Dense Matter and Neutrinos

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- Neutron Stars and QCD phase diagram
- Nuclear Interactions
- Quantum Monte Carlo
- Low-Density Equation of State
- High-Density Equation of State
- Neutron Star Matter
  (protons, hyperons, etc.)
- Mass/Radius relations and observations
- Neutrinos - neutron star cooling
- Future

Collaborators:
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- A. Gezerlis (Guelph)
- D. Lonardoni (ANL)
- A. Lovato (ANL)
- S. Reddy (UW/INT)
Neutron Stars

1-2 Solar Masses
~12 km radius

Outer Crust
nuclei + electrons

Inner Crust
nuclei + neutrons + electrons

Core
neutrons + protons + electrons + …

We will concentrate on the core: bulk of the star dominates the M/R curve important for neutrino cooling

charge neutrality + small electron mass \(\rightarrow\) \(~10\%\) electrons, protons

Predicted by Baade and Zwicky 1 year after discovery of the neutron
QCD phase diagram (minimal)

from FAIR, new facility in Darmstadt

high density and cold very difficult to reach in experiments
Color superconductor at very high density; important for neutron stars?
Neutron Star Mass/Radius Relations

For many years only ~1.4-1.5 solar mass neutron stars observed
Recently several observed with ~ 2 solar masses!

Transitions to superconducting quark matter
Wide range of predictions for mass/radius relationship

see Demorest, et al
Nature 467: 2081 (2010)
Nuclear Interactions

Up to $\sim 2-3 \times$ nuclear densities, matter can be described as a system of interacting nucleons.

Phase shifts for NN scattering - simple model (AV8’)
compared to experiment

At $r=2 \rho_0: k_F \sim 2 \text{ fm}^{-1}$
implies 2 nucleons at Fermi surface have $E_{\text{CM}} = 160 \text{ MeV}; E_{\text{lab}} \sim 320 \text{ MeV}$. 

\[ \uparrow \iff \rho_0 \implies 2 \text{ nucleons at Fermi surface have } E_{\text{CM}} = 160 \text{ MeV}; E_{\text{lab}} \sim 320 \text{ MeV} \]
Nuclear Interactions

Very low densities dominated by $^1S_0$ interaction

Very similar to cold atomic Fermi Gases

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_0 \delta(r_{ij})$$

Neutron-Neutron Scattering length $\sim -18$ fm

pion + 2-pion + short-range repulsion
Quantum Monte Carlo Methods

\[
H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij} + \ldots
\]

\[
V_{ij} = \sum_k V_{ij}^k(r_{ij}) O_{ij}^k
\]

\[
O_{ij}^k = [1, \sigma_i \cdot \sigma_j, \sigma_i \cdot r_{ij} \sigma_j \cdot r_{ij}, L \cdot S_{ij}] \times [1, \tau_i \cdot \tau_j]
\]

\[
H \Psi = E \Psi
\]

\[
2^A = 7 \times 10^{19} \text{ amplitudes for } 66 \text{ neutrons}
\]

\[
\Psi = \sum_{i=1}^{2^A(A/Z)} \psi(i)(R)
\]

in 3A=198 dimensions
Quantum Monte Carlo (Auxiliary Field Diffusion Monte Carlo)

\[ \Psi_0 = \exp[-H\tau] \Psi_T \]

\[ \exp[-H\tau] \approx \exp[-V\tau/2] \exp[-T\tau] \exp[-V\tau/2] \]

Kinetic Term is a diffusion process in 3A coordinates
Spin-dependent potential terms rewritten as coupled
to an auxiliary field which is sampled by Monte Carlo,
giving rotations of spins (and isospins)

\[ \exp[-V\sigma_i \cdot \sigma_j \tau] = \sum_{x=\pm1} \exp[-V^{1/2} \tau^{1/2} \sigma_i \cdot x] \exp[-V^{1/2} \tau^{1/2} \sigma_j \cdot x] \]

The simulation is a branching random walk in 3A coordinates and A spins and isospins.
Equation of State (E/A) for neutrons and cold Fermi atoms

Figure 3. Comparison of the equation of state of cold atoms and neutron matter at low density. Neutron matter calculations are from Ref. 14. Differences at low density are primarily due to the effective range of the neutron-neutron interaction. The solid line is a fit to the cold atom results, the dashed line includes an estimate of effective range effects (see text).

Figure 4 shows the DMC calculations of \( \langle \varepsilon \rangle \) for different mass ratios. Initial calculations for different mass ratios were reported in Ref. 49. From Eq. 11 we can see that the energy change can be evaluated in perturbation theory near \( r = 1 \).

\[
\left( \frac{E}{N} \right) = \frac{1}{2} \left( \frac{P^2}{4m} \right)_{ij} \left( \frac{r_1}{(r_1 + 1)^2} \right),
\]

where in the last line the particles have arbitrarily been divided into \( N/2 \) spin up - spin down pairs.
Superfluidity (s-wave)

Spin up, down densities in a trap

\[ \delta = 0.45(05) \]

JC and Reddy, PRL 2007
analyzing MIT data

\[ \Delta = \delta \frac{\hbar^2 k^2}{2m} \]

\[ (k_{min}/k_f)^2 = 0.80(10) \]

JC and Reddy, PRL 2005
Superfluid Pairing Gap

The fact that the pairing gap is so large, a sizable fraction of the Fermi energy, makes it possible to use QMC methods to accurately calculate the gap by separately calculating the energies of the even and odd particle systems. In addition, the fact that the energy per particle shows no significant shell effects for reasonably small systems ($N > 30$) makes it much easier to approach the continuum limit. Though there is an upper bound principle for the even and odd systems, there is no specific bound on the pairing gap.

The original calculations of the pairing gap in cold atoms at unitarity found $\Delta / E_F \approx 0.9$ or $\approx 0.55(5)$. Subsequent improvements to the wave function found a slightly reduced value for the gap, $\approx 0.50(5)$. These results can be compared to an extraction of the pairing gap from the measured density distributions in partially spin-polarized trapped cold atoms and measurements of the RF response in such systems, who find $\approx 0.45(5)$ and $\approx 0.44(3)$, respectively.

The pairing gap in neutron matter has historically been the subject of a great deal of interest and theoretical activity. QMC calculations of the pairing gap were performed in $^{13}$ and $^{14}$. These calculations used the $s$-wave and $s$-$p$-wave components of the AV18 interaction, respectively. A summary of the results are shown in Fig. 6.

In the figure, BCS results are given by solid lines. In the weak-coupling limit, the pairing gap is expected to be reduced from the BCS value by $(1/4 \pi)^{1/3}$.

Cold Atoms have highest superfluid gap / $E_F$ of any system; Neutrons have highest pairing gap / $E_F$ in nature.
Equation of State at Higher Densities: near nuclear saturation

From experiments:

\[ E_{SNM}(\rho_0) = -16\, MeV, \quad \rho_0 = 0.16\, fm^{-3}, \quad E_{sym} = E_{PNM}(\rho_0) + 16 \]

The symmetry energy is accessible (indirectly) by experiment
At higher densities three-nucleon interactions start to become important.

+ short-range correlations (spin/isospin independent).

Calibrated to light nuclei
We consider different forms of three-neutron interaction by only requiring a particular value of $E_{\text{sym}}$ at saturation.

Consider a wide range of three-nucleon forces that give the same symmetry energy and then see how they extrapolate to high density.
Equations of state with a fixed symmetry energy

\[ E_{\text{sym}} = 35.1 \text{ MeV (AV8'+UIX)} \]
\[ E_{\text{sym}} = 33.7 \text{ MeV} \]
\[ E_{\text{sym}} = 32 \text{ MeV} \]
\[ E_{\text{sym}} = 30.5 \text{ MeV (AV8')} \]

Gandolfi, Carlson, Reddy (2012)

Gandolfi, Carlson, Reddy 2012
From the EOS, we can fit the symmetry energy around $\rho_0$ using
$$E_{\text{sym}}(\rho) = E_{\text{sym}} + L \rho_0 \rho^3 + \cdots$$

$E_{\text{sym}} = 33.7 \text{ MeV}$

$E_{\text{sym}} = 32.0 \text{ MeV}$

Gandolfi et al., EPJ (2014)

Strong Correlation between Symmetry Energy and its Derivative

New chiral interaction models give very similar results
Fits to nuclear masses

Fig. 1.— Comparison of confidence intervals for nuclear mass fitting. Solid figures are the UNEDF0 68% and 95% confidence intervals of Kortelainen et al. (2010) assuming $\text{UNEDF}_0 = 2 \text{ MeV}$. The dashed and dotted figures denote the 68% confidence intervals for a liquid droplet fit assuming $\text{LD} = 0.5 \text{ MeV}$ and a Thomas-Fermi finite-range fit assuming $\text{FRTF} = 1.6 \text{ MeV}$, respectively. Circles mark values of $S_v$ and $L$ at the respective $2 \text{ minima}$. The solid line is the correlation of Farine, Pearson and Rouben (1978) and the dashed line is the correlation of Oyamatsu and Iida (2003). The diamond (Myers and Swiatecki 1990) and triangle (M"oller et al. 2012) show finite-range liquid droplet mass fits.
Fig. 2.— Summary of constraints on symmetry energy parameters. The filled ellipsoid indicates joint $S_v$ and $L$ constraints from nuclear masses (Kortelainen et al. 2010). Filled bands show constraints from neutron skin thicknesses of Sn isotopes (Chen et al. 2010), the dipole polarizability of $^{208}$Pb (Piekarewicz et al. 2012), giant dipole resonances (GDR) (Trippa, Colò and Vigezzi 2008), and isotope diffusion in heavy ion collisions (HIC) (Tsang et al. 2009). The hatched rectangle shows constraints from fitting astrophysical $M_R$ observations (Steiner, Lattimer and Brown 2010, 2013). The two closed regions show neutron matter constraints (H is Hebeler et al. (2010) and G is Gandolfi, Carlson and Reddy (2012)). The enclosed white area is the experimentally-allowed overlap region.
TOV equations:

\[
\frac{dP}{dr} = - \frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]},
\]

\[
\frac{dm(r)}{dr} = 4\pi \epsilon r^2,
\]

from Lattimer

Tolman Oppenheimer Volkov equations: 1939

used free neutron gas to estimate upper bound of 0.7 solar masses

see Silbar and Reddy: arXiv:nucl-th/0309041 for an introduction
EOS used to solve the TOV equations.

Causality: $R > 2.9 \ (GM/c^2)$

$\rho_{\text{central}} = 2 \rho_0$

$\rho_{\text{central}} = 3 \rho_0$

$E_{\text{sym}} = 30.5 \text{ MeV (NN)}$

Neutron stars

Observations of the mass-radius relation are becoming available:

<table>
<thead>
<tr>
<th>Star</th>
<th>Mass (M_\odot)</th>
<th>Radius (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4U 1608-52</td>
<td>1.5</td>
<td>13</td>
</tr>
<tr>
<td>EDO 1745-248</td>
<td>1.6</td>
<td>13.5</td>
</tr>
<tr>
<td>4U 1820-30</td>
<td>2.2</td>
<td>14</td>
</tr>
<tr>
<td>M13</td>
<td>1.5</td>
<td>12.5</td>
</tr>
<tr>
<td>a Cen</td>
<td>1.4</td>
<td>13</td>
</tr>
<tr>
<td>X7</td>
<td>1.6</td>
<td>14</td>
</tr>
</tbody>
</table>

Mass radius constraints subject to assumptions

Observations - still controversial

constraints from individual stars
observations from
3 X-ray bursars
plus 3 low-mass X-ray binaries

Comparison of theory and observations

32 < \( E_{\text{sym}} \) < 34 MeV, 43 < \( L \) < 52 MeV

Steiner, Gandolfi, PRL (2012).
What about other particles? protons

Quadratic dependence of $E$ versus $n/p$ imbalance

Asymmetric nuclear matter $E(\rho, x) = E_{SNM}(\rho) + E^{(2)}_{sym}(\rho)(1 - 2x)^2$ -

Gandolfi, Lovato, Carlson, Schmidt, arXiv:1406.3388

proton fraction also important for neutrino processes
What about other particles? hyperons, ...

Hyperons are bound in nuclei by $\sim 30$ MeV. What happens in dense matter?

Hyperons in Neutron Matter


Best model gives no hyperons up to $3-4 \times$ saturation density
Neutrinos in neutron stars and proto-neutron stars

Figure 1. Observational limits of surface temperatures for several isolated NSs compared with the basic theoretical cooling curve of a non-superfluid NS model.

Figure 2. Internal and surface temperatures; neutrino, photon and total luminosities (redshifted for a distant observer) for the same NS model as in Fig. 1.

For the two youngest sources only upper limits on the surface temperature $T_s$ have been established [5, 6]. The surface temperatures of the next five sources, with ages $10^3 < t < 10^5$ years, have been obtained [7, 8, 9, 10, 11] by fitting their thermal radiation spectra with hydrogen atmosphere models. Such models are more consistent with other information on these sources (e.g., Ref. [12]) than the blackbody model. On the contrary, for Geminga and PSR B1055–52 we present the values of $T_s$ inferred using the blackbody spectrum because this spectrum is more consistent for these sources. The surface temperature of RX J1856.4–3754 is still uncertain. Following [4] we adopt the upper limit $T_s < 0.65$ MK. Finally, $T_s$ for RX J0720.4–3125 is taken from Ref. [13], where the observed spectrum is interpreted with a model of a hydrogen atmosphere of finite depth.

As seen from Fig. 1, observational limits scatter in the $T_s - t$ plane. What can be learnt on dense matter in NS interiors from this scatter?

3. THEORY VERSUS OBSERVATIONS

An neutron star consists of a crust (of mass $< 10^{-2} M_\odot$, where $M_\odot$ is the solar mass) and a core (e.g., Ref. [1]). The core-crust interface is placed at the mass density $\rho \sim \rho_0/2$, where $\rho_0 \approx 2.8 \times 10^{14}$ g cm$^{-3}$ is the density of saturated nuclear matter. The crustal matter contains atomic nuclei, electrons, and (at $\rho > 4 \times 10^{11}$ g cm$^{-3}$) free neutrons. The core is further divided into the outer ($\rho < 2 \rho_0$) and inner parts. The outer core consists of neutrons, with an admixture of protons, electrons, and muons. The composition of the inner core is still unknown. It may be the same composition as the...
Neutron Star Cooling Introduction

Sensitive to:
- Equation of state
- Neutrino Emission
- Superfluidity
- Magnetic Fields
- Surface

Direct Urca:
Lattimer, Pethick, Prakash, Haensel (1991)

\[
\begin{align*}
 n & \rightarrow p + e + \bar{\nu}_e \\
 p + e & \rightarrow n + \nu_e
\end{align*}
\]

threshold associated with Fermi surfaces limit this to \(\rho > 2 \rho_0\)
Requires \(\sim 15\%\) proton fraction
to satisfy energy and momentum conservation

modified Urca works throughout the core
\[
n + N \rightarrow p + e + N + \bar{\nu}_e
\]
much slower
Superfluidity

suppresses familiar neutrino processes
creates new process: production through Cooper pairing
3P2 - 3F2 pairing particularly important but not well constrained

3x10^8 K ~ .026 MeV
typical s-wave pairing gaps ~ 1 MeV
angle average 3P2 ~ .01 MeV
log(nuclear saturation density in g/cm^3) ~ 14.4

Yakovlev, et al, 2004
also see Gezerlis, Pethick, Schwenk arXiv:1406.6109
Neutron and Proto-Neutron Star Cooling

Neutron star cooling depends upon
  Equation of State
  Neutrino Emission and Propagation
  Neutron (and proton) Superfluidity
+ ...

Supernovae neutrino emission also depends upon
  weak response of matter
  interesting regime at low densities ($0.1 \rho_0$)
  and moderate temperatures (non-degenerate matter)

Rapid progress in theory and observations
Summary/Outlook

Rapid progress in our understanding of cold dense matter

Excellent connections to
Theory of strongly-correlated matter
Experiments in cold atom physics
Astrophysical observations
Future measurements of gravitational waves
Supernovae physics and neutrino physics