Kepler's elliptic orbits in wave mechanics, and problems with the de Broglie - Bohm interpretation of Schrödinger's wave function

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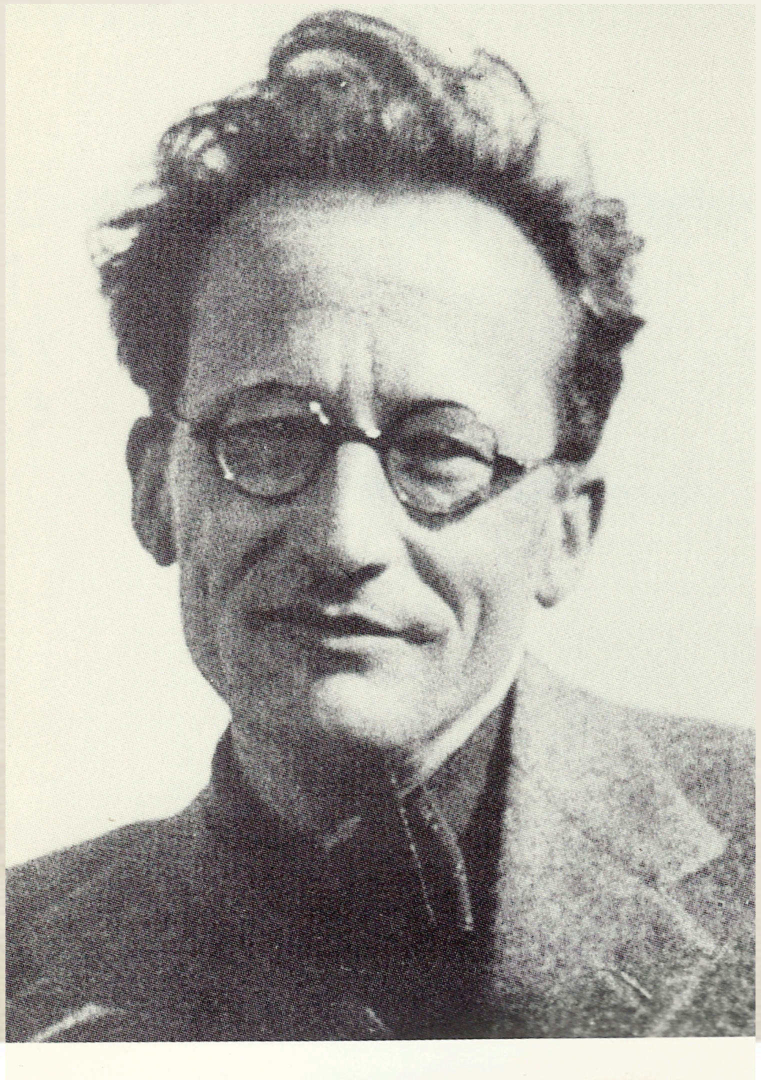
Erwin with his psi can do
Calculations quite a few
But one thing has not been seen:
Just what does psi really mean

Verses by Erich Hückel, Felix Bloch and other physicists attending a 1926 summer conference on Schrödinger's wave mechanics held in Zurich.
Physics Today, Dec. 1976, p. 24

"We emphasize not only that our view is that of a minority, but also that current interest in such questions [about the foundations of quantum mechanics] is small. The typical physicist feels that they have been answered, and that he will fully understand just how if ever he can spare twenty minutes to think about it. (footnote 8)"

J. Bell and M. Nauenberg, " The moral aspects of quantum mechanics"
in Preludes in Theoretical Physics, ed. by A. de Shalit, H. Feshbach and L. van Hove (North Holland, Amsterdam 1966) pp. 279-286
Reprinted in J. Bell, " Speakable and unspeakable in quantum mechanics (Cambridge Univ. Press, 1987) pp. 22-28

Peter Debye
1884-1966



Erwin Schroedinger
1887-1961

Louis de Broglie
1892-1987

$p=h/\lambda$

$\int dq/\lambda=n$
or
$\int dq\ p=nh$

Bohr -Sommerfel
quantization (1916)

Lorentz to Schrōdinger, May 27, 1926 in "Letters on Wave Mechanics", edited by K. Przibram, (Philosophical Library, New York 1967)

Your conjecture that the transformation which our dynamics will have to undergo will be similar to the transition from ray optics to wave optics sounds very tempting, but I have some doubts about it. If I have understood you correctly, then a "particle", an electron for example, would be comparable to a wave packet which moves with the group velocity. But a wave packet can never stay together and remained confined to a small volume in the long run. The slightest dispersion in the medium will pull it apart in the direction of propagation, and even without that dispersion it will always spread more and more in the transverse direction. Because of this unavoidable blurring a wave packet does not seem to me to be very suitable for representing things to which we want to ascribe a rather permanent individual existence...

Schrōdinger to Lorentz, June 6, 1926

Allow me to send you, in an enclosure, a copy of a short note in which something is carried through for the simple case of the oscillator which is also an urgent requirement for the more complicated cases, where however one encounters great computational difficulties. (It would be nicest if it could be carried through in general, but for the present that is hopeless.) It is a question of really establishing the wave packet which mediate the transition to macroscopic mechanics when one goes to large quantum numbers. You see from the text of the note, which was written *before* I received your letter, how much I too was concerned about the "staying together" of these wave packets. I am very fortunate that now I can at least point to a simple example where, contrary to all reasonable conjectures, it still proves right. I hope that this is so, in any event for all those cases where ordinary mechanics speaks of quasi-periodic motion.

Lorentz to Schrōdinger, June 10, 1926

You gave me a great deal of pleasure by sending me your note, "The continuous transition from micro-to macro-mechanics and as soon as I read it my first thought was: one must be on the right track with a theory that can refute an objection in such a surprising and beatiful way. Unfortunately my joy immediately dimmed again; namely, I can not comprehend how, e.g. in the case of the hydrogen atom, you can construct wave-packet that move like the electron (I am now thinking of the very high Bohr orbits). The short waves for doing this are not at your disposal . . . This is the reason why it seems to me that in the present form of your theory you will be unable to construct wave-packets that can represent electrons revolving in very Bohr orbits.

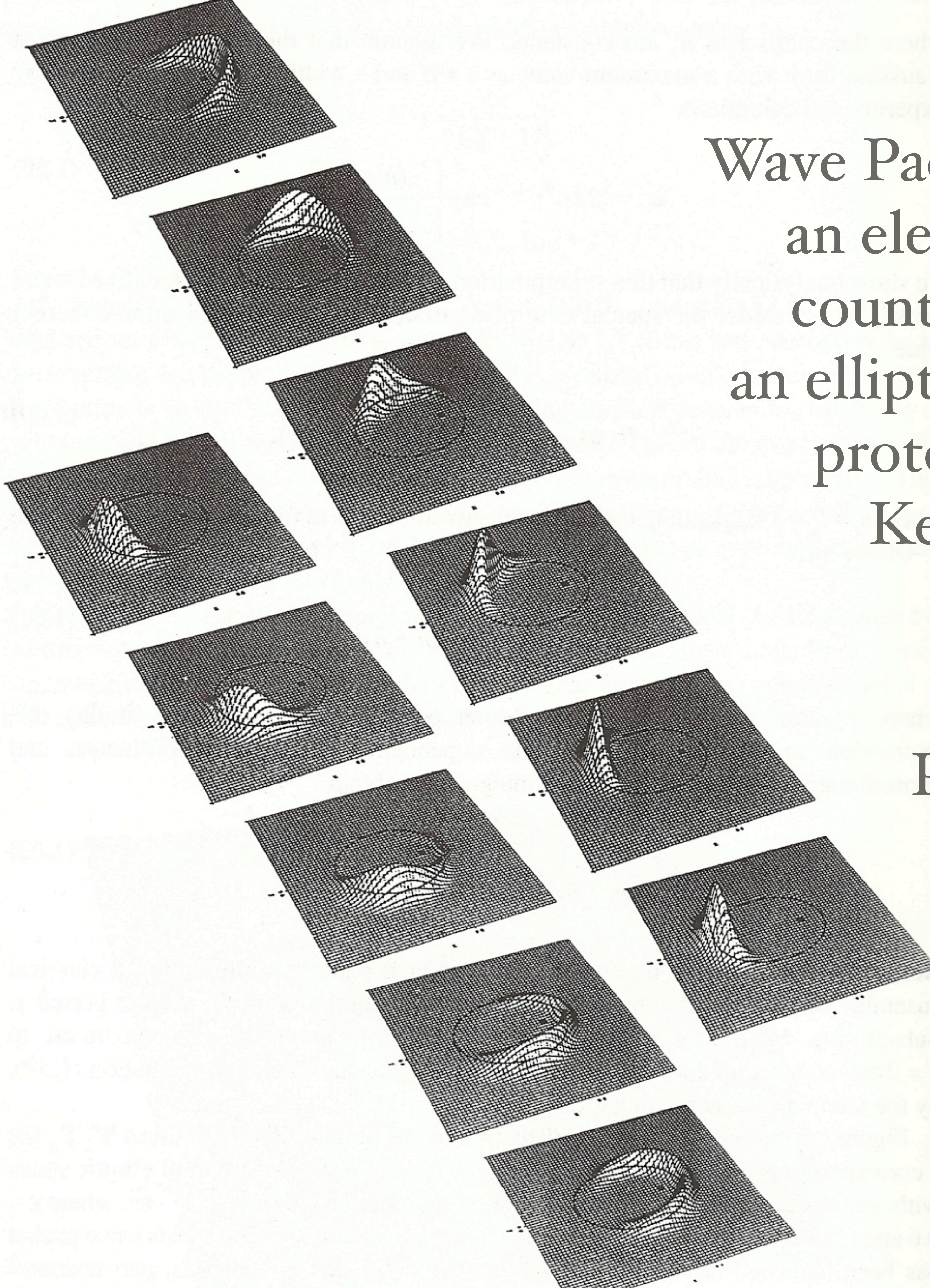
...One can foresee with certainty that similar wave packets can be constructed which will travel along Keplerian ellipses for high quantum numbers; however technical computational difficulties are greater than in the simple example given here...

Schrōdinger, “ The continuous transition from micro - to macro - Mechanics”
Die Naturwissenschaften 14 (1926) 664-666

It is misleading to compare quantum mechanics with deterministically formulated classical mechanics; instead one should first reformulate the classical theory, even for a single particle, in an indeterministic, statistical manner. Then some of the distinctions between the two theories disappear, others emerge with great clarity... The essential quantum effects are of two kinds: the reciprocal relation between the maximum of sharpness for coordinate and velocity in the initial and consequently in any later state (uncertainty relations), and the interference of probabilities whenever two (coherent) branches of the probability function overlap. For macro-bodies both these effects can be made small in the beginning and then remain small for a long time; during this period the individualistic description of traditional classical mechanics is a good approximation. But there is a critical moment tc where this ceases to be true and the quasi-individual is transforming itself into a genuine statistical ensemble. Max Born

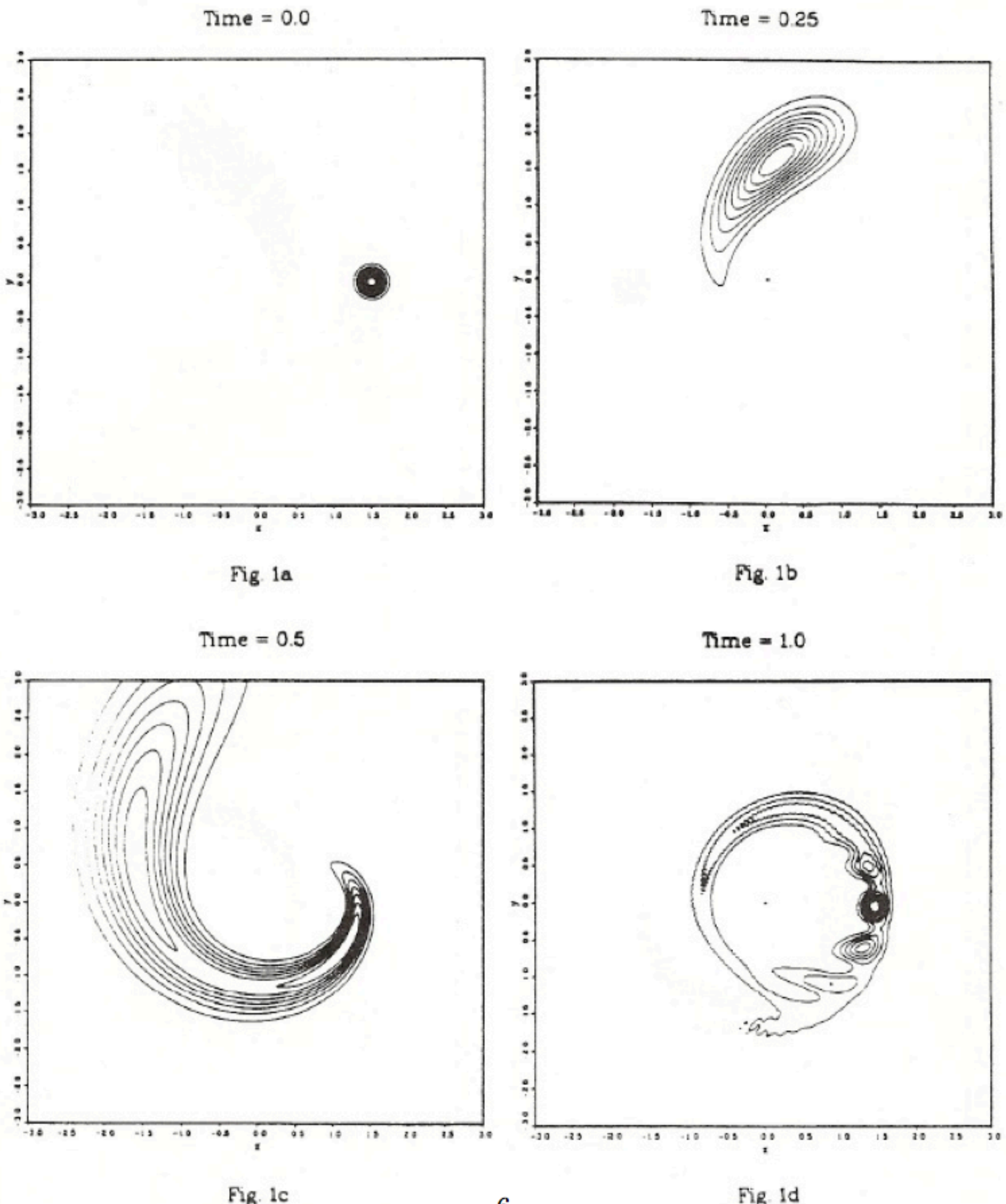
God knows I am no friend of the probability theory, I have hated it from the first moment our dear friend Max Born gave it birth. For it could be seen how easy and simple it made everything, in principle, every thing ironed out and the true problems concealed. . .

Schrōdinger



Wave Packet representing an electron rotating counterclockwise in an elliptic orbit around a proton during one Kepler period

M. Nauenberg,
Phys. Rev. A 40 , (1989) 1133



Contours for the evolution of the absolute square of an initial Gaussian wave-packet propagating in a Coulomb field. The initial mean momentum and position correspond to a particle traveling in a circular orbit with Bohr radius $a=a_B n^2$ for principal quantum number $n=40$.

The evolution is shown for times $t=0.,.25,.50$ and 1.0 in units of the mean Kepler period $\tau=2\pi\sqrt{ma^3/e^2}$

M. Nauenberg and
A. Keith

Quantum Chaos and
Quantum Measurement
ed. P. Cvitanovic et al.
(Kluwer Academic 1992)

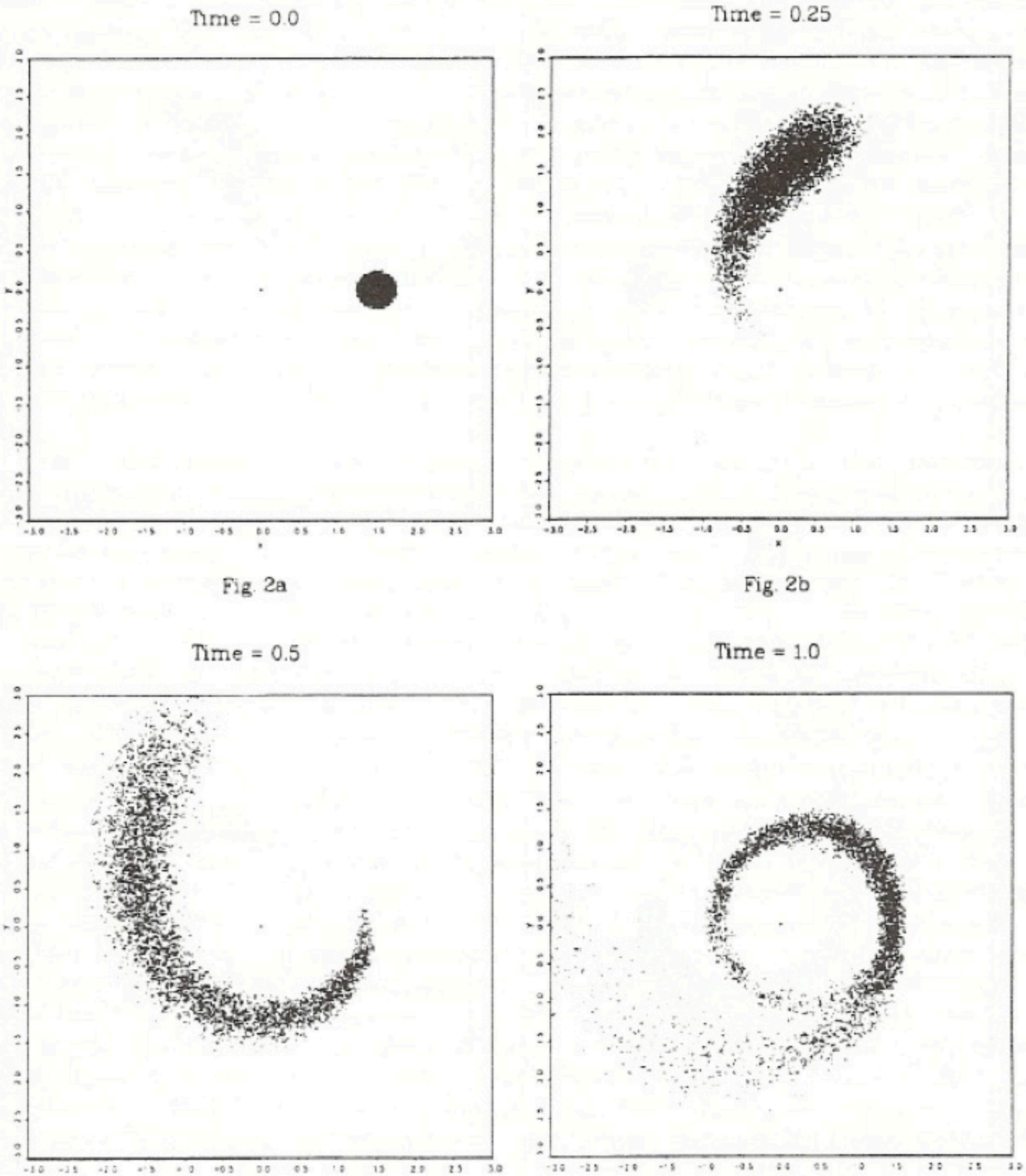


Fig. 2a

Fig. 2b

Fig. 2c

Fig. 2d

Corresponding classical evolution of an ensemble of 6000 particles distributed initially according to the Wigner distribution associated with the Gaussian distribution in the previous slide. The coordinates of these particles are shown at the same time intervals of 1/4 of the mean Kepler period

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Quantum distribution

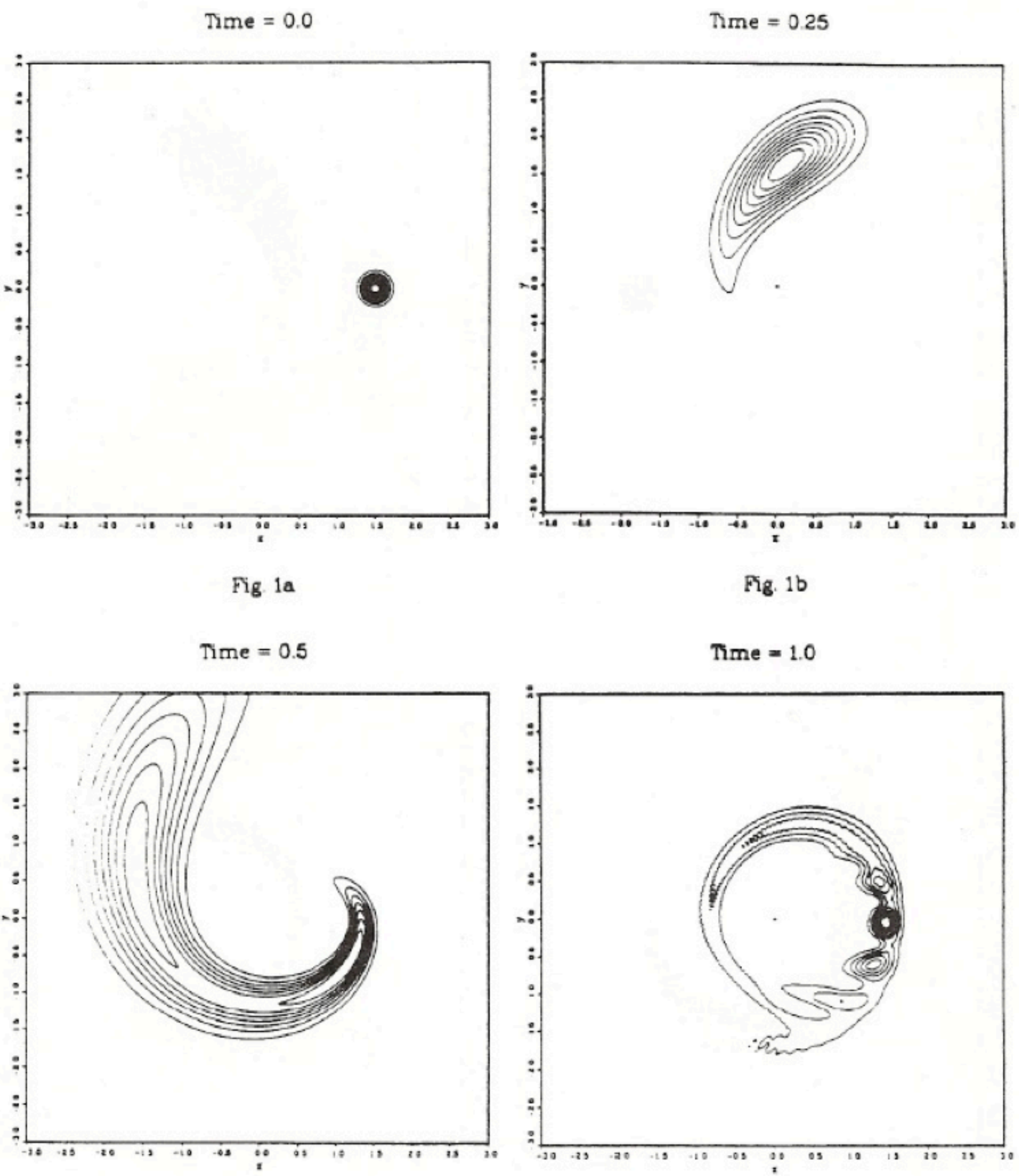


Fig. 1a Fig. 1b Fig. 1c Fig. 1d

6

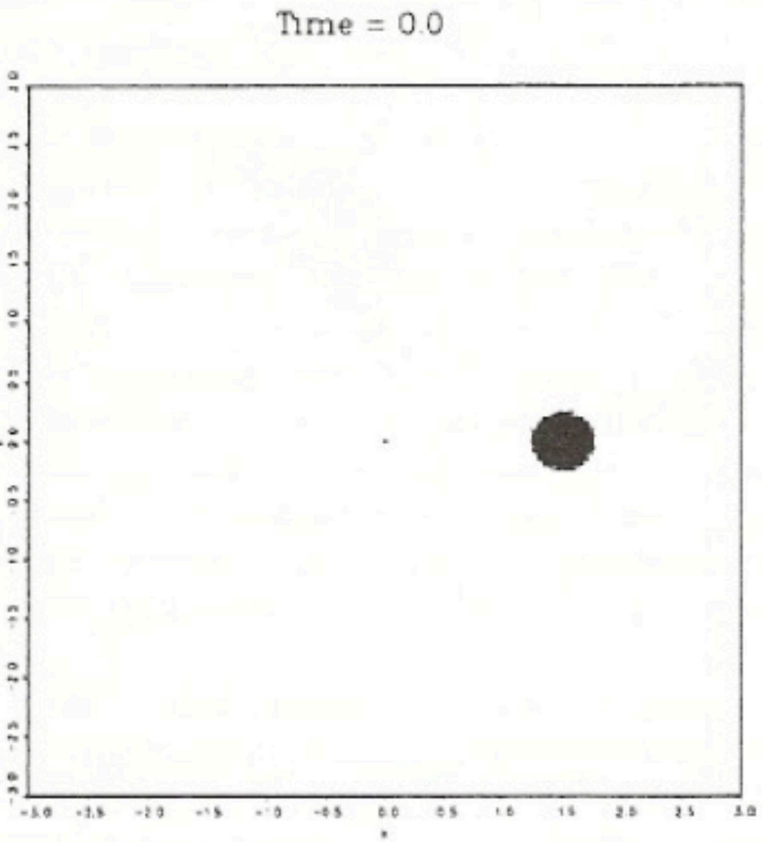


Fig. 2a

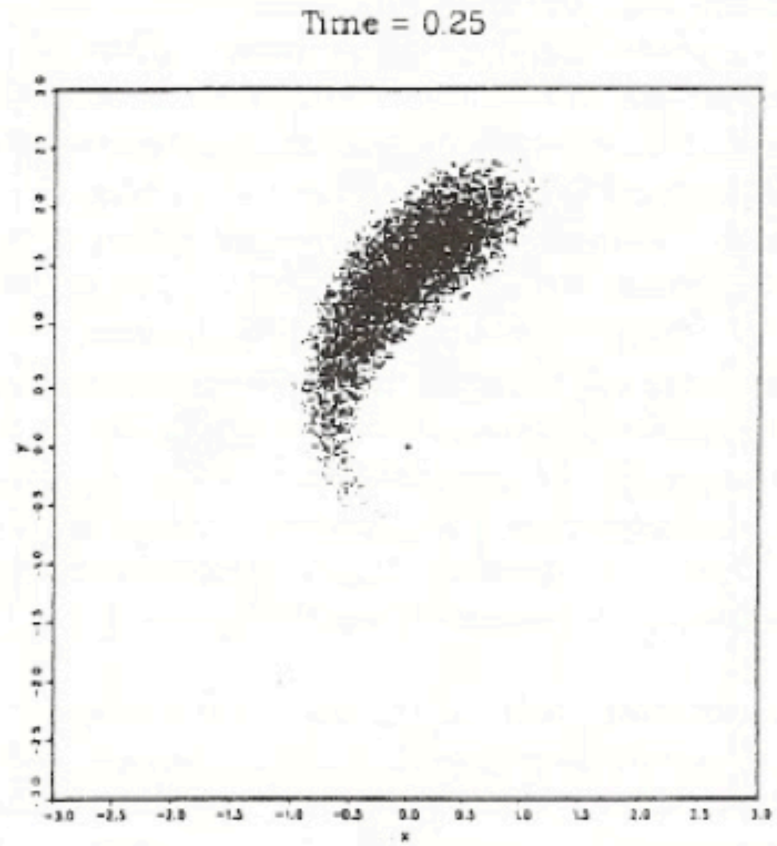


Fig. 2b

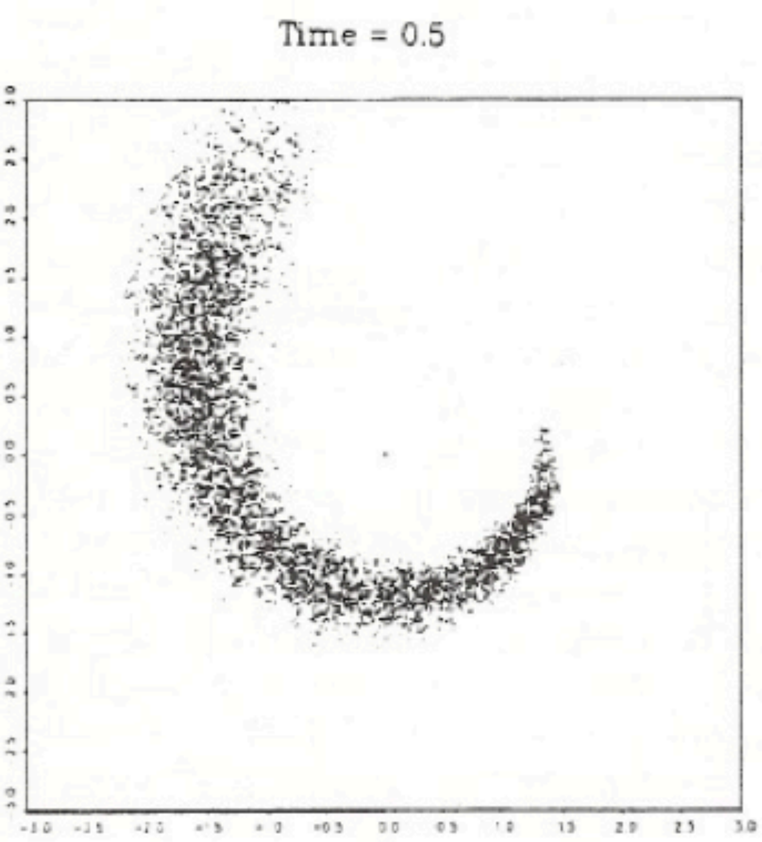


Fig. 2c

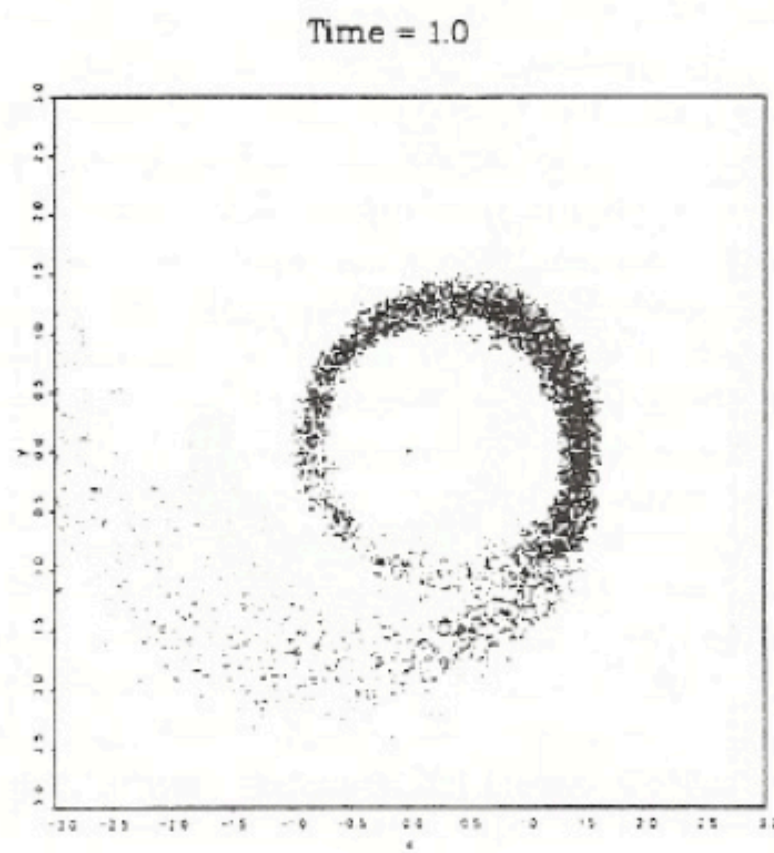


Fig. 2d

Classical distribution

"The Classical Atom", Nauenberg, Stroud, Yeazell, Scientific American, June 1995

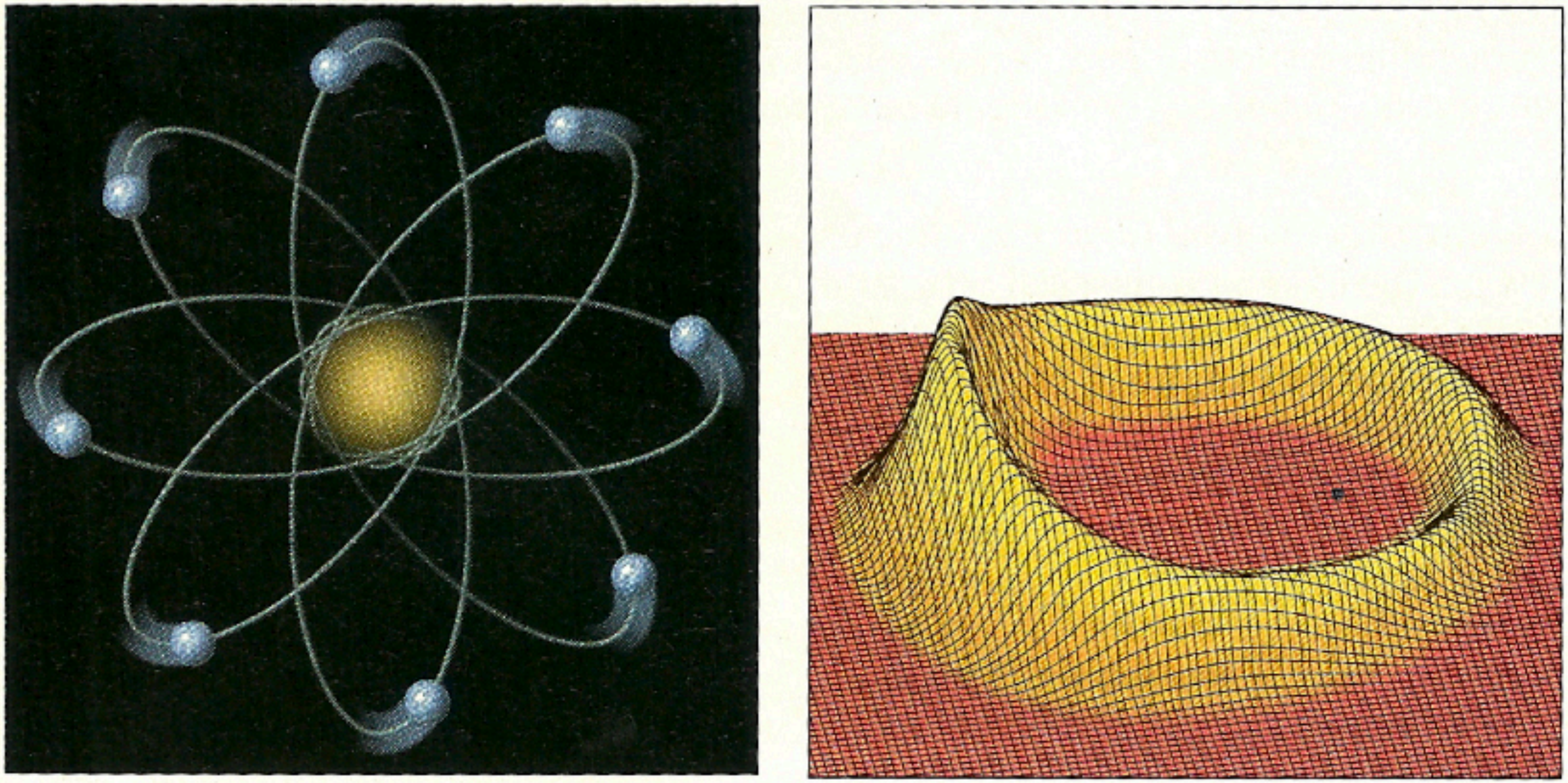
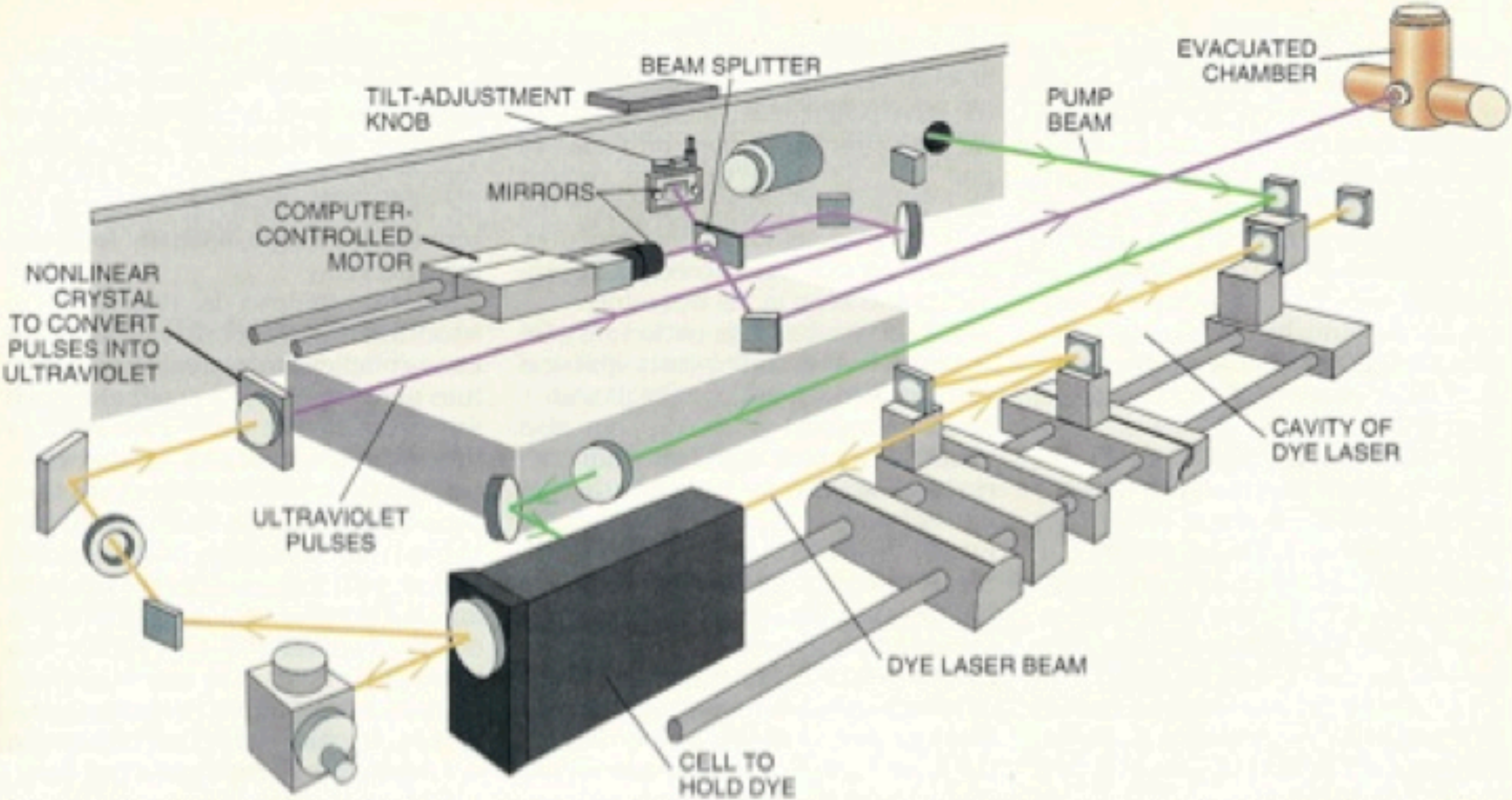


Figure 3: Left figure - Elliptic orbits in the Bohr-Sommerfeld model for an electron orbiting around a proton located at the focus of these ellipses. Right figure - Probability distribution for finding the electron in a stationary quantum elliptic state for a mean principal quantum number $n = 40$.



REACHING THE CLASSICAL LIMIT demands the excitation of atoms by brief pulses of laser light. A green laser beam emerges from behind the right side of the partition. It "pumps" a dye laser, which then produces yellow pulses (it appears faint green in the photograph on the opposite page). The nonlinear crystal converts the yellow light into ultraviolet (*invisible in photograph*). A beam splitter separates each ultraviolet pulse into two parts that move along different paths. A computer-controlled motor can alter the length of one path by shifting a mirror. Such adjustments allow one pulse to lag behind the other: a 0.3-millimeter increase produces a one-picosecond delay. The beams are recombined and directed at atoms in an evacuated chamber. The first pulse excites the atoms; the second pulse probes the result. The red and orange beams, used to maintain mirror alignments, and some components have been omitted from the diagram for clarity.

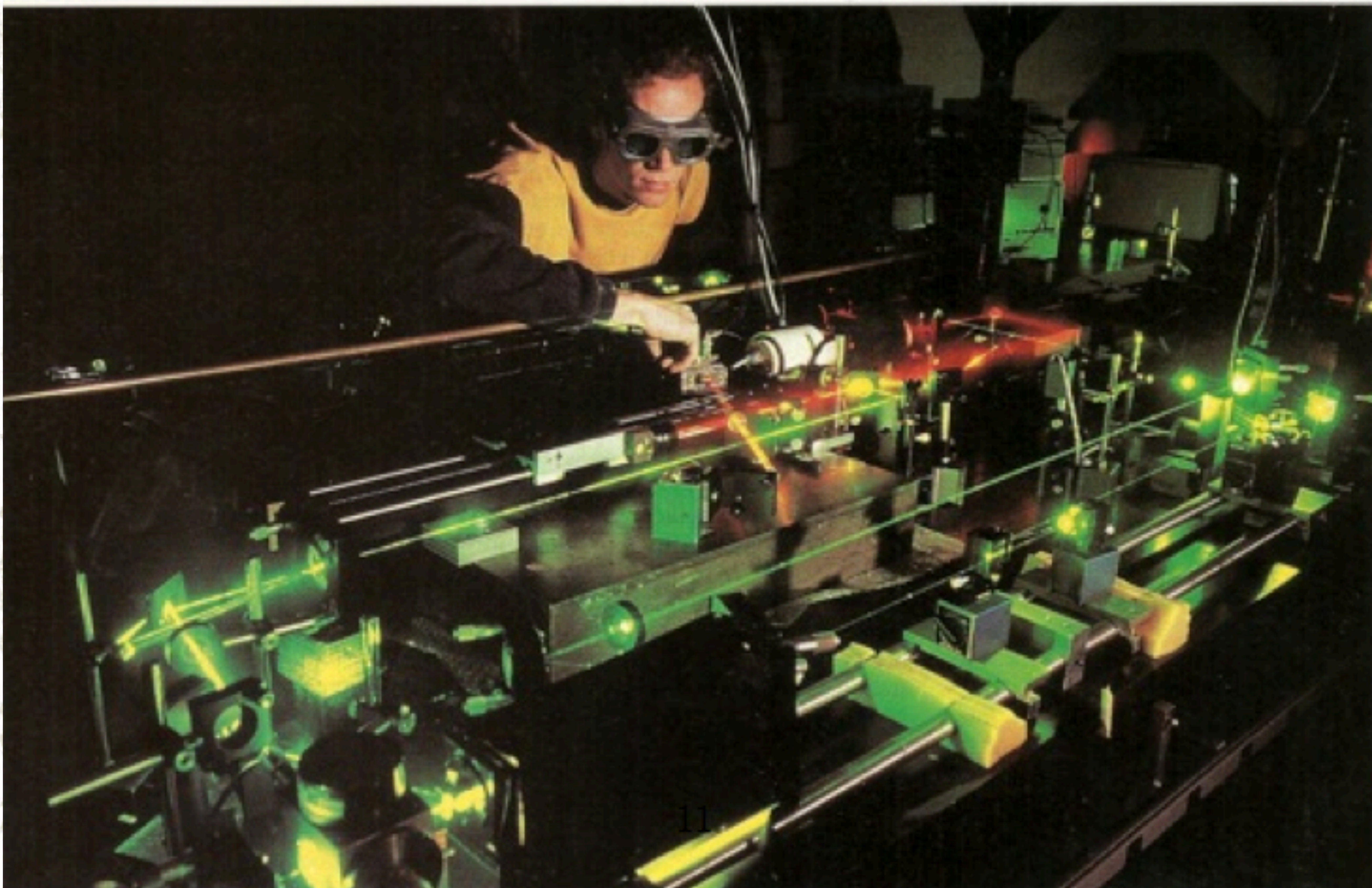


Figure 5: A "pump - probe" experiment to demonstrate the elliptic orbit of an electron in a Rydberg atom as shown in Scientific American, June 1994.

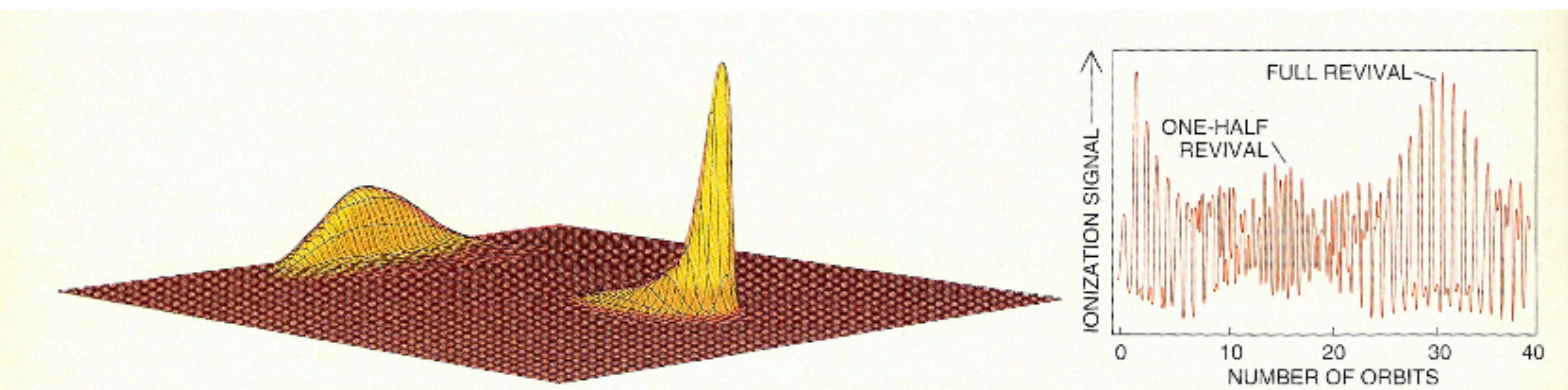
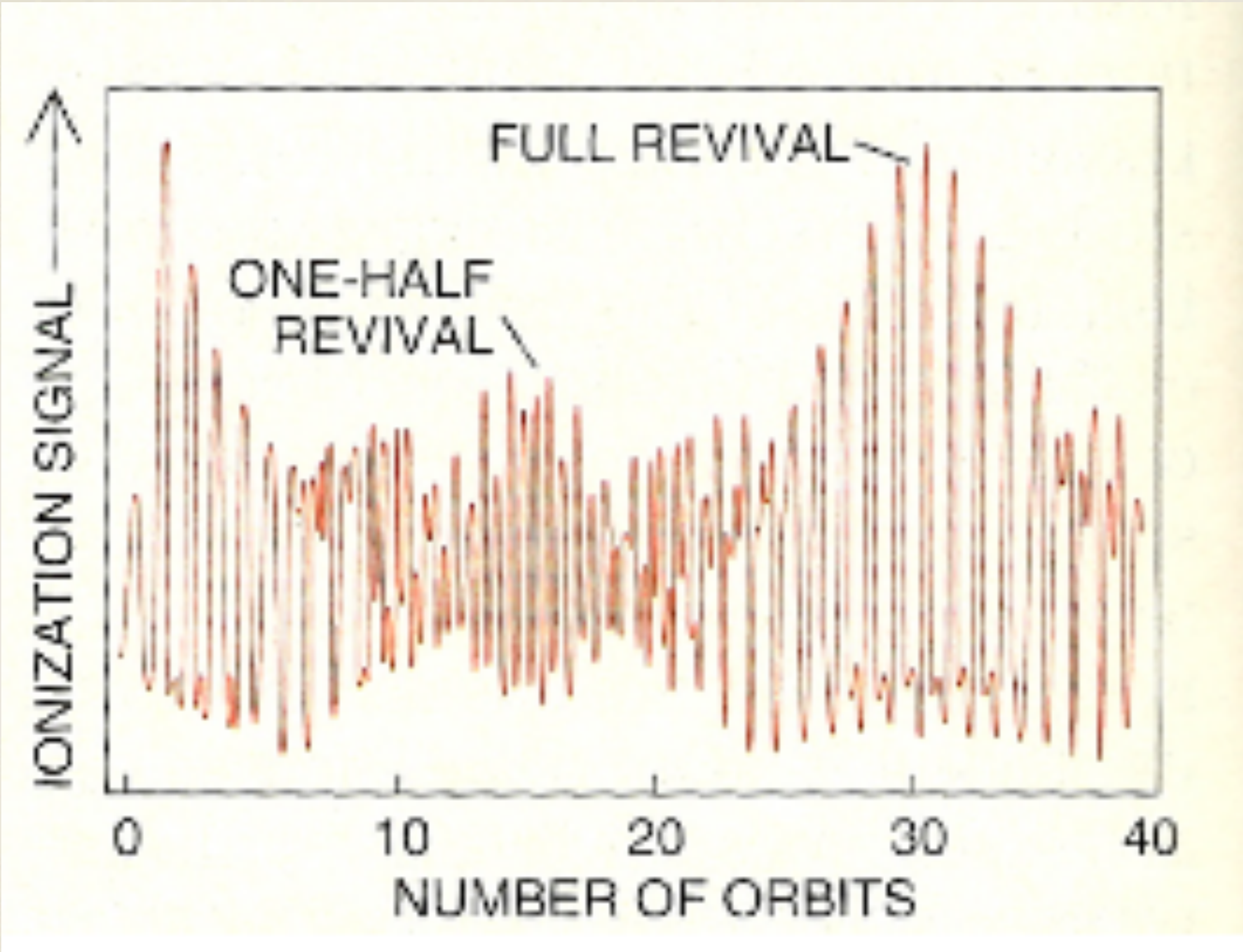


Figure 6: Left figure shows a revival of the initial wave packet into two wave packets. Rigth figure shows the observed ionization signal for the one-half revival, seen as a doubling of the oscillation frequency, and a subsequent full revival of the initial wave packet.


FULL REVIVAL
ONE-HALF REVIVAL
IONIZATION SIGNAL
0
10
20
30
40
NUMBER OF ORBITS

David Bohm, "A suggested interpretation of the quantum theory in terms of `hidden' variables. I
Phys. Rev. 85 (1952) 166

(1) That the ψ-field satisfies Schroedinger's equation.
(2) That the particle momentum is restricted to $p = \nabla S(x)$.
(3) That we do not predict or control the precise location of the particle, but have, in practice, a statistical ensemble with probability density $P(x) = |\psi|^2$. The use of statistics is, however, not inherent in the conceptual structure, but merely a consequence of our ignorance of the precise initial conditions of the particle.

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$(\hbar/m)\nabla R$ = random osmotic velocity,

Motion of a free particle in Bohmian mechanics

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The phase S of the Schrödinger function for a Gaussian wave-packet representing a particle moving with mean velocity v_0

$$S(x,t)/m = v_0(x - v_0 t/2) + (x - v_0 t)^2 t/2\tau^2(1 + (t/\tau)^2) + (1/2i)\ln(1 + it/\tau)(1 - it/\tau)$$

where $\tau = 2m\,\Delta^2/\hbar$, and Δ is the mean square width of the wave packet

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Such weird behaviour for a free particle is supposed to be explained in Bohmian mechanics by a ficticious non-local potential