Radiative Transfer in a Clumpy Universe: the UVB

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The cosmic UVB originates from the integrated emission of star-forming galaxies and QSOs. It determines the thermal and ionization state of the IGM, the repository of most of the baryons in the Universe at high redshift.

It is a crucial yet most uncertain input parameters for cosmological simulations of LSS and galaxy formation, for interpreting QSO absorption-line data and derive information on the distribution of primordial gas -- traced by HI, HeI, HeII transitions -- and of the nucleosynthetic products of star formation -- CIII, CIV, SiIII, SiIV, OVI, etc.
Outline

Hydrogen recombination
The dark ages
Cosmic structure formation
The reionization equation
Quasars or galaxies?
The Gunn-Peterson trough
Quasar absorbers along the LOS
Effective optical depth of the Universe
Cosmological radiative transfer
CUBA (the code)
Hydrogen recombination

Two factors delay recombination:
1. \( \eta^{-1} = 1.5 \times 10^9 \gg I \)
2. inability to maintain equilibrium as \( H^{-1} \ll t_{\text{rec}} \equiv x/\dot{x} \)

\[ \frac{x^2}{1 - x} = 2.5 \times 10^6 \eta^{-1} \left( \frac{I}{k_B T} \right)^{3/2} \exp \left( -\frac{I}{k_B T} \right) \]

Big coefficient

\( I = 13.6 \text{ eV} \)
\( \eta^{-1} = n_Y/n_b \)

\[ \log \frac{x = n_e/n_H}{(1+z)} \]

\( e^- + p \rightarrow H + \gamma \) @ \( z = 1100 \) marks the end of the plasma era

Equilibrium (Saha) eq. for \( x = n_e/n_H \)
Hydrogen recombination (a digression)

When an e⁻ is captured to the ground state of HI, it produces a photon that immediately ionizes another atom, leaving no net change.

When it is captured to an excited state, the allowed decay to the ground state produces a resonant Lyman series photon, which has a large capture cross-section → puts another atom in a high energy state that is easily photoionized again, thereby annulling the effect.

Two main routes to the production of atomic hydrogen:

1) two-photon decay from the 2s level to 1s.
2) loss of Lyα resonance photons by the cosmological redshift.
The “dark ages”...

Timescale for relaxation of matter temperature is

\[ t_{\text{comp}} = \frac{3m_ec}{4\sigma_T a_B T^4} \frac{1+x}{2x} \]

\[ z_{\text{th}} \simeq 580(\Omega_b h^2)^{2/5} \simeq 130. \]

..and their end

Universe becomes semi-opaque after reionization

\[ \tau_T(z) = \int_0^z n_e(z)\sigma_T |c dt/ dz| dz = 0.087 \]

\[ \rightarrow \text{Universe becomes semi-opaque after reionization} \]
The “dark ages”...

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Cosmic structure formation: I

Clumpiness boosts H recombination rate:
\[ \langle n_e n_p \rangle \alpha_B(T) = C \langle n_p \rangle^2 \alpha_B(T) \]
\[ t_{\text{rec}} = (n_p \alpha_B C)^{-1} \]

Example: ionized gas of density \( n_e \) filling uniformly a fraction \( f \) of the available volume, rest is empty space.

Then
\[ \langle n_e^2 \rangle = f n_e^2; \quad \langle n_e \rangle = f n_e \]
\[ \rightarrow \langle n_e^2 \rangle = \langle n_e \rangle^2 / f \rightarrow C = 1 / f \]
Cosmic structure formation: II

HII region in homogeneous ISM (Stromgren analysis):

\[ n_H \frac{dV}{dt} = \dot{N}_\gamma - V \alpha_B n_H^2 \]

\[ \rightarrow V = \frac{\dot{N}_\gamma t_{\text{rec}}}{n_H} \left( 1 - e^{-t/t_{\text{rec}}} \right) \]

HII region in expanding clumpy IGM (Shapiro & Giroux 1987):

\[ n_H(t) \left( \frac{dV}{dt} - 3HV \right) = \dot{N}_\gamma - V \alpha_B \langle n_H(t) \rangle^2 C \]

\[ V=\text{proper volume} \]
The reionization equation

Reionization @ milliFLOP speed (PM, Haardt, & Rees 1999)

\[ Q_I(t) = \text{volume filling factor of HII regions at } t \]

\[ Q_I(t) = \int_0^t \frac{\dot{n}_\gamma(t')}{{\langle n_H(t') \rangle}} \, dt' - \int_0^t \frac{Q_I(t')}{{t_{rec}}} \, dt' \]

(no redshifting, ionizing photons absorbed locally). Differentiating:

\[ \frac{dQ_I}{dt} = \frac{\dot{n}_\gamma}{{\langle n_H \rangle}} - \frac{Q_I}{{t_{rec}}} \]

Contrary to the static case, cosmological HII regions will always percolate in an expanding universe with constant comoving ionizing emissivity....

simple diff. eq. statistically describes transition from a neutral Universe to a fully ionized one!
The reionization equation

\[ t_{\text{rec}} \ll t \rightarrow Q_I \approx \frac{\dot{n}_\gamma}{\langle n_H \rangle} t_{\text{rec}} \]

Because of hydrogen recombinations, only a fraction \( t_{\text{rec}}/t \) of the photons emitted above 13.6 eV is actually used to ionize new IGM material.

The universe is completely reionized when \( Q_I = 1 \), i.e. when

\[ \dot{n}_\gamma t_{\text{rec}} = \langle n_H \rangle \]

Numerical simulation of stellar reionization (Gnedin 2000)
Quasars or galaxies?

(Bright) QSOs are not responsible for the reionization of cosmic hydrogen!
radiation emitted at $\nu_e$ and $z_e$ becomes resonant (Ly$\alpha$) at $(l+z) = (l+z_e) \nu_\alpha/\nu_e \rightarrow$ scattered off the los with cross-section:

$$\sigma(\nu) = \frac{\pi e^2}{m_e c} f \phi(\nu)$$

total optical depth for resonant scattering (Gunn-Peterson)

$$\tau_{GP}(z_e) = \int_{0}^{z_e} \sigma n_{HI}(z) |c dt/dz| dz = \left( \frac{\pi e^2 f}{m_e \nu_\alpha} \right) \frac{n_{HI}}{H}.$$ 

**Hydrogen is highly ionized at $z<5.7$**
We can now quantify the degree of attenuation of UV radiation in a clumpy Universe by introducing the concept of an effective continuum optical depth $\tau_{\text{eff}}$ along the line-of-sight to redshift $z$: 

$$\langle e^{-\tau} \rangle = e^{-\tau_{\text{eff}}}$$

where the average is taken over all lines-of-sight.
Effective optical depth

Assume random distribution of absorbers in column density and redshift space, then:

\[ \tau_{\text{eff}}(\nu_o, z_o, z) = \int_{z_o}^{z} dz' \int_{0}^{\infty} f(N_{\text{HI}}, z)(1 - e^{-\tau}) \]

\[ \tau = N_{\text{HI}}\sigma_{\text{HI}}(\nu) \left[ \ldots + N_{\text{HeI}}\sigma_{\text{HeI}} + N_{\text{HeII}}\sigma_{\text{HeII}} \right] \]

\[ \nu = \nu_0(1 + z)/(1 + z_0); \quad \sigma_i = \text{photoionization cross section} \]

⇒ Poissonian probability of encountering a total optical depth \( k\tau_0 \) is:

\[ p(k\tau_0) = e^{-\Delta N} \Delta N^k / (\tau_0 k!) \]

\[ \langle e^{-\tau} \rangle = e^{-k\tau_0} p(k\tau_0) = \exp[-\Delta N(1 - e^{-\tau_0})] \]

\[ \langle \tau \rangle = \Delta N\tau_0 > \tau_{\text{eff}} = \Delta N(1 - e^{-\tau_0}) \]
The equation of cosmological radiative transfer describes the time evolution of the space- and angle-averaged monochromatic intensity $J_\nu$:

$$
\left( \frac{\partial}{\partial t} - \nu H \frac{\partial}{\partial \nu} \right) J_\nu + 3H J_\nu = -c \kappa_\nu J_\nu + \frac{c}{4\pi} \epsilon_\nu
$$

* $J_{\nu_o}(z_o) = \frac{c}{4\pi} \int_{z_o}^{\infty} |dt/dz| \, dz \frac{(1+z_o)^3}{(1+z)^3} \epsilon_\nu(z) e^{-\tau_{\text{eff}}}$

$$
\tau_{\text{eff}}(\nu_o, z_o, z) = \int_{z_o}^{z} dz' \int_{0}^{\infty} f(N_{\text{HI}}, z')(1 - e^{-\tau})
$$

$$
\tau = N_{\text{HI}} \sigma_{\text{HI}}(\nu) \left[ ... + N_{\text{HeI}} \sigma_{\text{HeI}} + N_{\text{HeII}} \sigma_{\text{HeII}} \right]
$$

$$
\nu = \nu_0 (1+z)/(1+z_0)
$$

$\sigma_i =$ photoionization cross section

\begin{itemize}
  \item \textbf{equation} * must be solved by iteration since
  \item $\tau = \tau(J)$
\end{itemize}
Two important effects must be included:

1) absorbers are not only sinks but also sources of ionizing radiation. In particular, HeII reprocesses soft X-rays He-ionizing photons into UV H-ionizing ones.

\[
\begin{align*}
\epsilon(\nu, z) &= \epsilon_{\text{QSO}} + \epsilon_{\text{Gal}} + \epsilon_{\text{rec}} \\
\epsilon(z) &= (1 + z)^3 \int dLL\phi(L,z)
\end{align*}
\]

Ionizing recombination radiation includes:

- recombinations to ground state of H\text{I}, H\text{II}, H\text{III}
- Hell Balmer and 2-photon continuum
- Hell Lyman-alpha
2) besides photoelectric absorption, resonant absorption by H and He Lyman series will produce a sawtooth modulation of the spectrum.

\[ \nu = \nu_\alpha \]

\[ \nu = \nu_\beta \]

Continuum for free electrons

Balmer Lines

Lyman Lines

Ground State = Lowest Energy Level

\( H_\alpha \), \( H_\beta \), \( H_\gamma \)

Lyman Lines

\( \text{Ly} \alpha \), \( \text{Ly} \beta \), \( \text{Ly} \gamma \)

13.6 eV

\( \log J_\nu, \text{(arbitrary units)} \)

photon energy (eV)
J-solution flow chart

**ABSORBERS**
- HI distribution

**SOURCES**
- QSO/GAL LF
- SED

**cosmological radiative transfer**

\( \rightarrow J \)
J-solution flow chart

**ABSORBERS**
- HI distribution
- local radiative transfer ➔ H/He ionization state

**SOURCES**
- QSO/GAL LF
- SED

- cosmological radiative transfer ➔ J
J-solution flow chart

**ABSORBERS**

- HI distribution
- local radiative transfer $\rightarrow$ H/He ionization state
- $\tau_{eff}$, $\varepsilon_{rec}$

**SOURCES**

- QSO/GAL LF
- SED
- cosmological radiative transfer $\rightarrow$ J
HI distribution ➔ local radiative transfer ➔ H/He ionization state

\( \tau_{\text{eff}}, \varepsilon_{\text{rec}} \) ➔ cosmological radiative transfer ➔ \( J \)

J-solution flow chart

ABSORBERS

HI distribution

SOURCES

QSO/GAL LF

SED
**J-solution flow chart**

**ABSORBERS**

- HI distribution

**SOURCES**

- QSO/GAL LF
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**Local radiative transfer ➔ H/He ionization state**

- $\tau_{\text{eff}}$, $\varepsilon_{\text{rec}}$

**Cosmological radiative transfer ➔ J**

**UVB**
HI distribution

local radiative transfer $\rightarrow$ H/He ionization state

$\tau_{\text{eff}}, \varepsilon_{\text{rec}}$

cosmological radiative transfer $\rightarrow$ J

UVB

QSO/GAL LF

SED

J-solution flow chart
Haardt & PM 2020, in preparation
http://pism.ucolick.org/CUBA

THE END