Outline

Lectures

• Motivation: neutrinos and the cosmos

(1) • Neutrinos in hot and dense media

• Structure of QKEs from quantum field theory

(II) • Anatomy of the QKEs
  • Coherent evolution: flavor and spin
  • Inelastic collisions

• Comparison to other approaches & future challenges

Talk by A. Vlasenko

• Neutrino-antineutrino transformation in astrophysical environments
Structure of the QKEs

- $F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$
- $\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$

- $iDF = [H, F] + iC$
- $iD\bar{F} = [\bar{H}, \bar{F}] + i\bar{C}$

Derivative along world line: drift & force term

Coherent evolution: vacuum mass & forward scattering (refractive potential)

Inelastic collisions

- $F, H, C$: $2n_f \times 2n_f$ matrices, all components coupled in general
- $D, H, C$ are functionals of $F, \bar{F}$: non-linear system
Interlude on kinematics

- For ultra-relativistic $\gamma$’s of 3-momentum $p$, express all Lorentz tensors in terms of the following basis:

$$n^\mu(p) = (1, \hat{p})$$
$$\bar{n}^\mu(p) = (1, -\hat{p})$$
$$x_{1,2}(p)$$

- light-like
- light-like
- transverse

$$n \cdot n = \bar{n} \cdot \bar{n} = 0$$
$$n \cdot \bar{n} = 2$$
$$n \cdot x_i = \bar{n} \cdot x_i = 0$$
$$x_i \cdot x_j = -\delta_{ij}$$
Interlude on kinematics

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x_{1,2}(p)
\end{align*}
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\begin{align*}
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n \cdot \bar{n} &= 2 \\
n \cdot x_i &= \bar{n} \cdot x_i = 0 \\
x_i \cdot x_j &= -\delta_{ij}
\end{align*}
\]

• Four-vector components along basis vectors:

\[
\begin{align*}
V^\mu &\rightarrow V^\kappa \equiv n \cdot V \\
V^i &\equiv x_i \cdot V \\
\partial^\kappa &\equiv n \cdot \partial \\
\partial^i &\equiv x_i \cdot \partial
\end{align*}
\]
Interlude on potentials

- Neutrino self-energy diagrams → in-medium $4$-vector potential (time-
  and space-like components in non-isotropic medium)
Interlude on potentials

- Neutrino self-energy diagrams → in-medium 4-vector potential (time- and space-like components in non-isotropic medium)

- Computed from neutrino interactions in the Standard Model. Ex: neutrino-matter interaction at low-energy can be put in the form

\[
\mathcal{L}_{\nu\psi} = -G_\psi \bar{\nu} \gamma_\mu P_L Y_\psi \nu - \bar{\psi} \Gamma_\psi^{\mu} \psi
\]

\[
G_\psi = n_\psi \times G_F \sim g^2/M_W^2
\]

\[
P_L = (1 - \gamma_5)/2
\]

\[
\Gamma_\psi^{\mu} = (g_V)_\psi \gamma_\mu - (g_A)_\psi \gamma_\mu \gamma^5
\]

\[
\nu = \begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}, \quad Y_\psi = \begin{pmatrix}
Y_{e\psi} & 0 & 0 \\
0 & Y_{\mu\psi} & 0 \\
0 & 0 & Y_{\tau\psi}
\end{pmatrix}
\]
Interlude on potentials

- Neutrino self-energy diagrams → in-medium 4-vector potential (time- and space-like components in non-isotropic medium)

- $2n_f \times 2n_f$ matrix structure:

$$
\Sigma^\mu (x) = \begin{pmatrix}
\Sigma^\mu_R & 0 \\
0 & \Sigma^\mu_L
\end{pmatrix}
$$

- Induced interaction

$$
\mathcal{L}_{int} = -\bar{\nu}_L \Sigma_R \nu_L - \bar{\nu}_R \Sigma_L \nu_R + h.c.
$$
Interlude on potentials

- Neutrino self-energy diagrams → in-medium 4-vector potential (time-
  and space-like components in non-isotropic medium)

- $2n_f \times 2n_f$ matrix structure:

$$\Sigma^\mu (x) = \begin{pmatrix} \Sigma^\mu_R & 0 \\ 0 & \Sigma^\mu_L \end{pmatrix}$$

- Induced interaction

$$\mathcal{L}_{\text{int}} = - \bar{\nu}_L \Sigma_R \nu_L - \bar{\nu}_R \Sigma_L \nu_R + \text{h.c.}$$
Interlude on potentials

• Neutrino self-energy diagrams → in-medium 4-vector potential (time- and space-like components in non-isotropic medium)

• $2n_f \times 2n_f$ matrix structure:

$$\sum^\mu (x) = \begin{pmatrix} \sum^\mu_R & 0 \\ 0 & \sum^\mu_L \end{pmatrix}$$

Potential for L-handed $\nu$'s:
Potential for R-handed $\nu$'s:
Dirac: $\sum_L \propto G_F m^2 \sim O(\alpha^3)$
Majorana: $\sum_L = -\sum_R^T$

• Induced interaction

$$\mathcal{L}_{\text{int}} = -\bar{\nu}_L \sum_R^{} \nu_L - \bar{\nu}_R \sum_L^{} \nu_R + \text{h.c.}$$
Interlude on potentials

- Neutrino self-energy diagrams → in-medium 4-vector potential (time- and space-like components in non-isotropic medium)

- $2n_f \times 2n_f$ matrix structure:

$$\sum^\mu (x) = \begin{pmatrix} \sum^\mu_R & 0 \\ 0 & \sum^\mu_L \end{pmatrix}$$

- Potential for L-handed $\nu$'s:
- Potential for R-handed $\nu$'s:
  - Dirac: $\Sigma_L \propto G_F m^2 \sim O(e^3)$
  - Majorana: $\Sigma_L = -\Sigma^T_R$

- For a test $\nu$ of momentum $p$, get components

$$\Sigma^k \equiv n(p) \cdot \Sigma$$
$$\Sigma^i \equiv x^i(p) \cdot \Sigma$$

approximately along $\nu$ trajectory

approximately transverse to $\nu$ trajectory
Interlude on potentials

- Neutrino self-energy diagrams → in-medium 4-vector potential (time- and space-like components in non-isotropic medium)

- Explicit form of neutrino-induced $\Sigma_R$:

$$
\left. \Sigma_R^\mu \right|_\nu = \sqrt{2} G_F \left( J_{(\nu)}^\mu + 1 \left( \text{tr} \ J_{(\nu)}^\mu \right) \right)
$$

$$
J_{(\nu)}^\mu (x) = \int \frac{d^3q}{(2\pi)^3} \ n^\mu (q) \left( f_{LL}(\vec{q}, x) - f_{RR}(\vec{q}, x) \right)
$$

$$
n^\mu (q) = (1, \hat{q})$$
Anatomy of the QKEs
Drift & force terms

\[ F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix} \]
\[ \tilde{F} = \begin{pmatrix} \tilde{f}_{RR} & \tilde{f}_{RL} \\ \tilde{f}_{LR} & \tilde{f}_{LL} \end{pmatrix} \]

\[ iD F = [H, F] + iC \]
\[ i\tilde{D} \tilde{F} = [\tilde{H}, \tilde{F}] + i\tilde{C} \]

Derivative along world line:
drift & force term

"Vlasov"
Drift & force terms

\[ DF = \partial^\kappa F + \frac{1}{2|\vec{p}|} \{\Sigma^i, \partial^i F\} - \frac{1}{2} \left\{ \frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial F}{\partial \vec{p}} \right\} \]

\[ \bar{D} \bar{F} = \partial^\kappa \bar{F} - \frac{1}{2|\vec{p}|} \{\Sigma^i, \partial^i \bar{F}\} + \frac{1}{2} \left\{ \frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial \bar{F}}{\partial \vec{p}} \right\} \]

- Simple interpretation if one notes that \( \nu(+) \) and \( \bar{\nu}(-) \) dispersion relations are:

\[ \omega_{\pm} = |\vec{p}| \pm \Sigma^\kappa \]

\[ \Sigma^\kappa \equiv n(p) \cdot \Sigma \]
Drift & force terms

\[ DF = \partial^\kappa F + \frac{1}{2|\vec{p}|} \left\{ \Sigma^i, \partial^i F \right\} - \frac{1}{2} \left\{ \frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial F}{\partial \vec{p}} \right\} \]

\[ \bar{D}F = \partial^\kappa \bar{F} - \frac{1}{2|\vec{p}|} \left\{ \Sigma^i, \partial^i \bar{F} \right\} + \frac{1}{2} \left\{ \frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial \bar{F}}{\partial \vec{p}} \right\} \]

- Simple interpretation if one notes that \( v(+) \) and \( \bar{v}(-) \) dispersion relations are:

\[ \omega_{\pm} = |\vec{p}| \pm \Sigma^\kappa \]

\( \Sigma^\kappa \equiv n(p) \cdot \Sigma \)

- Then one finds:

\[ D = \partial_t + \frac{1}{2} \left\{ \partial_{\vec{p}} \omega_+, \partial_{\vec{x}} \right\} - \frac{1}{2} \left\{ \partial_{\vec{x}} \omega_+, \partial_{\vec{p}} \right\} \]

\[ \bar{D} = \partial_t + \frac{1}{2} \left\{ \partial_{\vec{p}} \omega_-, \partial_{\vec{x}} \right\} - \frac{1}{2} \left\{ \partial_{\vec{x}} \omega_-, \partial_{\vec{p}} \right\} \]

- Generalization of familiar

\[ d_t = \partial_t + \dot{\vec{x}} \cdot \partial_{\vec{x}} + \dot{\vec{p}} \cdot \partial_{\vec{p}} \]

\[ \dot{\vec{x}} = \partial_{\vec{p}} \omega \quad \dot{\vec{p}} = -\partial_{\vec{x}} \omega \]
**Coherent evolution**

\[
\begin{align*}
F & = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix} \\
\bar{F} & = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
iD F & = [H, F] + iC \\
i\bar{D} \bar{F} & = [\bar{H}, \bar{F}] + i\bar{C}
\end{align*}
\]

Coherent evolution: vacuum mass & forward scattering (refractive potential)

“MSW”
Coherent evolution

\[
F = \begin{pmatrix}
  f_{LL} & f_{LR} \\
  f_{RL} & f_{RR}
\end{pmatrix}
\]
\[
\bar{F} = \begin{pmatrix}
  \bar{f}_{RR} & \bar{f}_{RL} \\
  \bar{f}_{LR} & \bar{f}_{LL}
\end{pmatrix}
\]

\[
i D F = [H, F] + i \mathcal{C}
\]
\[
i D \bar{F} = [\bar{H}, \bar{F}] + i \mathcal{C}
\]

- Often written in the equivalent form of a Schrodinger-like equation for “ν flavor wave-function”

- Mapping of the two approaches: off-diagonal entries in \( f_{LL} \) encode information about relative QM phases

- Not clear how to include inelastic collisions in wave-function approach
Coherent evolution

- Controlled by $2n_f \times 2n_f$ Hamiltonian-like operators

\[
H = \begin{pmatrix} H_R & H_m \\ H_m^\dagger & H_L \end{pmatrix} \quad \bar{H} = \begin{pmatrix} \bar{H}_R & H_m \\ H_m^\dagger & \bar{H}_L \end{pmatrix}
\]

\[
\bar{H}_R = \Sigma_R^\kappa \pm \frac{1}{2|\vec{p}|} \left( m^\dagger m - \epsilon_{ij} \partial^i \Sigma_R^j + 4 \Sigma_R^+ \Sigma_R^- \right)
\]

\[
\bar{H}_L = \Sigma_L^\kappa \pm \frac{1}{2|\vec{p}|} \left( m m^\dagger + \epsilon_{ij} \partial^i \Sigma_L^j + 4 \Sigma_L^- \Sigma_L^+ \right)
\]
Coherent evolution

- Controlled by $2n_f \times 2n_f$ Hamiltonian-like operators

\[
H = \begin{pmatrix} H_R & H_m \\ H_m^\dagger & H_L \end{pmatrix} \quad \bar{H} = \begin{pmatrix} \bar{H}_R & H_m \\ H_m^\dagger & \bar{H}_L \end{pmatrix}
\]

\[
\bar{H}_R = \Sigma_R^\kappa \pm \frac{1}{2|p|} \left( m^\dagger m - \epsilon^{ij} \partial^i \Sigma_R^j + 4 \Sigma_R^+ \Sigma_R^- \right)
\]

\[
\bar{H}_L = \Sigma_L^\kappa \pm \frac{1}{2|p|} \left( m m^\dagger + \epsilon^{ij} \partial^i \Sigma_L^j + 4 \Sigma_L^- \Sigma_L^+ \right)
\]

Standard vacuum mass term + medium refraction (included in all analyses)

\[
\Sigma_R^\kappa(x) = \sqrt{2} G_F \int \frac{d^3 q}{(2\pi)^3} n(p) \cdot n(q) \left( f_{LL}(\vec{q}, x) - \bar{f}_{RR}(\vec{q}, x) \right) 1 - \cos \theta_{\vec{p}\vec{q}}
\]
Coherent evolution

- Controlled by $2n_f \times 2n_f$ Hamiltonian-like operators

$$H = \begin{pmatrix} H_R & H_m \\ H_m^\dagger & H_L \end{pmatrix} \quad \bar{H} = \begin{pmatrix} \bar{H}_R & H_m \\ H_m^\dagger & \bar{H}_L \end{pmatrix}$$

$$\bar{H}_R = \Sigma_R^\kappa \pm \frac{1}{2|\vec{p}|} \begin{pmatrix} m^\dagger m \\ mm^\dagger \end{pmatrix} + \epsilon^{ij} \partial^i \Sigma_R^j + 4\Sigma_R^+ \Sigma_R^-$$

$$\bar{H}_L = \Sigma_L^\kappa \pm \frac{1}{2|\vec{p}|} \begin{pmatrix} mm^\dagger \\ m^\dagger m \end{pmatrix} + \epsilon^{ij} \partial^i \Sigma_L^j + 4\Sigma_L^- \Sigma_L^+$$

Standard vacuum mass term + medium refraction (included in all analyses)

Additional $O(\varepsilon^2)$ terms if potential has space-like components

$$\Sigma_{L,R}^\pm \equiv \frac{1}{2} (\Sigma_{L,R}^1 \pm i\Sigma_{L,R}^2)$$
Coherent evolution

- Controlled by $2n_f \times 2n_f$ Hamiltonian-like operators

$$H = \begin{pmatrix} H_R & H_m \\ H_m^\dagger & H_L \end{pmatrix} \quad \tilde{H} = \begin{pmatrix} \tilde{H}_R & H_m \\ H_m^\dagger & \tilde{H}_L \end{pmatrix}$$

$$H_m = -\frac{1}{|\vec{p}|} \left( \sum_R^+ m^\dagger - m^\dagger \sum_L^+ \right)$$

- Qualitatively new $O(\xi^2)$ effect: coherent conversion of LH ↔ RH $\nu$'s
  - Need anisotropic environment (transverse component of $\Sigma$)
  - Need axial components, coupling to spin
    \[ 1\text{-flavor} \quad H_m \sim m_\nu/p (\Sigma_R - \Sigma_L)^+ \]
  - Potentially big impact: Dirac (active-sterile) vs Majorana ($\nu$-$\bar{\nu}$)
More on spin-mixing term

- Effect can be derived using effective hamiltonian approach

\[ \langle i, \vec{p}', h' | j, \vec{p}, h \rangle \equiv -i(2\pi)^2 \frac{2|\vec{p}|}{4!} \delta^{(4)}(p - p') \mathcal{H}^{ij}_{h'h}(p) \]

- Use medium-modified neutrino Lagrangian in perturbation theory

\[ \mathcal{L}_{\text{int}} = -\bar{\nu}_L m \nu_R - \bar{\nu}_L \Sigma_R \nu_L - \bar{\nu}_R \Sigma_L \nu_R + \text{h.c.} \]
More on spin-mixing term

- Effect can be derived using effective hamiltonian approach

\[
\langle i, p', h' \mid j, p, h \rangle \equiv -i(2\pi)^2 2|p| \delta^{(4)}(p - p') \mathcal{H}^{ij}_{h' h}(p)
\]

- Use medium-modified neutrino Lagrangian in perturbation theory

\[
\mathcal{L}_{\text{int}} = -\bar{\nu}_L m \nu_R - \bar{\nu}_L \Sigma_R \nu_L - \bar{\nu}_R \Sigma_L \nu_R + \text{h.c.}
\]

* \( \Sigma_{L,R} \): medium-induced vector potentials

* Even in simple “bulb” model for SN:

\[
\tilde{\Sigma}_R(x) \neq 0
\]
More on spin-mixing term

- Effect can be derived using effective hamiltonian approach

\[
\langle i, \vec{p}', h' | j, \vec{p}, h \rangle \equiv -i(2\pi)^2 \frac{2}{2|\vec{p}|} \delta^{(4)}(p - p') \mathcal{H}_{h', h}^{ij}(p)
\]

- Use medium-modified neutrino Lagrangian in perturbation theory

\[
\mathcal{L}_{\text{int}} = -\bar{\nu}_L m \nu_R - \bar{\nu}_L \bar{\Sigma}_R \nu_L - \bar{\nu}_R \Sigma_L \nu_R + \text{h.c.}
\]

- 1-flavor result

\[
\begin{align*}
\mathcal{H}_{LL}(p) &= \Sigma_0^0 - \bar{\Sigma}_R \cdot \hat{p} \\
\mathcal{H}_{RR}(p) &= \Sigma_0^0 - \bar{\Sigma}_L \cdot \hat{p} \\
\mathcal{H}_{LR}(p) &= -\frac{m}{2|\vec{p}|} \bar{\Sigma}_A \cdot \vec{x}_+(p)
\end{align*}
\]

Axial potential

\[
\Sigma_A^\mu = \Sigma_L^\mu - \Sigma_R^\mu
\]

\downarrow

medium birefringence +

mixing (transverse part)
More on spin-mixing term

- Effect can be derived using effective hamiltonian approach

\[ \langle i, \vec{p}', h' \mid j, \vec{p}, h \rangle \equiv -i(2\pi)^2 2|\vec{p}| \delta^{(4)}(p-p') \mathcal{H}_{h',h}^{ij}(p) \]

- Similar mixing is induced by magnetic moment (Dirac for simplicity)

\[ \Delta \mathcal{L} = \left( \frac{\mu_\nu}{2} \right) \bar{\nu}_R \sigma_{\mu\nu} F^{\mu\nu} \nu_L + \text{h.c.} \]

- 1-flavor result

\[ \mathcal{H}_{LR}(p) = \mu_\nu \vec{B} \cdot \vec{x}_+(p) \]

See de Gouvea & Shalgar for impact on SN neutrino collective oscillations
Inelastic collisions

\[ F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix} \]

\[ \bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix} \]

\[ i\mathcal{D}F = [H, F] + i\mathcal{C} \]

\[ i\bar{\mathcal{D}}\bar{F} = [\bar{H}, \bar{F}] + i\bar{\mathcal{C}} \]
Inelastic collisions

• Controlled by $2n_f \times 2n_f$ gain and loss potentials $\Pi^\pm[F, \bar{F}, f_{e,n,p,\ldots}]$

\[
C = \frac{1}{2} \{\Pi^+, F\} - \frac{1}{2} \{\Pi^-, I - F\}
\]

\[
\bar{C} = \frac{1}{2} \{\bar{\Pi}^+, \bar{F}\} - \frac{1}{2} \{\bar{\Pi}^-, I - \bar{F}\}
\]

• $\Pi^\pm$ are non-diagonal in both flavor and spin ($\rightarrow$ decoherence)
- Example: \( C_{\text{LL}} \) (upper \( n_f \times n_f \) block) induced by neutrino scattering off medium particles (e,p,n,...) in isotropic environment

\[
Y = \begin{pmatrix}
Y_e & 0 & 0 \\
0 & Y_\mu & 0 \\
0 & 0 & Y_\tau
\end{pmatrix}
\]

Medium response function (knows about medium particle distributions and their interactions)

\[
C_{\text{LL}}(\vec{p}) = - \int d^3\vec{p}' \, R(\vec{p}, \vec{p}') \left\{ Y \left(1 - f_{\text{LL}}(\vec{p}')\right) Y, f_{\text{LL}}(\vec{p}) \right\} + \text{gain}
\]
Example: $C_{LL}$ (upper $n_f \times n_f$ block) induced by neutrino scattering off medium particles (e,p,n,...) in isotropic environment

$$V_\alpha \, V_\alpha$$

$$G_F \, Y_\alpha$$

$$Y = \begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \end{pmatrix}$$

Medium response function (knows about medium particle distributions and their interactions)

$$C_{LL}(\vec{p}) = -\int d^3p' \, R(\vec{p}, \vec{p}') \left\{ Y \left(1-f_{LL}(\vec{p}')\right) Y \right\} + \text{gain}$$

$$\neq \begin{pmatrix} C_e & 0 & 0 \\ 0 & C_\mu & 0 \\ 0 & 0 & C_\tau \end{pmatrix}$$
Comparison with other QKEs

\[ F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix} \]

\[ iDF = [H, F] + iC \]

- Restricting to \( f_{LL} \) and isotropic media, equivalent to Sigl-Raffelt

NPB 406, 423 (1993)
Comparison with other QKEs

\[ F = \left( \begin{array}{ll} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{array} \right) \]

\[ iDF = [H, F] + iC \]

- Restricting to \( f_{LL} \) and isotropic media, equivalent to Sigl-Raffelt

- Similar in form to Strack-Burrows and Zhang-Burrows

\[ \frac{\partial \mathcal{F}}{\partial t} + \vec{v} \cdot \frac{\partial \mathcal{F}}{\partial \vec{r}} + \vec{p} \cdot \frac{\partial \mathcal{F}}{\partial \vec{p}} = -i[H, \mathcal{F}] + C \]

But \( H, C, \vec{p} \) are quite different

NPB 406, 423 (1993)

hep-ph/0504035
Comparison with other QKEs

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But \( H, C, \mathbf{p} \) are quite different

- Quite different from Volpe et al., who include “abnormal densities” (correlations of \( \mathbf{v} \) and \( \mathbf{\bar{v}} \) of opposite momentum) and discuss their coherent evolution coupled to “normal densities”. We do not include this, based on \( L_{\text{gradients}} >> L_{\text{deBroglie}} \)
Summary & future challenges

• Neutrino QKEs can be formulated from QFT + power counting in ratio of length scales \((L_{\text{osc}}, L_{\text{mfp}}, L_{\text{gradients}} \gg L_{\text{deBroglie}})\)

• Many expected features, some surprising ones (spin oscillations). See A. Vlasenko’s talk for first applications to astrophysics

• Challenges:
  • Explicit form of the collision term (in progress)
  • Computational implementation
Summary & future challenges

- Neutrino QKEs can be formulated from QFT + power counting in ratio of length scales (\(L_{\text{osc}}, L_{\text{mfp}}, L_{\text{gradients}} \gg L_{\text{deBroglie}}\))

- Many expected features, some surprising ones (spin oscillations). See A. Vlasenko’s talk for first applications to astrophysics

- Challenges:
  - Explicit form of the collision term (in progress)
  - Computational implementation

\[
\begin{align*}
\text{Early Universe: } & F(x, p) \rightarrow F(t, |p|) \rightarrow F_{|p|}(t) \\
\text{isotropy: } & \text{no L-R coherence} \\
\text{binning: } & \\
2 \ast (n_f)^2 \ast n_{|p|} & \text{coupled ODEs, initial value problem}
\end{align*}
\]
Summary & future challenges

- Neutrino QKEs can be formulated from QFT + power counting in ratio of length scales \((L_{osc}, L_{mfp}, L_{gradients} >> L_{deBroglie})\)

- Many expected features, some surprising ones (spin oscillations). See A. Vlasenko’s talk for first applications to astrophysics

- Challenges:
  - Explicit form of the collision term (in progress)
  - Computational implementation

Supernovae with spherical symmetry:

\[
\begin{pmatrix}
  f \\
  \phi \\
  f^T
\end{pmatrix}
\]

\[F(x,p) \rightarrow F(r,|p|,\theta) \rightarrow F_{|p|,\theta}(r)\]

\[4*(n_f)^2*|p|*n_\theta\] coupled ODEs, boundary value problem