Sunrise:
Panchromatic SED Models of Simulated Galaxies

Lecture 3:
Monte Carlo Radiation Transfer

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Lecture outline


Lecture 2: Sunrise work flow. Parameters, convergence, other subtleties.

Lecture 3: Radiation transfer theory. Monte Carlo. Polychromatic MC.

The equation of radiative transfer

\[
\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{k} \cdot \nabla I_\nu = \frac{1}{4\pi} \rho j_\nu - \rho \kappa_\nu^{\text{abs}} I_\nu - \rho \kappa_\nu^{\text{sca}} I_\nu + \rho \kappa_\nu^{\text{sca}} \int \phi_\nu(\hat{k}, \hat{k}') I_\nu(\hat{k}') d\Omega'
\]

where

\[
dE = I_\nu(\hat{k}, x, t) \hat{k} \cdot dA \, d\Omega \, d\nu \, dt
\]

Ouch... Simplify by ignoring time dependence and only looking at the intensity in a specific direction:

\[
\frac{dI_\nu}{dx} + \rho \kappa_\nu I_\nu = \rho \left( \frac{j_\nu}{4\pi} + \kappa_\nu^{\text{sca}} \Phi_\nu \right)
\]
from
\[ \frac{dI_\nu}{dx} + \rho \kappa_\nu I_\nu = \rho \left( \frac{j_\nu}{4\pi} + \kappa_\nu^{\text{sca}} \Phi_\nu \right) \]
define the optical depth
\[ d\tau = \rho \kappa_\nu \, dx \]
and we get
\[ \frac{dI_\nu}{d\tau} + I_\nu = \frac{j_\nu}{4\pi \kappa_\nu} + \frac{\kappa_\nu^{\text{sca}}}{\kappa_\nu} \Phi_\nu \equiv S_\nu \]
which looks better...

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\[ \frac{dI_\nu}{d\tau} + I_\nu = \frac{j_\nu}{4\pi\kappa_\nu} + \frac{\kappa_\nu^{\text{sca}}}{\kappa_\nu} \Phi_\nu \equiv S_\nu \]

the “source function”

without sources, we quickly see that

\[ I(\tau) = I_0 e^{-\tau} \]

the canonical result that the intensity decreases exponentially with optical depth
with sources and absorbers:

it's like each source is independently attenuated according to

\[ I(\tau) = I_0 e^{-\tau} \]

(superposition of solution from different sources)
seems pretty simple
what’s the big deal then?

that’s in 1D, monochromatic...

\[
\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \mathbf{\hat{k}} \cdot \nabla I_\nu = \frac{1}{4\pi} \rho j_\nu - \rho \kappa_\nu^{\text{abs}} I_\nu - \rho \kappa_\nu^{\text{sca}} I_\nu + \rho \kappa_\nu^{\text{sca}} \int \phi_{\nu}(\mathbf{\hat{k}}, \mathbf{\hat{k}'}) I_{\nu}(\mathbf{\hat{k}'}) d\Omega'
\]

emissivity can depend on intensity at other wavelengths (like a heated blackbody...)

scattering couples different directions

This makes it hard! do it numerically
\[ \frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{k} \cdot \nabla I_\nu = \frac{1}{4\pi} \rho_j \sigma T^4 - \rho \kappa_{\text{abs}} I_\nu - \rho \kappa_{\text{sca}} I_\nu + \rho \kappa_{\text{sca}} \int \phi_\nu(\hat{k}, \hat{k}') I_\nu(\hat{k}') d\Omega' \]

The intensity depends on 6 independent variables - position, direction, and wavelength! (and time too, in some cases)

If you try to solve it with a normal finite-difference scheme (like a hydro code), you’ll get nowhere!

Must be smarter...
\[
\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{k} \cdot \nabla I_\nu = \frac{1}{4\pi} \rho j_\nu - \rho \kappa_\nu^{\text{abs}} I_\nu - \rho \kappa_\nu^{\text{sca}} I_\nu + \rho \kappa_\nu^{\text{sca}} \int \phi_\nu(\hat{k}, \hat{k}') I_\nu(\hat{k}') d\Omega'
\]

- if the optical depth is large, the mean free path is small and photons scatter so many times they forget where they came from
- the problem then reduces to a diffusion problem ("diffusion approximation")
- if the radiation is absorbed and re-emitted repeatedly, the radiation field at any point looks like the emission at that point
- this is called "local thermal equilibrium" (LTE)
- now 3-D, one variable problem
- this is the case in, e.g., the interior of stars

\[dT/dr \sim K r^{-2} T^{-3}\]

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Other Approaches

\[ \frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{k} \cdot \nabla I_\nu = \frac{1}{4\pi} \rho j_\nu - \rho \kappa_\nu^{\text{abs}} I_\nu - \rho \kappa_\nu^{\text{sca}} I_\nu + \rho \kappa_\nu^{\text{sca}} \int \phi_\nu(\hat{k}, \hat{k}') I_\nu(\hat{k}') d\Omega' \]

- Discretize directions and integrate along rays
- one grid cell ("Short Characteristics")
- to the edge ("Long Characteristics")
- Use moments of the intensity and write equations in terms of mean intensity and flux
- "Variable Eddington Tensor" methods
- Sample the radiation field statistically
- "Monte Carlo" methods

(note how suspiciously many papers are from Los Alamos...)

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solve RTE “like nature does”
randomly emit “photons” from sources
scatter/absorb them according to opacity
make an image from rays that reach the observer
as rays traverse the volume, they sample the radiation intensity distribution
Monte Carlo Radiation Transfer

Advantages:
- very general
- easily handles arbitrary geometries or complicated media (scattering characteristics)

Disadvantages:
- solution contains Poisson noise
- converges as $\sqrt{N}$, slowly
- fails in the limit of very large $\tau$
- computationally expensive
MC example: photon propagation

Remember $I(\tau) = I_0 e^{-\tau}$?

this means that

$$\left| \frac{dI}{I_0}(\tau) \right| = e^{-\tau} d\tau$$

the probability of absorption is

$P(\text{absorption between } \tau, \tau + d\tau) = e^{-\tau}$

i.e.: the length a photon goes before it interacts is a random variable distributed as $e^{-\tau}$
Other processes

- in this way, sample the relevant processes
  - position and direction of emission
  - length of propagation
  - direction of scattering

- If you can apply an analytic solution instead of sampling it with MC, do it
  - example: use grain albedo to change the statistical “weight” instead of separately sampling absorption and scattering
“next event estimator” or “peel-off”

Problem: if we randomly sample the ray random walk, practically none will reach the “camera”

increase the efficiency (by a lot) by calculating the probability that a ray will reach the camera

Example: scattering

\[ F_{i,1} = L_i I_{i,1} e^{-\tau_{i,1}^{\text{obs}}} \Phi_s(\hat{r}_{i,0}, \hat{r}_{i,1}^{\text{obs}}) \frac{1}{d_{i,1}^2} \]

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"Forced scattering"

If medium is very optically thin, most rays pass through without scattering

- Poor signal in the scattered light

Can calculate analytically what fraction of the ray will leave and which will scatter *somewhere* on the way

- The location of the scattering event is then drawn from $[0, \tau_{\text{exit}}]$
“Russian Roulette”

If a ray scatters many times, its intensity becomes low.

Don’t want to keep tracking a bunch of rays that won’t make much contribution.

But to preserve energy conservation, we can’t just drop the ray.

Ray with $I < 0.01$

$P = \frac{5}{6}$

$I = I \times 6$

$P = \frac{1}{6}$

carry on

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All these distributions depend on wavelength, so a separate random walk is necessary for each wavelength.

In Sunrise, the computational cost of tracing the ray is dominated by walking the ray through the octree. This means:

- wavelength resolution is expensive!
- Uncorrelated noise in spectra
- Can we do something more efficient?
Biased sampling

**Biasing** – drawing from a different distribution than that sampled

Suppose we want to sample \( f(x) \)

We can do that while drawing from \( g(x) \)

IF we also weight every sample \( x_i \) by \( w_i = \frac{f(x_i)}{g(x_i)} \)

only requirement is that \( g(x) > 0 \) \( \forall x \) where \( f(x) > 0 \)

Can draw numbers from a gaussian distribution

**trivial**

but: what if we need to sample the wings??

we can also draw numbers from a **uniform** distribution

by giving each sample a gaussian weight
Polychromatic ray tracing

- Mean free path varies with wavelength
- Can only draw scattering point correctly for one wavelength - $\lambda_{ref}$
- The other wavelengths are weighted according to the probability of them interacting at the drawn point
- Converges to correct distribution for all wavelengths
Polychromatic ray tracing

Probability of wavelength $\lambda$ interacting at $\tau(\lambda)$ is

$$dP[\tau(\lambda)] = e^{-\tau(\lambda)} \, d\tau(\lambda) = e^{-(\tau(\lambda)/\tau_{\text{ref}}) \tau_{\text{ref}}} \left[ \frac{\tau(\lambda)}{\tau_{\text{ref}}} \right] \, d\tau_{\text{ref}}$$

so if we sample it like $e^{-\tau_{\text{ref}}} \, d\tau$ we need to weight it by

$$w_\lambda = \frac{P[\tau(\lambda)]}{P[\tau_{\text{ref}}]} = e^{\tau_{\text{ref}} - \tau(\lambda)} \left[ \frac{\tau(\lambda)}{\tau_{\text{ref}}} \right]$$
Polychromatic ray tracing

- Now each wavelength is not a separate random walk but instead just a vector operation – much faster!
- No (uncorrelated) noise between wavelengths
- Makes the very high wavelength resolution feasible

Nothing’s for free though...
Polychromatic ray tracing

Drawback:

\[ w_\lambda = \frac{P[\tau(\lambda)]}{P[\tau_{\text{ref}}]} = e^{\tau_{\text{ref}} - \tau(\lambda)} \left[ \frac{\tau(\lambda)}{\tau_{\text{ref}}} \right] \]

if \( \tau(\lambda) \) very different from \( \tau_{\text{ref}} \)

\( w \) can be large \( \rightarrow \) increased noise

Bad situations:
- very large optical depths
- rapidly changing opacity (e.g. lines!)
- Mitigated by splitting rays
Does this all work?

Pascucci et al. 2004
2D RT benchmark

The other codes did 50 calculations, polychromatic Sunrise did 1...

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Intensity estimator

- Want to estimate the mean radiative intensity in the grid cells (for determining dust temperatures)
- Use the “path length estimator” (Lucy 99)
  \[ J = \sum_{i} (I_i \, dL_i) / (4\pi V) \]
- \( I_i \) is the luminosity carried by ray \( i \), \( dL_i \) the path length through the cell, \( V \) the cell volume
Parallelization

- This method is trivial to parallelize.
- Each processor shoots its own ray, independent of every other.
- Only need to worry about locking shared outputs: camera images and radiation intensities in cells.
- With distributed memory, very different approach is needed: domain needs to be decomposed, rays need to be shifted from processor to processor as they travel.
References

Variable Tensor Methods:
- Gnedin & Abel 2001, New Astronomy, 6, 437

Long Characteristics/ray tracing

Short Characteristics:

Flux-limited diffusion

Monte Carlo
- Dupree & Fraley 2002: “A Monte Carlo Primer”
- Lux & Koblinger 1991: “Monte Carlo Transport Methods”

Radiation Transfer Test cases