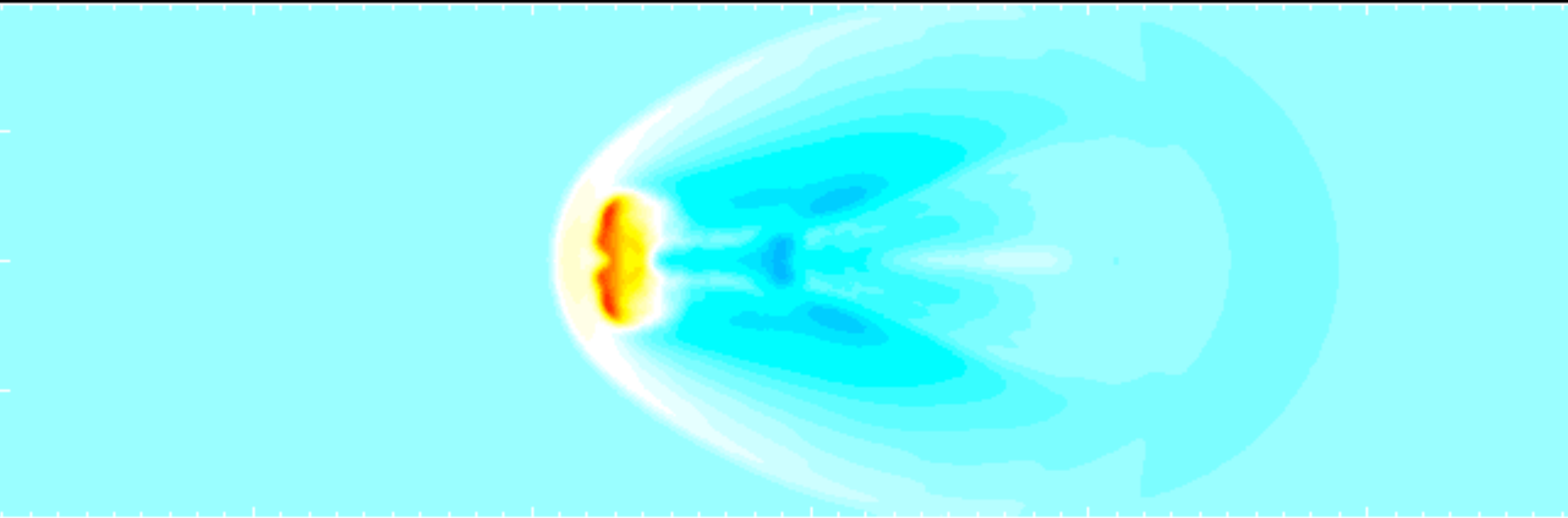
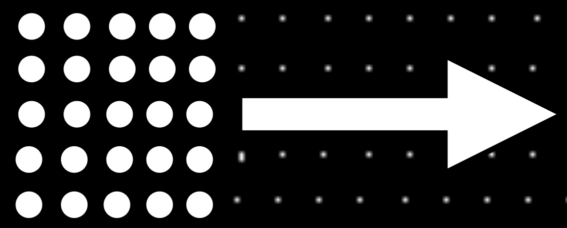
Galaxy formation in SPHS

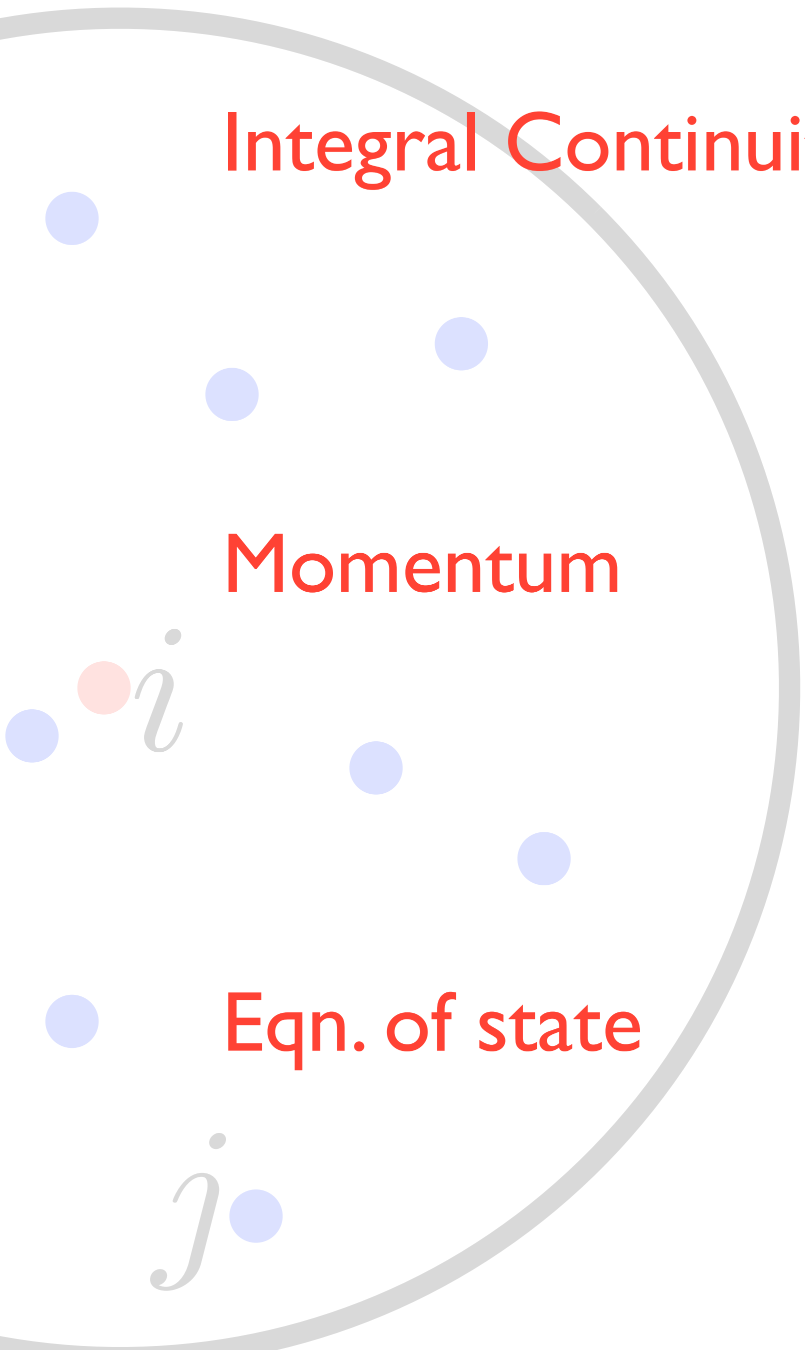


Justin Read
ETH Zürich | University of Leicester

With: T. Hayfield, A. Hobbs, C. Power



Background | The Euler equations (Lagrangian 'entropy' form)



Integral Continuity

$$\rho_i = \sum_j^N m_j W_{ij}(|\mathbf{r}_{ij}|, h_i)$$

Momentum

$$\frac{d\mathbf{v}_i}{dt} = \sum_j^N m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i \overline{W}_{ij}$$

Eqn. of state

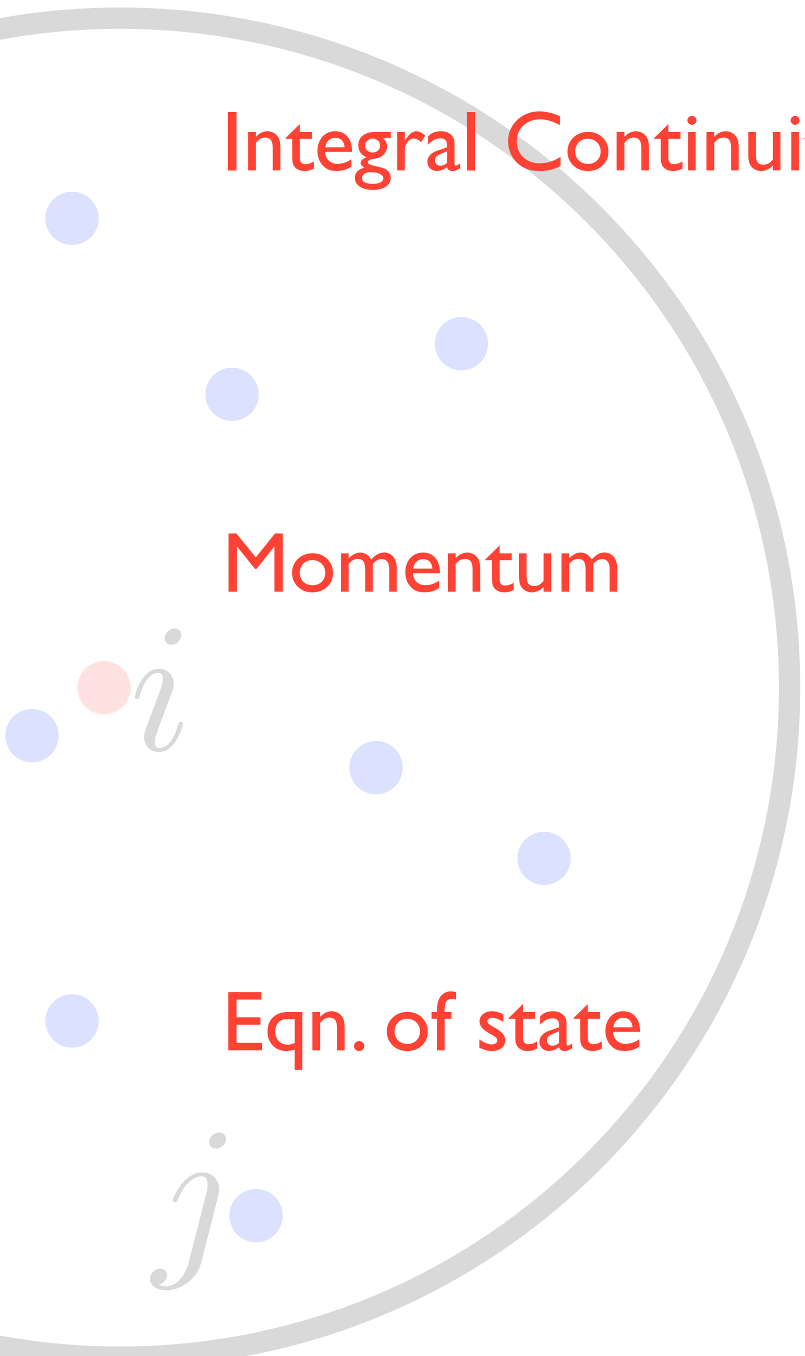
$$P_i = A_i \rho_i^\gamma \qquad ; A_i = \mathrm{const.}$$

'classic' SPH

[inc. 'energy' form and similar]

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

Background | The Euler equations (Lagrangian ‘entropy’ form)



Integral Continuity

$$\rho_i = \sum_j^N m_j W_{ij}(|\mathbf{r}_{ij}|, h_i)$$

Momentum

$$\frac{d\mathbf{v}_i}{dt} = \sum_j^N \frac{m_j}{\rho_i \rho_j}(P_i + P_j)\nabla_i \overline{W}_{ij}$$

Eqn. of state

$$P_i = A_i \rho_i^\gamma \qquad ; A_i = \text{const.}$$

Improved force error

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

1. Lagrangian

2. Galilean invariant

3. Manifestly conservative

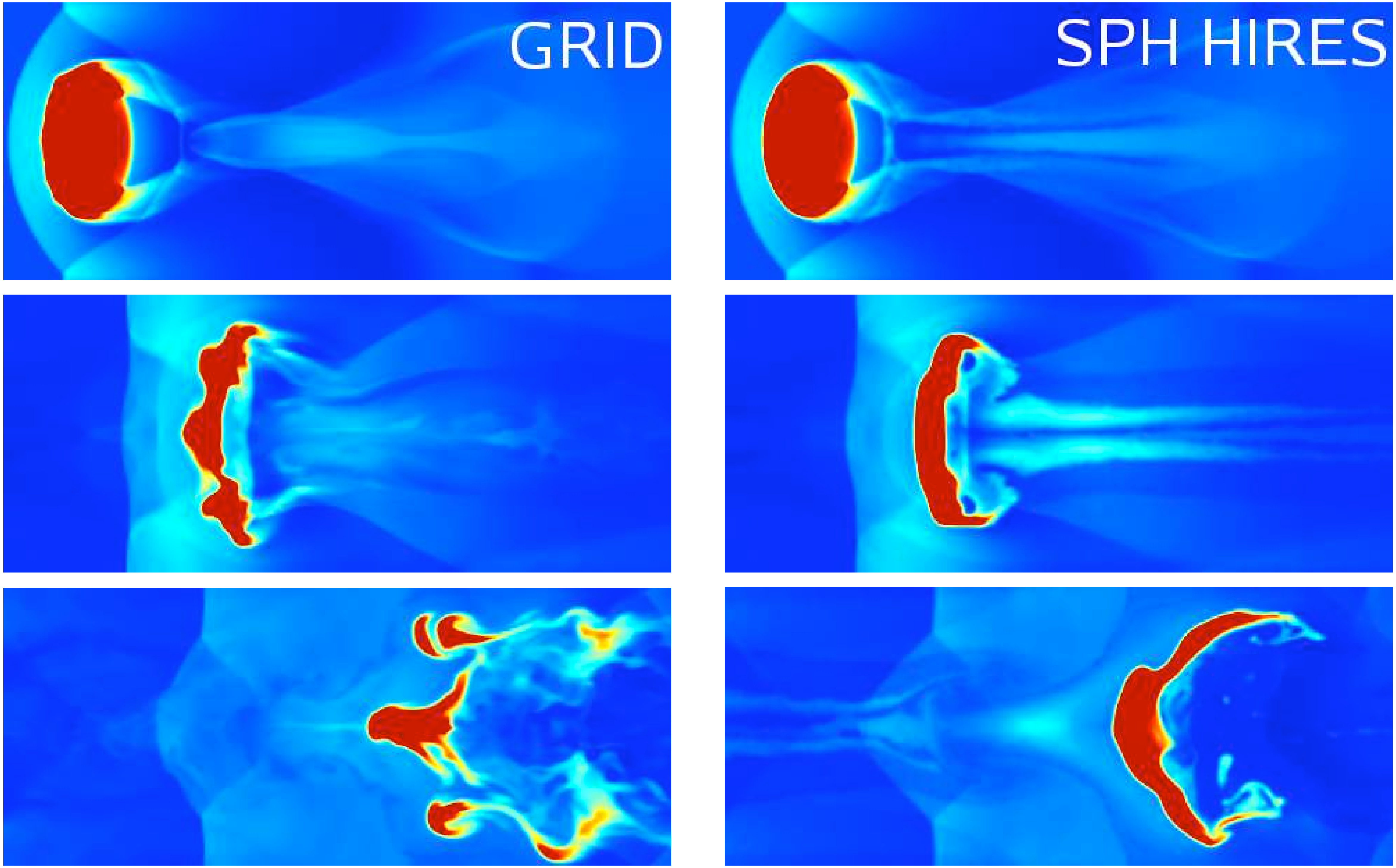
4. Easy to implement

5. Couples to O(N) FMM gravity

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

Background | Some problems with 'classic' SPH

The "blob test"



A 1:10 density ratio gas sphere in a wind tunnel (Mach 2.7), initially in pressure eq.

Agertz et al. 2007

Background | Some problems with 'classic' SPH

1. The 'E0' error
2. Multivalued pressures
3. Overly viscous
4. Noisy

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

1. The 'E0 error' | Taylor expanding the momentum equation

Momentum
$$\frac{d\mathbf{v}_i}{dt} = \sum_j^N \frac{m_j}{\rho_i \rho_j} (P_i + P_j) \nabla_i \overline{W}_{ij}$$

$$P_j \simeq P_i + h\underline{x}_{ij} \cdot \underline{\nabla} P_i + O(h^2)$$

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

1. The 'E0 error' | Taylor expanding the momentum equation

Momentum
$$\frac{d\mathbf{v}_i}{dt} = \sum_j^N \frac{m_j}{\rho_i \rho_j} \left(P_i + P_j\right) \nabla_i \overline{W}_{ij}$$

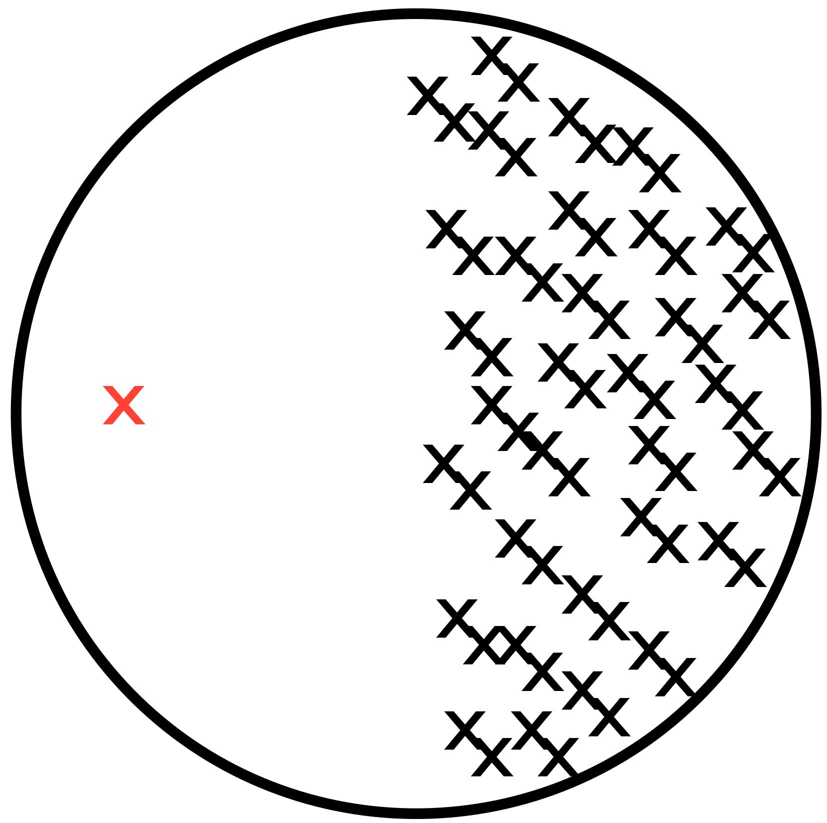
$$\frac{d\mathbf{v}_i}{dt} \simeq -\frac{P_i}{h_i \rho_i} \boxed{2\sum_j^N \frac{m_j}{\rho_j} \nabla_i^x \overline{W}_{ij}} \boxed{-\frac{\mathbf{M_i} \nabla_i P_i}{\rho_i}} + O(h)$$

E_0 $\Rightarrow$ Euler eqn.

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

1. The 'E0 error' | Minimising E0 - raising the kernel sampling

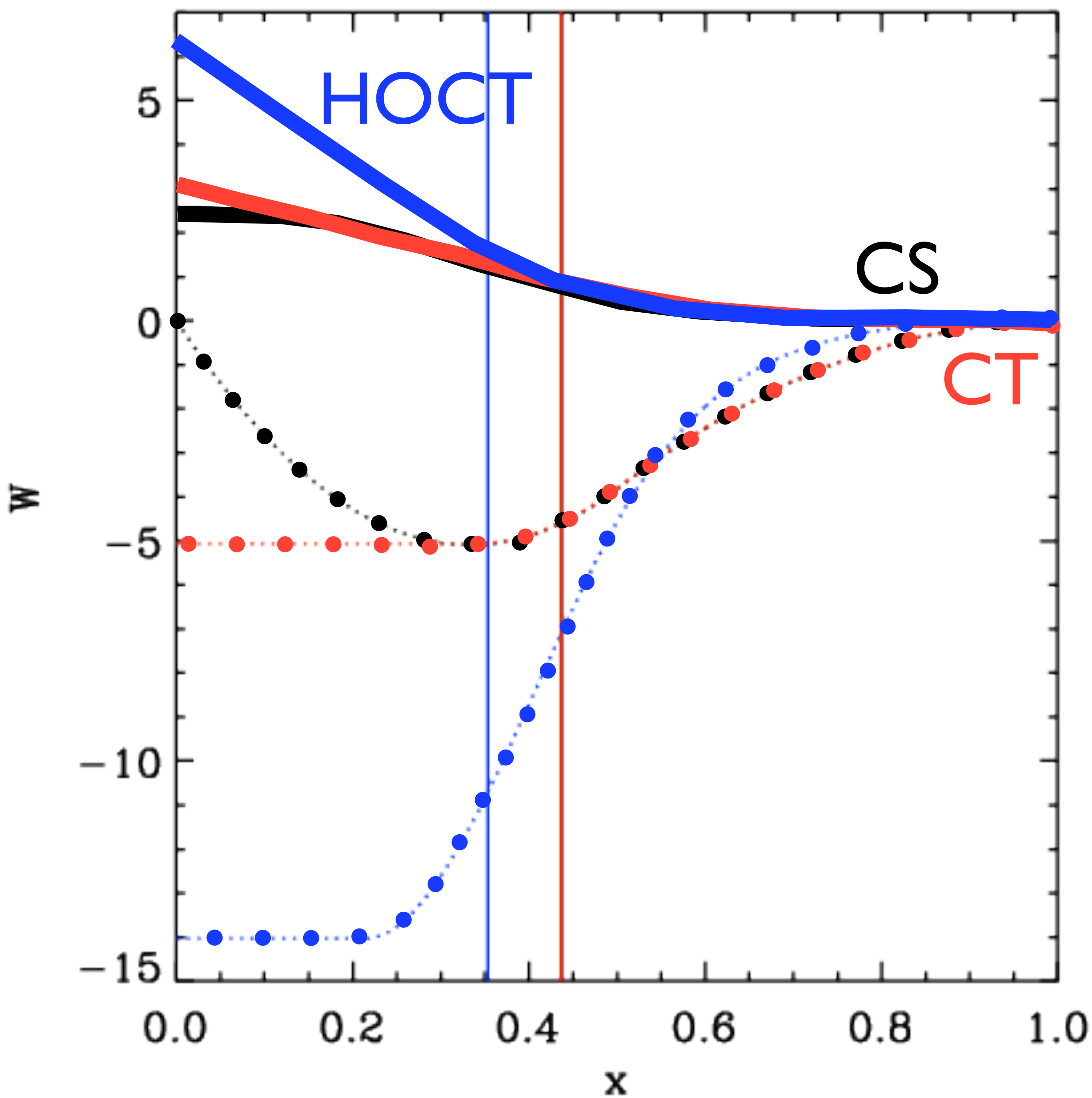
$$\mathbf{E}_0 = 2\sum_j^N \frac{m_j}{\rho_j}\nabla_i^x \overline{W}_{ij} \simeq 2\int_V dV \nabla^x W$$



1. Smooth on kernel scale (stable kernel)
2. Larger neighbour number
3. More power in kernel wings

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

1. The 'E0 error' | Minimising E0 - raising the kernel sampling



Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

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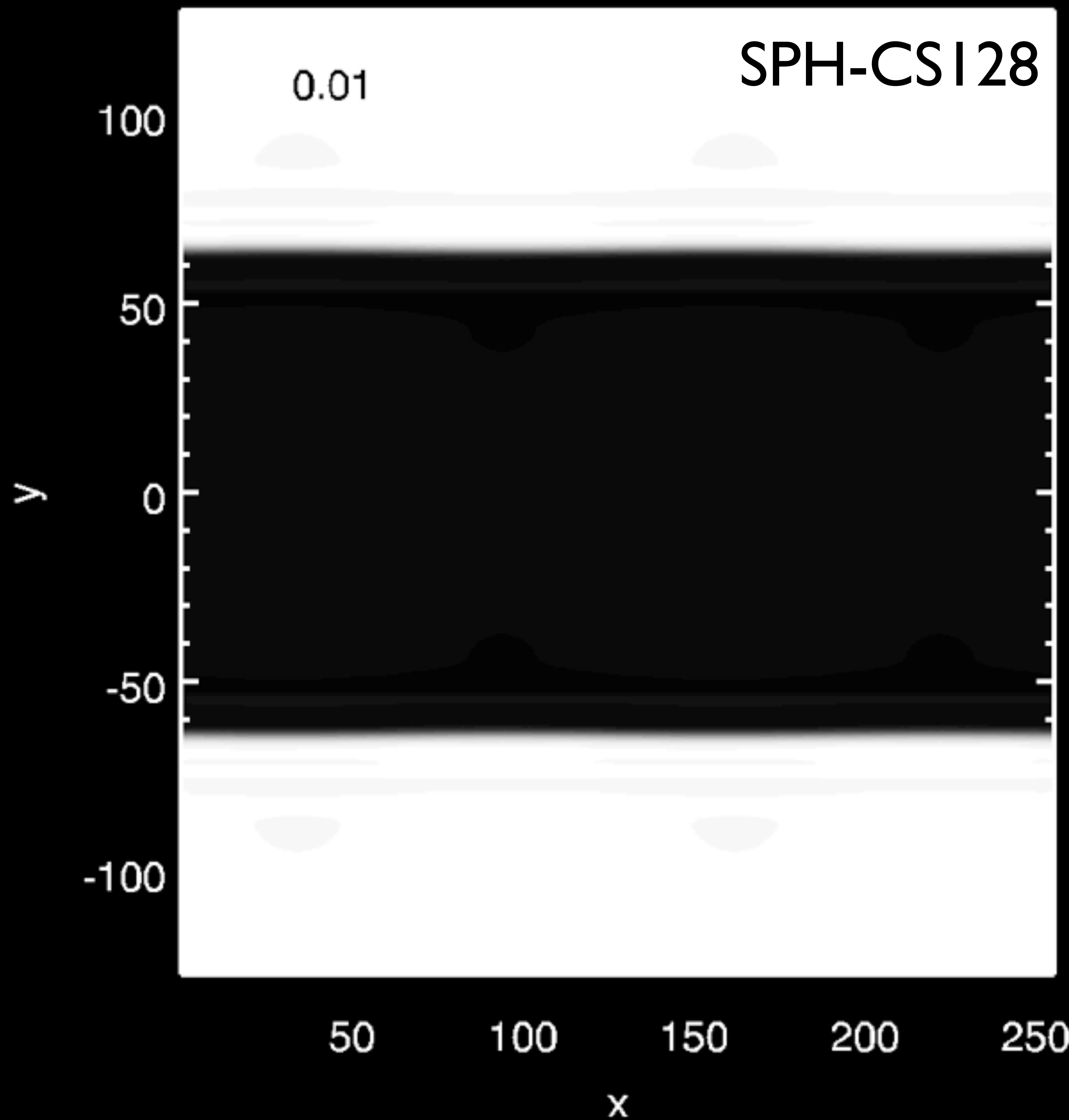

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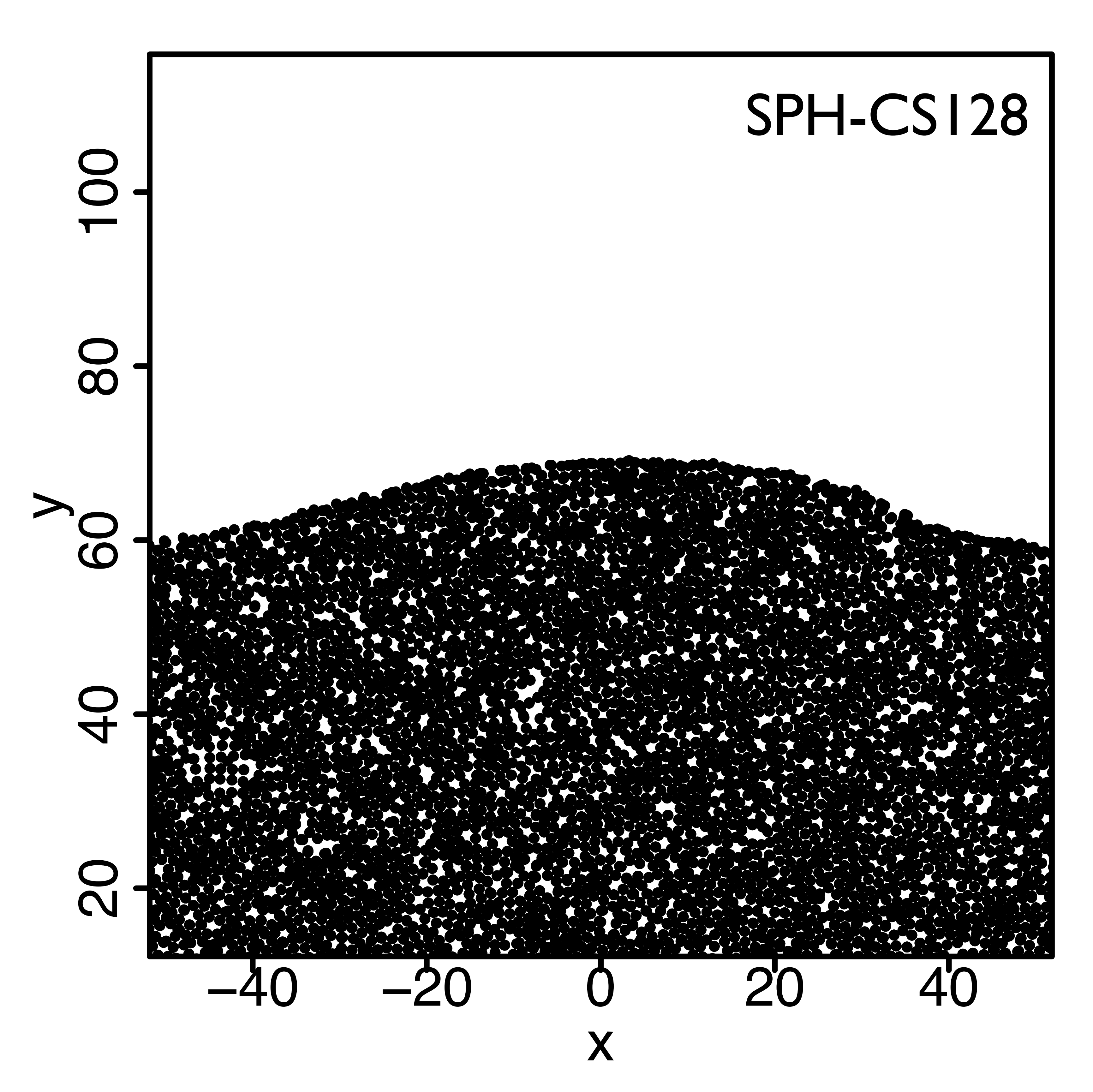

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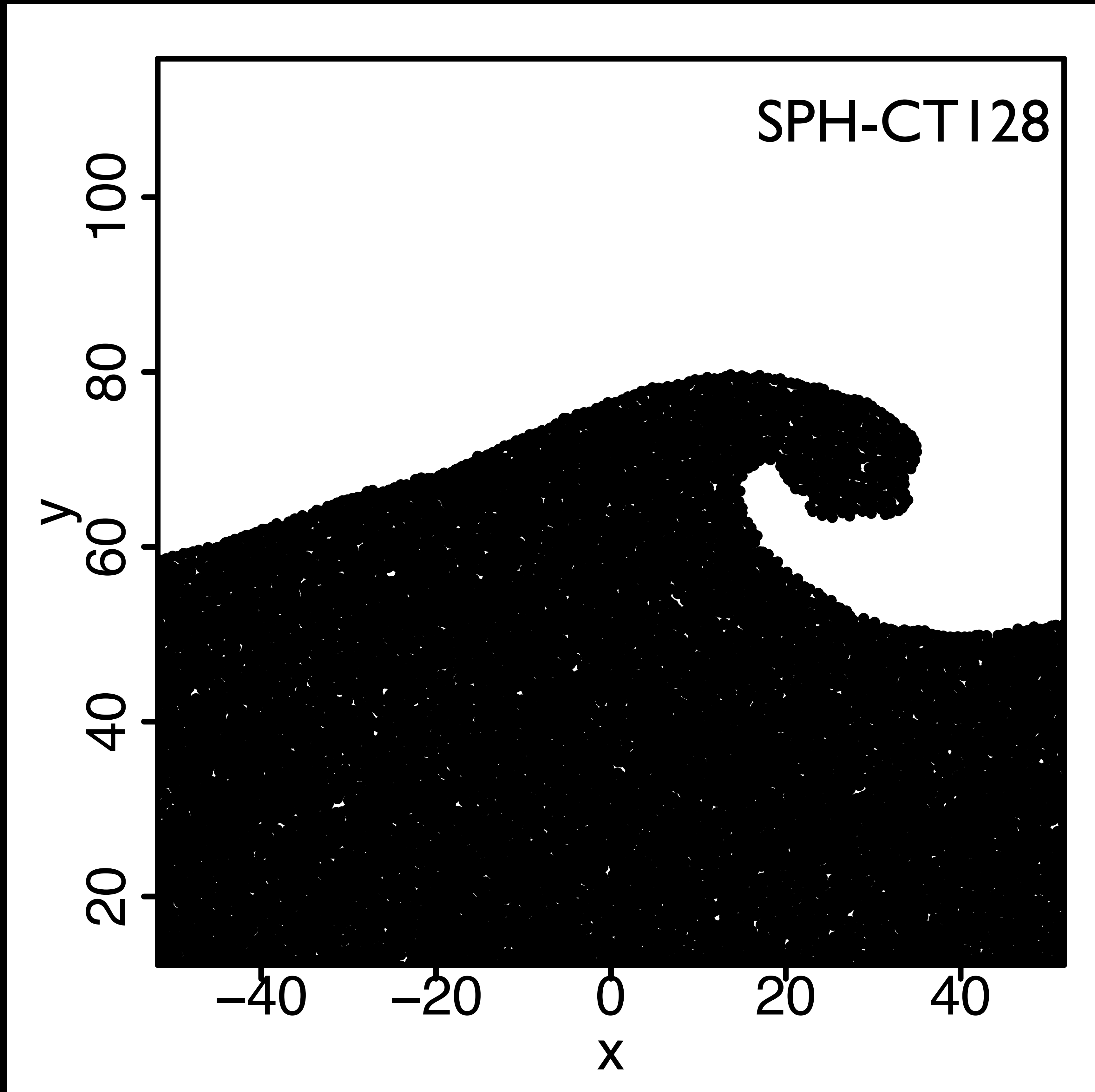

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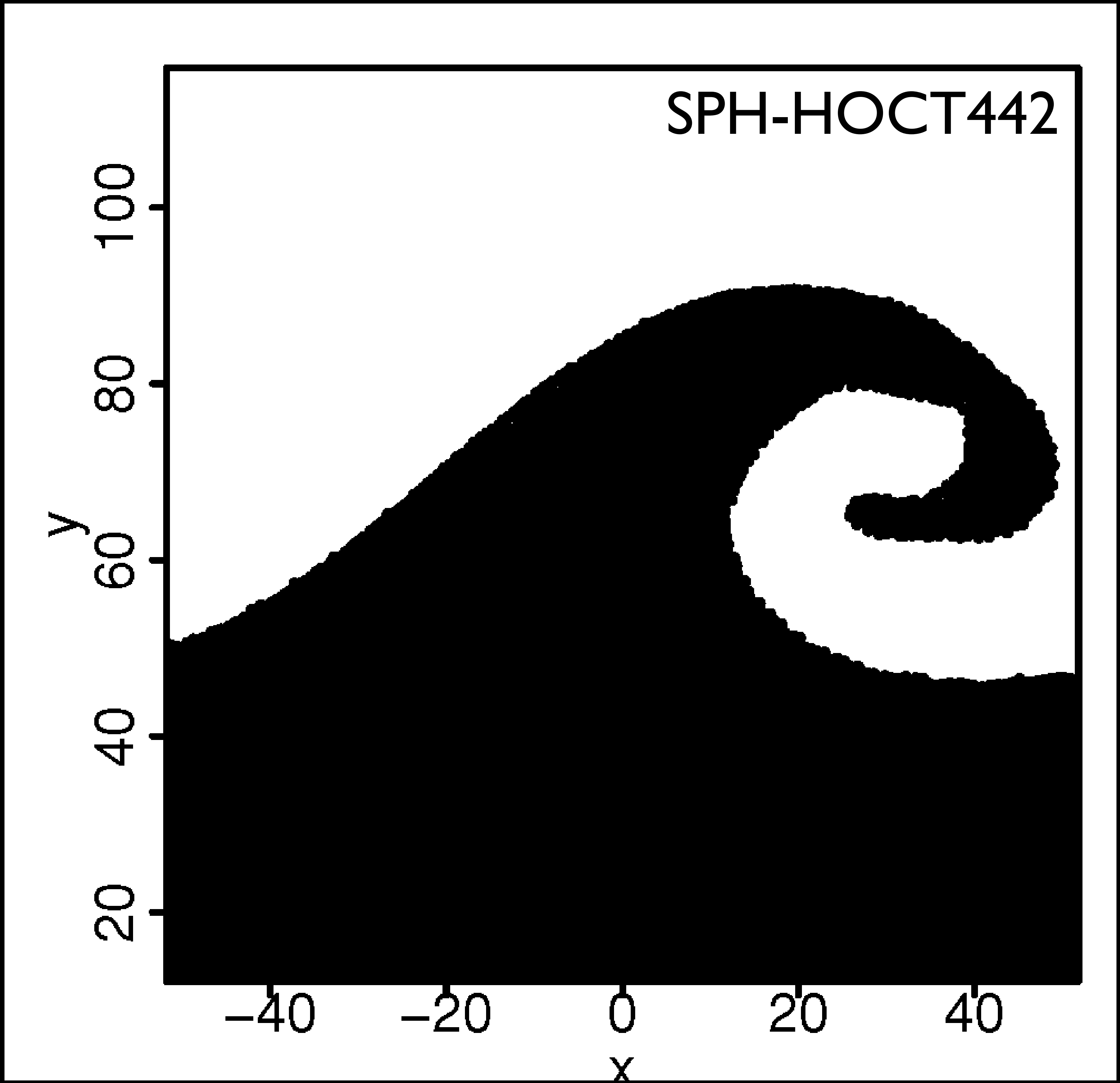

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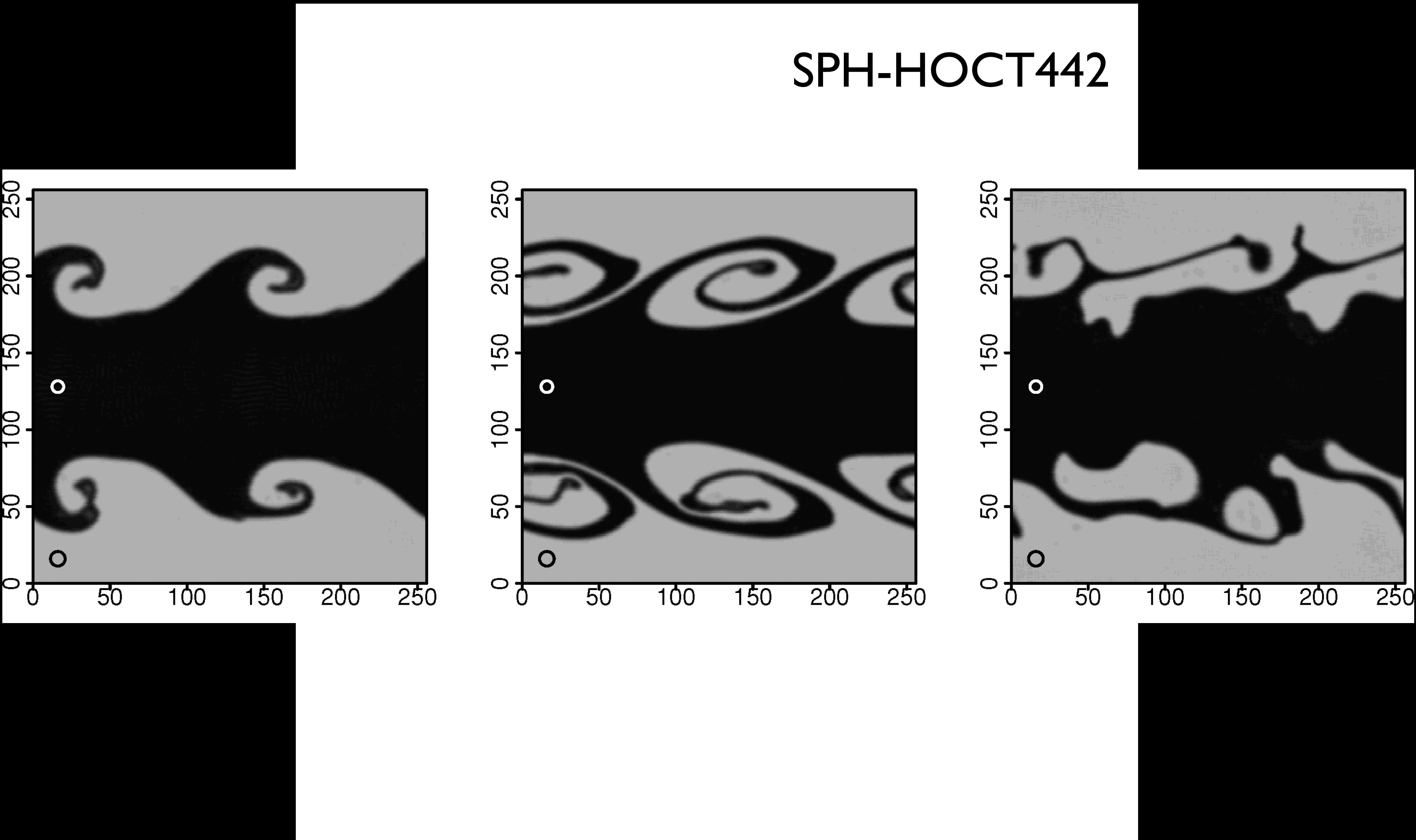
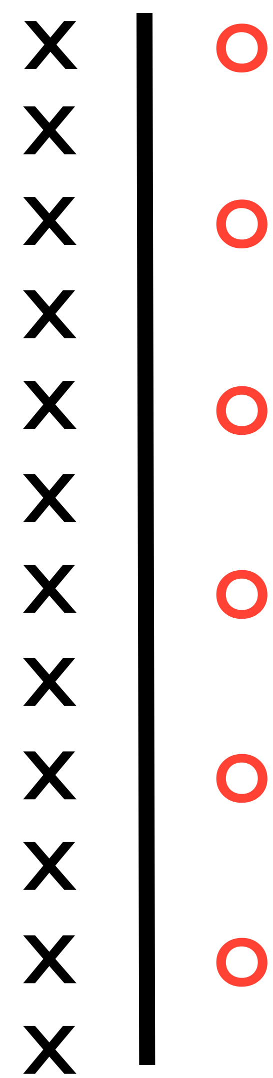


Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

2. Multivalued pressures | The problem

$$P_1 = A_1 \rho_1^\gamma$$



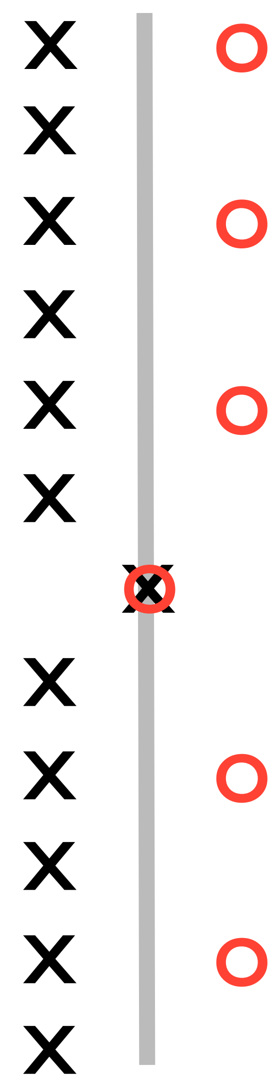
$$P_2 = A_2 \rho_2^\gamma$$

$$P_1 = P_2$$

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

2. Multivalued pressures | The problem

$$P_1 = A_1 \rho^\gamma$$

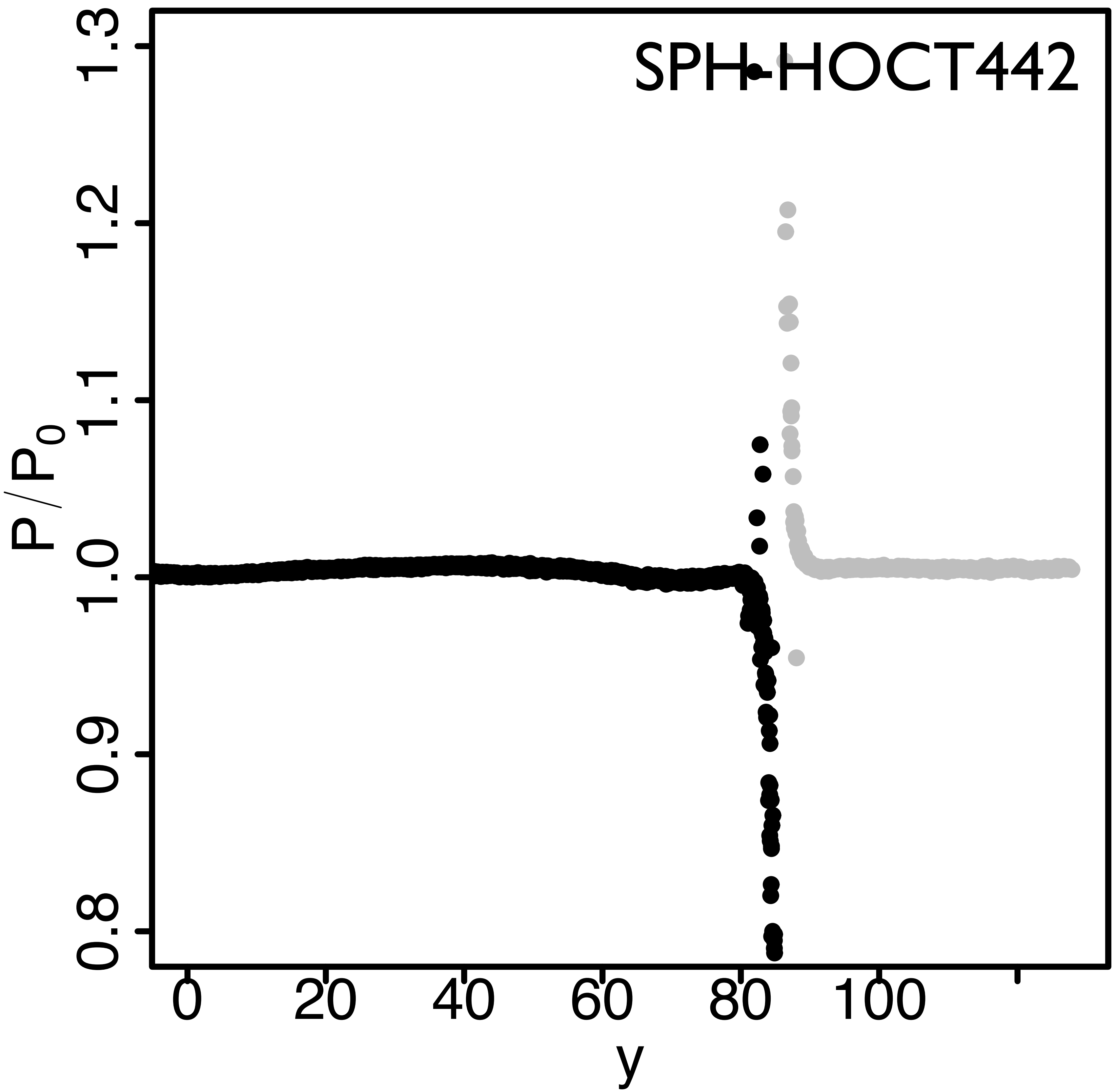


$$P_2 = A_2 \rho^\gamma$$

$$P_1 \neq P_2$$

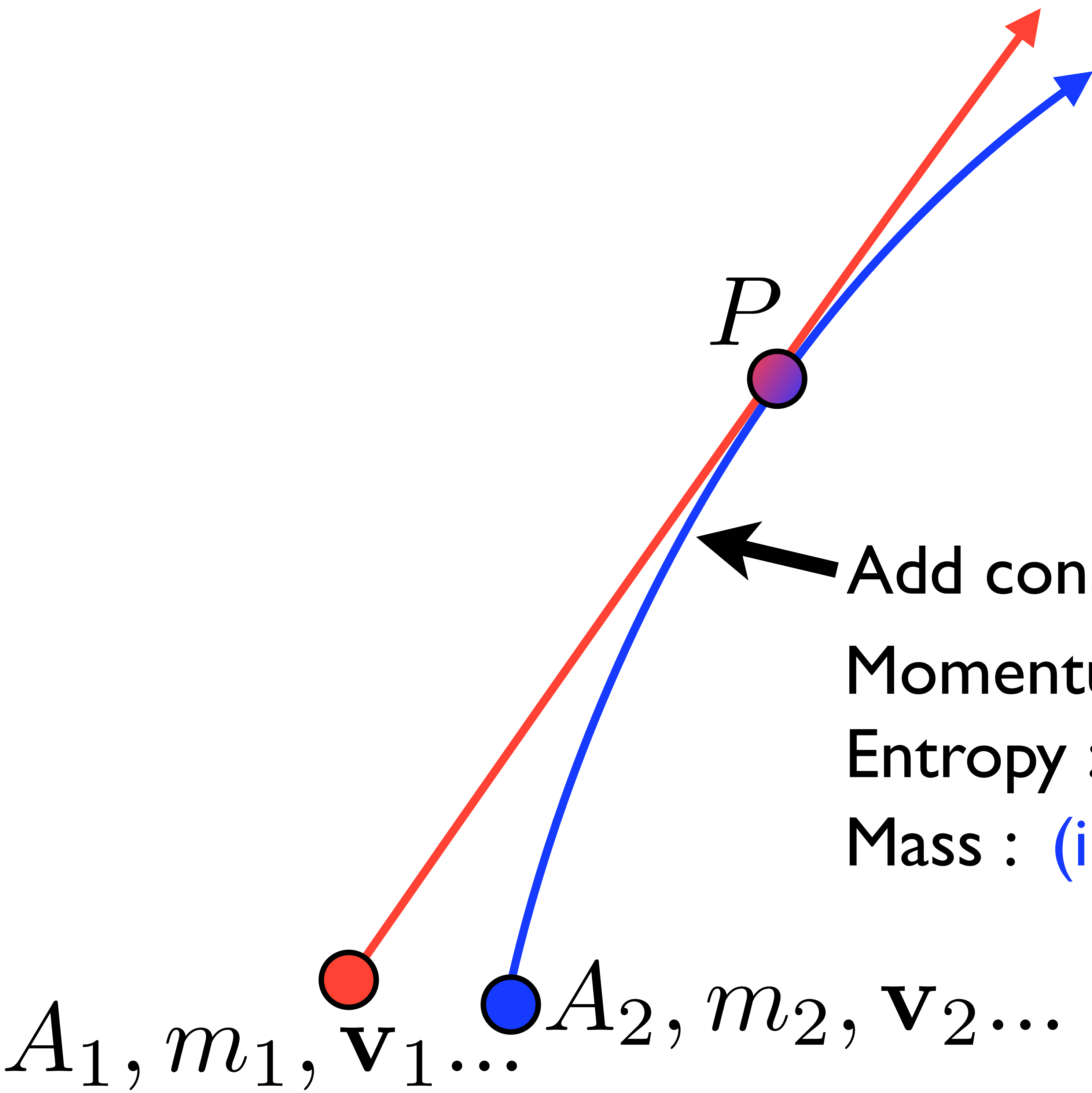
Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

2. Multivalued pressures | The problem



Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011

2. Multivalued pressures | An 'early warning' switch



Add conservative dissipation:

Momentum : Artificial viscosity

Entropy : Artificial thermal conductivity

Mass : (i.e. for multimass applications)

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011; Ritchie & Thomas 2001; Price 2008; Wadsley et al. 2008; Cullen & Dehnen 2010

2. Multivalued pressures | An 'early warning' switch

$$\alpha_{\mathrm{loc},i} = \begin{cases} \dfrac{h_i^2 |\nabla(\nabla\cdot\mathbf{v}_i)|}{h_i^2 |\nabla(\nabla\cdot\mathbf{v}_i)| + h_i |\nabla\cdot\mathbf{v}_i| + n_s c_s}\alpha_{\mathrm{max}} & \nabla\cdot\mathbf{v}_i < 0 \\ 0 & \text{otherwise} \end{cases}$$

[i.e. converging]

[i.e. going to converge]

[Requires high order gradient estimator]

$$\alpha_i = \alpha_{\mathrm{loc},i} \quad \alpha_i < \alpha_{\mathrm{loc},i}$$

otherwise, α_i smoothly decays back to zero:

$$\dot{\alpha}_i = (\alpha_{\mathrm{loc},i} - \alpha_i)/\tau_i \quad \alpha_{\mathrm{min}} < \alpha_{\mathrm{loc},i} < \alpha_i$$
$$\dot{\alpha}_i = (\alpha_{\mathrm{min}} - \alpha_i)/\tau_i \quad \alpha_{\mathrm{min}} > \alpha_{\mathrm{loc},i}$$

Read, Hayfield & Agertz 2010 (RHA10); Read & Hayfield 2011; Ritchie & Thomas 2001; Price 2008; Wadsley et al. 2008; Cullen & Dehnen 2010

SPHS | Putting it all together

1. 'E0' error reduced using 442 neighbours and stable higher order HOCT kernel. Also much lower noise (4).

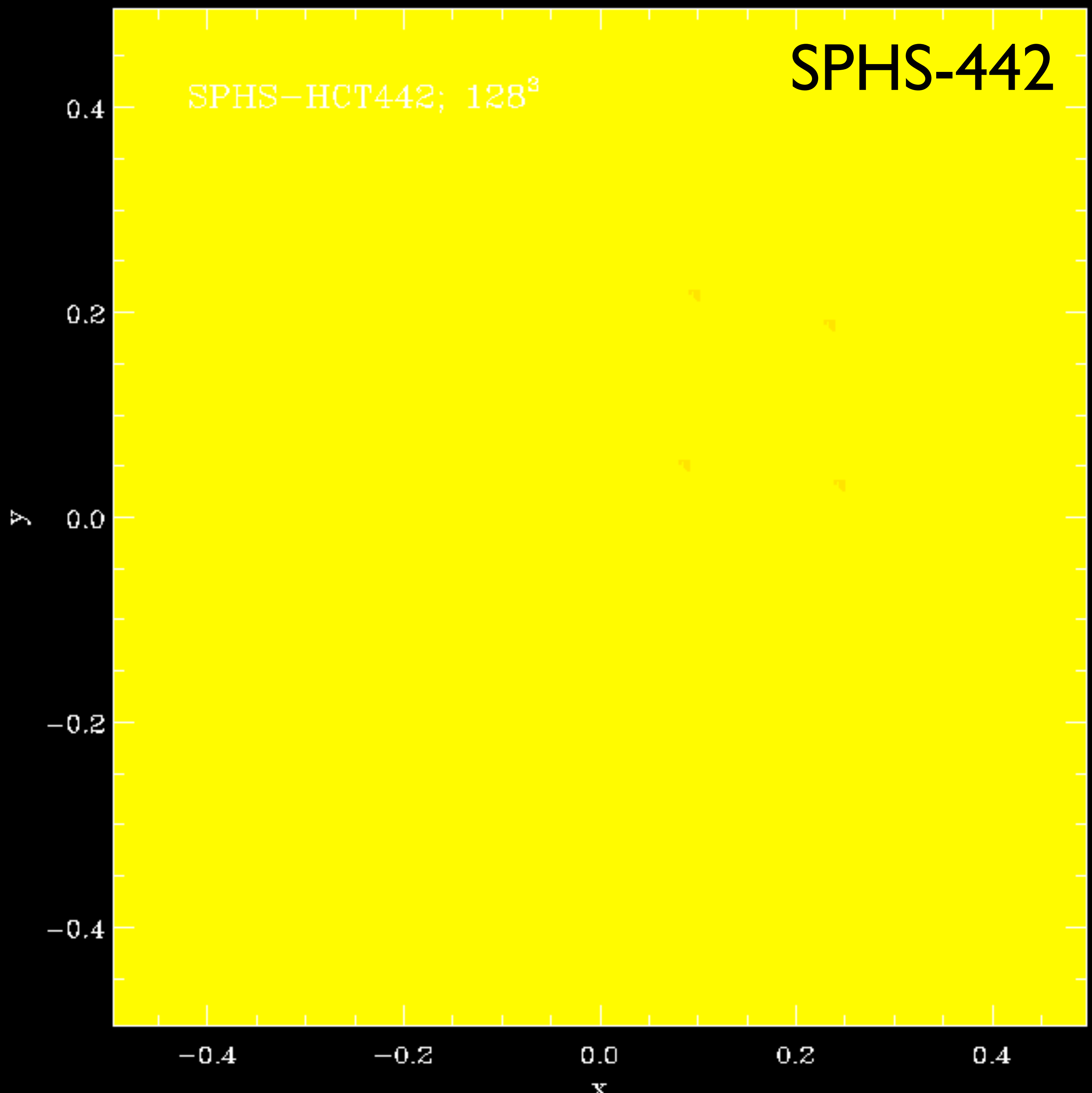
2. Multivalued pressures eliminated using advance warning high order switch and conservative dissipation. Lower viscosity away from shocks (3); multimass particles now possible.

3. Timestep limiter [Saitoh & Makino 2009] => strong shocks correctly tracked.

4. Implementations in GADGET2 & 3.

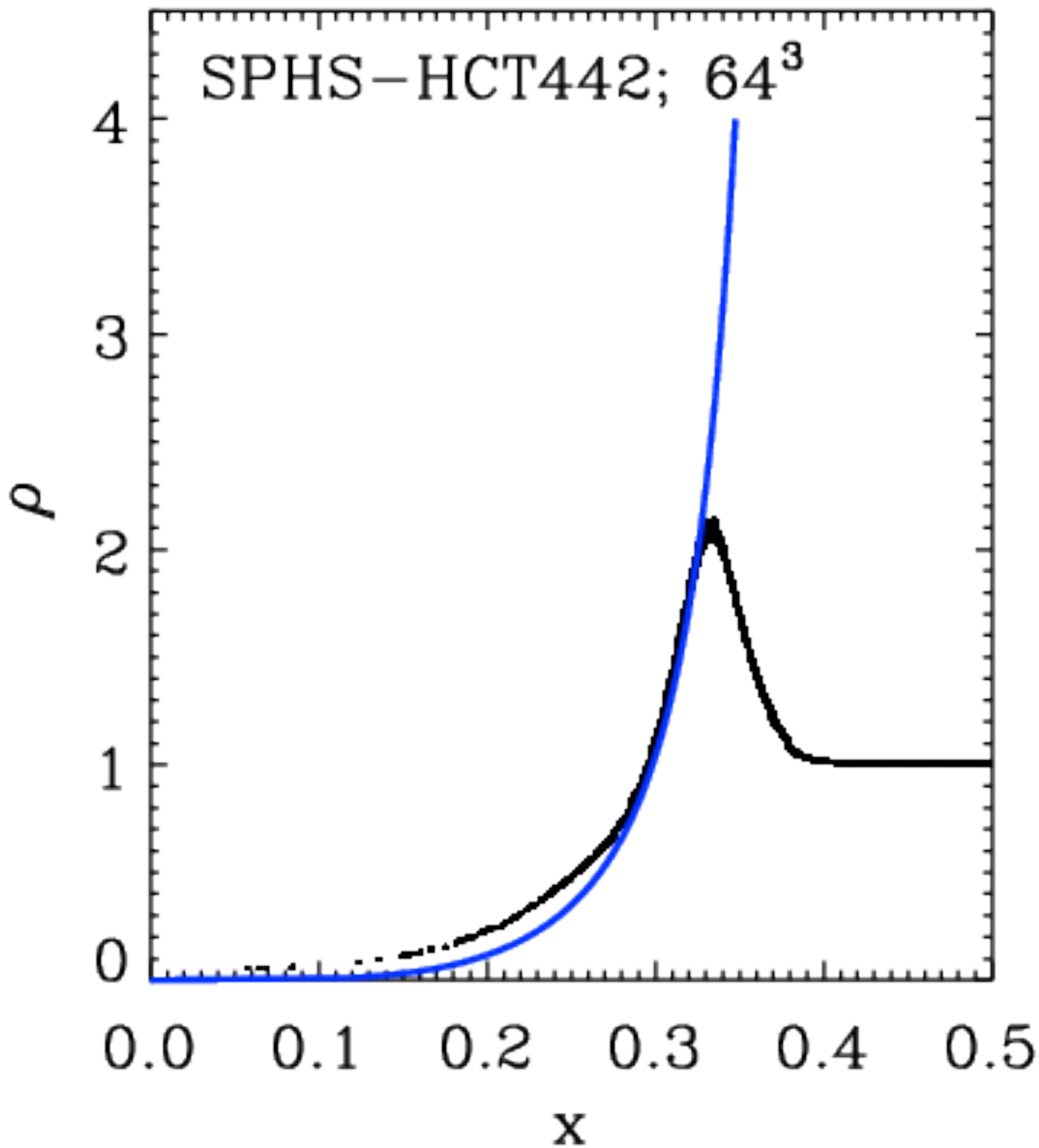
Read & Hayfield 2011

SPHS tests | Sedov-Taylor blast wave



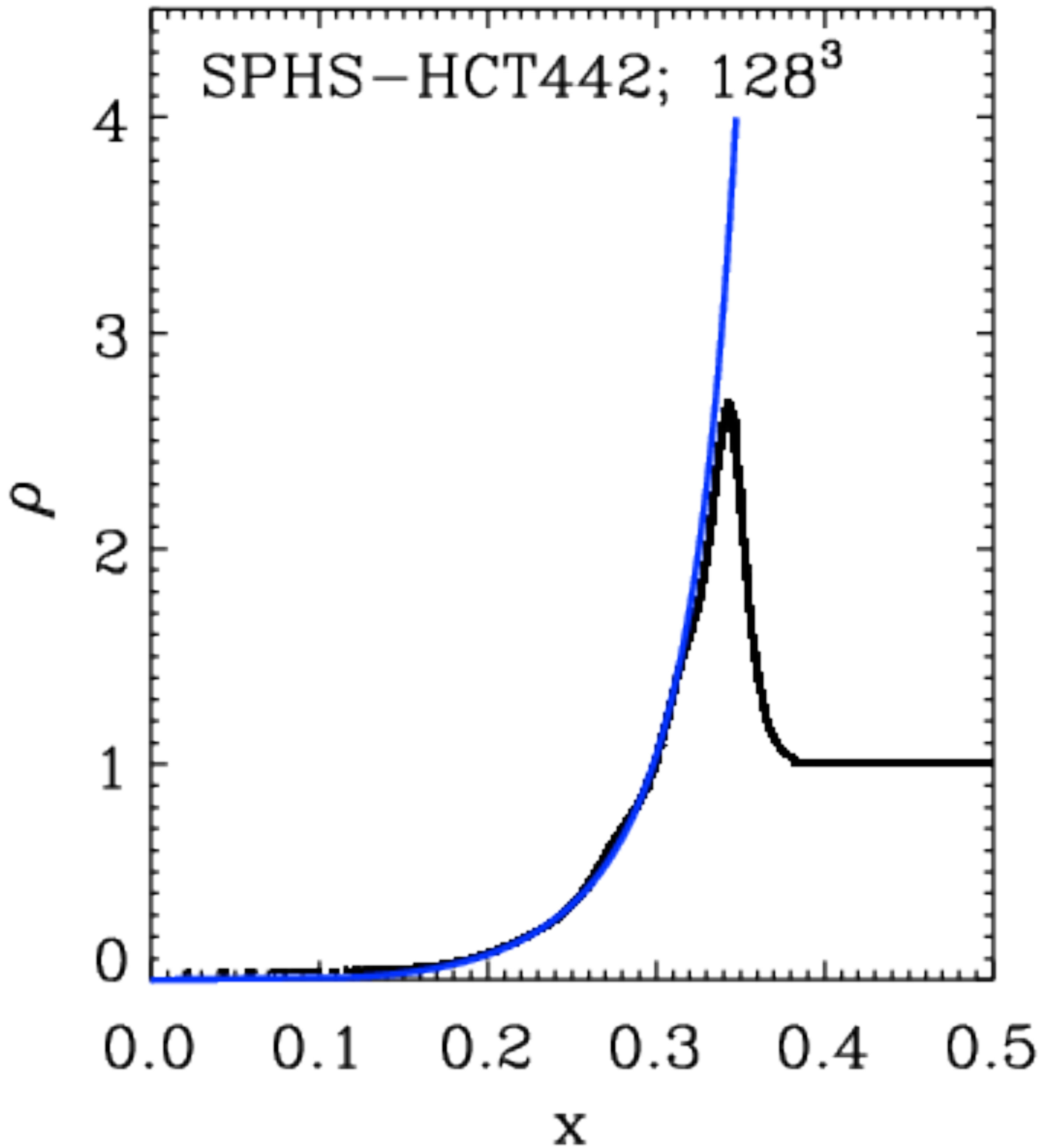
Read & Hayfield 2011

SPHS tests | Sedov-Taylor blast wave



Read & Hayfield 2011

SPHS tests | Sedov-Taylor blast wave



Read & Hayfield 2011

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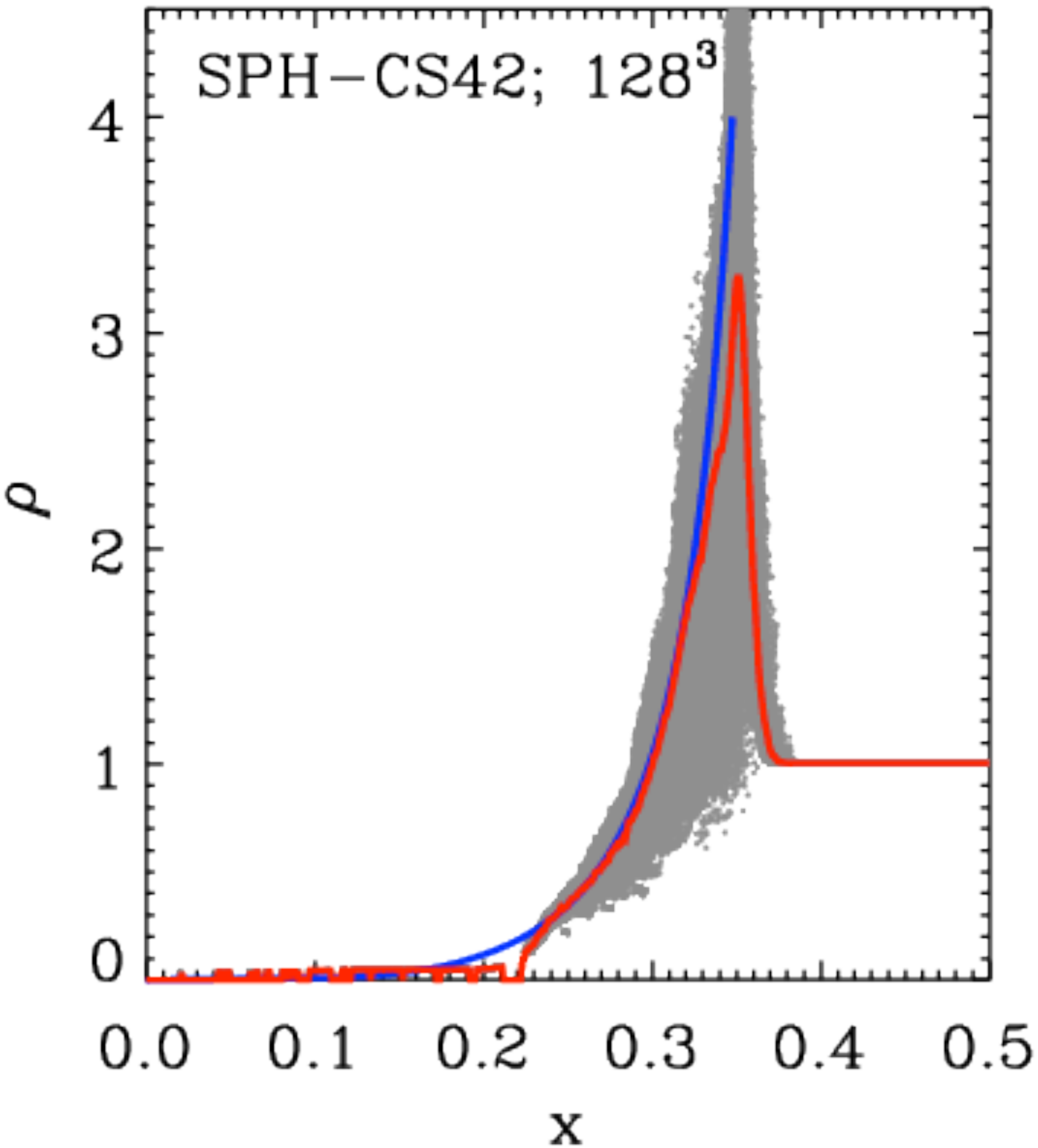
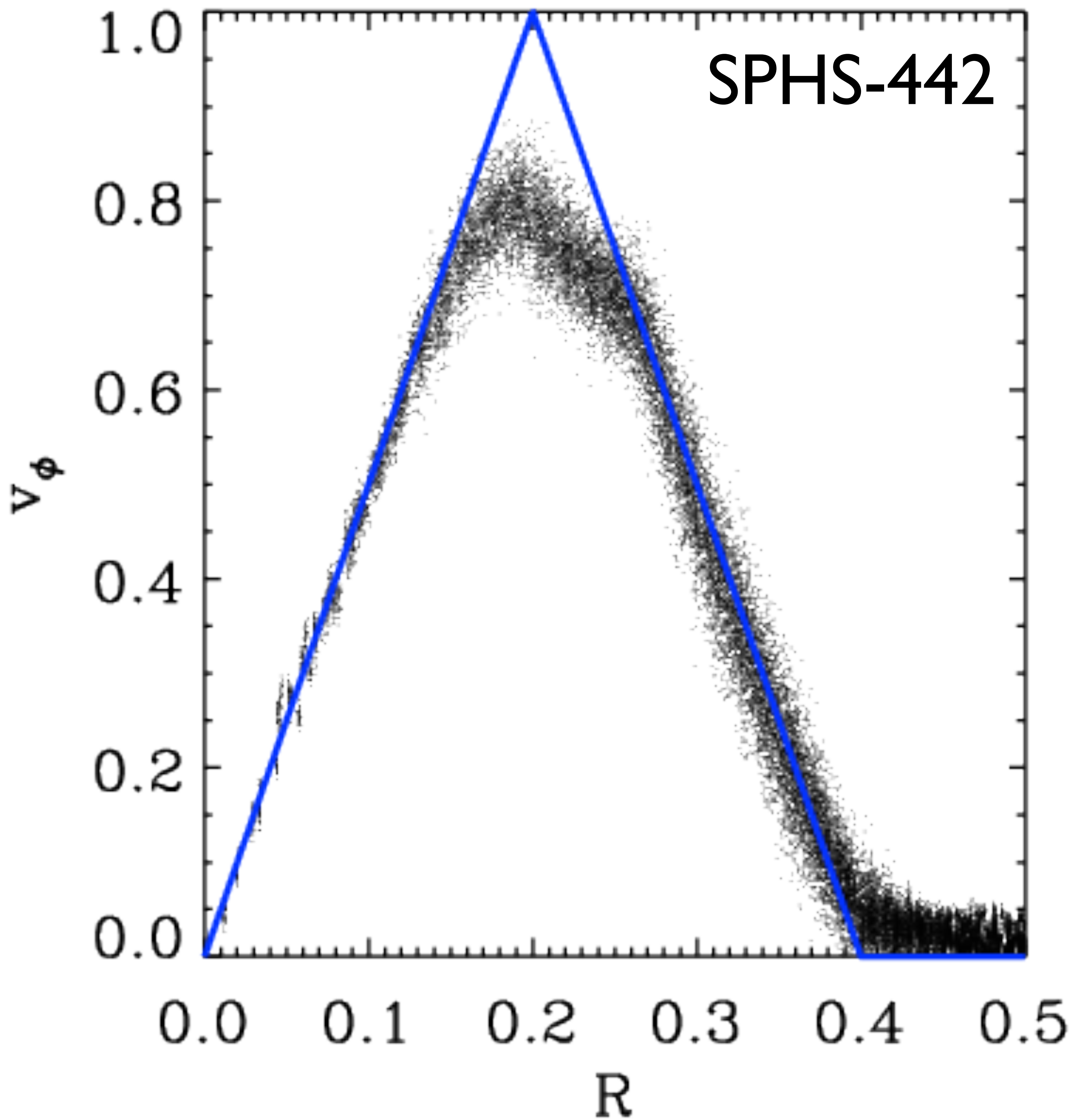


Read & Hayfield 2011

SPHS tests | Gresho vortex



Read & Hayfield 2011

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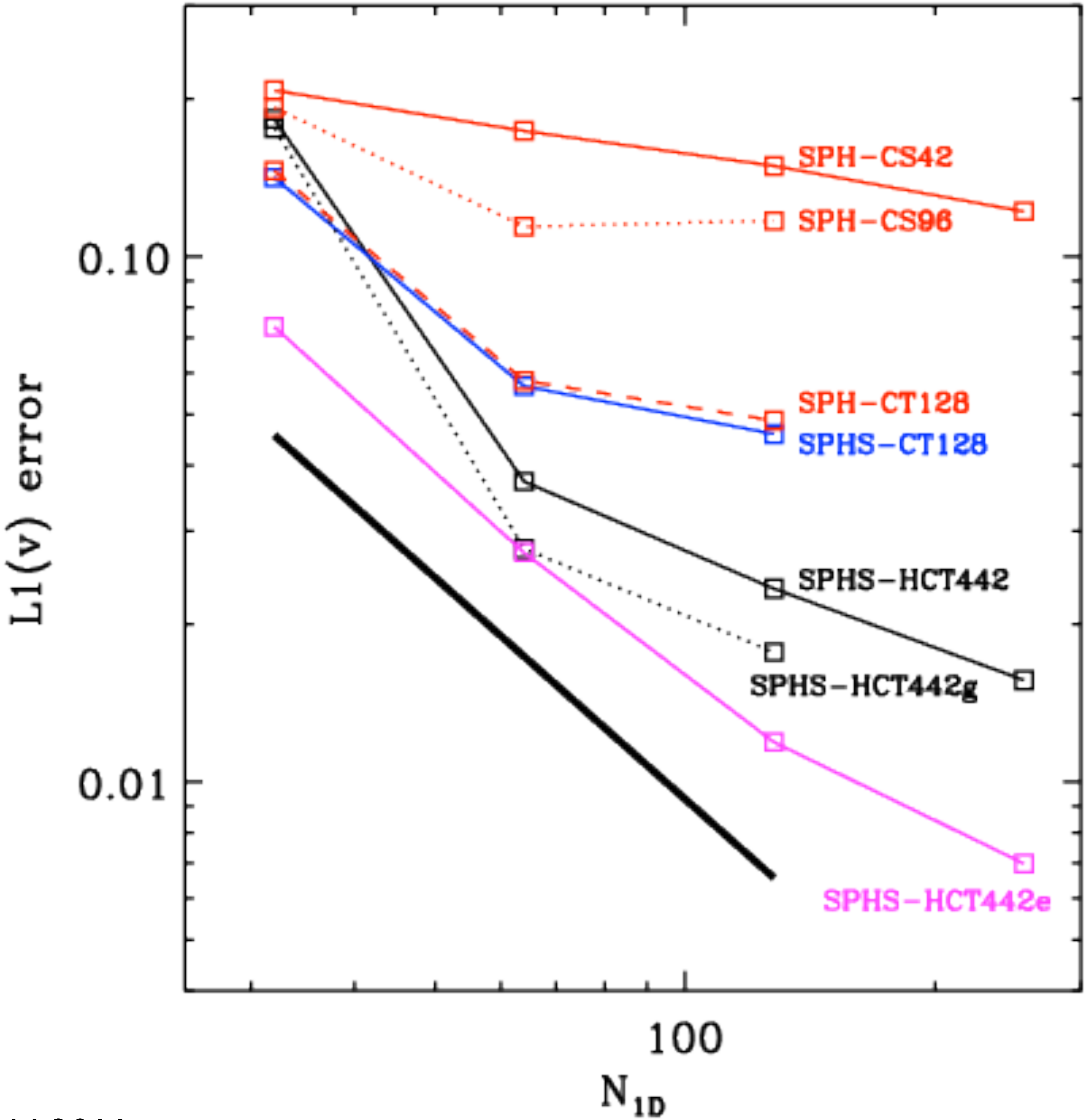
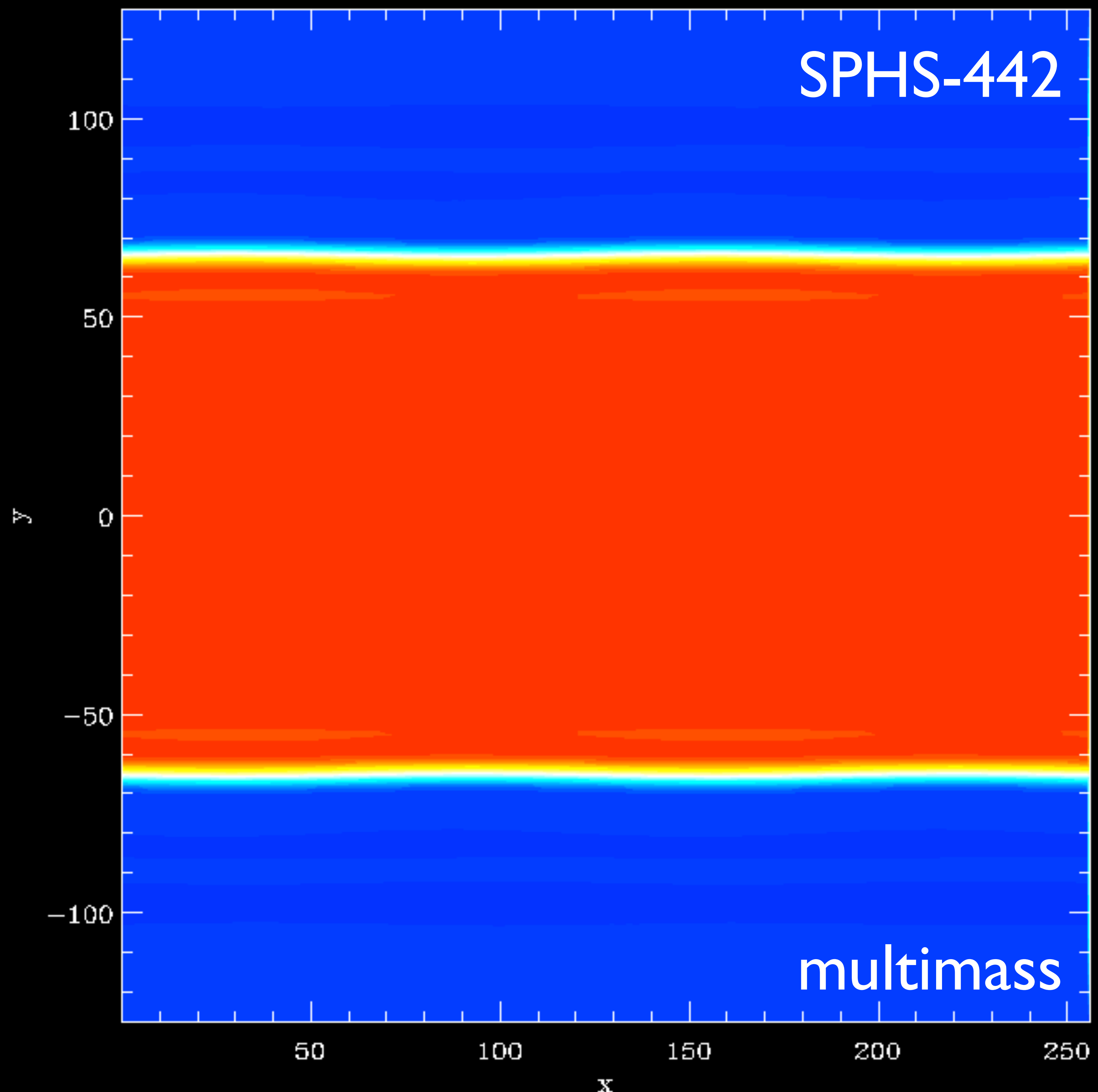


Read & Hayfield 2011

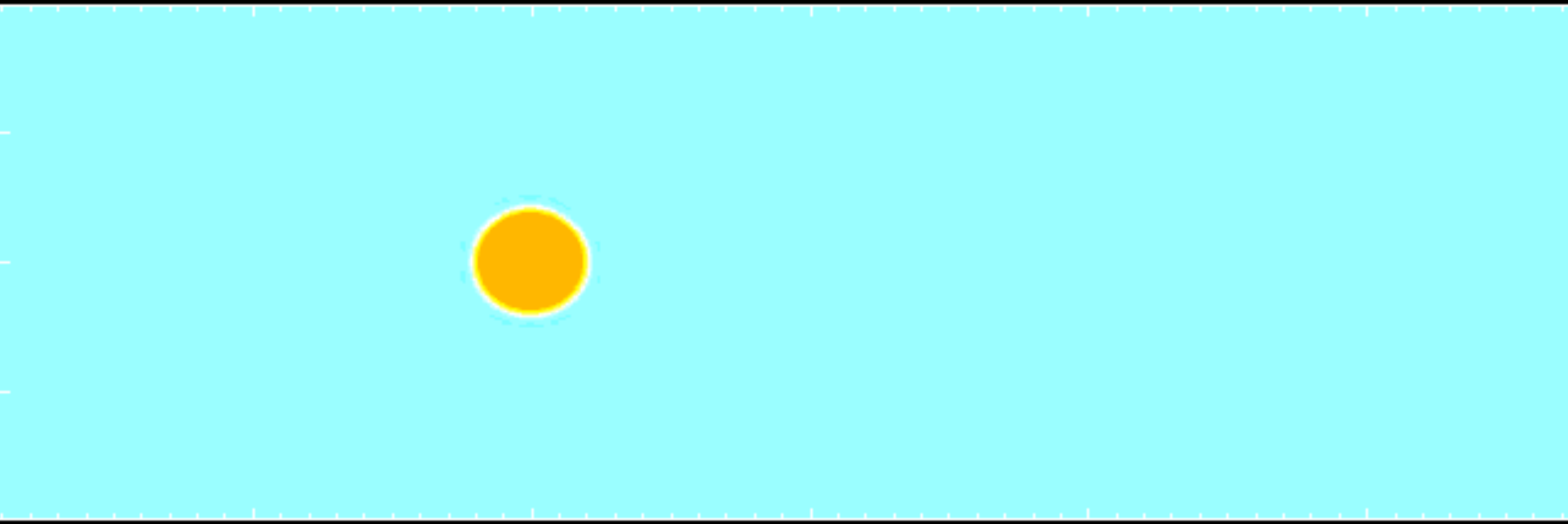
SPHS tests | KH instability 1:8 density contrast ... multimass



Read & Hayfield 2011

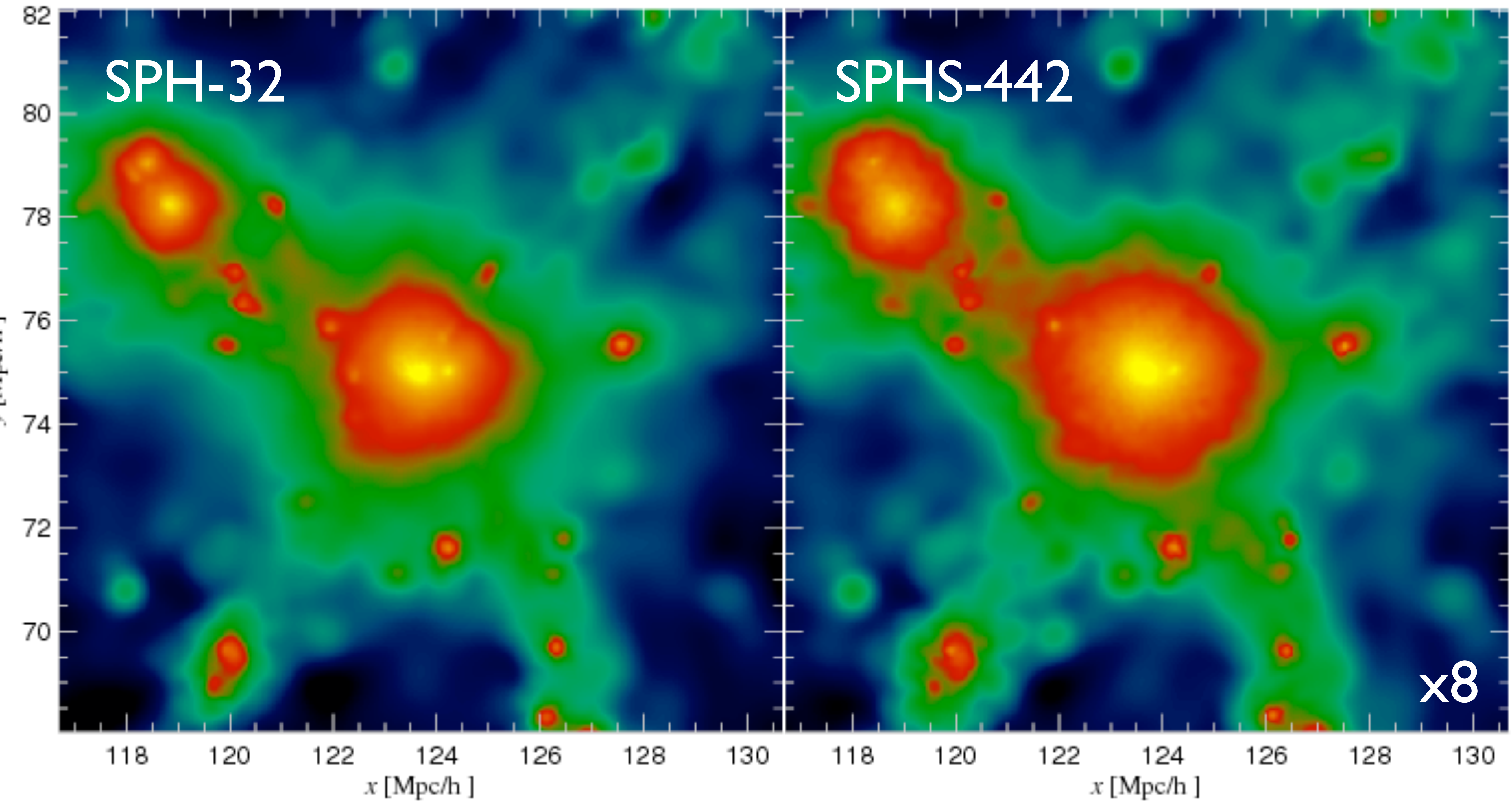
SPHS tests | Blob test

SPHS-442



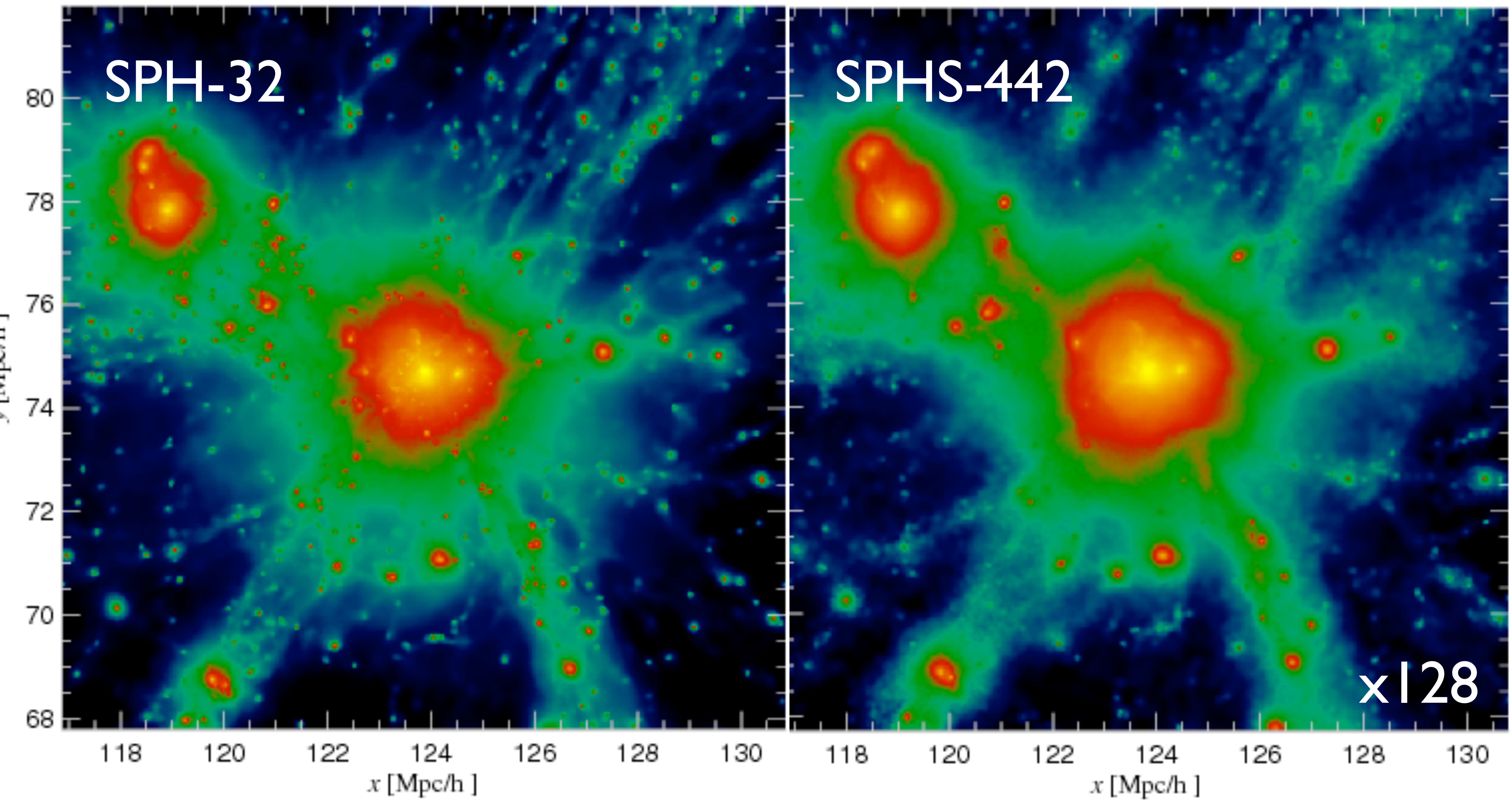
Read & Hayfield 2011

SPHS tests | Santa Barbara test



Power, Read & Hobbs in prep. 2012

SPHS tests | Santa Barbara test



Power, Read & Hobbs in prep. 2012

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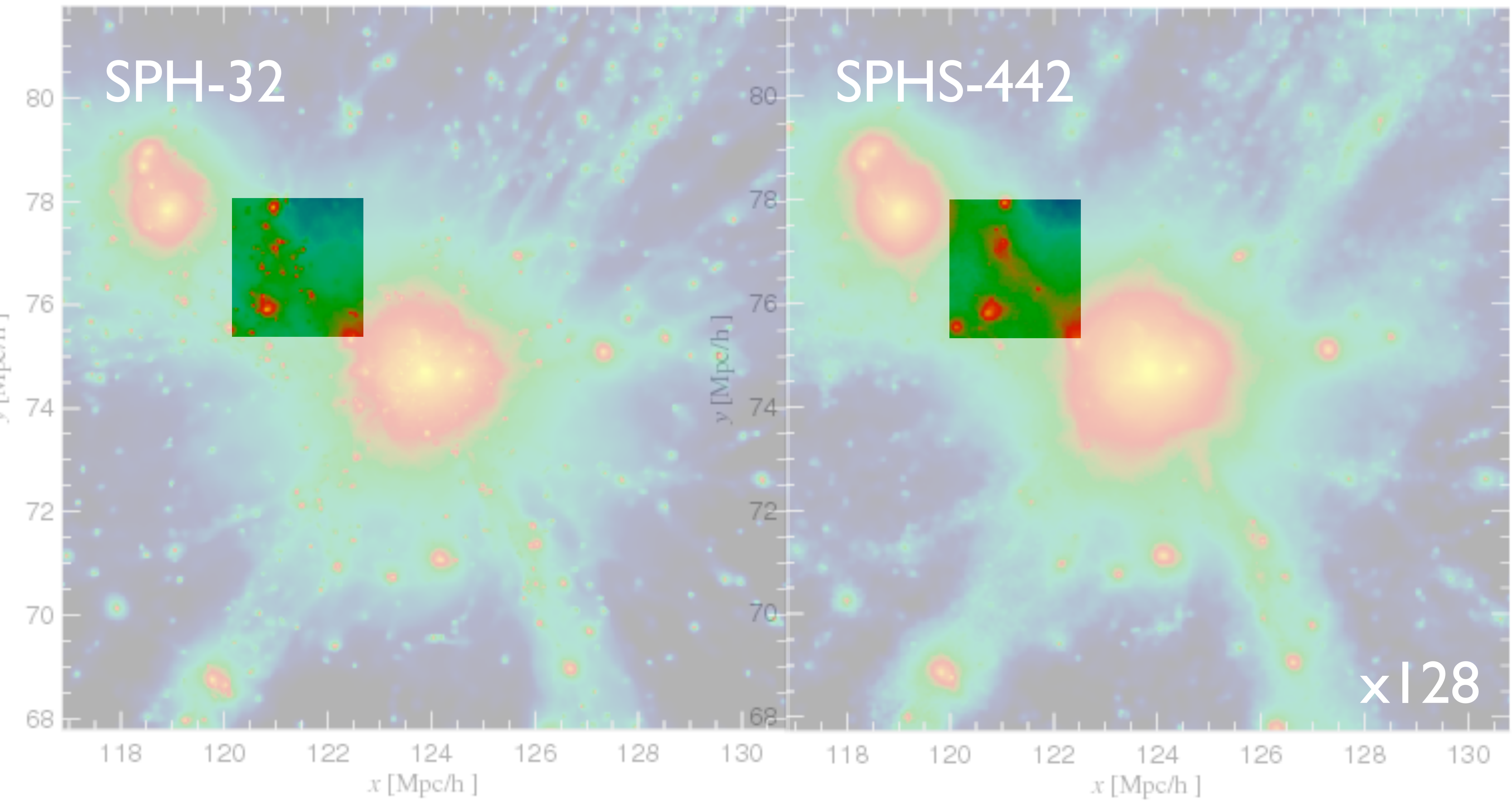

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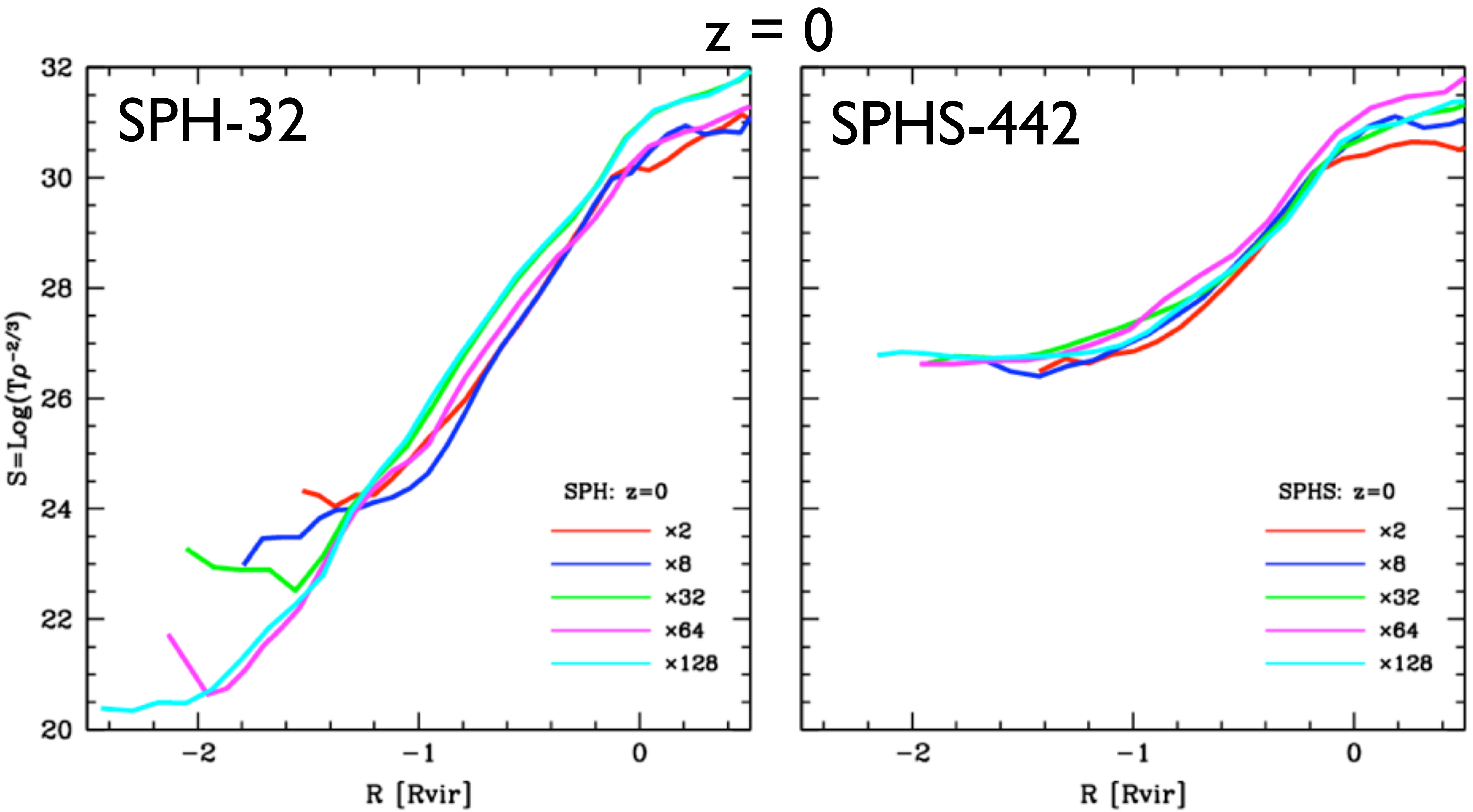

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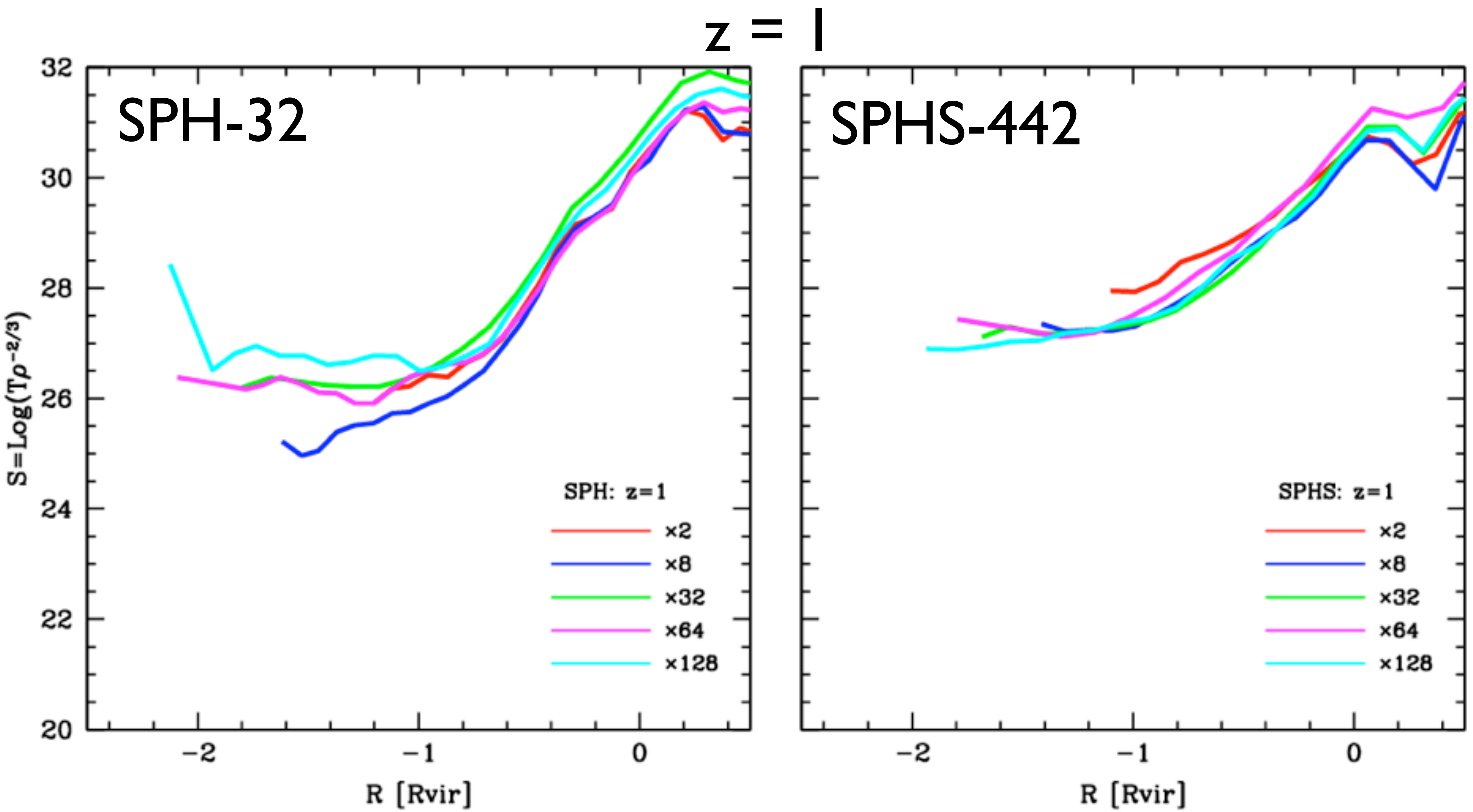

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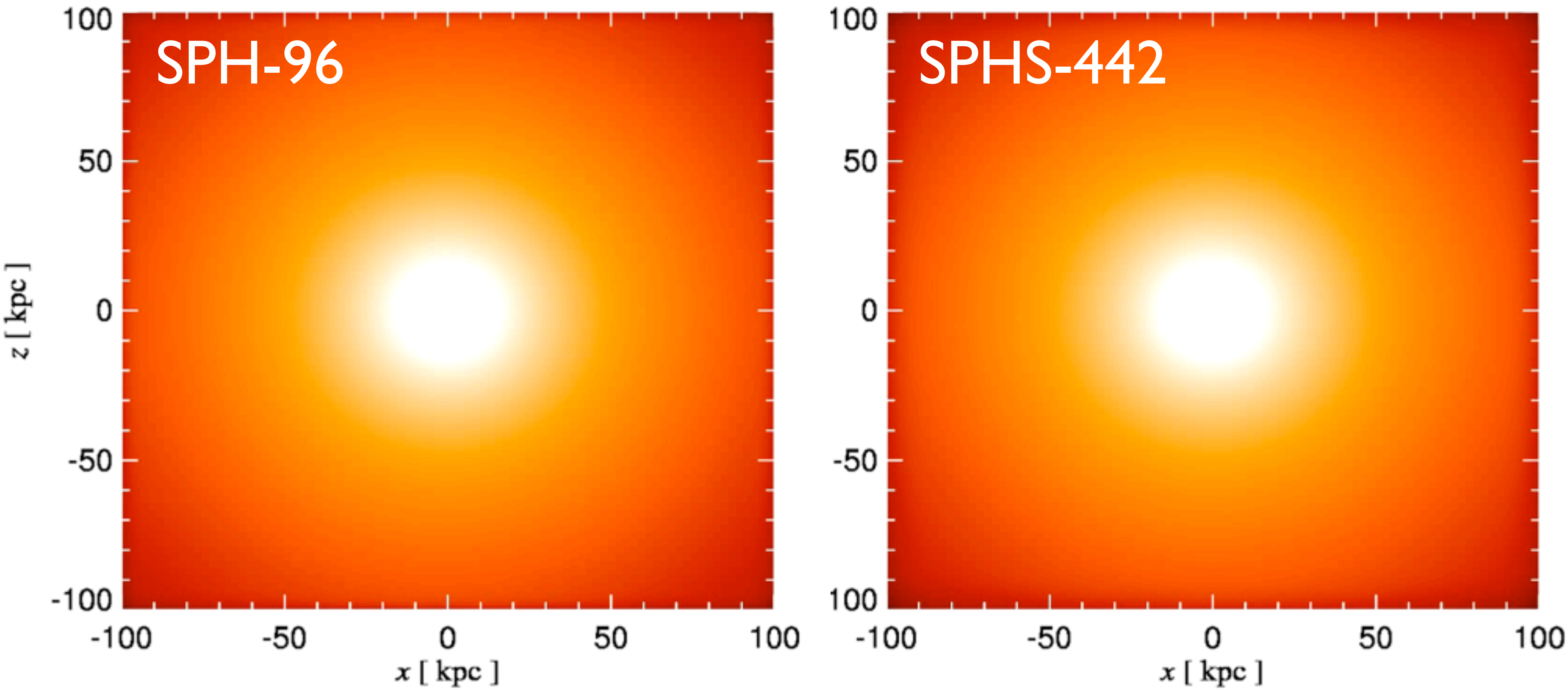
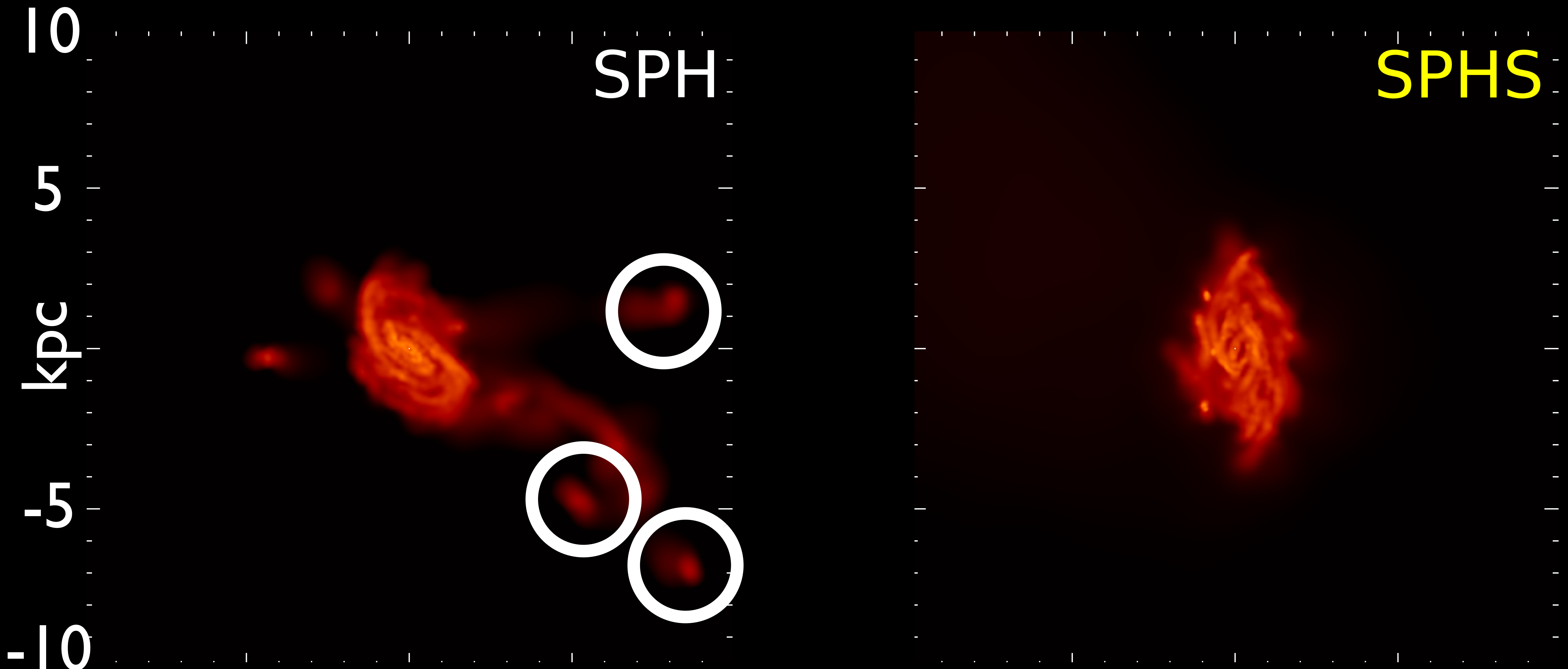
SPHS | Cooling halos



Hobbs, Read & Cole 2012; http://arxiv.org/abs/1207.3814



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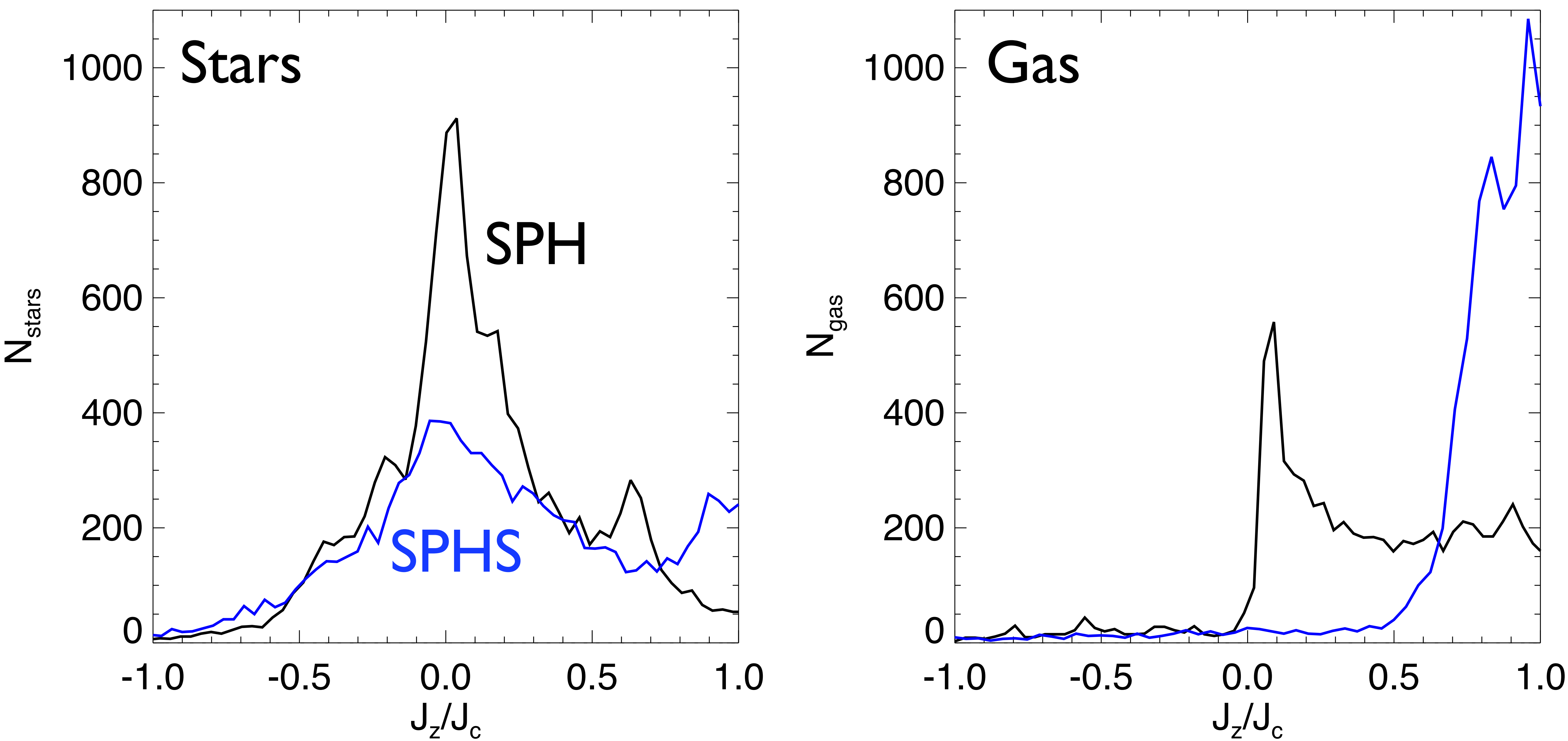

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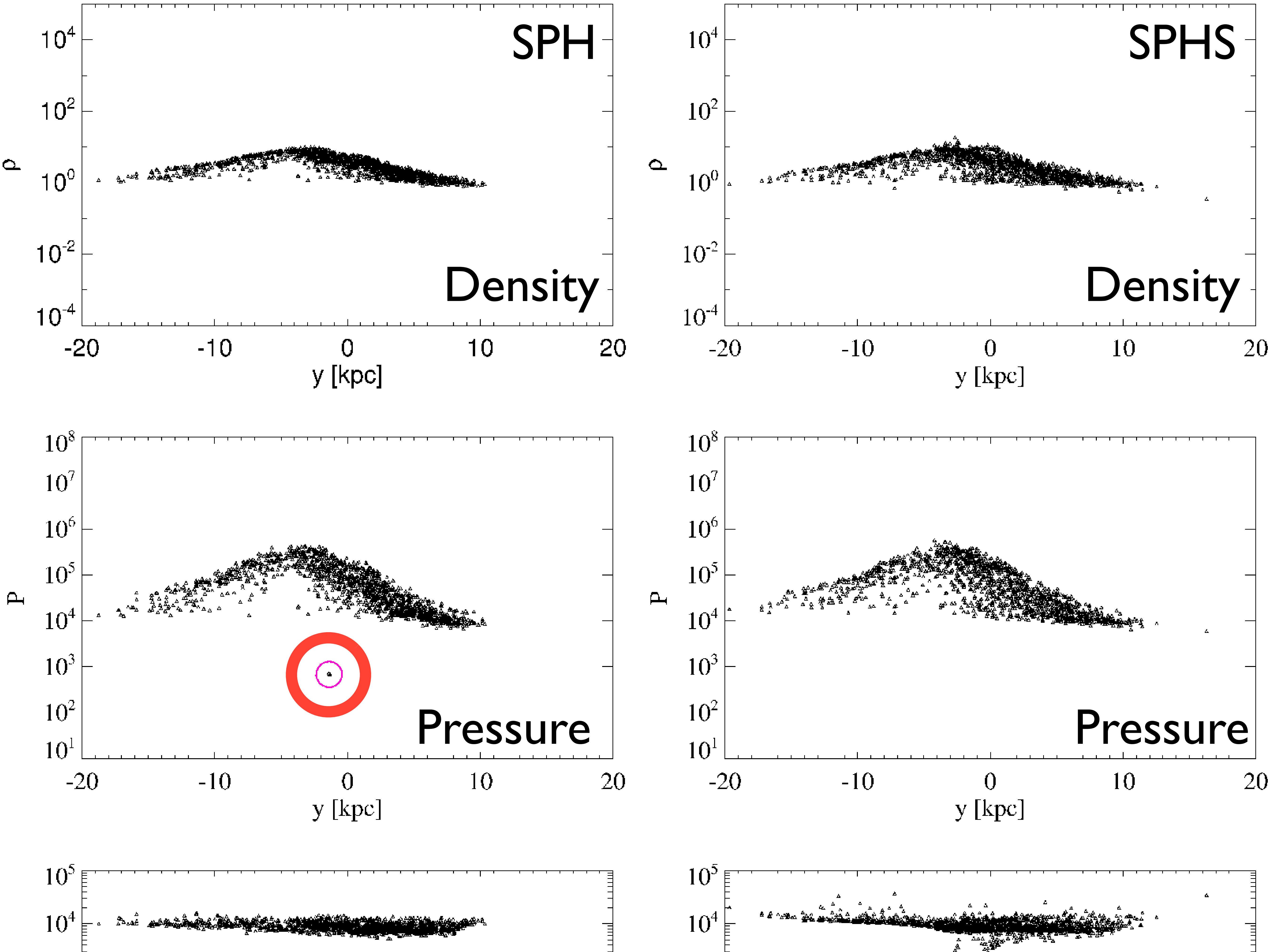
SPH
SPHS
Density
Density
Pressure
Pressure
ρ
P
y [kpc]
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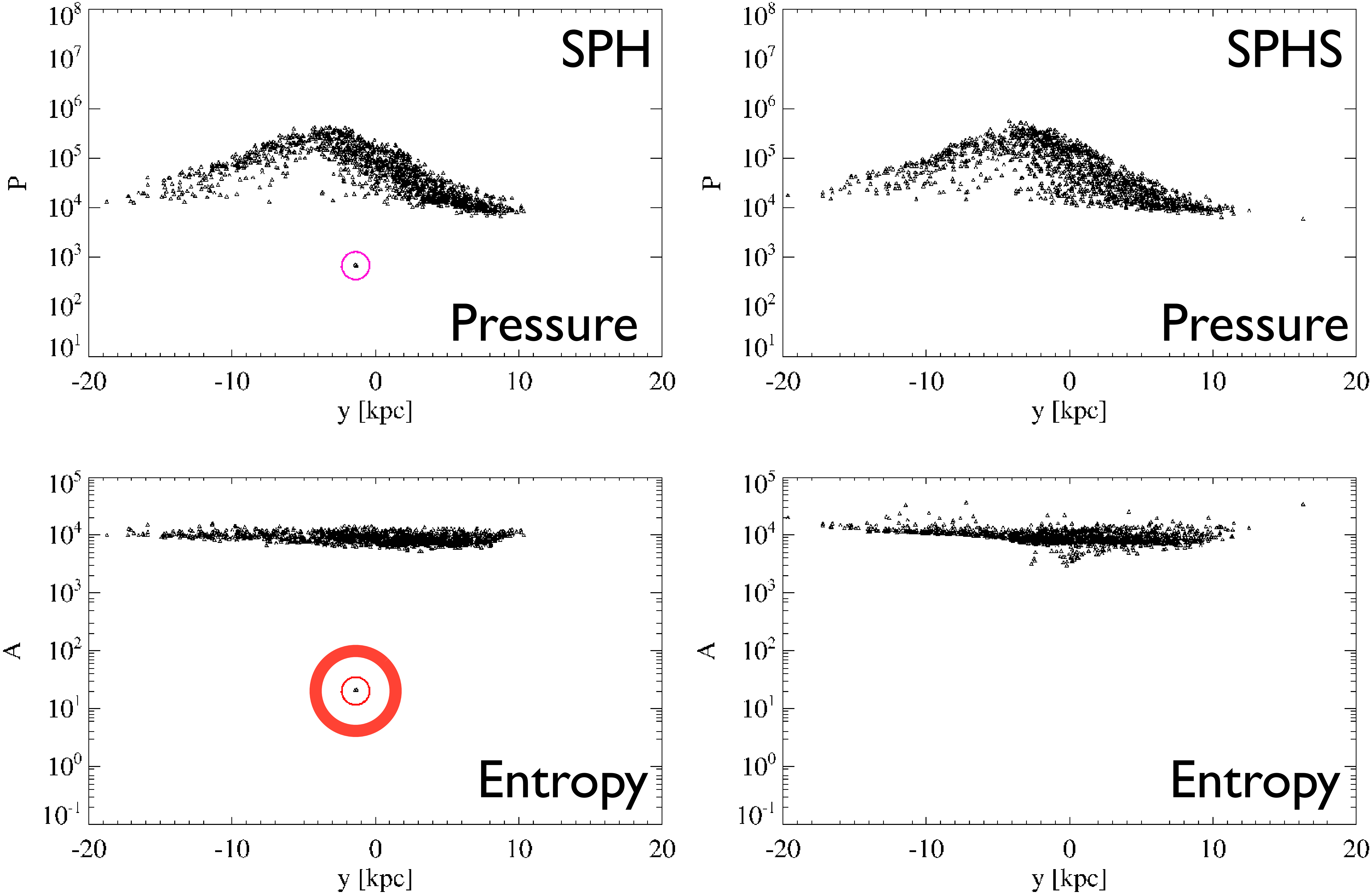

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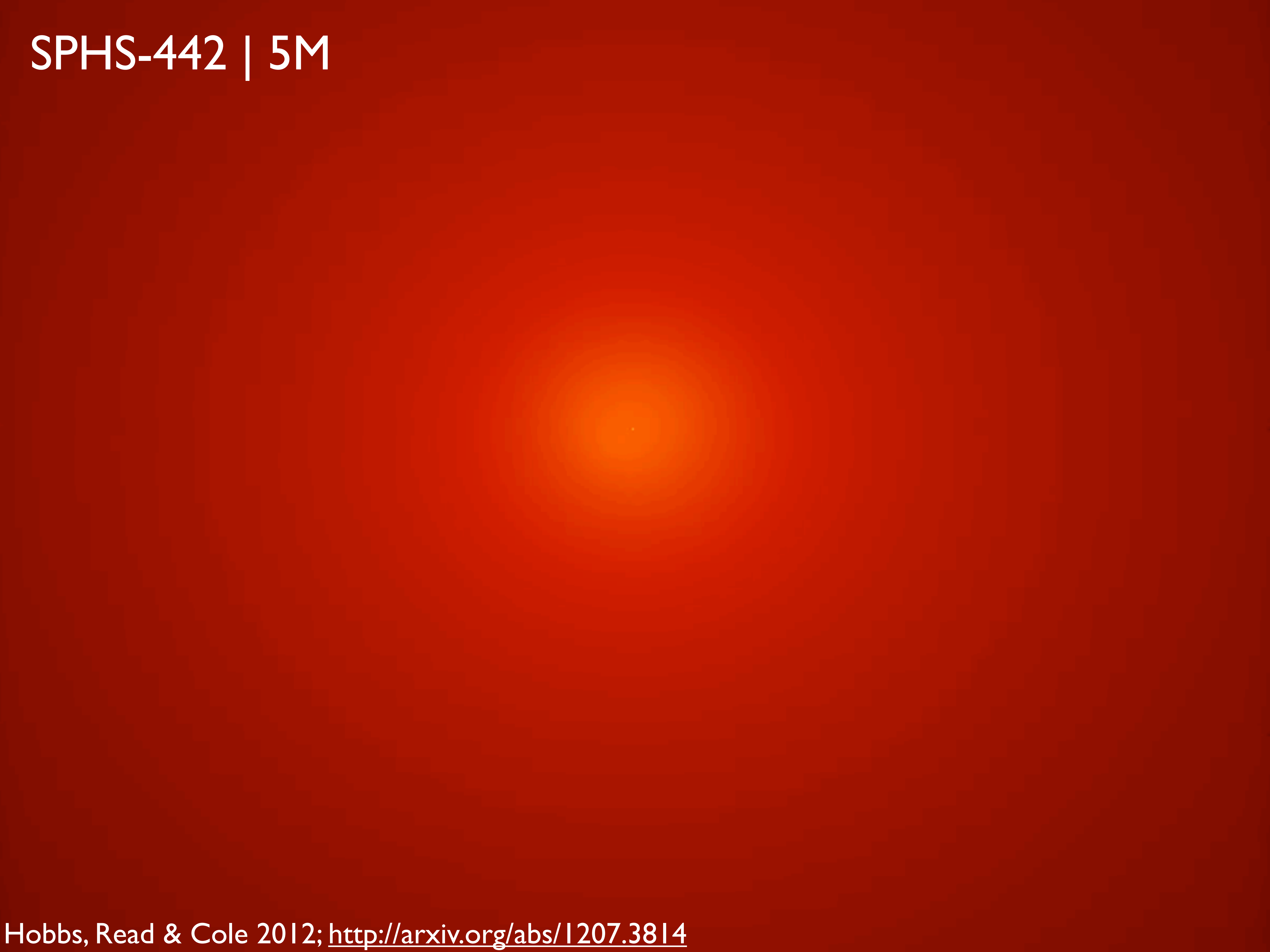
SPHS-442 | 5M



Hobbs, Read & Cole 2012; http://arxiv.org/abs/1207.3814

SPHS | Conclusions

- 'E0' error reduced using 442 neighbours and stable higher order HOCT kernel. Much lower noise.
- Multivalued pressures eliminated using advance warning high order switch and conservative dissipation. Lower viscosity away from shocks; multimass particles now possible.
- Timestep limiter => strong shocks correctly tracked.
- Good performance and convergence to >1% accuracy on a wide range of test problems.
- Santa Barbara test => entropy profile core
- Cooling halos => no SPH blobs