The Architecture of Fundamental Physical Theories.

The Idea of Typicality

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I too have many reasons to believe that the present quantum theory, inspite of its many successes, is far from the truth. This theory reminds me a little of the system of delusion of an exceedingly intelligent paranoiac concocted of incoherent elements of thought. (Einstein, 1952; letter to Daniel Lipkin)

… conventional formulations of quantum theory, and of quantum field theory in particular, are unprofessionally vague and ambiguous. Professional theoretical physicists ought to be able to do better. Bohm has shown us a way. (John Stewart Bell)

The Point of Bohmian Mechanics

1. Quantum Mechanics

2. Quantum Properties

3. QTWO

4. The Measurement Problem

5. Ontology

6. Bohmian Mechanics

7. BM and the Problems with QM

Quantum Mechanics (Part 1)

- N-particle system $\leftrightarrow$ Hilbert space $\mathcal{H}$ $\quad [\, = L^2(\mathbb{R}^{3N})\,]$
- state $\leftrightarrow$ $\psi \in \mathcal{H}$ $\quad [\, \psi = \psi(q) = \psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)\,]$
- evolution $\leftrightarrow$ Schrödinger's equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi\,,$$

$$[\ H = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k}\Delta_k + V, \qquad \Delta_k = \nabla_k^2\ ]$$

Quantum Mechanics (Part 2)

"Measurement" Postulates

- Observables $\leftrightarrow$ self-adjoint operators A on $\mathcal{H}$

- measurement of $A \leftrightarrow$ spectral measures $\text{Prob}^{\psi}_{A}(da)$

$$E^{\psi}(A) = \langle \psi, A\psi \rangle$$

- measurement of $\underline{A} \leftrightarrow$ spectral measures $\text{Prob}^{\psi}_{\underline{A}}(d\underline{a})$

$\underline{A} = (A_1, \ldots, A_m), \quad [A_i, A_j] = 0$

$$\text{Prob}^{\psi}_{q}(dq) = |\psi(q)|^2$$

- Collapse of the wave function:

$A|\alpha\rangle = \alpha|\alpha\rangle$,

$$\psi = \sum_{\alpha} c_{\alpha}|\alpha\rangle$$

"Measure" A and find a (with probability $|c_a|^2$)

$\Rightarrow$

$$\psi \rightarrow |a\rangle$$

What's with the quotes?

"Measurement", "Measure"

Quantum Properties

NRAO: Naive Realism About Operators

A final moral concerns terminology. Why did such serious people take so seriously axioms which now seem so arbitrary? I suspect that they were misled by the pernicious misuse of the word ‘measurement’ in contemporary theory. This word very strongly suggests the ascertaining of some preexisting property of some thing, any instrument involved playing a purely passive role. Quantum experiments are just not like that, as we learned especially from Bohr. The results have to be regarded as the joint product of ‘system’ and ‘apparatus,’ the complete experimental set-up. But the misuse of the word ‘measurement’ makes it easy to forget this and then to expect that the ‘results of measurements’ should obey some simple logic in which the apparatus is not mentioned. The resulting difficulties soon show that any such logic is not ordinary logic. It is my impression that the whole vast subject of ‘Quantum Logic’ has arisen in this way from the misuse of a word. I am convinced that the word ‘measurement’ has now been so abused that the field would be significantly advanced by banning its use altogether, in favour for example of the word ‘experiment.’ (page 166)

QTWO

Quantum Theory Without Observers

The concept of ‘measurement’ becomes so fuzzy on reflection that it is quite surprising to have it appearing in physical theory at the most fundamental level. … [D]oes not any analysis of measurement require concepts more fundamental than measurement? And should not the fundamental theory be about these more fundamental concepts? (John Stewart Bell, 1981)

It would seem that the theory is exclusively concerned about "results of measurement", and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of "measurer"? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system... with a Ph.D.? If the theory is to apply to anything but highly idealized laboratory operations, are we not obliged to admit that more or less "measurement-like" processes are going on more or less all the time, more or less everywhere. Do we not have jumping then all the time? (John Stewart Bell, 1990)

The Measurement Problem

Does the wave function of a system provide a complete description of that system?

$$\Psi_{alive} + \Psi_{dead}$$

$$\Psi_{left} + \Psi_{right}$$

Ontology

What is missing?

- a clear ontology

- an adequate ontology

- that does the job (correct predictions, explaining observed facts)

Bohmian Mechanics

Bohmian Mechanics

$$\psi = \psi(\mathrm{q}_1, \ldots, \mathrm{q}_N)$$

$$Q: \qquad \mathrm{Q}_1, \ldots, \mathrm{Q}_N$$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi\,,$$

$$H = -\sum_{k=1}^{N}\frac{\hbar^2}{2m_k}\nabla_k^2 + V,$$

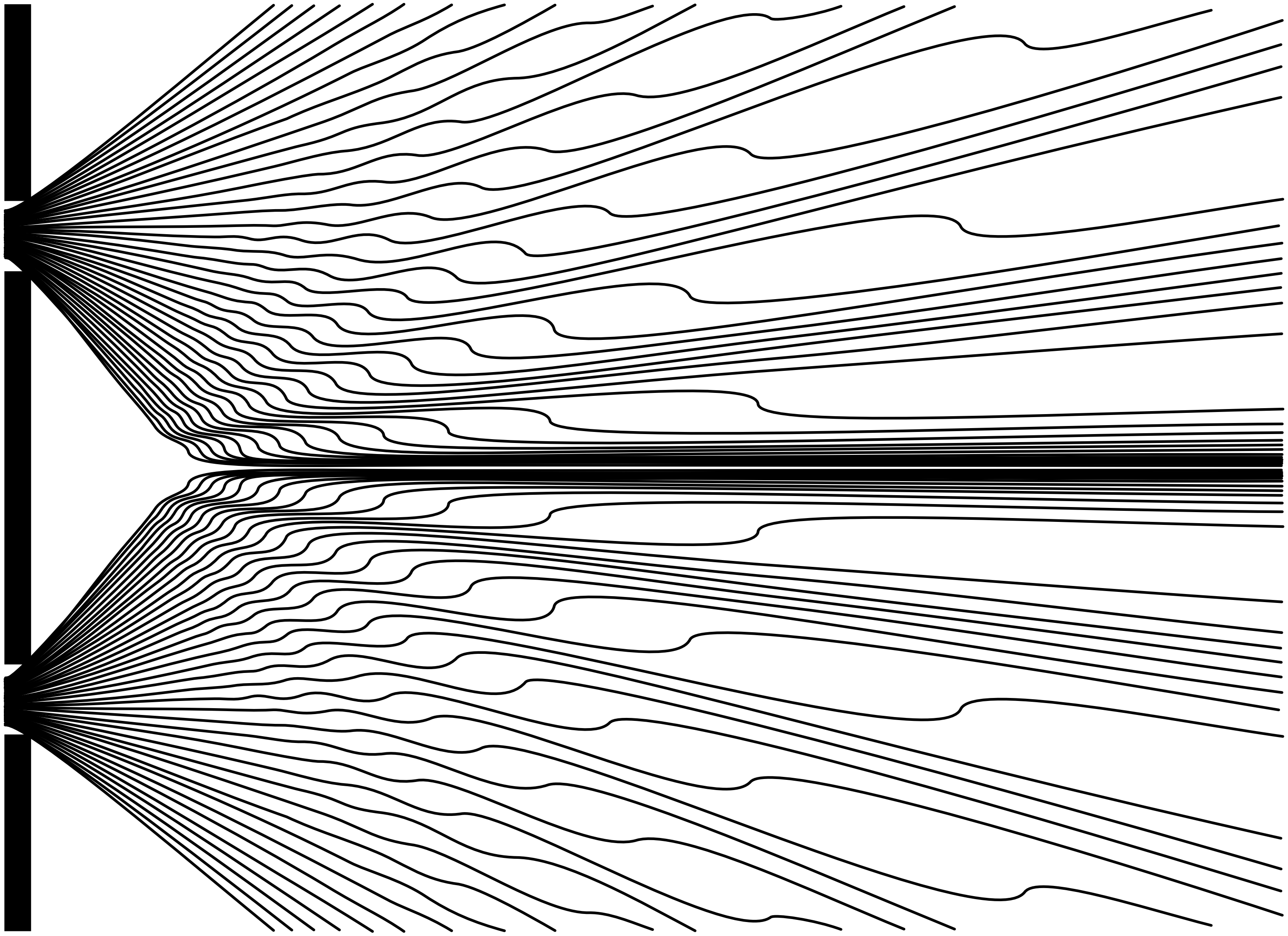
$$\frac{d\mathbf{Q}_k}{dt} = \frac{\hbar}{m_k}\mathrm{Im}\frac{\psi^*\nabla_k\psi}{\psi^*\psi}(\mathbf{Q}_1\ldots,\mathbf{Q}_N)$$

time evolution for ψ

$$p = \hbar k$$

time evolution for Q

$$dQ/dt = \nabla S/m$$



Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored. (John Stewart Bell, 1986)

Implications of Bohmian mechanics:

- familiar (macroscopic) reality
- quantum randomness
- absolute uncertainty
- operators as observables
- the wave function of a (sub)system
- collapse of the wave packet
- quantum nonlocality

BM $\leftrightarrow$ 2–4; QP, MP, QTWO

Bohmian Mechanics versus Bohmian Approach

- There is a clear primitive ontology (PO), and it describes matter in space and time.

- There is a state vector ψ in Hilbert space that evolves according to Schrödinger's equation.

- The state vector ψ governs the behavior of the PO by means of (possibly stochastic) laws.

- The theory provides a notion of a *typical* history of the PO (of the universe), for example by a probability distribution on the space of all possible histories; from this notion of typicality the probabilistic predictions emerge.

- The predicted probability distribution of the macroscopic configuration at time t determined by the PO agrees with that of the quantum formalism.

The Architecture of Fundamental Physical Theories

Ontology

Law

Adequate ontology?

For example:

a “decoration” of space-time

Can the ontology be too abstract: not involving something like local beables in more or less familiar space-time?

Note: The issue is not whether objects of very high abstraction can be a part of the ontology of a physical theory, but whether such objects can constitute the entire ontology.

Example: pure wave function ontology versus wave function and particles [Bohmian mechanics]

Levels of abstraction

- decoration of [quasi-familiar] space-time (particles, fields, etc.)
- decoration of very high dimensional space-time [regarded as the fundamental space] (David Albert)
- decoration of a completely abstract space
- no decoration but on space-time (operators on a Hilbert space of wave functions on configuration space)
- no decoration, no space-time (abstract operators or more general noncommutative objects)

classical ontology (BM, GRW, SL)

quantum ontology (CH?)

macroscopic ontology (?QTWO?)

Hilbert space ontology

Properties are associated with subspaces of Hilbert space, or with projection operators. (NRAO?)

Law

- differential equations (ordinary or partial)

- stochastic process / SDEs

- variational principle

- something else?

1. PO: Primitive ontology / local beables [Q]

2. $\mathscr{X}$: set of (kinematically possible) space-time histories of PO / decorations of space-time [$Q(t)$]

3. $\mathscr{L} \subset \mathscr{X}$: Law for space-time history / (additional) theoretical entities [$Q(t)$ / Ψ]

4. Typical space-time history ($\in \mathscr{L}$) / P on $\mathscr{L}$ [$|\Psi_0|^2$]

For a stochastic law there may be no separate $\mathscr{L}$. Rather

$$P \text{ on } \mathscr{L}$$

would be replaced by

$$P \text{ on } \mathscr{X}$$

Typicality

- History
- Statistical mechanics
- Roles of probability
- Typicality beyond probability
- The method of appeal to typicality

probability

chance

likelihood

distribution

measure

Notions of probability

Subjective chance (Bayesian?)

Objective chance (propensity?)

Relative frequency, empirical (pattern)

A mathematical structure providing a measure of the size of sets (Kolmogorov)

Typicality

One should not forget that the Maxwell distribution is not a state in which each molecule has a definite position and velocity, and which is thereby attained when the position and velocity of each molecule approach these definite values asymptotically. . . . It is in no way a special singular distribution which is to be contrasted to infinitely many more non-Maxwellian distributions; rather it is characterized by the fact that by far the largest number of possible velocity distributions have the characteristic properties of the Maxwell distribution, and compared to these there are only a relatively small number of possible distributions that deviate significantly from Maxwell's. Whereas Zermelo says that the number of states that finally lead to the Maxwellian state is small compared to all possible states, I assert on the contrary that by far the largest number of possible states are "Maxwellian" and that the number that deviate from the Maxwellian state is vanishingly small. (Boltzmann)

Ancient History (< 1950)

Glen Shafer, "Why did Cournot's principle disappear?" and "The Sources of Kolmogorov's *Grundbegriffe*"

Jakob Bernoulli, Ars Conjectandi (1713): "Because it is only rarely possible to obtain full certainty, necessity and custom demand that what is merely morally certain be taken as certain."

Antoine Cournot (1843): "A physically impossible event is one whose probability is infinitely small. This remark alone gives substance—an objective and phenomenological value—to the mathematical theory of probability." (Cournot's principle)

Paul Levy ($\approx$1919), Cournot's principle is the only connection between probability and the empirical world

The principle of the very unlikely event (Levy); The principle of the negligible event (Hadamard)

Kolmogorov, Foundations (1933), Chapter 1, §2, The Relation to Experimental Data: Only Cournot's principle connects the mathematical formalism with the real world.

Borel ($\approx$1948): The principle that an event with very small probability will not happen is the only law of chance.

Modern History (> 1950)

In order to establish quantitative results, we must put some sort of measure (weighting) on the elements of a final superposition. This is necessary to be able to make assertions which hold for almost all of the observer states described by elements of the superposition. We wish to make quantitative statements about the relative frequencies of the different possible results of observation—which are recorded in the memory—for a typical observer state; but to accomplish this we must have a method for selecting a typical element from a superposition of orthogonal states. (Everett 1957)

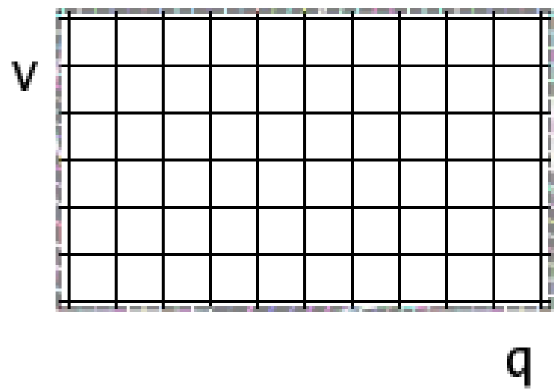
The situation here is fully analogous to that of classical statistical mechanics, where one puts a measure on trajectories of systems in the phase space by placing a measure on the phase space itself, and then making assertions . . . which hold for "almost all" trajectories. This notion of "almost all" depends here also upon the choice of measure, which is in this case taken to be the Lebesgue measure on the phase space. . . . Nevertheless the choice of Lebesgue measure on the phase space can be justified by the fact that it is the only choice for which the "conservation of probability" holds, (Liouville's theorem) and hence the only choice which makes possible any reasonable statistical deductions at all. (Everett 1957)

Then for instantaneous macroscopic configurations the pilot-wave theory gives the same distribution as the orthodox theory, insofar as the latter is unambiguous. However, this question arises: what is the good of *either* theory, giving distributions over a hypothetical ensemble (of worlds!) when we have only one world. (Bell 1981)

. . . a single configuration of the world will show statistical distributions over its different parts. Suppose, for example, this world contains an actual ensemble of similar experimental set-ups. . . . it follows from the theory that the 'typical' world will approximately realize quantum mechanical distributions over such approximately independent components. The role of the hypothetical ensemble is precisely to permit definition of the word 'typical.' (Bell 1981)

Then there is the surprising contention of Everett and De Witt that the theory ‘yields its own interpretation’. The hard core of this seems to be the assertion that the probability interpretation emerges without being assumed. In so far as this is true it is true also in the pilot-wave theory. In that theory our unique world is supposed to evolve in deterministic fashion from some definite initial state. However, to identify which features are details crucially dependent on the initial conditions (like whether the first scattering is up or down in an α track) and which features are more general (like the distribution of scattering angles over the track as a whole) it seems necessary to envisage a comparison class. This class we took to be a hypothetical ensemble of initial configurations with distribution $|\psi|^2$. In the same way Everett has to attach weights to the different branches of his multiple universe, and in the same way does so in proportion to the norms of the relevant parts of the wave function. Everett and De Witt seem to regard this choice as inevitable. I am unable to see why, although of course it is a perfectly reasonable choice with several nice properties. (Bell 1981)

1872: $X = (\mathbf{q}_1, \mathbf{v}_1, \ldots, \mathbf{q}_N, \mathbf{v}_N)$



$$f_{emp}(\mathbf{q}, \mathbf{v}) \equiv f_X(\mathbf{q}, \mathbf{v})$$

$$f_X(\mathbf{q},\mathbf{v}) = \frac{|X \cap \mathbf{\Delta}(\mathbf{q},\mathbf{v})|}{|\mathbf{\Delta}(\mathbf{q},\mathbf{v})|N}$$

$$= \frac{n_X(\mathbf{\Delta}(\mathbf{q},\mathbf{v}))/N}{|\mathbf{\Delta}(\mathbf{q},\mathbf{v})|}$$

$$f_{emp}(\mathbf{q},\mathbf{v},t) \equiv f_{X(t)}(\mathbf{q},\mathbf{v})$$

At low density

$$f_{X(t)}(\mathbf{q}, \mathbf{v}) \approx f_t(\mathbf{q}, \mathbf{v})$$

where f_t obeys Boltzmann's equation

$$\frac{\partial f_t}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{q}} f_t = Q(f_t)$$

$$f_t(\mathbf{q}, \mathbf{v}) \to f_{eq}(\mathbf{q}, \mathbf{v}) \sim e^{-\frac{1}{2} m \mathbf{v}^2 / kT}$$

$$H(f_t) = \int f_t(\mathbf{q}, \mathbf{v}) \log f_t(\mathbf{q}, \mathbf{v}) \mathbf{dqdv}$$

$$S(X) = -NH(f_X)$$

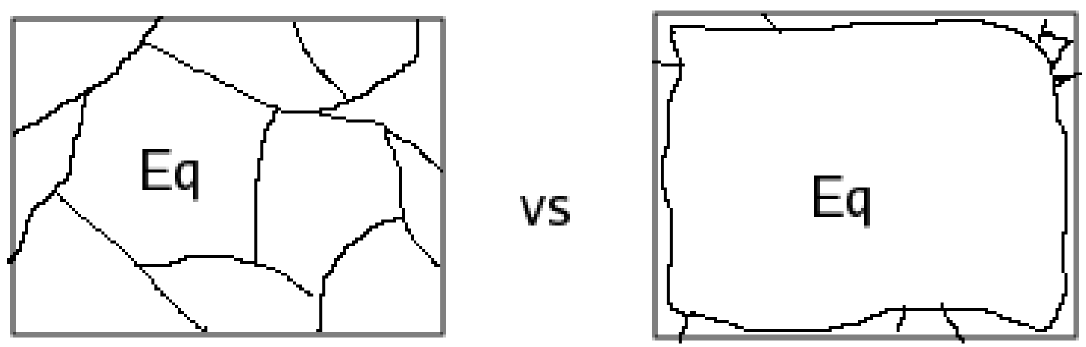
1877: Macrostates

$$\Gamma_f = \{X \in \Gamma_E \,|\, f_X(\mathbf{q},\mathbf{v}) \approx f(\mathbf{q},\mathbf{v})\}$$

1877↔1872: At low density

$$|\Gamma_f| \sim e^{-NH(f)}$$

$N \sim 10^{20}$: most of Γ_E is $\Gamma_{f_{eq}}$



Γ_E $\qquad\qquad$ Γ_E

$$\log |\Gamma_f| = -NH(f)$$

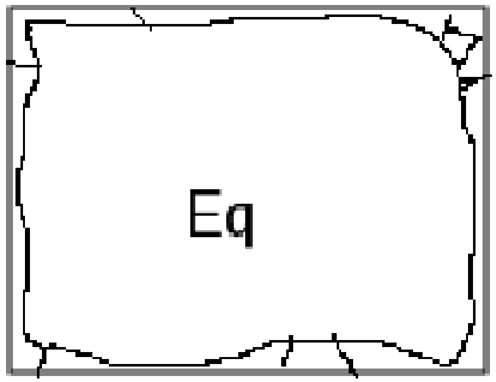
$$S(X) = \log |\Gamma_{f_X}|$$

($S(X) = S_G(\varrho)$ when ϱ is uniform on Γ_{f_X}.)

Smallness of Atypical Events

$$10^{-10^{20}} \text{ (for } N = 10^{20}\text{)}$$

$\Gamma_{f_{eq}}$ $\qquad$ f_{eq}



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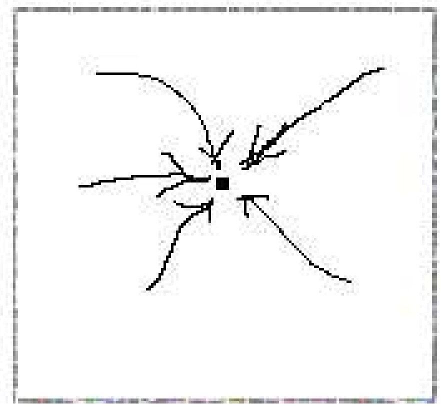


$X \in \Gamma_E \quad f_X \quad f - \text{space}$

microscopic picture $\qquad$ macroscopic picture

Some Consequences of Typicality

Maxwellian velocities

Boltzmann's equation (Lanford)

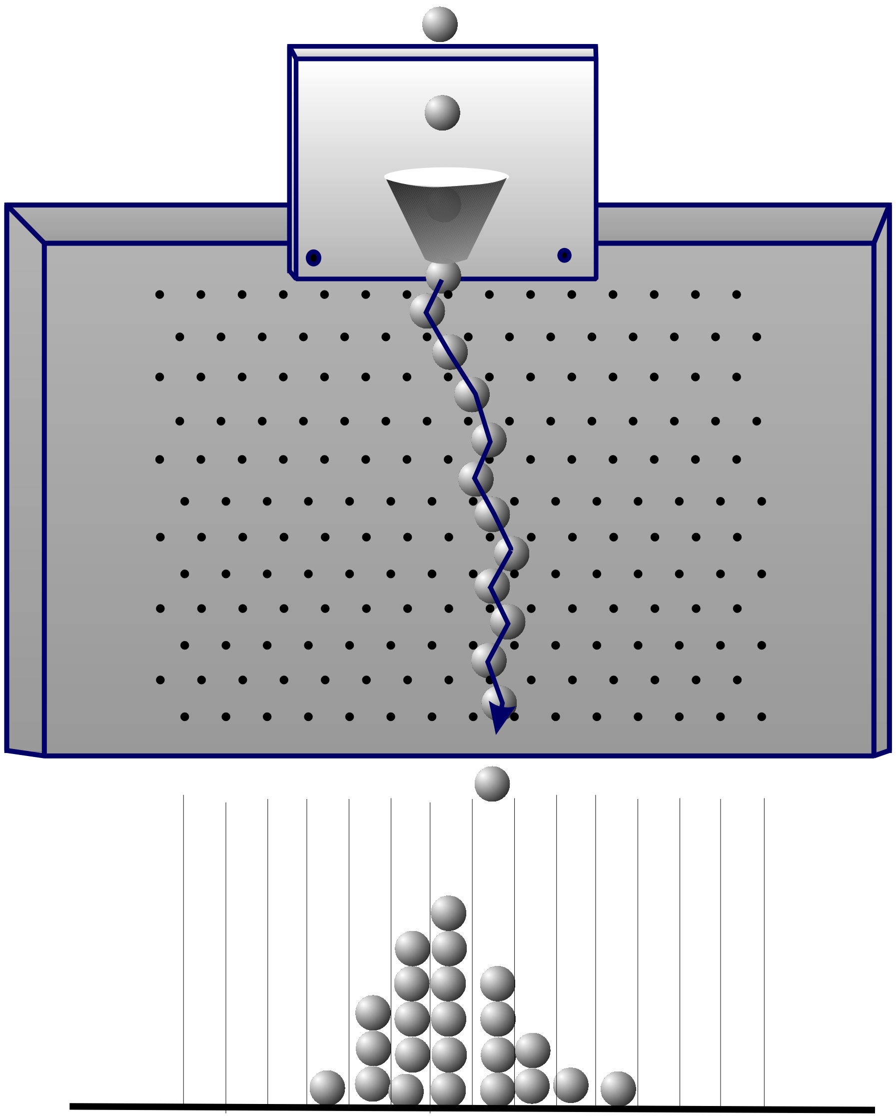
Second law of thermodynamics

Hydrodynamic equations

Canonical ensemble for quantum mechanics (Canonical typicality)

Quantum randomness (in Bohmian mechanics)

Approach to equilibrium in quantum mechanics



Relevant Roles of Probability

- ρ_{emp}: Empirical distribution (relative frequency)

- ρ_{th}: Theoretical distribution (idealization, $N \to \infty$)

- P: Measure for typicality

$$\rho_{emp}(\mathbf{x}) \approx \rho_{th}(\mathbf{x})$$

Common situation:

$$\rho_{th}(\mathbf{x})d\mathbf{x} = \rho^P(\mathbf{x})d\mathbf{x} = \text{``}P(\mathbf{X} \in d\mathbf{x})\text{''}$$

$$\rho_{emp} \leftrightarrow \rho^{P} \leftrightarrow P$$

$$\rho_{emp} \leftrightarrow \rho_{th}^{\psi} \leftrightarrow P^{\Psi}$$

Good (simplicity)

Bad (misleading simplicity)

ψ versus Ψ

Struyve and G (2007):

Quantum equilibrium is “unique.”

Typicality not given by probability:

Murray Gell-Mann and James Hartle, decoherent histories

Bruno Galvan, "trajectory-based formulation of quantum mechanics only based on the standard formalism of quantum mechanics"

Rafael Sorkin and Fay Dowker, co-event formulation of quantum mechanics

Remark on typicality (with J. L. Lebowitz, R. Tumulka, and N. Zanghì, The European Physical Journal H: Historical Perspectives on Contemporary Physics 35, 173-200 (2010), arXiv:1003.2129v1)

When employing the method of appeal to typicality, one usually uses the language of probability theory. But that does not imply that any of the objects considered is random in reality. Rather, it means that certain sets (of wave functions, of orthonormal bases, etc.) have certain sizes (e.g., close to 1) in terms of certain natural (normalized) measures of size. That is, one describes the behavior that is *typical* of wave functions, orthonormal bases, etc.. However, since the mathematics is equivalent to that of probability theory, it is convenient to adopt that language. For this reason, using a normalized measure μ does not mean making an "assumption of equal probability," even if one uses the word "probability." Rather, it means that, if a condition is true of most . . . , or most H, this fact may suggest that the condition is also true of a concrete given system, unless we have reasons to expect otherwise.

Of course, a theorem saying that a condition is true of the vast majority of systems does not *prove* anything about a concrete given system; if we want to know for sure whether a given system is normal for every initial wave function, we need to check the relevant condition . . . . Nevertheless, a typicality theorem is, as we have suggested, illuminating; at the very least, it is certainly useful to know which behaviour is typical and which is exceptional. . . .

The method of appeal to typicality belongs to a long tradition in physics, which includes also Wigner's work on random matrices of the 1950s. In the words of Wigner . . . :

> One [. . .] deals with a specific system, with its proper (though in many cases unknown) Hamiltonian, yet pretends that one deals with a multitude of systems, all with their own Hamiltonians, and averages over the properties of these systems. Evidently, such a procedure can be meaningful only if it turns out that the properties in which one is interested are the same for the vast majority of the admissible Hamiltonians.

This method was used by Wigner to obtain specific new and surprising predictions about detailed properties of complex quantum systems in nuclear physics.

If we know of a given system that its Hamiltonian H belongs to a particular small subset S_0 of the set S of all self-adjoint operators on the appropriate Hilbert space, then two kinds of typicality theorems are of interest: one saying that the relevant behavior occurs for most H in S_0, the other saying that it occurs for most H in S. Note that the former does not follow from the latter when S_0 is very small compared to S, as it would then be consistent with the latter for S_0 to consist exclusively of exceptional Hs. Nor does the latter follow from the former, so the two statements are logically independent. In fact, both are of interest because each statement has its merits: The typicality theorem about S_0 gives us more certainty that the given system, whose Hamiltonian belongs to S_0, will behave in the relevant way. The typicality theorem about S gives us a deeper understanding of why the relevant behavior occurs, as it indicates that the behavior has not much to do with S_0 but is widespread all over S. That is, there is a reciprocal relation: The greater the degree of certainty that a typicality theorem confers, the less its explanatory power.