Measures, Self-Selection, Anthropics, and other conundrums: a tale told in paradoxes

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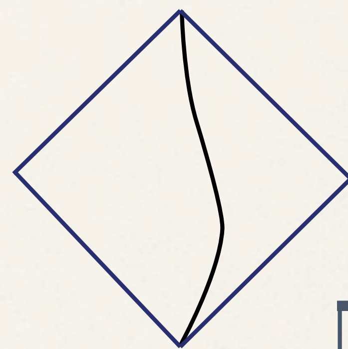
UCSC Summer School in Philosophy and Cosmology, 2013

Systems and states

- **Closed systems:**
  - Full specification of system at any time leads to full specification at all other times.
  - Often assumed; generally only a convenient approximation
  - Spacetime and causality can create truly closed systems: physics in region with boundary at infinity, null, or non-existent



- **Open systems:**
  - Spacetime regions with timelike boundary

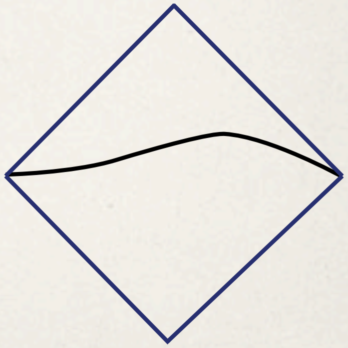
Systems and states

- **Phase space:**
  - Can be
    - continuous [e.g. particles in a box]
    - discrete (or discretized) [e.g. quantum particles in a box]
  - Can be
    - compact [e.g. finite-energy particles in a box]
    - non-compact [e.g. unrealistic particles in a box]
- **Hilbert space**
  - finite or
  - infinite dimensional

finite maximum entropy

Entropy bounds

- **Bekenstein bound** (Bekenstein 81)**:**
  - Saves second law from black holes $S \leq \dfrac{2\pi k R E}{\hbar c}$
  - Saturated by Bekenstein-Hawking entropy of BH
  - Suggests *finite number of states* (or *finite-dimension Hilbert space) for finite regions of finite energy.* (Not true in classical physics)
- **Bousso bound** (Bousso 99)**:**
  - Consider area A of boundary of some volume, and converging lightsheets from A. Integral of entropy flux though either sheet is S < A/4.
  - Derivable from version of Bekenstein bound: trying to pack entropy leads to mass, and spacetime curvature. (Flanagan et al. 2000)



Evolution and entropy

- **Closed systems** we generally assume to have a unitary evolution operator, often preserving phase space volume (e.g. Hamiltonian systems). Open systems may or may not approximate this.
  - Fine-grained entropy preserved.
  - Coarse-grained entropy generally non-decreasing.
- **'Hamiltonian' closed systems with finite maximum entropy:**
  - Poincare recurrence theorem applies.
  - If system 'lasts' a recurrence time, will return arbitrarily close to initial state.

‘**Boltzmann’s Brain**’ Paradox 1

- Consider a Hamiltonian (H) system of finite entropy S that starts away from equilibrium in macrostate A_0, and let it evolve.
- Suppose at some time data D is observed. Would like to predict using D, A_0, and H.
- **Problem**: nearly all instances of D will correspond to macrostates that:

  A. Are part of fluctuations away from equiliubrium (like Poincare recurrences)

  B. Are maximal entropy subject to constraints D.

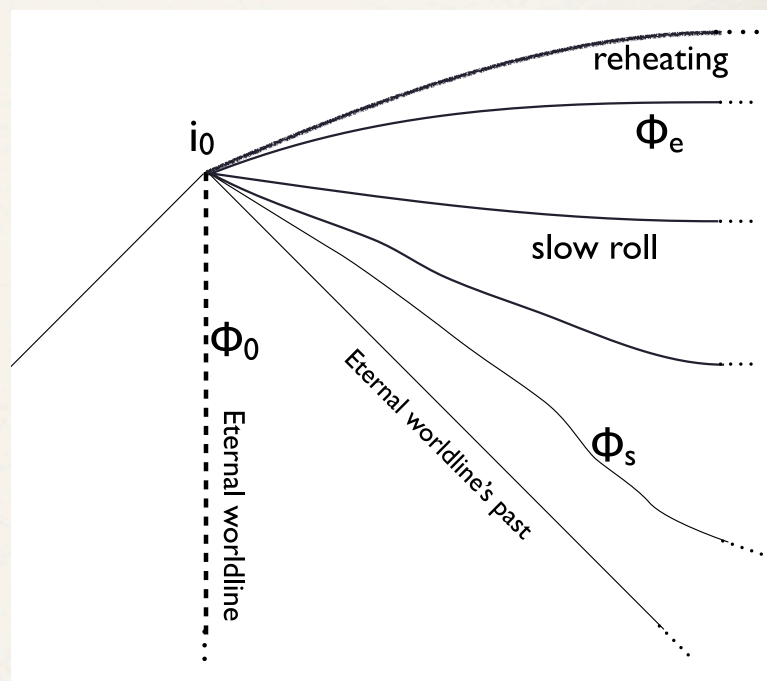
- This makes incorrect predictions, thus it seems *we do not inhabit a finite-entropy Hamiltonian system.*

‘Boltzmann’s Brain’ paradox 2

- Conclusion holds for entropy S *arbitrarily large* but finite.
- Does not (apparently) hold if S is infinite.

Infinite statistically uniform spaces

- Eternal inflation *produces* such spaces as post-inflationary reheating surfaces.*
  - Reheating surfaces are generically infinite
  - Properties are determined by field evolution, which can be same classically everywhere.
  - Randomness provided by thermal/ quantum fluctuations with uniform statistics.
- Because physical laws obey FLRW symmetries, **later universe is also statistically uniform**.



* Some subtleties about the uniformity; see ATL.

Duplicate semi-paradox: a given local configuration will have infinite replicas distributed uniformly throughout the space.

- A configuration (including one we create in a lab) is something that evolved from our initial cosmic state.
- Those initial data (and variations of it) are part of a finite state-space, and should thus be replicated infinitely often throughout a statistically uniform space.
- Thus our configuration should also arise elsewhere.
- The *preponderance* is something quite difficult to calculate, and involves many subtle questions; but it is not relevant here.
- No link between this evolution and cosmic 'location' thus these replicas should arise with a (statistically) uniform distribution.

Duplicate semi-paradox: a given local configuration will have infinite replicas distributed uniformly throughout the space. *Do we care?*

- One might argue as to whether duplicates are *different* or *same* system. Can't reduce to 'periodic' universe, as period differs for different-size systems.

- Weird improbable things happen, but we can't see/interact with them. Somewhat like MWI of QM.

Consider a prototypical quantum experiment, plus macroscopic measuring apparatus.

- e.g., measurement of z-component of single particle's spin

$$\psi_1 = \alpha|\uparrow\rangle + \beta|\downarrow\rangle,\ (|\alpha|^2 + |\beta|^2 = 1)$$

- Apparatus has 'ready' state and states* corresponding to outcomes. (Pre)-measurement as per Von Neumann:

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|a_r\rangle \longrightarrow \alpha|\uparrow\rangle|a_\uparrow\rangle + \beta|\downarrow\rangle|a_\downarrow\rangle$$

- We could also include an environment, human observer, etc., along similar lines.

- From previous argument: *There are replicas of our setup distributed throughout the space.* ***We don't know which one 'we' are measuring.***

*Realistically, many, many microstates for each outcome.

(aside): This exhibits the measurement problem.

- The apparatus has just made the superposition larger, not collapsed it. Both outcomes are still there.
- Decoherence via interaction with random environment can remove any practical possibility of interference between device outcome states, but does not remove the superposition.
  - **Copenhagen (and related):** at some point the superposition must be replaced by one of its elements.
  - **Many-worlds:** The superposition always remains, and grows to include observer, environment, etc.
- Where do probabilities enter?
  - **Copenhagenesque**: Born rule postulate specifies that in repeated sequence of identical trials, relative frequencies given by $|\alpha|^2$ and $|\beta|^2$.
  - **Many-worlds:** more subtle, since *both* 'happen.' Can't naively compare relative frequencies of (sequences of) outcomes: in long series most observers will see 50-50, regardless of α and β.

Quantum duplicate paradox: if we consider the joint system, the standard Born rule is insufficient to produce probabilities.

$$\psi = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes (\alpha|\uparrow\rangle + \beta|\downarrow\rangle)$$

$$= \alpha^3|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle + \alpha^2\beta|\uparrow\rangle|\uparrow\rangle|\downarrow\rangle + \ldots + \beta^3|\downarrow\rangle|\downarrow\rangle|\downarrow\rangle$$

Don Page, arXiv:0903:4888:
"This isn't the square modulus of a quantum amplitude"

$$P_\uparrow = \sum_{n=0}^{N} \binom{N}{n} (\alpha^*\alpha)^n (\beta^*\beta)^{(N-n)} \frac{n}{N} = \alpha^*\alpha = p$$

Quantum probability

Classical probability

Partial resolution

- Accept classical probabilities, look at N → ∞ limit. The classical probabilities take over!
- All terms look like random strings with relative frequencies given by $|\alpha|^2$ and $|\beta|^2$, representing a spatial ensemble in accord with Born rule.
- Except those that don't - but these have total Hilbert measure *zero*.

$$+$$
$$\ldots|\downarrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle\ldots$$
$$+$$
$$\ldots|\downarrow\rangle|\uparrow\rangle|\uparrow\rangle|\downarrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle\ldots$$
$$+$$

these are indistinguishable

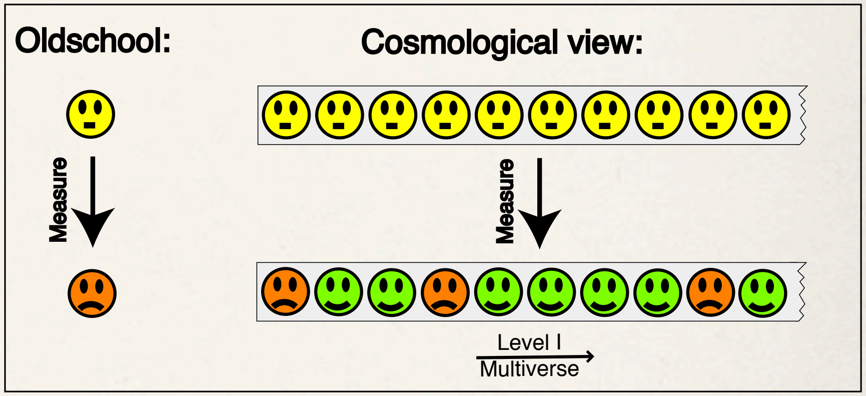
Proof:

Define confusion operator as in arXiv:1008:1066, show that $\|\widehat{\smiley}|\psi\rangle\|^2 \leq \; 2e^{-2\epsilon^2 N}$

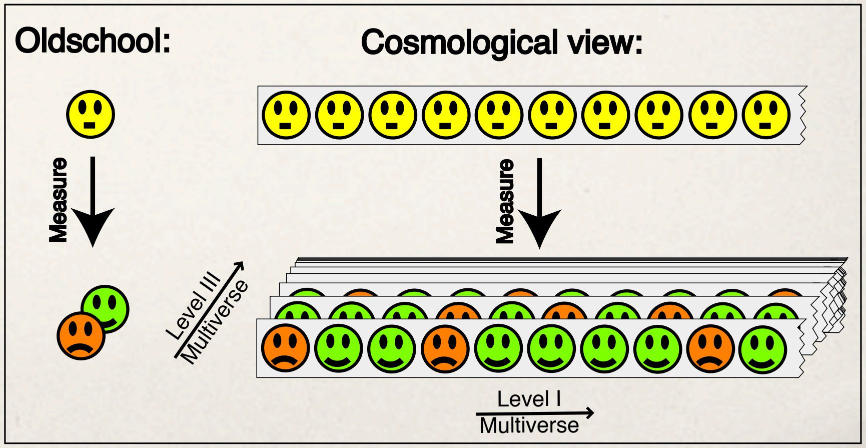
This puts quantum interpretation in a different light.

- Infinite set of equally-valid observers, measuring both outcomes, w/Born freq.
- Two lenses:
  - **Copenhagen**: difference between terms is questionable; collapse is irrelevant.
  - **Everett**: the many worlds are redundant; No observers are 'more real' than others.
- 'Born rule' probabilities not really relevant: probabilities determined by relative spatial frequencies.
- Randomness from inability to 'self-identify' amongst indistinguishable systems.

COPENHAGEN (WAVEFUNCTION COLLAPSES)



EVERETT (NO WAVEFUNCTION COLLAPSE)

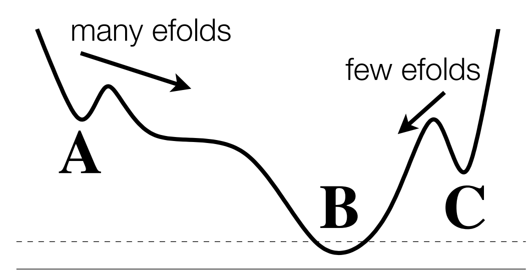


Cosmological prediction conundrum: if all possible local Universes are created, how do we test the underlying theory?

Little problem if all 'pocket universes' are equivalent. But what if they are not?

- Random-valued fields (e.g. axion) $\xi_{\rm axion} = \xi_* \sin^2 \frac{\theta}{2}, \; 0 \leq \theta \leq \pi$

- Different transitions into minima ⇒ different inflationary predictions.



What question do we want to ask ?

- ~~What values of the observables will we observe~~?
- **More well posed***: given* that I am a randomly chosen ***X***, what will I observe? (see AA & Tegmark; Bostrom; Hartle)
  - What values would be observed in a randomly chosen **universe**?
  - What values would be seen from a random **point in space**?
  - What values would be seen by a random **observer**?
- Then: *assume that our observations are like those of a typical* **X.**

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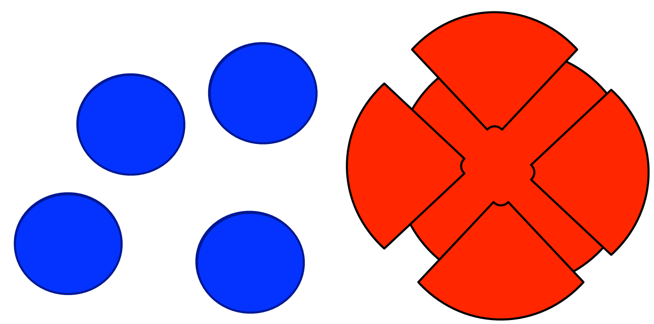

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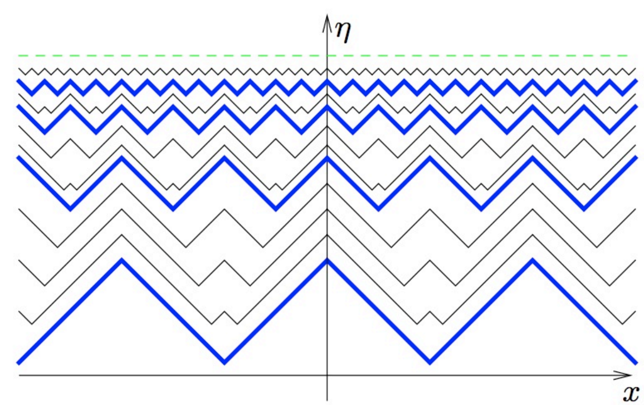

Winitzki 05

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  - What values would be seen by a random **observer**?

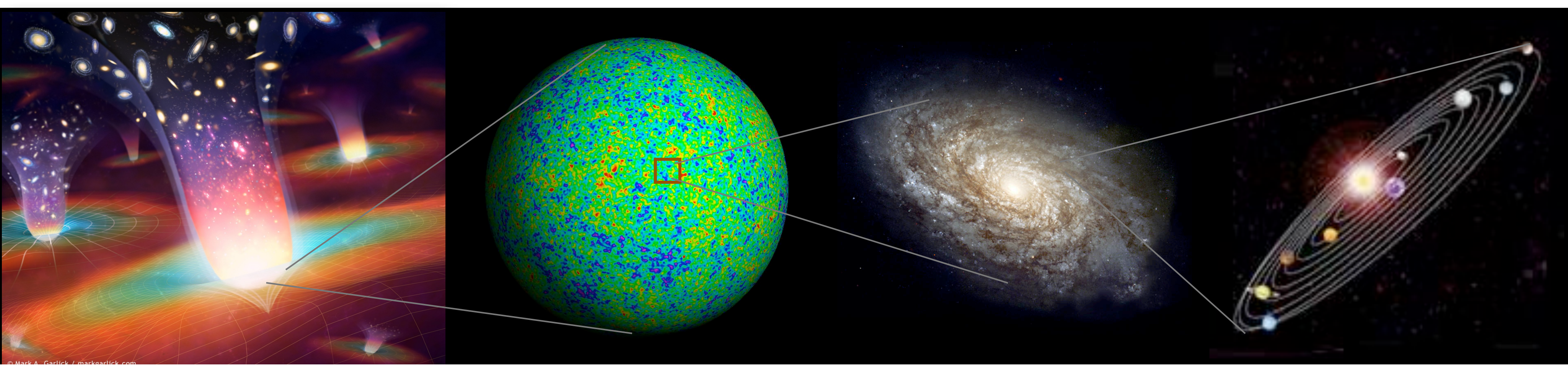
Let $p_X(o_i)$ = *probability of randomly chosen X measuring o_i.* How might we compute $p_X(o_i)$?

1. Choose ***X*** *(e.g. “observer”: proxied by a stable solar mass, solar metallicity star.)*

2. Choose ***p*** useful in calculating $p_X(o_i) = p_p(o_i) \times n_{x,p}(o_i)$. *(e.g., a cm³ of physical volume at the time of reheating)*

3. Calculate $p_p(o_i)$ using inflationary dynamics.

4. Calculate $n_{x,p}(o_i)$ *(e.g. the number of solar-mass, solar-metallicity stars per cm³)*

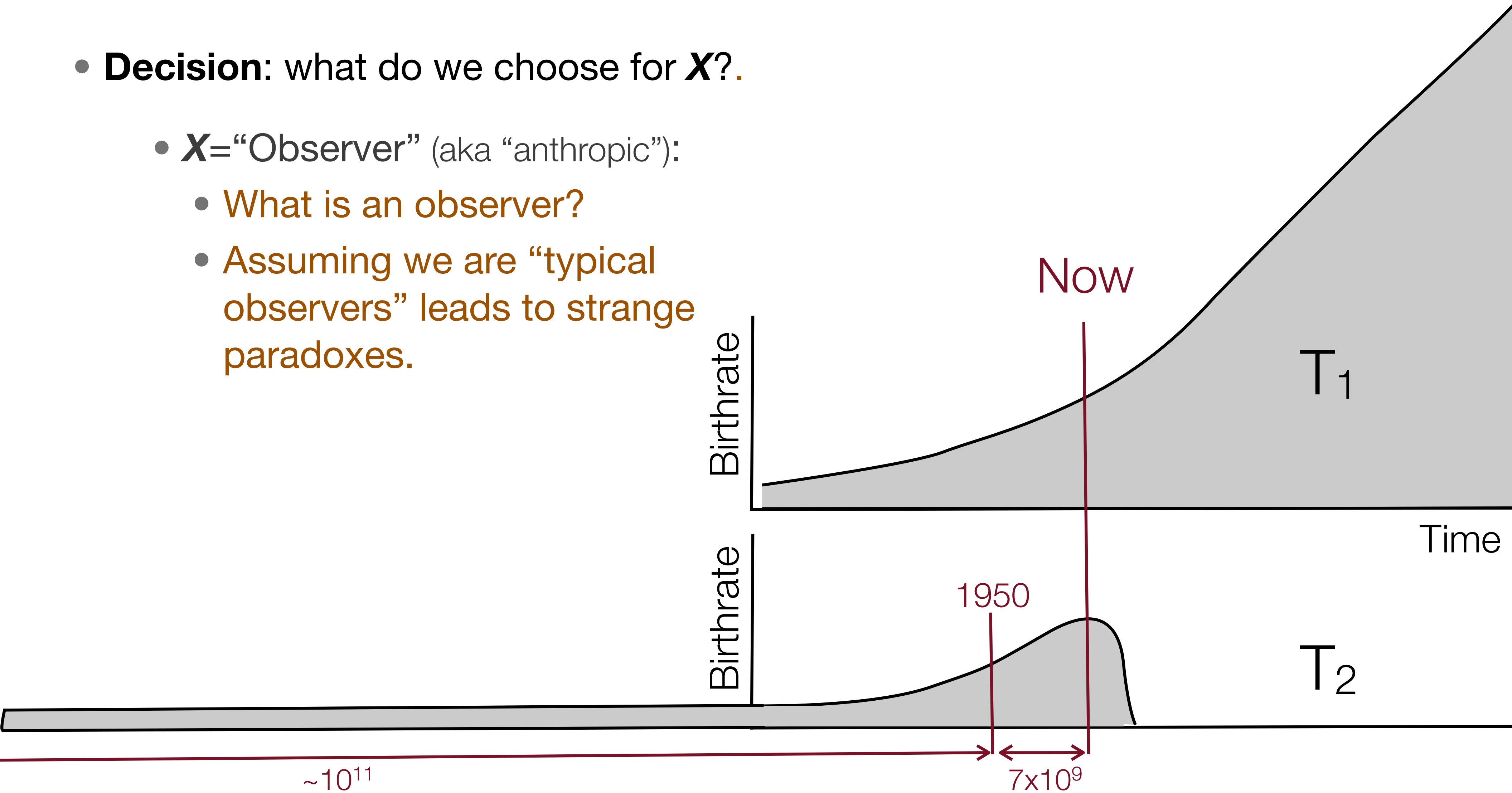


Case study: the Weinberg/Banks/Vilenkin Λ argument.

- Assume:
  - p=baryon, and $p_p(\Lambda)$ = const.
  - Only Λ varies: Q (pert. amplitude) and ξ (matter/photon ratio) etc. fixed.
  - X=galaxy of 10^{12} $M_{\odot}$
- Then:
  - Exponential cutoff in $\Lambda/\xi^4 Q^3$
  - For observed ξ, Q, find $p_X(\Lambda)$ peaks at (few)~ Λ_{obs}.
  - Weinberg on this basis predicted a small but nonzero Λ before it was observed.
- (See Tegmark, Aguirre, Wilczek & Rees 06 for an axion case study).

Choosing *X*: what question do we want to ask?

- **Decision**: what do we choose for ***X***?.
    - ***X***="Observer" (aka "anthropic"):
        - What is an observer?
        - Assuming we are "typical observers" leads to strange paradoxes.



Observable: N: number of observers born before us.

Choosing *X*: what question do we want to ask?

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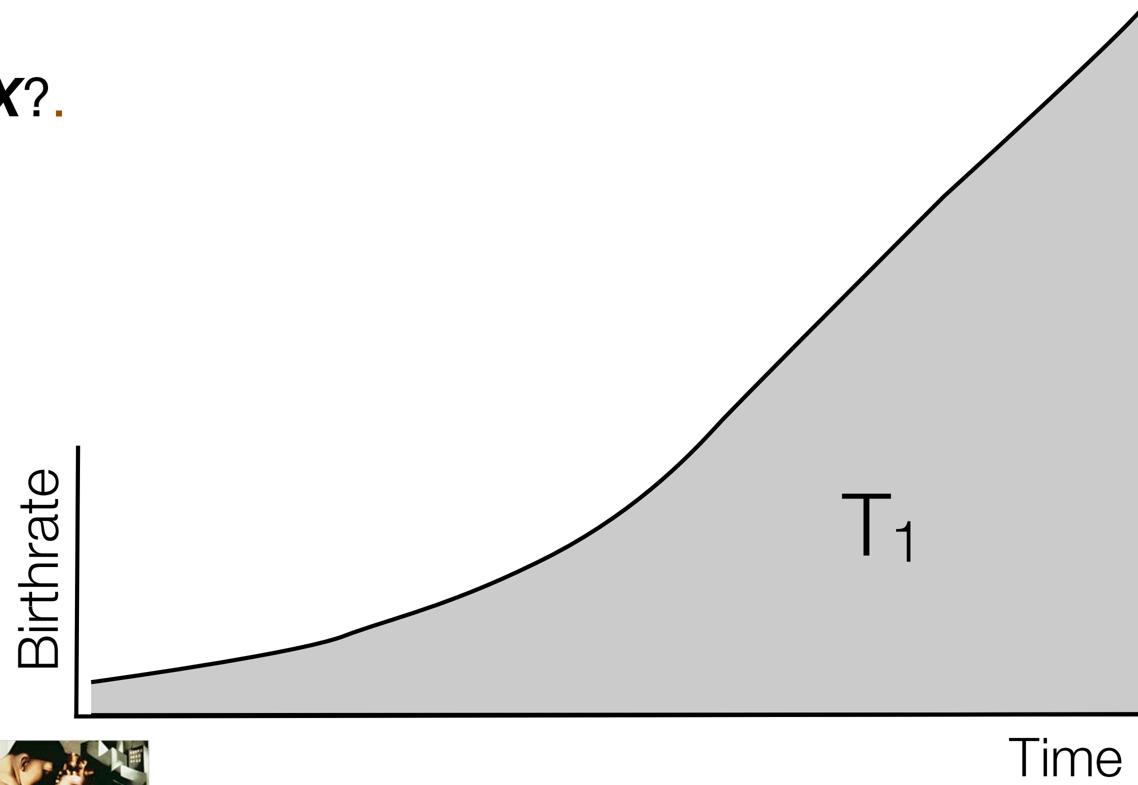
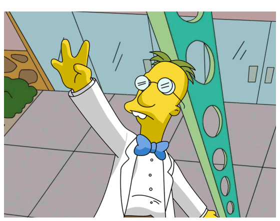


Choosing *X*: what question do we want to ask?

- **Decision**: what do we choose for ***X***?.
  - ***X***="Observer with all our observations" (aka "top-down"):
    - Cannot rule theories out!

Monday -- Theorist A: *"according to my doubly-quantum supertorus theory, with p=0.9999999 confidence, the universe will be red and right-spinning. There is a tiny chance 1-p that it is blue and left-spinning."*

Tuesday -- The universe is observed to be blue. Theorist A: *"Oh well."*



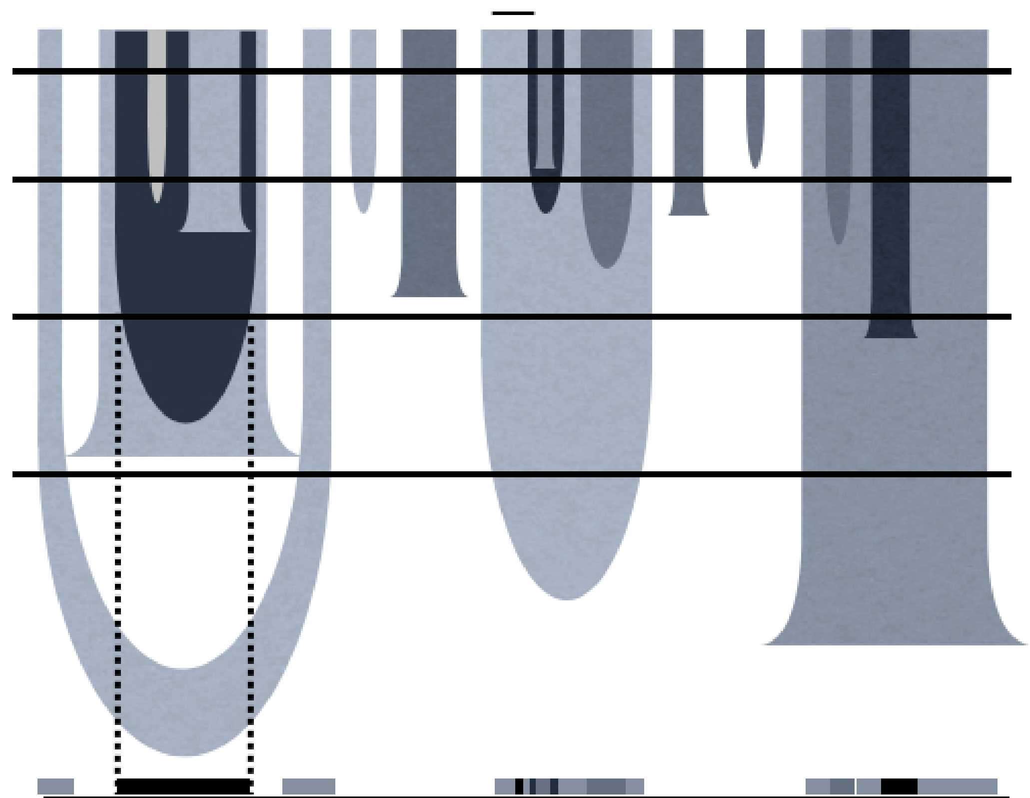
Wednesday -- Theorist B: *"Don't despair! Using top-down reasoning, a blue universe is given. According to supertorus theory, the universe is left-spinning."*

Thursday -- The universe is observed to be right-spinning.

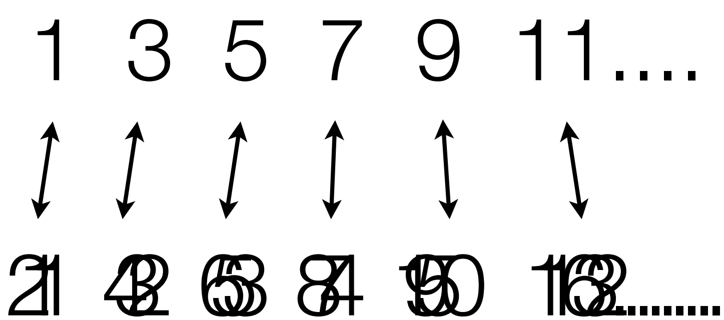
etcetera...

How do we calculate $p_p(o_i)$?

- Bad news: regularization required.
    - Infinitely many bubbles.
    - Each is spatially infinite inside.



Galilean paradox and the ordering ambiguity

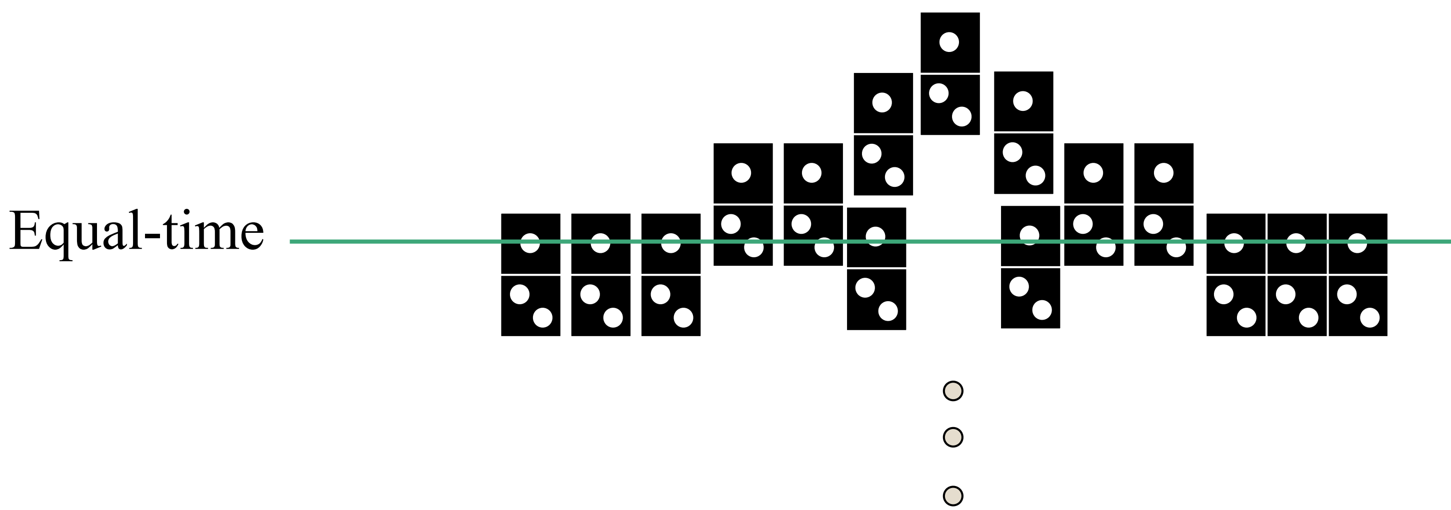


Galileo 1638

- ‘Obvious’ answer is to order by ‘proximity’, i.e. take space or spacetime volumes, compute ratio, send volume to infinity
- **Problem**: in eternal inflation the answer completely depends upon the manner in which this is done.

Galilean paradox and the ordering ambiguity

(Guth): Imagine that universe is dominos. At each instance, you line up 2 1's for each 2. Twice as many 1s as 2s at each time. 1s are twice as probable!



Galilean paradox and the ordering ambiguity

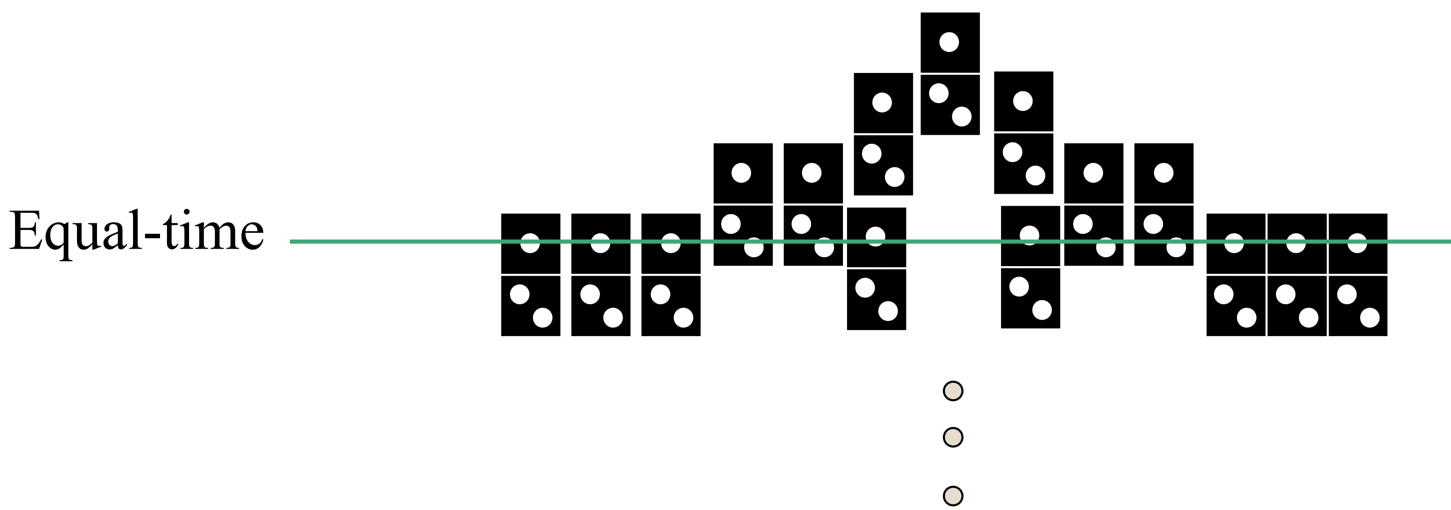
(Guth): Imagine that universe is dominos. At each instance, you line up 2 1's for each 2. Twice as many 1s as 2s at each time. 1s are twice as probable!

(Aguirre): But you know by construction that each 2 comes with a 1: the probabilities must be equal!

(Guth): no.

(Aguirre): yes.

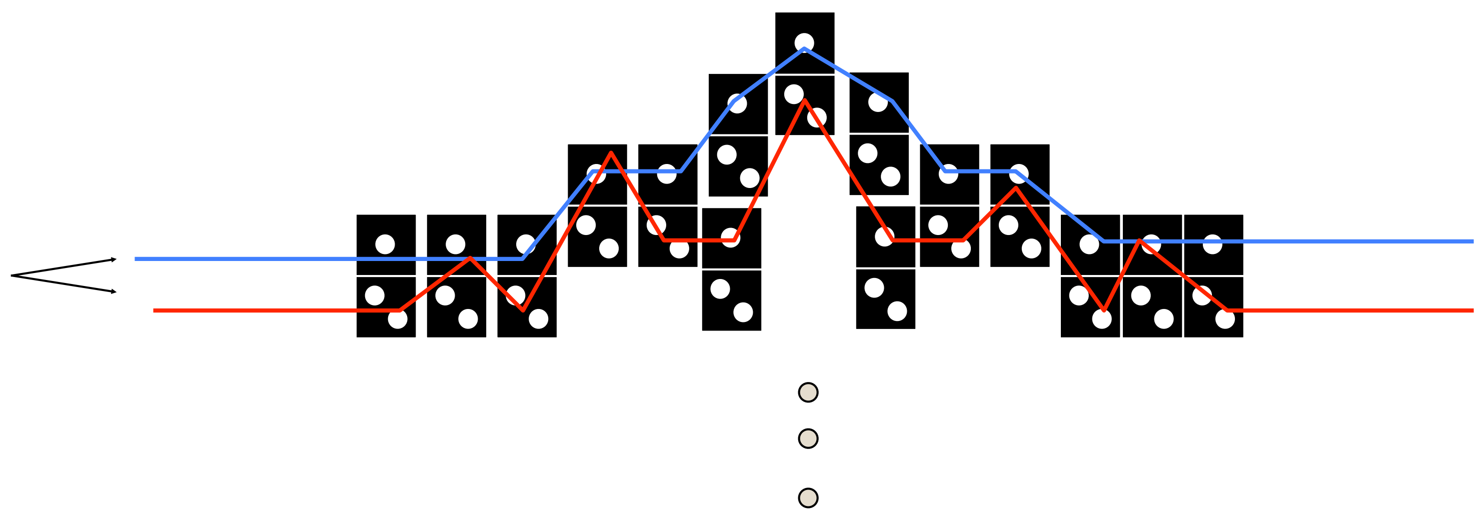
⋮



Galilean paradox and the ordering ambiguity

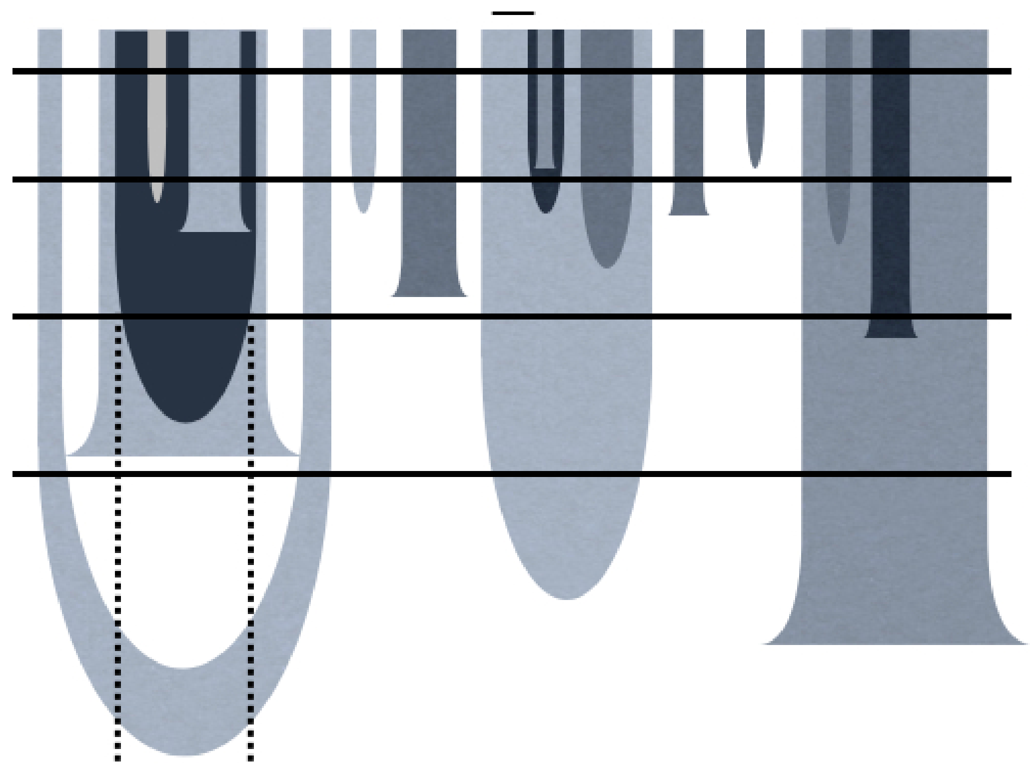
Also: time slices can be drawn to
include all 1's, or all 2's, or a mix

Equal-time choices



How do we calculate $p_p(o_i)$?

- Bad news: regularization required.
  - Infinitely many bubbles.
  - Each is spatially infinite inside.
- Yet: we can put sensible-seeming measures on huge but finite regions of the spacetime, which converge; surely these mean something?
- But many choices



Methods of regularization

- Many choices, a few basic philosophies

Youngness paradox

Boltzmann brains

Domino dilemma

comoving

at fixed

Q-catastrophe

pocket

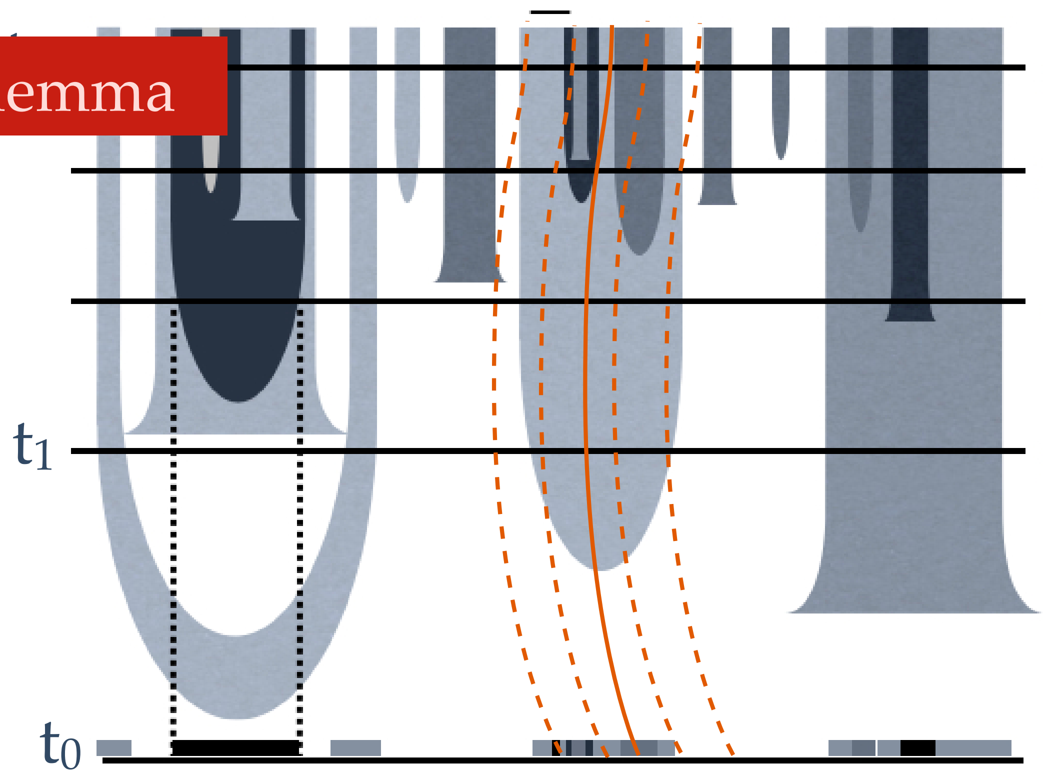
Fast-transition fiasco

- C … g a W

Up-transition upside-downness

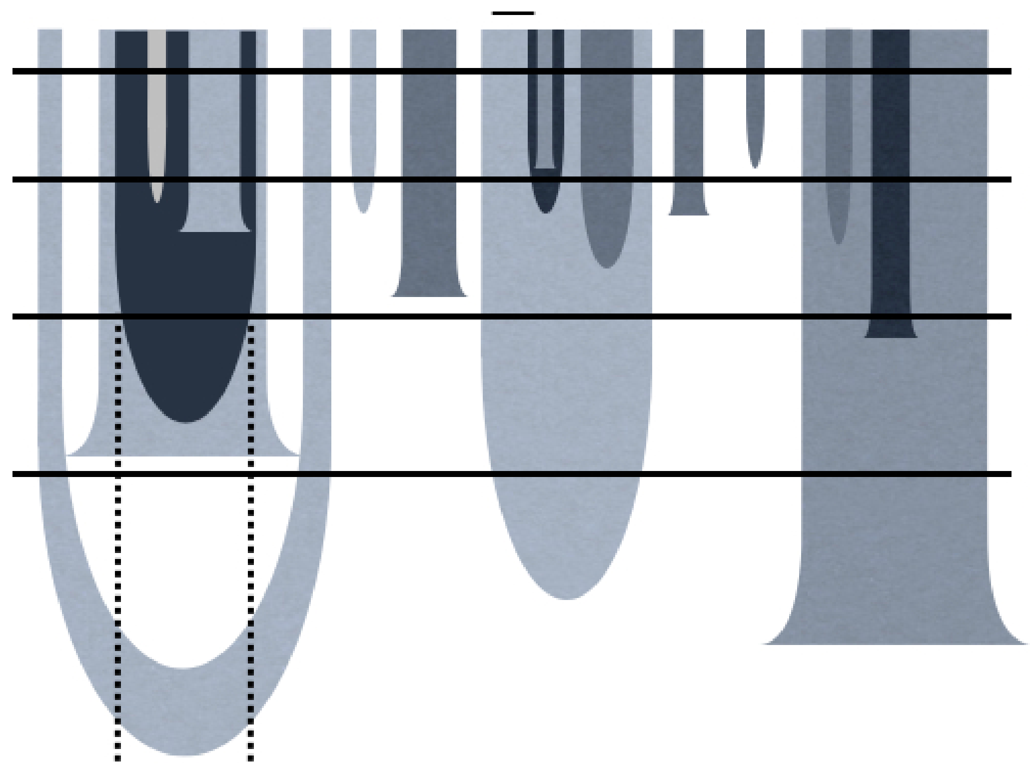
ting)

Observational obstacles



How do we calculate $p_p(o_i)$?

- Which is “correct”? How might we choose? Unclear!
- But:
  - Many arguably ruled out
  - Many turn out to be the same!
    - e.g., Worldline transition frequency measure from comoving volume.
    - e.g., Shadow-counting from worldline entries.
    - e.g. light-cone time cutoff and causal patch (Bousso & Katz ’12)
- Perhaps we are running out of ideas? (or not: ideas are $\aleph_2$)



Computing $n_{x,p}(o_i)$

- Suppose ***X*** = stable star w/ metallicity.
- Good news: It's hard, but w much do this (for *cosmolog* parameters).

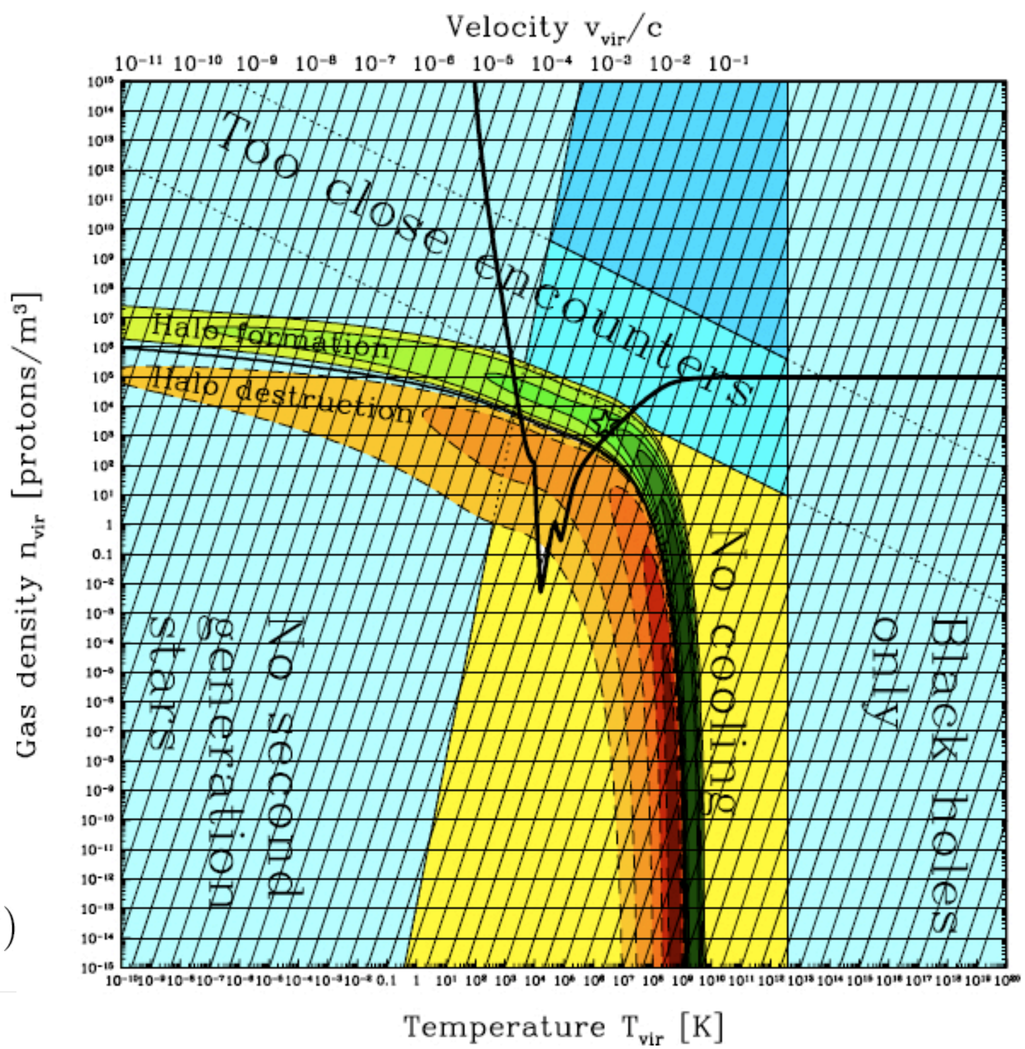
G_μ	Muon Yukawa coupling	0.000607
G_τ	Tauon Yukawa coupling	0.0102156233
G_u	Up quark Yukawa coupling	0.000016 ± 0.000007
G_d	Down quark Yukawa coupling	0.00003 ± 0.00002
G_c	Charm quark Yukawa coupling	0.0072 ± 0.0006
G_s	Strange quark Yukawa coupling	0.0006 ± 0.0002
G_t	Top quark Yukawa coupling	1.002 ± 0.029
G_b	Bottom quark Yukawa coupling	0.026 ± 0.003
$\sin\theta_{12}$	Quark CKM matrix angle	0.2243 ± 0.0016
$\sin\theta_{23}$	Quark CKM matrix angle	0.0413 ± 0.0015
$\sin\theta_{13}$	Quark CKM matrix angle	0.0037 ± 0.0005
δ_{13}	Quark CKM matrix phase	1.05 ± 0.24
$\theta_{\rm qcd}$	CP-violating QCD vacuum phase	$< 10^{-9}$
G_{ν_e}	Electron neutrino Yukawa coupling	$< 1.7 \times 10^{-11}$
G_{ν_μ}	Muon neutrino Yukawa coupling	$< 1.1 \times 10^{-6}$
G_{ν_τ}	Tau neutrino Yukawa coupling	< 0.10
$\sin\theta'_{12}$	Neutrino MNS matrix angle	0.55 ± 0.06
$\sin 2\theta'_{23}$	Neutrino MNS matrix angle	≥ 0.94
$\sin\theta'_{13}$	Neutrino MNS matrix angle	≤ 0.22
δ'_{13}	Neutrino MNS matrix phase	?
ρ_Λ	Dark energy density	$(1.25 \pm 0.25) \times 10^{-123}$
$\xi_{\rm b}$	Baryon mass per photon $\rho_{\rm b}/n_\gamma$	$(0.50 \pm 0.03) \times 10^{-28}$
$\xi_{\rm c}$	Cold dark matter mass per photon $\rho_{\rm c}/n_\gamma$	$(2.5 \pm 0.2) \times 10^{-28}$
ξ_ν	Neutrino mass per photon $\rho_\nu/n_\gamma = \frac{3}{11}\sum m_{\nu_i}$	$< 0.9 \times 10^{-28}$
Q	Scalar fluctuation amplitude δ_H on horizon	$(2.0 \pm 0.2) \times 10^{-5}$
$n_{\rm s}$	Scalar spectral index	0.98 ± 0.02
α_n	Running of spectral index $dn_{\rm s}/d\ln k$	$\lvert\alpha\rvert \lesssim 0.01$
r	Tensor-to-scalar ratio $(Q_t/Q)^2$	$\lesssim 0.36$
$n_{\rm t}$	Tensor spectral index	Unconstrained
w	Dark energy equation of state	-1 ± 0.1
κ	Dimensionless spatial curvature k/a^2T^2 [85]	$\lvert\kappa\rvert \lesssim 10^{-60}$

Tegmark, Aguirre, Rees & Wilczek 06

Computing $n_{x,p}(o_i)$

- Suppose ***X*** = stable star w/solar metallicity.
- Good news: It's hard, but we can hope to do this (for *cosmological* parameters).
- First step (Tegmark et al. 06): let *F*=fraction of dark matter collapsed into halos above virial temperature $T_{vir.}$

$$F = \mathrm{erfc}\left[\frac{A\left(\frac{m_\mathrm{p}n_\mathrm{vir}}{\xi_\mathrm{b}\xi^3Q^3}\right)^{1/3}}{(18\pi^2)^{1/3}\,s\left[\left(\frac{T_\mathrm{vir}}{m_\mathrm{p}Q}\right)^{3/2}\left(\frac{m_\mathrm{p}n_\mathrm{vir}}{\xi_\mathrm{b}\xi^3Q^3}\right)^{-1/2}\right]}\right]$$
$$= B\left(\frac{m_\mathrm{p}n_\mathrm{vir}}{\xi_\mathrm{b}\xi^3Q^3}, \frac{T_\mathrm{vir}}{m_\mathrm{p}Q}\right), \quad (32)$$



Computing $n_{x,p}(o_i)$

- Suppose ***X*** = stable star w/solar metallicity.
- Good news: It's hard, but we can pretty much do this (for *cosmological* parameters).
- Second step: try to convert halos into galaxies with stars.

Computing $n_{x,p}(o_i)$

- Suppose ***X*** = stable star w/solar metallicity.
- Good news: It's hard, but we can pretty much do this (for *cosmological* parameters).
- Second step: try to convert halos into galaxies with stars. Note:
    - Cutoff at high Λ from structure suppression.
    - Cutoffs at high density (encounters) and low density (cooling and metals), but soft.

Issues in computing $n_{x,p}(o_i)$

- Bad news: ("counterfactual cosmology conundrum"?)Many regions of parameters space may support ***X***s:
  - Degeneracies exist in cosmological parameters:

$$F(\mu, x) = \mathrm{erfc}\left[\frac{A(\rho_\Lambda/\rho_*)^{1/3}}{G_\Lambda(x)s(\mu)}\right], \quad \rho_* \equiv \xi^4 Q^3.$$

  - Qualitatively new physics can change simple reasoning, e.g.
    - "Cold Big-Bang" (AA 01)
    - "Weakless universe" (Harnik et al. 06)
  - Thus, even successful prediction may not survive when additional parameters are allowed to vary.
    - E.g., Weinberg argument falls apart if p_p for $\xi^4 Q^3$ is rising.
  - Bottom line: many anthropic "successes" are fragile and provisional -- we need to do the whole problem.

Summary

- Either infinitely *or finitely* many states creates uncomfortable issues.
- The infinite, statistically uniform spaces many claim are created be various cosmologies produce uncomfortable issues.
- The infinite production of different cosmologies with different properties creates uncomfortable issues.

We probably need to get comfortable with the discomfort --
we may well learn a lot!