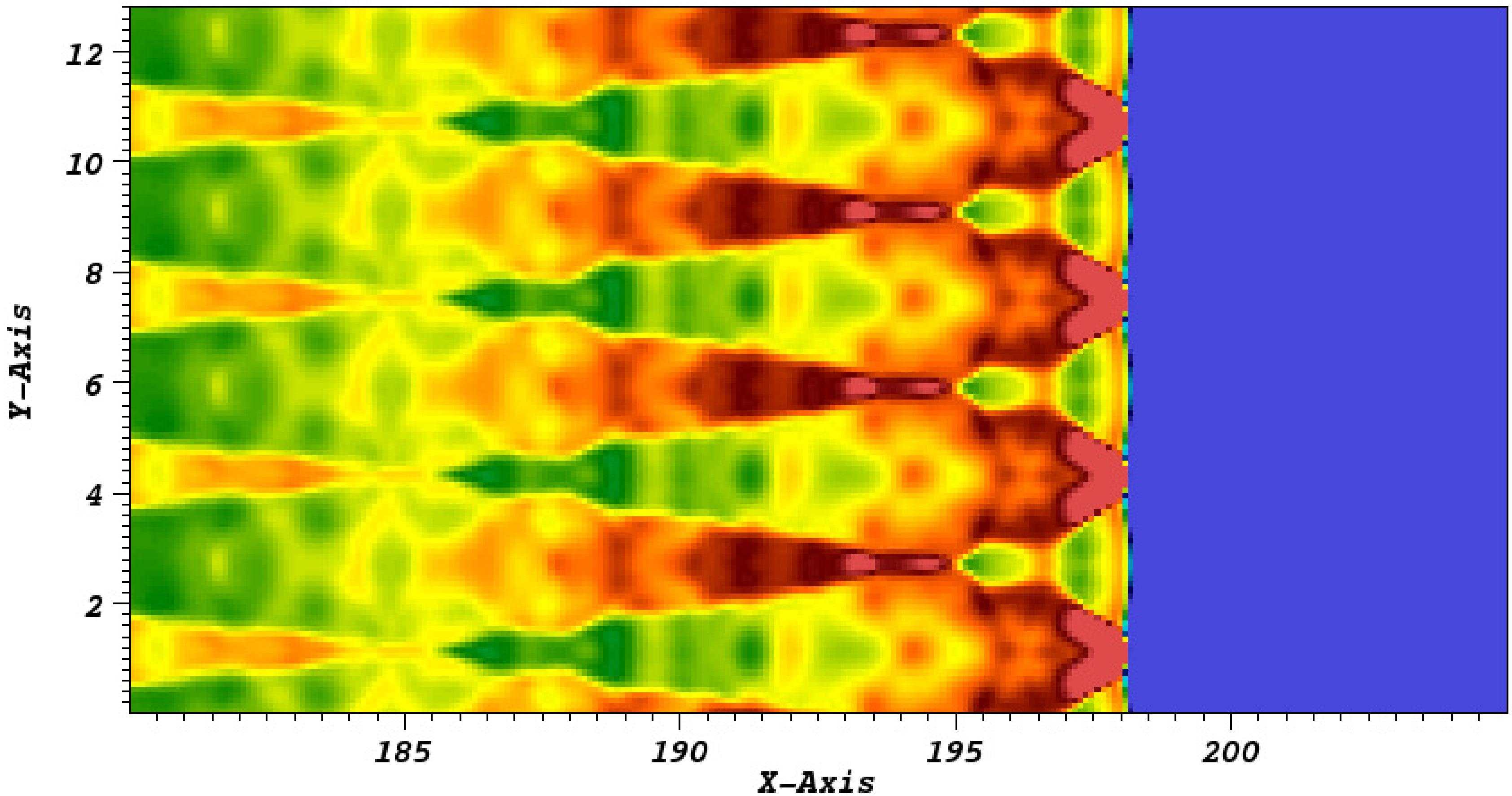
Detonations



Direction of propagation

Fickett & Davis “Detonation”

Cellular Detonation



Direction of propagation

Timmes et al. ApJ 543 938 (2000)

Flash simulations of nuclear burning

- Cellular detonation (in distribution): a resolved 2-d detonation
- Thermonuclear flame (homework assignment): a resolved deflagration
- Both have small length scales (< 1 cm.). What about simulating a type Ia supernova with ~1000 km length scales?
- The thermal diffusion time scale limits the resolution of a deflagration. Deflagrations must be resolved or one must use a model or thickened flame. More about a model flame in Flash later.
- Flash with PPM has special algorithms for shocks. Can it capture unresolved detonations? Yes, but there are some issues. What does one do?
- SnDet detonating white dwarf (in distribution): an unresolved detonation in a WD model.

Parameters for Nuclear Burning

From the SnDet flash.par file:

```
# burn, but not in a shock
useBurn = .true.
useShockBurn = .false.

# threshold to cut off burning.
nuclearNI56Max = 0.7

#maximum fraction of eint to release by burning with a
   time step.
enucdtfactor = 0.1
```

Simulating detonations:

Fryxell, Müller, & Arnett, MPI Astrophys. Rep. 449 (1989)

Townsley et al. (2011 in prep)

Running the SnDet setup

- No promises! Not as debugged as I had hoped. Feel encouraged to test and improve.

- Output from a run:

```
*** Wrote checkpoint file to snd_hdf5_chk_0000 ****
*** Wrote plotfile to snd_hdf5_plt_cnt_0000 ****
 Initial plotfile written
 Driver init all done
n          t          dt        (      x,            y,            z)     |  dt_hydro  dt_Burn
 1 2.0000E-16 1.2000E-16  (1.800E+06, -2.000E+05,  0.000E+00) |  1.211E-04 6.792E-11
 2 4.4000E-16 1.4400E-16  (1.800E+06, -2.000E+05,  0.000E+00) |  1.211E-04 6.792E-11
 3 7.2800E-16 1.7280E-16  (1.800E+06, -2.000E+05,  0.000E+00) |  1.211E-04 6.791E-11
 4 1.0736E-15 2.0736E-16  (1.800E+06, -2.000E+05,  0.000E+00) |  1.211E-04 6.791E-11
 5 1.4883E-15 2.4883E-16  (1.800E+06, -2.000E+05,  0.000E+00) |  1.211E-04 6.790E-11

69 1.2398E-10 3.1847E-12  (1.800E+06, -1.000E+06,  0.000E+00) |  1.211E-04 4.496E-12
70 1.3035E-10 2.6483E-12  (6.000E+05, -6.000E+05,  0.000E+00) |  1.211E-04 2.648E-12
71 1.3564E-10 3.1779E-12  (6.000E+05, -1.000E+06,  0.000E+00) |  1.211E-04 4.390E-12
```

SN Ia Picture We Will Explore

- Smoldering phase gradually heats the core and produces considerable turbulence.
- Eventually a patch stagnates and gets hot enough that the energy generation exceeds convective cooling and a flame is born.
- A period of deflagration (subsonic burning) ensues. The flame consumes some of the star, but it has time to react and it expands some.
- A transition to a detonation (supersonic burning) occurs, incinerating the star and producing ~0.6 M_{solar} ^{56}Ni, which powers the light curve.
- Note that much of what we will see applies to other pictures as well.

Evolution Equations

Euler:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla P = \rho \mathbf{g}$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho E + P)\, \mathbf{v} \ = \ \rho \mathbf{v} \cdot \mathbf{g} + S$$

$$P = P\,(\rho, E)$$

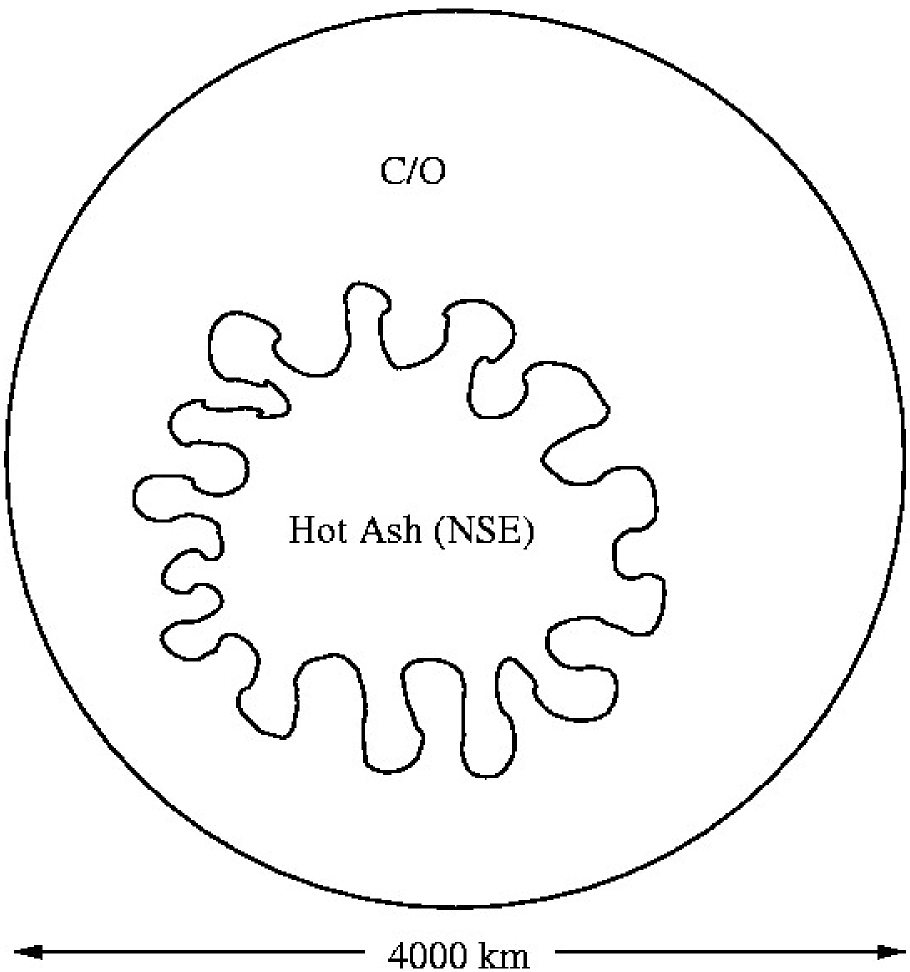
Gravity:

$$\mathbf{g} = -\nabla \Phi, \quad \text{where} \ \ \nabla^2 \Phi\,(\mathbf{r}) = 4\pi G \rho\,(\mathbf{r})$$

Advection of scalars:

$$\frac{\partial X \rho}{\partial t} + \nabla \cdot (X \rho \mathbf{v}) = 0$$

Fluid Instability in a Type Ia Supernova



Fluid dynamics are very important. The simmering progenitor and Rayleigh-Taylor instabilities (RTI) generate turbulence.

Even with AMR, the disparate scales of Ia necessitate use of a model flame and a sub-grid-scale model for turbulent combustion.

Subgrid model should capture effects of RTI and the flame-turbulence interaction on unresolved scales.

Flame Model Implemented in Flash

- "Thick flame" based on an advection-reaction-diffusion equation model (Khokhlov 1995) $\Delta = 4$ zones
  - Flame speed is input parameter to the model
  - Input flame speed is the maximum of the laminar or the turbulent model speed, $S = \max(S_{lam},S_{sub})$
  - S_{lam} from Timmes and Woosley (1992) and Chamulak et al. (2008)
  - S_{sub} accounts for unresolved R-T instability and TFI.
- Energetics of the flame described using the results of previous detailed calculations (Calder et al. 2007, Townsley et al. 2007).
- Evolution of the NSE ash similarly described using results of prior calculations (Seitenzahl et al. 2009)

Evolution Equations

ADR:

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = \kappa \nabla^2 \phi + \frac{1}{\tau} R(\phi)$$

κ and τ set to produce the input flame speed.

Reactions:

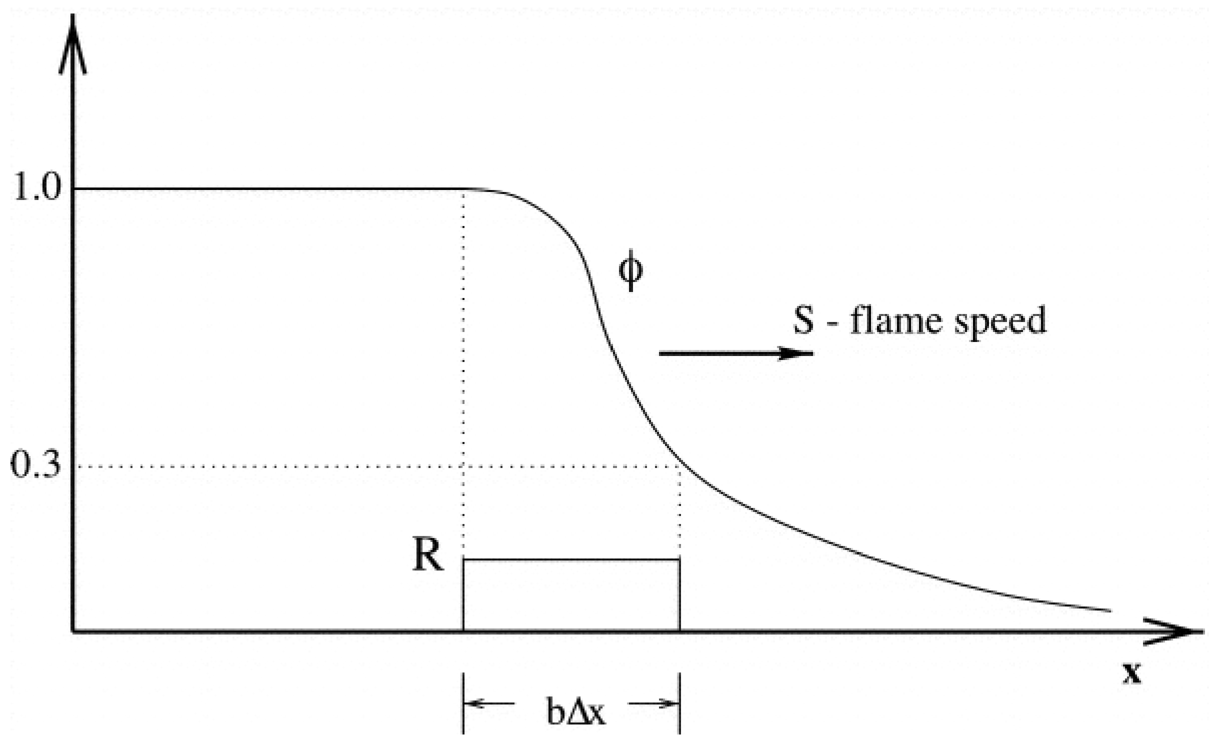
$$R(\phi) = \begin{cases} R_0 = \text{const.}, & \text{if } \phi_0 \leq \phi \leq 1 \\ 0, \text{otherwise} \end{cases} \qquad \text{top hat}$$

$$R(\phi) = \tfrac{1}{4}\phi(1-\phi) \qquad \text{KPP}$$

$$R(\phi) = \tfrac{f}{4}(\phi - \epsilon_1)(1 - \phi + \epsilon_2) \qquad \text{sKPP}$$

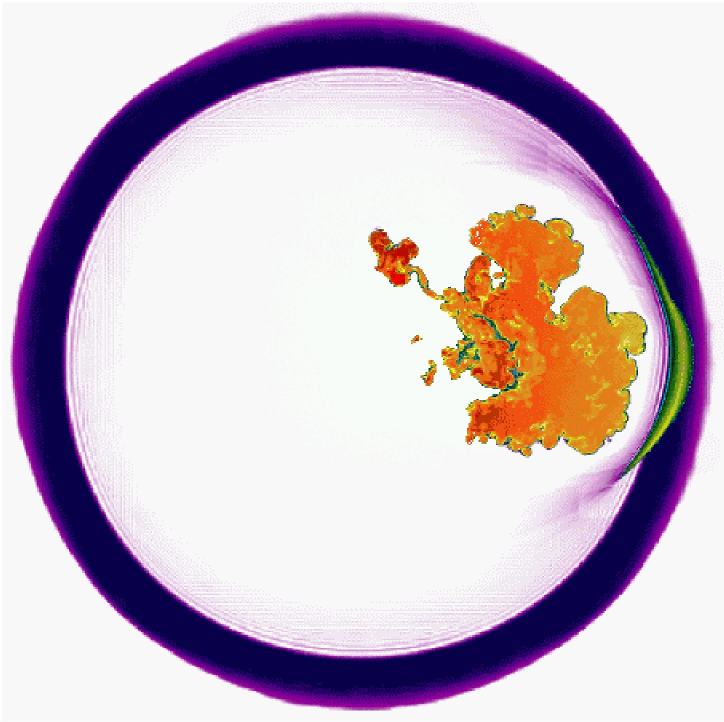
where $0 < \epsilon_1, \epsilon_2 < 1$ and $f = f(\epsilon_1, \epsilon_2)$

One-stage ADR scheme



Role of Flame and Ash Energetics

- Buoyancy of bubble is the key – depends on composition and energy produced in flame and in “ash”
  - Binding energy of NSE state at end of flame determines the composition and energy release (temperature)
  - Binding energy of NSE state continues to change as density decreases and composition changes in rising bubble
  - Weak interactions (neutronization) also produce composition changes and gain/loss of energy
- Accurate treatment of composition and energy are therefore essential

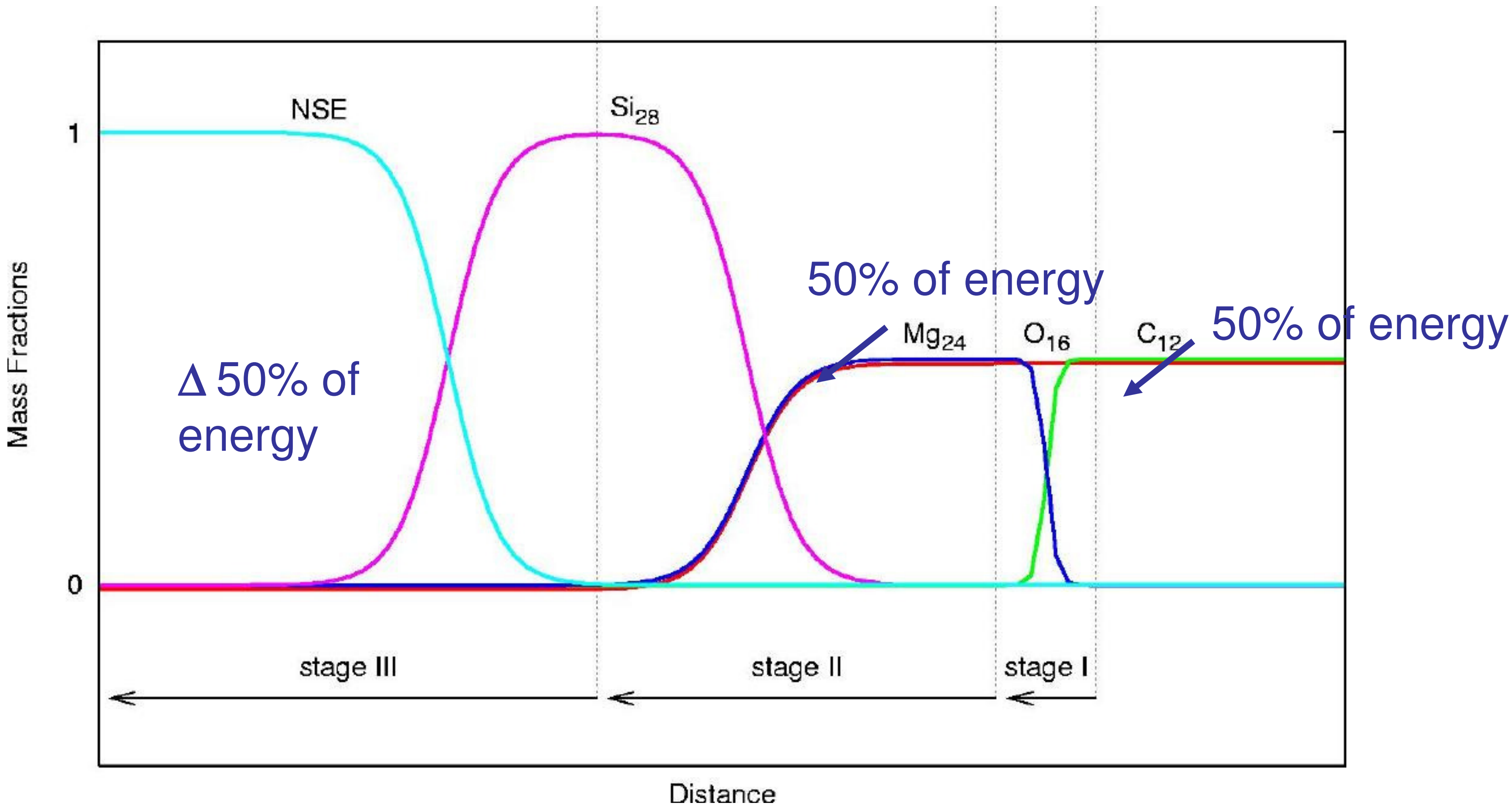


Energetics Procedure

- Perform self-heating (one-zone) network calculations with contemporary reaction rates (including weak reactions) and Coulomb effects.
  - Energy release
  - Time scales for stages of burning
  - Compare to DNS flames where possible for verification.
- Describe long-term evolution of NSE (binding energy and neutronization) with NSE code consistent with network calculations.
- Incorporate both into multi-stage flame model and dynamic NSE ash.
- Test, test, test.
  - ADR scheme (verify and quantify noise and curvature effects)
  - Subgrid turbulence model

Three-Stage Flame Model

- Energy released in flame and ash are both important



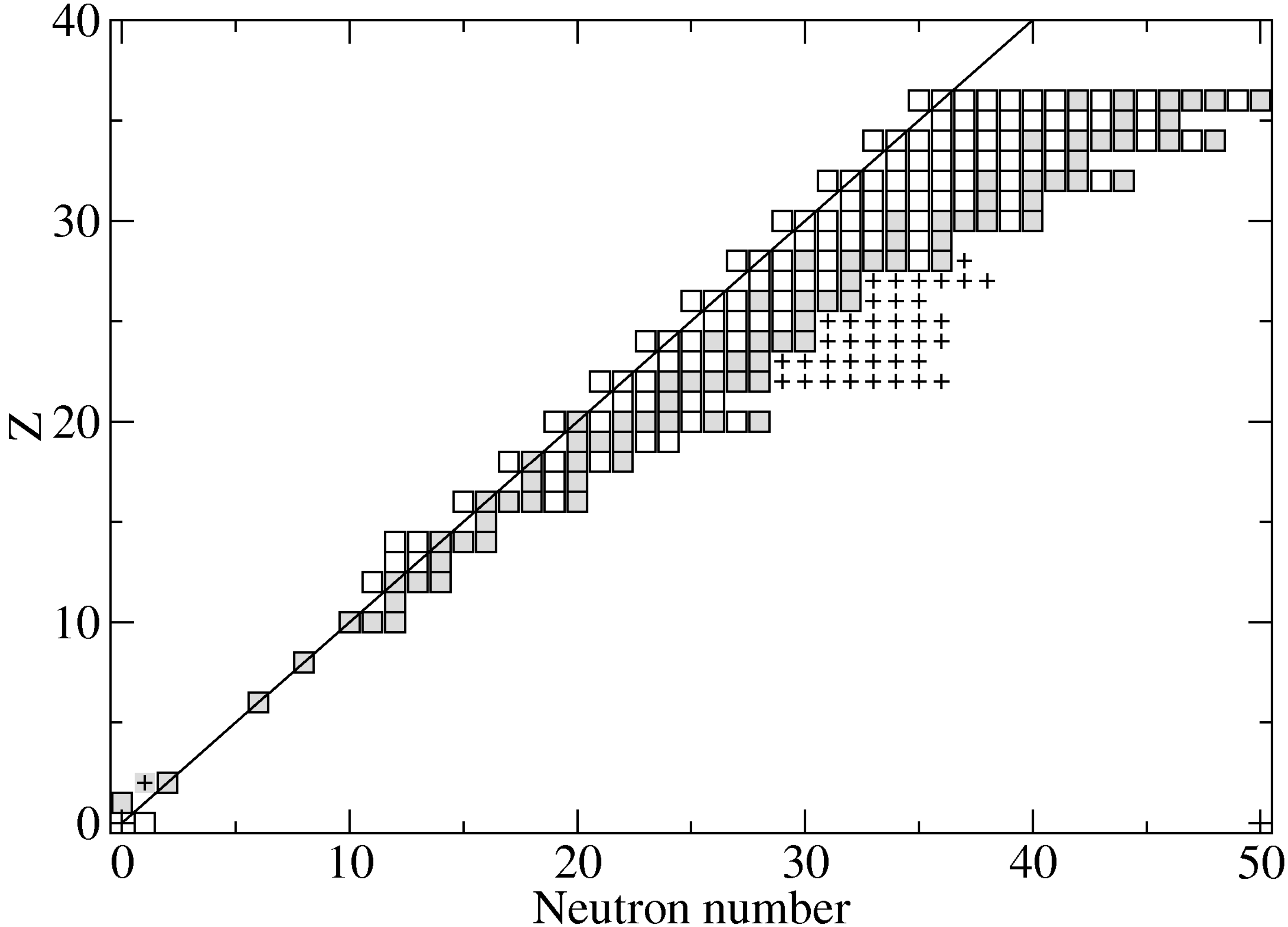
Flame propagation →

NSE and Self-Heating Calculations

- Nuclear Statistical Equilibrium code:
  - Solves NSE equations for 238 nuclides
  - Recent work has more (443)
  - Includes excited states (Rauscher et al. 1997)
  - Includes Coulomb corrections to Helmholtz free energy
  - Calculates energy, ν loss rates, and neutronization rates
  - Details in Seitenzahl, et al. (2009)

- Self-heating network code: Isochoric (constant volume) and isobaric (constant pressure) burning
  - 200 nuclide network
  - Temperature dependent nuclear partition functions from Rauscher and Thielemann (2000)
  - Reverse rates derived for first time self-consistently from forward rates with Coulomb effects included
  - Include electron screening (Wallace et al.1982)
  - Isobaric and isochoric results

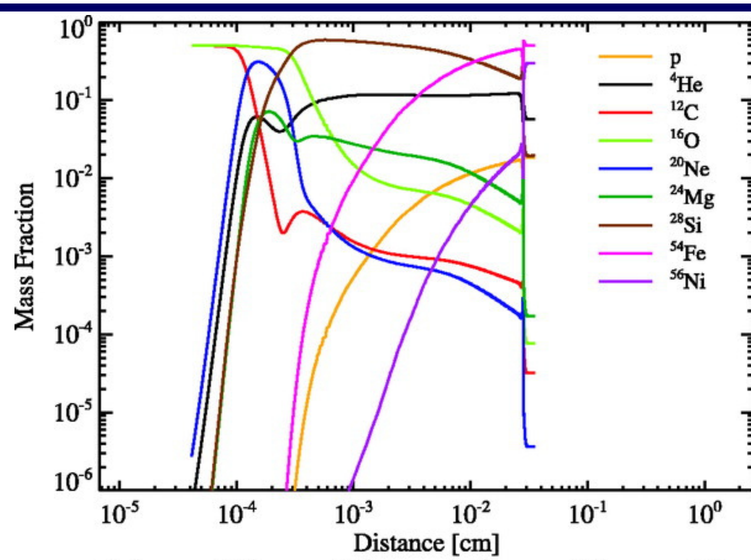
Nuclides involved



DNS of Nuclear Flames and Self-heating

DNS

C/O Fuel



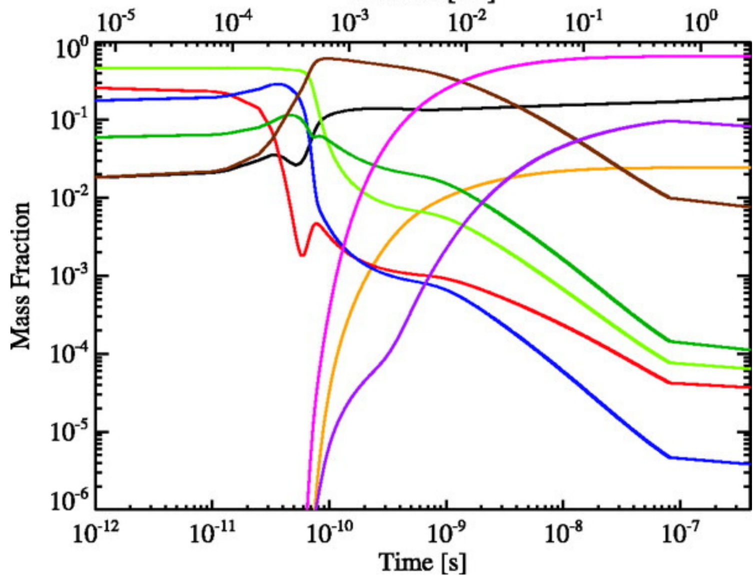
$\rho = 10^9$ g/cm^3

Ash (→ NSE)

Both with aprox19 network

Self-heating

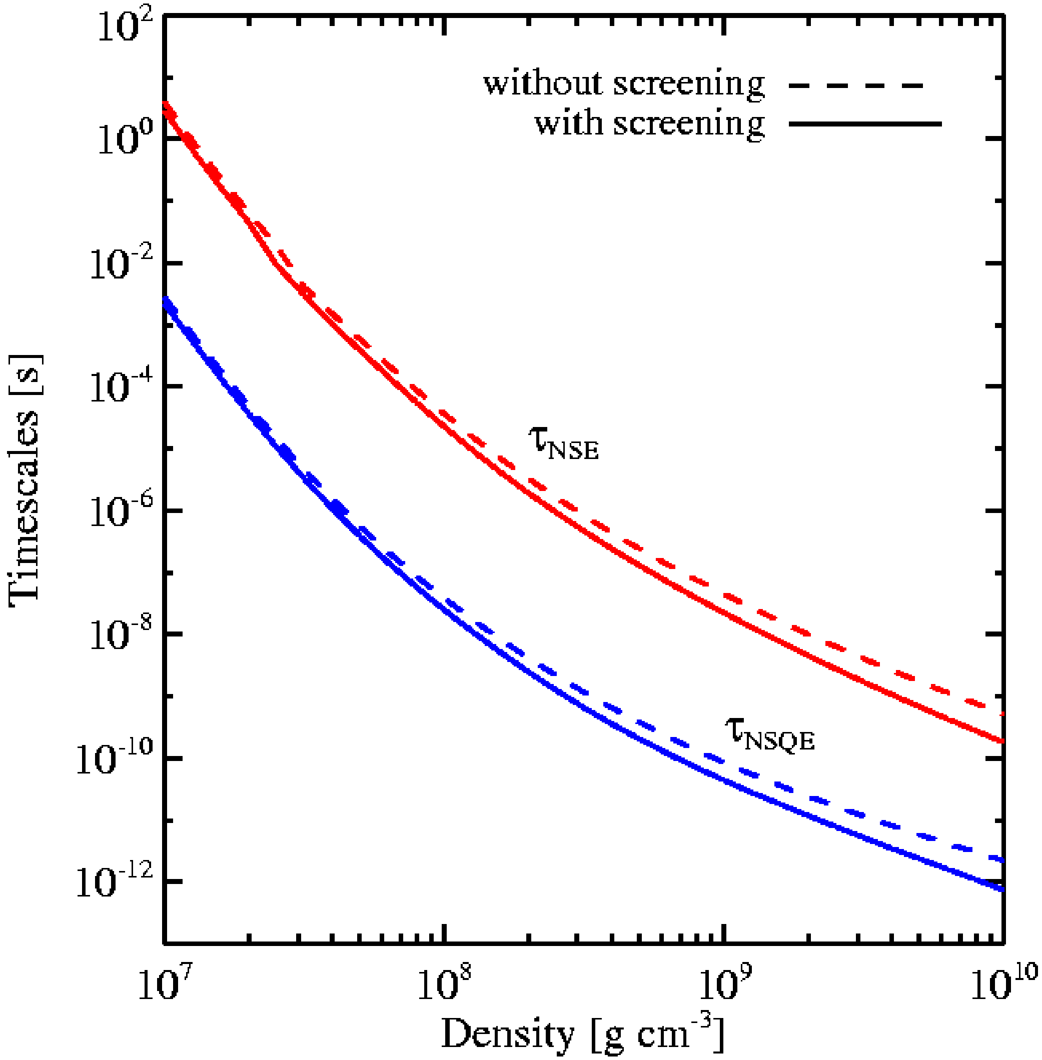
C/O Fuel



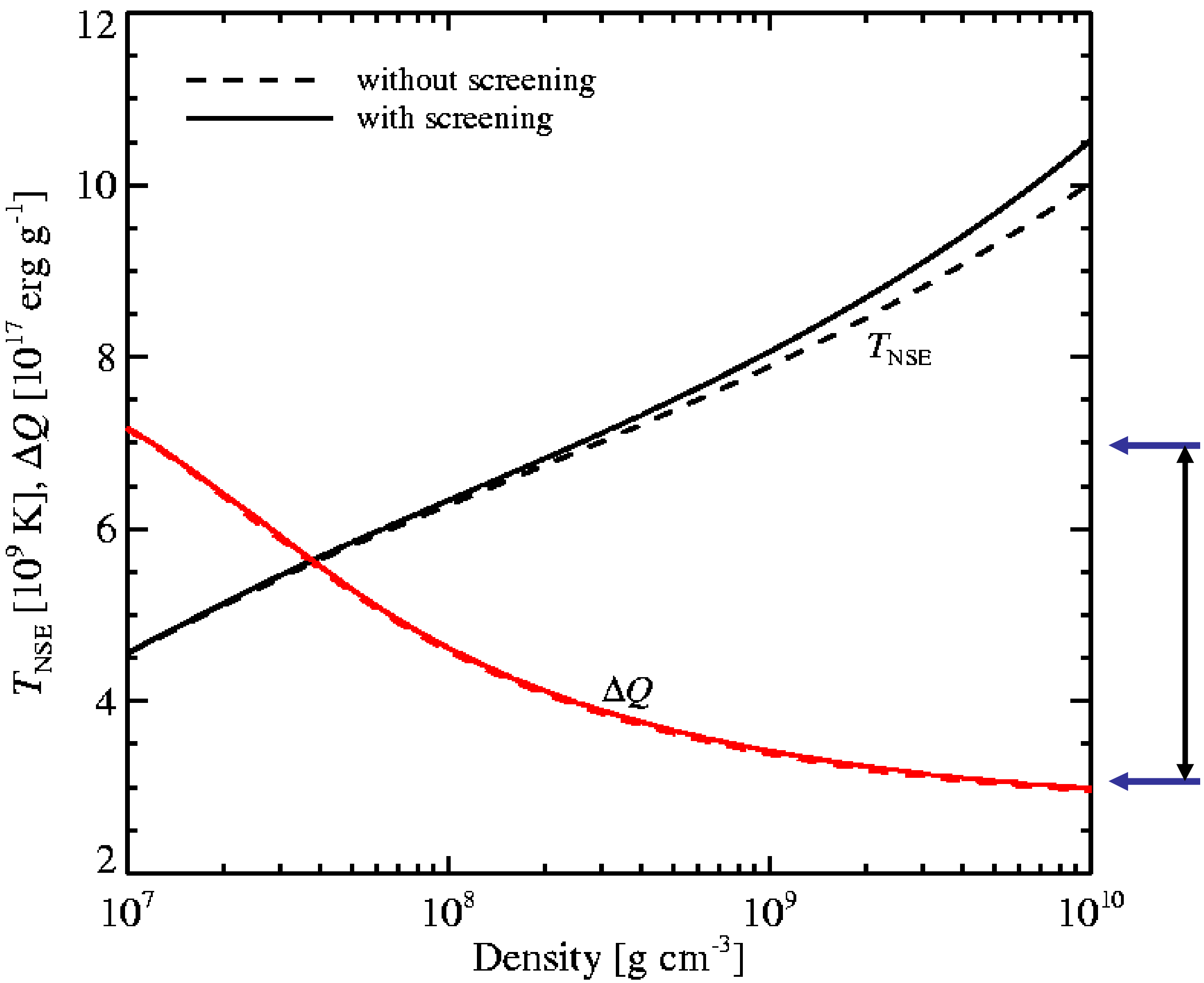
Ash (→ NSE)

← Flame propagation

Self-Heating Network Study

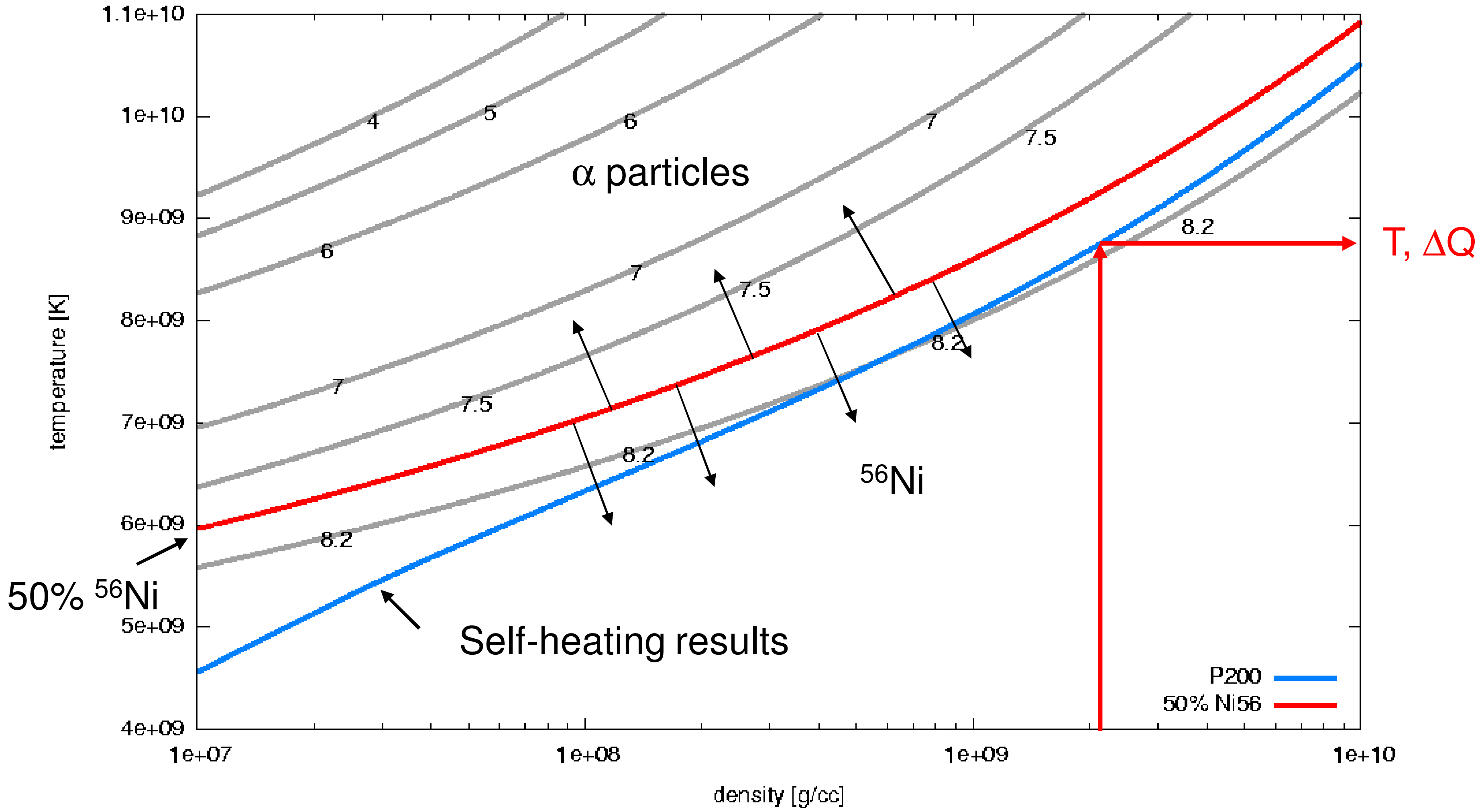


Self-Heating Network Study

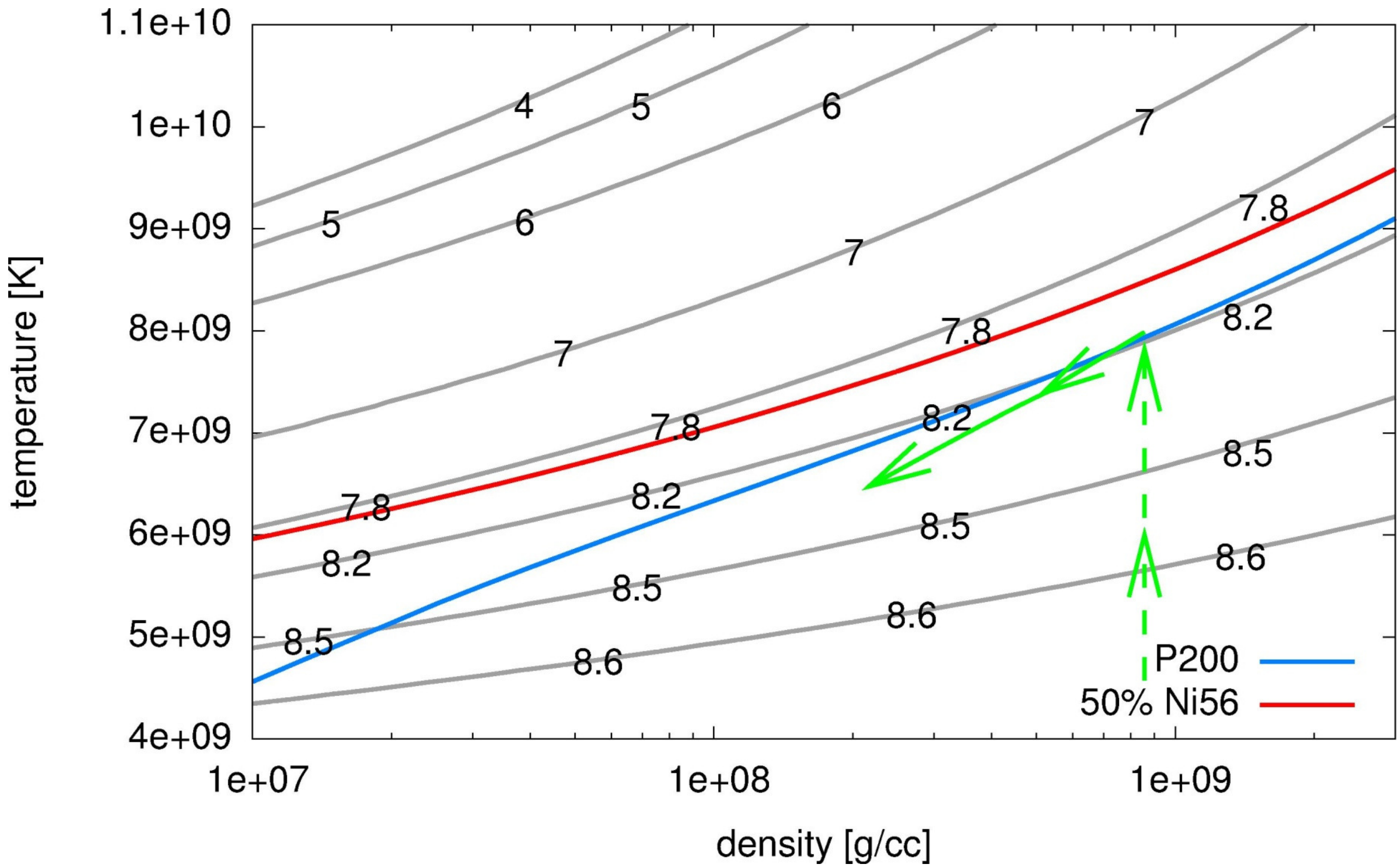


Binding Energy to Tap

Average Binding Energy per Nucleon

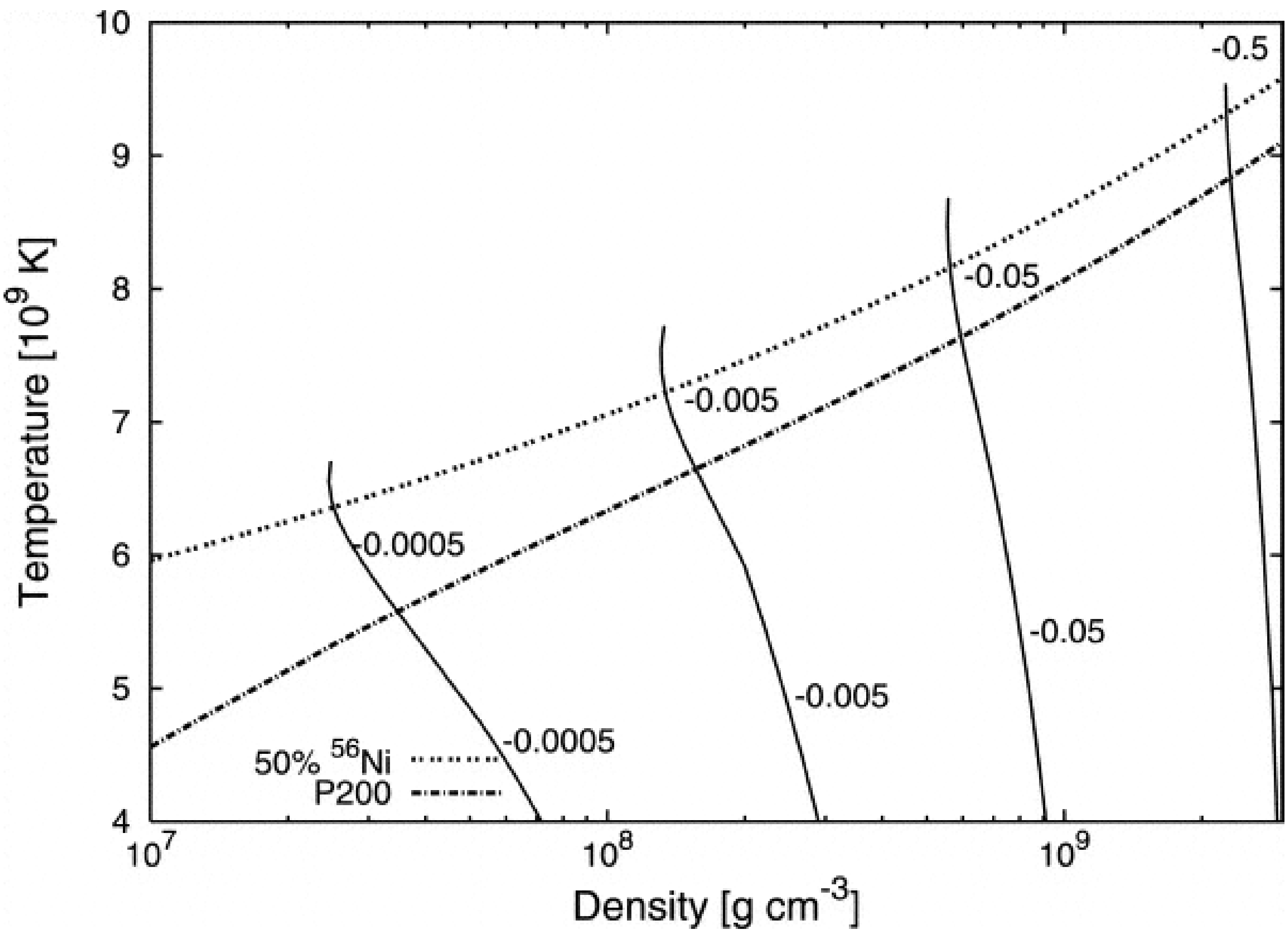


Post Flame Energy Release



Neutronization Rates

$Y_e = 0.5$



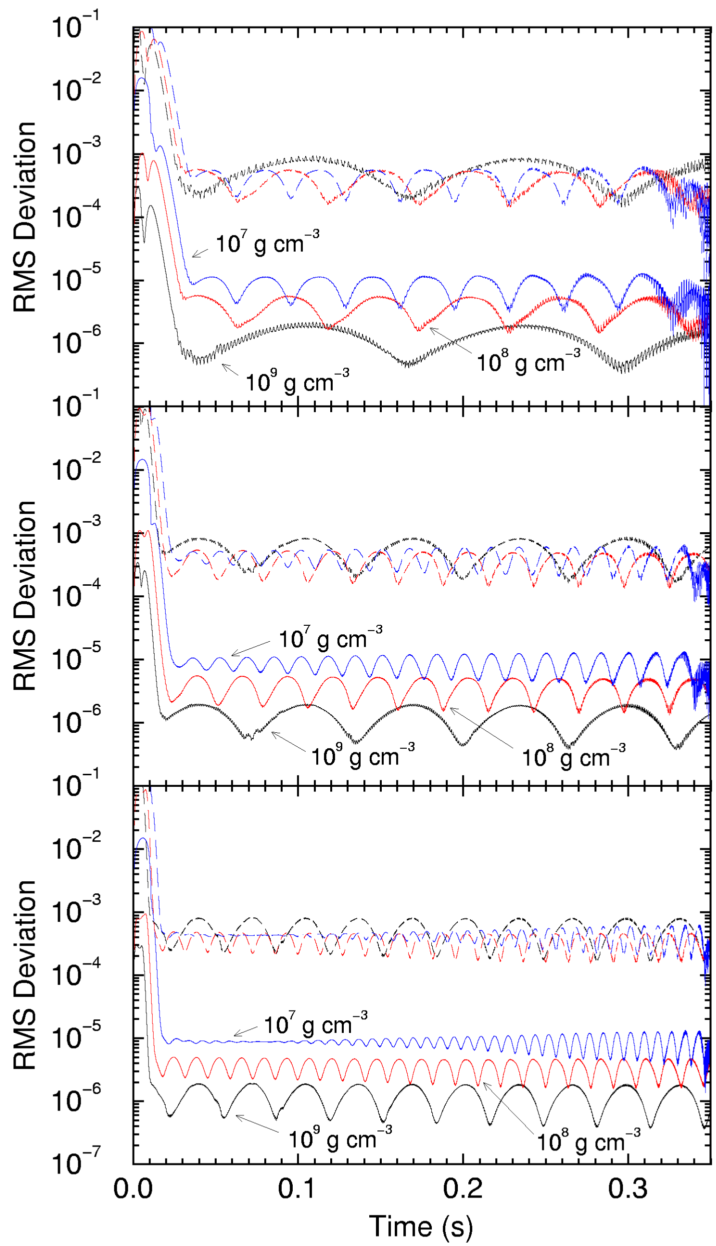
Noise quantification

s = 6 X 10^6 cm/s

256 zones

512 zones

1024 zones



← velocity results

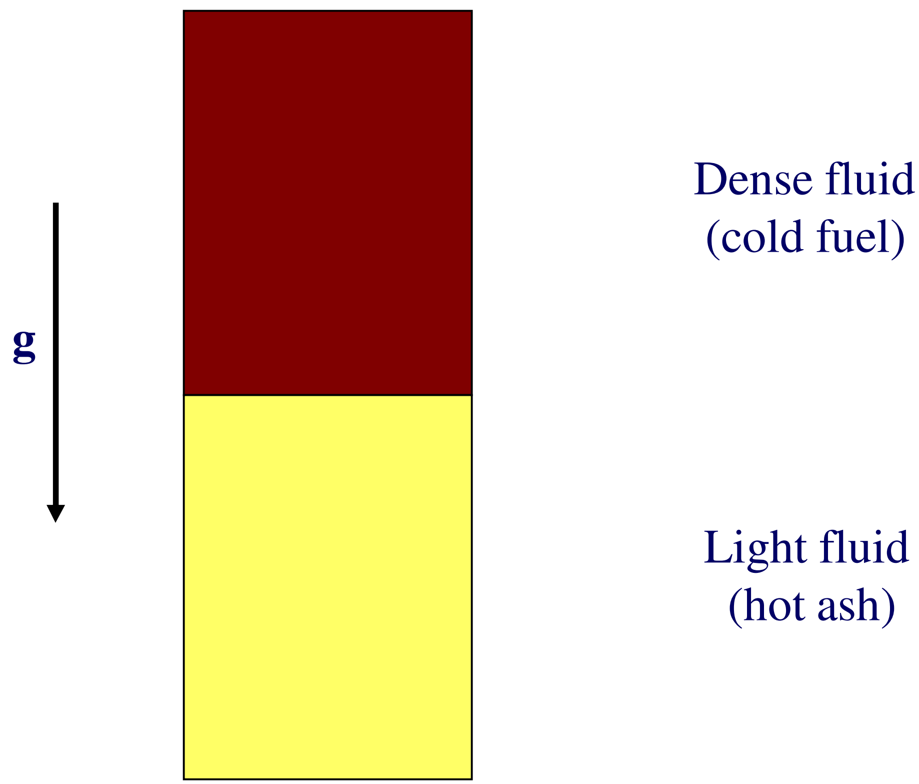
← pressure results

$$\mathrm{dev}_{\mathrm{RMS}}(x) = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

Townsley , et al. (2007)

Rayleigh-Taylor Instabilities

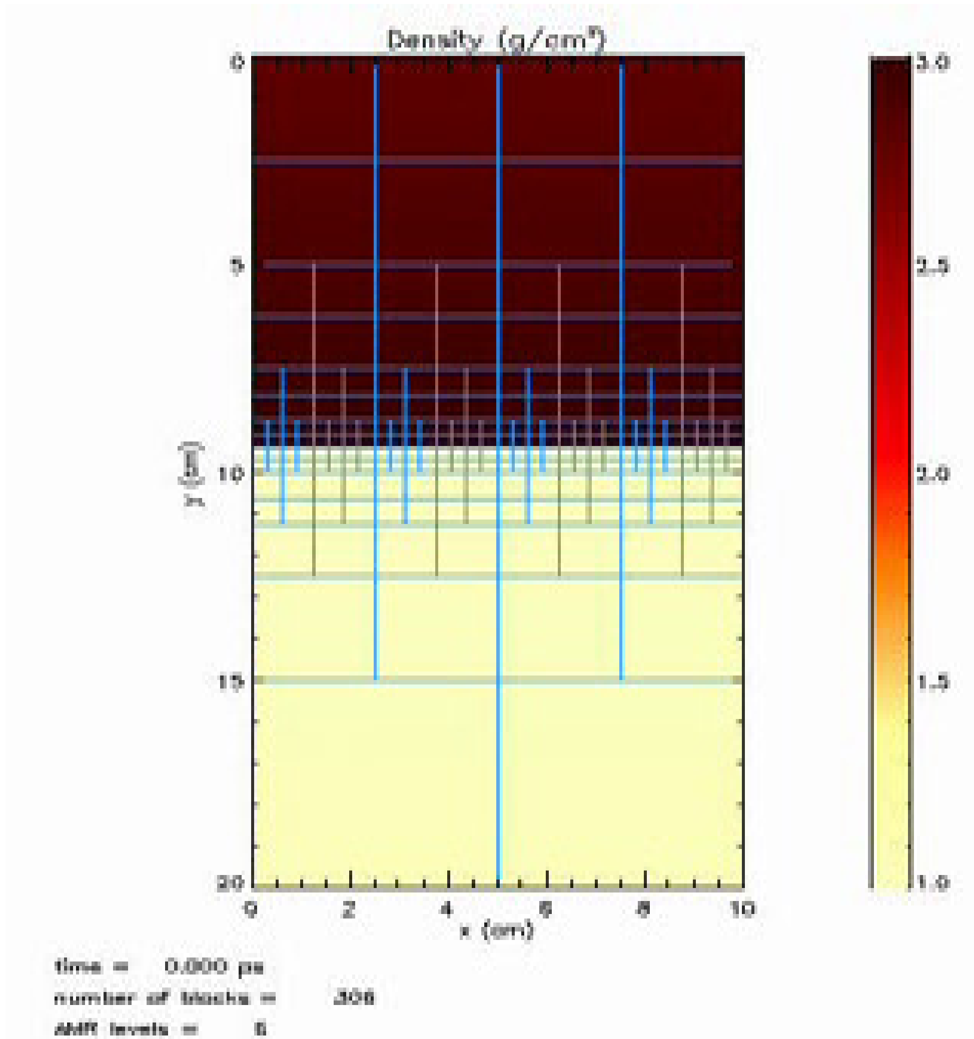
Density schematic:



Aside: Mesh Adaptivity and RTI

AMR allows an increased range of scales in a simulation by adding resolution where it is needed.

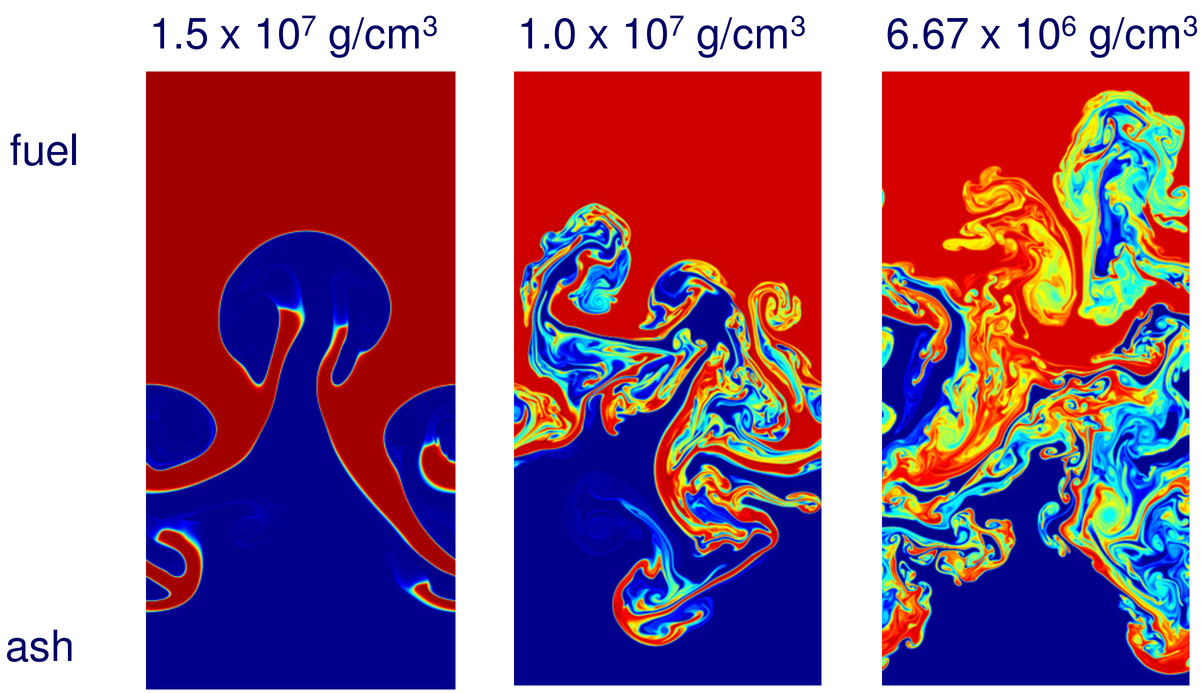
RTI increases the area of the flame, thereby boosting the burning rate.



Type Ia Supernovae as a Combustion Problem

- Physics of turbulent flames (deflagration)
- Transition to detonation (if any)
- Ignition (initial conditions)
- Effects of shear (local and global – rotation)

Carbon mass fraction

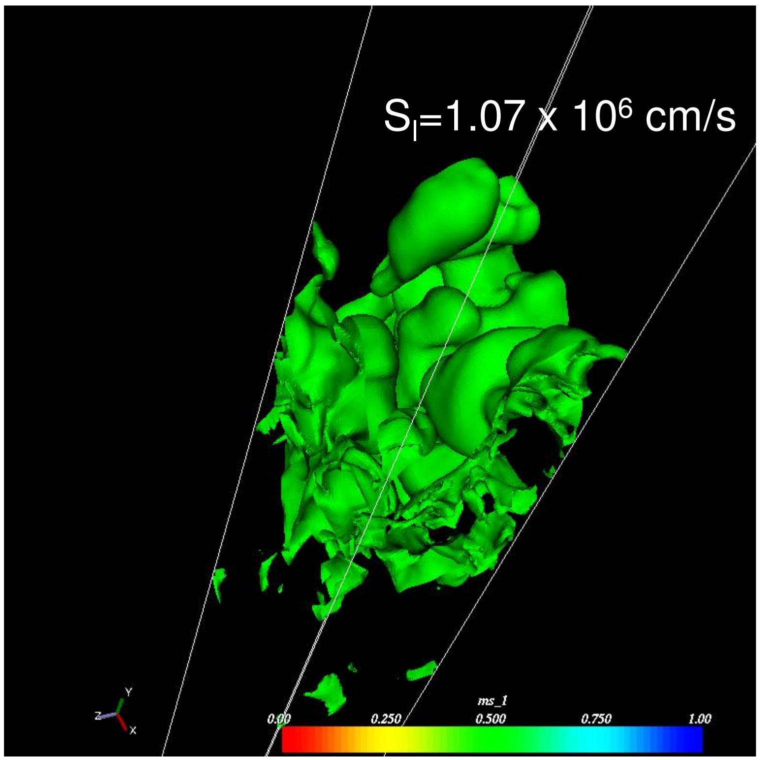


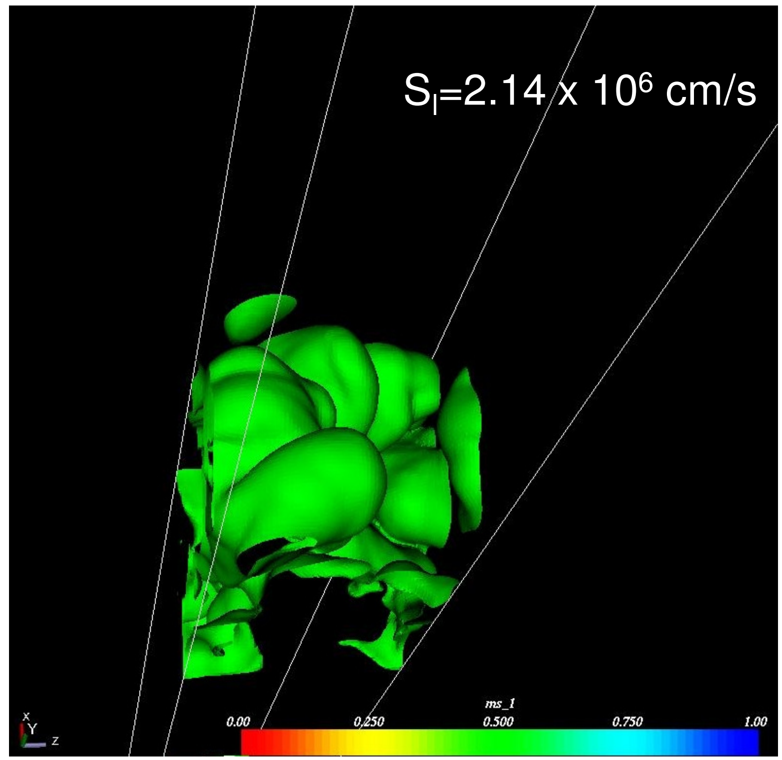
Three-dimensional reactive flow modeling needed to get correct physical behavior of the system.

M. Zingale (2003)

Self-regulation of Flame

Two simulations that differ only in choice of input flame speed.





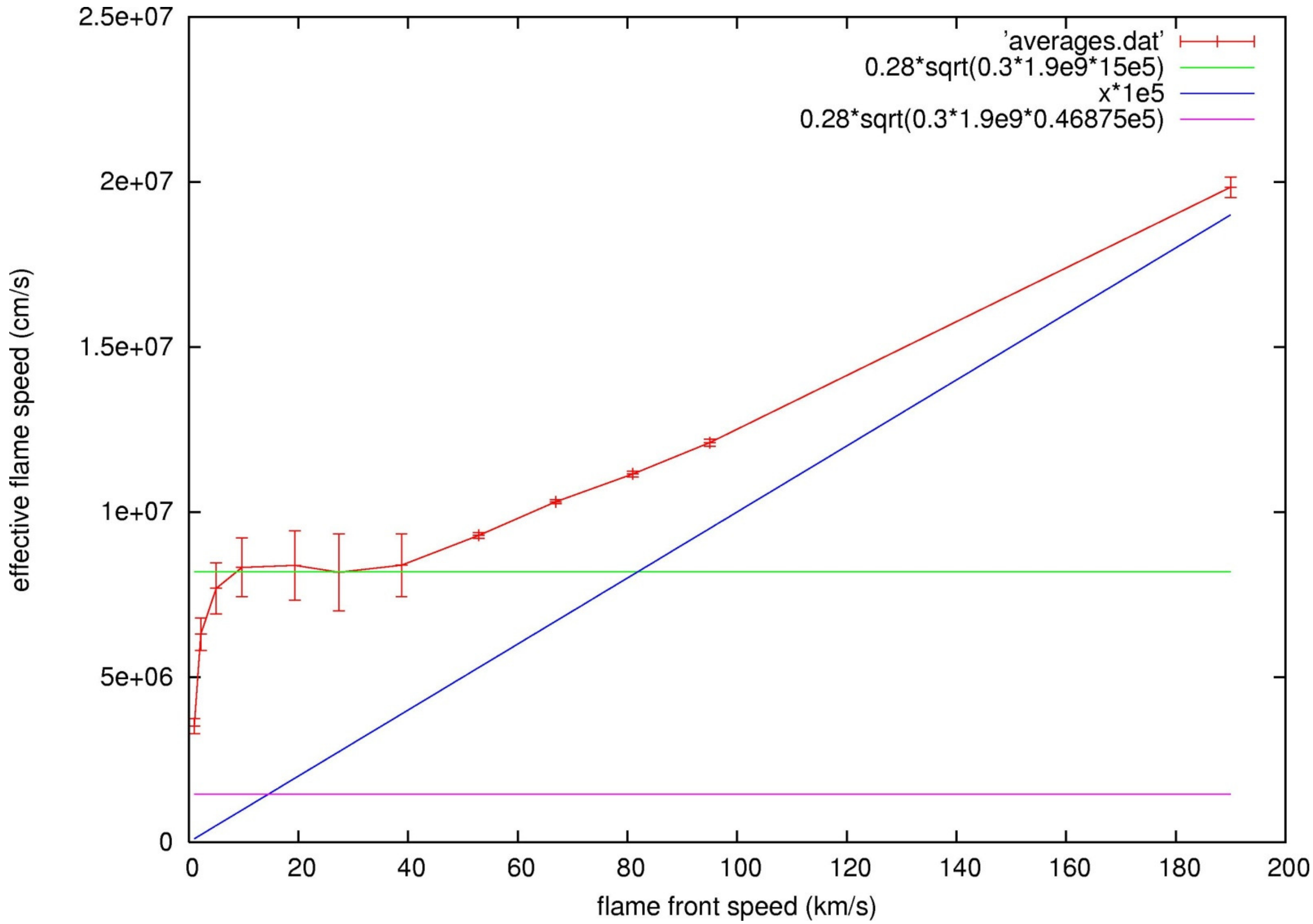
Flame surface area = 2.84 x 10^{13} cm^2

Area = 1.41 x 10^{13} cm^2

$$\boxed{S_t \approx S_l * Area}$$

Messer et al. (2004)

Confirmation of Scaling Law S $\sim(AgL)^{1/2}$



S $\sim(AgL)^{1/2}$
(Khokhlov 1995)

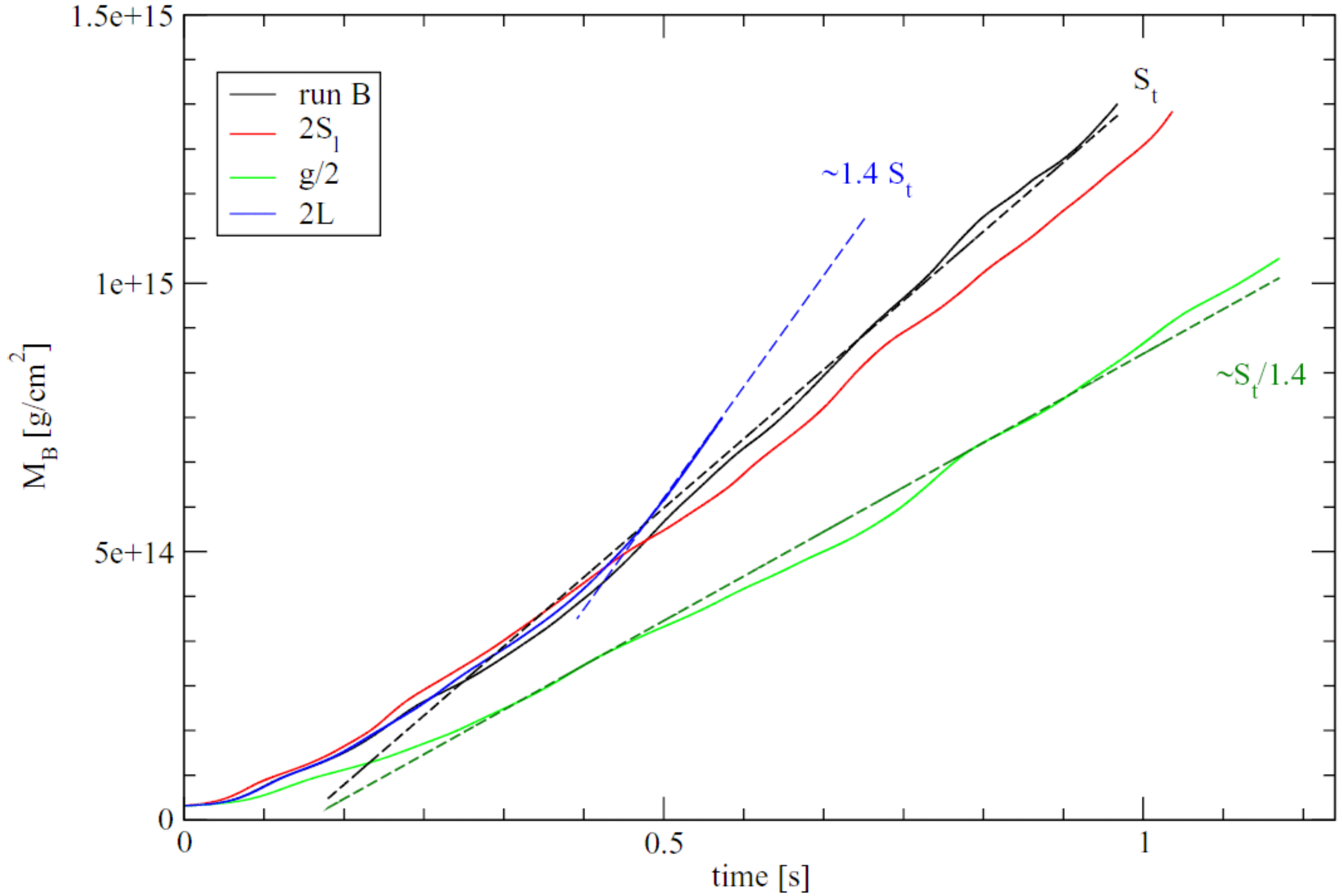
$A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$

Input turbulent flame speed for model flame

Confirmation of Scaling Law

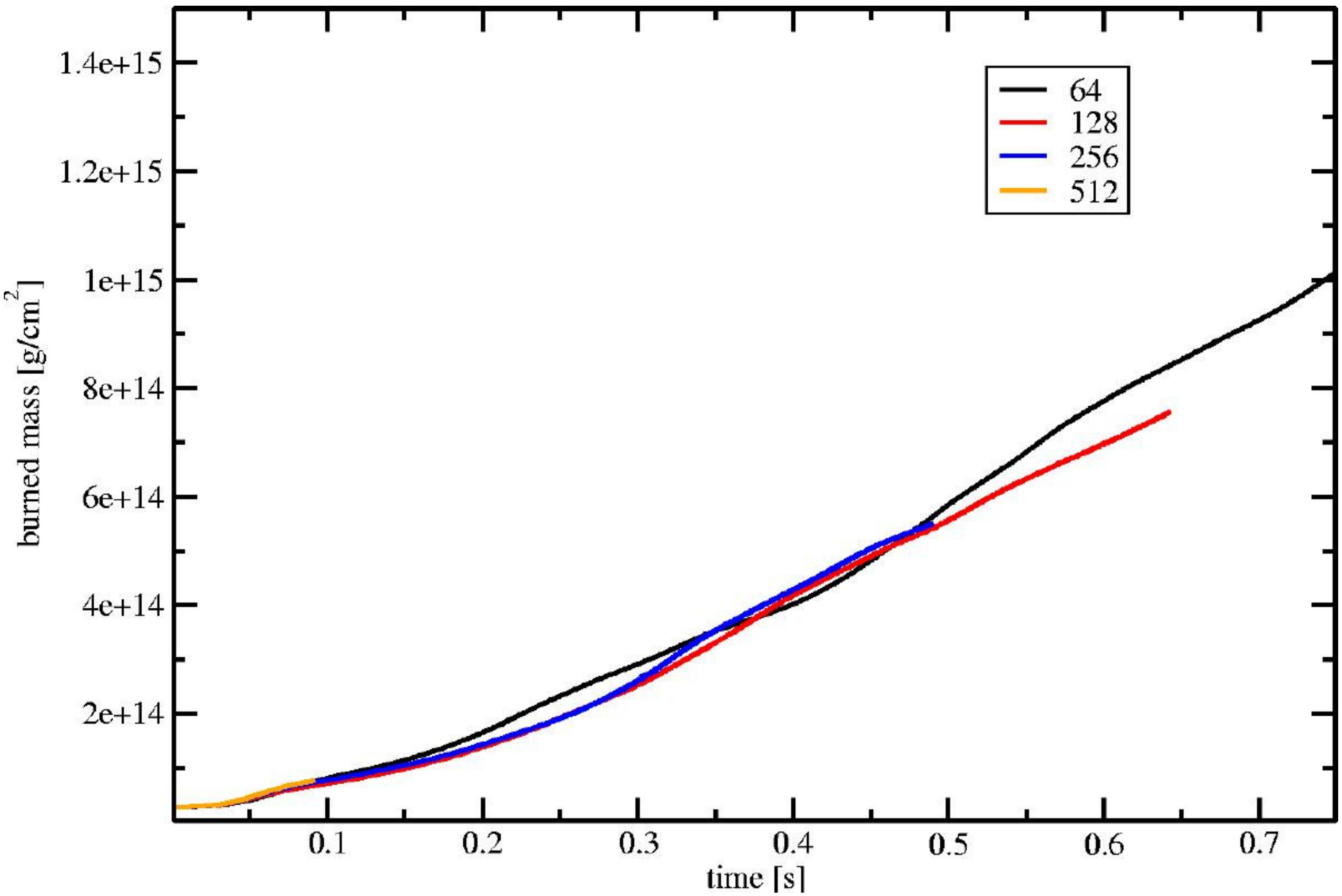
Steady-state turbulent flame speed does not depend on small-scale physics:

$$\boxed{S_t = \alpha\sqrt{AgL}}$$



Zhang, et al. (2006)

Flame Model Resolution Study



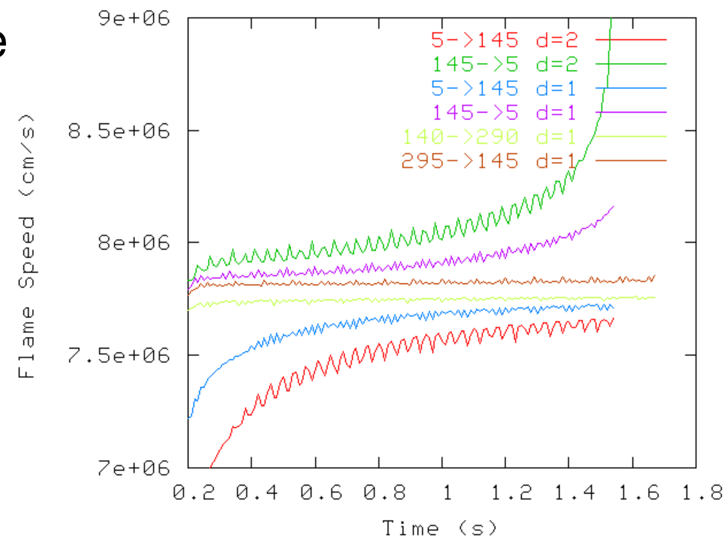
Results must demonstrate convergence with resolution!

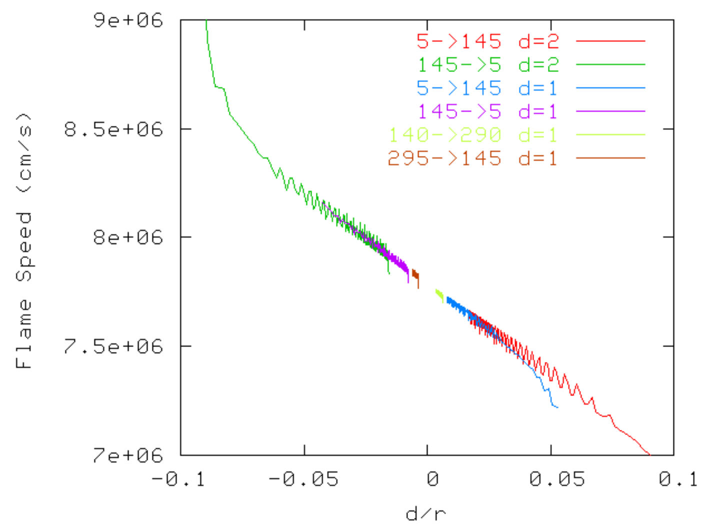
Curvature effects on flame speed

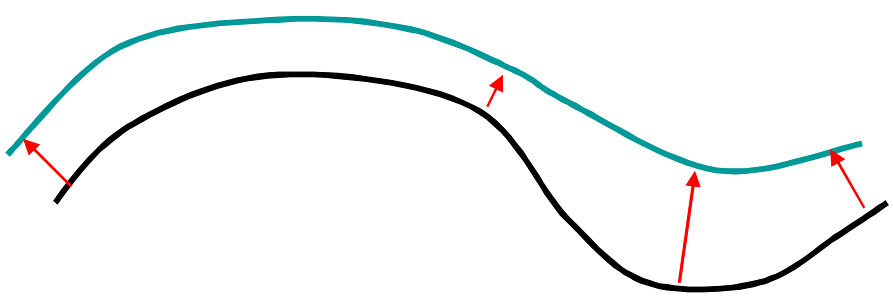
Flames described by an ARD model have curvature effects: flame speed depends on the local curvature – the speed is increased when the flame converges and decreased when the flame diverges.

The effect is larger when the flame is broad. As the width of the flame in the model is unphysically large, the effect is magnified.

This effect stabilizes the flame. As a result, lower resolution models are more stable.







Shimon Asida

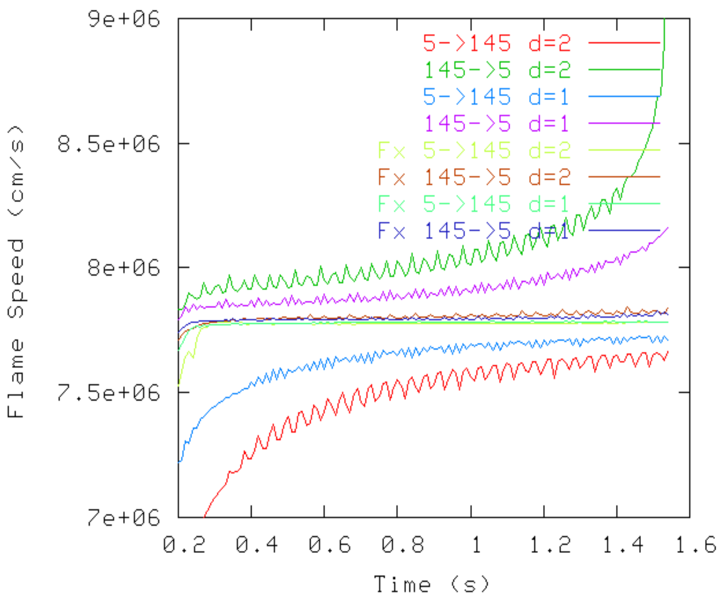
Correction for curvature effect for Top Hat

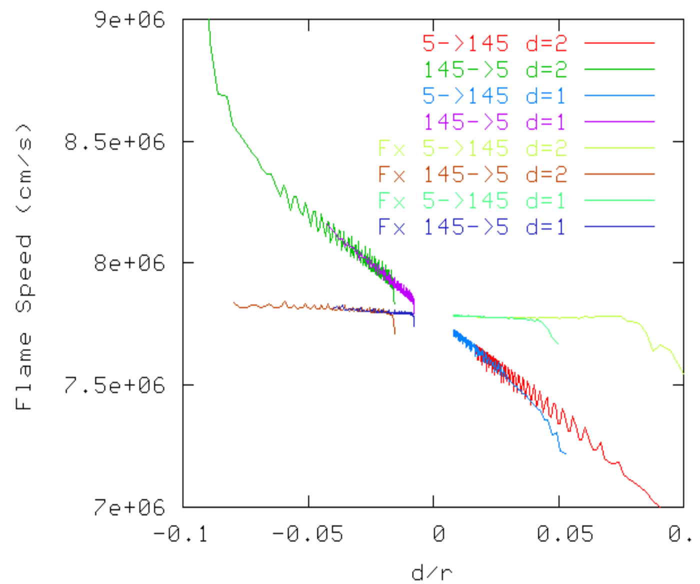
A straightforward solution is to adjust the model according to local curvature.

$$\partial_t \phi + \mathbf{v}\cdot\nabla\phi = s_0\delta_0\nabla^2\phi + \frac{s_0}{\delta_0}R(\phi)$$

$$s\delta = (s_0\delta_0)\Big/\left(1-\delta_0/r_c\right) \quad , \quad \delta/s = (\delta_0/s_0)\Big/\left(1-\delta_0/r_c\right)$$

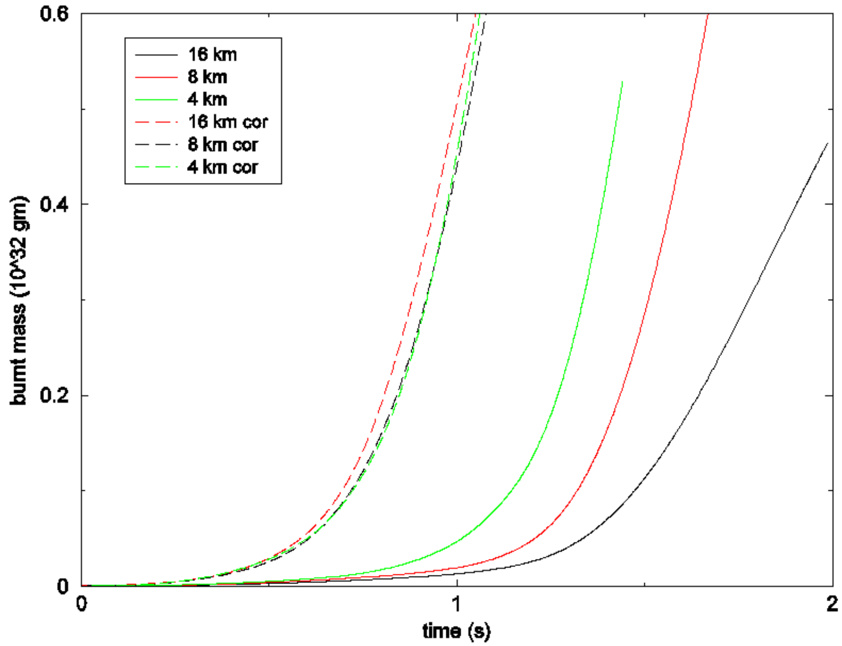
$$r_c = -(\nabla\cdot\mathbf{n})^{-1} \quad , \quad \mathbf{n} = \nabla\phi\big/\|\nabla\phi\|$$

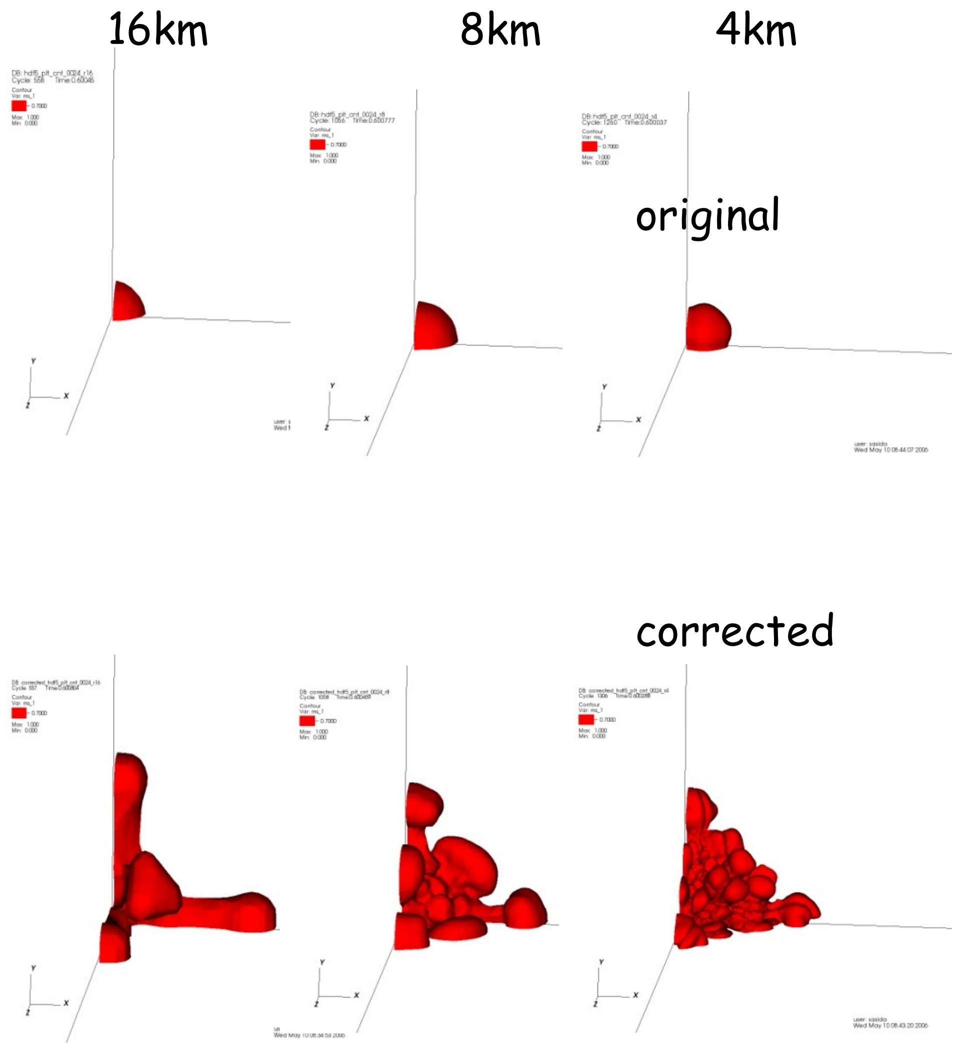




Shimon Asida

Results of Curvature Corrected Top Hat



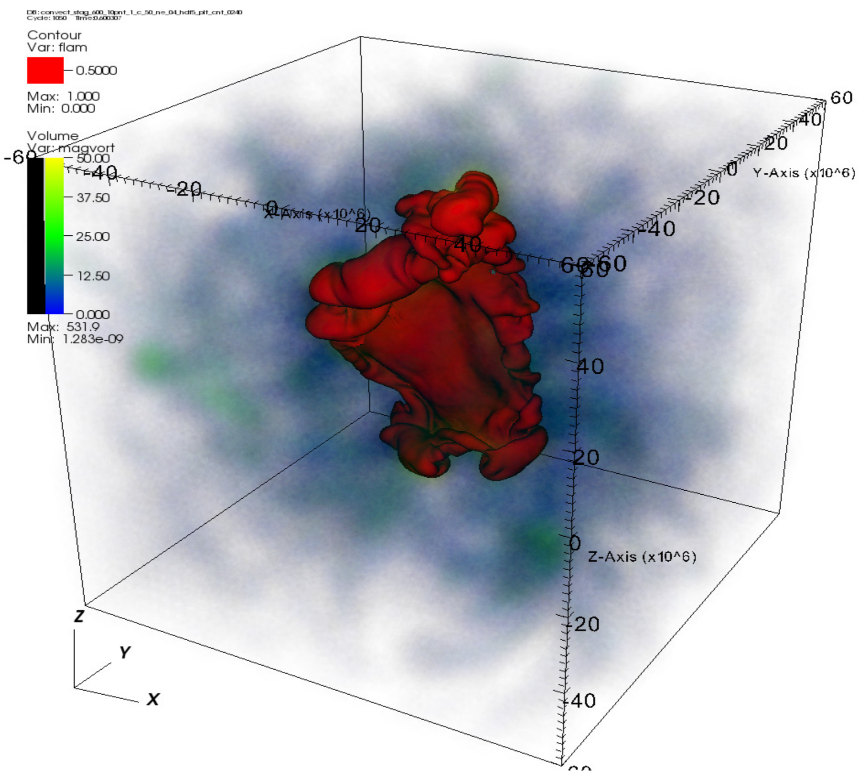


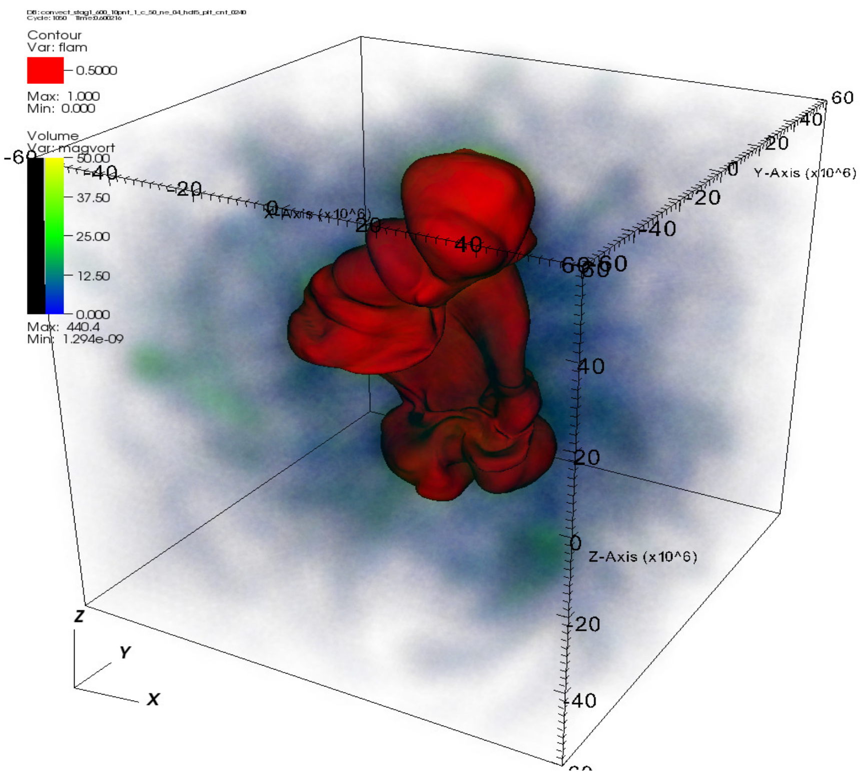
Curvature corrections have a large effect.

The estimate of curvature is problematic.

Shimon Asida

Turbulence-Flame Interaction





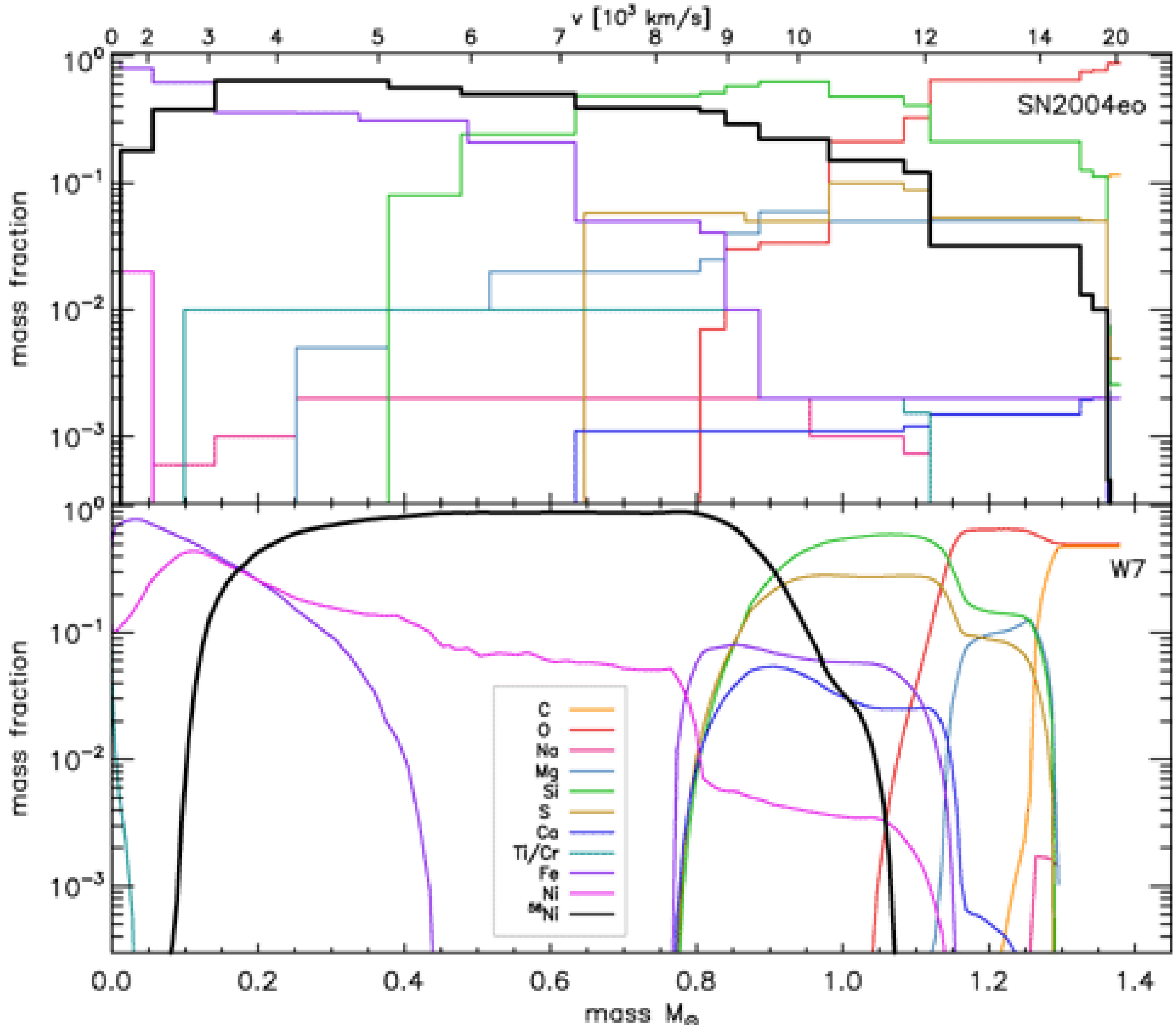
Summary of Flame Model

- Use ADR scheme to propagate a thickened flame with a specified input flame speed. This in a modified version of the Flash code.
- Laminar flame speed from detailed nuclear combustion calculations.
- Model flame captures R-T instability on large scales
- Subgrid model captures R-T instability and TFI on unresolved scales → turbulent flame speed (input)
- Flame model is coupled to appropriate energy release for the C flame, burn to NSQE, burn to NSE, and subsequent evolution of NSE.
- Model and subgrid model verified (and validated) as possible. Timescales for burning calculated and effect of incorporation of screening investigated. sKPP flame is quiet (Townsley et al. 2007).

Current research

- We are in a golden age of SNe Ia observing. Observations suggest (among many other things)
  - Brightness variations → considerable intrinsic scatter in ^{56}Ni yield
  - There may be two populations of SNe Ia.
- Questions: Can we find theoretical evidence for these? Can we estimate the intrinsic scatter of these events?
- Model SNe Ia in the deflagration to detonation paradigm- rising plumes from a central ignition transition to a detonation near the surface of the white dwarf. DDT models produce results consistent with observations and are readily parameterized.
- Models allow us to investigate role of metallicity, central density, etc., of the progenitor to look for systematic effects on the ^{56}Ni yield.
- Study these issues with a well-controlled statistical sample (Townsley, et al. 2009)

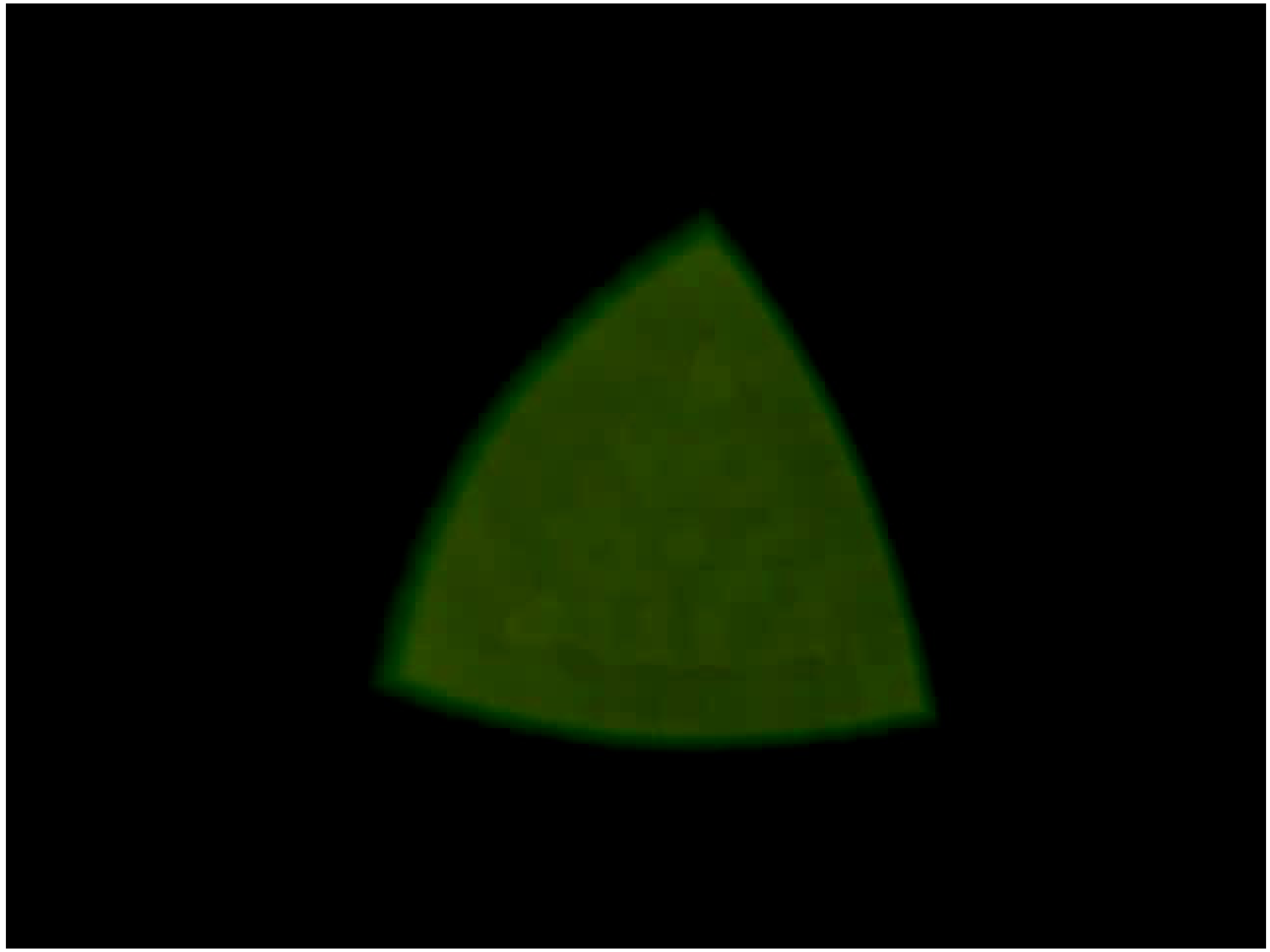
Observation compared with W7 model



Mazzali et al. (2008)

Octant (3-d)

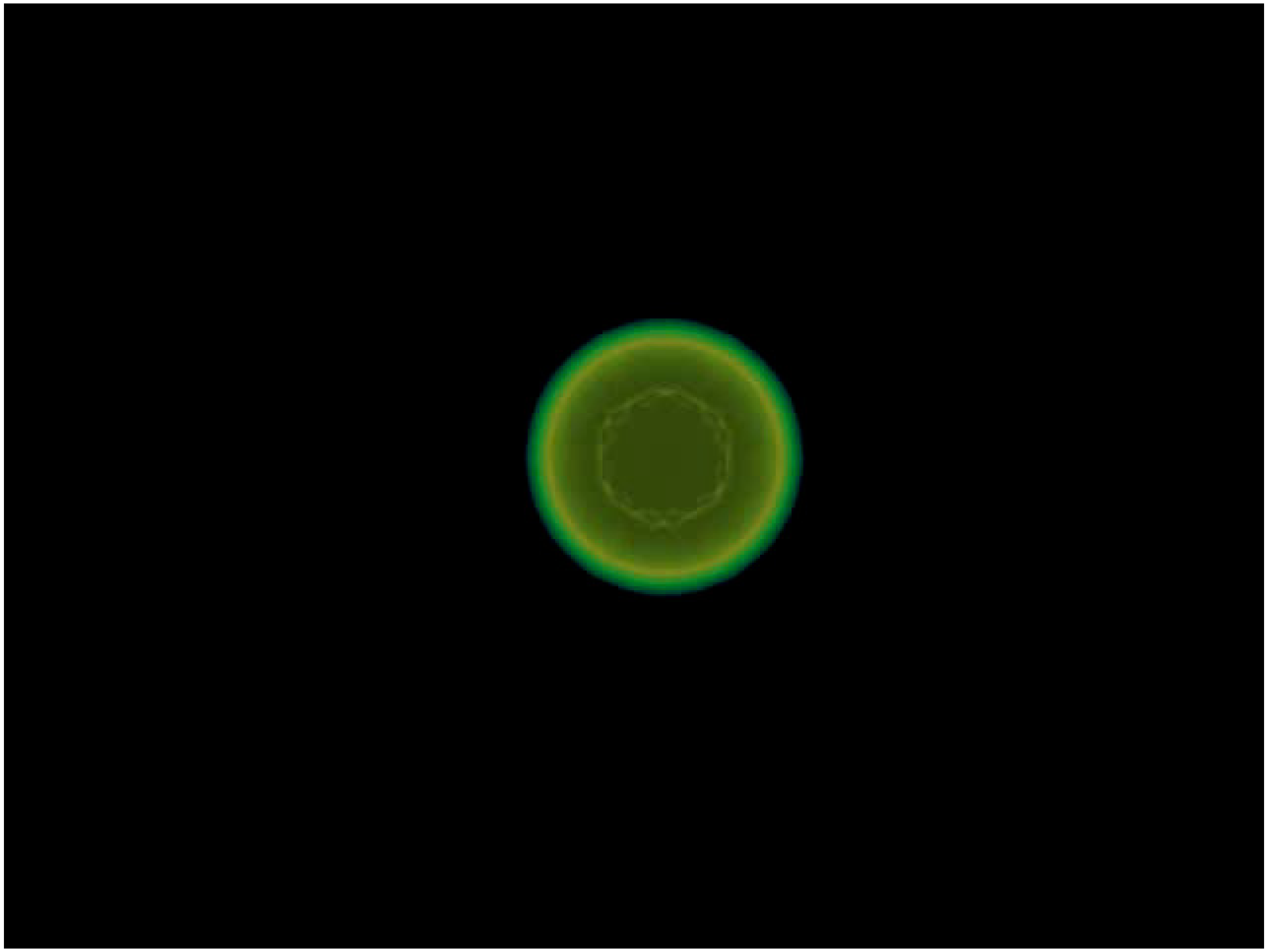
Volume rendering of flame front



INCITE

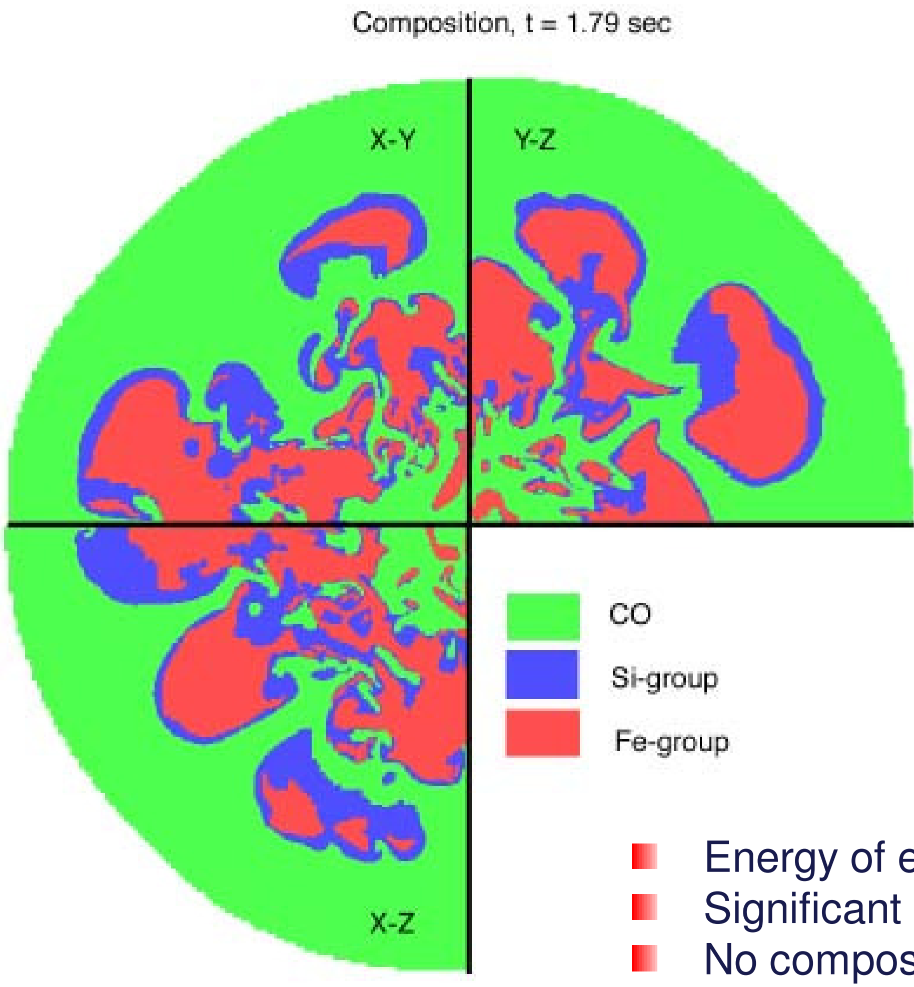
Whole Star (3-d) deflagration

Volume rendering of flame front



INCITE

Deflagration Models: Incomplete Burning

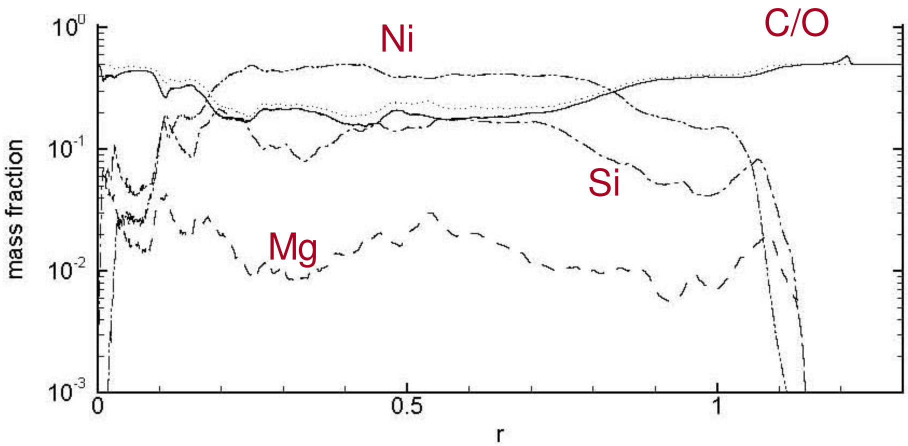


Khokhlov (2001)

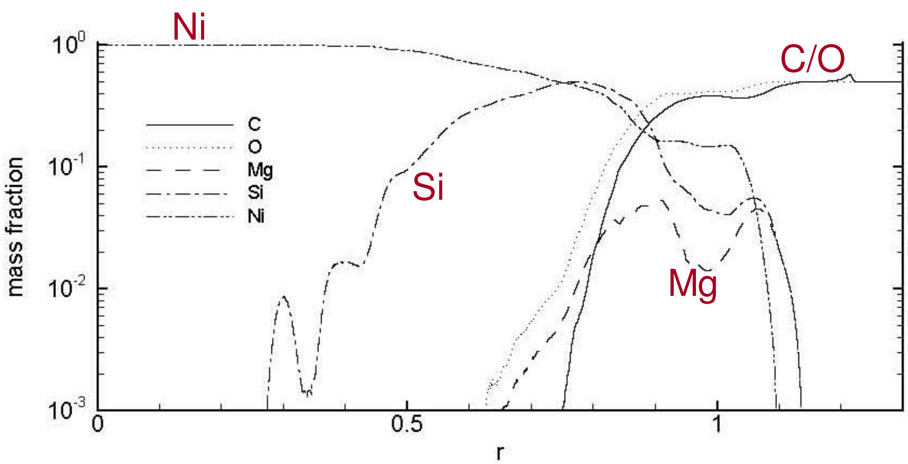
- Energy of explosion is too small
- Significant mass of unburned C+O
- No composition stratification: complete mixing of Ni, Si, C+O throughout the star

3-D Delayed Detonation Model

Average chemical composition as function of radius



3-D pure deflagration

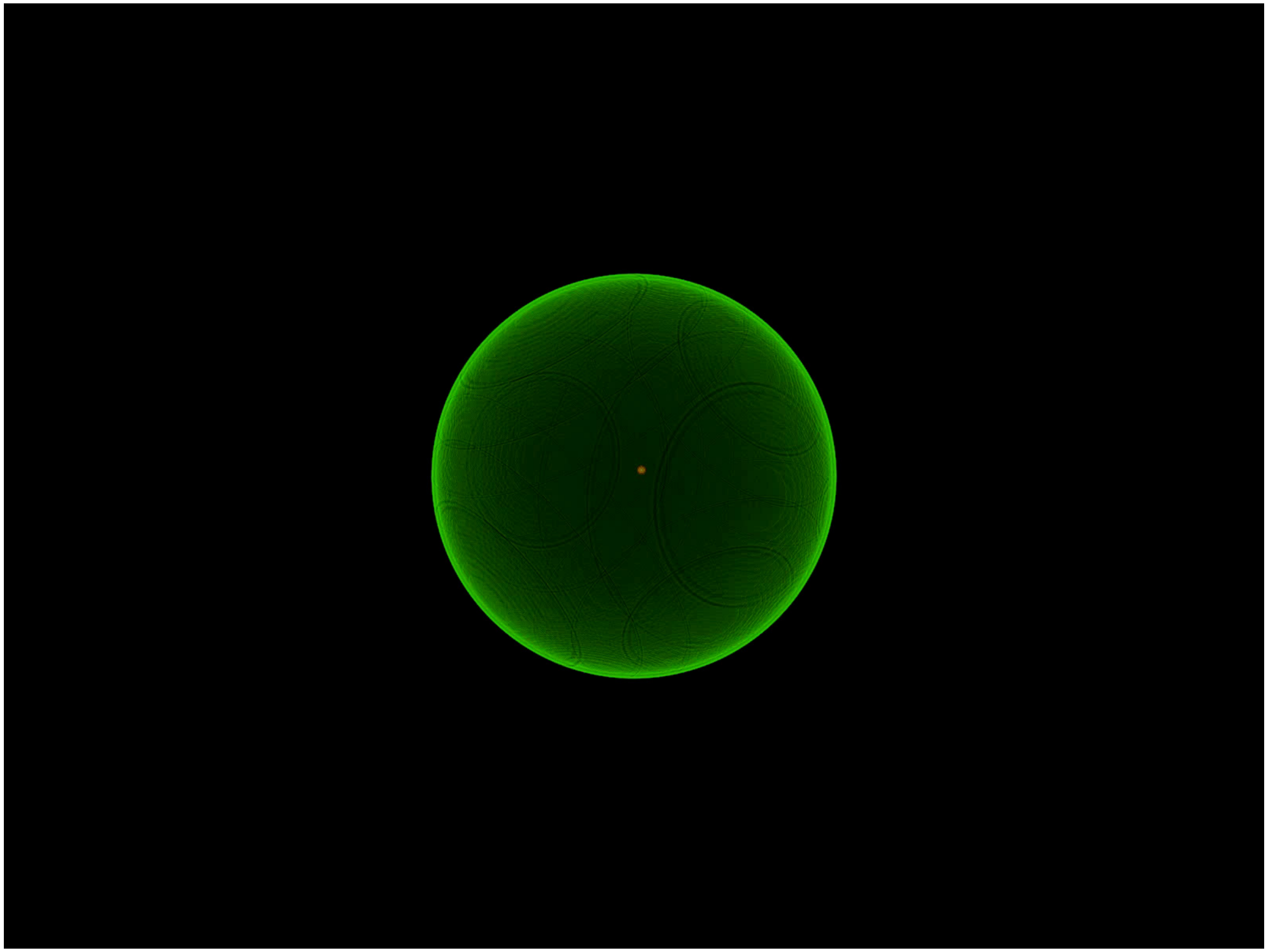


Gamezo et al. (2003)

3-D deflagration followed by detonation
Ignited "by hand" at the center of the pre-expanded star.

Resulting stratified compositions are in better agreement with observations! "Classic" DDT scenario

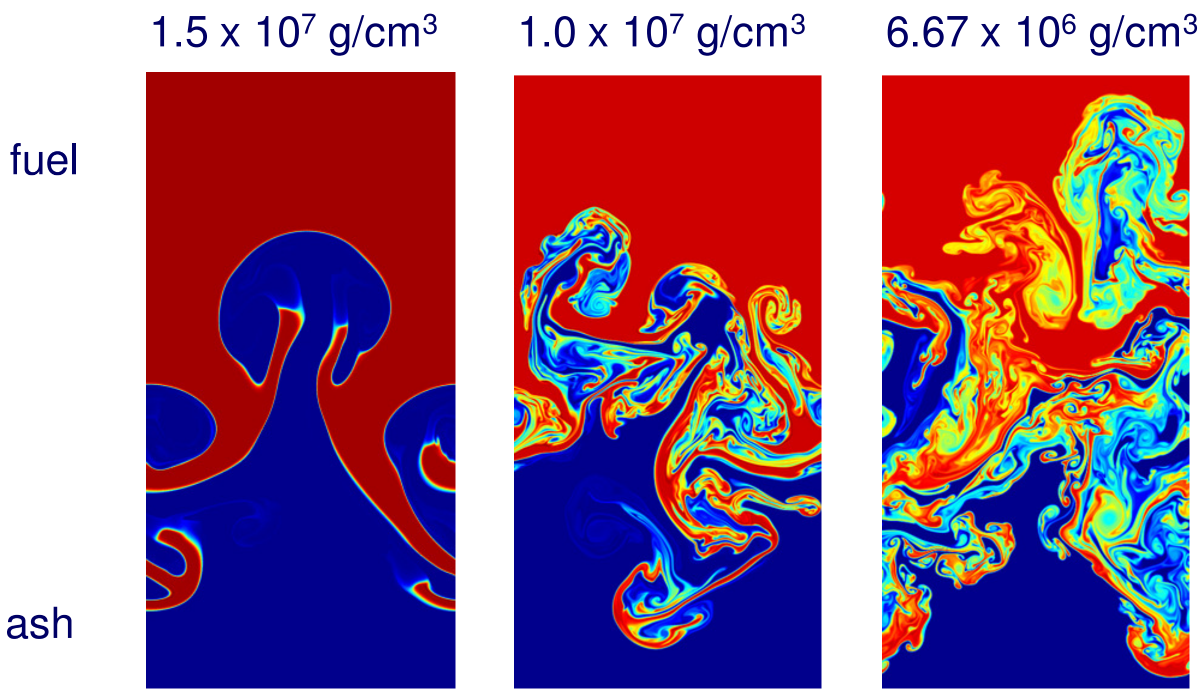
Gravitationally Confined Detonation



Jordan, et al. 2008

DDT mechanism

- The mechanism by which a DDT might occur is not well understood!
- One proposed way follows from the wrinkling of the flame with decreasing density.
- At some point, the net burning rate is fast enough that the equivalent flame would be supersonic → DDT!



Carbon mass fraction

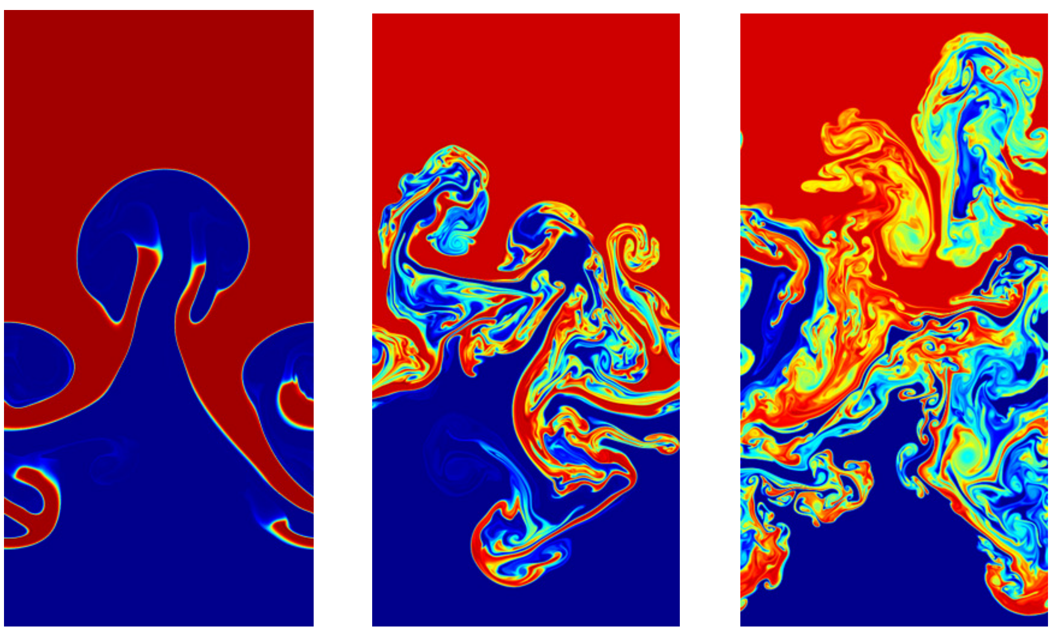
M. Zingale

DDT mechanism

- Note that one way to think of this is a race between the flame and instability growth.
- The composition of the material determines the flame speed. So if the speed changes, the race result changes.
- One way that that the composition affects the DDT density.

1.5×10^7 g/cm^3 1.0×10^7 g/cm^3 6.67×10^6 g/cm^3

fuel

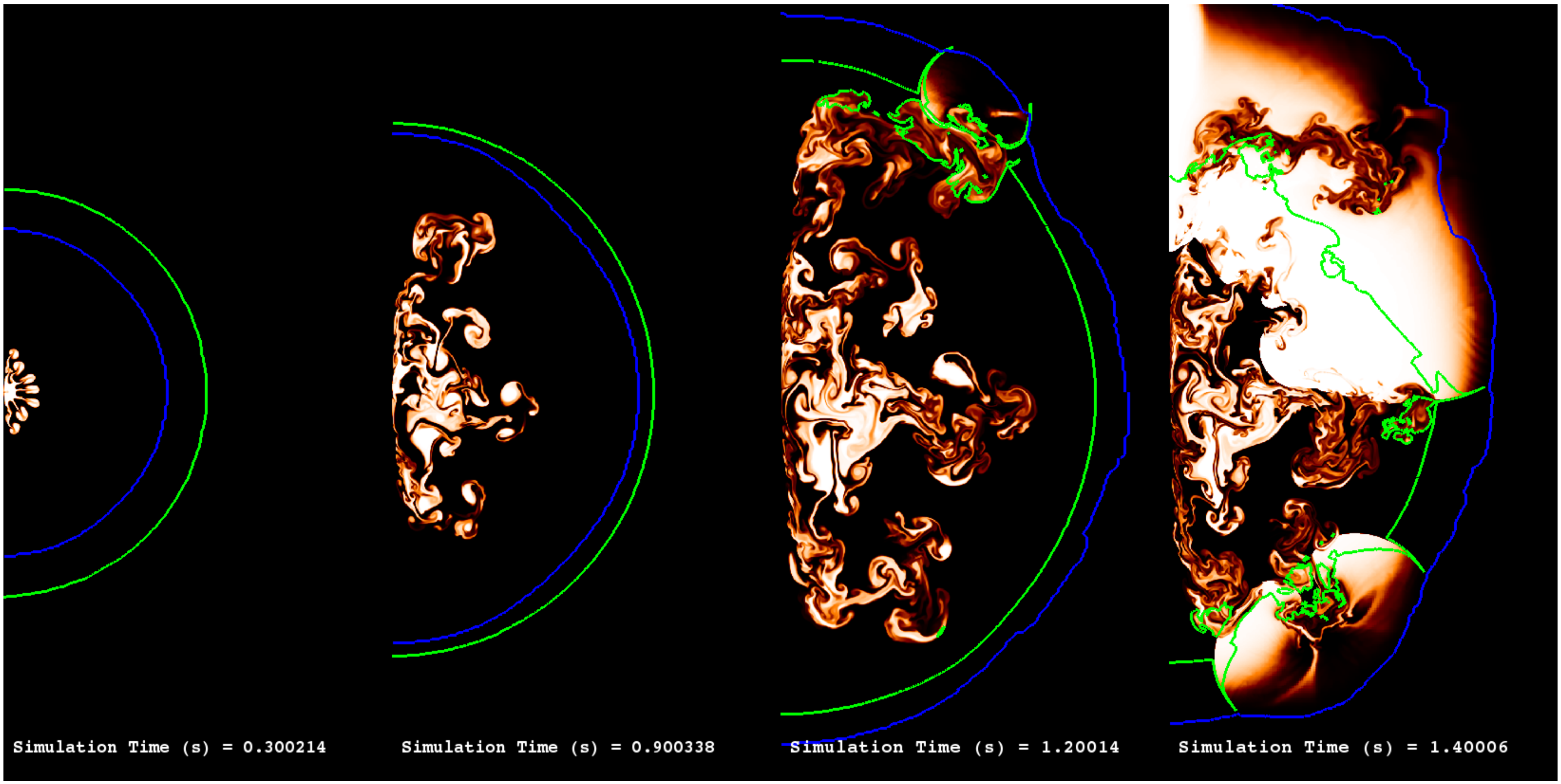


ash

Carbon mass fraction

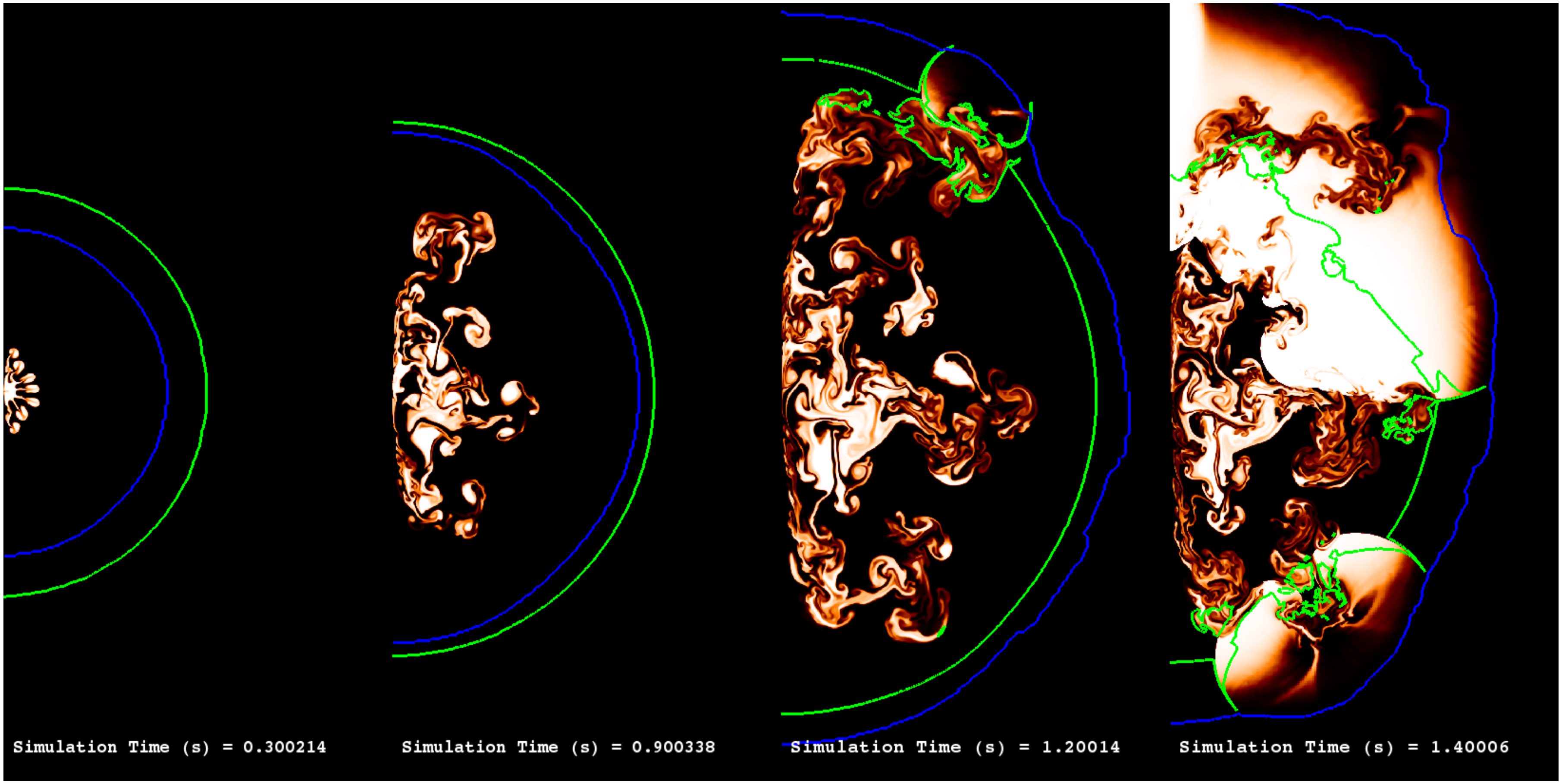
M. Zingale

Simulations in the DDT paradigm



Jackson, et al. 2010

Simulations in the DDT paradigm

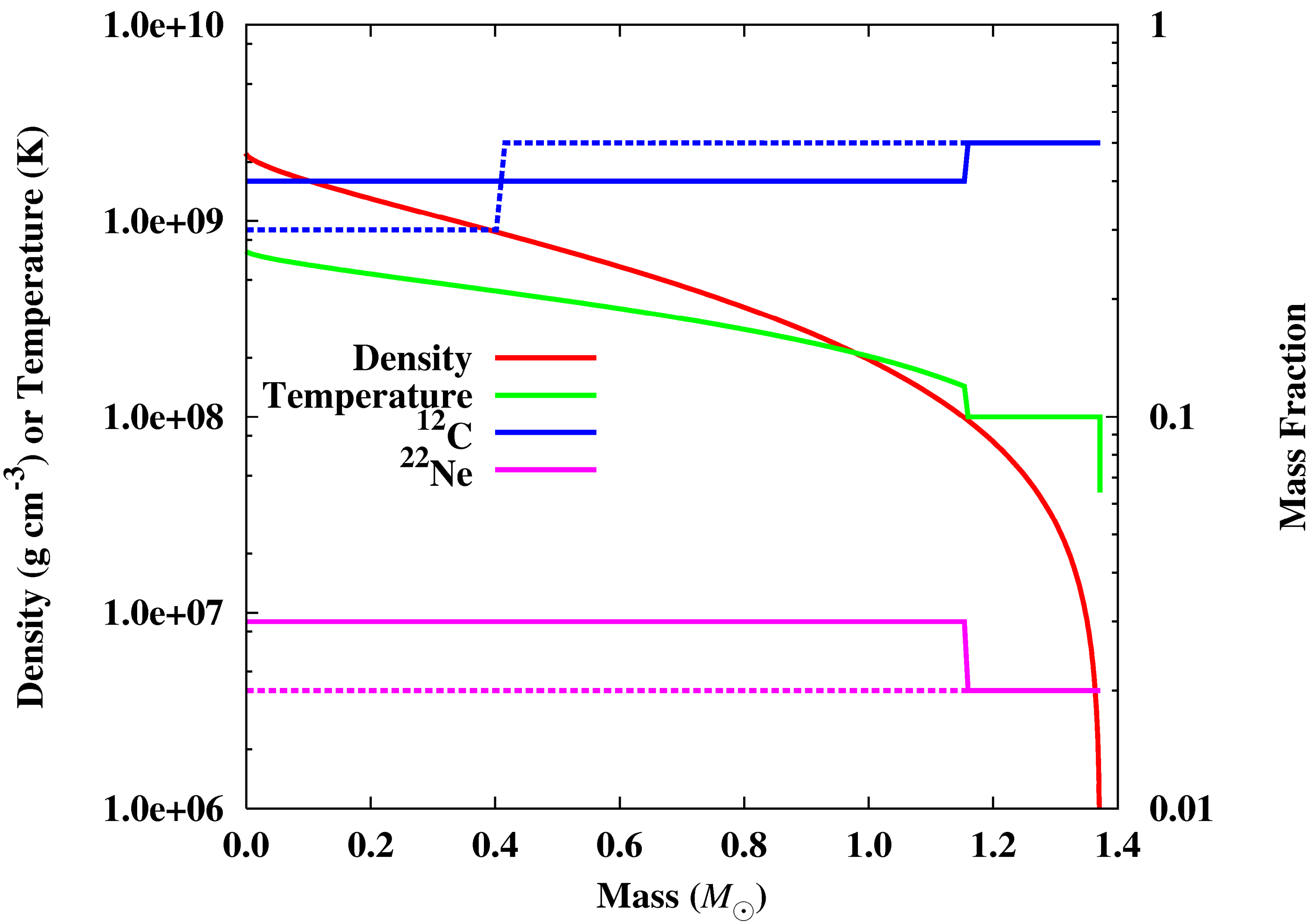


Jackson, et al. 2010

DDT simulations

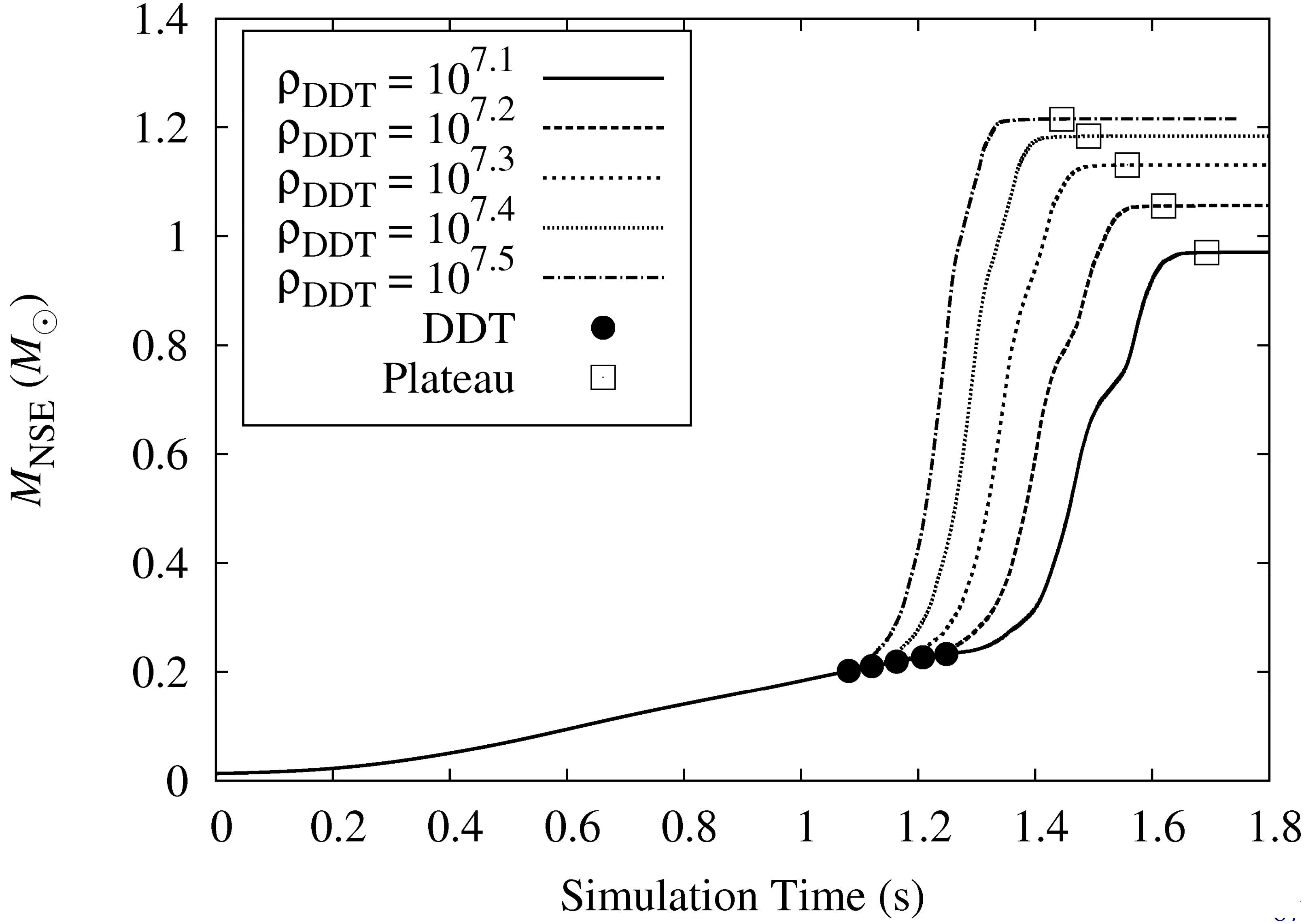
- Developed a framework for the statistical analysis of thermonuclear supernova simulations from randomized initial conditions. For each study, perform an ensemble of simulations and analyze its properties.
- Investigated the role of ^{22}Ne, which is known to be directly influenced by the progenitor stellar population's metallicity.
- Found that ^{22}Ne does not greatly influence the evolution of the explosion prior to detonation, suggesting that other parameters such as the ignition conditions are the more dominant influence on the mass of ^{56}Ni synthesized (Townsley, et al. 2009).
- New results on the role of the DDT transition density and the central density of the WD on explosion outcome.

Evolved progenitor



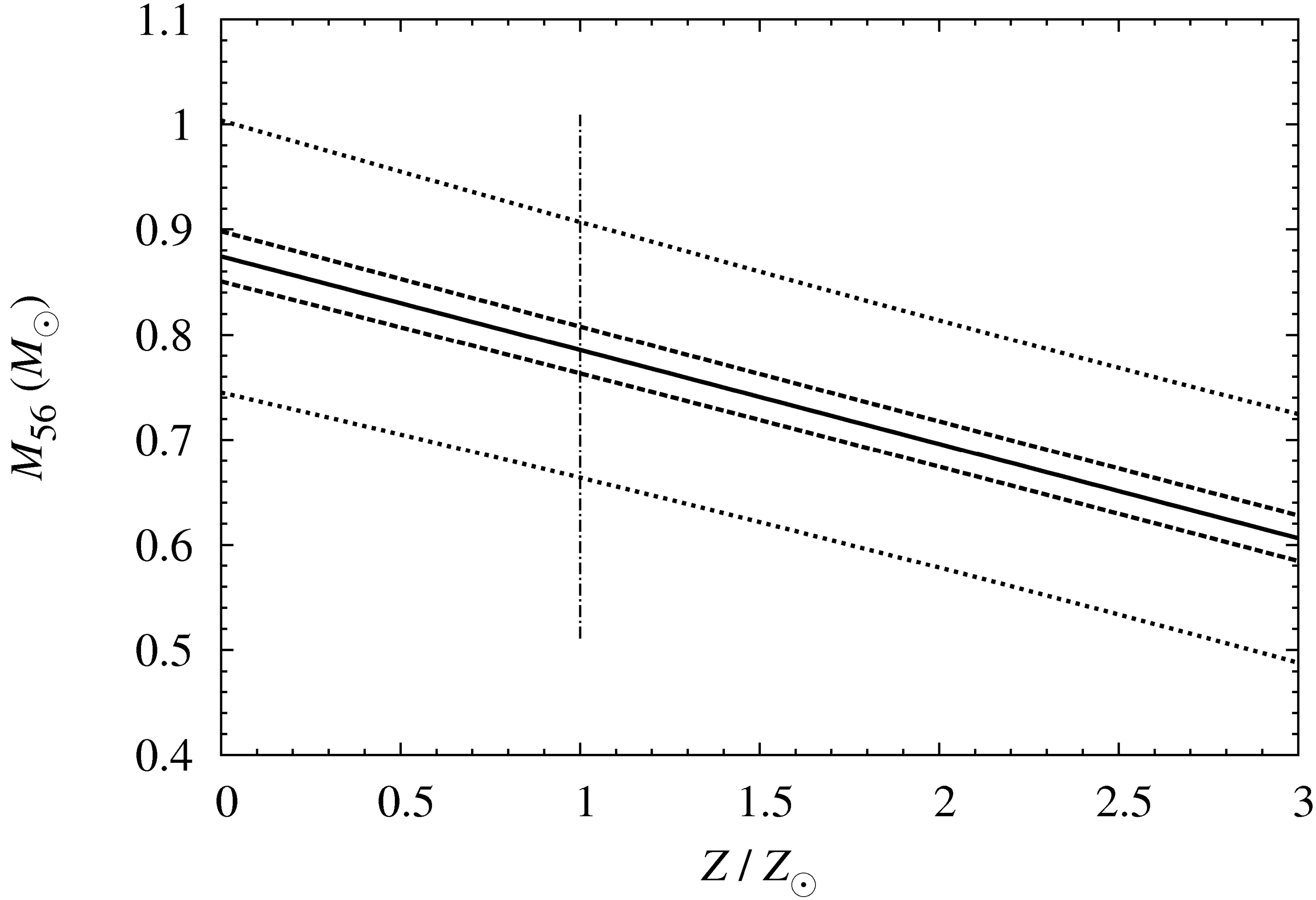
Inspired by Piro & Chang (2008)

DDT Density Study



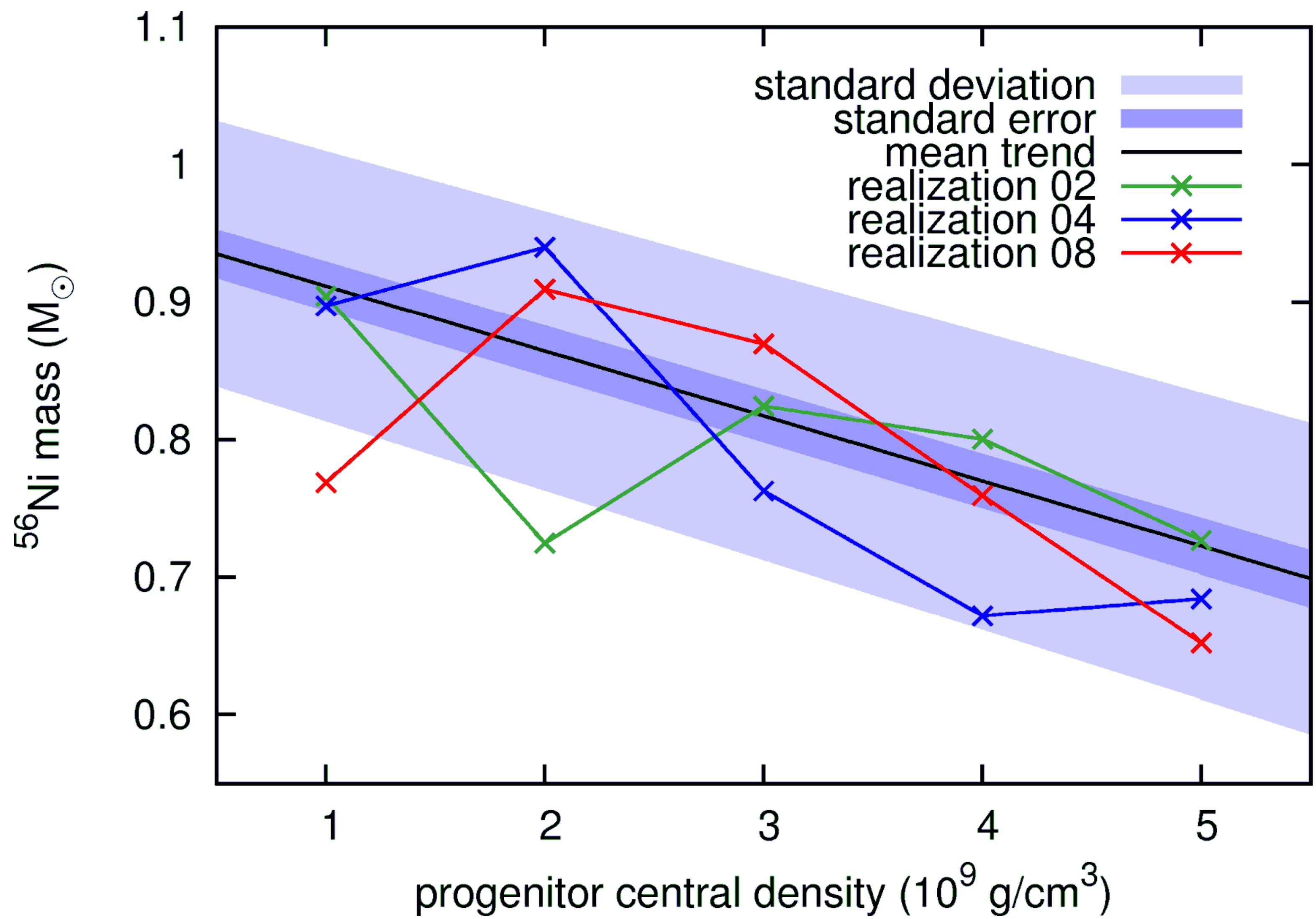
Jackson et al. (2010)

DDT Density Study



Jackson et al. (2010)

Central Density Study

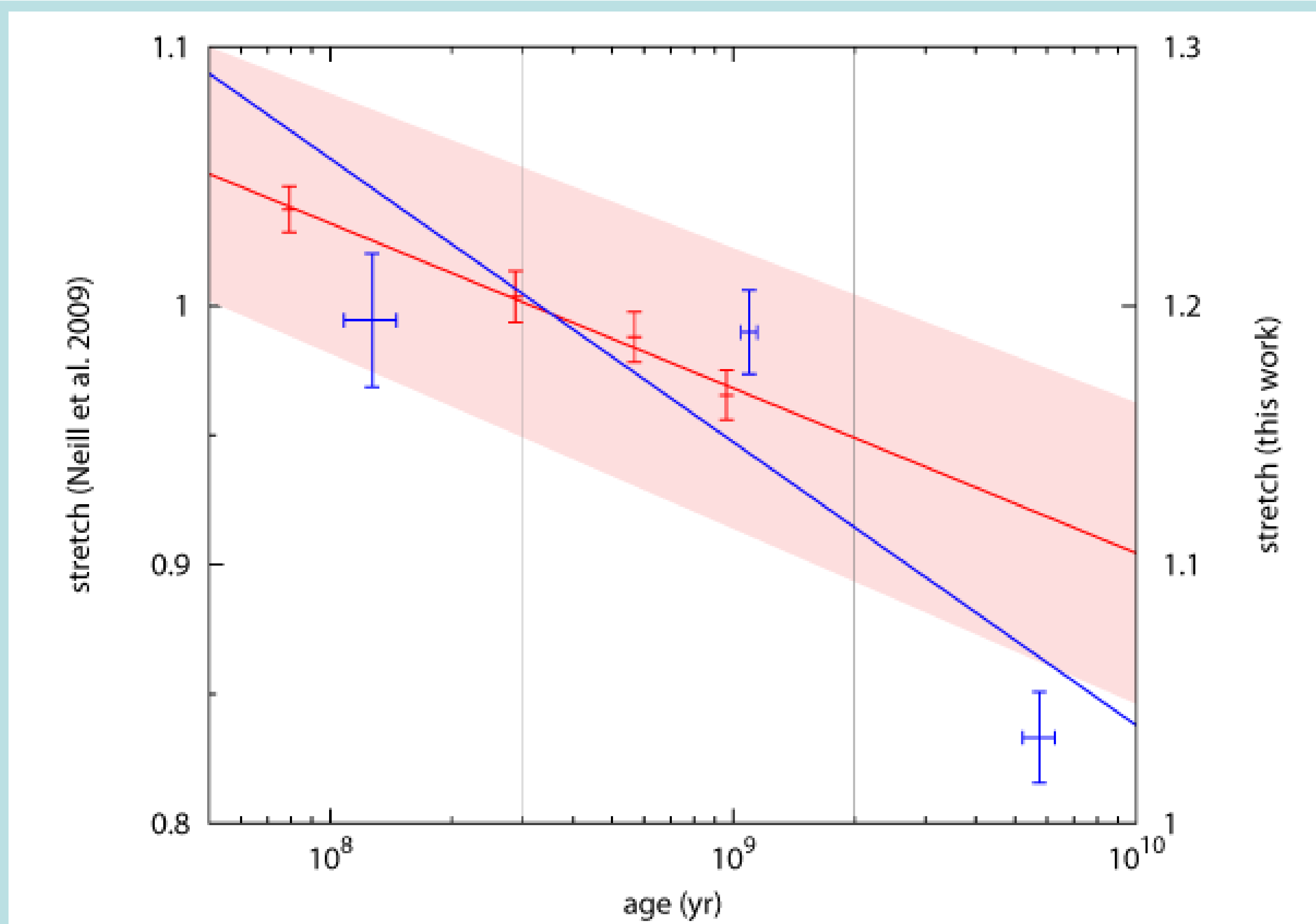


Krueger et al. (2010)

Relationship between central density and age

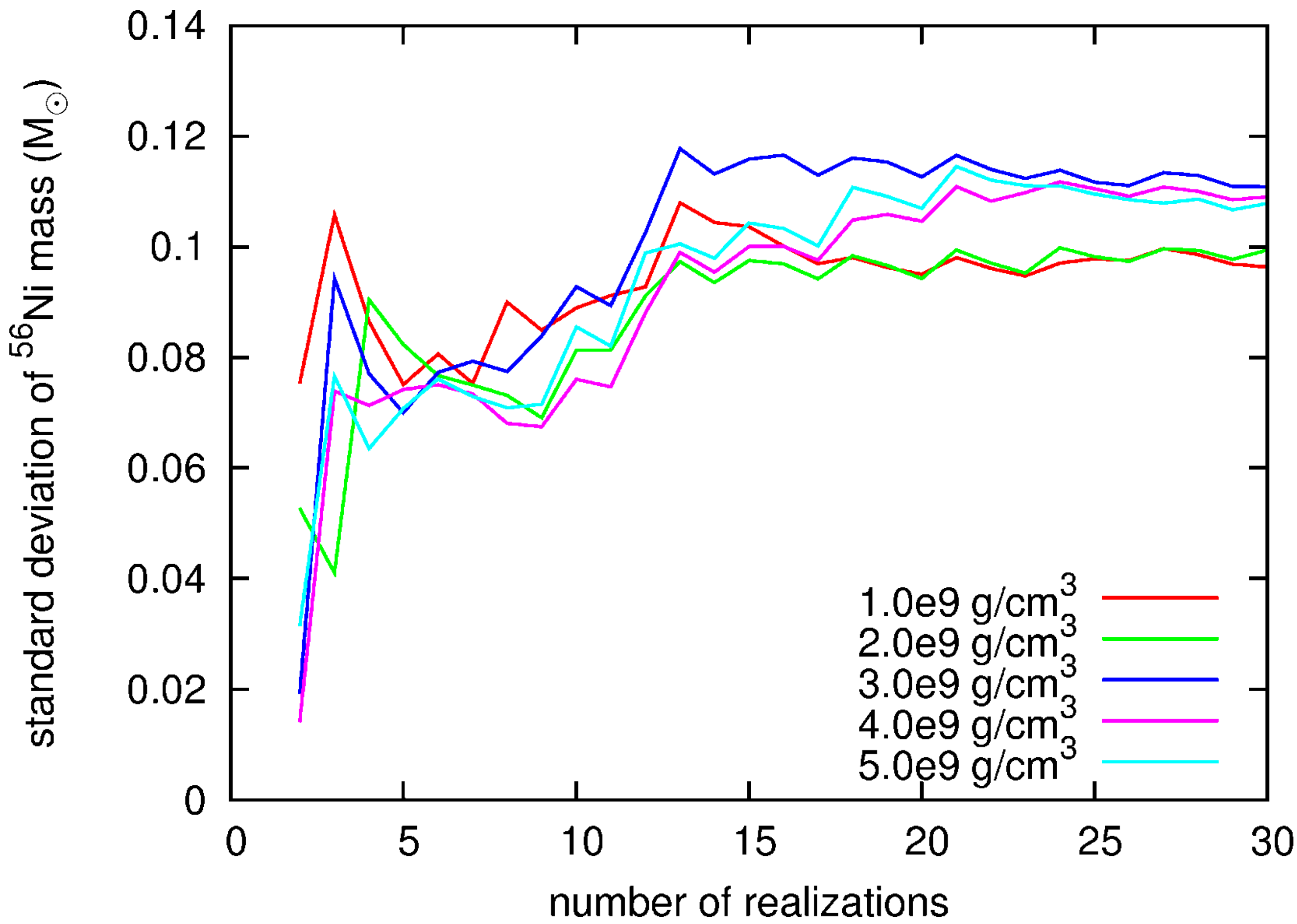
- A WD cools after it forms until the onset of accretion.
- Once accretion starts, the core temperature begins to rise.
- An initially cooler WD has a higher central density when the core reaches the ignition temperature (7-8 X 10^9 K). (Lesaffre 2006)
- We find the increased rates of weak interactions (neutronization) at higher densities produce less ^{56}Ni and thus a dimmer event.
- A SN Ia in an older population may have undergone a longer period of isolation, leading to a higher central density.
- Therefore, we study the effect of central density on ^{56}Ni yield as a proxy for the relationship between age and brightness.
- (Some) observations indicate older stellar populations have dimmer SN Ia.

Trend confronted with observations.



Krueger et al. (2010)

Central Density Study



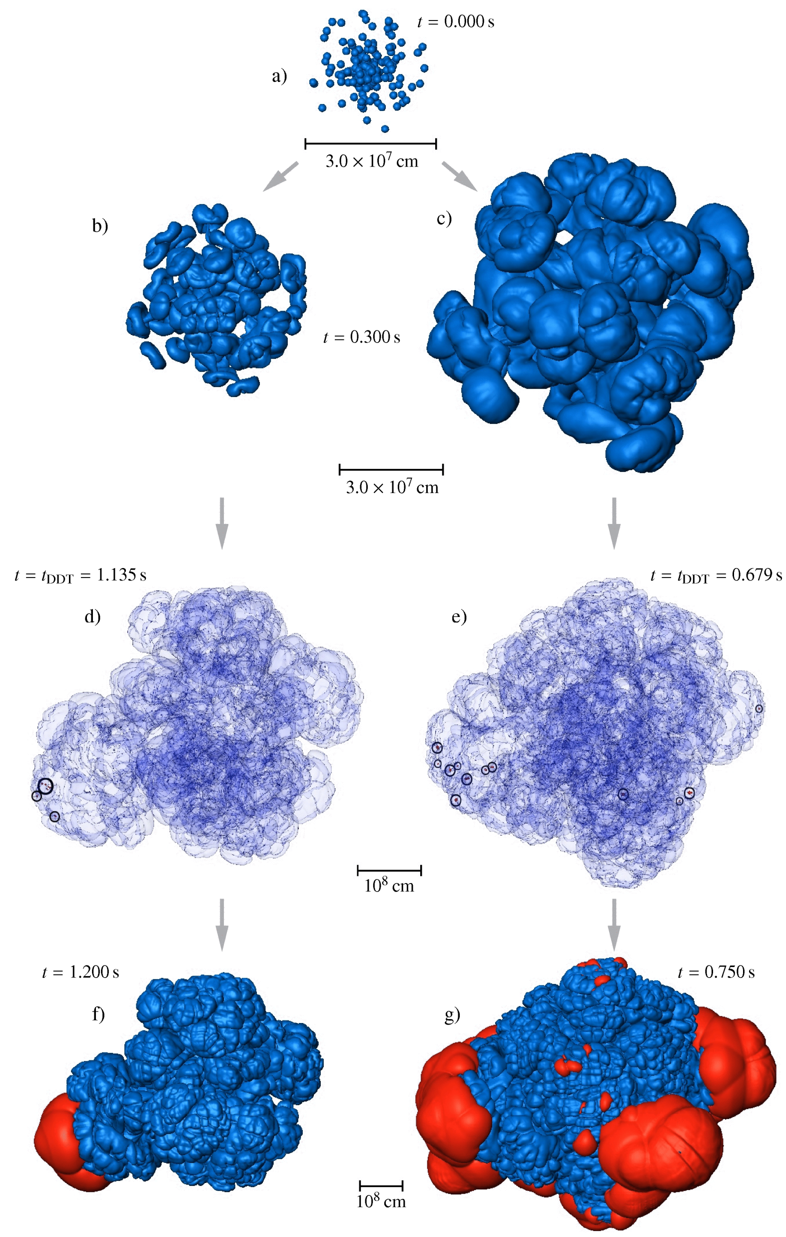
Krueger et al. (2010)

Conclusions

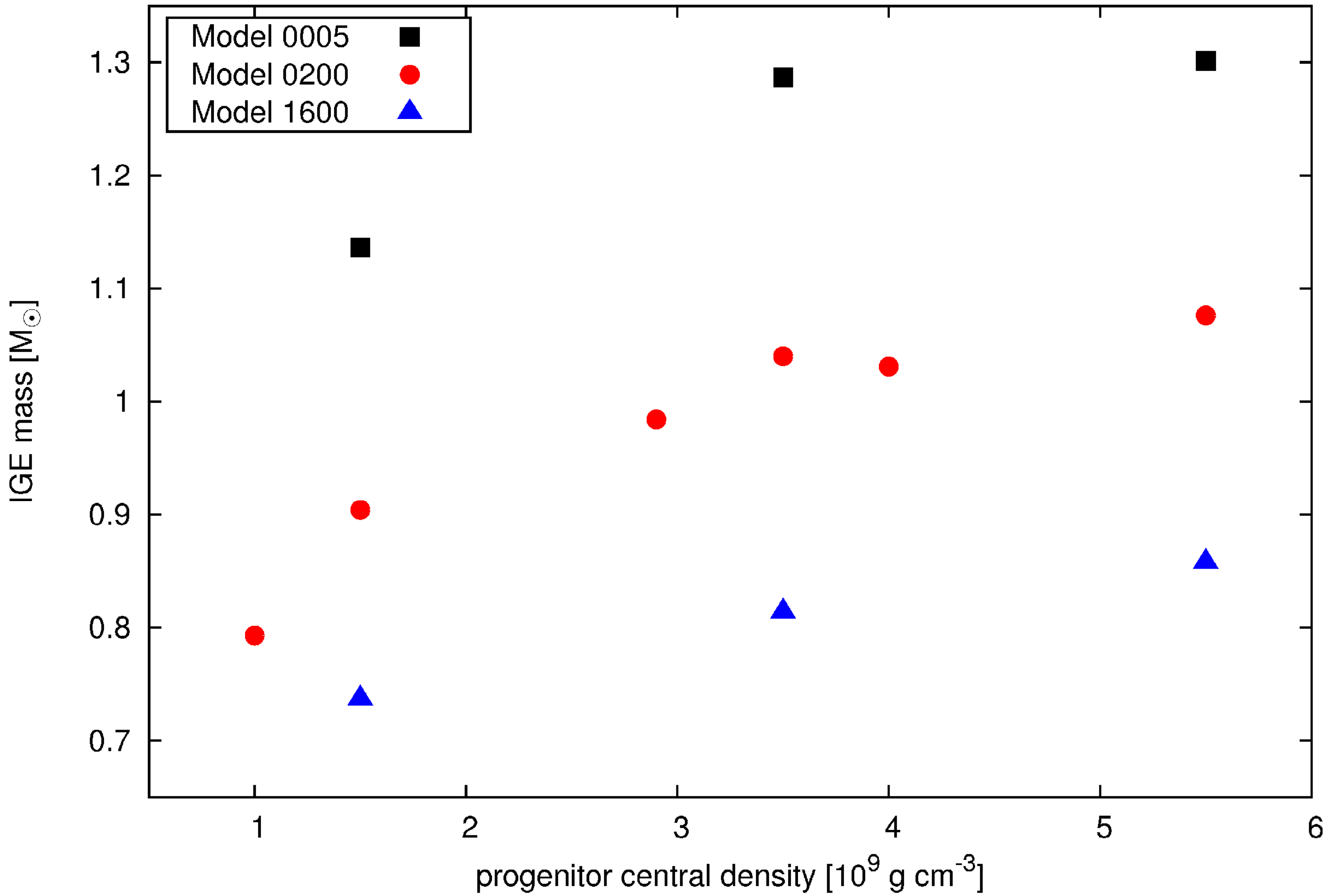
- This is a fun time to be observing or modeling SNe Ia!
- Models are increasing in sophistication and are now able to explore systematic effects such as properties of host galaxy (active vs. passive, metallicity).
- Many questions remain and models still rely on un-validated assumptions.
- We find little effect from including ^{22}Ne as a proxy for metallicity in DDT simulations beyond the direct modification by neutron excess described in Timmes, Brown, & Truran (2003).
- But, by considering the DDT density, we find the change in ^{56}Ni yield with metallicity to be a decrease 0.09 M_sol for a 1 Z_sol increase. This result is about twice that of TBT.
- We find a significant dependence of ^{56}Ni yield on progenitor density, suggesting a cooling time/age dependence.

Recent Similar Study

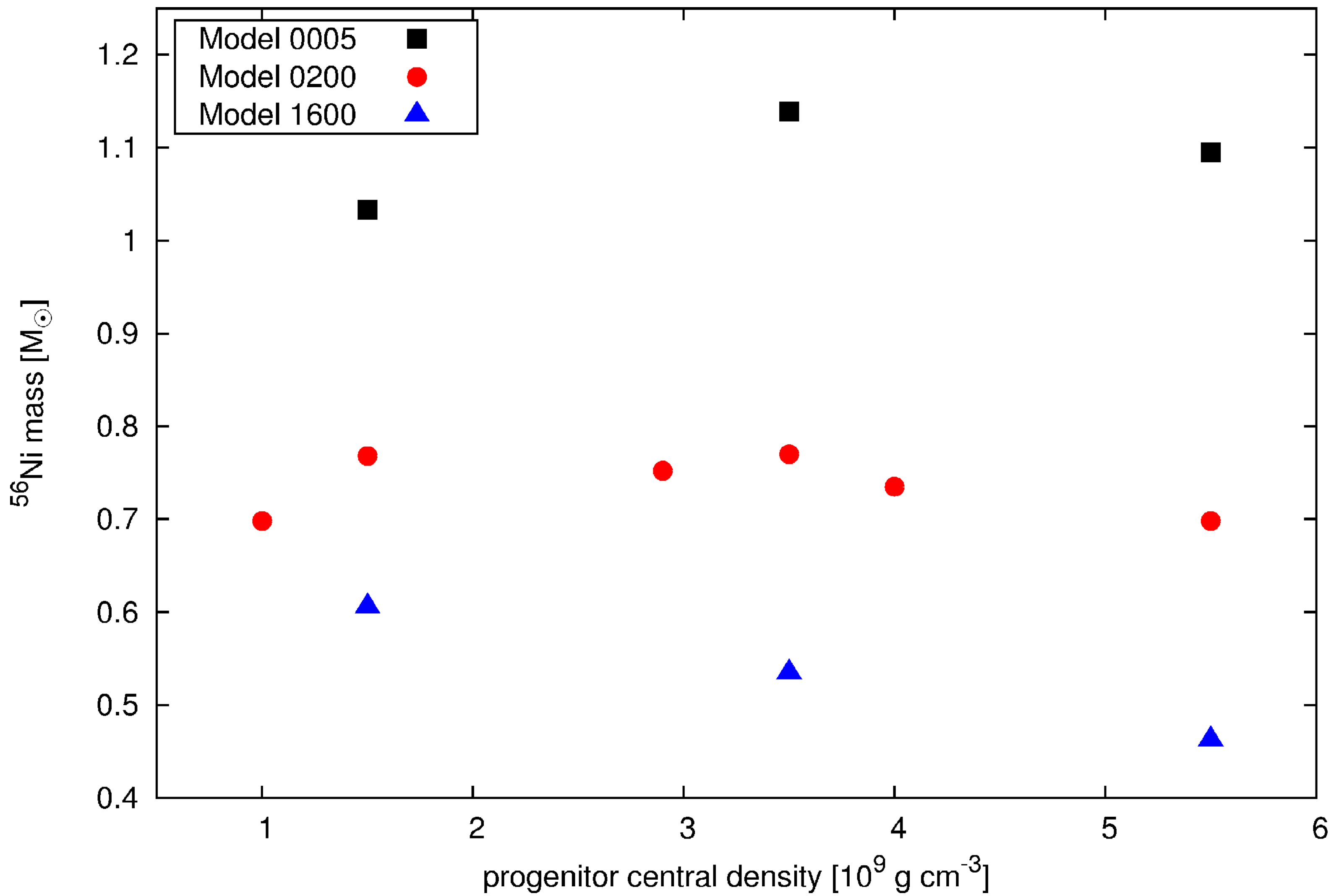
- Seitenzahl et al. (2011) recently performed a similar study in 3-d.
- Found a proportional decrease in the relative amount of ^{56}Ni, but also found an increase in NSE elements.
- 3-d simulations more "believable", but performed a far smaller number.



Seitenzahl et al. (2011)



Seitenzahl et al. (2011)



…and that leads us to

QUESTIONS AND DISCUSSION

Bibliography

- Fryxell, et al. ApJS 131, 273 (2000) [Flash Code]
- Lesaffre, et al. MNRAS, 368, 187 (2006)
- Calder, et al. ApJ 635, 313 (2007)
- Townsley, et al. ApJ 688, 1118 (2007)
- Jordan et al. ApJ 681 1448 (2008)
- Townsley, et al. ApJ 701, 1582 (2009)
- Seitenzahl, et al. ADNDT , 95, 96 (2009)
- Seitnezahl, et al. MNRAS 414, 2709 (2011)
- Krueger et al. ApJ 719, L5 (2010)
- Jackson, et al. ApJ 720, 99 (2010)
- Gilfanov & Bogdán Nature 463 924 (2010)
- Hachisu, Kato, and Nomoto ApJ 724 L212 (2010)