Collective Neutrino Oscillations

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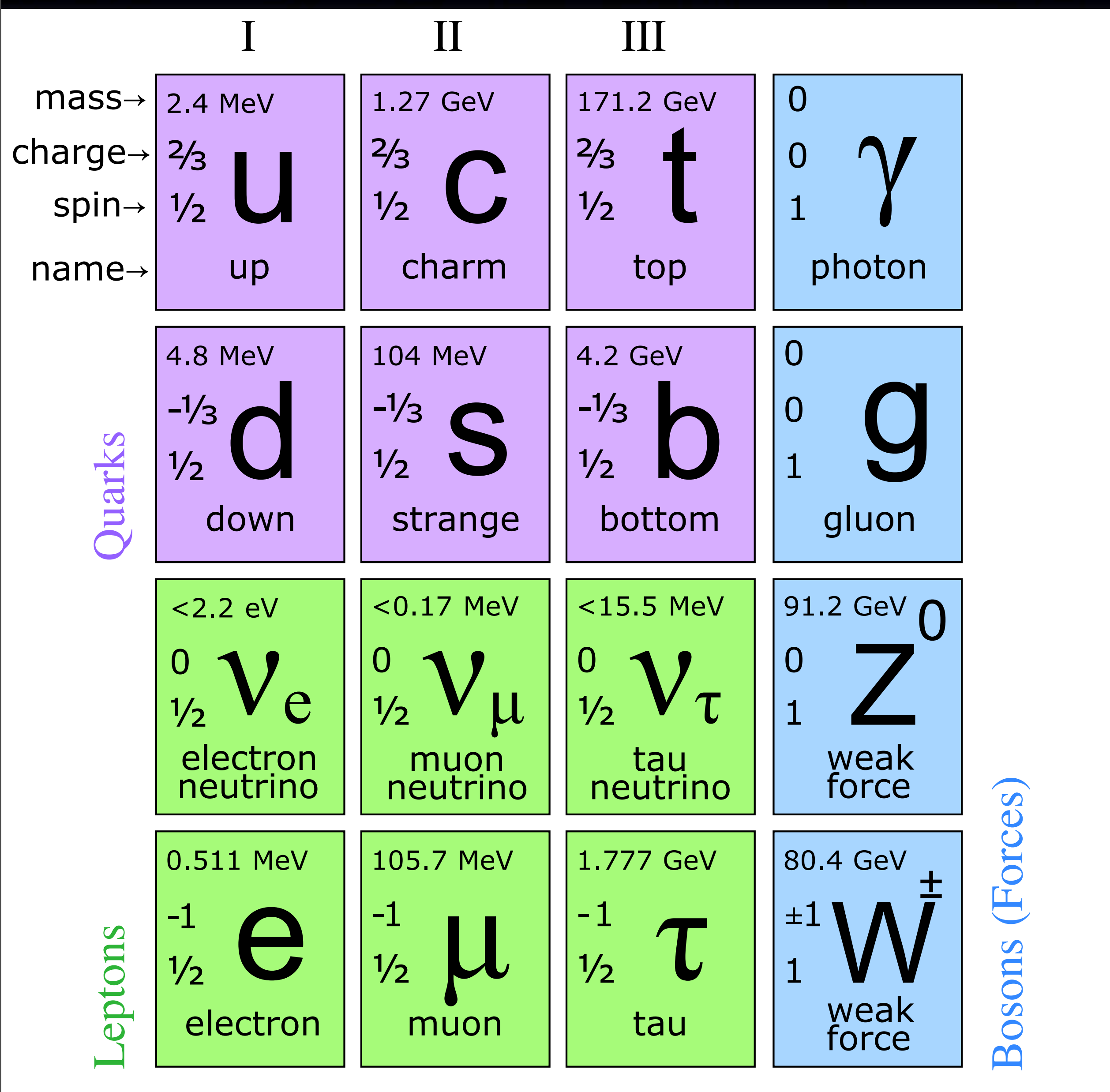
THE UNIVERSITY of NEW MEXICO

International Summer School on AstroComputing 2014
Neutrino & Nuclear Astrophysics

Outline

- ✦ Introduction & overview
- ✦ Understandings & insights
- ✦ New developments & challenges

Ghostly particle



Wikimedia: Standard Model of Elementary Particles

Cross Section (low energy):

$$\sigma \sim G_{\rm F}^2 E_\nu^2$$

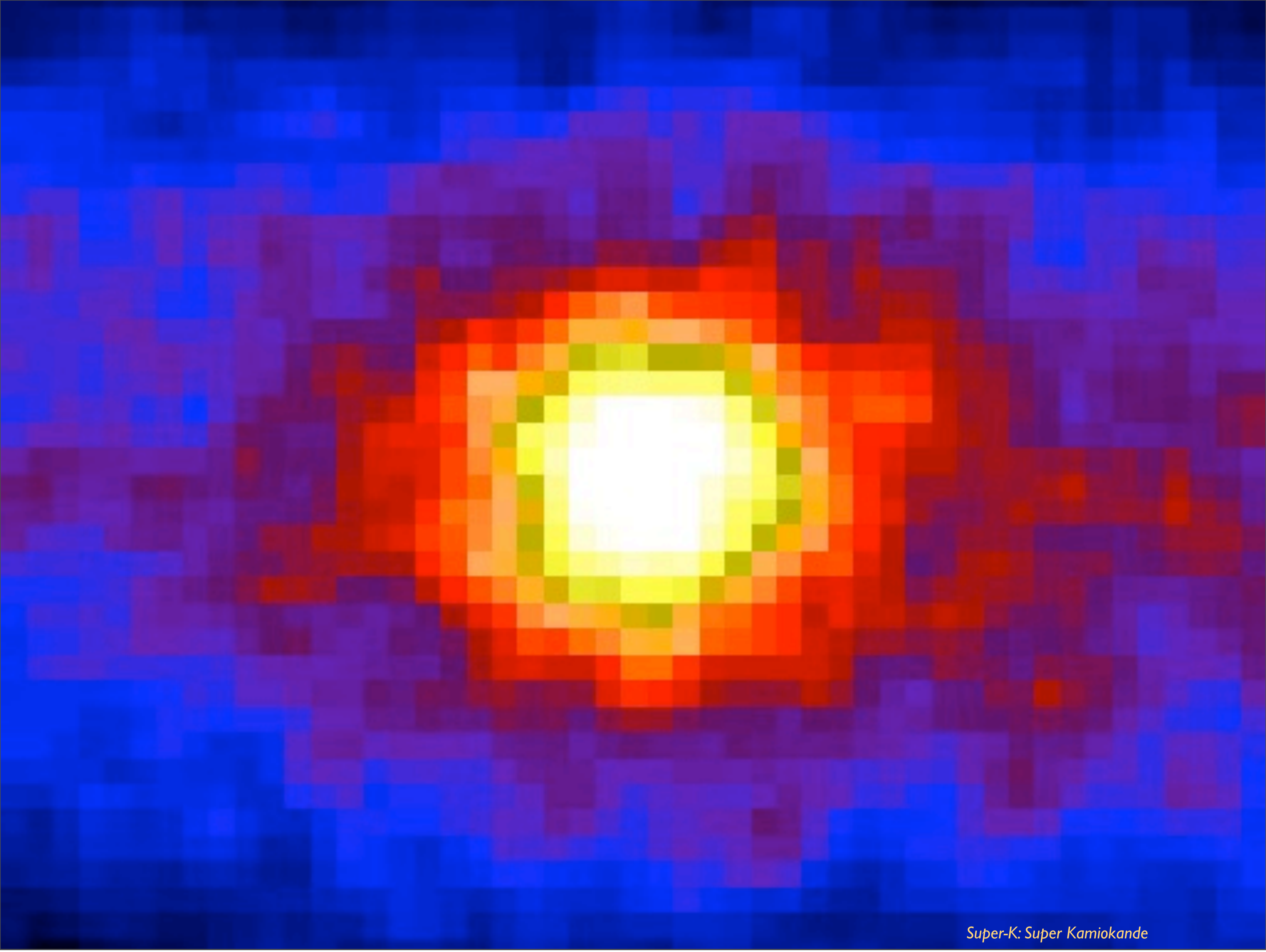
$$\simeq 10^{-44} \left(\frac{E_\nu}{1\,\mathrm{MeV}}\right)^2 \mathrm{cm}^2$$

Mean Free Path:

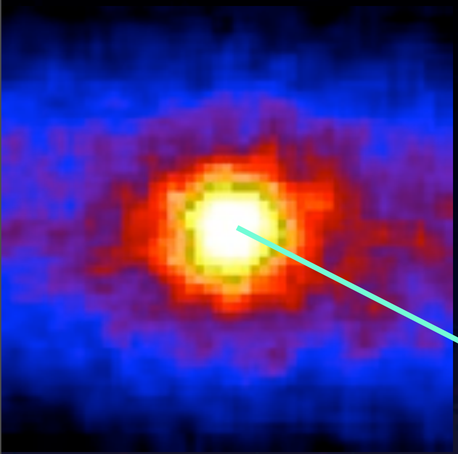
$$\lambda = \frac{1}{\sigma \rho_{\rm water} N_{\rm A}}$$

$$\simeq 10^{19} \left(\frac{E_\nu}{1\,\mathrm{MeV}}\right)^{-2} \mathrm{cm}$$

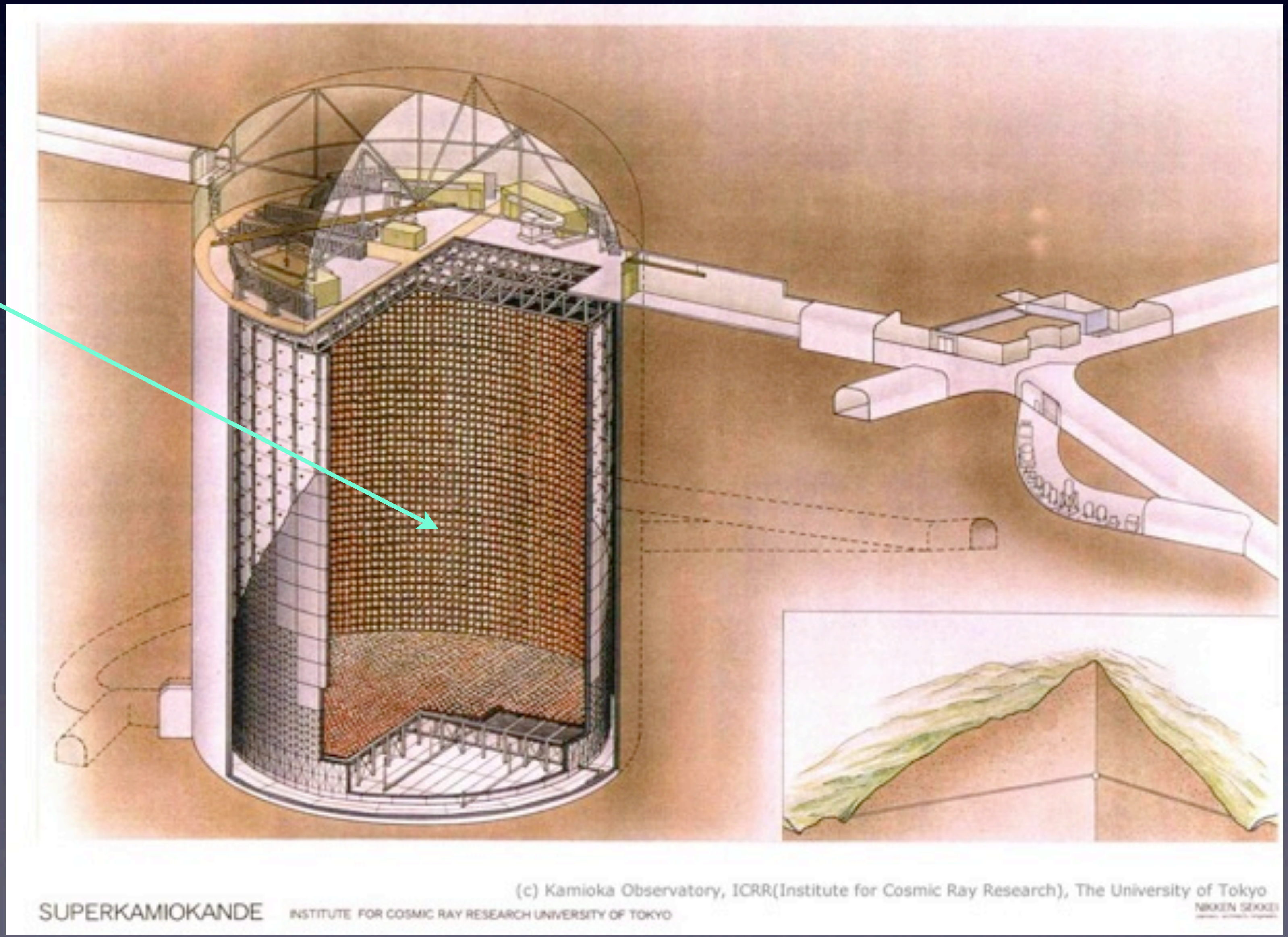
(1 AU = 1.5×10^{13} cm)


Super-K: Super Kamiokande

Neutrino Astronomy

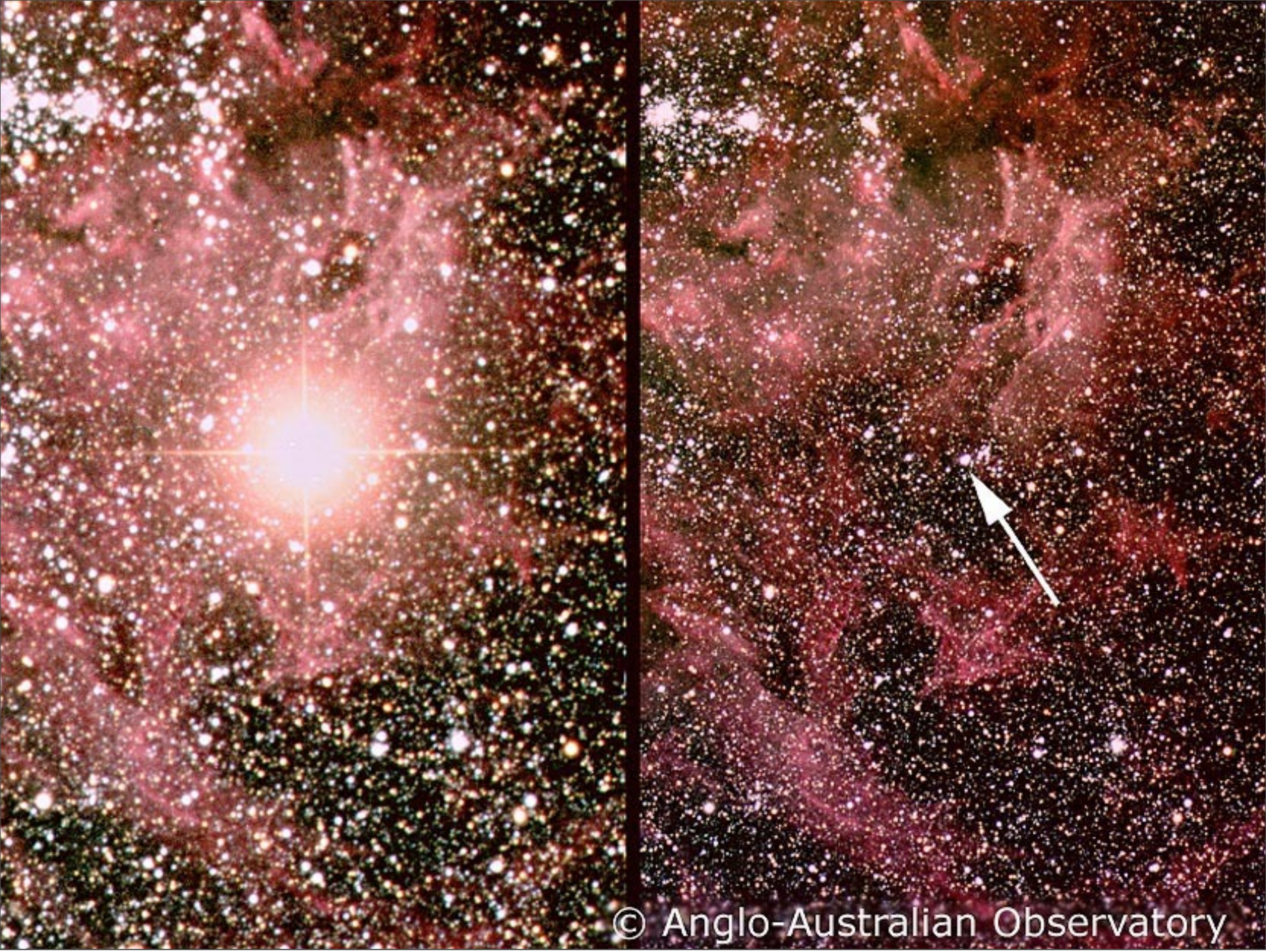


$$4p + 2e^- \longrightarrow {}^4\mathrm{He} + 2\nu_e + \mathrm{energy}$$



ES: Elastic Scattering

$$\nu_x + e^- \rightarrow \nu_x + e^-$$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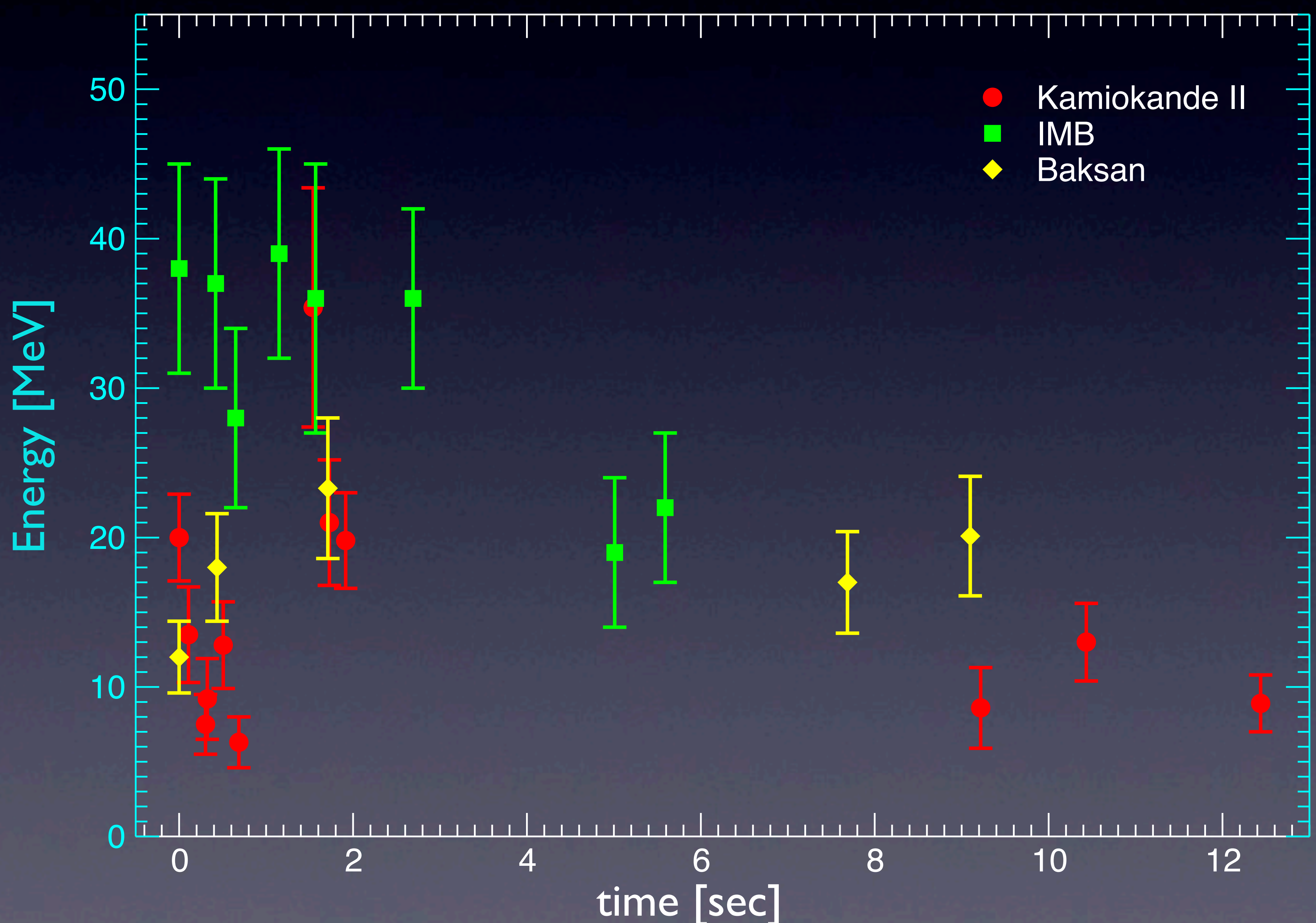
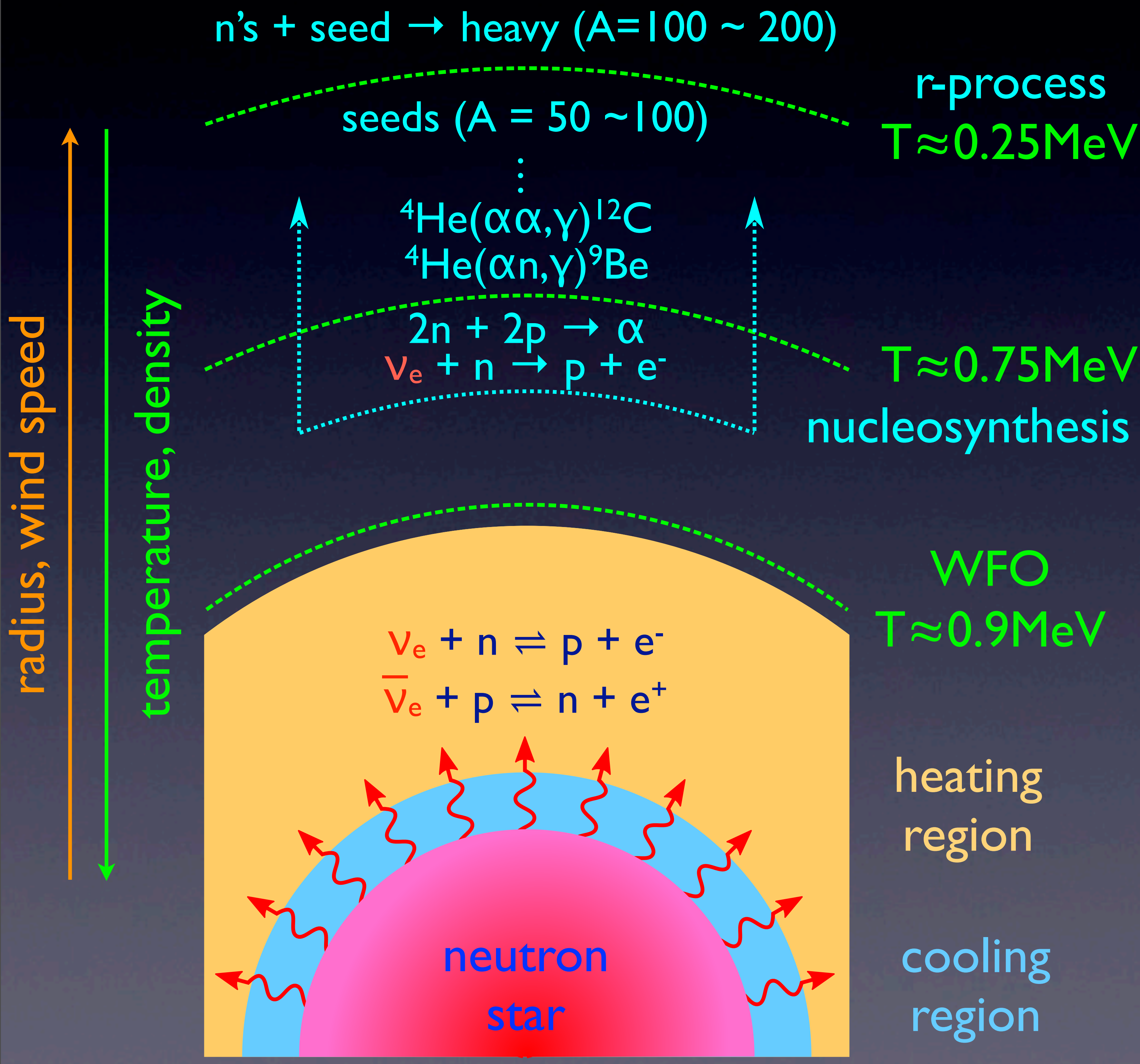
Neutrino Astronomy



Neutrinos in Supernovae



- ~10^{53} ergs, 10^{58} neutrinos in ~10 seconds
- All neutrino species, 10~30 MeV
- Dominate energetics
- Influence nucleosynthesis
- Probe into SNe

Neutrinos in SNe

- ν_e and $\bar{\nu}_e$ affect supernova dynamics and nucleosynthesis
- $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau} \rangle$
- What if $\nu_e \rightleftharpoons \nu_{\mu,\tau}$ and/or $\bar{\nu}_e \rightleftharpoons \bar{\nu}_{\mu,\tau}$?

Vacuum Oscillations

neutrinos are generated/detected in flavor states

neutrino mass eigenstates ≠ neutrino flavor states

$$|\nu_1\rangle = \cos\theta_\text{v}|\nu_e\rangle + \sin\theta_\text{v}|\nu_\mu\rangle \quad \text{with mass } m_1$$

$$|\nu_2\rangle = -\sin\theta_\text{v}|\nu_e\rangle + \cos\theta_\text{v}|\nu_\mu\rangle \quad \text{with mass } m_2$$

θ_v: vacuum mixing angle

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-\omega\cos 2\theta_\text{v} & \omega\sin 2\theta_\text{v}\\ \omega\sin 2\theta_\text{v} & \omega\cos 2\theta_\text{v}\end{bmatrix}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}$$

vac. osc. freq. $\omega = \dfrac{\delta m^2}{2E_\nu}$

$$\delta m^2 = m_2^2 - m_1^2$$

Vacuum Oscillations

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-\omega\cos 2\theta_\mathrm{v} & \omega\sin 2\theta_\mathrm{v}\\ \omega\sin 2\theta_\mathrm{v} & \omega\cos 2\theta_\mathrm{v}\end{bmatrix}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}$$

vac. osc. freq. $\omega = \frac{\delta m^2}{2E_\nu}$

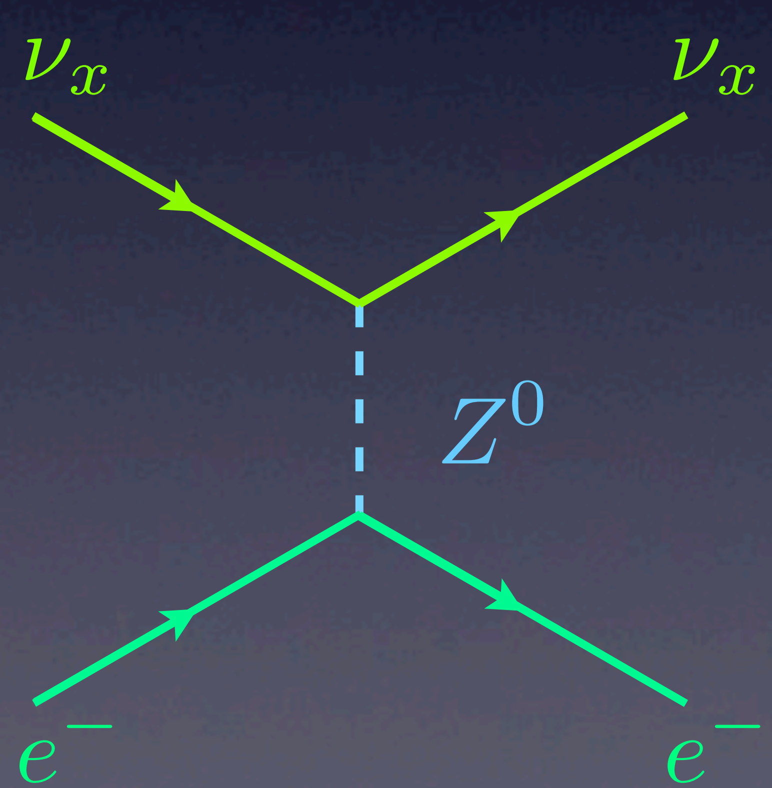
initially $|\psi(x=0)\rangle = |\nu_e\rangle$

$$P_{\nu_e\nu_e}(x) \equiv |\langle\nu_e|\psi(x)\rangle|^2 = 1 - \sin^2 2\theta_\mathrm{v}\sin^2\left(\frac{\delta m^2 x}{4E_\nu}\right)$$

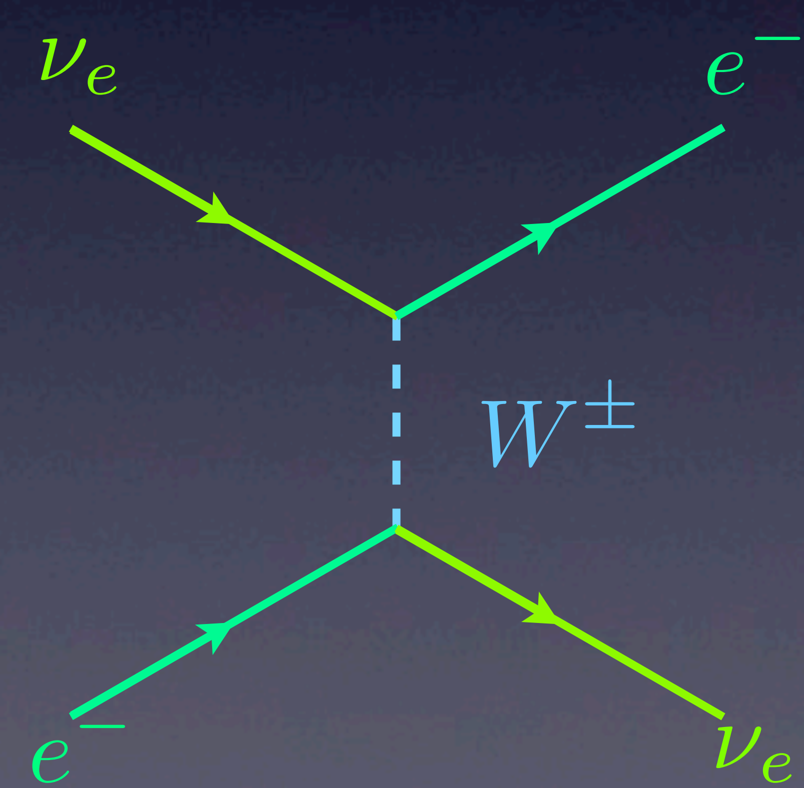
neutrino survival probability

MSW Effect

does not affect
neutrino oscillations



Wolfenstein (1978)

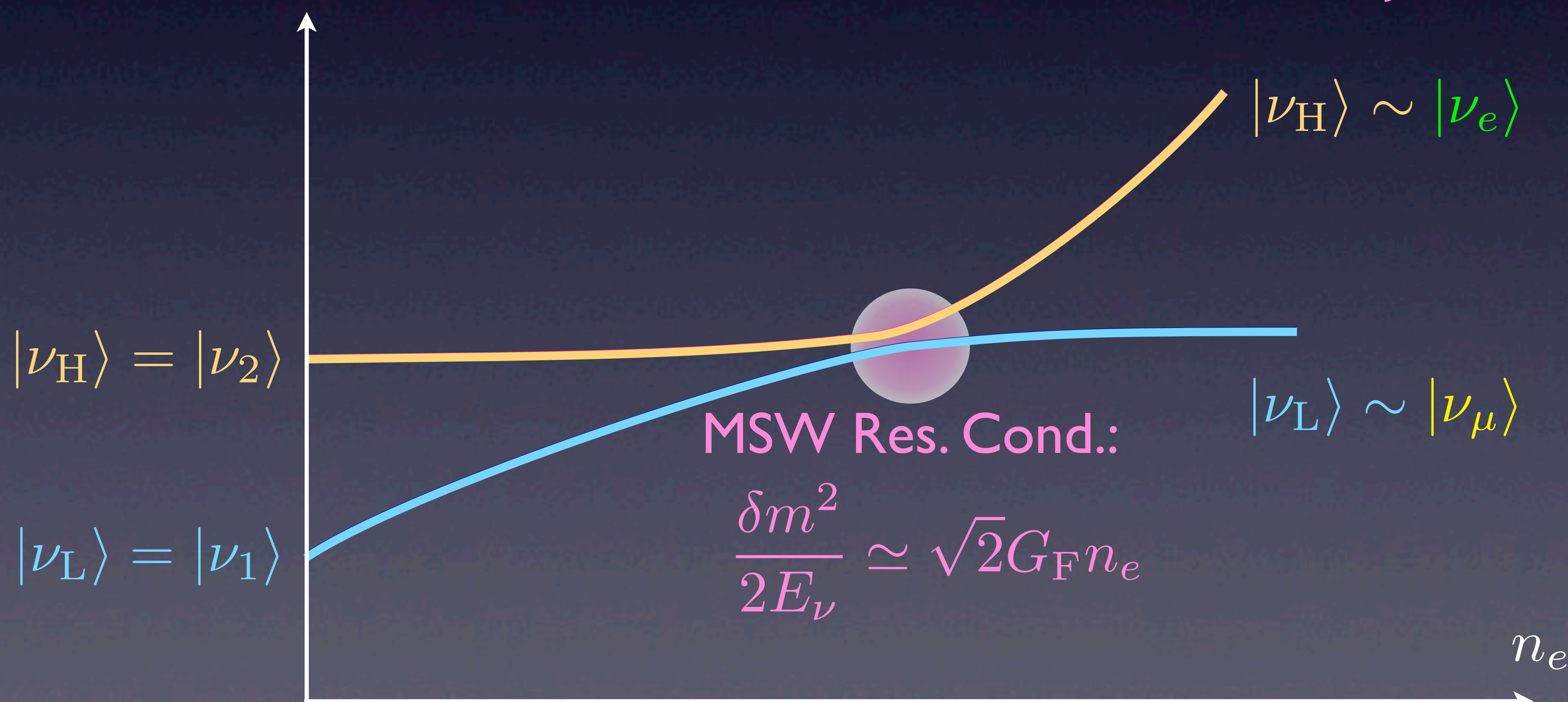


MSW Effect

electron number density

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\sqrt{2}G_\mathrm{F}n_e - \omega\cos 2\theta_\mathrm{v} & \omega\sin 2\theta_\mathrm{v}\\ \omega\sin 2\theta_\mathrm{v} & \omega\cos 2\theta_\mathrm{v}\end{bmatrix}\begin{bmatrix}\langle\nu_e|\psi_\nu\rangle\\ \langle\nu_\mu|\psi_\nu\rangle\end{bmatrix}$$

vac. osc. freq. $\omega = \frac{\delta m^2}{2E_\nu}$



Mikheyev, Smirnov (1985)

Three Flavor Mixing

WEAK FLAVOR STATES — VACUUM MASS EIGENSTATES

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

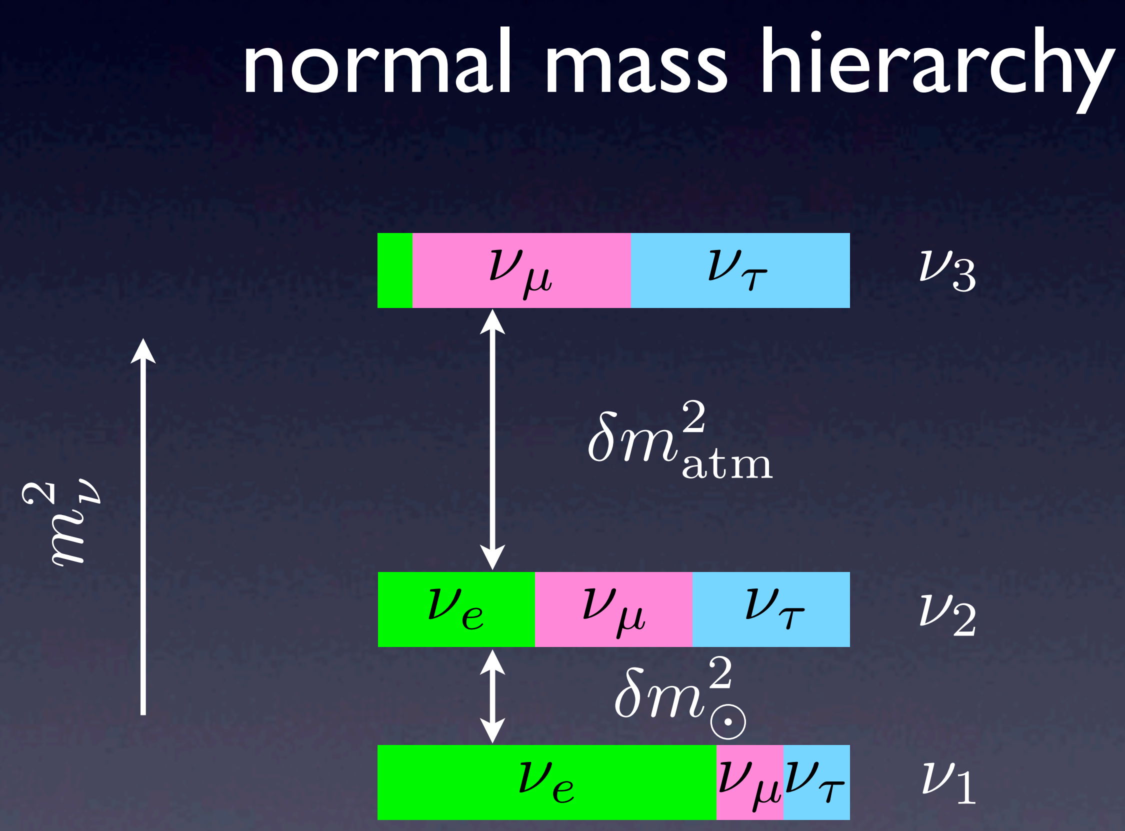
$$\delta m^2_{12} \simeq \delta m^2_{\odot} \simeq 7\text{–}8 \times 10^{-5}\mathrm{eV}^2, \quad \theta_{12} \simeq \theta_{\odot} \simeq 0.6$$

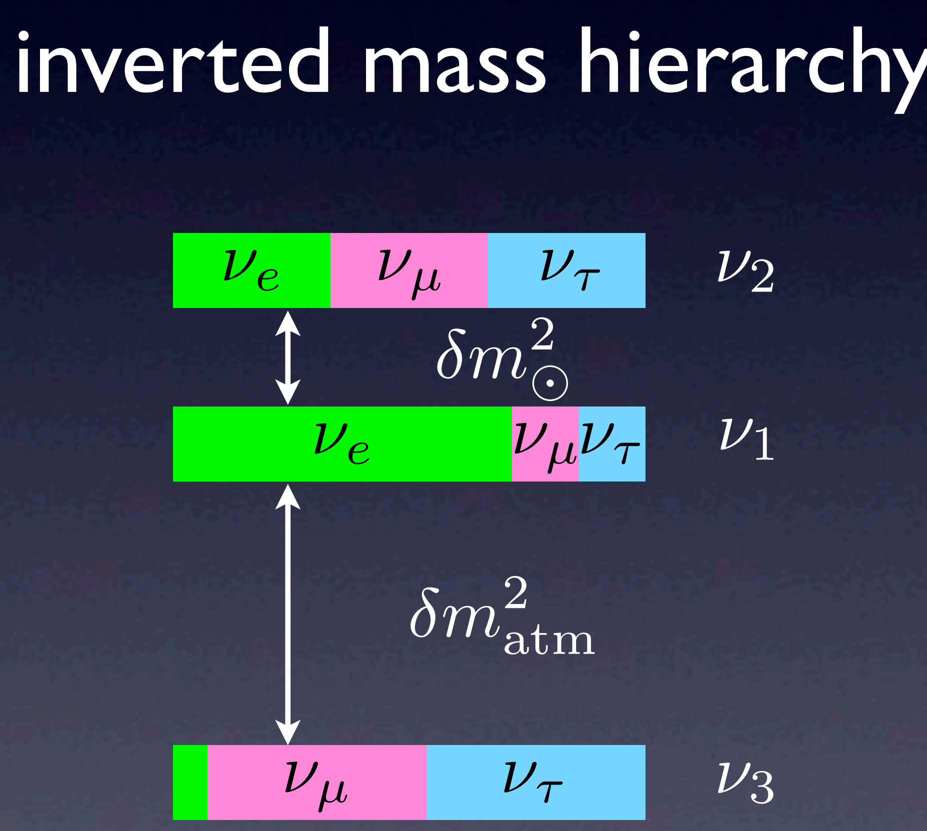
$$|\delta m^2_{23}| \simeq \delta m^2_{\mathrm{atm}} \simeq 2\text{–}3 \times 10^{-3}\mathrm{eV}^2, \quad \theta_{23} \simeq \theta_{\mathrm{atm}} \simeq \frac{\pi}{4}$$

$$|\delta m^2_{13}| \simeq |\delta m^2_{23}| \simeq 2\text{–}3 \times 10^{-3}\mathrm{eV}^2, \quad \theta_{13} \simeq 0.15$$

ϕ is unknown ⟵ CP VIOLATION PHASE

Mass Hierarchy





Oscillations in SN

$$i\frac{d}{d\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

mass matrix → M^2; electron density → n_e

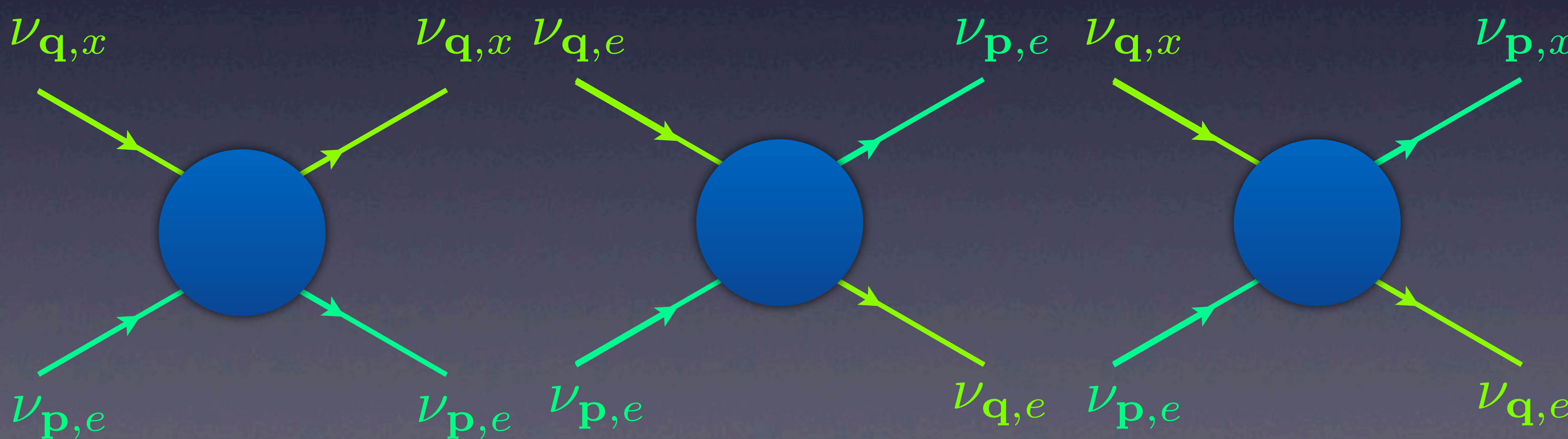
$$H = \frac{M^2}{2E} + \sqrt{2}G_F\,\mathrm{diag}[n_e, 0, 0] + H_{\nu\nu}$$

neutrino energy → E

ν–ν forward scattering (self-coupling) → $H_{\nu\nu}$

Neutrino Self-Coupling

No oscillation effect | Fuller et al (1987) | Pantaleone (1992)



Density Matrix

Pure State:

$$|\psi\rangle \Longrightarrow \rho = \begin{bmatrix} \langle\nu_e|\psi\rangle\langle\psi|\nu_e\rangle & \langle\nu_e|\psi\rangle\langle\psi|\nu_x\rangle \\ \langle\nu_x|\psi\rangle\langle\psi|\nu_e\rangle & \langle\nu_x|\psi\rangle\langle\psi|\nu_x\rangle \end{bmatrix}$$

$$\text{Example: } |\nu_e\rangle \Longrightarrow \rho = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Mixed State:

$$\rho \propto \begin{bmatrix} n_{\nu_e} & 0 \\ 0 & n_{\nu_x} \end{bmatrix}$$

Neutrino Self-Coupling

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

mass matrix → M^2

electron density → n_e

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$

neutrino energy → E

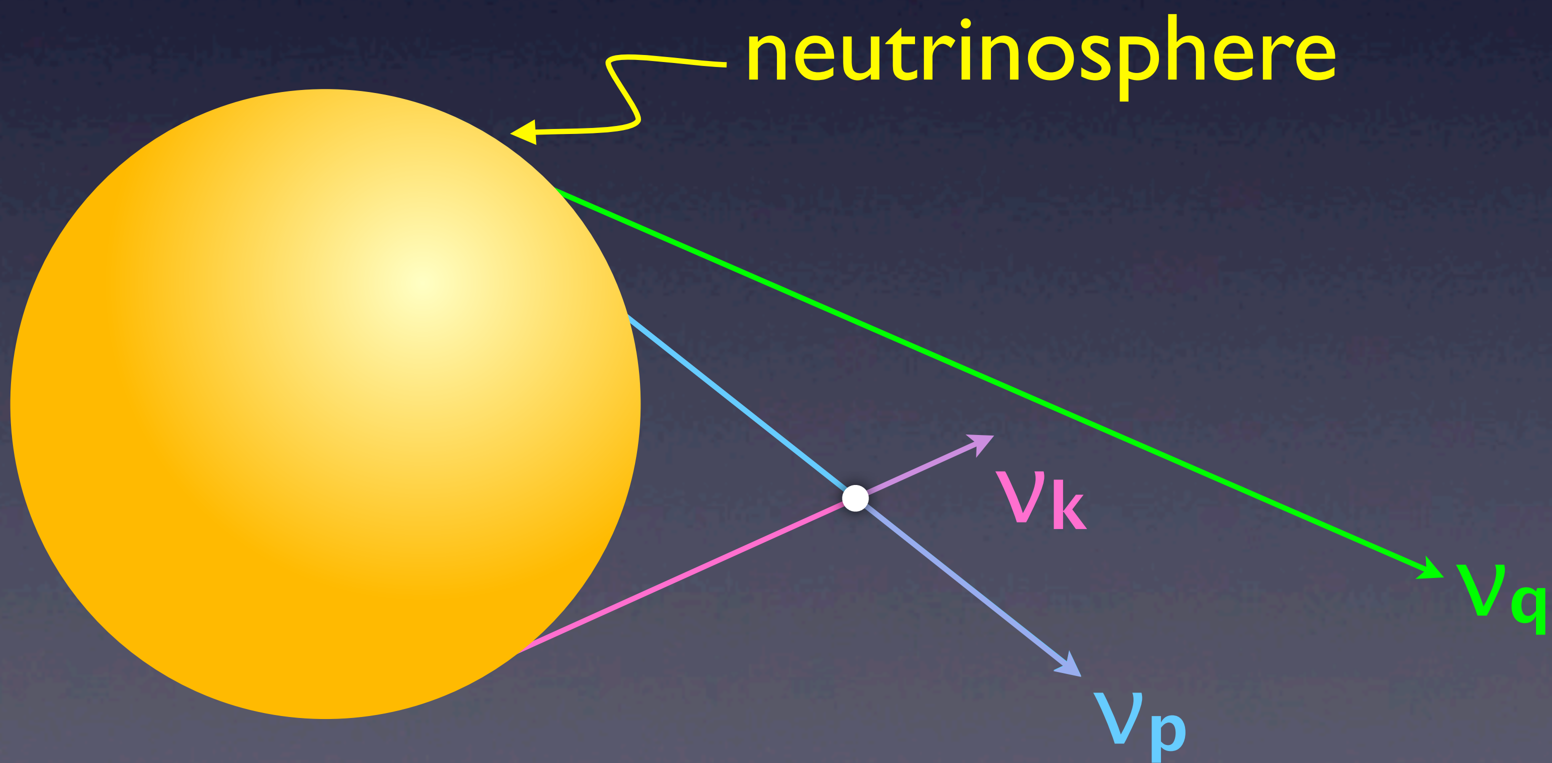
ν–ν forward scattering (self-coupling) → $\mathsf{H}_{\nu\nu}$

$$\mathsf{H}_{\nu\nu} = \sqrt{2}G_\mathrm{F}\int \mathrm{d}\mathbf{p}'(1-\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\rho_{\mathbf{p}'} - \bar{\rho}^*_{\mathbf{p}'})$$

ν oscillations in SN

$$i\frac{d}{d\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

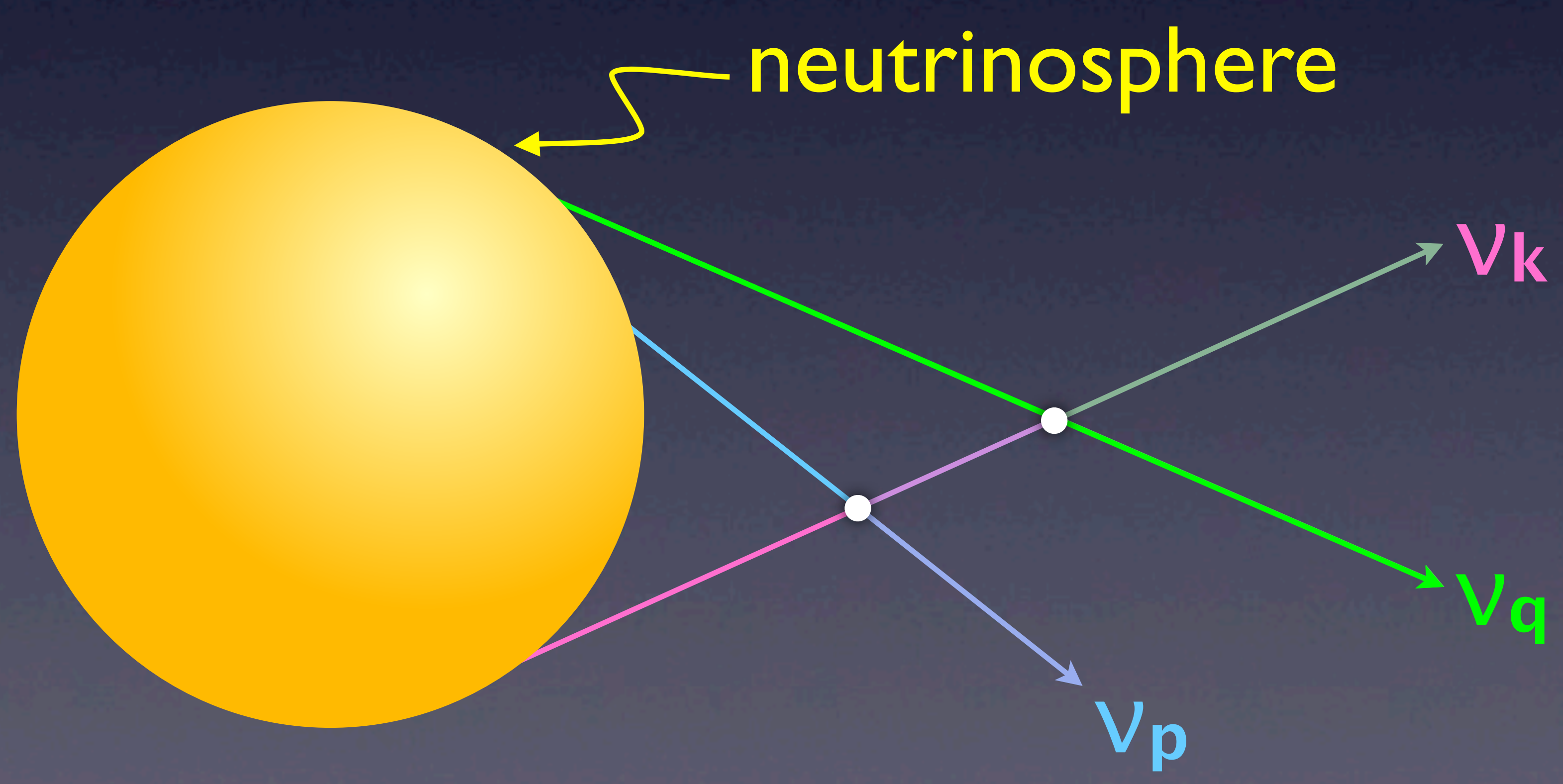
$$H = \frac{M^2}{2E} + \sqrt{2}G_F\,\mathrm{diag}[n_e, 0, 0] + H_{\nu\nu}$$



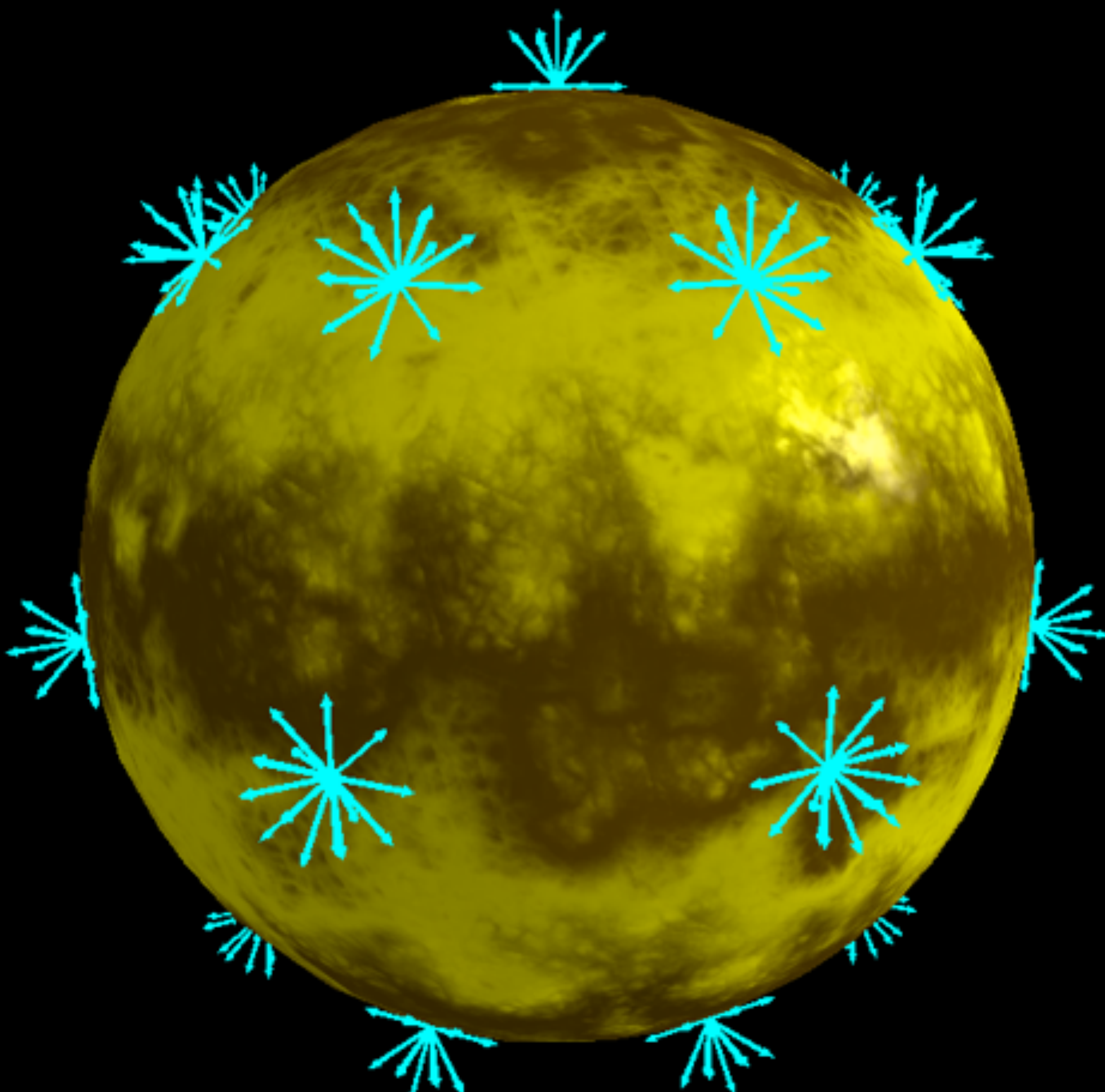
ν oscillations in SN

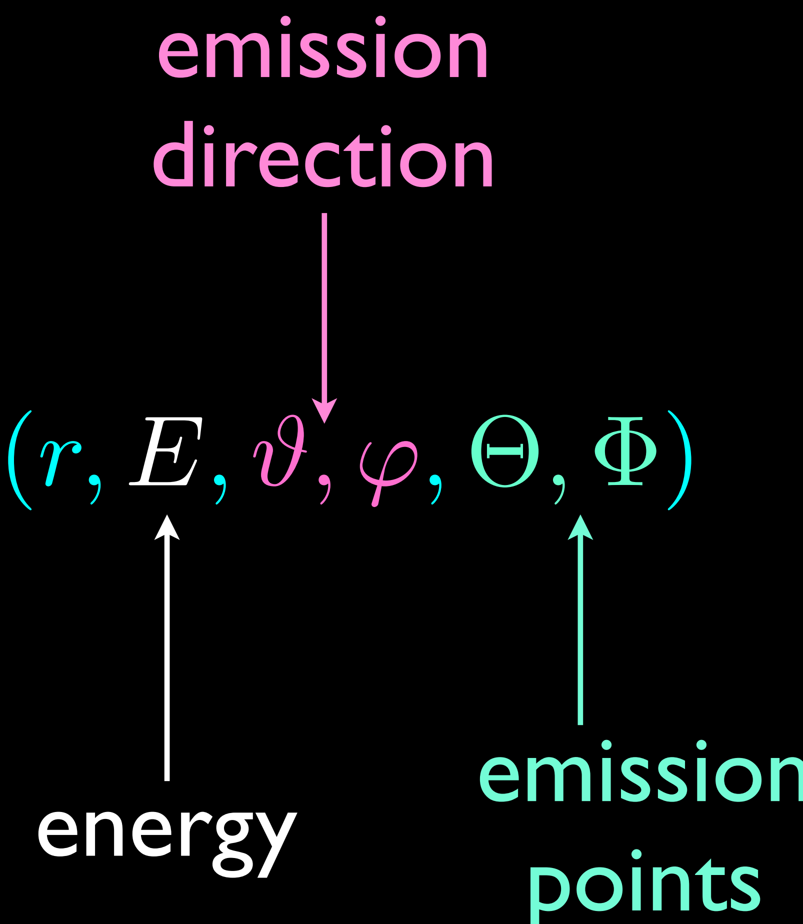
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$



(3+3)D

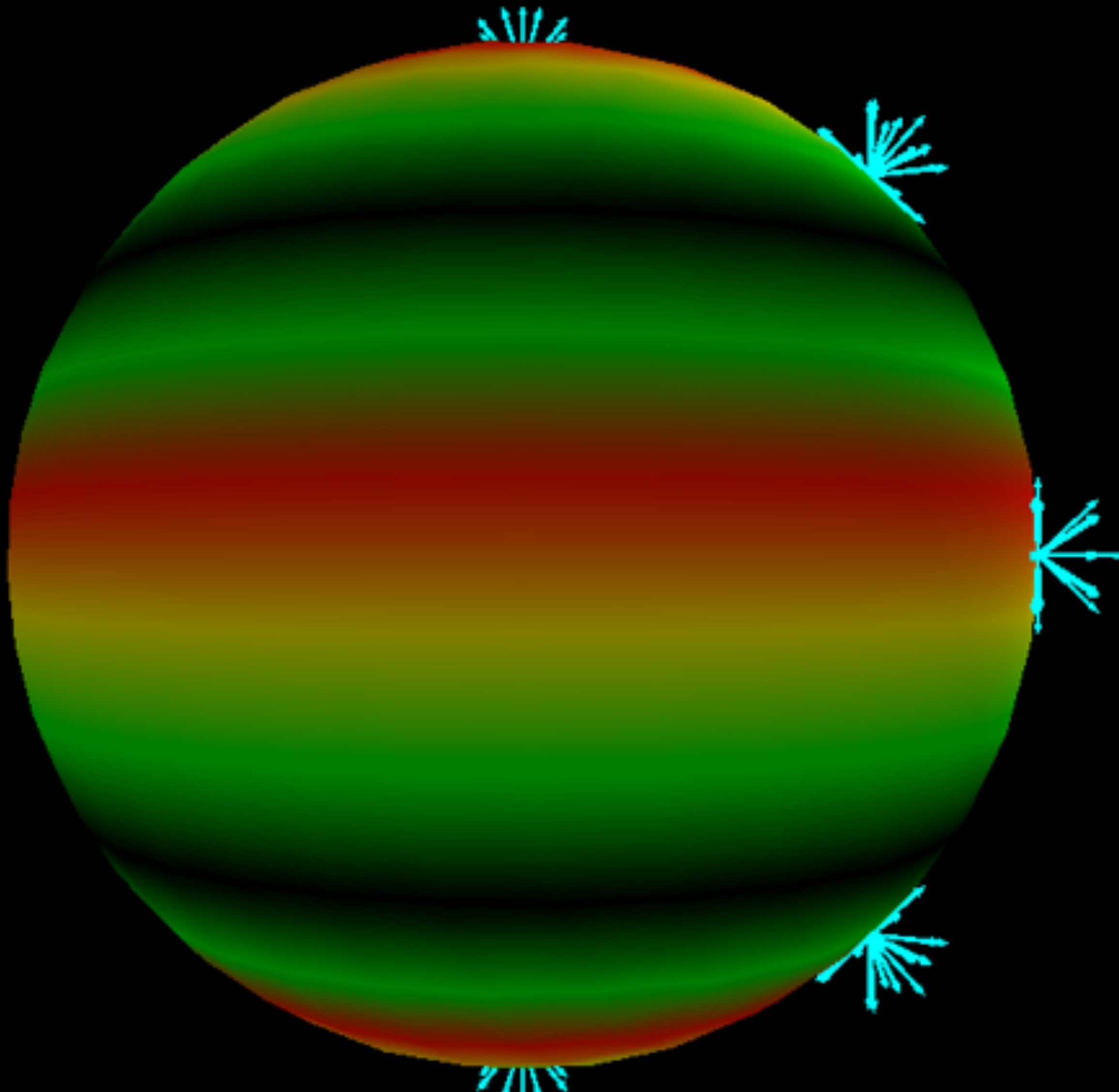




$$\psi(r, E, \vartheta, \varphi, \Theta, \Phi)$$

Coherent forward scattering only outside neutrino sphere.

(2+3)D



propagation direction

$\psi(r, E, \vartheta, \varphi, \Theta)$

energy

emission points

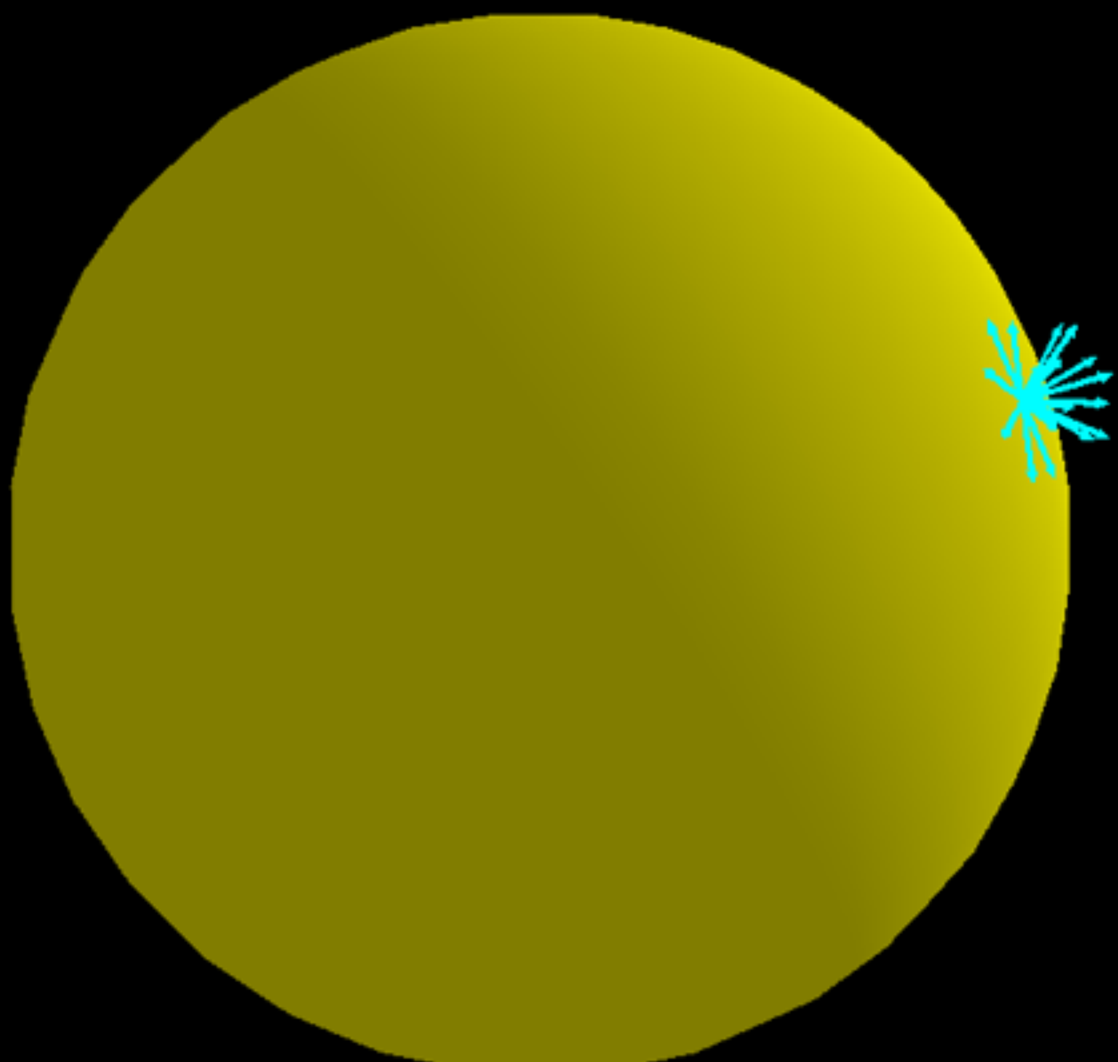
previous assumptions + Axial symmetry around the Z axis.

(1+3)D

propagation direction

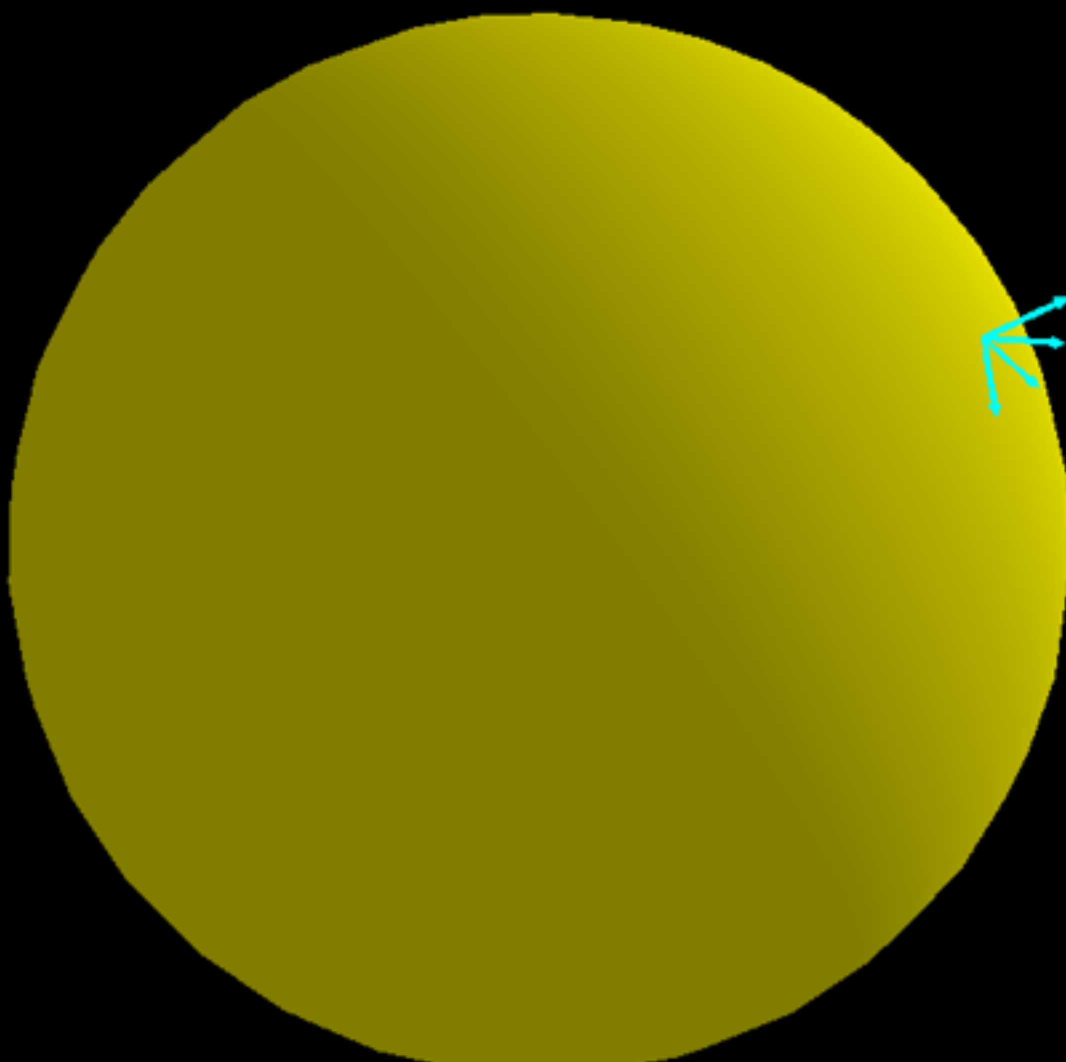
$$\psi(r, E, \vartheta, \varphi)$$

energy



previous assumptions + Spherical symmetry about the center (Consistency?)

(1+2)D
Multi-Angle/Bulb Model



propagation direction

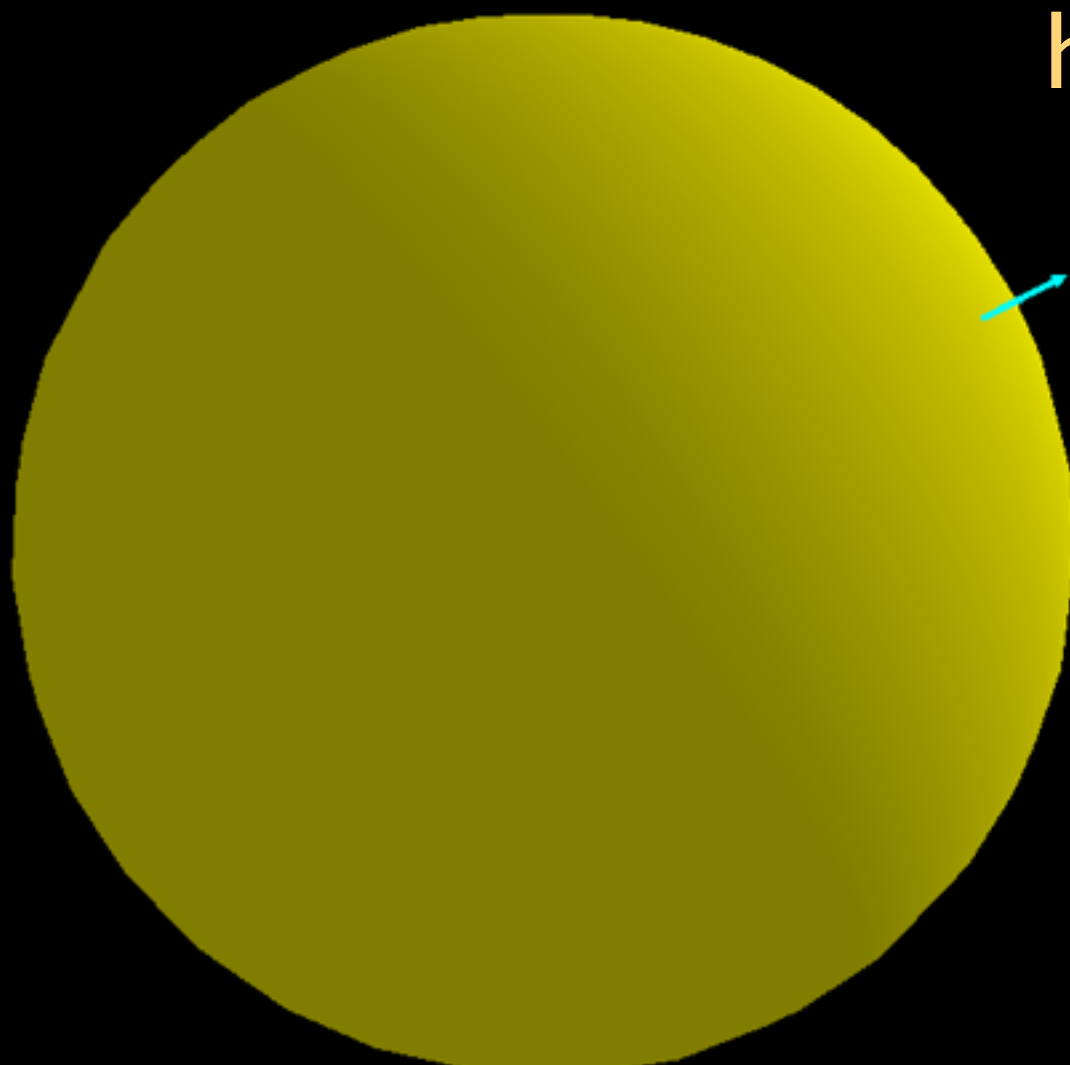
$$\psi(r, E, \vartheta)$$

energy

previous + assumptions + Azimuthal symmetry around any radial direction

(1+1)D
Single-Angle

Equivalent to an expanding homogeneous neutrino gas



$\psi(r, E)$

energy

previous assumptions +
Trajectory independent
neutrino flavor evolution

Semi-Analytic Treatment

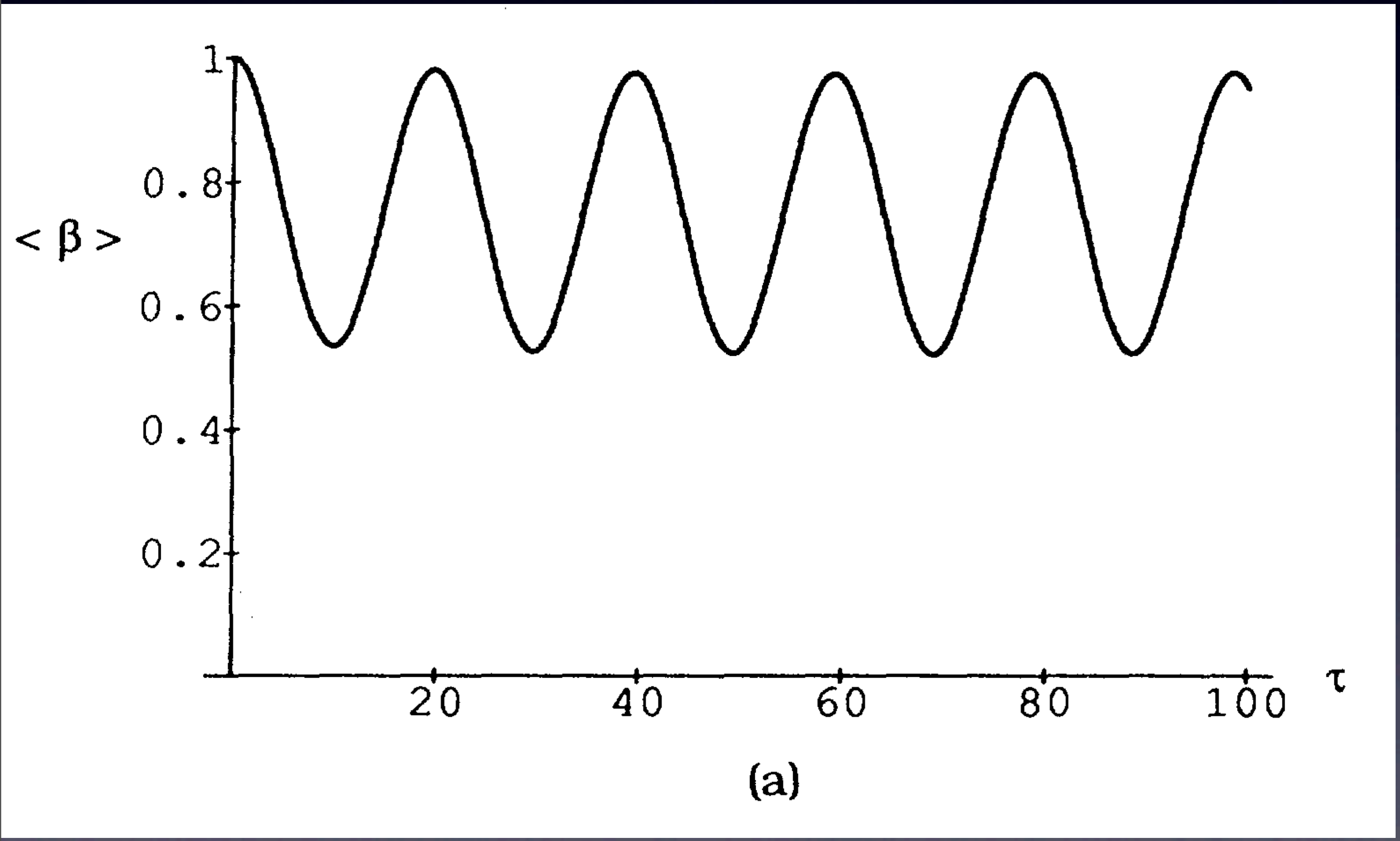
$$\mathrm{H}_{\nu\nu} = \sqrt{2} G_{\mathrm{F}} \int \mathrm{d}\mathbf{p}' (1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')(\rho_{\mathbf{p}'} - \bar{\rho}^*_{\mathbf{p}'})$$

Qian & Fuller (1995)

- Single-angle approximation.
- Assume that the off-diagonal elements of ρ are 0.
  - They are 0 in the adiabatic MSW flavor evolution.
  - They average to 0 in the non-adiabatic case.

Numerical Treatment

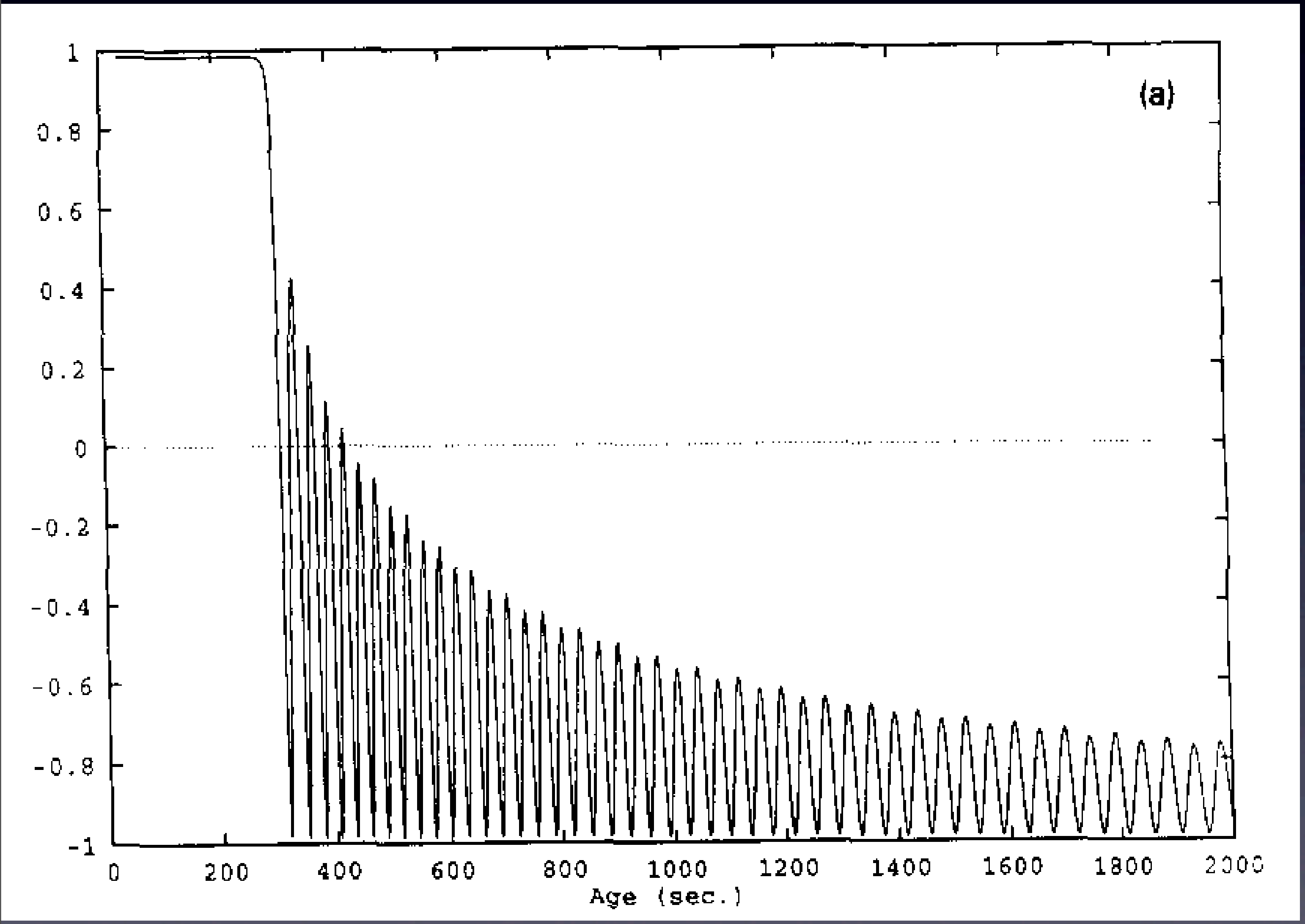
Samuel (1993)



- Homogeneous & isotropic neutrino gas.
- Small electron-neutrino excess.
- Self-maintained coherence.

Numerical Treatment

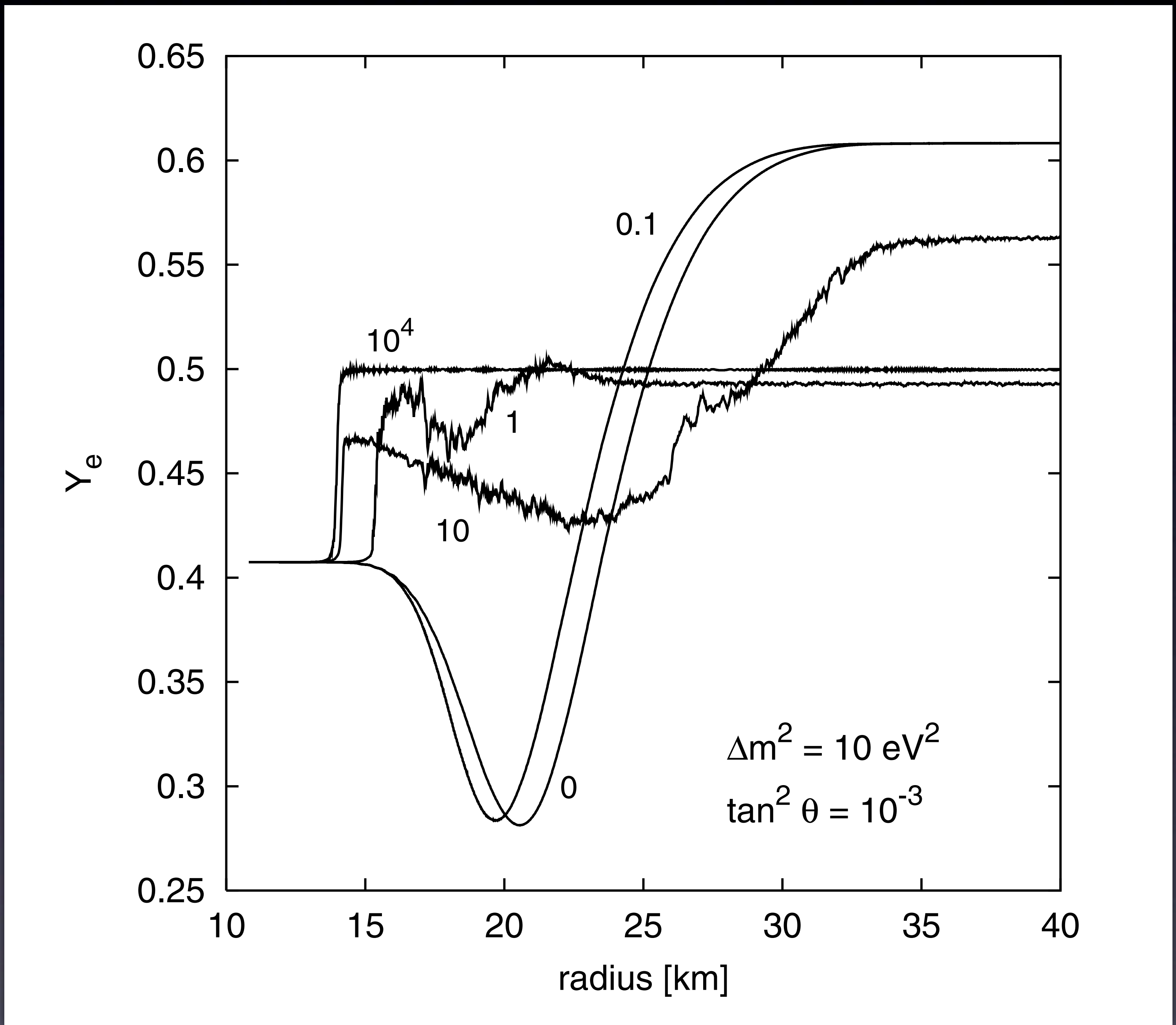
Kostelecky & Samuel (1993)



- Homogeneous & isotropic neutrino gas.
- Small electron-neutrino excess.
- Self-maintained coherence.

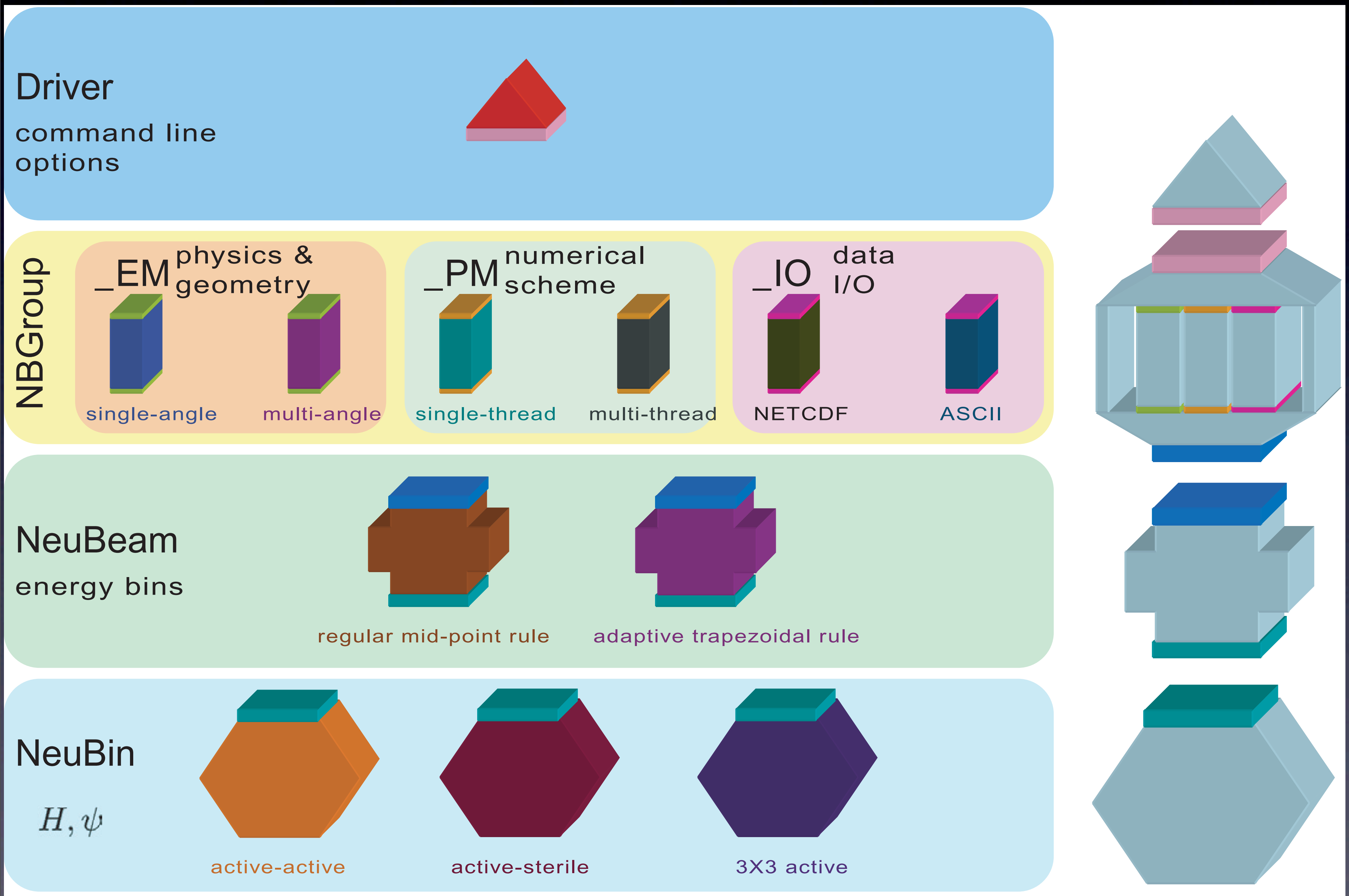
Numerical Treatment

Pastor & Raffelt (2002)



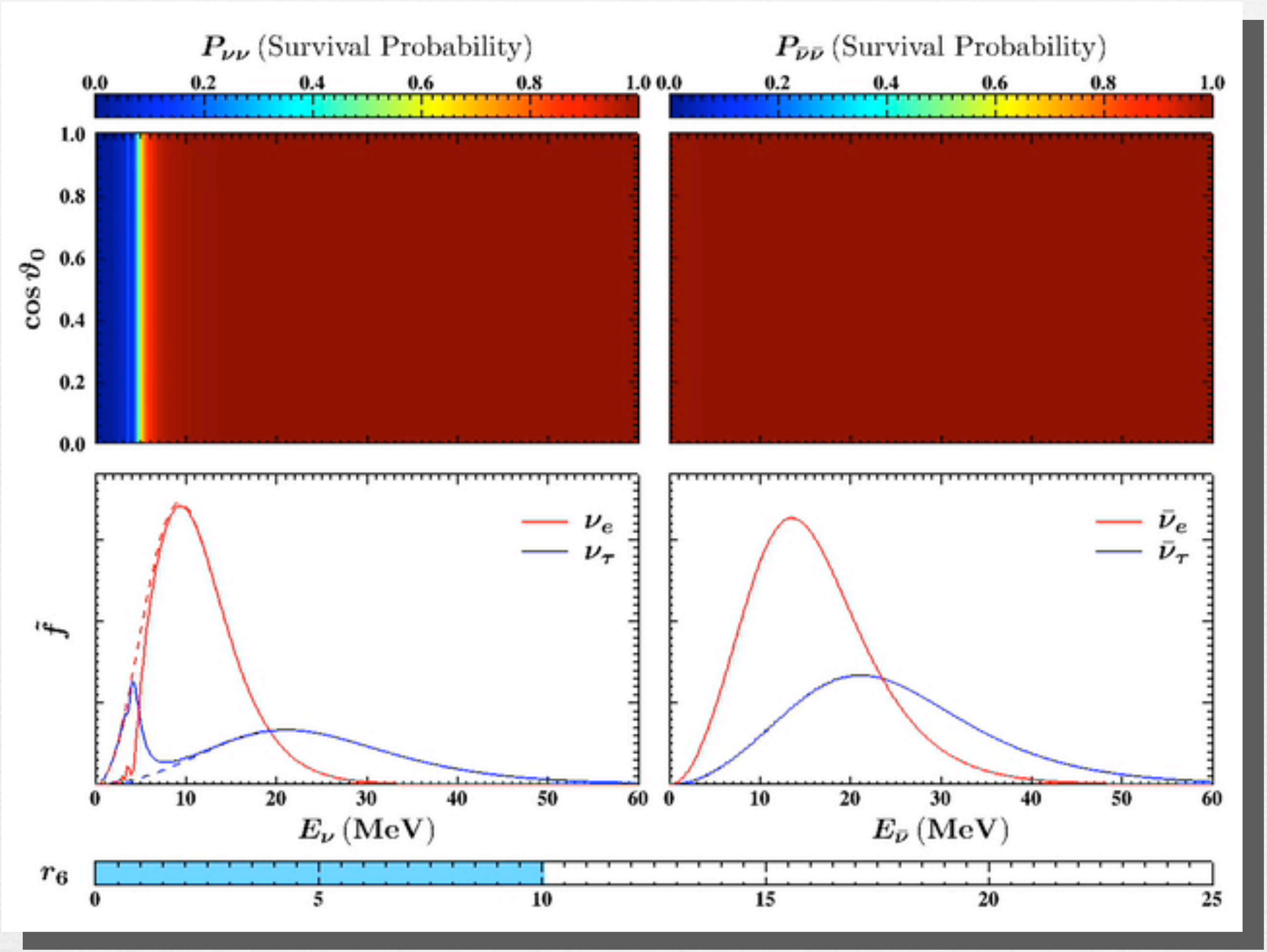
- Single-angle approximation
- Large mass-squared difference
- Synchronized neutrino oscillations

FLAT

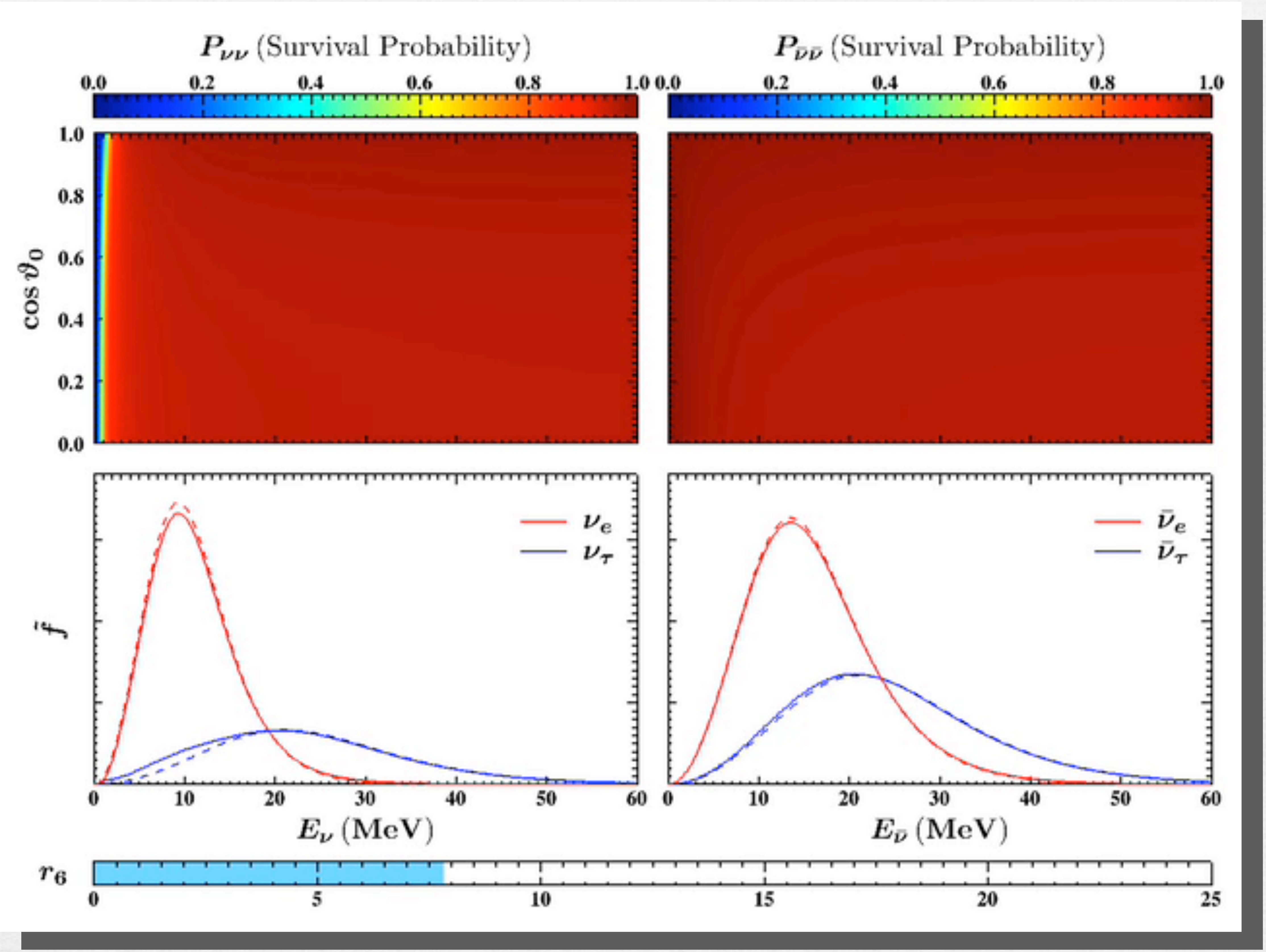


- Highly modularized program
- Multi-purpose
  - single-angle vs. multi-angle
  - 2 flavors vs. 3 flavors

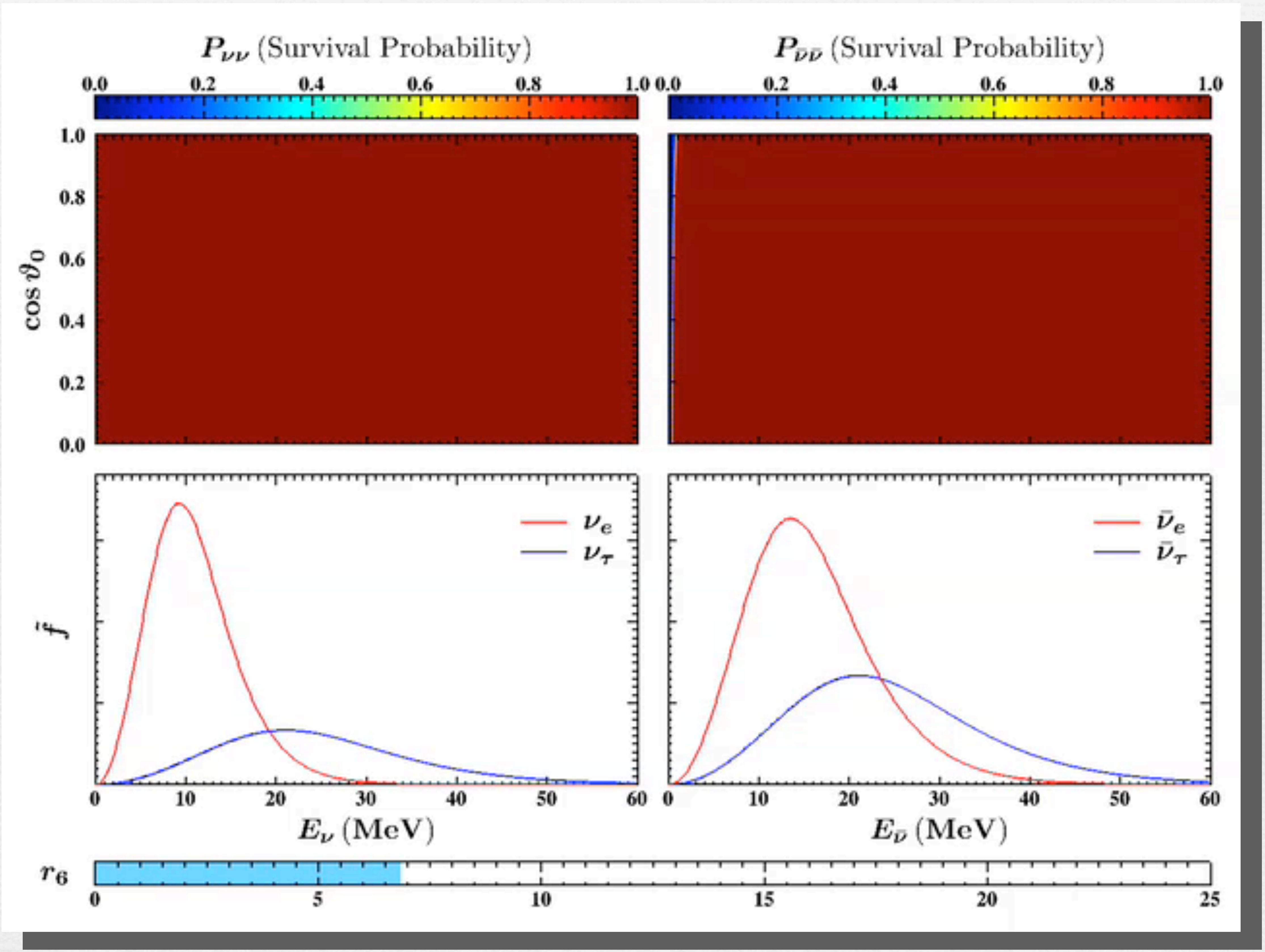
$$\delta m^2 = 3 \times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\mathrm{atm}},\ \theta_{\mathrm{v}} = 0.1,\ L_\nu = 0$$

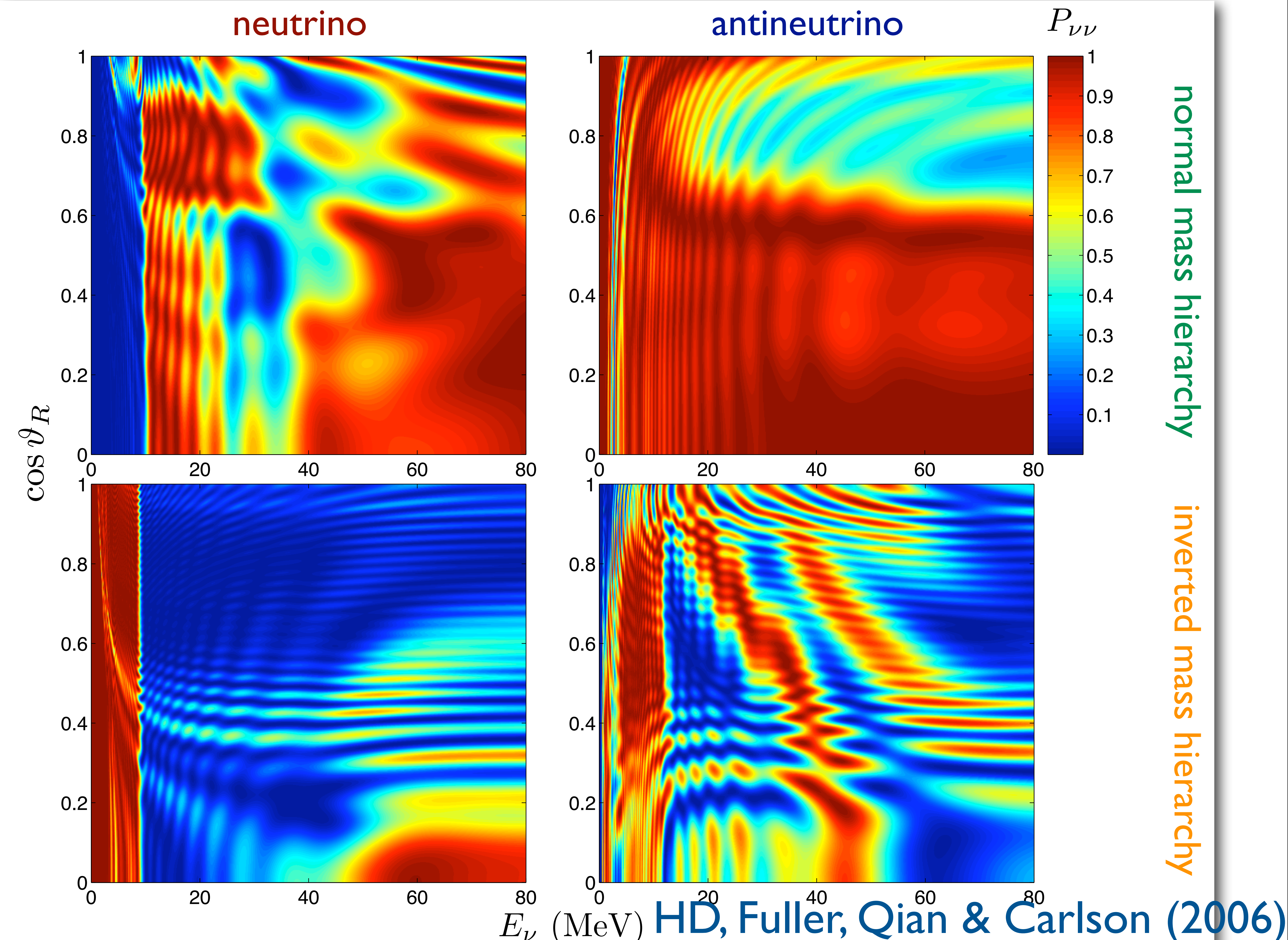


$$\delta m^2 = 3\times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\mathrm{atm}},\ \theta_{\mathrm{v}} = 0.1,\ L_\nu = 10^{51}\,\mathrm{erg/s}$$



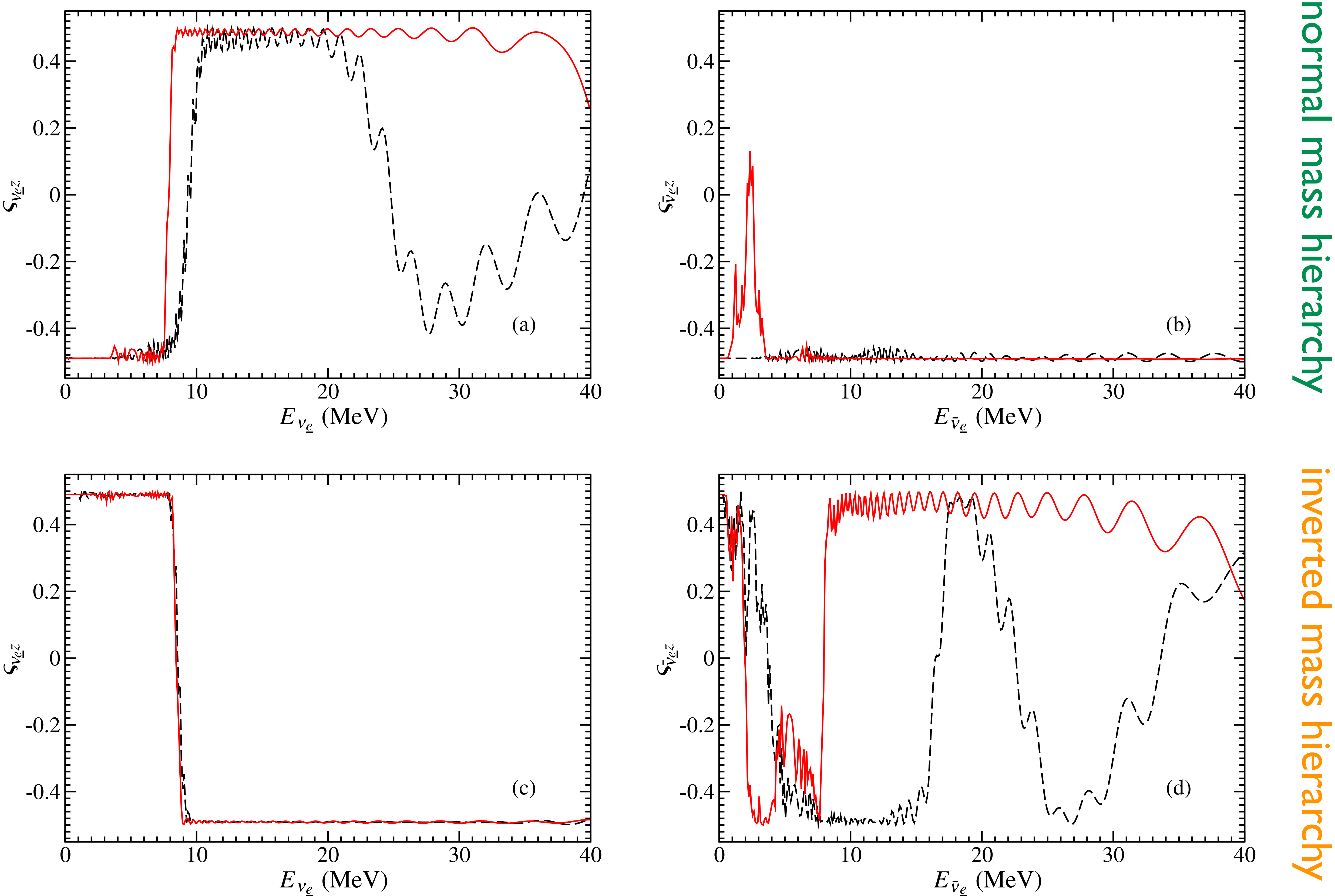
$$\delta m^2 = -3 \times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\mathrm{atm}},\ \theta_{\mathrm{v}} = 0.1,\ L_\nu = 10^{51}\,\mathrm{erg/s}$$




neutrino
antineutrino
$P_{\nu\nu}$
normal mass hierarchy
inverted mass hierarchy
$\cos\vartheta_R$
E_ν (MeV)
HD, Fuller, Qian & Carlson (2006)

neutrino

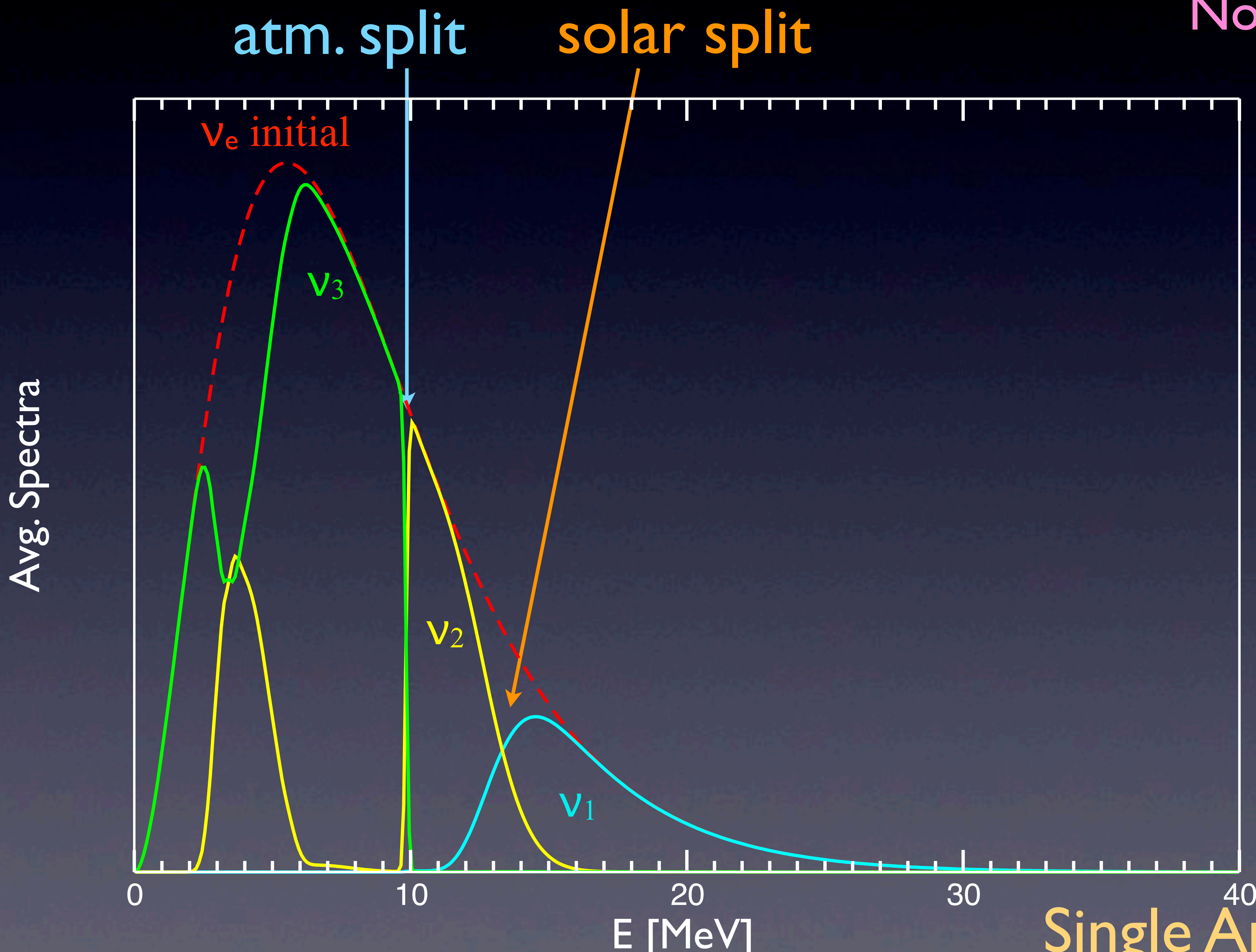
antineutrino



HD, Fuller, Qian & Carlson (2006)

3 Flavor Mixing

Normal Mass Hierarchy



O-Ne-Mg Core-Collapse
Neutronization Pulse

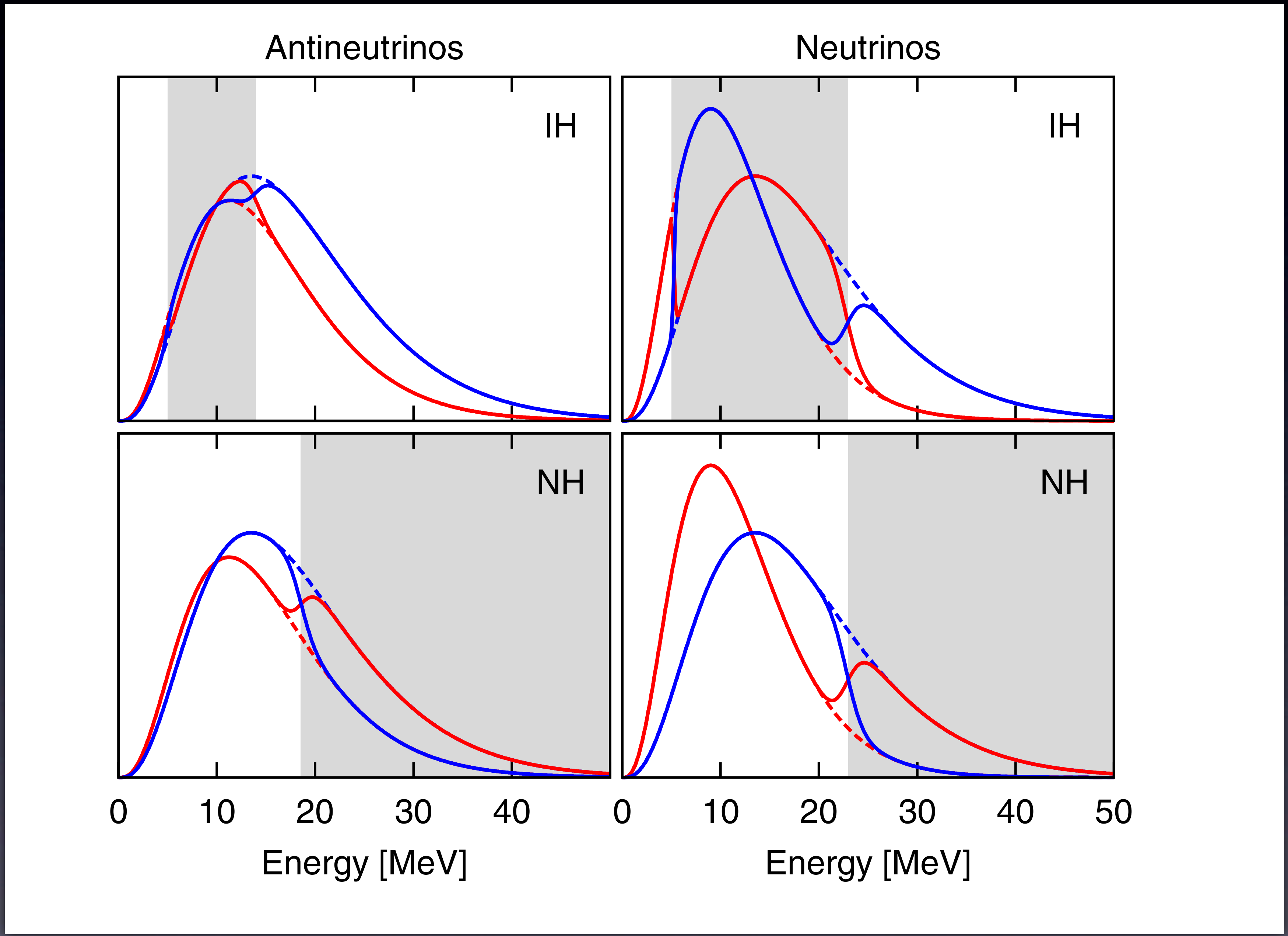
$L_{\nu_e} = 10^{53}\,\mathrm{erg/s}$

$\langle E_{\nu_e} \rangle = 11\,\mathrm{MeV}$

r = 5000 km

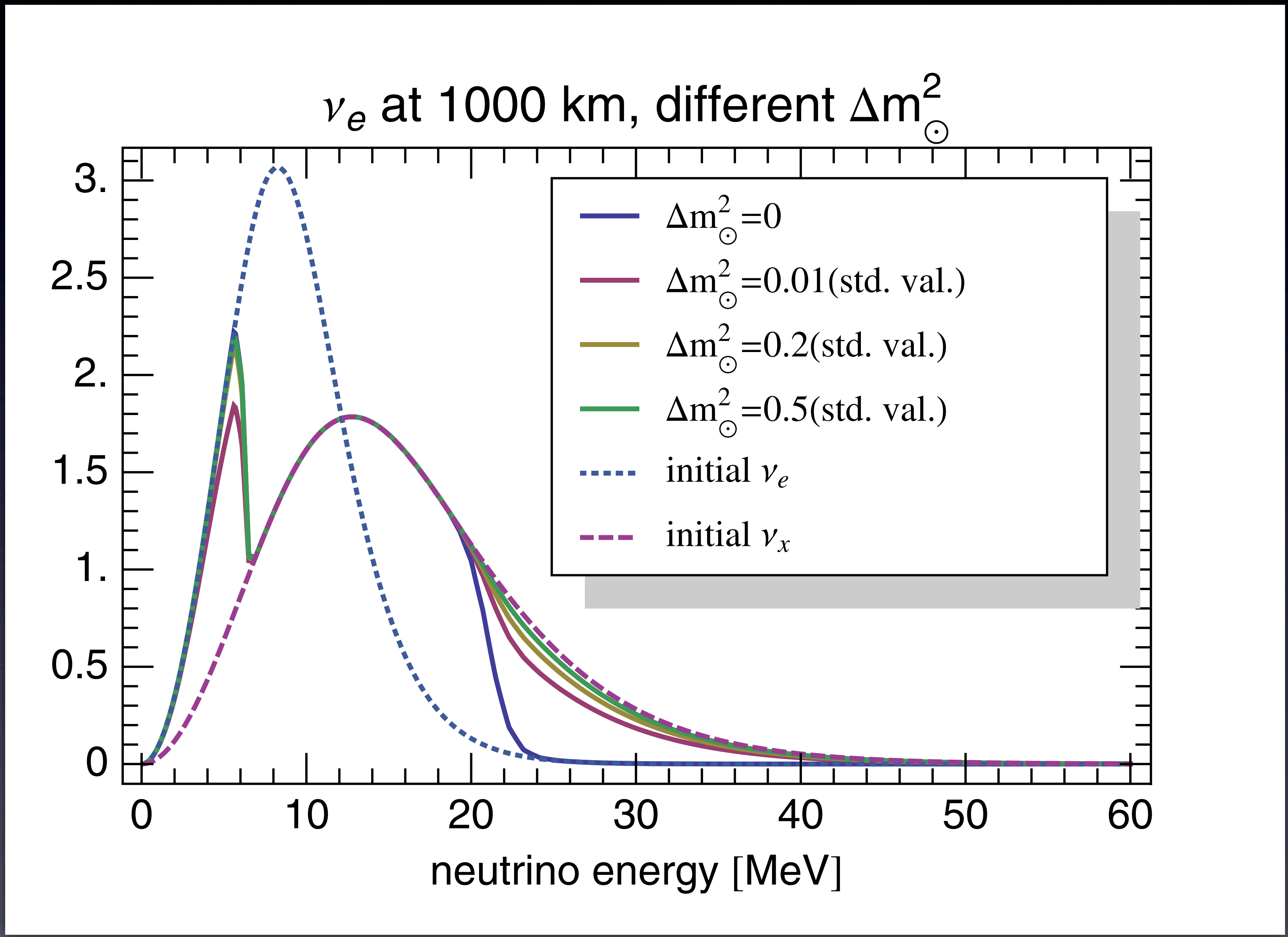
Single Angle: HD et al (2007)
Multi-Angle: Cherry et al (2010)

Multiple Spectral Splits



Dasgupta et al (2009)

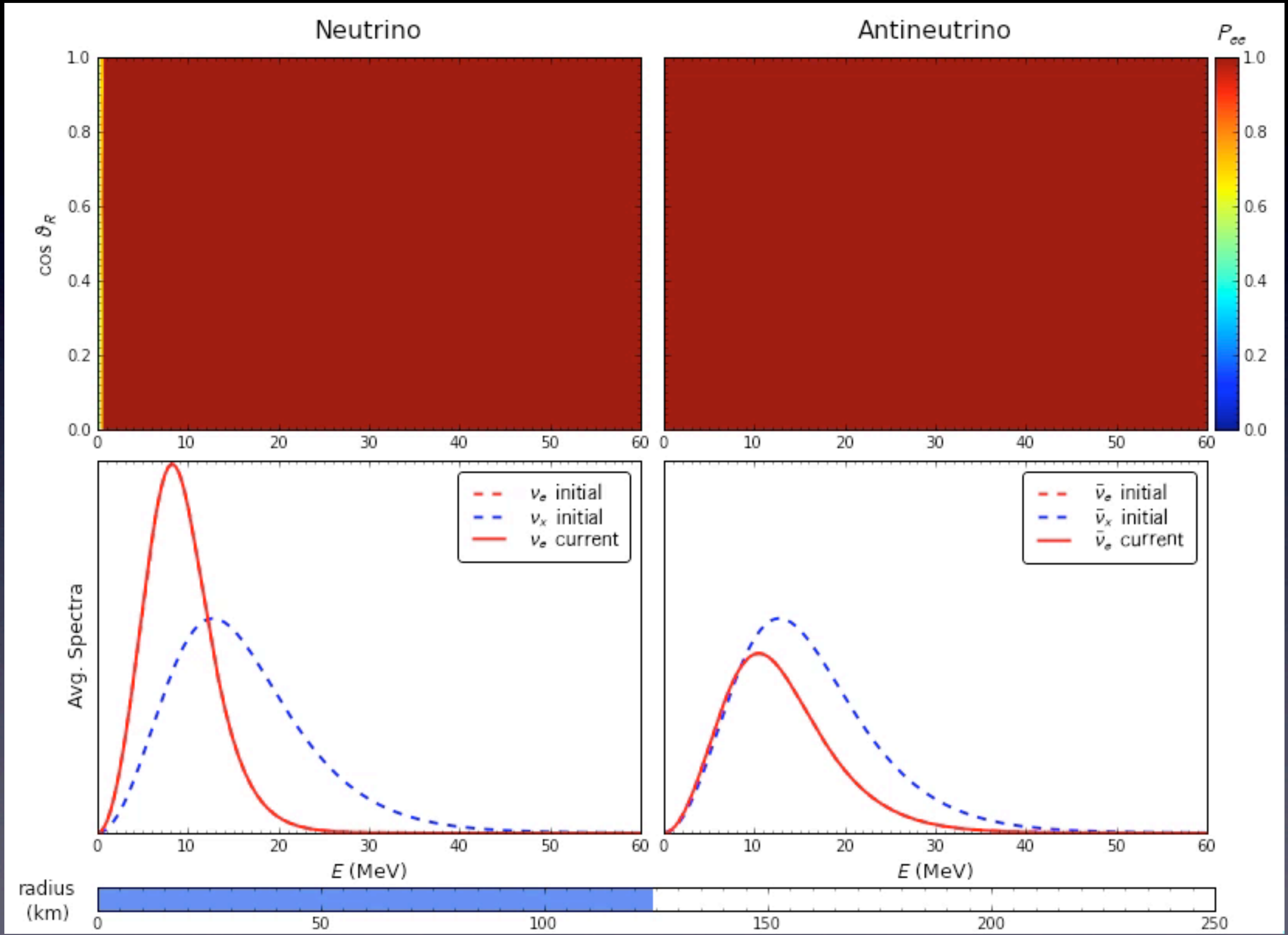
3 Flavor Instability



Friedland (2010)

$$\langle L_{\nu_e} \rangle = 4.1\,\text{foe},\ \langle L_{\bar{\nu}_e} \rangle = 4.3\,\text{foe},\ \langle L_{\nu_x,\bar{\nu}_x} \rangle = 7.9\,\text{foe}$$

$$\langle E_{\nu_e} \rangle = 9.4\,\text{MeV},\ \langle E_{\bar{\nu}_e} \rangle = 13.0\,\text{MeV},\ \langle E_{\nu_x,\bar{\nu}_x} \rangle = 15.8\text{MeV}$$



HD & Friedland (2010)

$$\langle L_{\nu_e} \rangle = 4.1\,\text{foe},\ \langle L_{\bar{\nu}_e} \rangle = 4.3\,\text{foe},\ \langle L_{\nu_x,\bar{\nu}_x} \rangle = 7.9\,\text{foe}$$

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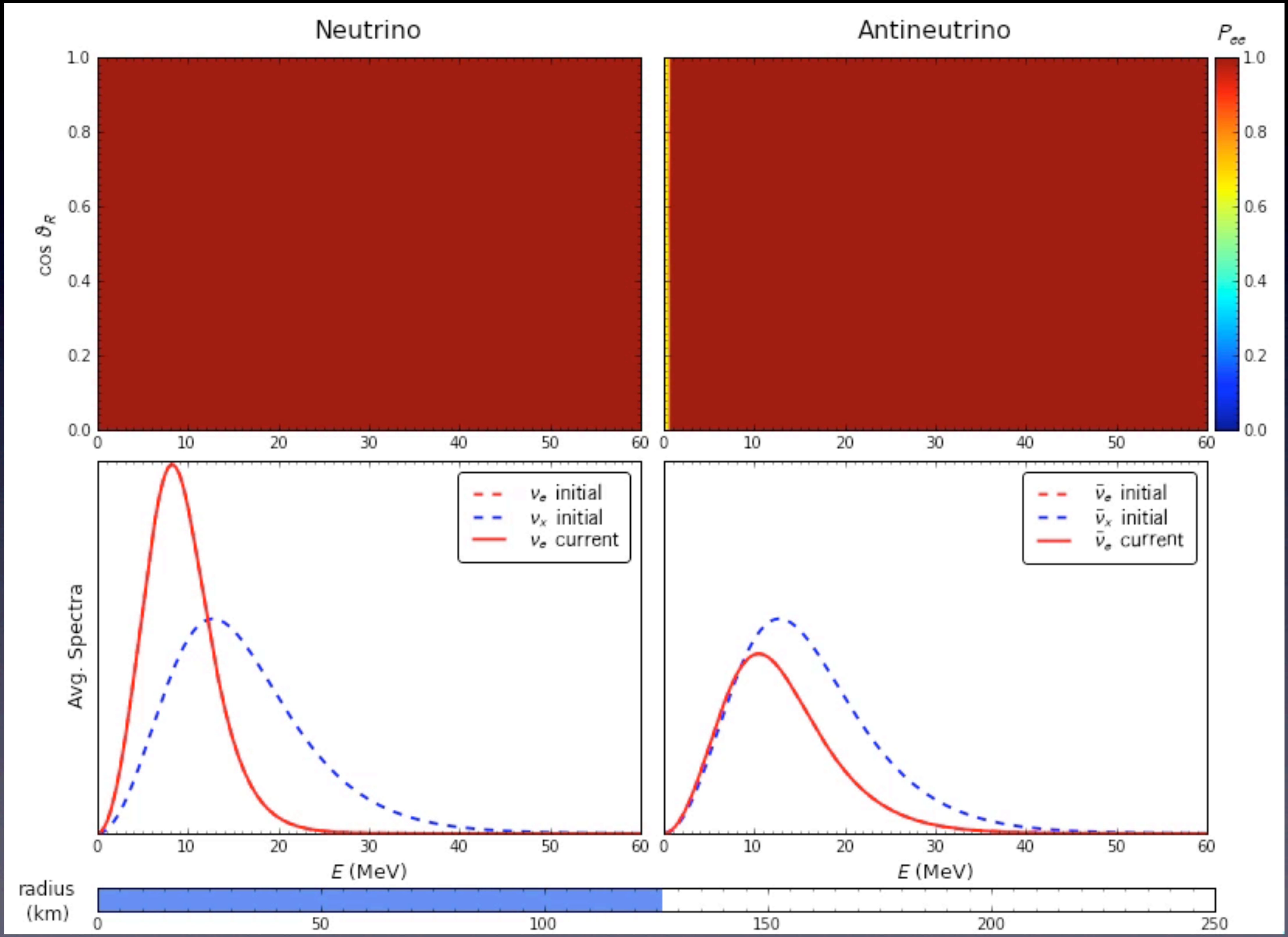
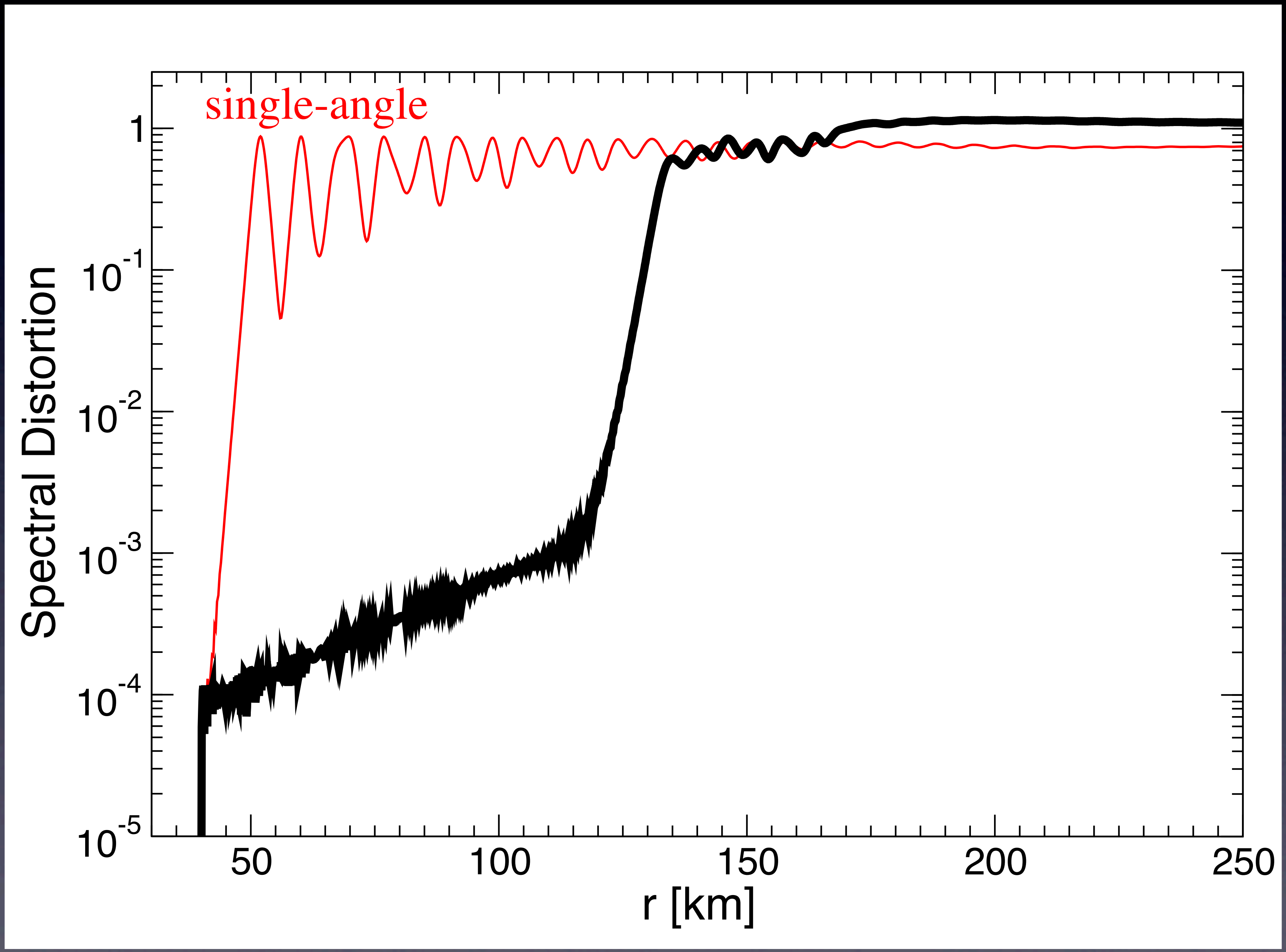


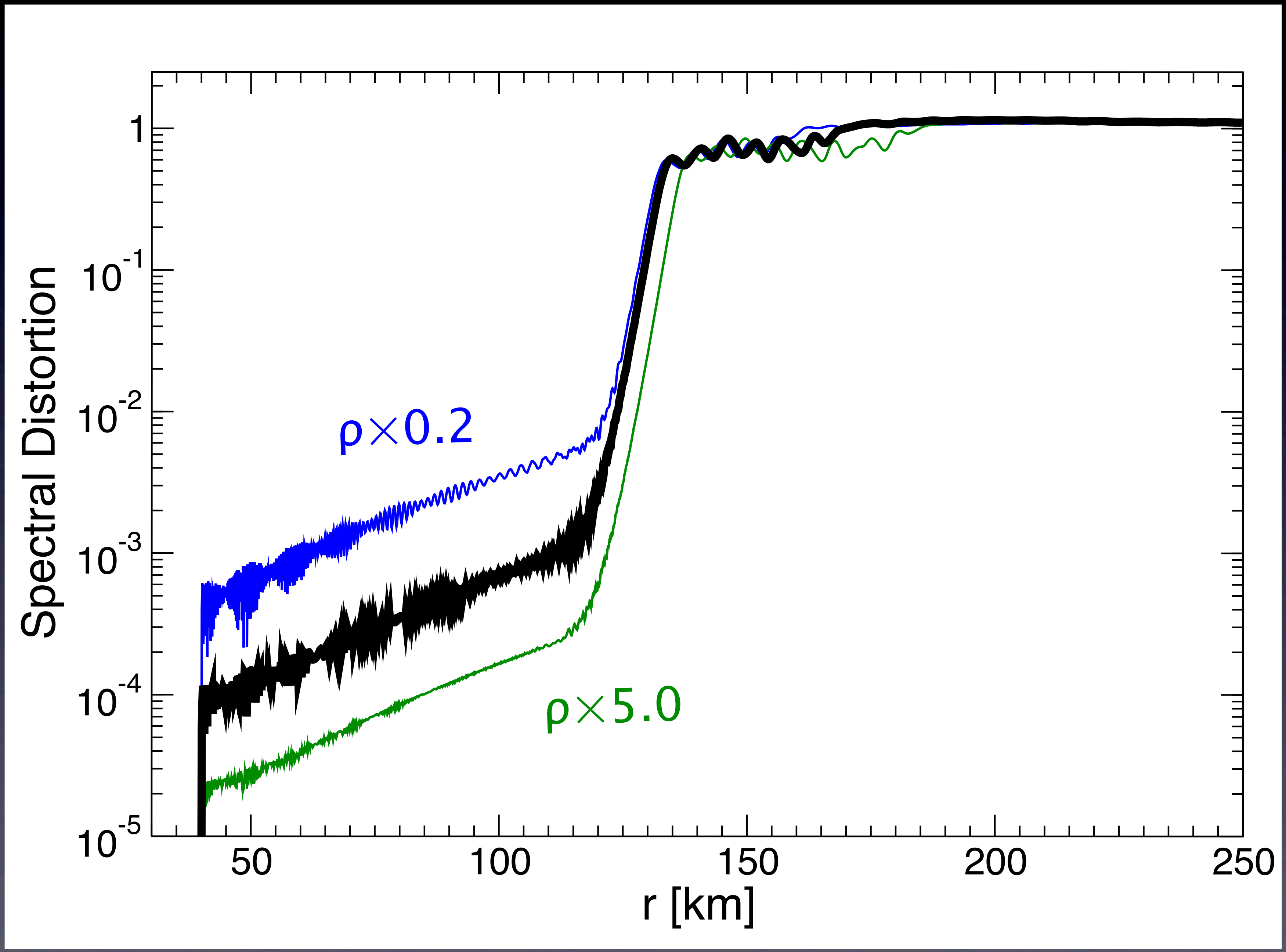
HD & Friedland (2010)

Multiangle Suppression



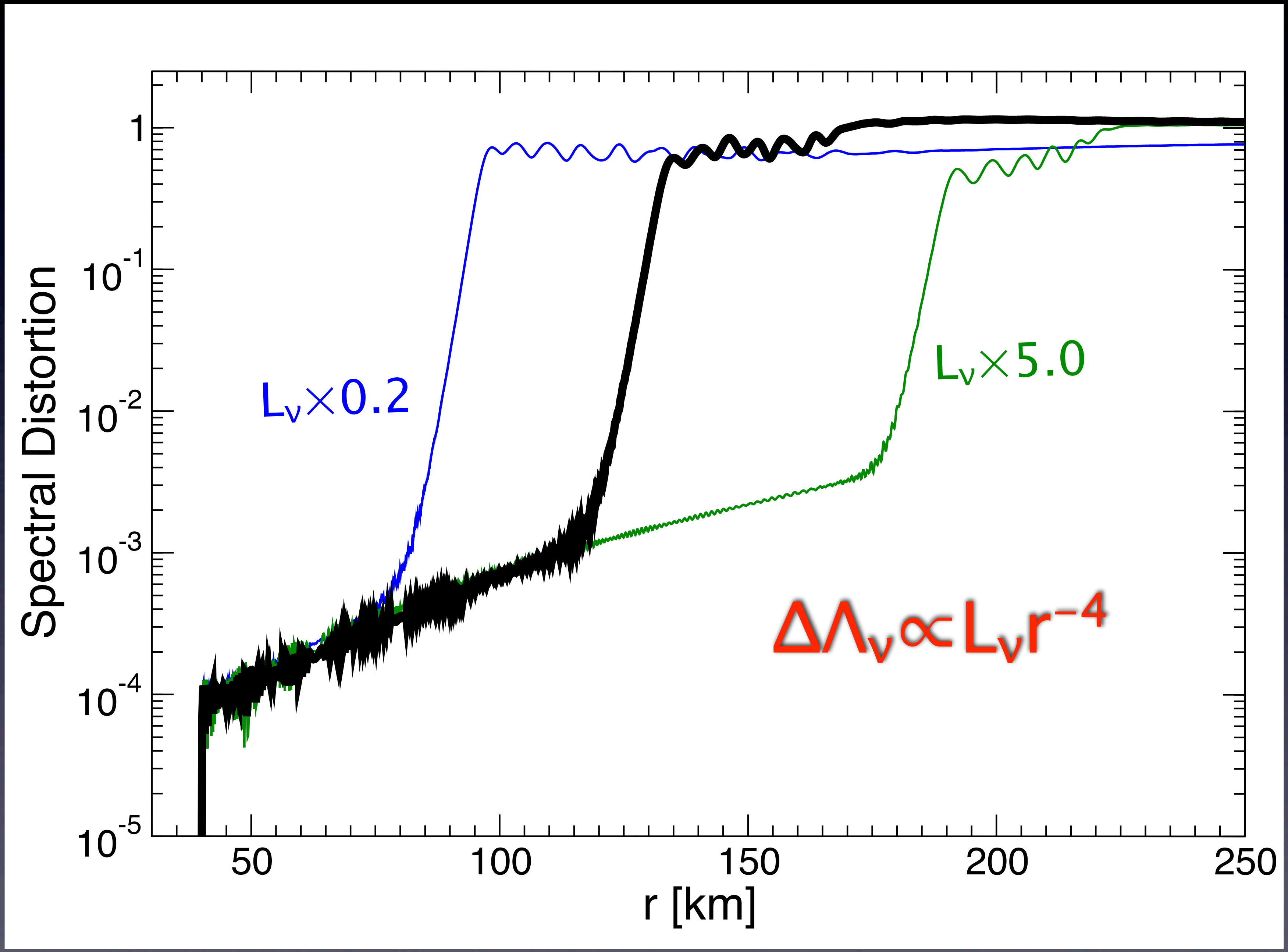
HD & Friedland (2010)

Multiangle Suppression



HD & Friedland (2010)

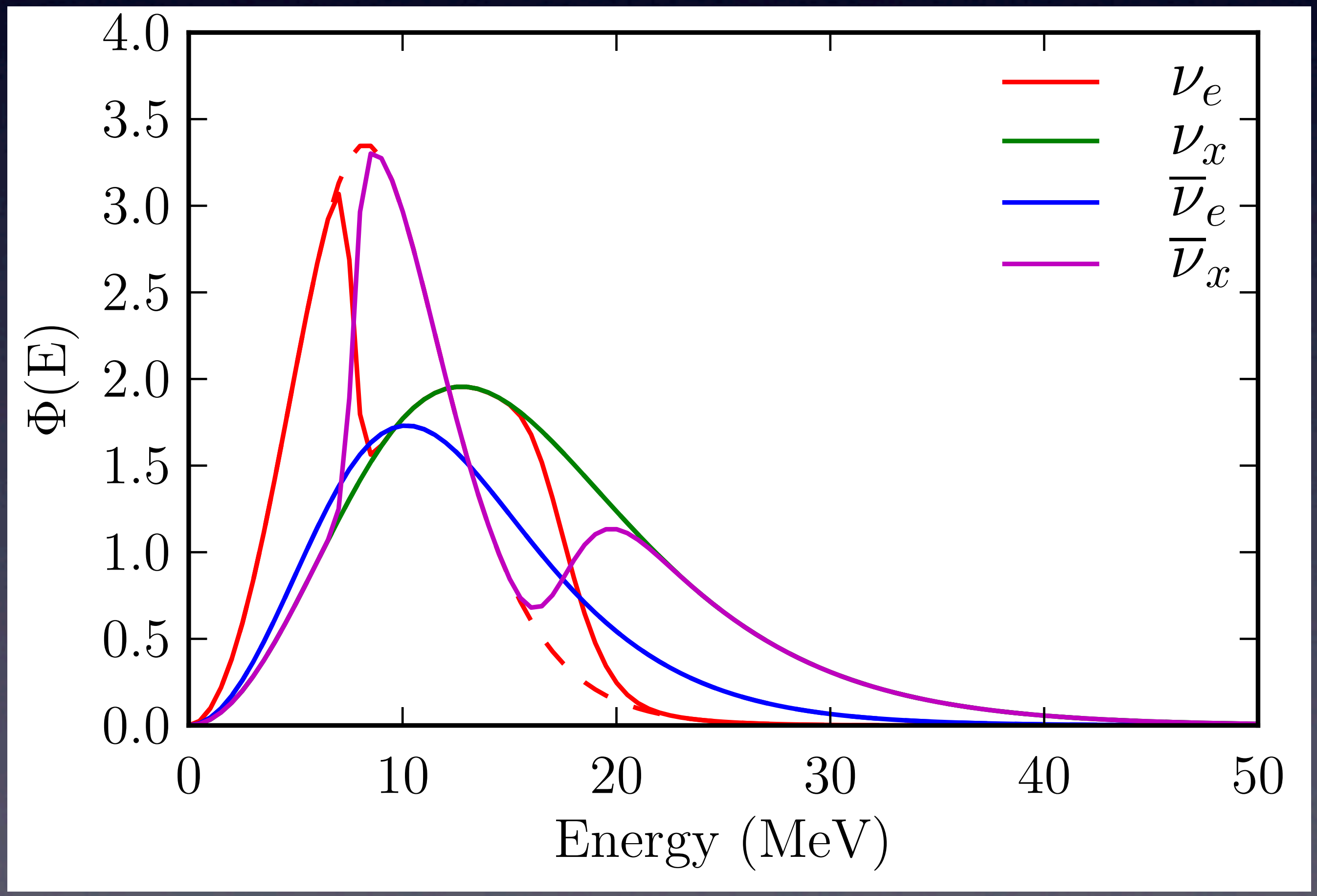
Multiangle Suppression

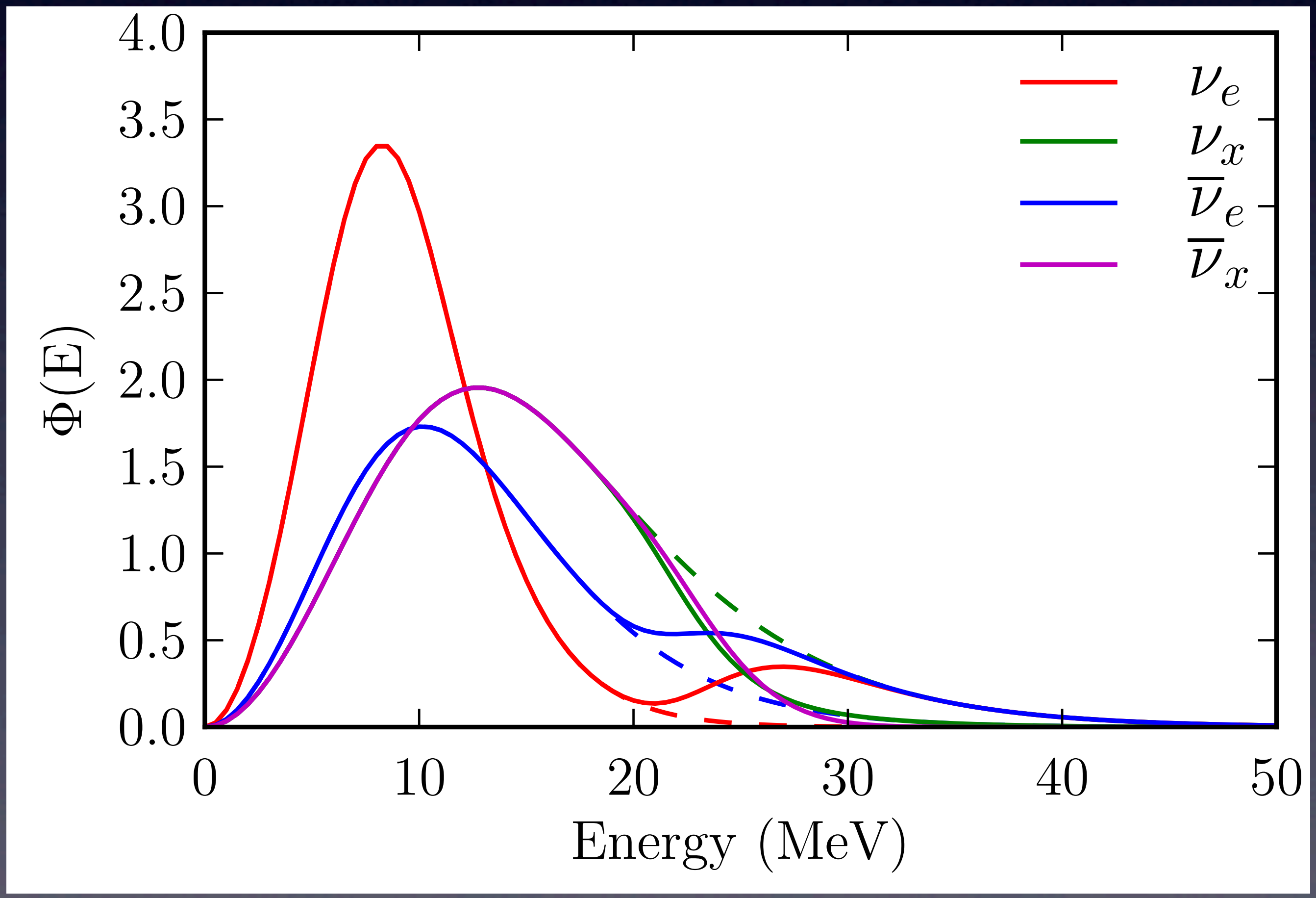


HD & Friedland (2010)

In Magnetic Field

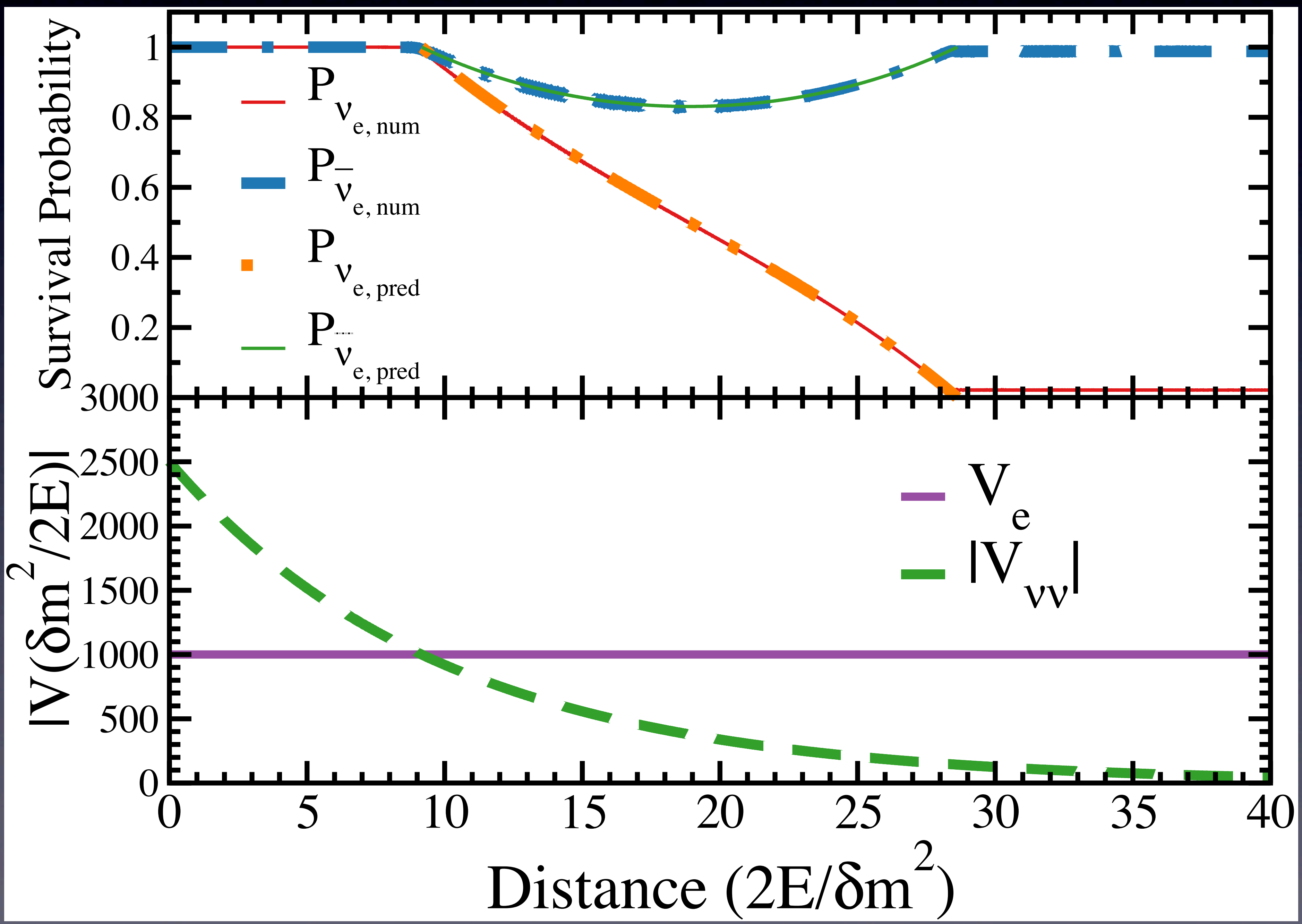
Assume neutrinos to be Majorana particles.





de Gouvêa & Shalgar (2012, 2013)

Matter-Neutrino Resonance



- Anti-neutrino excess
- Merging compact objects
- Single-angle

Malkus, Friedland, McLaughlin (2014)

YES!! You can make discoveries through numerical calculations.

But are you sure whether the numerical calculations are correct?