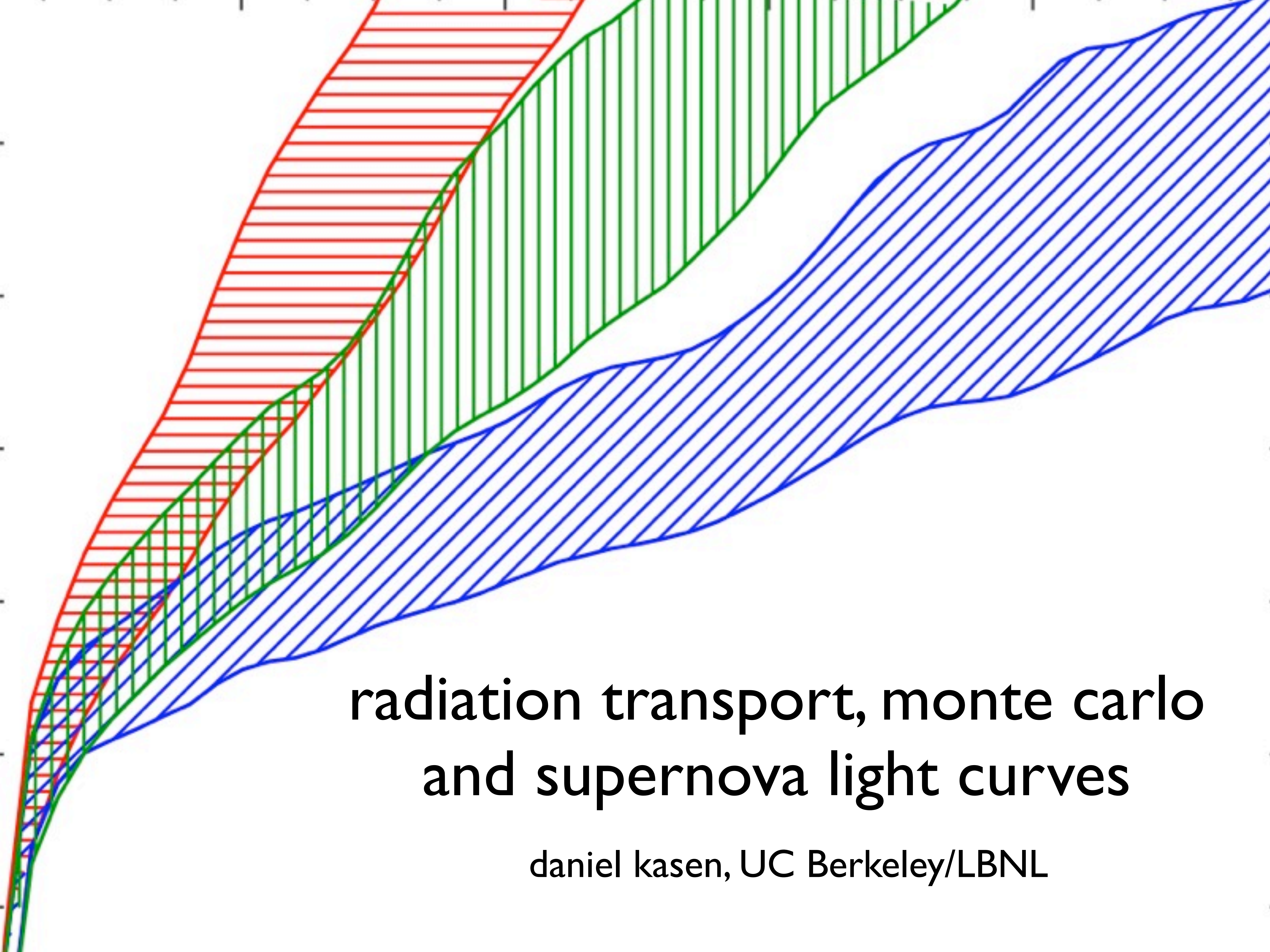
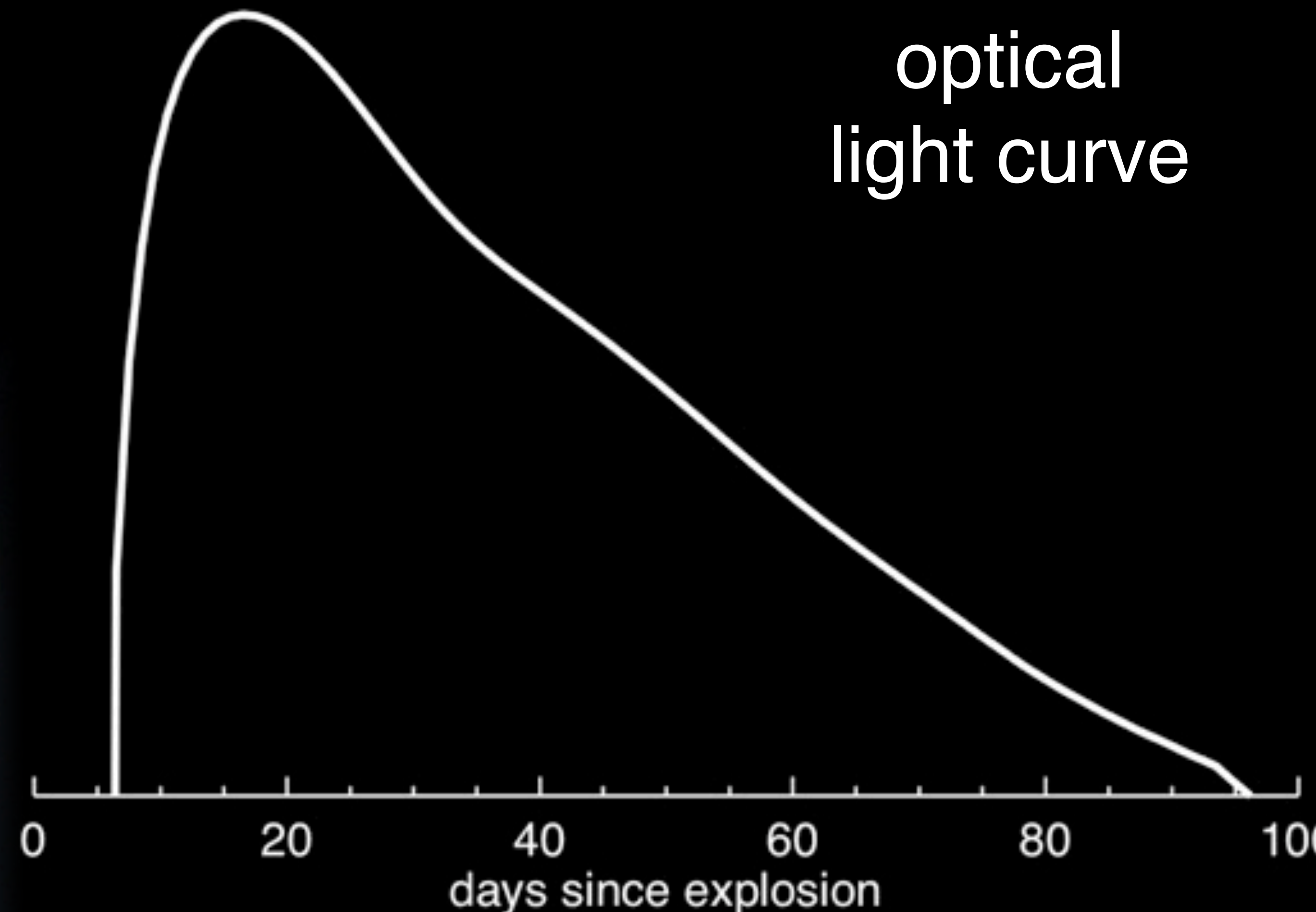


radiation transport, monte carlo and supernova light curves

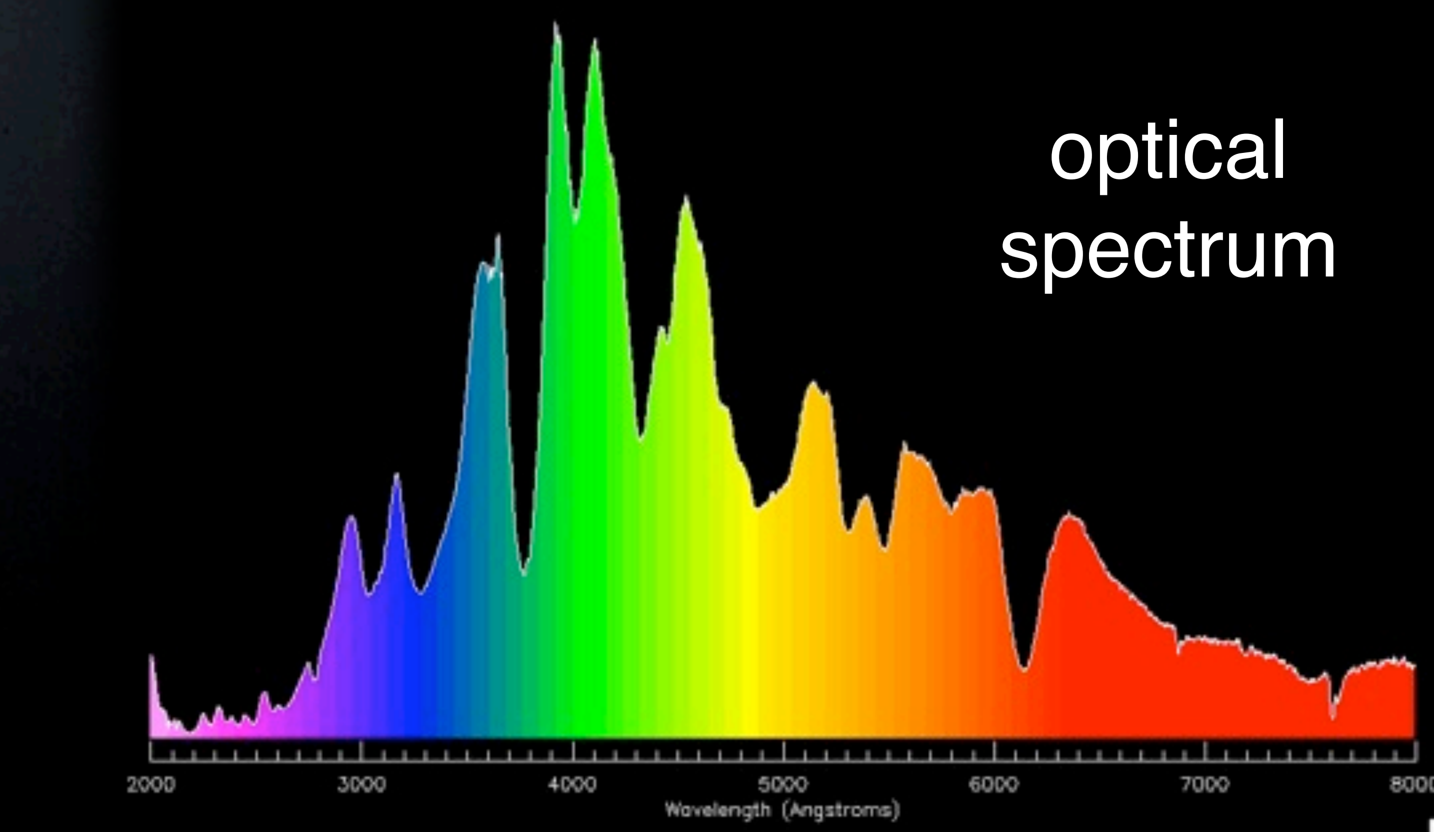
daniel kasen, UC Berkeley/LBNL

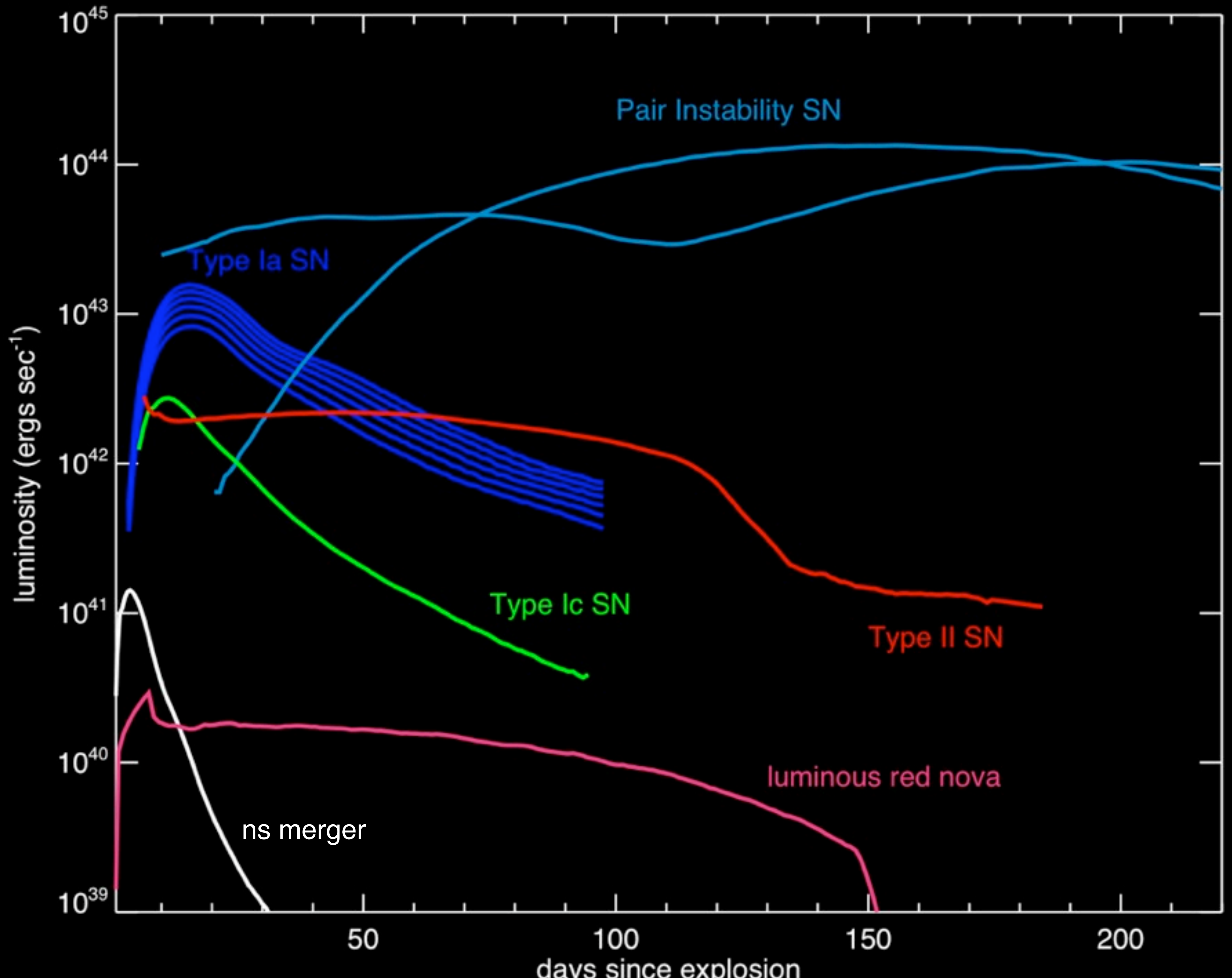
supernovae and the transient universe

optical light curve

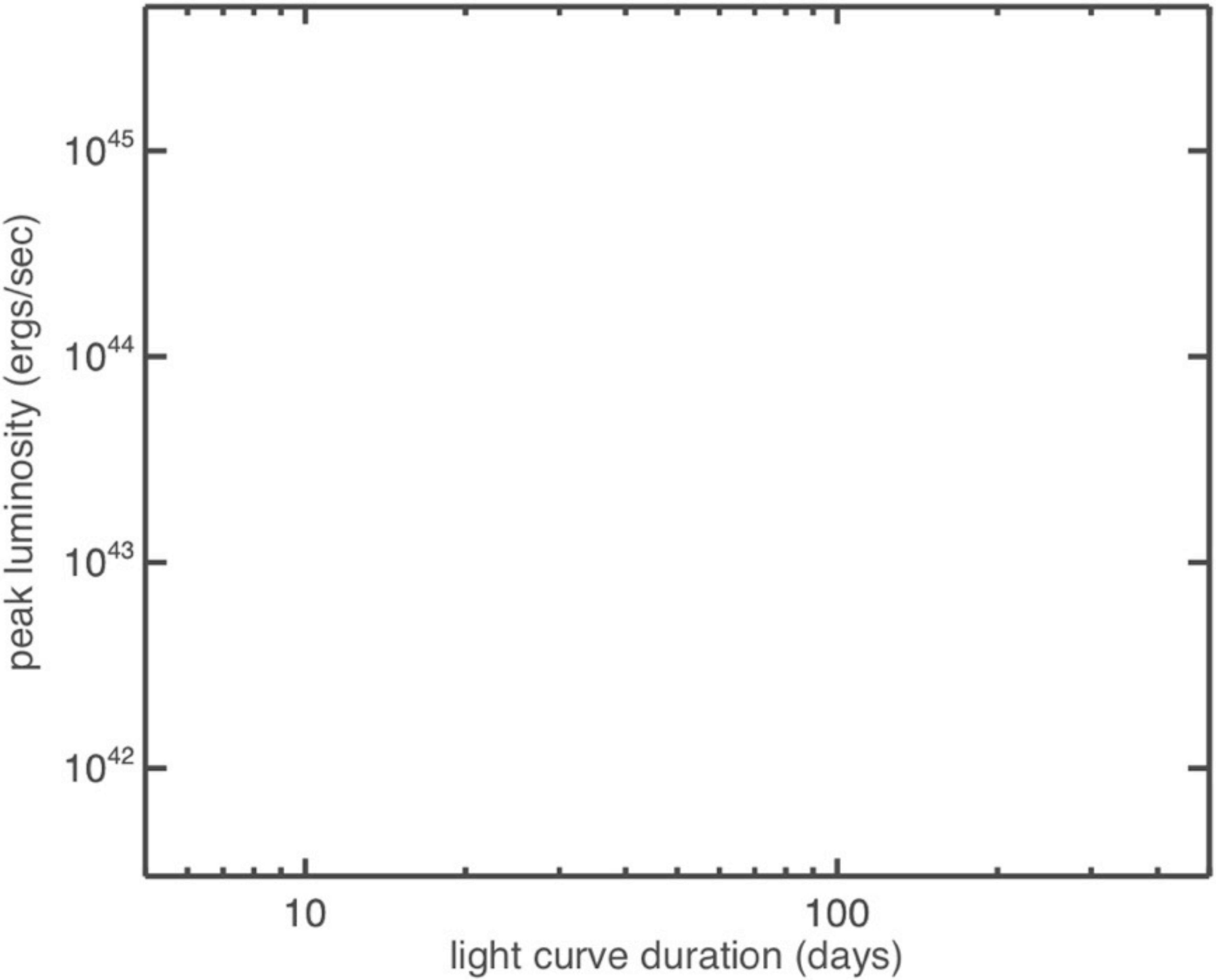


optical spectrum

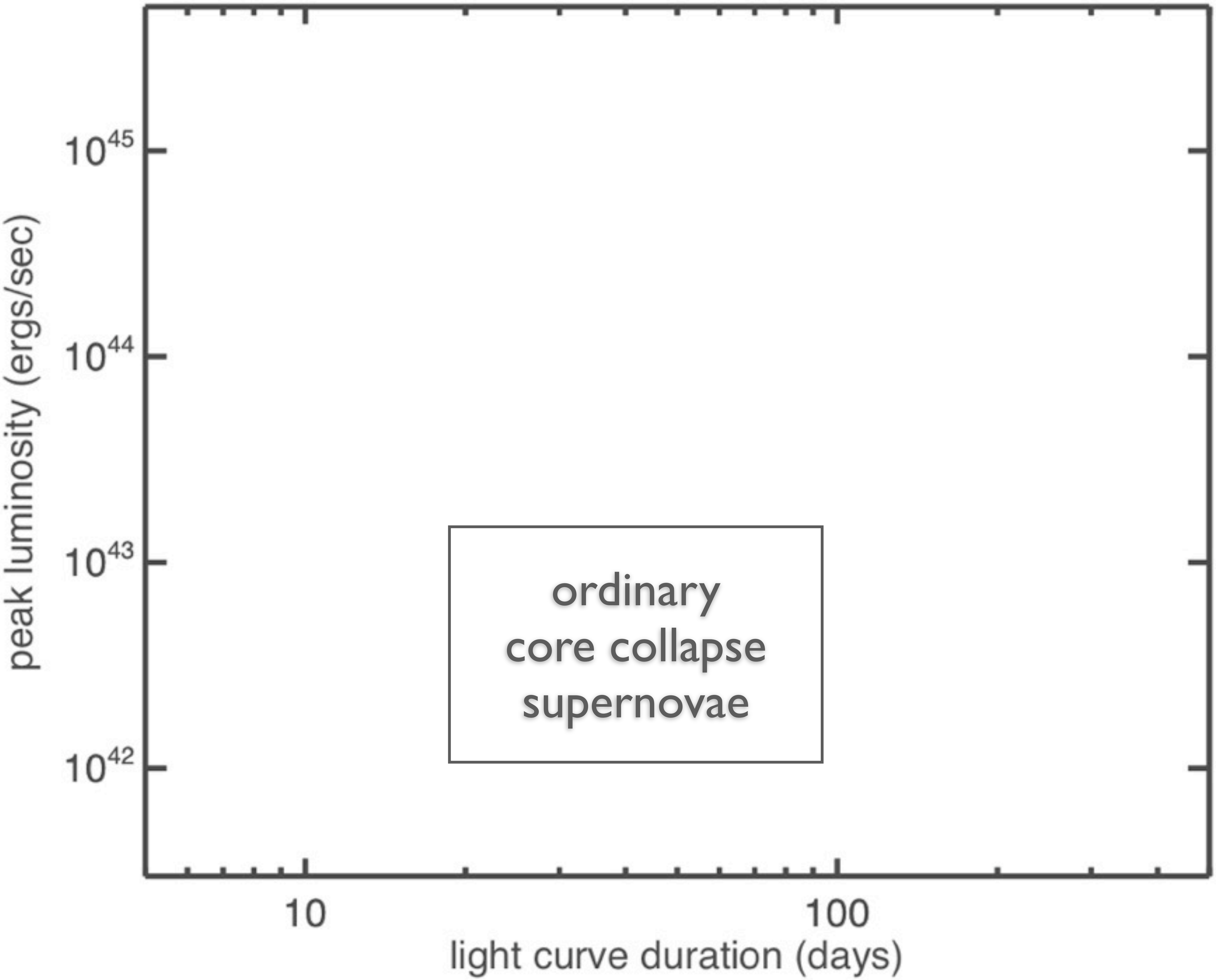




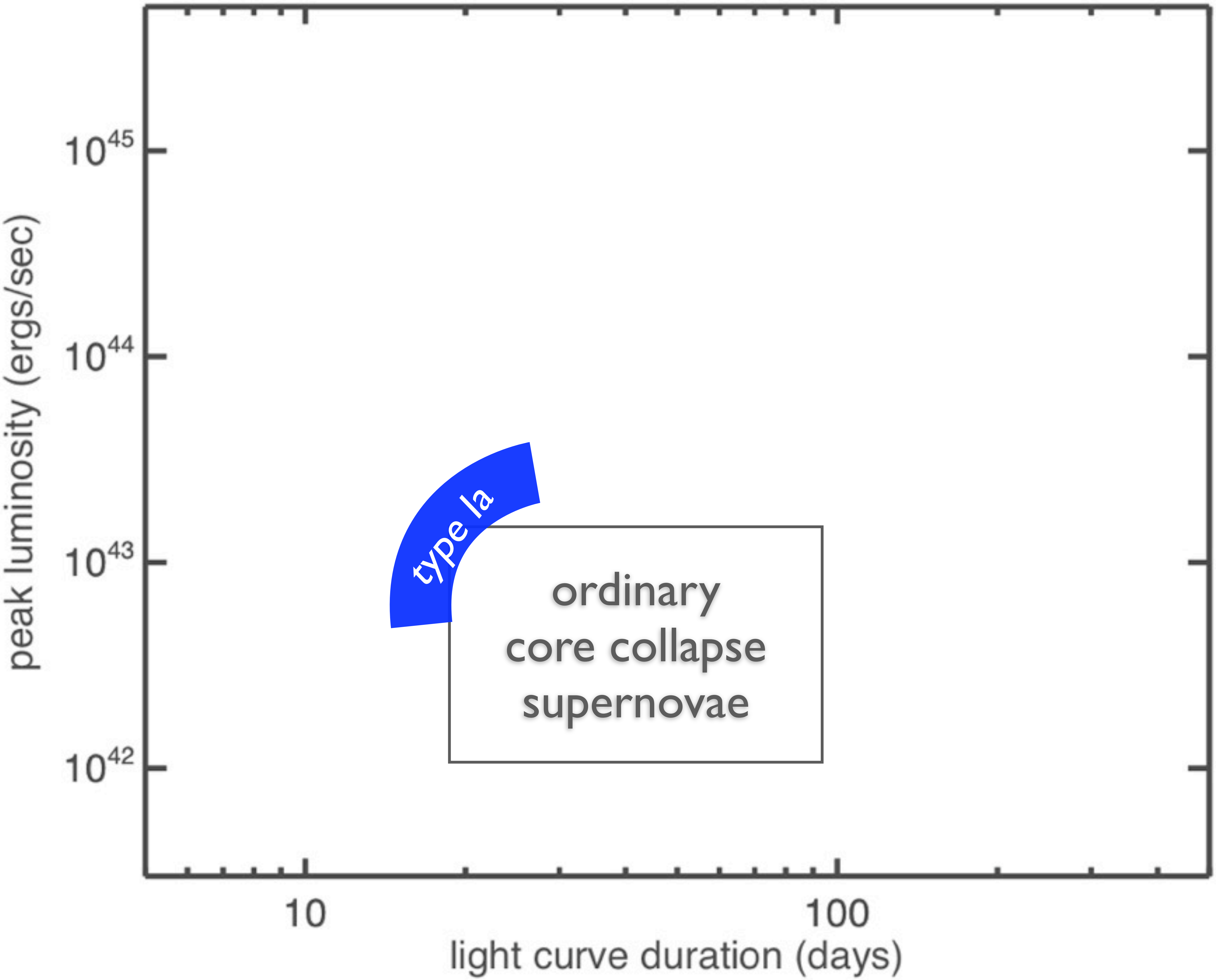
Pair Instability SN
Type Ia SN
Type Ic SN
Type II SN
luminous red nova
ns merger
luminosity (ergs sec^{-1})
days since explosion
10^{45}
10^{44}
10^{43}
10^{42}
10^{41}
10^{40}
10^{39}
50
100
150
200



peak luminosity (ergs/sec)
10^{45}
10^{44}
10^{43}
10^{42}
10
100
light curve duration (days)



peak luminosity (ergs/sec)
10^{45}
10^{44}
10^{43}
10^{42}
ordinary
core collapse
supernovae
10
100
light curve duration (days)



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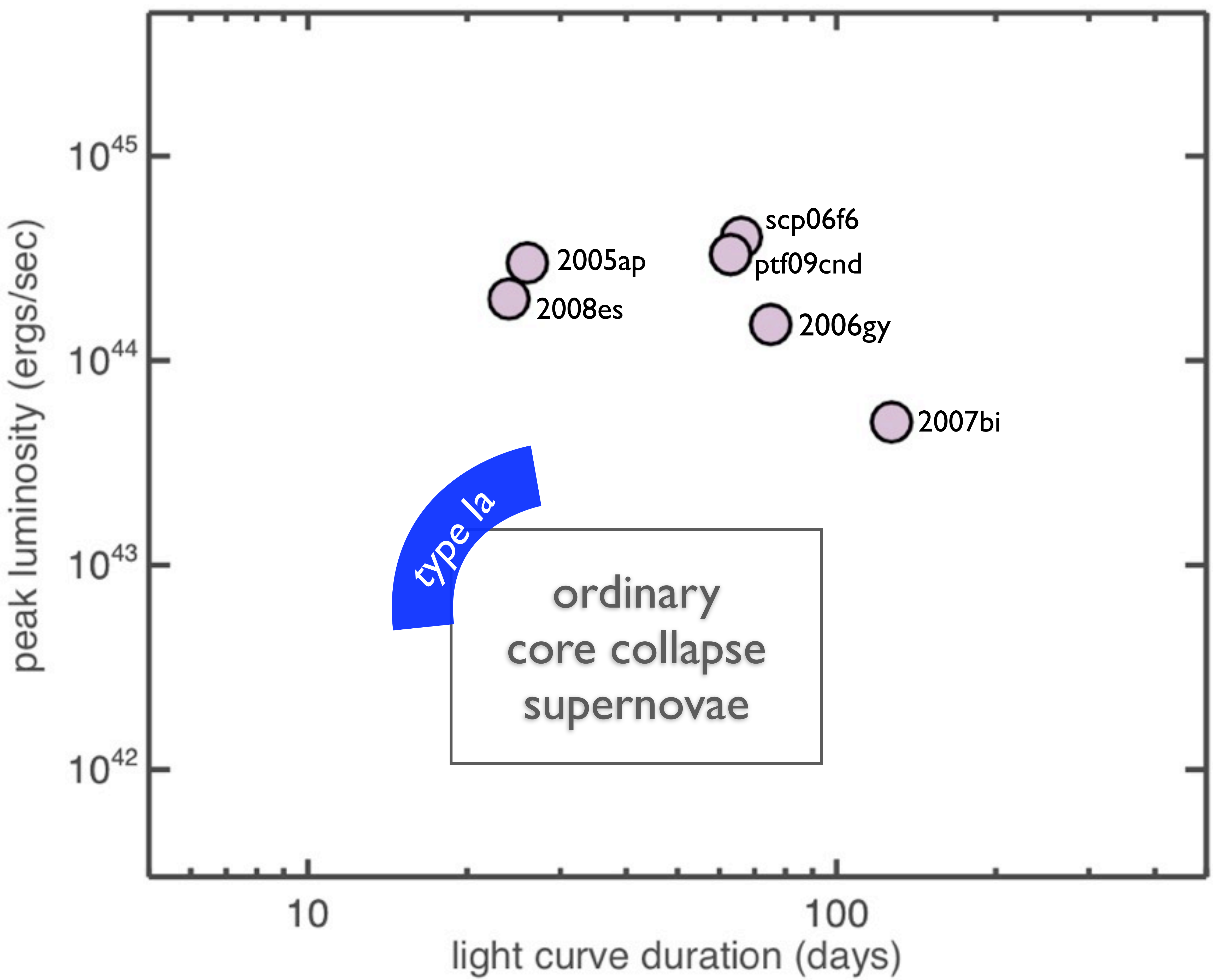

peak luminosity (ergs/sec)
10^45
10^44
10^43
10^42
scp06f6
2005ap
ptf09cnd
2008es
2006gy
2007bi
type Ia
ordinary
core collapse
supernovae
10
100
light curve duration (days)

supernova light curves

some basic physical scales

supernova light curves

some basic physical scales

assume (conservatively) blackbody emission at T ~ 10^4 K

$$L = 4\pi R^2 \sigma_{\rm SB} T^4 \longrightarrow R_{\rm sn} \sim 10^{15}\ {\rm cm} \approx 10^4\ R_\odot$$

supernova light curves

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$$v = \frac{R_{\mathrm{sn}}}{t} \approx 10^9\ \mathrm{cm\ s^{-1}} \approx 10,000\ \mathrm{km\ s^{-1}}$$

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the kinetic energy of the remnant is then (for M ~ M_{sun})

$$E \approx \frac{1}{2} M v^2 \approx 10^{51}\ {\rm ergs} \equiv 1\ B$$

the computational problem

stellar evolution (>10^6 years)

↓

$\rho(r)$, T(r), $A_i(r)$ at ignition/collapse

the computational problem

stellar evolution (>10^6 years)

$\rho(r), T(r), A_i(r)$ at ignition/collapse

explosion (seconds/hours)

hydrodynamics, equation of state
nuclear burning, <u>neutrino transport</u>

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→

neutrinos
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x-rays, γ-rays

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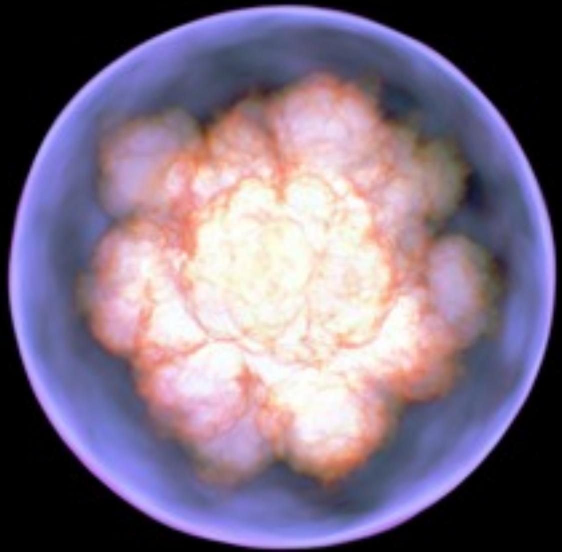

explosion (seconds/hours)

hydrodynamics, equation of state
nuclear burning, neutrino transport

→ neutrinos
grav. waves
x-rays, γ-rays

↓ $\rho(x,y,z)$, $v(x,y,z)$, $T(x,y,z)$, $A_i(x,y,z)$
in free expansion

expanding ejecta (months)

photon transport
matter opacity
thermodynamics
radioactive decay

the computational problem

stellar evolution ($>10^6$ years)

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expanding ejecta (months)

photon transport
matter opacity
thermodynamics
radioactive decay

→ optical spectra
light curves

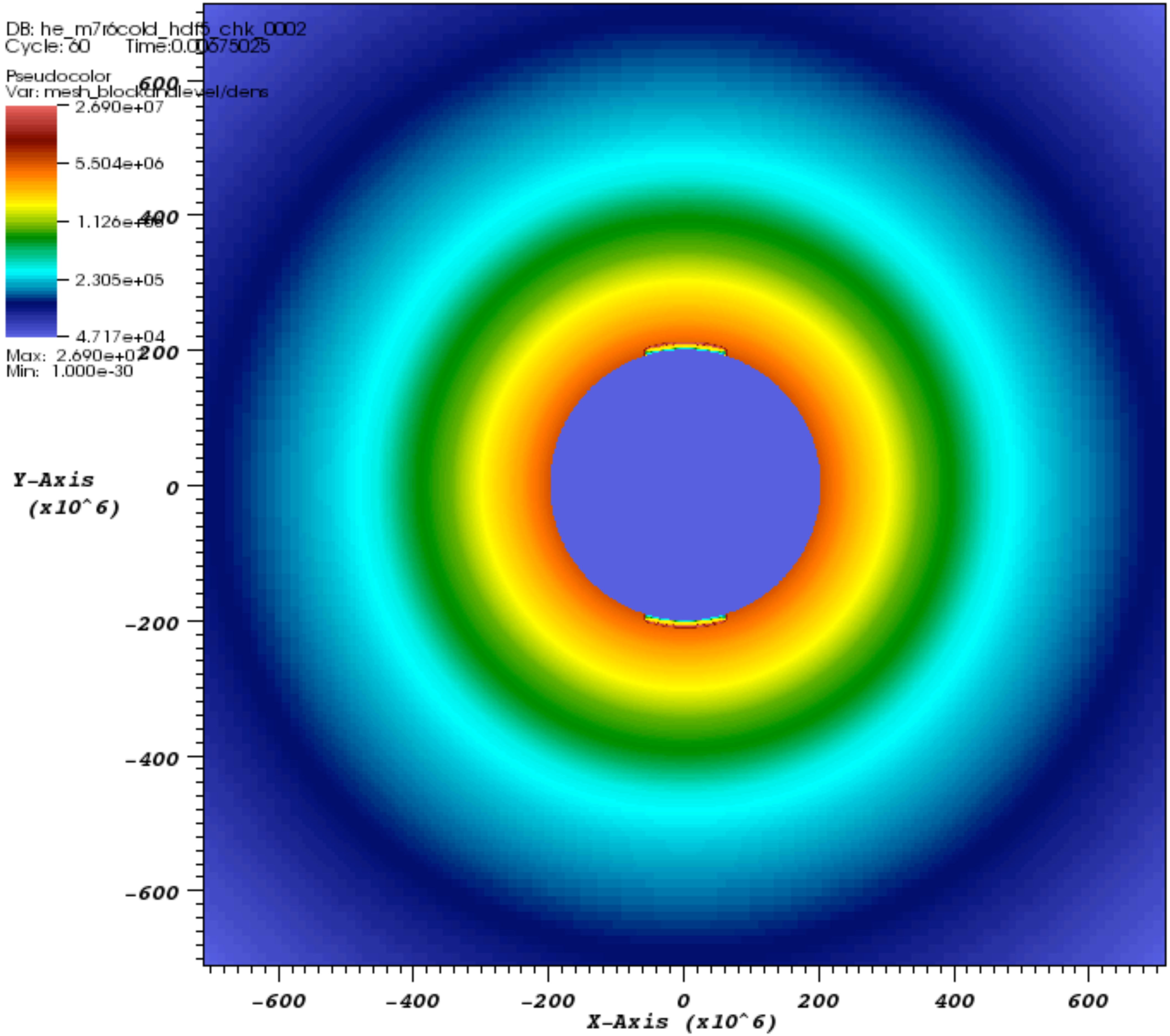
how to explode a supernova

simple description

take a	white dwarf	helium star	red giant
with a mass	$1.4\ M_{sun}$	$\sim 5\ M_{sun}$	$10\text{-}20\ M_{sun}$
and a radius	10^9 cm	10^{11} cm	10^{13} cm
dump in	$\sim 10^{51}$ ergs		
	hydro, burning, neutrinos, etc...		
get a	type Ia	type Ib/Ic	type II

jet
powered
supernova

sean
couch



jet
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energetics

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immediately after the explosion (e.g., strong shock) total energy is split between kinetic energy and radiation

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radiation energy dominates over gas thermal energy density
e.g., explode the sun, with E = 10^{51} ergs

$$\frac{\epsilon_{\rm rad}}{\epsilon_{\rm gas}} \simeq \frac{aT^4}{\frac{3}{2}nkT} \simeq 60 \left(\frac{T}{10^8 K}\right)^3 \left(\frac{1 \ {\rm g \ cm^{-3}}}{\rho}\right)$$

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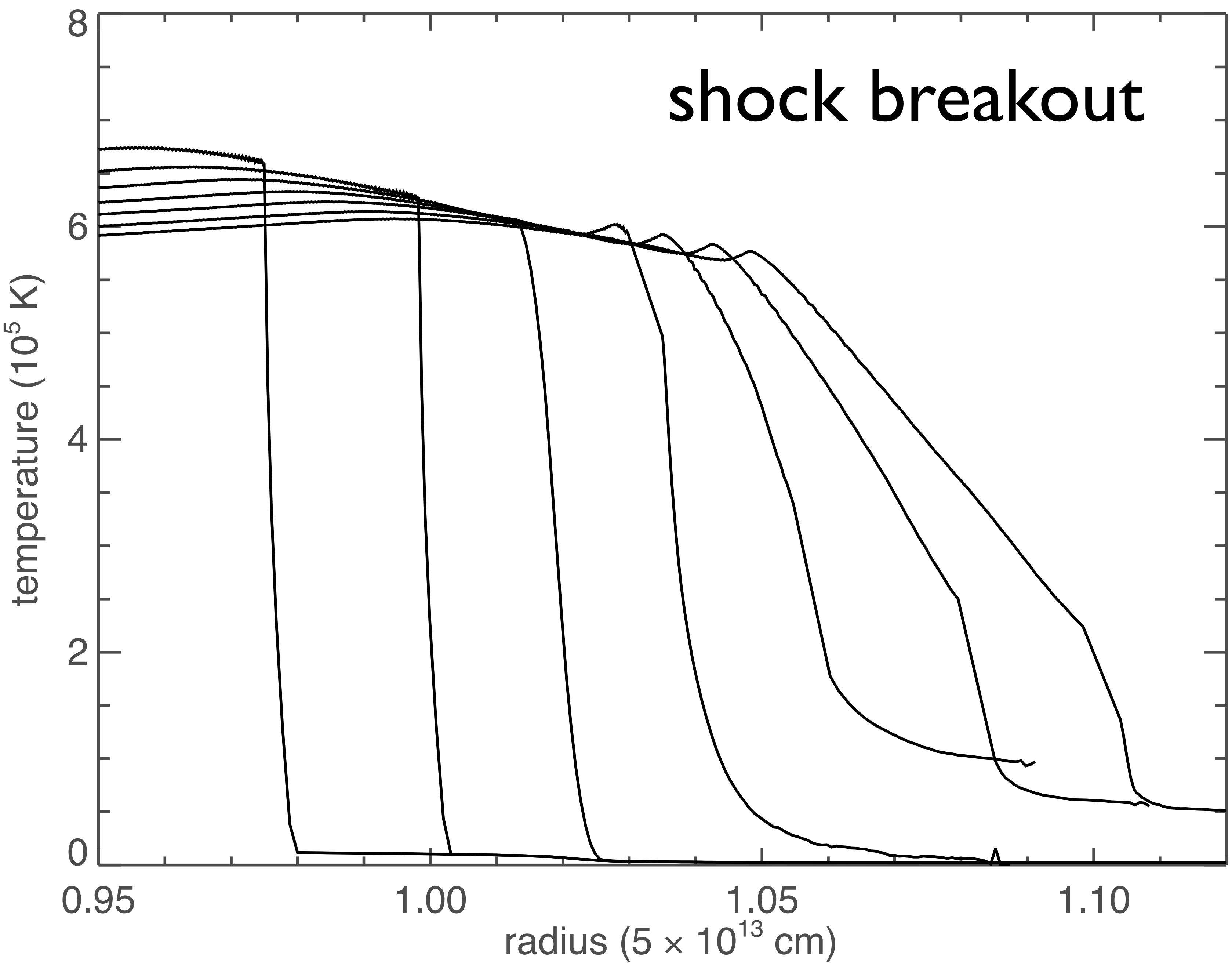
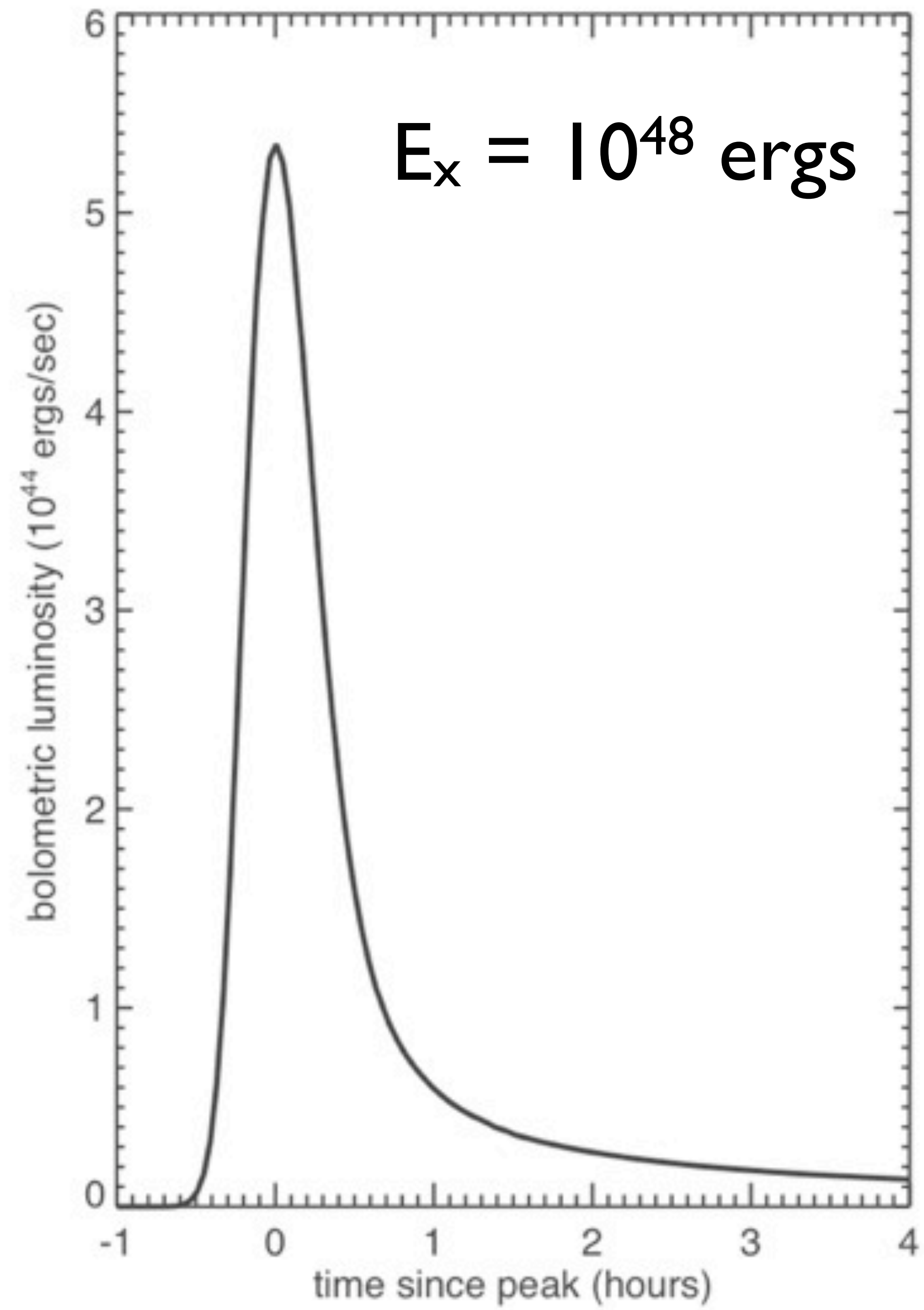
but the radiation can't escape because a star is opaque.
The ejecta expands by a factor of 10^2-10^6 in radius before the density drops enough to become translucent

initial radiation from supernova explosions

shock breakout x-ray burst from a red super-giant





adiabatic expansion

converts E_{thermal} into E_{kinetic} as the radiation does work

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first law of thermodynamics (with no heat transfer)

$$\frac{\partial(\epsilon_{\text{rad}} V)}{\partial t} = -p_{\text{rad}} \frac{\partial V}{\partial t}$$

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use chain rule and $\epsilon_{\rm rad} = 3p_{\rm rad}$

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$$\frac{\dot{\epsilon}_{\text{rad}}}{\epsilon_{\text{rad}}} = -\frac{4}{3} \frac{\dot{V}}{V}$$

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so the change in energy density as the radius expands is:

$$\frac{\dot{\epsilon}_{\text{rad}}}{\epsilon_{\text{rad}}} = -\frac{4}{3}\frac{\dot{V}}{V} \longrightarrow \boxed{\epsilon_{\text{rad}} \propto V^{-4/3} \propto R^{-4}}$$

adiabatic expansion

adiabatic expansion

basic thermodynamics

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$$\boxed{\epsilon_{\rm rad} \propto R^{-4} \quad \text{or} \quad \epsilon_{\rm rad} V = E_{\rm rad} \propto R^{-1}}$$

transition to free expansion

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$$\boxed{R(t) = vt + \cancel{R_0}}$$

negligible

homologous expansion

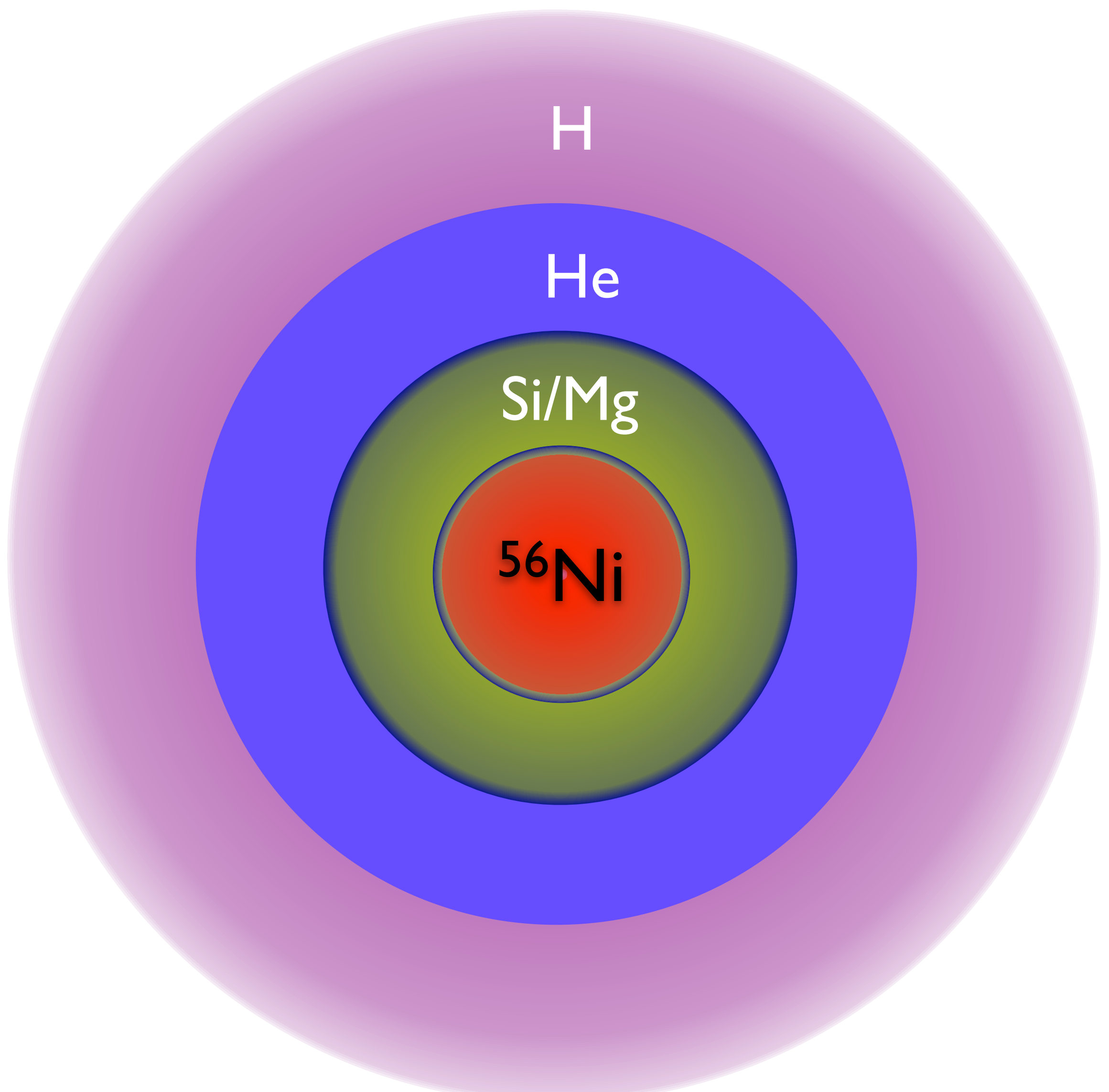
self-similar ejecta structure expands over time

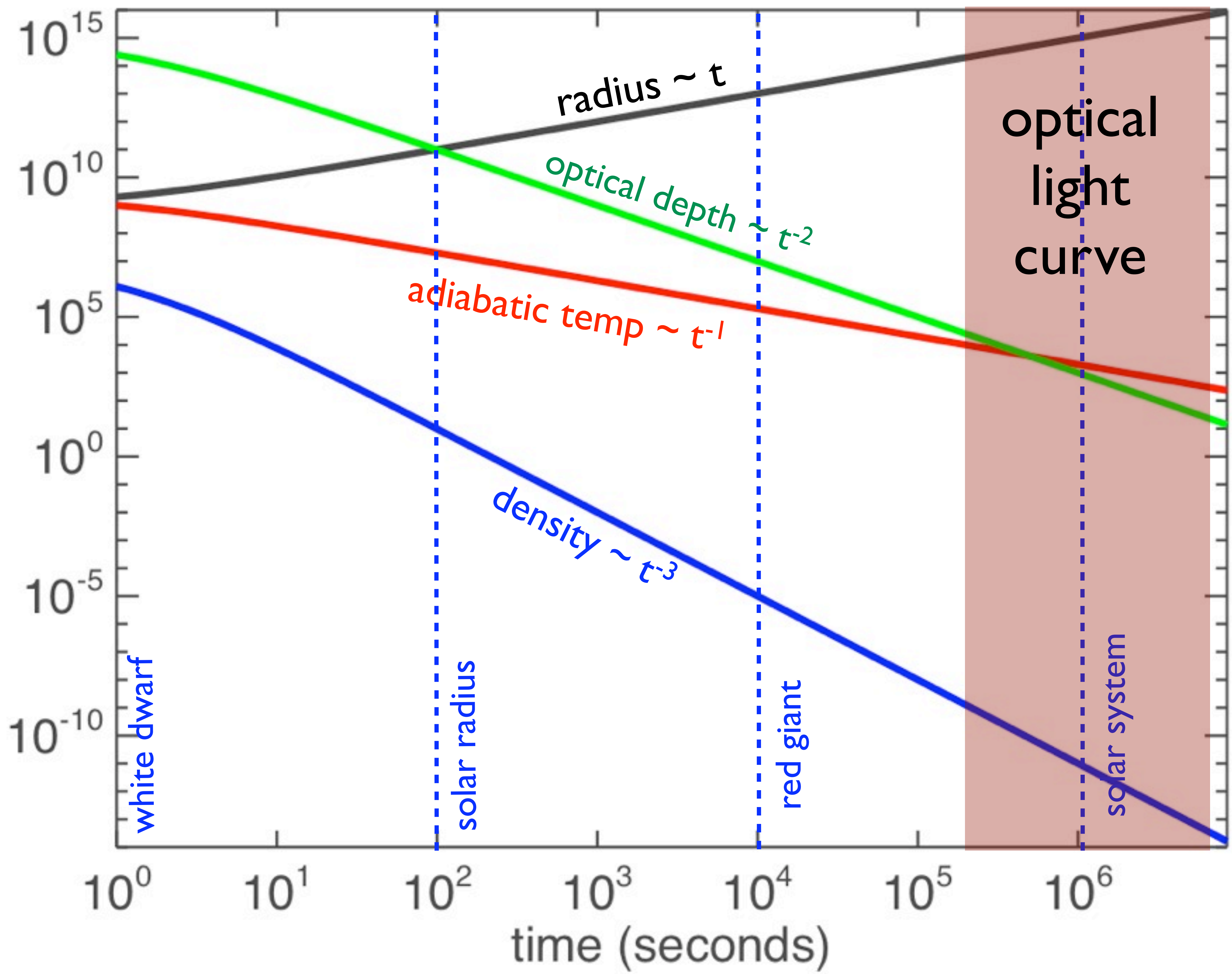
$$R(t) = vt$$

$$\rho(t) = \rho_0 (R_0/R)^3 \propto t^{-3}$$

rule of thumb:
to reach homology
run your hydrodynamics
simulations until
$R_{final} \gtrsim 10\ R_0$
better: $R_{final} \sim 100\ R_0$

check, $E_{thermal} \ll E_{kinetic}$





radius ~ t
optical depth ~ t^-2
adiabatic temp ~ t^-1
density ~ t^-3
optical
light
curve
white dwarf
solar radius
red giant
solar system
10^15
10^10
10^5
10^0
10^-5
10^-10
10^0
10^1
10^2
10^3
10^4
10^5
10^6
time (seconds)

duration of the light curve

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the *diffusion time* of photons through the optically thick remnant

$$t_{\rm d} = \tau \left[\frac{R}{c} \right]$$

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the *diffusion time* of photons through the optically thick remnant

$$t_{\rm d} = \tau\left[\frac{R}{c}\right] = \kappa\rho R\left[\frac{R}{c}\right]$$

duration of the light curve

the *diffusion time* of photons through the optically thick remnant

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since

$$\rho \sim M/R^3$$

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solving for time (i.e., diffusion time ~ elapsed time)

$$t_{\rm d} \sim \left[\frac{M\kappa}{vc}\right]^{1/2}$$

e.g., arnett (1979)

diffusion in an expanding medium

arnett 1979, 1980, 1982

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gives the scaling relation for light curve duration

$$t_{\rm d} \approx 29 \text{ days } M_{1,\odot}^{1/2} \kappa_{0.4}^{1/2} E_{51}^{-1/4}$$

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mass often tends to be the dominating factor

opacity terminology

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$$\sigma = \text{“cross section”} \ (\text{cm}^2)$$

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for example

$$\sigma_{\mathrm{t}} \simeq 0.6 \times 10^{-24} \ \mathrm{cm}^2 \text{ for thomson scattering}$$

$$\kappa_{\mathrm{es}} = \frac{x_{\mathrm{ion}} \sigma_{\mathrm{t}}}{m_{\mathrm{a}}} \approx 0.4 \text{ for ionized hydrogen}$$

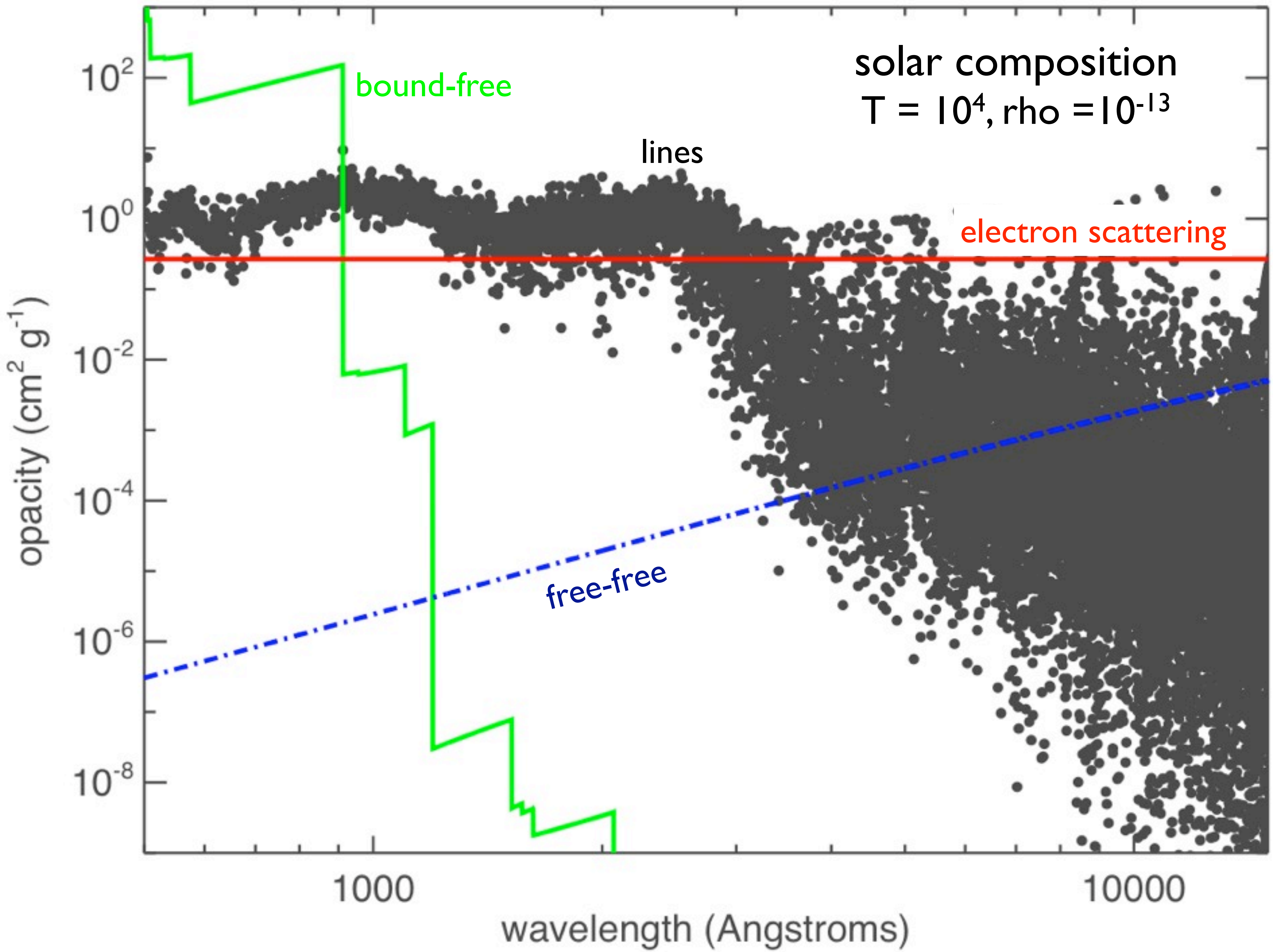
$$\approx 0.007 \text{ for singly ionized iron}$$

sources of supernova opacity

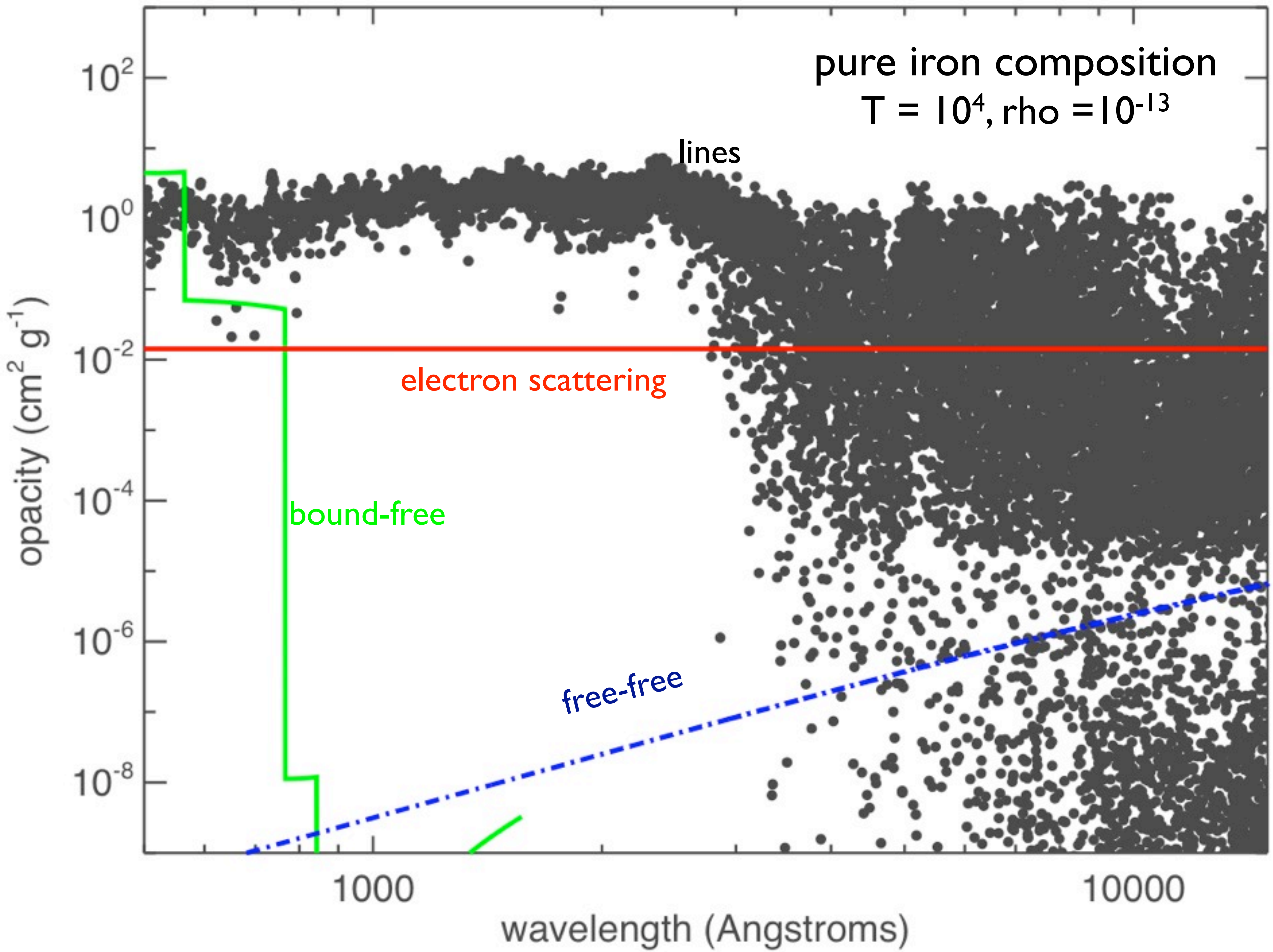
see karp (1977) pinto and eastman (2000)

thomson scattering	interaction with free electrons	optical
atomic lines	scattering/absorption from doppler broadened lines	UV/optical
bound-free	photo-ionization of atoms	UV
free-free	bremsstrahlung (free electron + nucleus)	infrared

all of these depend sensitively on the composition and **ionization state** of the ejecta!

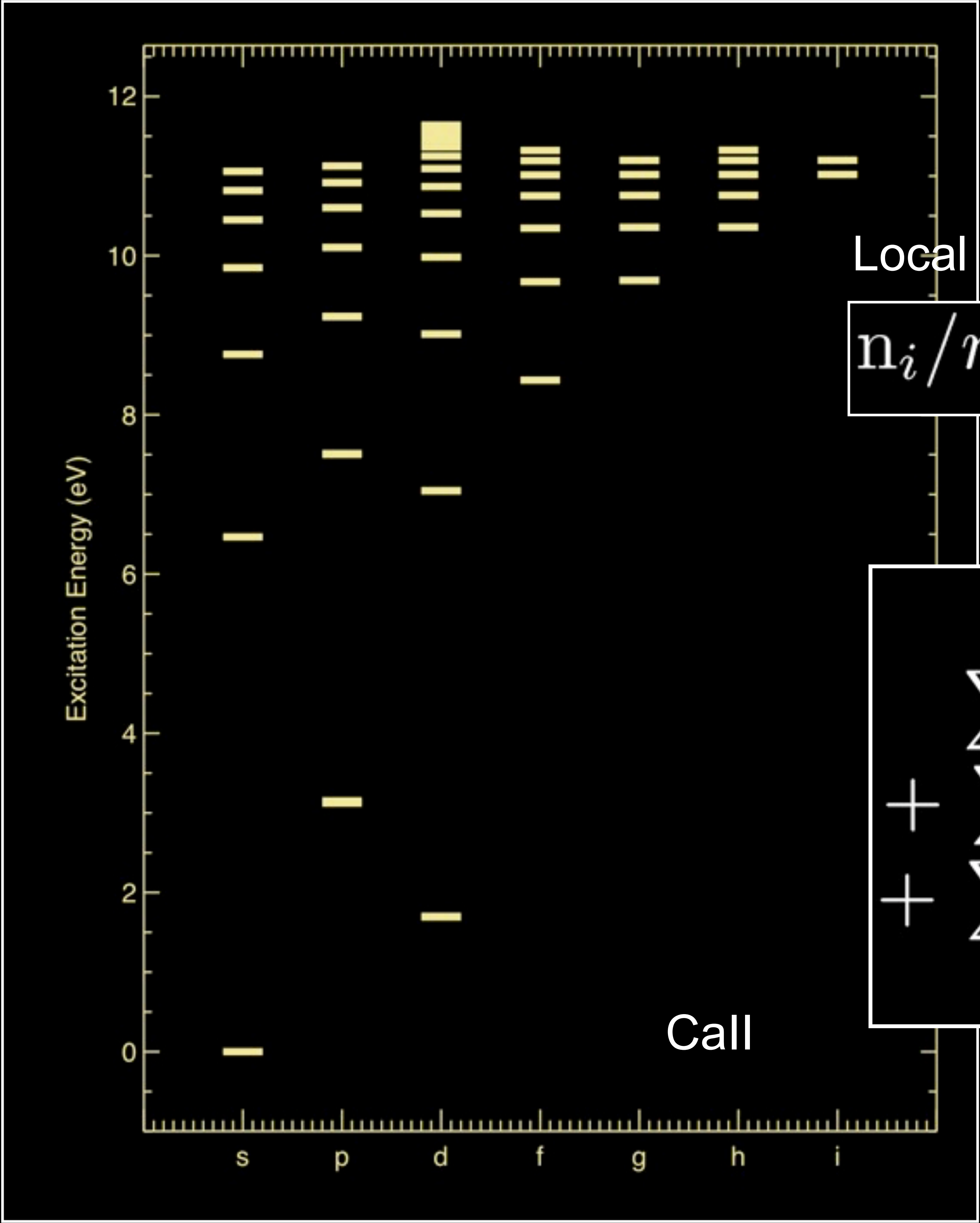


solar composition
T = 10^4, rho =10^-13
bound-free
lines
electron scattering
free-free
opacity (cm^2 g^-1)
wavelength (Angstroms)
10^2
10^0
10^-2
10^-4
10^-6
10^-8
1000
10000



pure iron composition
T = 10^4, rho =10^-13
lines
electron scattering
bound-free
free-free
opacity (cm^2 g^-1)
wavelength (Angstroms)
10^2
10^0
10^-2
10^-4
10^-6
10^-8
1000
10000

line interactions



~1/2 GB atomic data

Local Thermodynamic Equilibrium (LTE)

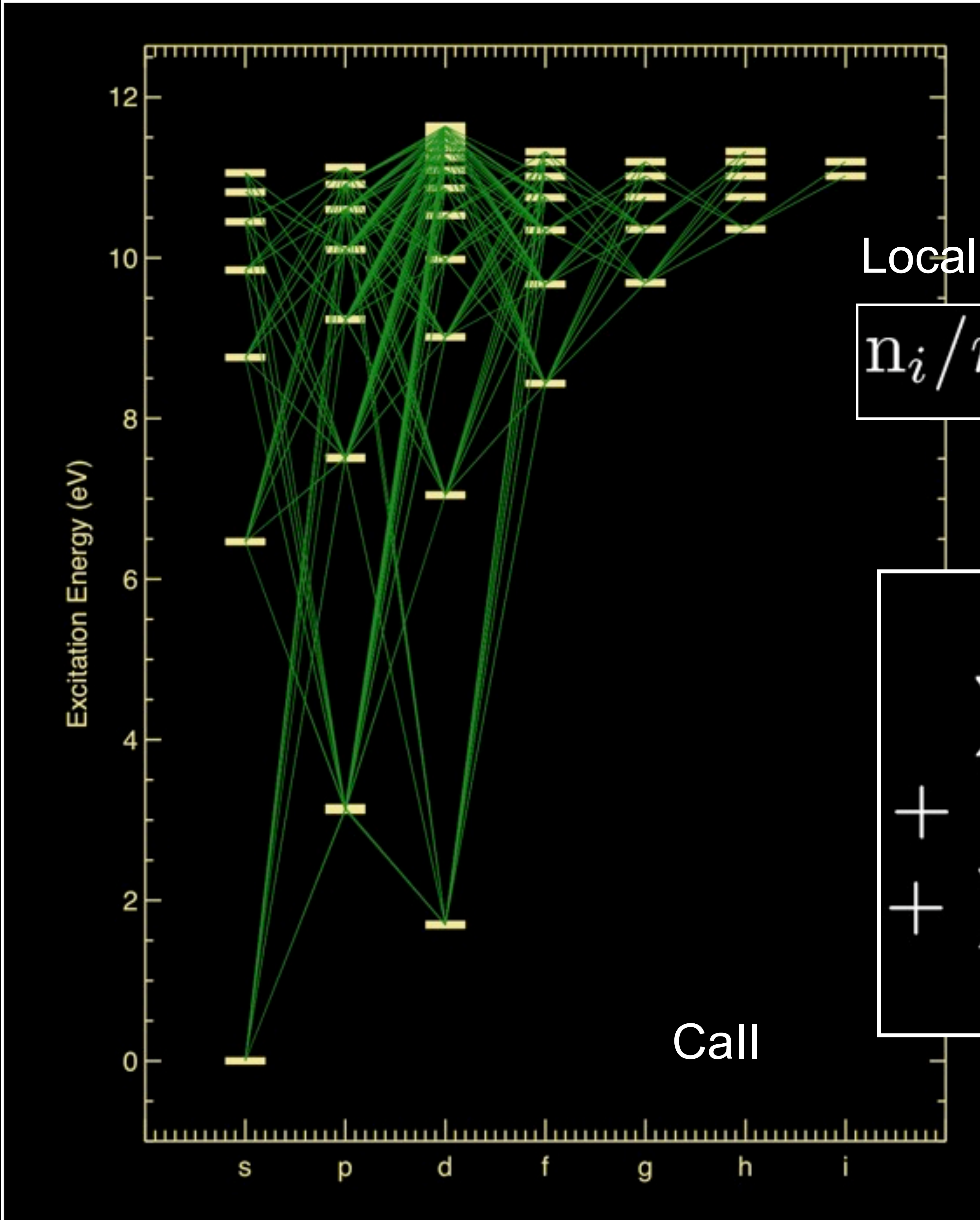
$$n_i/n_j = \frac{g_i}{g_j}\exp(-\Delta E/kT)$$

non-equilibrium (NLTE)

$$\partial n_i/\partial t = \sum_{j\neq i}(n_j R_{ji} - n_i R_{ij}) + \sum_{j\neq i}(n_j C_{ji} - n_i C_{ij}) + \sum_{j\neq i}(n_j G_{ji} - n_i G_{ij}) = 0$$

nxn matrix, where n = number of atomic levels (sparsity depends on number of transitions included)

line interactions



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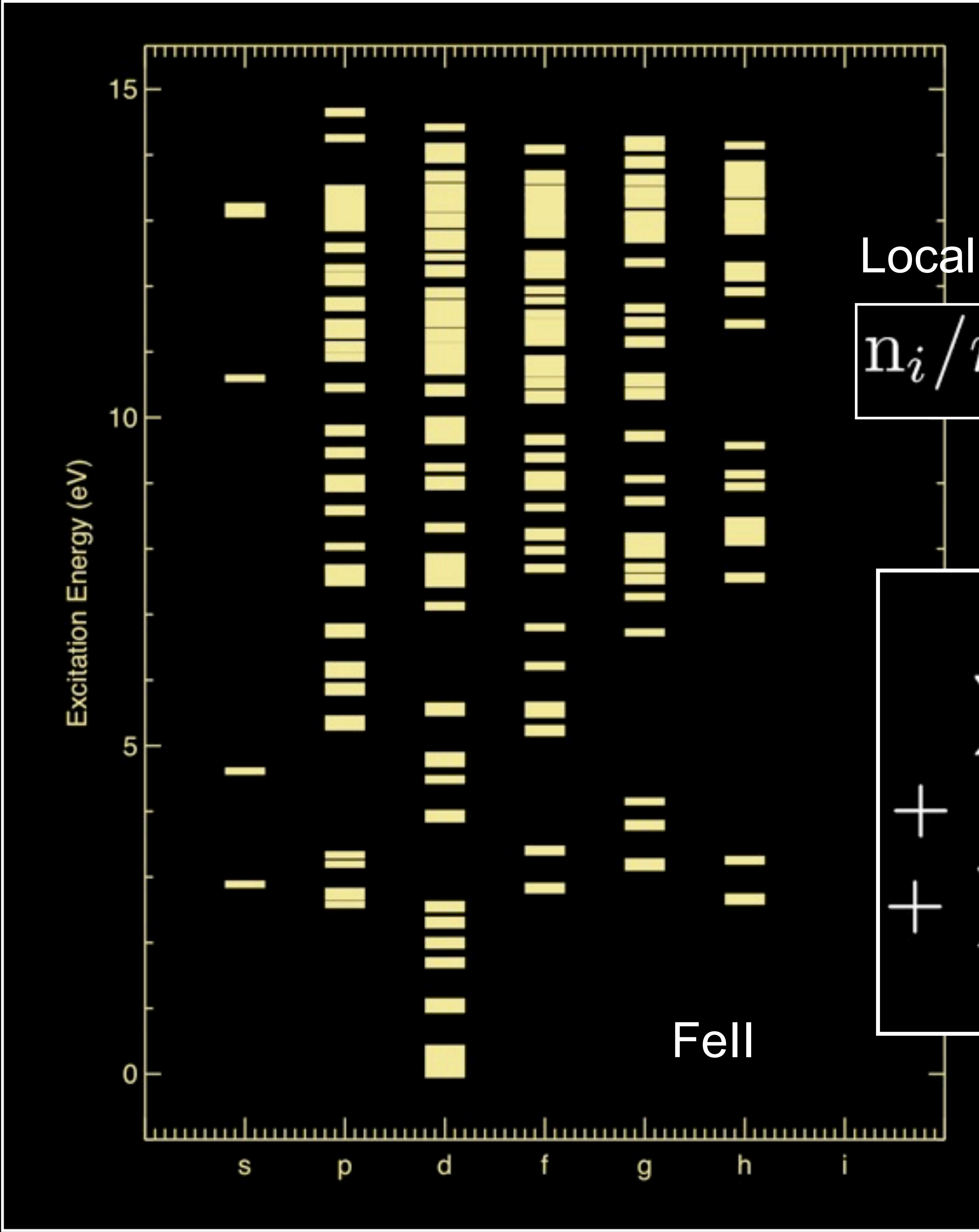
nxn matrix, where n = number of atomic levels (sparsity depends on number of transitions included)

line interactions



~1/2 GB atomic data

Local Thermodynamic Equilibrium (LTE)

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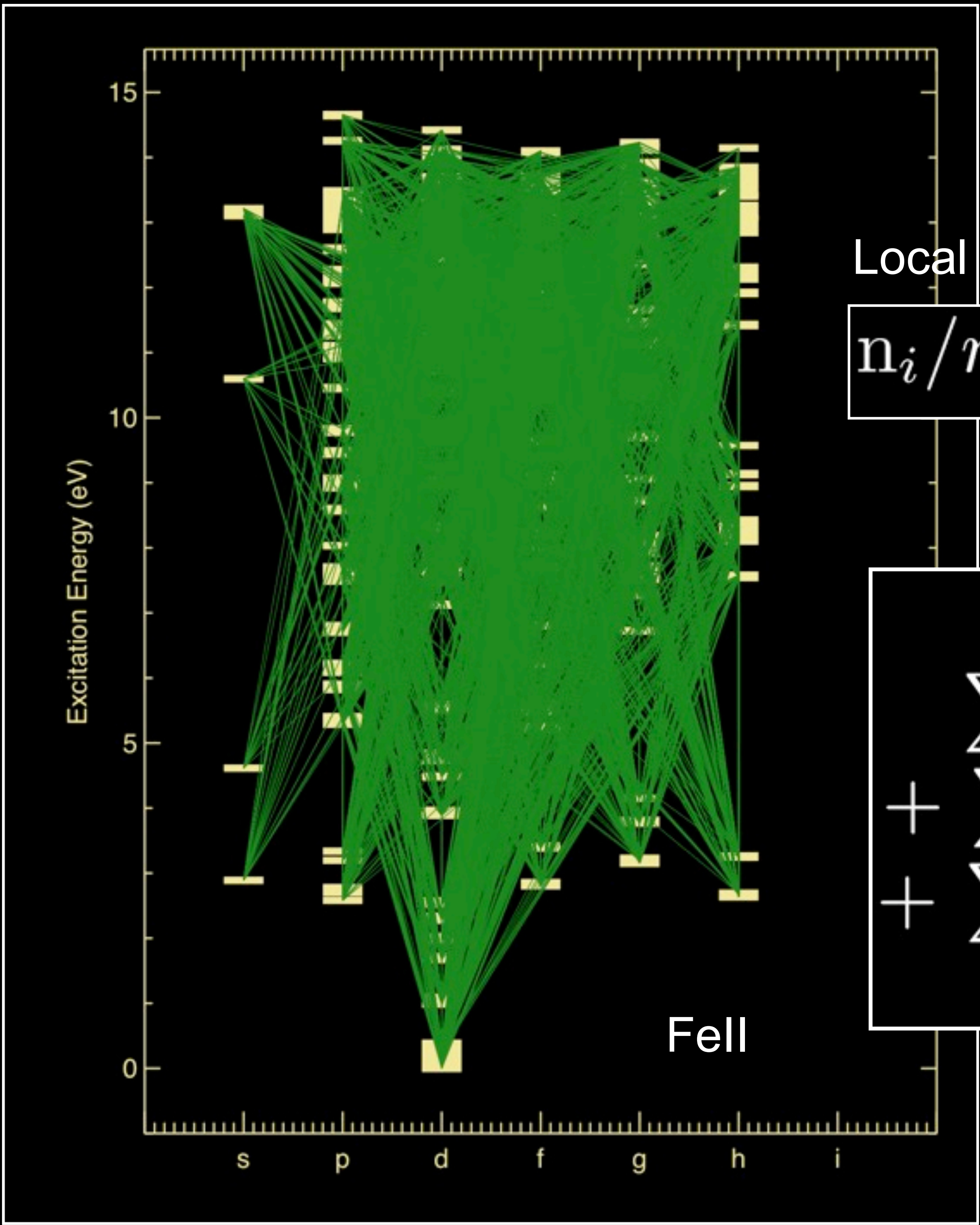

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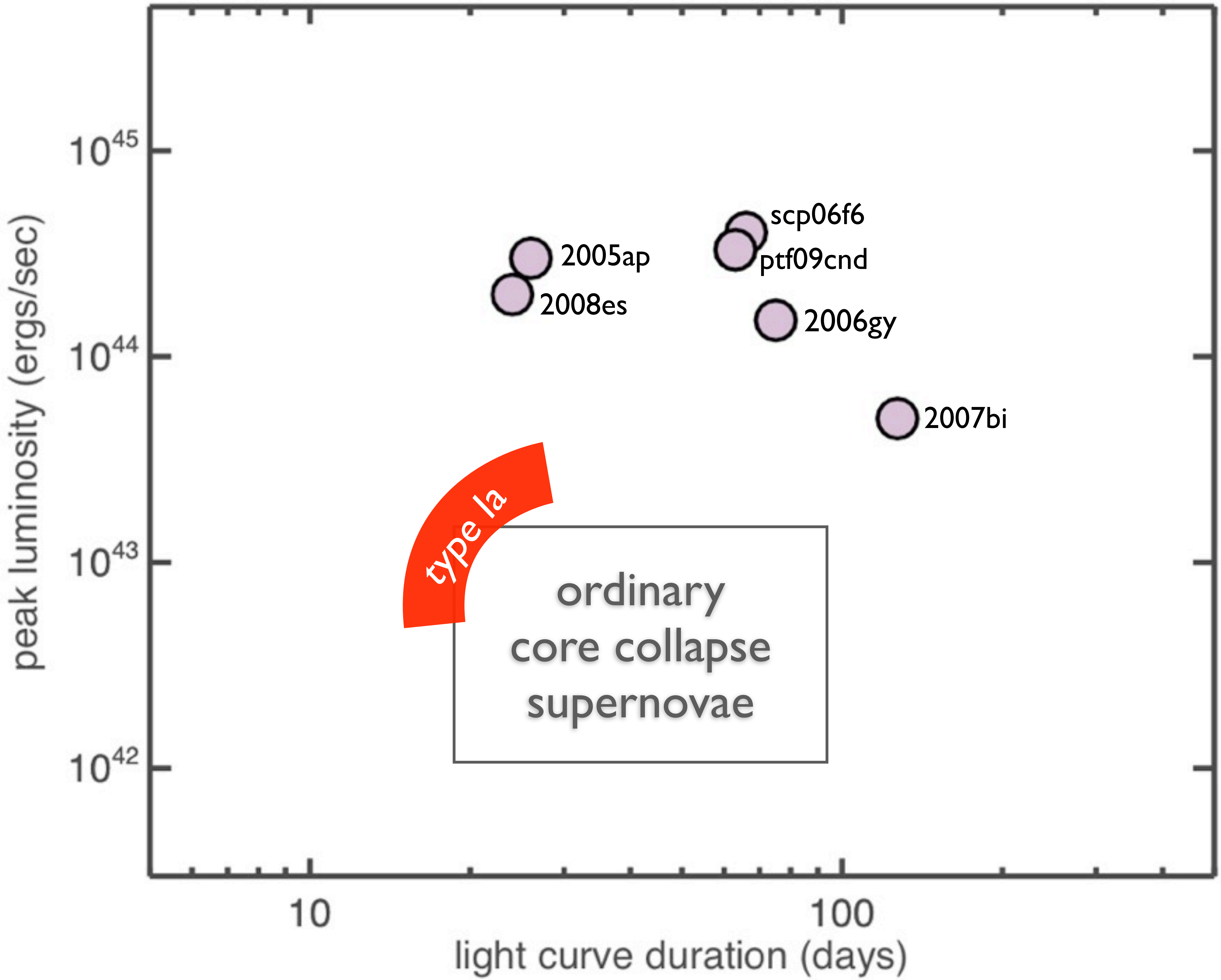
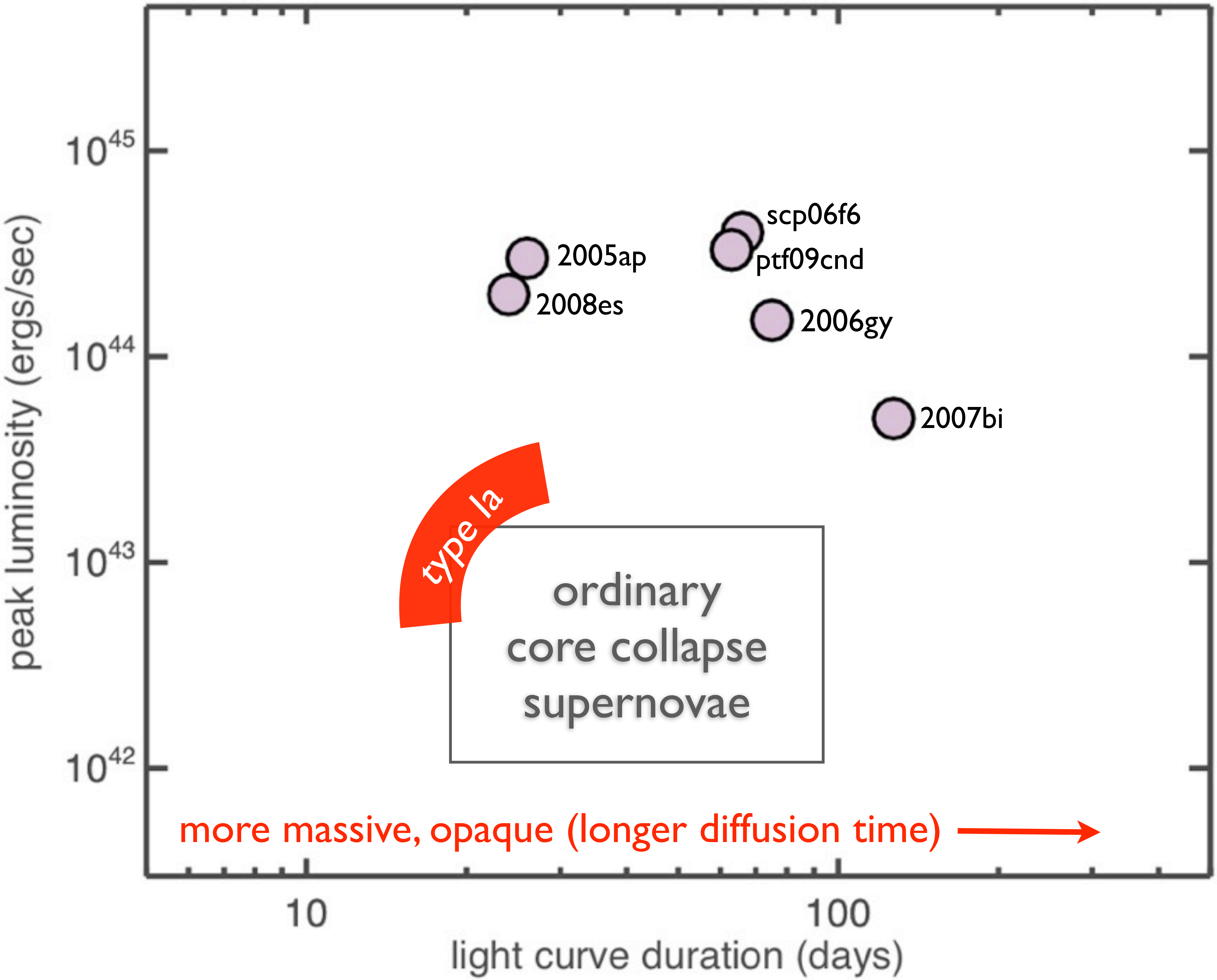


peak luminosity (ergs/sec)
10^45
10^44
10^43
10^42
scp06f6
2005ap
ptf09cnd
2008es
2006gy
2007bi
type Ia
ordinary
core collapse
supernovae
10
100
light curve duration (days)



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10^{43}
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10
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2005ap
ptf09cnd
2008es
2006gy
2007bi
type Ia
ordinary
core collapse
supernovae
more massive, opaque (longer diffusion time)

how to power a supernova light curve

how to power a supernova light curve

- thermal energy released in the explosion shock, nuclear burning

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how to power a supernova light curve

- thermal energy released in the explosion shock, nuclear burning
- radioactive decay of freshly synthesized isotopes: ^{56}Ni (^{52}Fe, ^{48}Cr, ^{44}Ti, R-process)
- interaction of the ejecta with a dense surrounding medium
- energy injection from a rotating, highly magnetized neutron star (magnetar)

thermally powered supernovae (Type IIP)

luminosity of thermal light curve

energy deposited by the explosion

luminosity of thermal light curve

energy deposited by the explosion

the radiation energy drops in the expanding gas

$$E_{\mathrm{rad}}(t) = E_0 \left[\frac{R_0}{R(t)} \right]$$

and takes a diffusion time to escape

$$t_{\mathrm{d}} \sim \left[\frac{M\kappa}{vc} \right]^{1/2}$$

luminosity of thermal light curve

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simple estimate of supernova luminosity

luminosity of thermal light curve

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simple estimate of supernova luminosity

$$L_{\rm sn} = \frac{E_0}{t_{\rm d}} \left[\frac{R_0}{v t_{\rm d}} \right] \approx 10^{41}\ {\rm ergs\ s^{-1}}\ R_{1,\odot} E_{51} \kappa_{0.4}^{-1} M_{1,\odot}^{-1}$$

luminosity of thermal light curve

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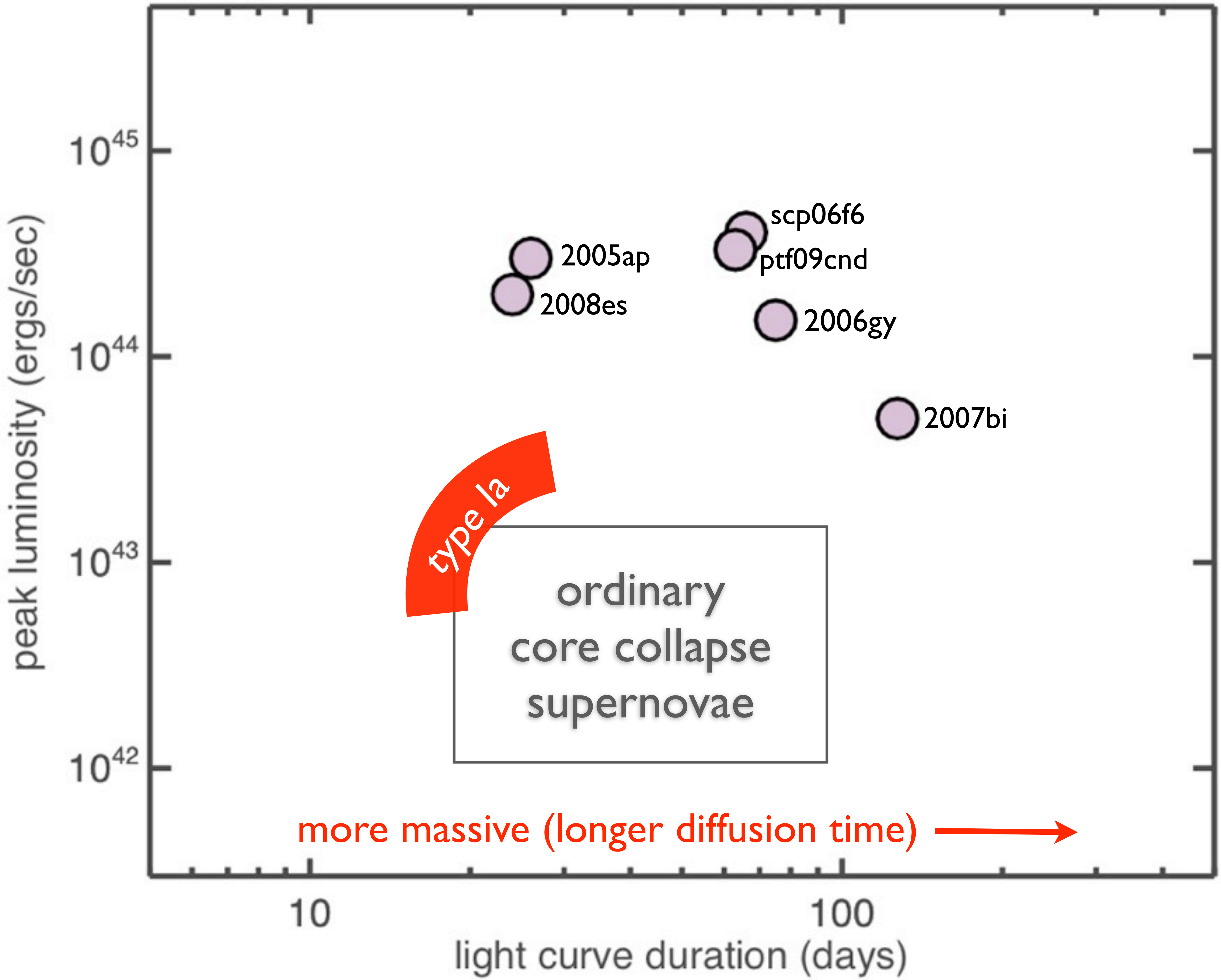
low radiative efficiency if initial radius is small!
for a bright thermally powered supernova, must have $R_0 >> R_{sun}$


peak luminosity (ergs/sec)
10^{45}
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2007bi
type Ia
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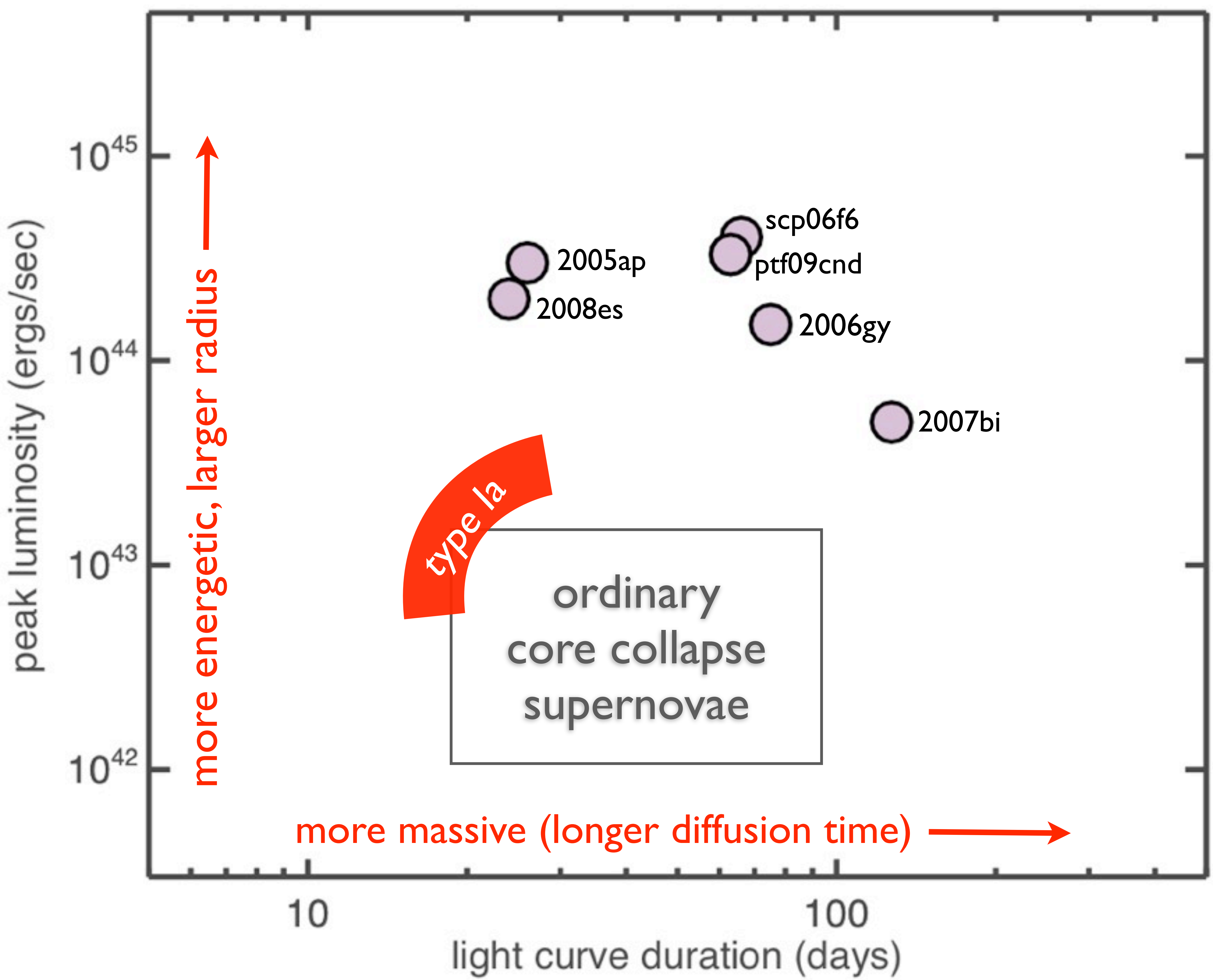
core collapse
supernovae
more massive (longer diffusion time)
10
100
light curve duration (days)



peak luminosity (ergs/sec)
10^{45}
10^{44}
10^{43}
10^{42}
more energetic, larger radius
scp06f6
2005ap
ptf09cnd
2008es
2006gy
2007bi
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10
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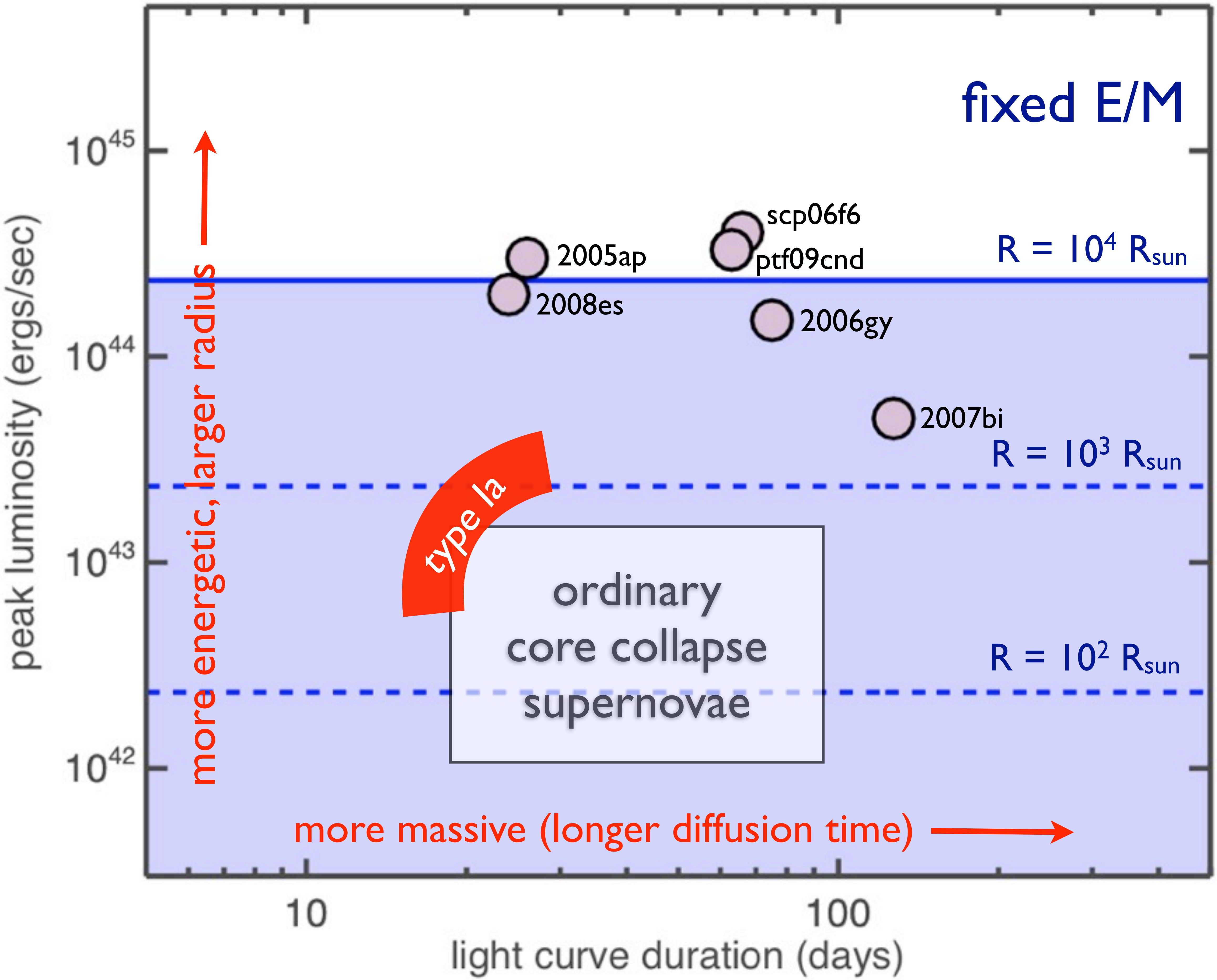

fixed E/M
$R = 10^4 R_{sun}$
$R = 10^3 R_{sun}$
$R = 10^2 R_{sun}$
scp06f6
ptf09cnd
2005ap
2008es
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peak luminosity (ergs/sec)
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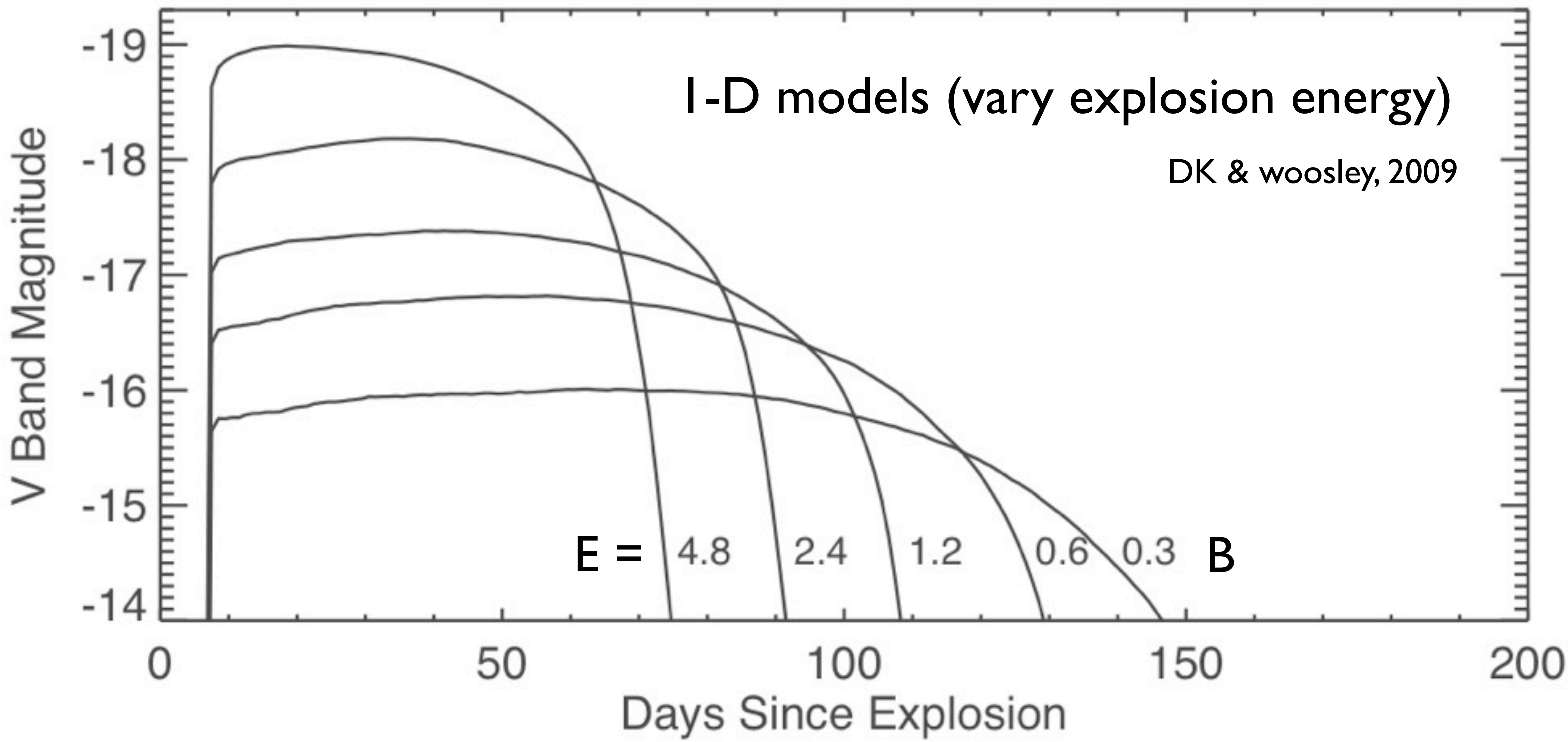
Type IIP core collapse supernovae

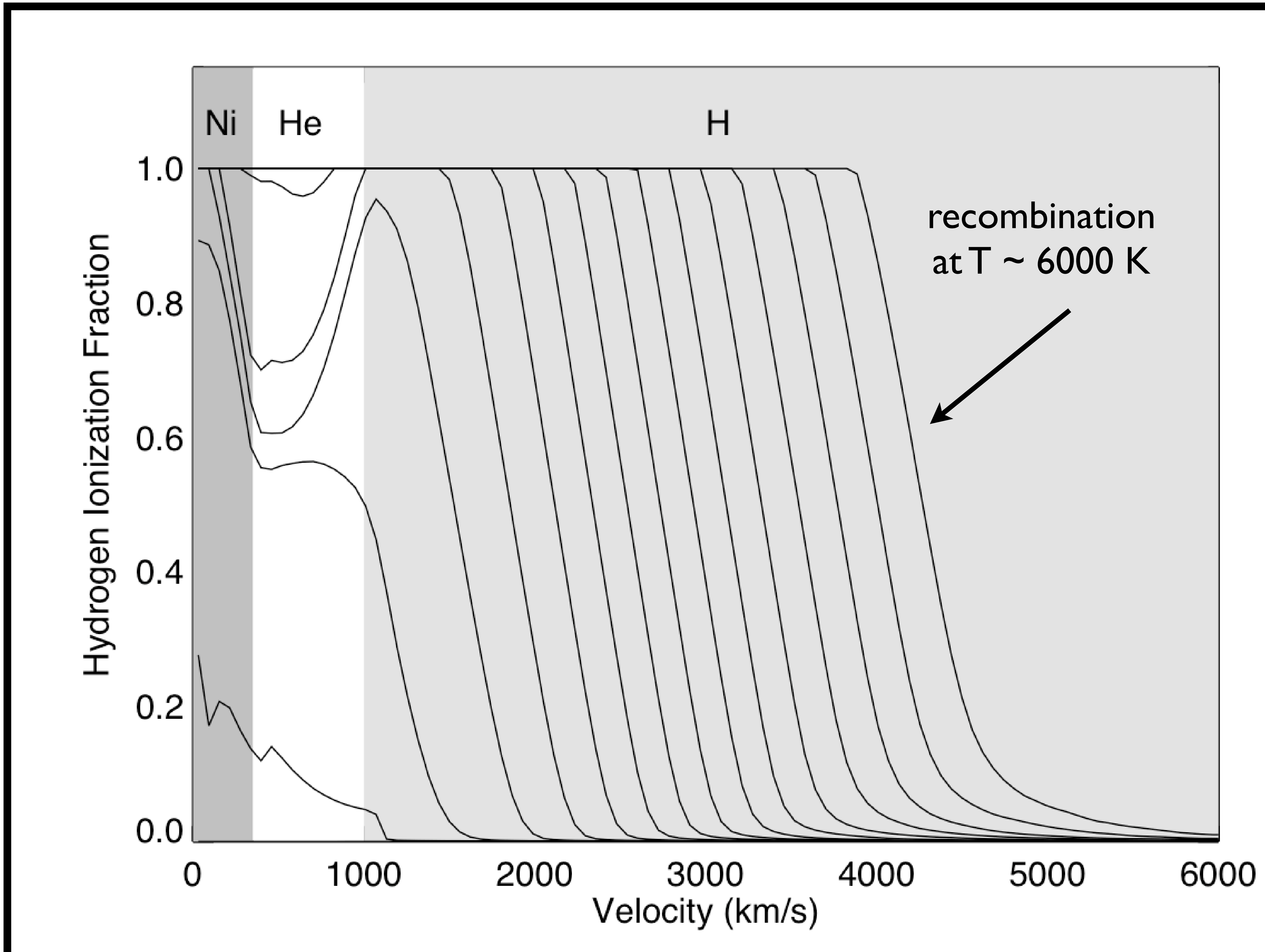
explosion of red supergiant stars

$$M = 15 M_\odot \qquad R_\star \approx 10^{13} \text{ cm}$$



recombination wave in Type IIP supernova

opacity from electron scattering drops as ejecta cool and become neutral



light curve scalings with recombination

gives a Type II plateau light curve

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the photosphere forms at the recombination front

$$R_{\rm I}^2 = \frac{L}{4\pi\sigma T_{\rm I}^4} = \frac{E_0 R_0}{v t_{\rm d}^2 \sigma T_{\rm I}^4}$$

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where the recombination temperature T_i =~ 6000 K.

light curve scalings with recombination

gives a Type II plateau light curve

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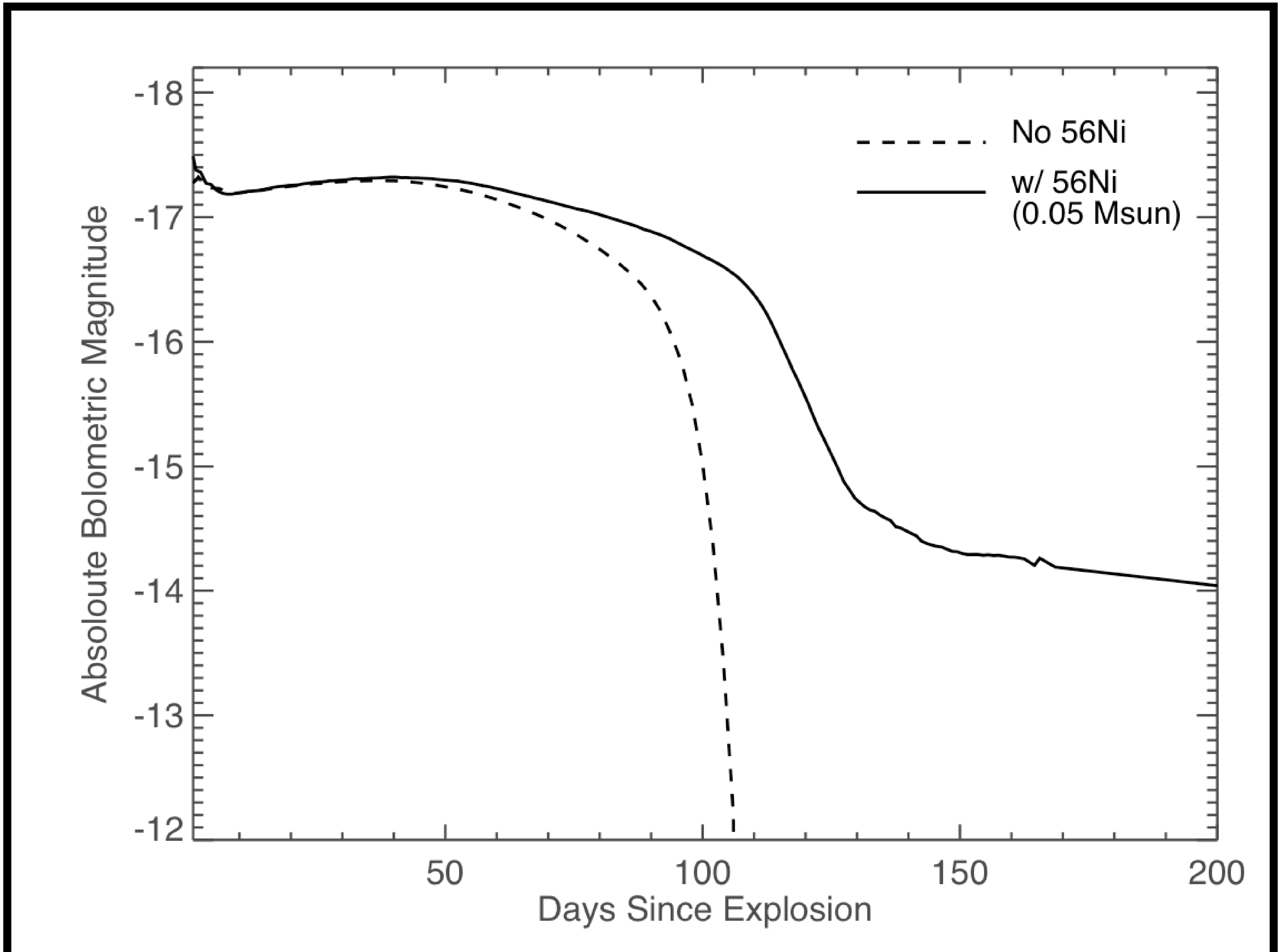
where the recombination temperature T_i =~ 6000 K.

Using previous results for diffusion time:

$$t_{\rm sn} \propto E^{-1/6} M_{\rm ej}^{1/2} R_0^{1/6} \kappa^{1/6} T_{\rm I}^{-2/3}$$

$$L_{\rm sn} \propto E^{5/6} M_{\rm ej}^{-1/2} R_0^{2/3} \kappa^{-1/3} T_{\rm I}^{4/3}.$$

see Popov (1993), DK & Woosley (2009)

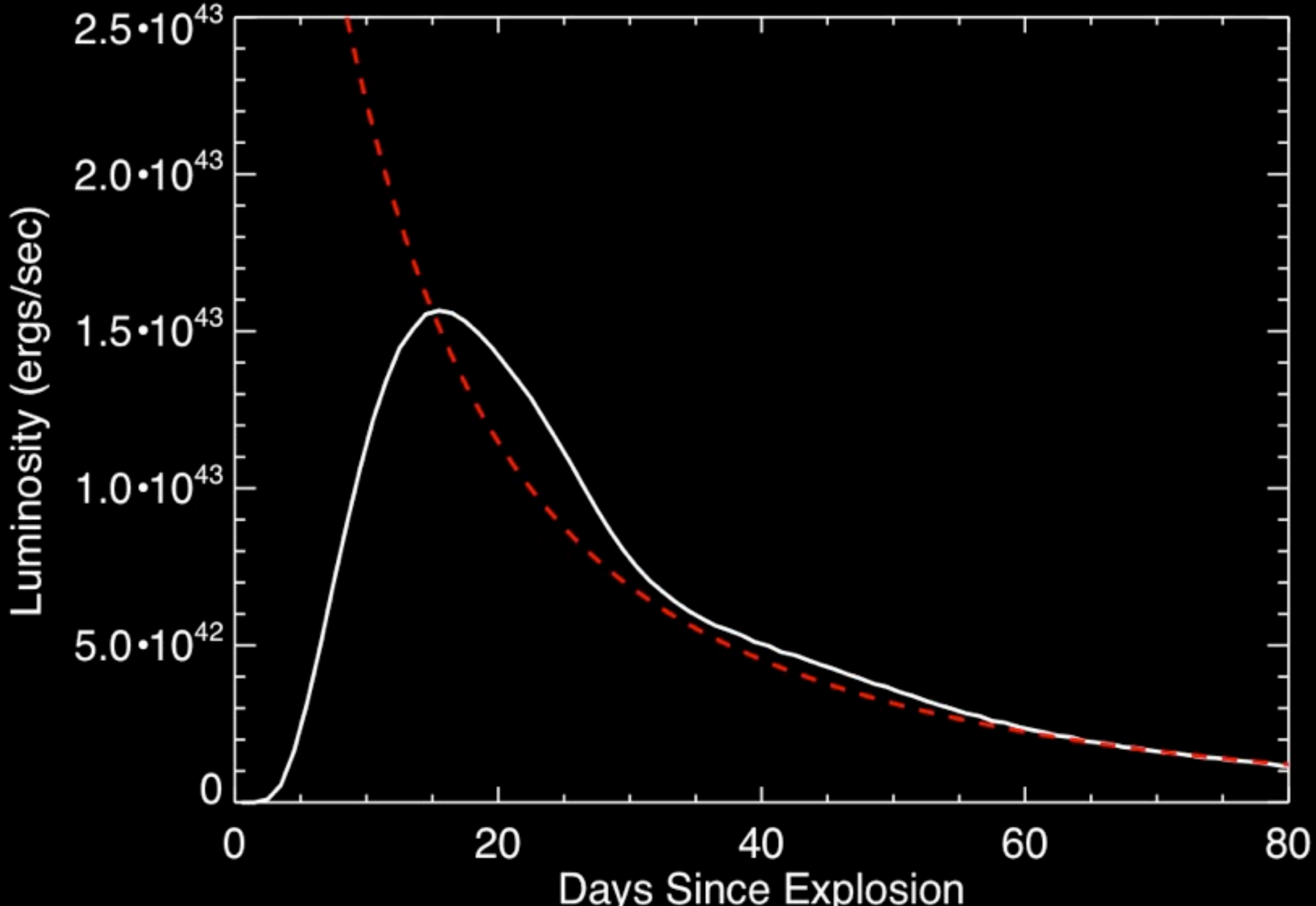


No 56Ni
w/ 56Ni
(0.05 Msun)
-18
-17
-16
-15
-14
-13
-12
Absoloute Bolometric Magnitude
50
100
150
200
Days Since Explosion

radioactivity powered supernovae

radioactively powered light curves

most important chain: $^{56}Ni \rightarrow {}^{56}Co \rightarrow {}^{56}Fe$



radioactive ^{56}Ni decay

$^{56}\mathrm{Ni} \rightarrow\ ^{56}\mathrm{Co}$ 8.8 days 100% electron capture

$^{56}\mathrm{Co} \rightarrow\ ^{56}\mathrm{Fe}$ 113 days 81% electron capture
19% positron production

Important Gamma-Ray Line for ^{56}Ni and ^{56}Co Decays

^{56}Ni Decay		^{56}Co Decay	
Energy (keV)	Intensity (photons/100 decays)	Energy (keV)	Intensity (photons/100 decays)
158...........	98.8	**847**	**100**
270...........	36.5	1038	14
480...........	36.5	**1238**	**67**
750...........	49.5	1772	15.5
812...........	**86.0**	2599	16.7
1562..........	14.0	3240[a]	12.5

milne et al. (2004)

gamma-ray deposition by compton scattering

since gamma-ray energies (MeV) are much greater than ionization potentials, all electrons (free + bound) contribute

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change in photon wavelength

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angle between incoming and outgoing photon directions

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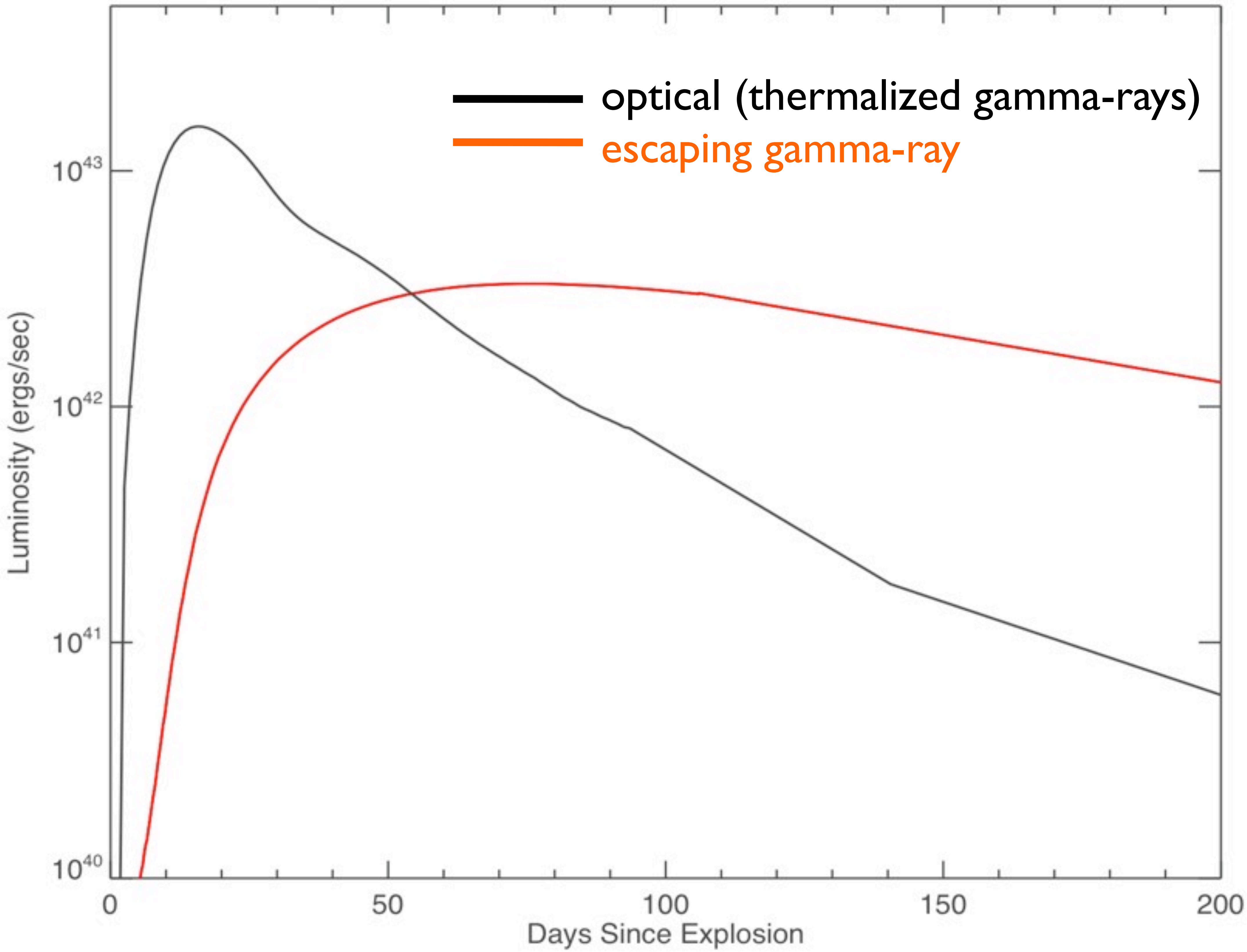
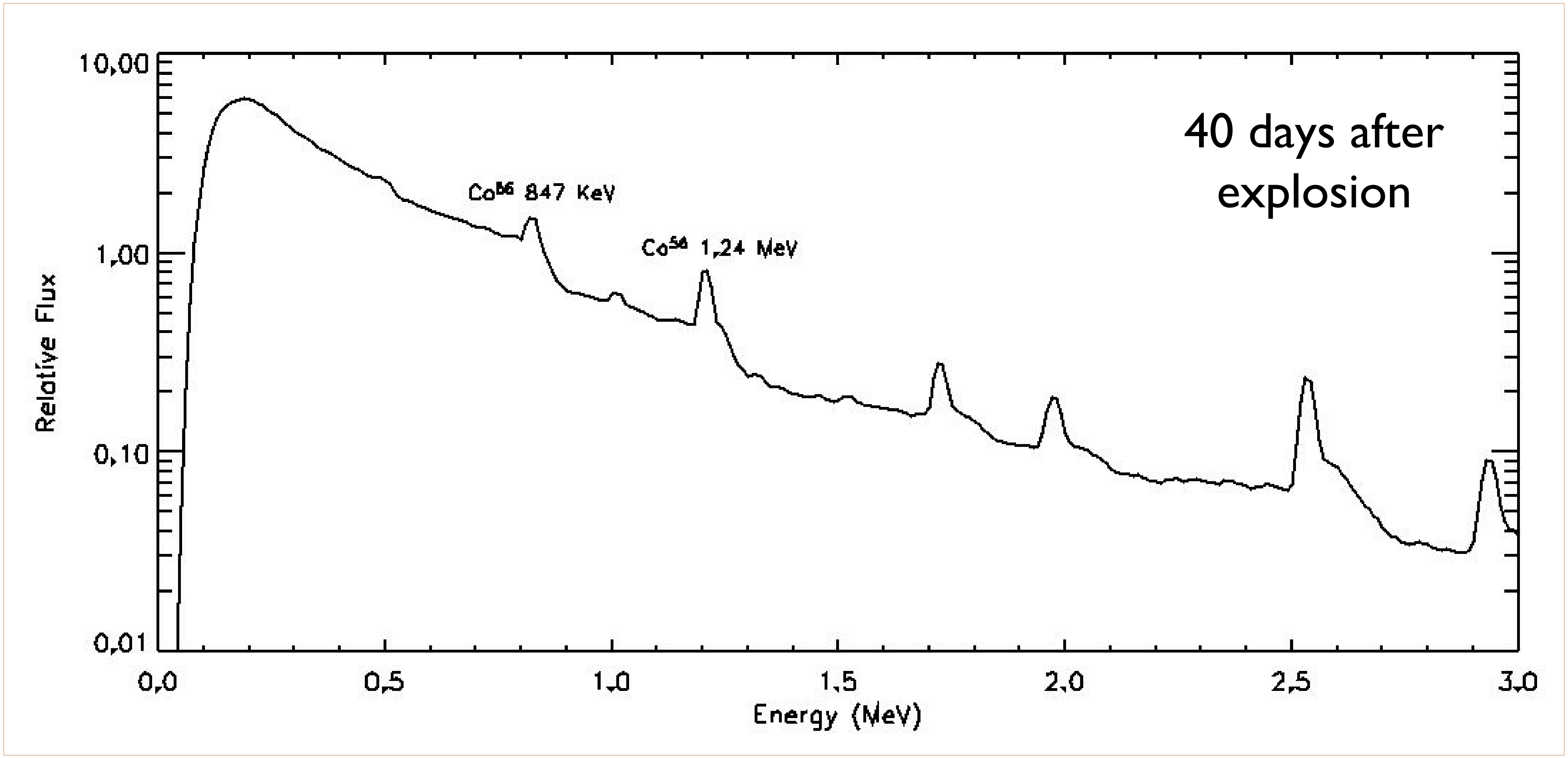
so an MeV (~2 $m_e c^2$) gamma-ray loses most of its energy after just a few compton scatterings (then it gets photo-absorbed)

type Ia supernova light curves

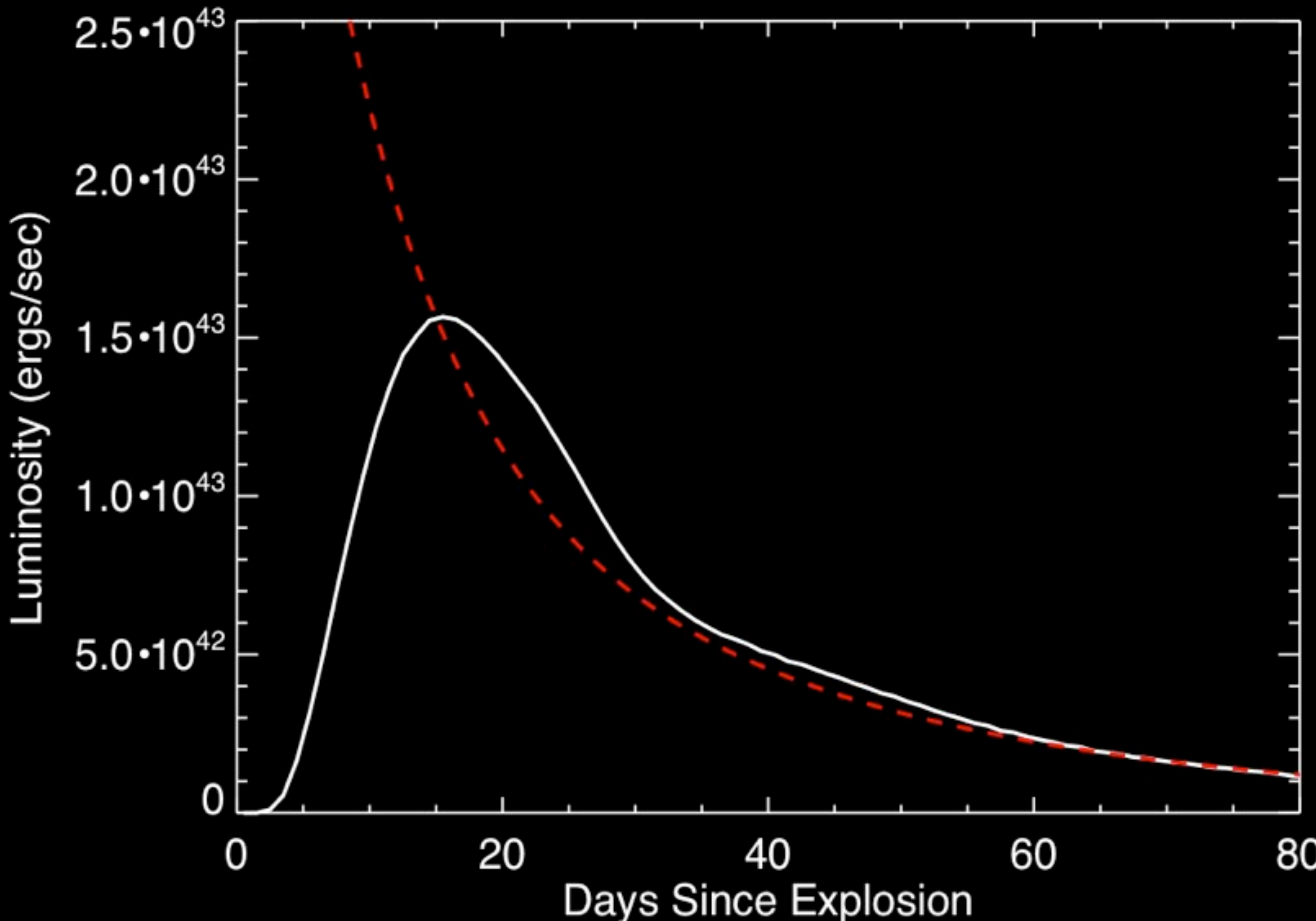


type Ia gamma-ray spectrum



radioactively powered light curves

most important chain: $^{56}Ni \rightarrow {}^{56}Co \rightarrow {}^{56}Fe$



radioactive supernovae

light curve estimates

radioactive supernovae

light curve estimates

light curve duration given by standard diffusion time

$$t_{\rm d} \sim \left[\frac{M\kappa}{vc} \right]^{1/2}$$

radioactive supernovae

light curve estimates

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luminosity estimate from radioactive energy deposition

$$L_{\rm ni} \approx \frac{M_{\rm ni}\epsilon_{\rm ni}}{t_{\rm ni}} e^{-t_{\rm d}/t_{\rm ni}}$$

arnett's law

arnett's law

$$\frac{\partial(\epsilon_{\mathrm{rad}} V)}{\partial t} = -p\frac{\partial V}{\partial t} - L_{\mathrm{diff}} + L_{\gamma}$$

arnett's law

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use $V \propto t^3,\ p_{\rm rad} = \epsilon_{\rm rad}/3$

$$\frac{1}{t}\frac{\partial(\epsilon_{\rm rad} V t)}{\partial t} = -L_{\rm diff} + L_\gamma$$

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$$L_{\rm diff} = 4\pi r^2 \frac{c}{3\kappa\rho}\frac{\partial \epsilon_{\rm rad}}{\partial r} \approx 4\pi r^2 \frac{c}{3\kappa\rho}\frac{\epsilon_{\rm rad}}{r}$$

arnett's law

$$\frac{\partial(\epsilon_{\rm rad} V)}{\partial t} = -p\frac{\partial V}{\partial t} - L_{\rm diff} + L_\gamma$$

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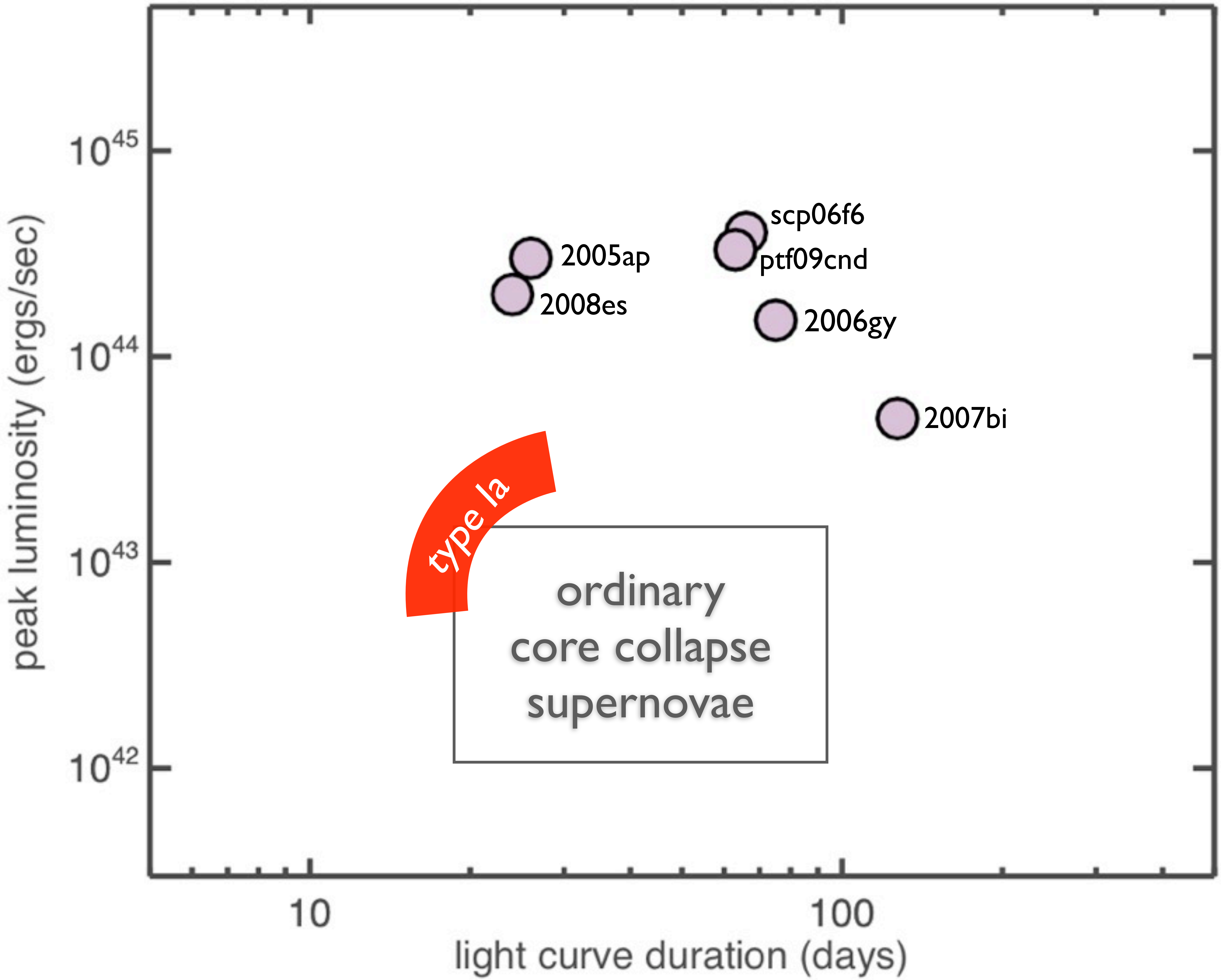
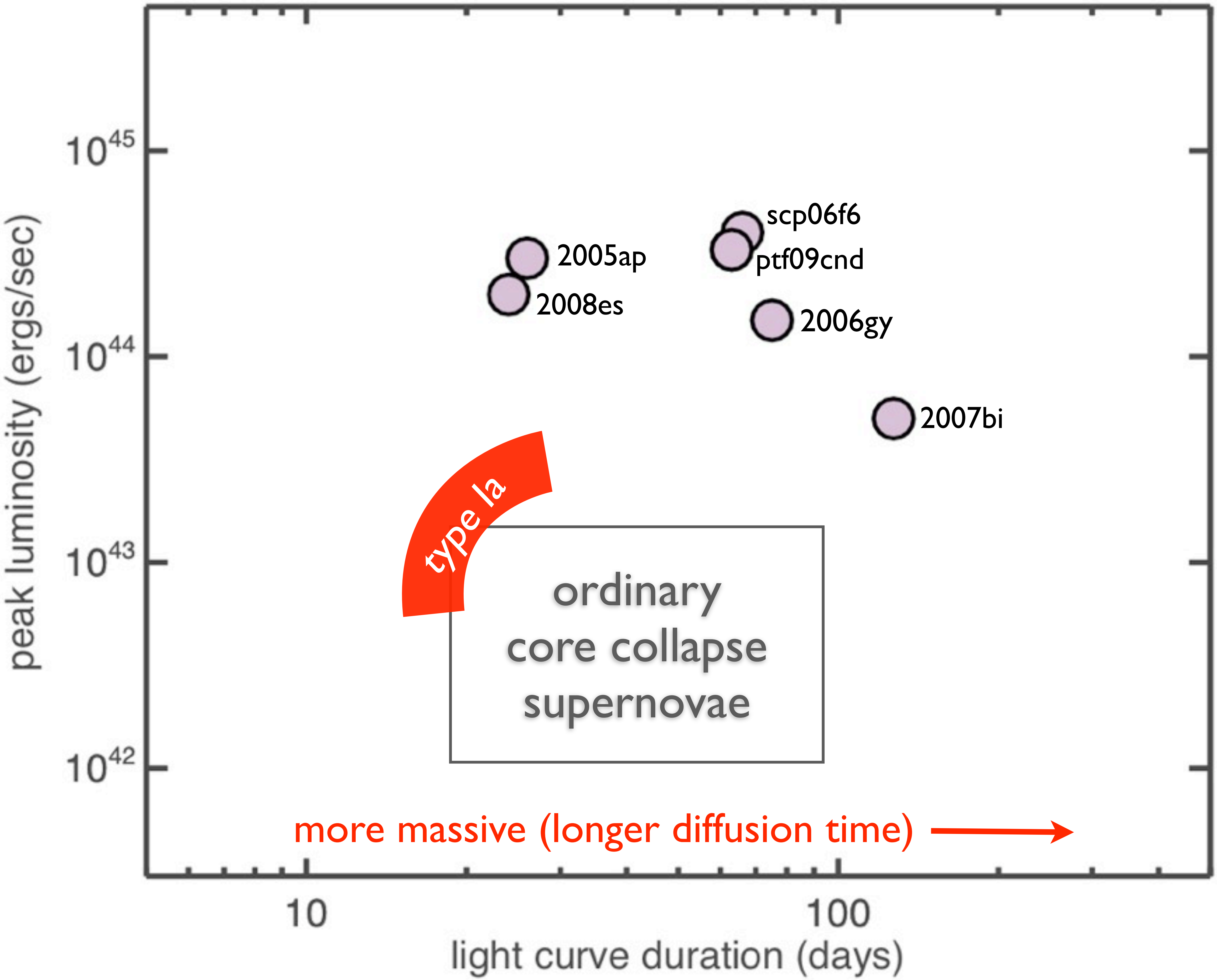
find $\quad L_{\rm diff} = \dfrac{\epsilon_{\rm rad} V t}{t_{\rm d}^2}$



peak luminosity (ergs/sec)
10^45
10^44
10^43
10^42
scp06f6
2005ap
ptf09cnd
2008es
2006gy
2007bi
type Ia
ordinary
core collapse
supernovae
10
100
light curve duration (days)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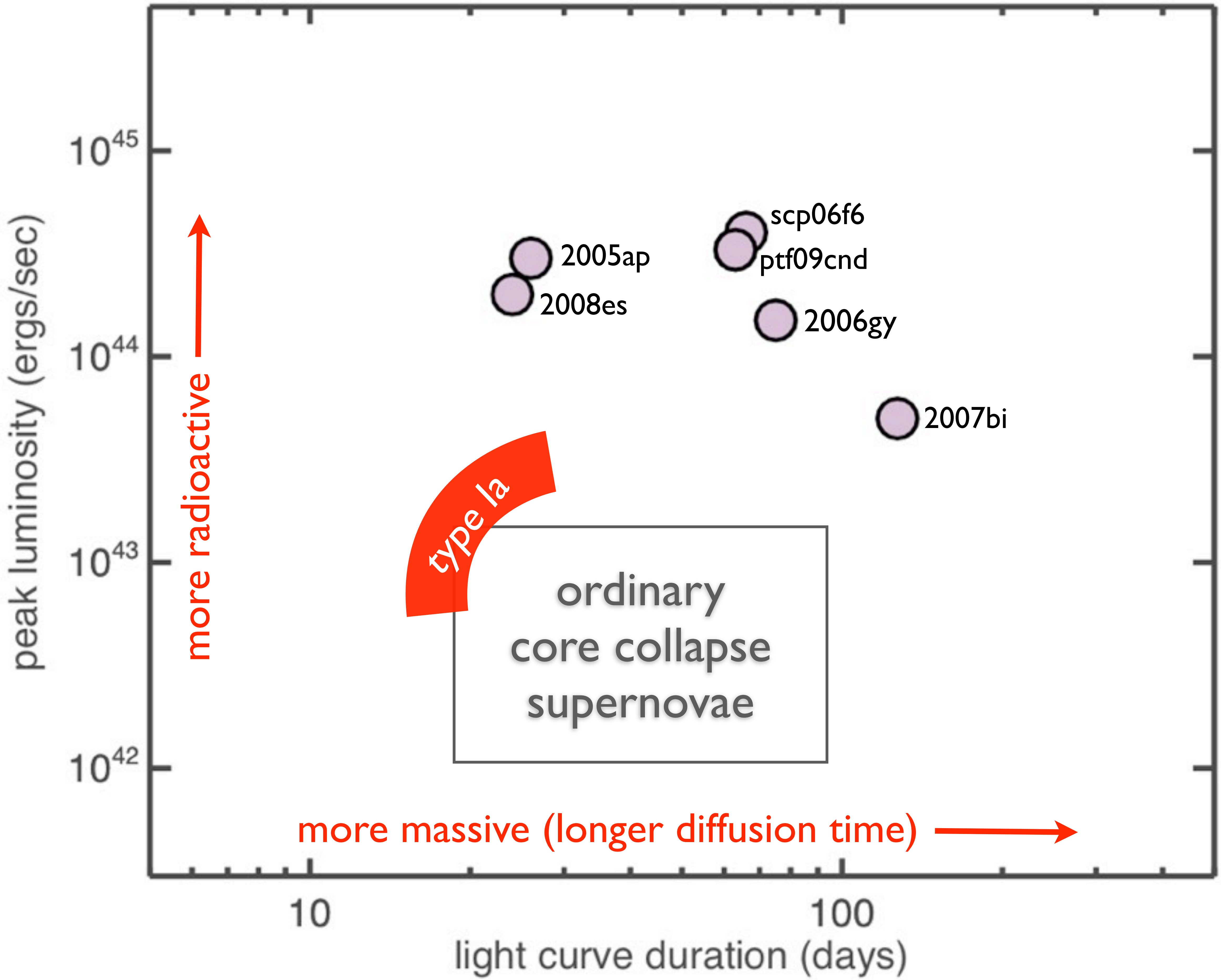

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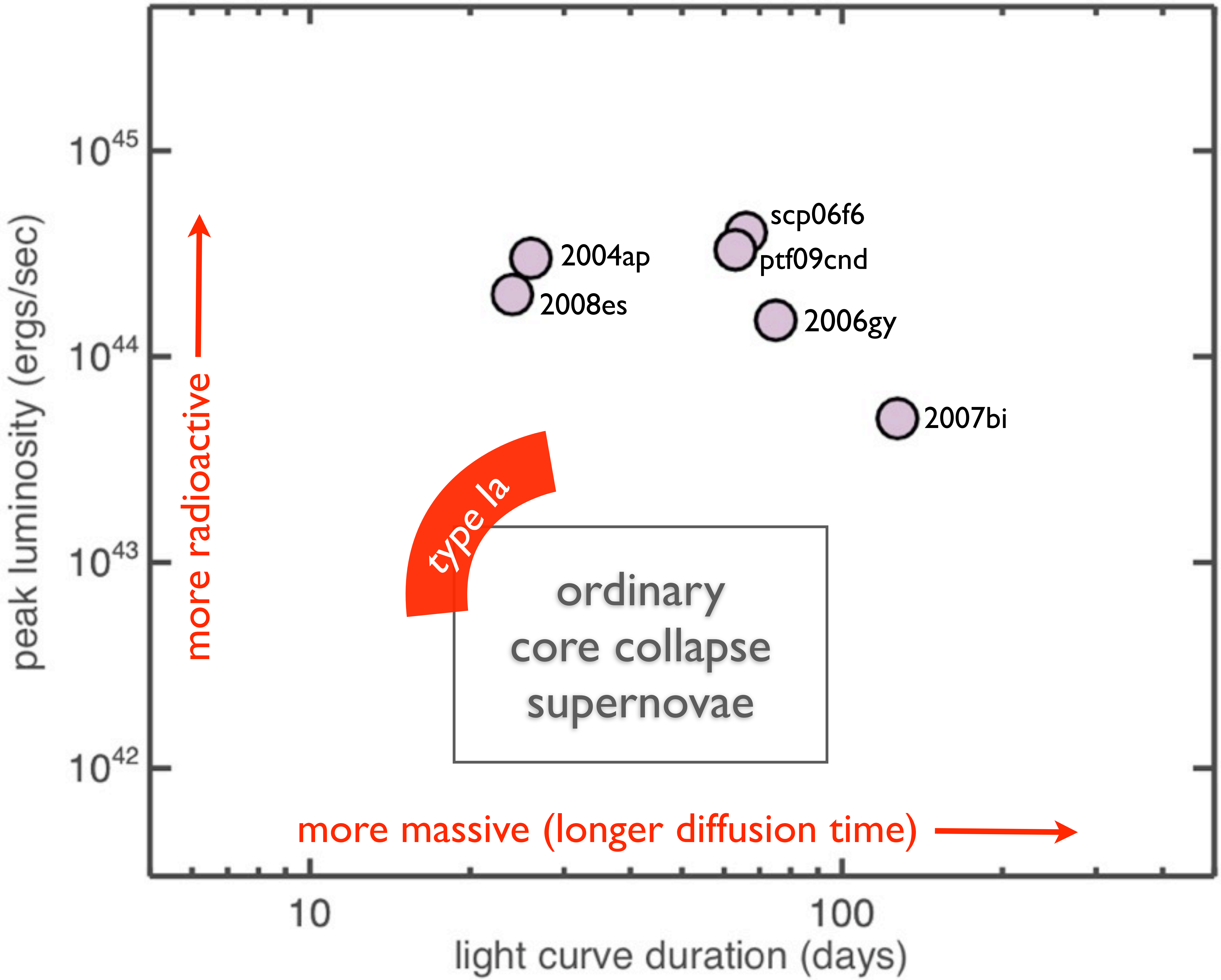

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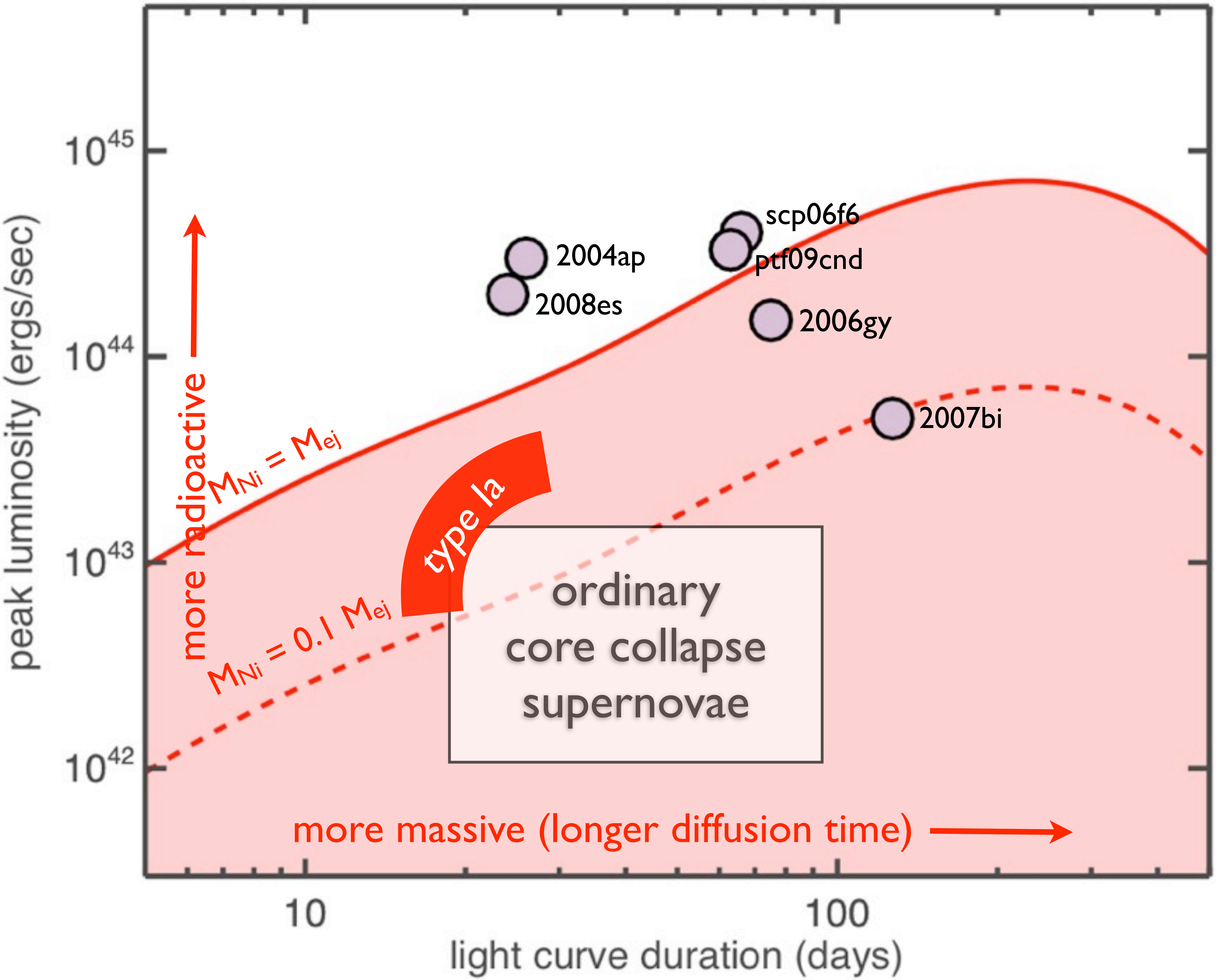
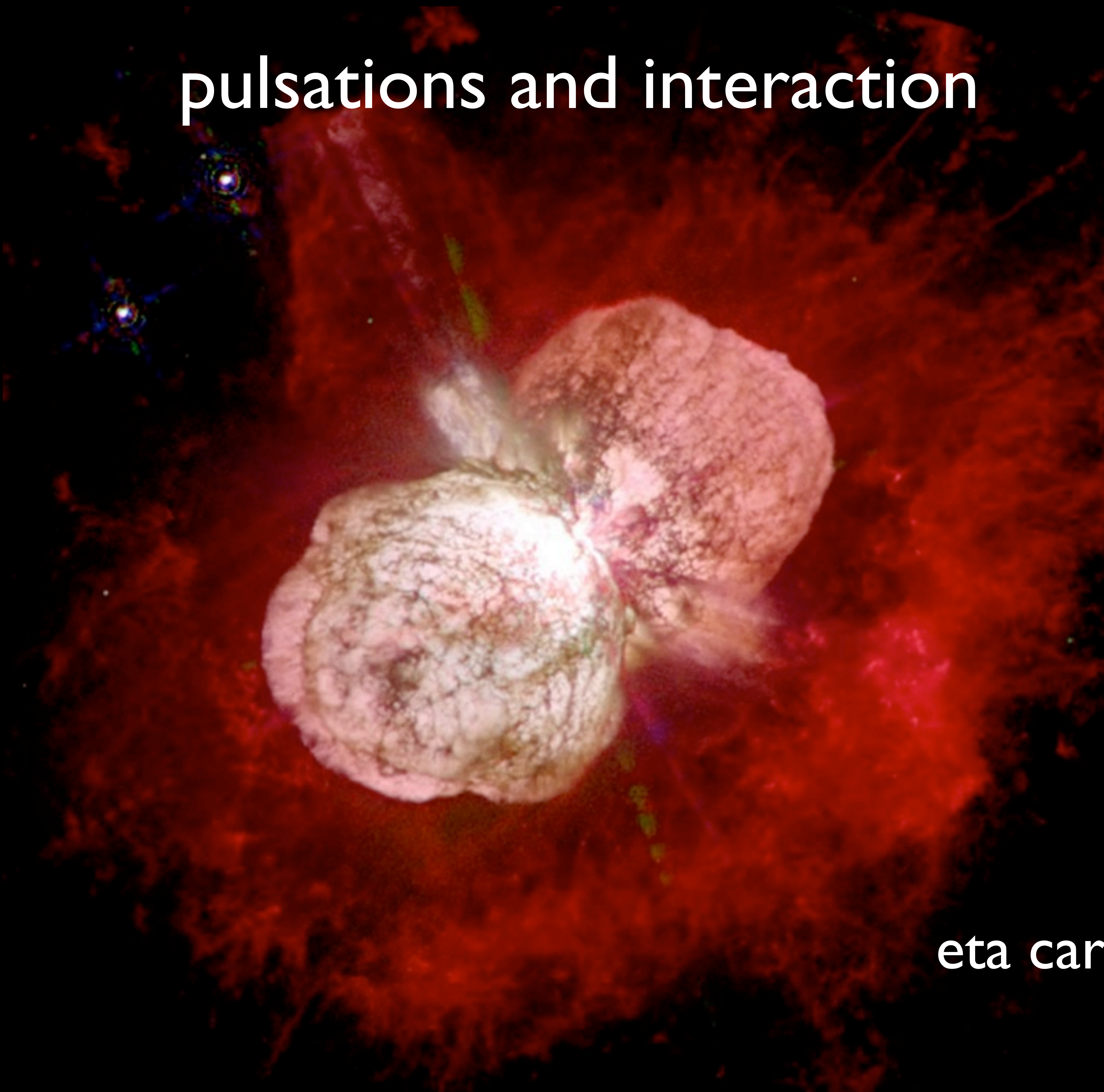


peak luminosity (ergs/sec)
10^{45}
10^{44}
10^{43}
10^{42}
more radioactive
$M_{Ni} = M_{ej}$
$M_{Ni} = 0.1\ M_{ej}$
type Ia
2004ap
2008es
scp06f6
ptf09cnd
2006gy
2007bi
ordinary
core collapse
supernovae
more massive (longer diffusion time)
10
100
light curve duration (days)

pulsations and interaction



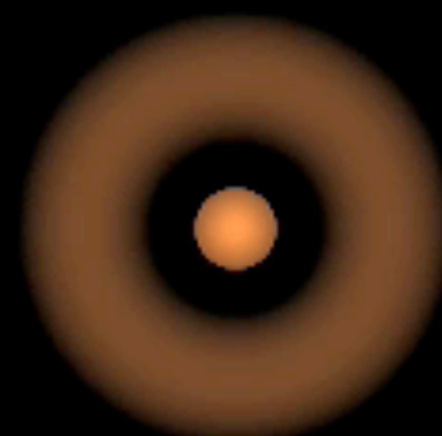
eta carinae

interacting supernovae

“tamped” supernova models

second ejection

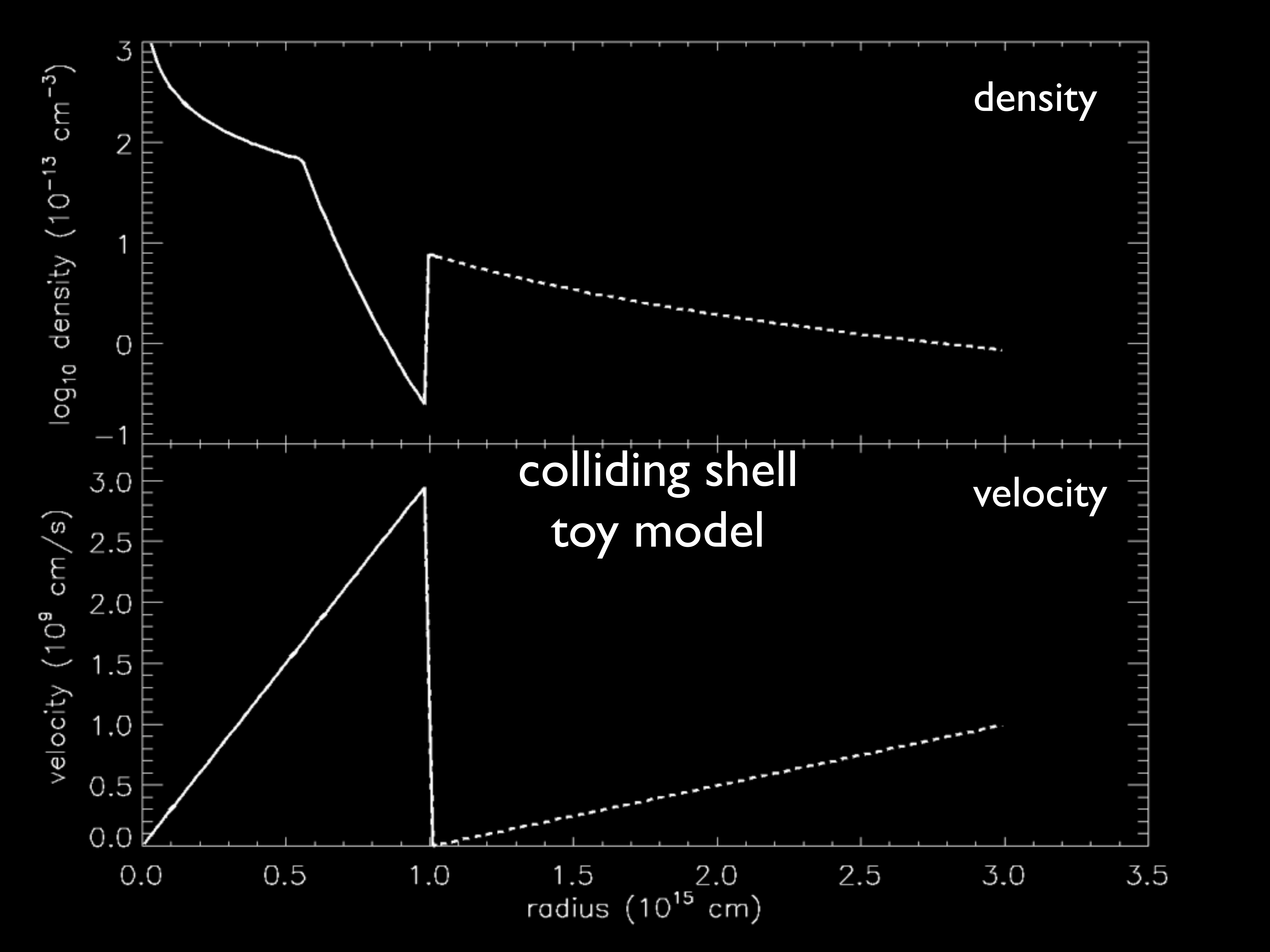
slow moving shell
of debris from
first ejection



density

colliding shell
toy model

velocity


density
colliding shell
toy model
velocity
log10 density (10^-13 cm^-3)
velocity (10^9 cm/s)
radius (10^15 cm)
3
2
1
0
−1
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

interacting supernovae

simple estimate of peak luminosity

interacting supernovae

simple estimate of peak luminosity

peak luminosity for shocked debris at shell radius

$$L_{\rm sn} \approx \frac{E_{\rm sn}}{t_{\rm d}} \left[\frac{R_{\rm sh}}{R_{\rm sn}}\right] \sim 10^{45}\ {\rm ergs\ s^{-1}} \left[\frac{R_{\rm sh}}{10^4\ R_\odot}\right]$$

interacting supernovae

simple estimate of peak luminosity

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to reach the highest luminosities, shell must be at radius

$$R_{\rm sh} \approx 10^4 \ R_\odot \approx 10^{15} \text{ cm}$$

interacting supernovae

simple estimate of peak luminosity

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to reach the highest luminosities, shell must be at radius

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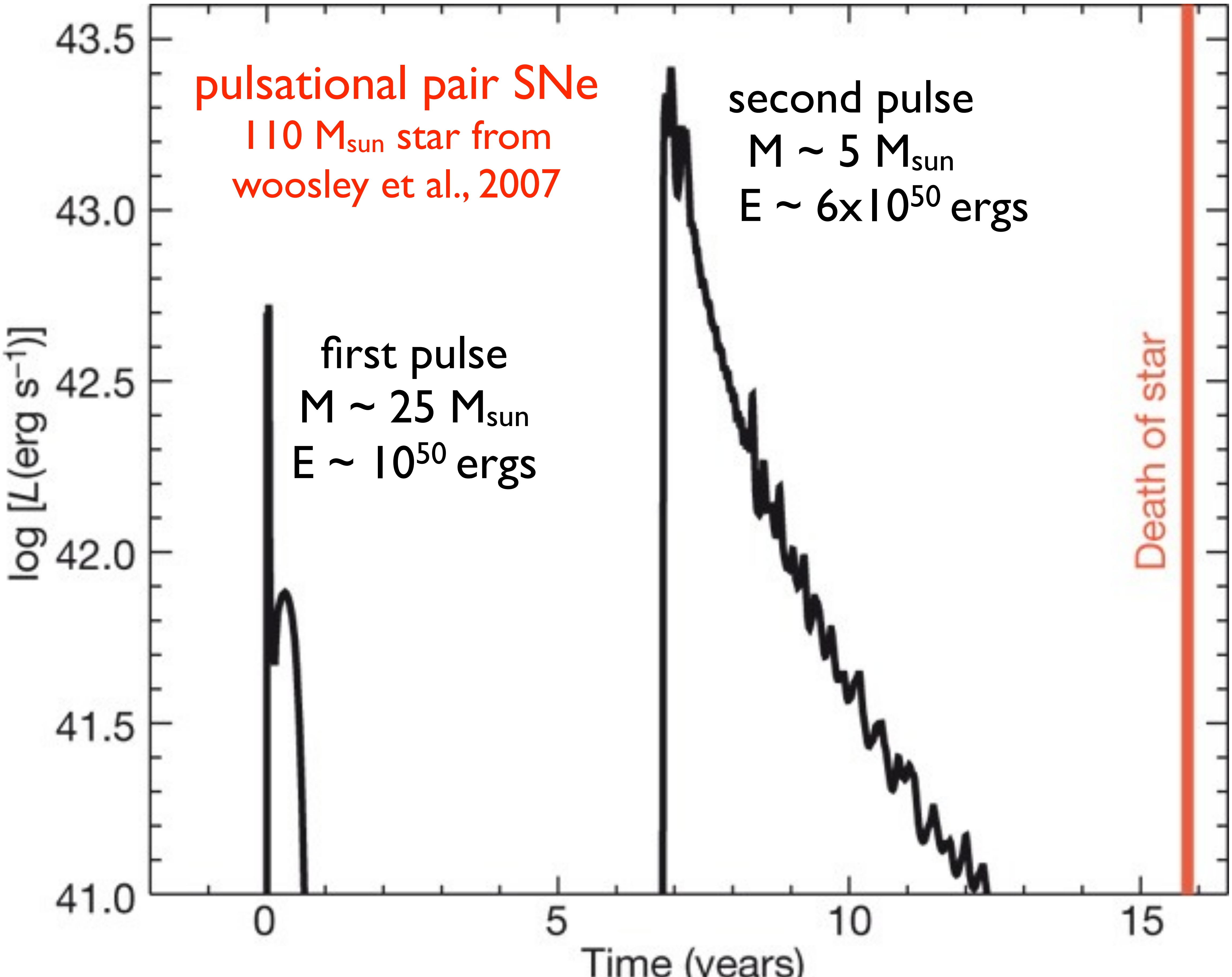
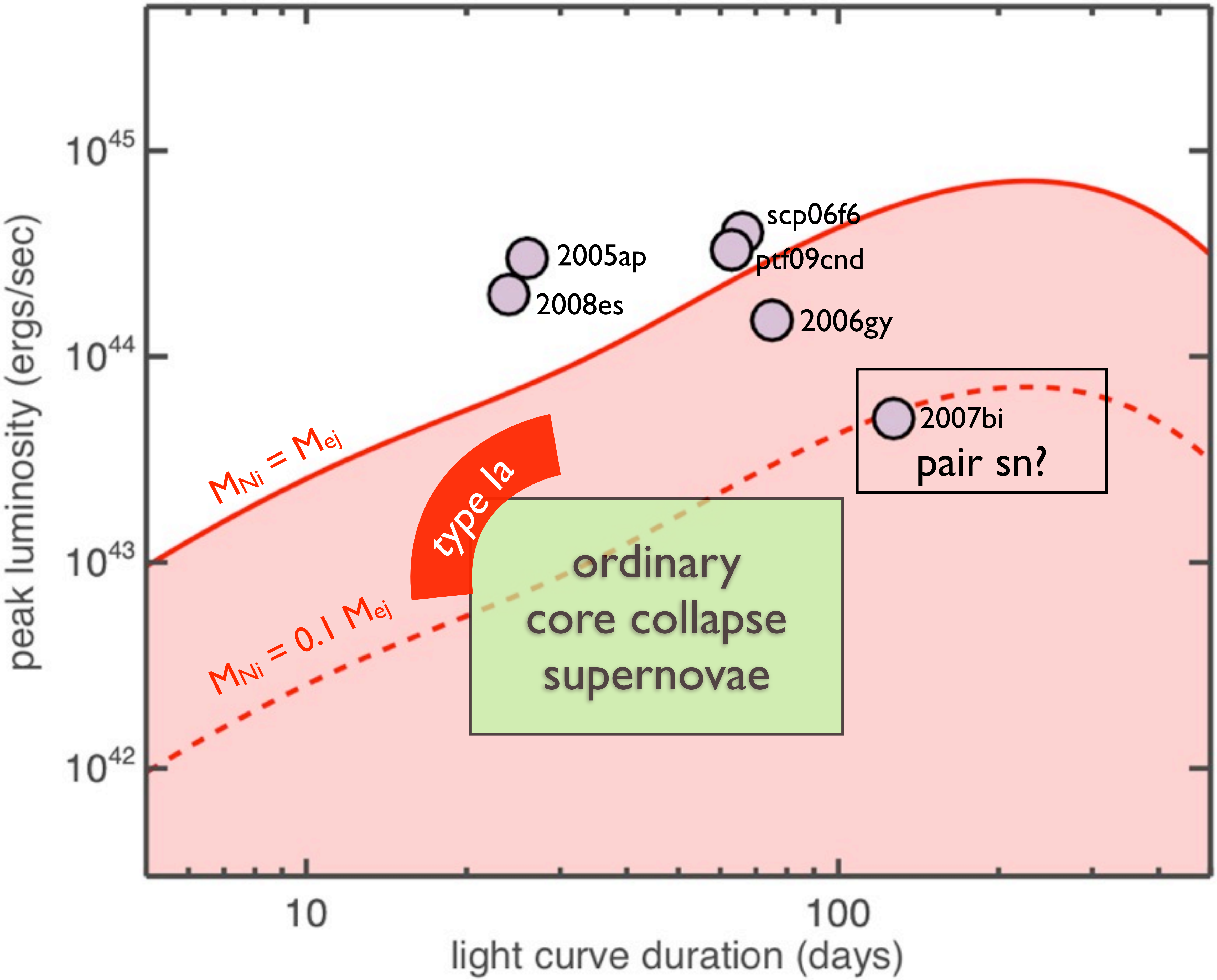
time between pulses of ejection

$$t_{\rm sh} = R_{\rm sh}/v_{\rm sh} = 2\ {\rm years} \left[\frac{100\ {\rm km\ s^{-1}}}{v_{\rm sh}}\right]$$



pulsational pair SNe
110 M_{sun} star from
woosley et al., 2007
second pulse
M ~ 5 M_{sun}
E ~ $6x10^{50}$ ergs
first pulse
M ~ 25 M_{sun}
E ~ 10^{50} ergs
Death of star
log [L(erg s^{-1})]
43.5
43.0
42.5
42.0
41.5
41.0
0
5
10
15
Time (years)



peak luminosity (ergs/sec)
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10^{44}
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10^{42}
10
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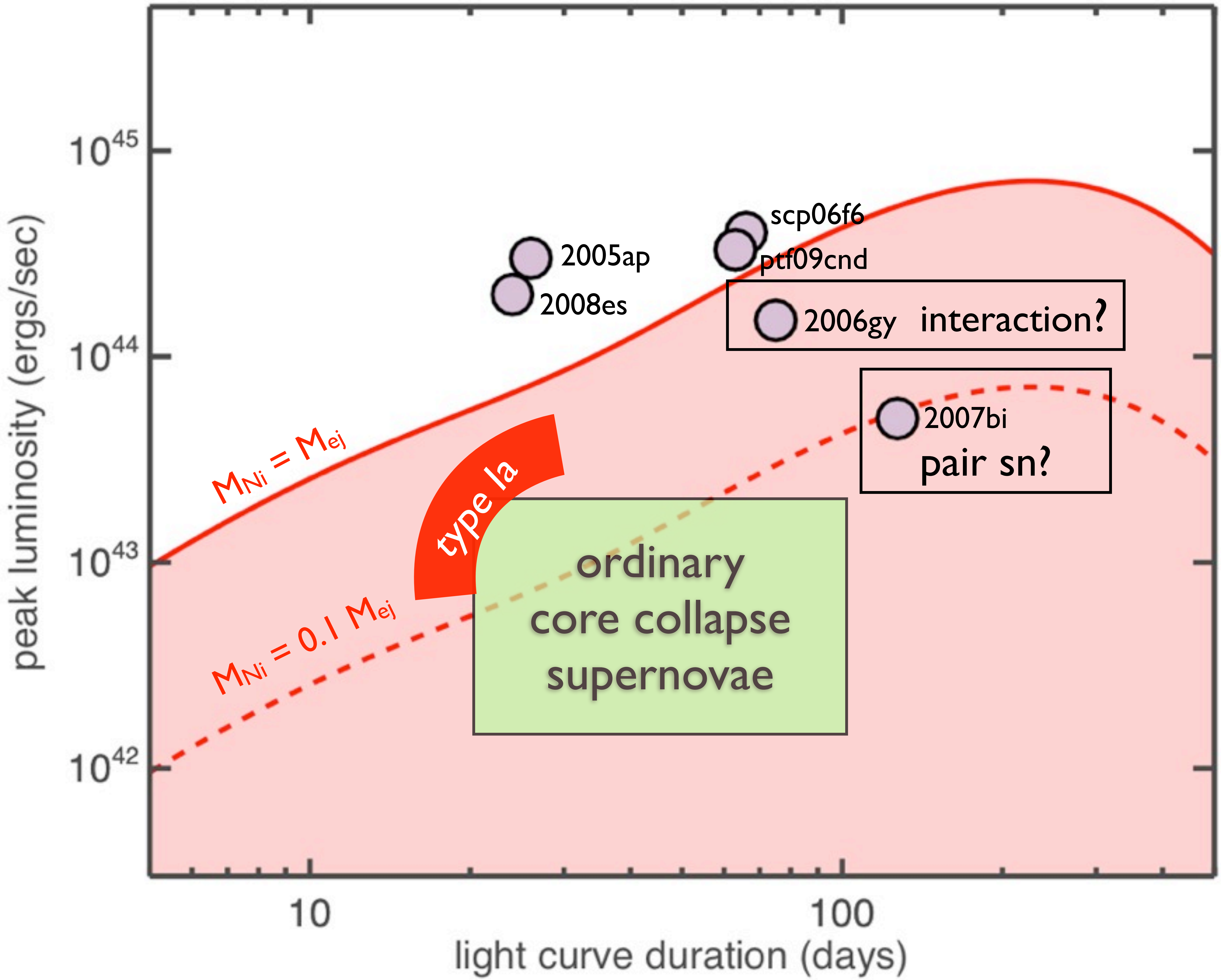
light curve duration (days)
scp06f6
2005ap
ptf09cnd
2008es
2006gy
2007bi
pair sn?
$M_{Ni} = M_{ej}$
type Ia
ordinary
core collapse
supernovae
$M_{Ni} = 0.1\ M_{ej}$



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light curve duration (days)
scp06f6
2005ap
ptf09cnd
2008es
2006gy interaction?
2007bi
pair sn?
$M_{Ni} = M_{ej}$
type Ia
$M_{Ni} = 0.1\ M_{ej}$
ordinary
core collapse
supernovae

power from neutron star spindown

crab nebula
$B \sim 5 \times 10^{12}$ g

neutron star spindown

~10% of neutron stars are born as magnetars,
with B ~ 10^{14} - 10^{15} g

rotational energy

$$E_{\rm rot} = \frac{1}{2} I_{\rm ns} \Omega^2 = 2 \times 10^{50} \text{ ergs} \left(\frac{P}{10 \text{ ms}} \right)^{-2}$$

spindown timescale

$$t_{\rm m} = \frac{6 I_{\rm ns} c^3}{B^2 R_{\rm ns}^6 \Omega^2} = 1.3 \text{ yrs} \left(\frac{B}{10^{14} \text{ g}} \right)^{-2} \left(\frac{P}{10 \text{ ms}} \right)^2$$

light curves from magnetars

roughly

$$L \sim \frac{E_\mathrm{m}}{t_\mathrm{d}} \left(\frac{t_\mathrm{m}}{t_\mathrm{d}} \right)$$

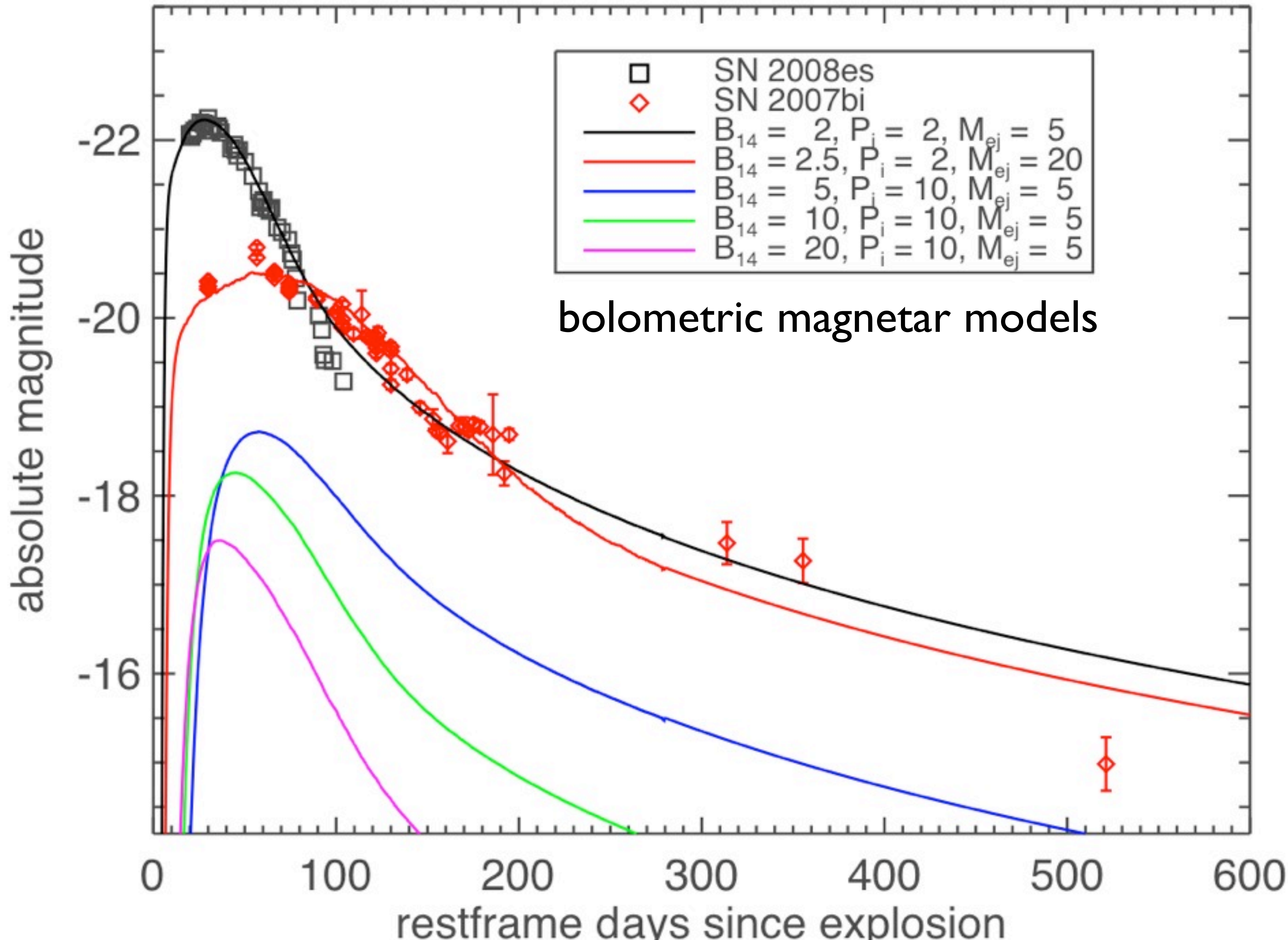
high radiative efficiency when B,P give $t_m \sim t_d$

better (for l = 2)

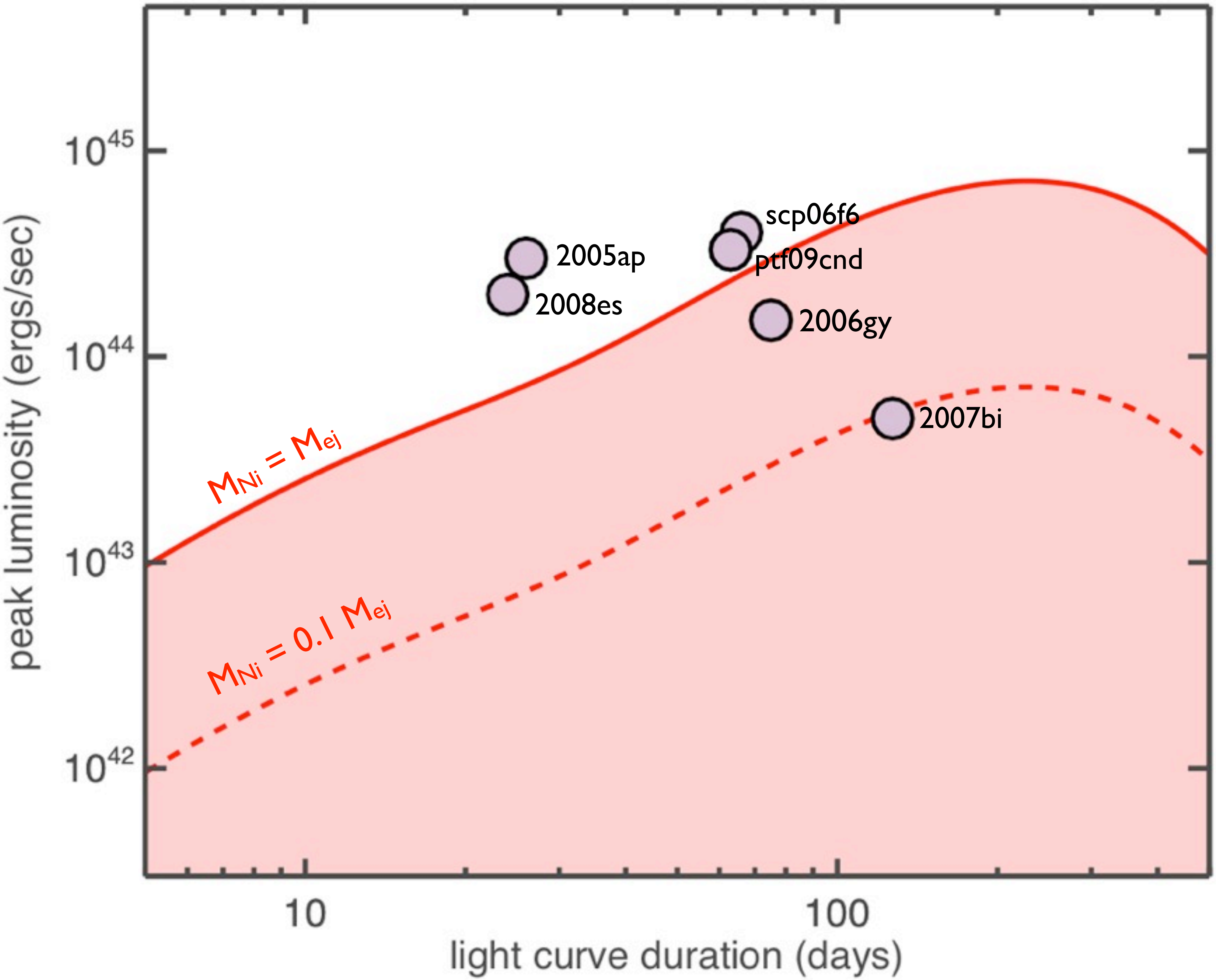
$$L_\mathrm{peak} \approx \frac{E_\mathrm{m} t_\mathrm{m}}{t_d^2} \left[\ln\left(1 + \frac{t_\mathrm{d}}{t_\mathrm{m}}\right) - \frac{t_\mathrm{d}}{t_\mathrm{d} + t_\mathrm{m}} \right]$$

$$t_\mathrm{peak} = t_\mathrm{m} \left(\left[\frac{E_\mathrm{m}}{L_\mathrm{peak}} t_\mathrm{m} \right]^{1/2} - 1 \right)$$

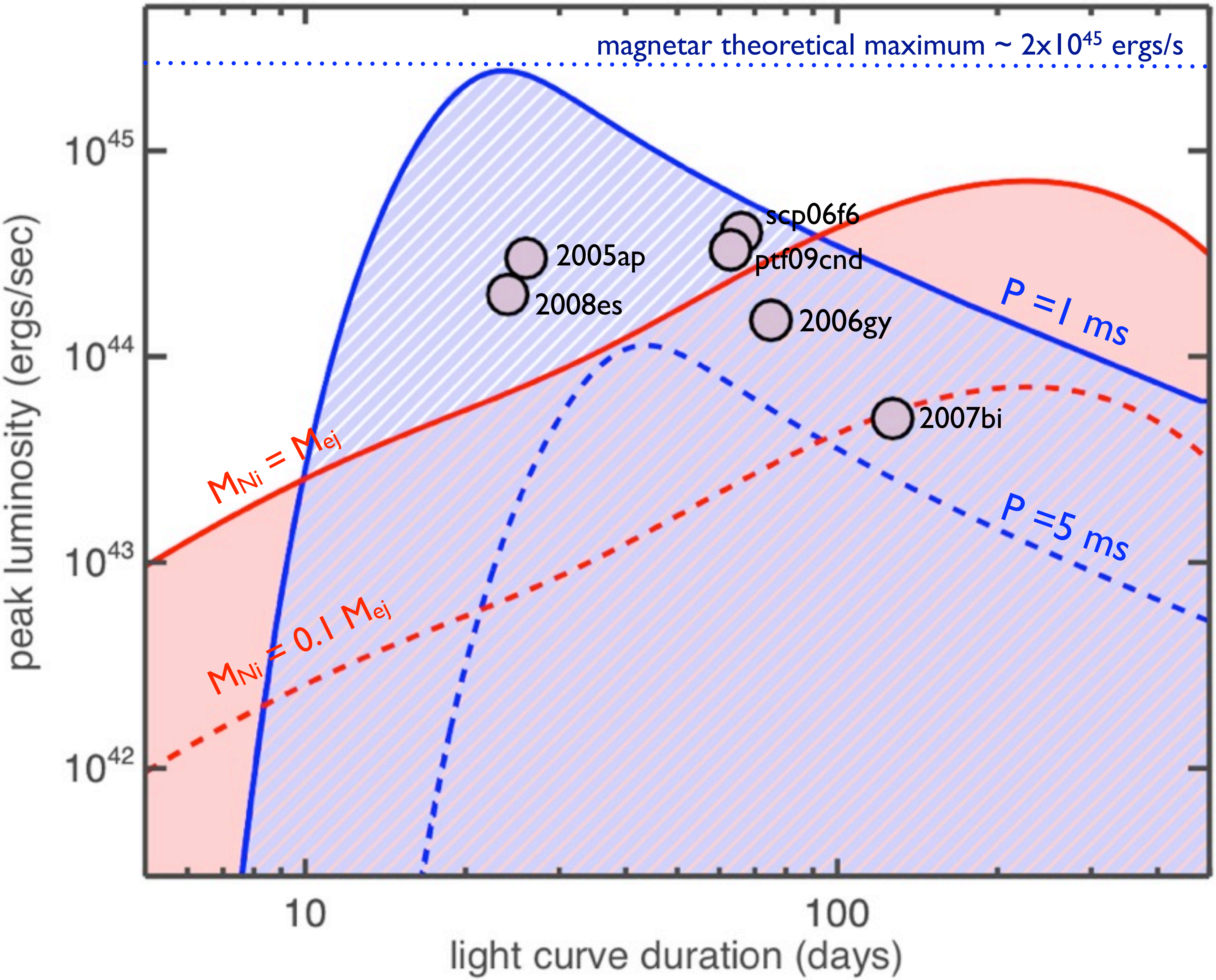
kasen&bildsten (2010)



SN 2008es
SN 2007bi
B14 = 2, Pi = 2, Mej = 5
B14 = 2.5, Pi = 2, Mej = 20
B14 = 5, Pi = 10, Mej = 5
B14 = 10, Pi = 10, Mej = 5
B14 = 20, Pi = 10, Mej = 5
bolometric magnetar models
absolute magnitude
-22
-20
-18
-16
0
100
200
300
400
500
600
restframe days since explosion



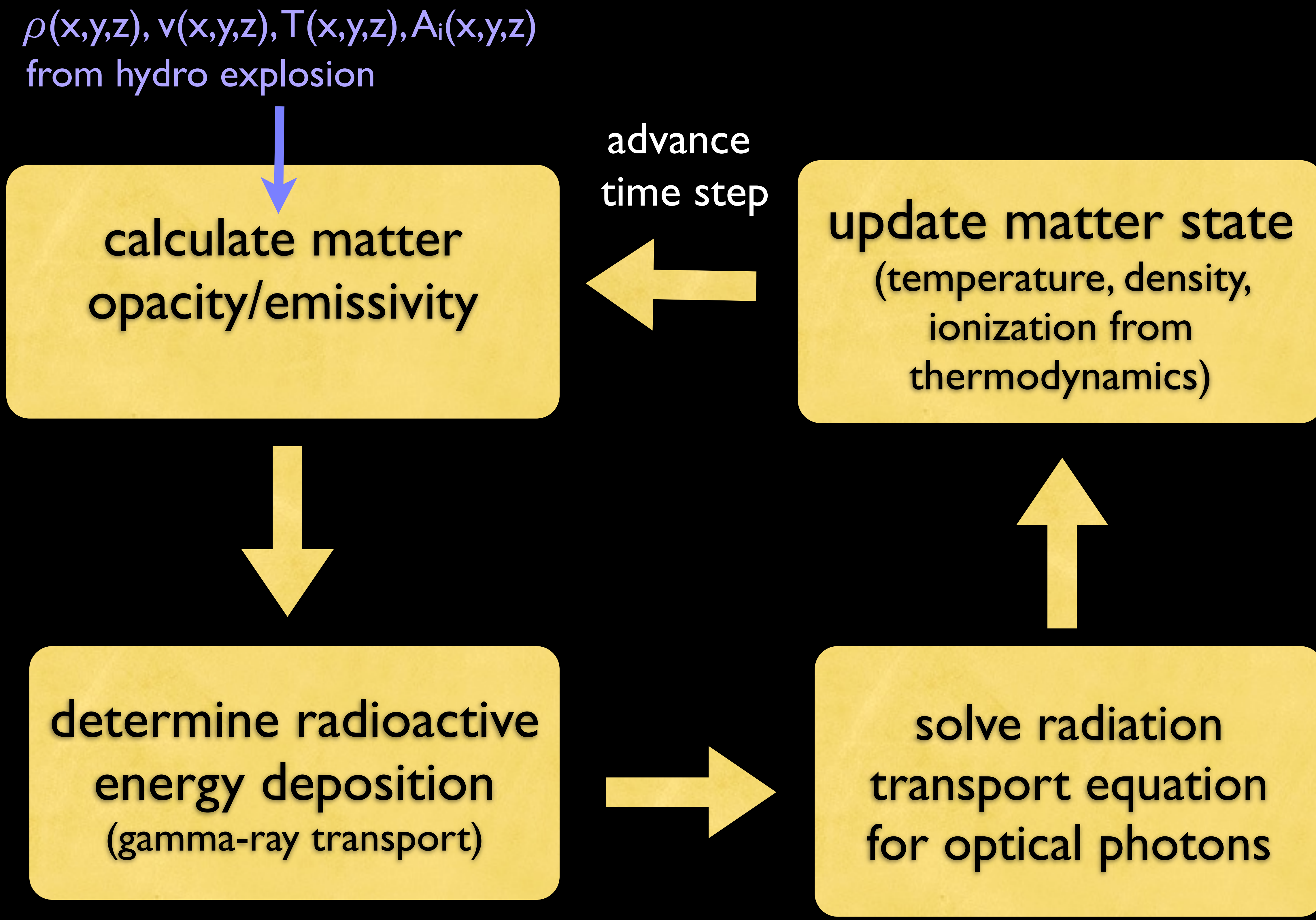
peak luminosity (ergs/sec)
10^{45}
10^{44}
10^{43}
10^{42}
10
100
light curve duration (days)
$M_{Ni} = M_{ej}$
$M_{Ni} = 0.1\ M_{ej}$
2005ap
2008es
scp06f6
ptf09cnd
2006gy
2007bi



magnetar theoretical maximum ~ 2x10^45 ergs/s
peak luminosity (ergs/sec)
10^45
10^44
10^43
10^42
scp06f6
2005ap
ptf09cnd
2008es
2006gy
2007bi
P =1 ms
P =5 ms
M_Ni = M_ej
M_Ni = 0.1 M_ej
10
100
light curve duration (days)

Monte Carlo and Numerical Radiation Transport

light curve computation



radiation transfer equation

$$\frac{dI}{ds} = -\chi I + \eta + \oint d\Omega \; \mathbf{R}(\hat{\mathbf{n}}, \hat{\mathbf{n}}')I$$

absorption ($-\chi I$), emission (η), scattering ($\mathbf{R}$)

where:

$I(x, y, z, \lambda, \theta, \phi)$ radiation specific intensity

$\eta(x, y, z, \lambda)$ matter emissivity

$\chi(x, y, z, \lambda)$ matter extinction coefficient

a 6 dimensional integro-differential equation
coupled through microphysics to matter energy equation

transport methods in astrophysics

grey flux limited diffusion

ignore θ,φ, λ dependence, solve diffusion equation
for "seeping" radiation fluid

multi-group flux limited diffusion (MGFLD)

ignore θ,φ, keep λ dependence, solve diffusion equation

ray tracing

follow individual trajectories; ignore scattering and diffusive terms

implicit monte carlo transport

mixed-frame stochastic particle propagation; retains
the full angle, wavelength, & polarization information

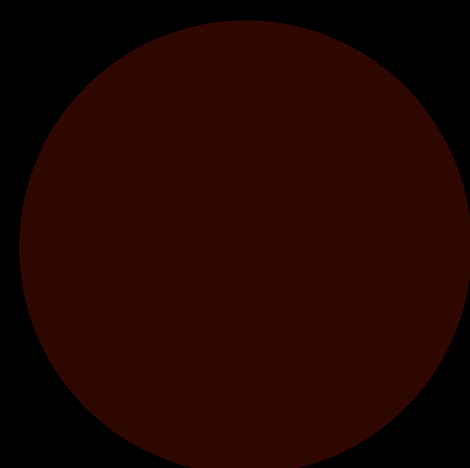
variable Eddington tensor

solve moments of the radiation transport equation with closure relation

S_n methods, etc....

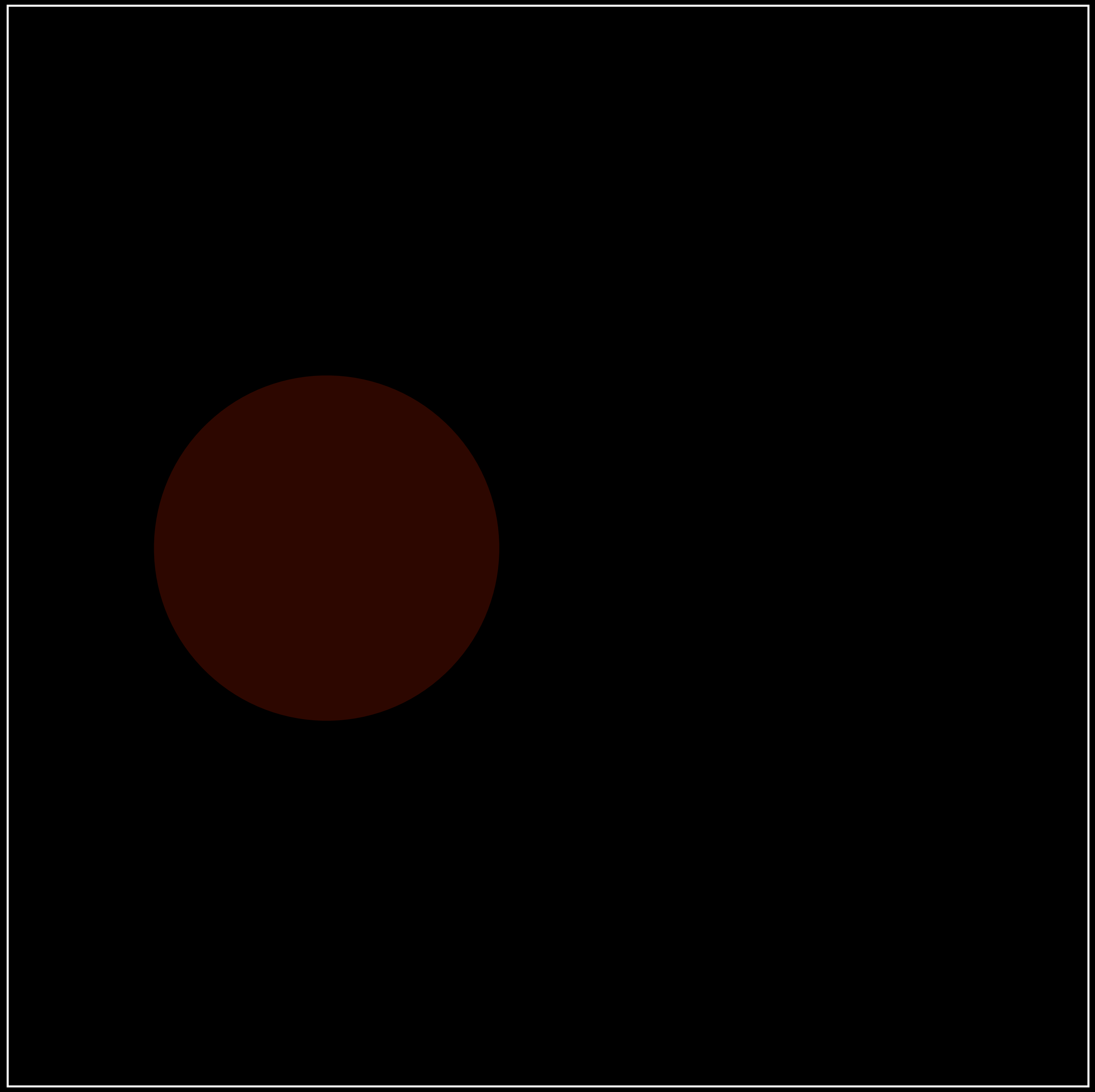
2-D shadow problem

multi-angle transport (monte carlo)



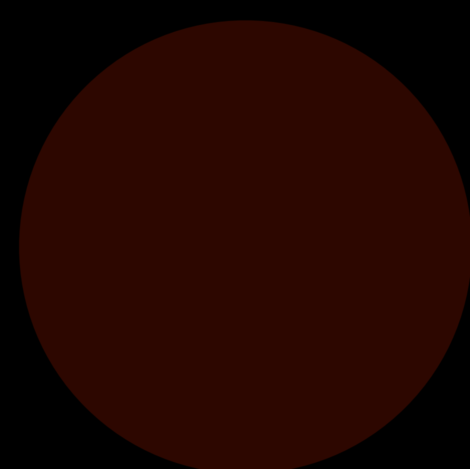
2-D shadow problem

multi-angle transport (monte carlo)



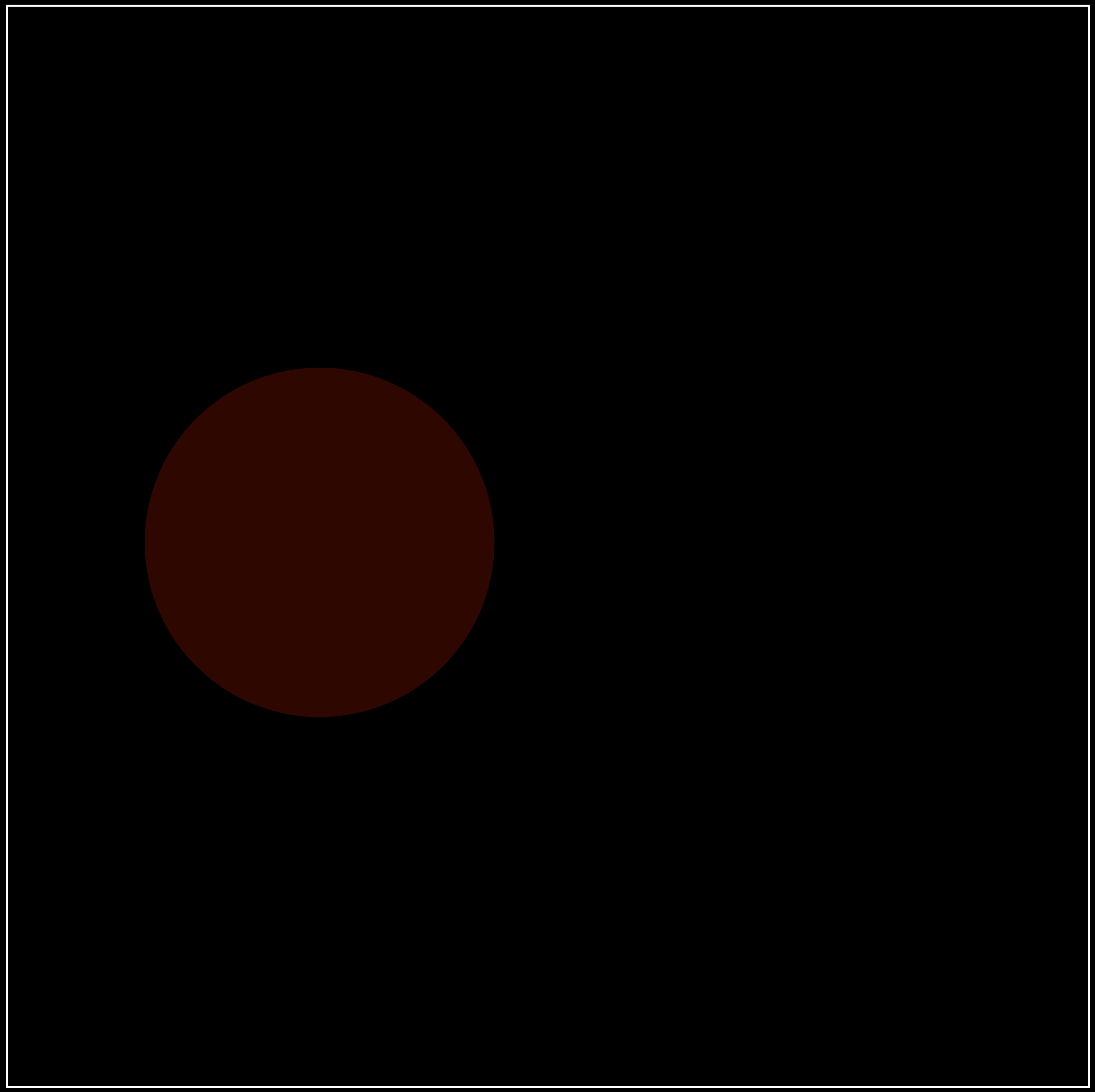
2-D shadow problem

diffusion approximation (DD monte carlo)



2-D shadow problem

diffusion approximation (DD monte carlo)



special relativistic transport in 1-D radiating flows

e.g., mihalas&mihalas

$$\frac{dI}{ds} = -\chi I + \eta + \oint d\Omega \, \mathbf{R}(\hat{\mathbf{n}}, \hat{\mathbf{n}'}) I$$

special relativistic transport in 1-D radiating flows

e.g., mihalas&mihalas

$$
\begin{aligned}
&\gamma(1+\beta\mu)\frac{\partial I_\nu}{\partial t}+\gamma(\mu+\beta)\frac{\partial I_\nu}{\partial r}\\
&\quad+\frac{\partial}{\partial\mu}\left\{\gamma(1-\mu^2)\left[\frac{1+\beta\mu}{r}-\gamma^2(\mu+\beta)\frac{\partial\beta}{\partial r}\right.\right.\\
&\quad\left.\left.-\gamma^2(1+\beta\mu)\frac{\partial\beta}{\partial t}\right]I_\nu\right\}-\frac{\partial}{\partial\nu}\left\{\gamma\nu\left[\frac{\beta(1-\mu^2)}{r}\right.\right.\\
&\quad\left.\left.+\gamma^2\mu(\mu+\beta)\frac{\partial\beta}{\partial r}+\gamma^2\mu(1+\beta\mu)\frac{\partial\beta}{\partial t}\right]I_\nu\right\}\\
&\quad+\gamma\left\{\frac{2\mu+\beta(3-\mu^2)}{r}+\gamma^2(1+\mu^2+2\beta\mu)\frac{\partial\beta}{\partial r}\right.\\
&\quad\left.+\gamma^2[2\mu+\beta(1+\mu^2)]\frac{\partial\beta}{\partial t}\right\}I_\nu=\eta_\nu-\chi_\nu I_\nu\,. \qquad (1)
\end{aligned}
$$

comoving frame spherical special relativistic transport eq.

monte carlo transport

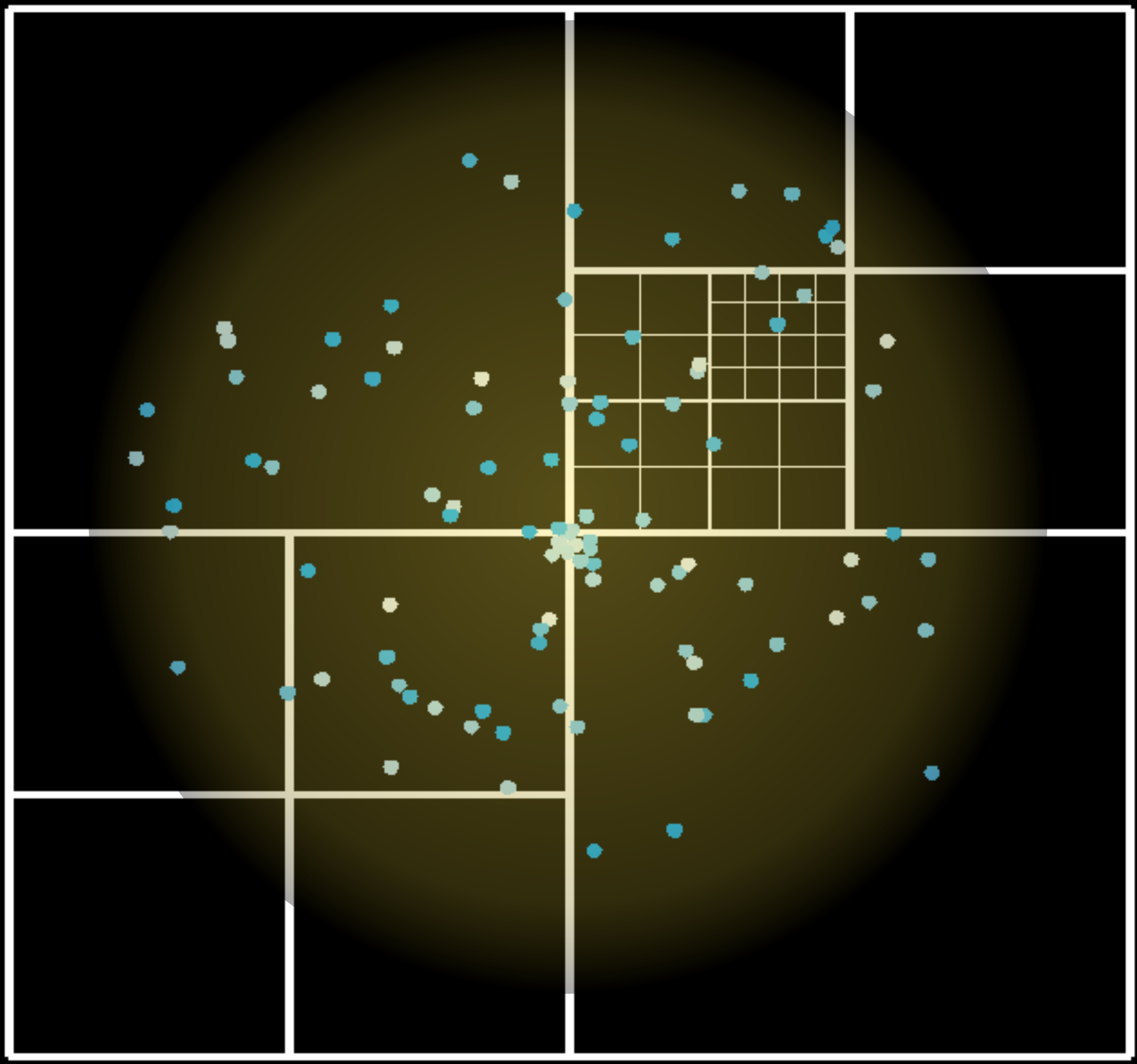
ulam

calculating pi at the bar

$$P_{\text{in}} = \frac{\pi R^2}{4R^2} = \frac{\pi}{4}$$

Signal to noise goes like $N^{-1/2}$
Need to throw $N = 10{,}000$ darts to get pi to two significant digits

Monte Carlo
Transport



Monte Carlo Transport

monte carlo transport

each particle has a position vector (x,y,z)
a direction vector (D_x, D_y, D_z), an energy, wavelength.
Evolution is sampled from appropriate probability distributions

monte carlo transport

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a direction vector (D_x, D_y, D_z), an energy, wavelength.
Evolution is sampled from appropriate probability distributions

probability of traveling a distance x before scattering

$$P = \exp(-\tau) = \exp(-\kappa \rho x) = \mathcal{R}$$

where R is a random number sampled
uniformly between (0, 1]

monte carlo transport

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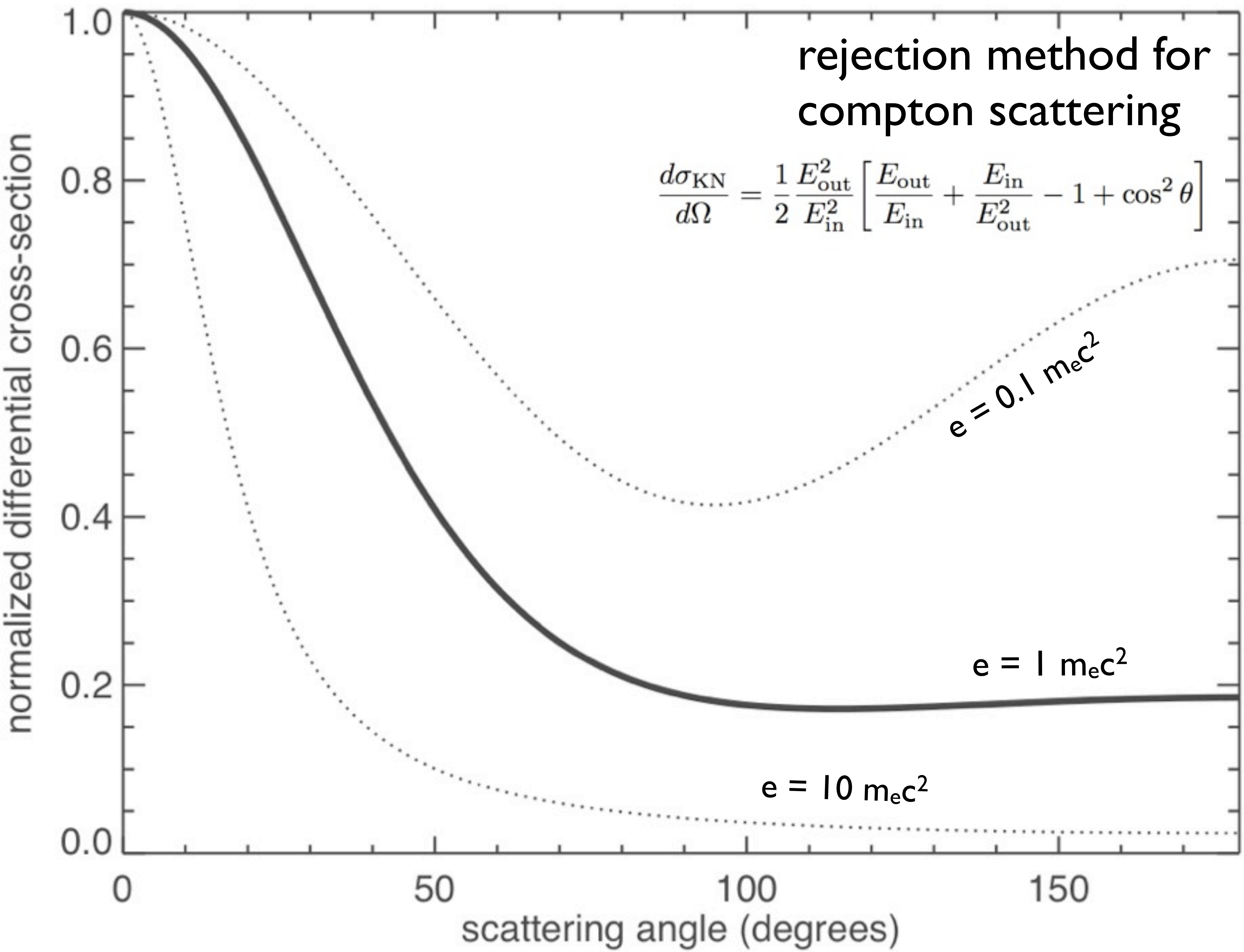
probability of traveling a distance x before scattering

$$P = \exp(-\tau) = \exp(-\kappa \rho x) = \mathcal{R}$$

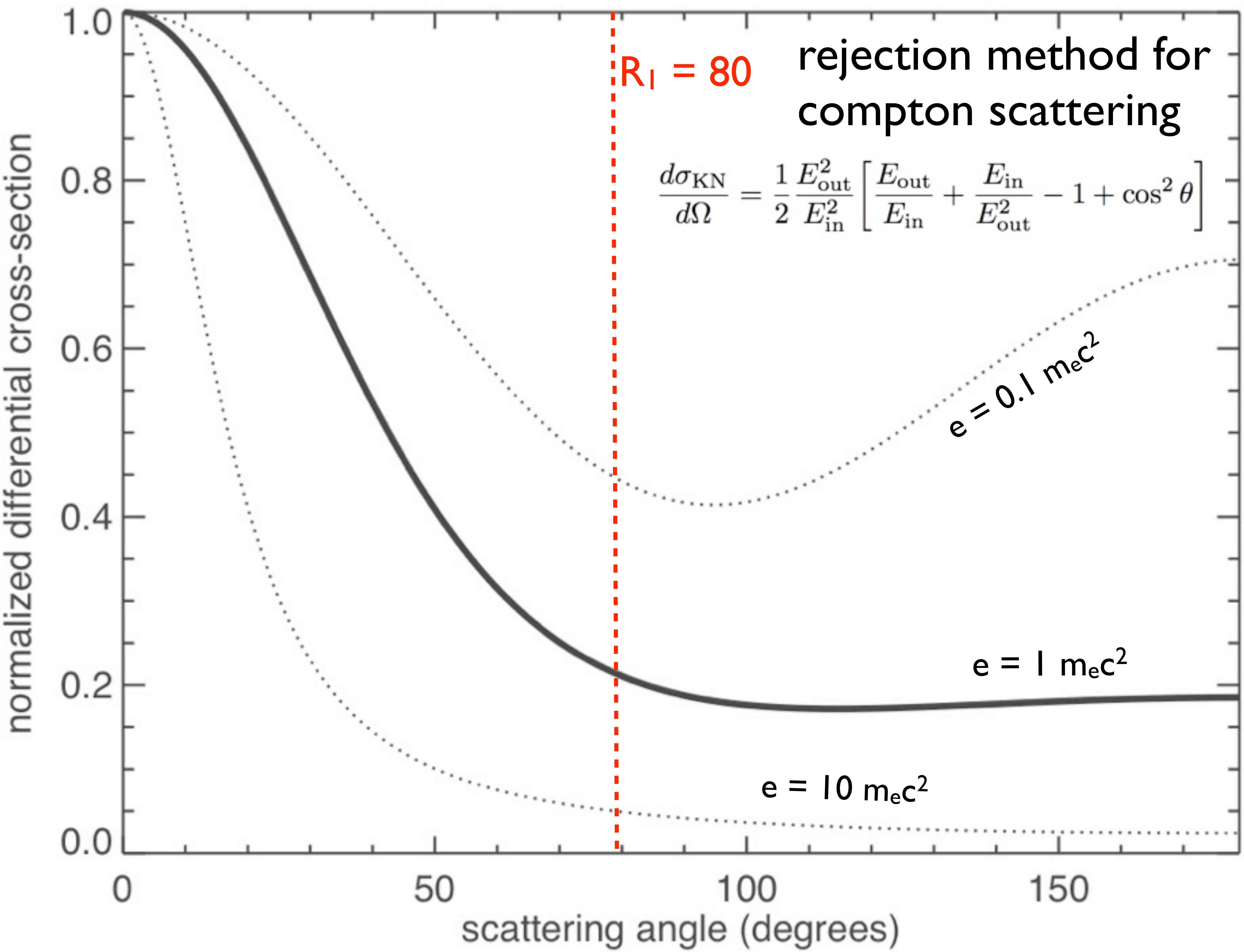
where R is a random number sampled
uniformly between (0, 1]

solve for x (distance traveled before scattering)

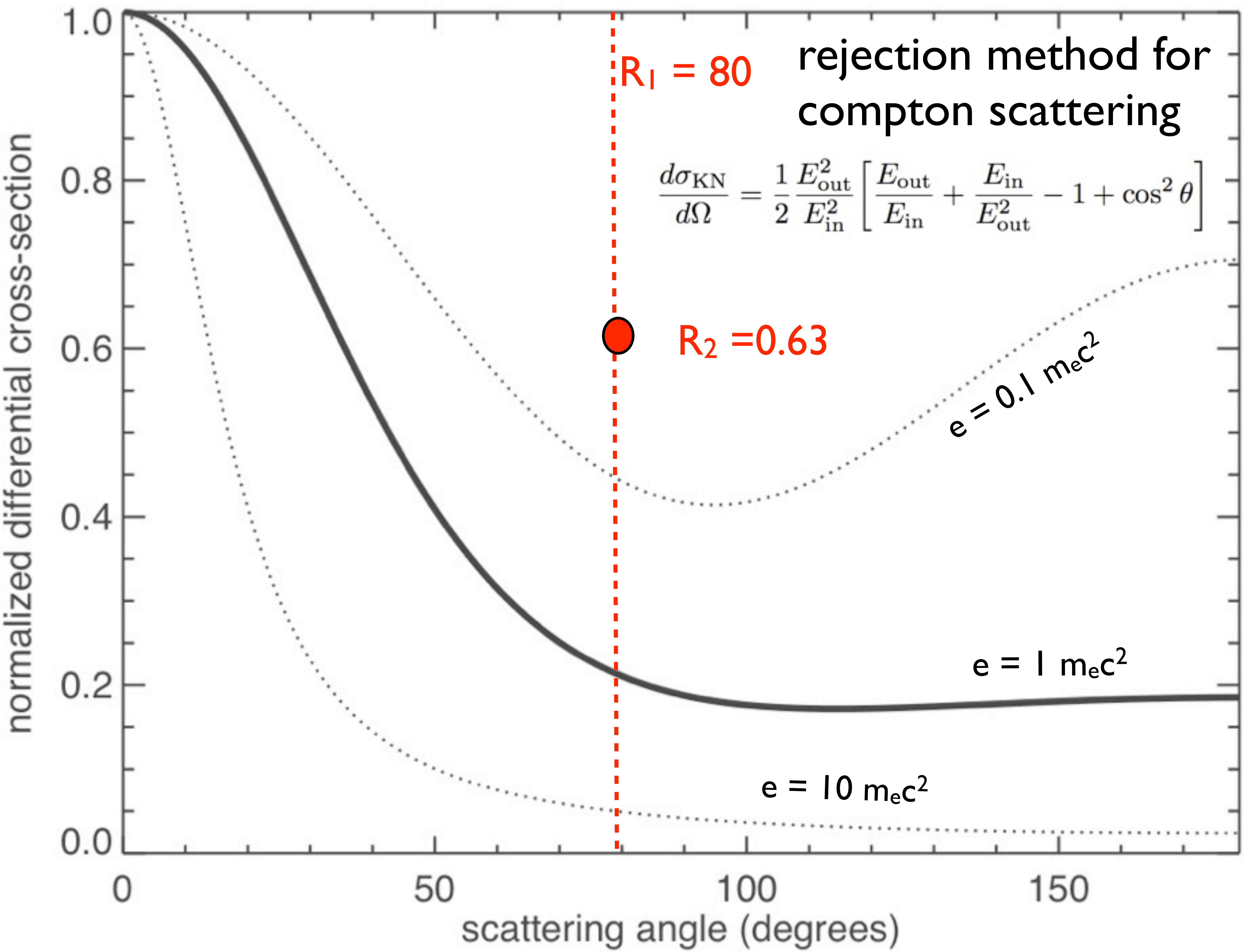
$$x = -(\kappa \rho)^{-1} \log(\mathcal{R})$$



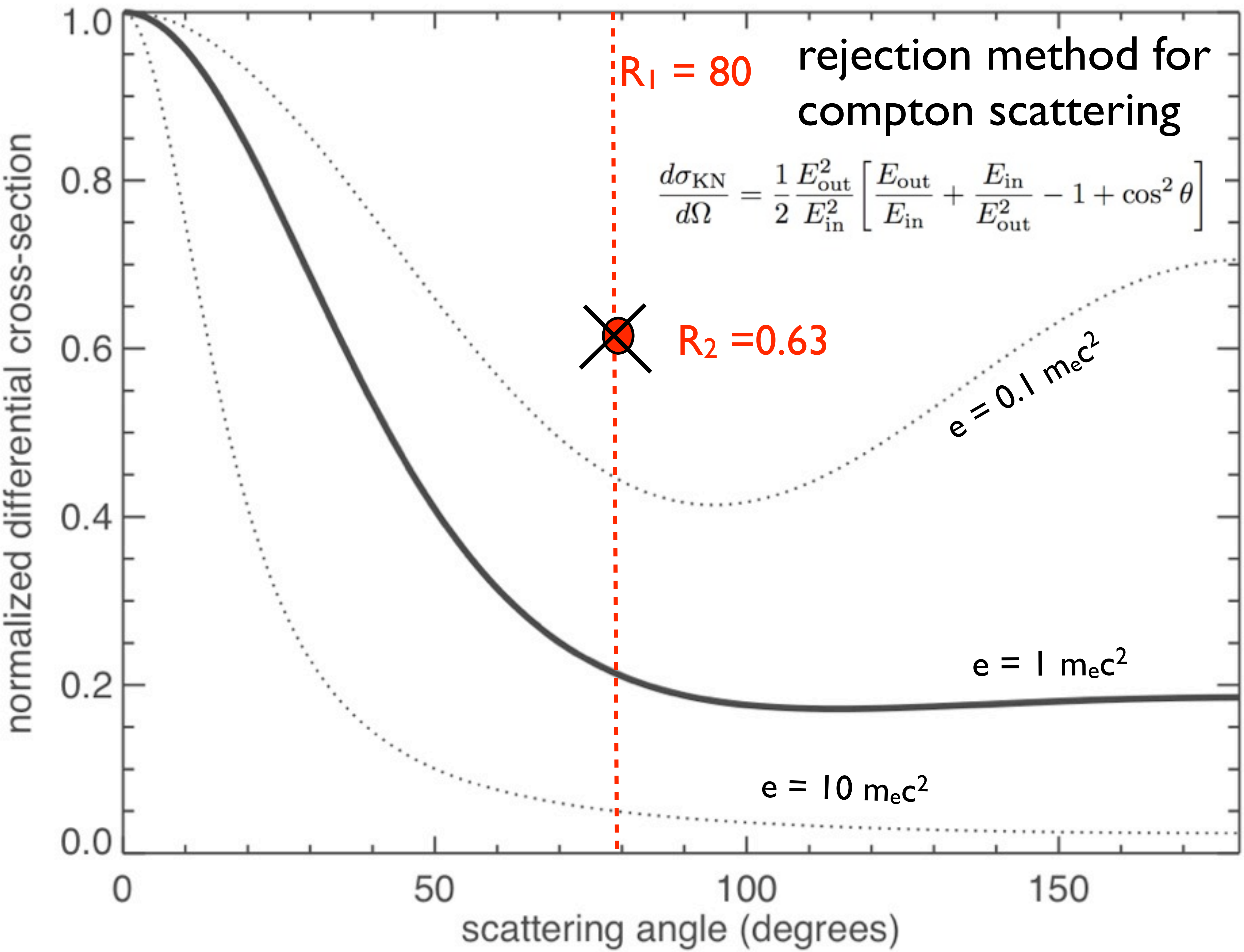
rejection method for
compton scattering
$$\frac{d\sigma_{\rm KN}}{d\Omega} = \frac{1}{2}\frac{E_{\rm out}^2}{E_{\rm in}^2}\left[\frac{E_{\rm out}}{E_{\rm in}} + \frac{E_{\rm in}}{E_{\rm out}^2} - 1 + \cos^2\theta\right]$$
e = 0.1 m_ec^2
e = 1 m_ec^2
e = 10 m_ec^2
normalized differential cross-section
scattering angle (degrees)
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150



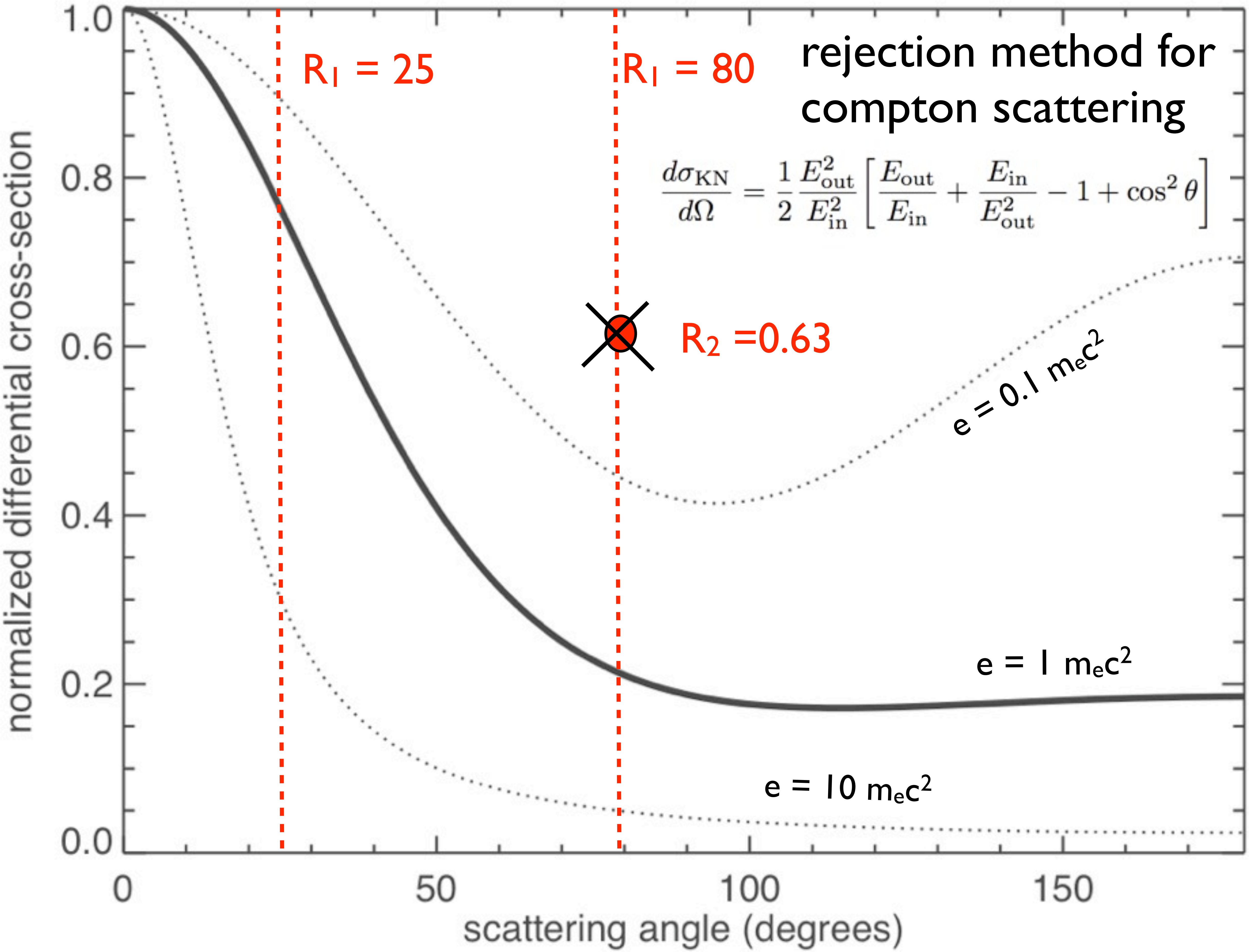
$R_1 = 80$
rejection method for
compton scattering
$$\frac{d\sigma_{\rm KN}}{d\Omega} = \frac{1}{2}\frac{E_{\rm out}^2}{E_{\rm in}^2}\left[\frac{E_{\rm out}}{E_{\rm in}} + \frac{E_{\rm in}}{E_{\rm out}^2} - 1 + \cos^2\theta\right]$$
$e = 0.1\ m_ec^2$
$e = 1\ m_ec^2$
$e = 10\ m_ec^2$
normalized differential cross-section
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150
scattering angle (degrees)



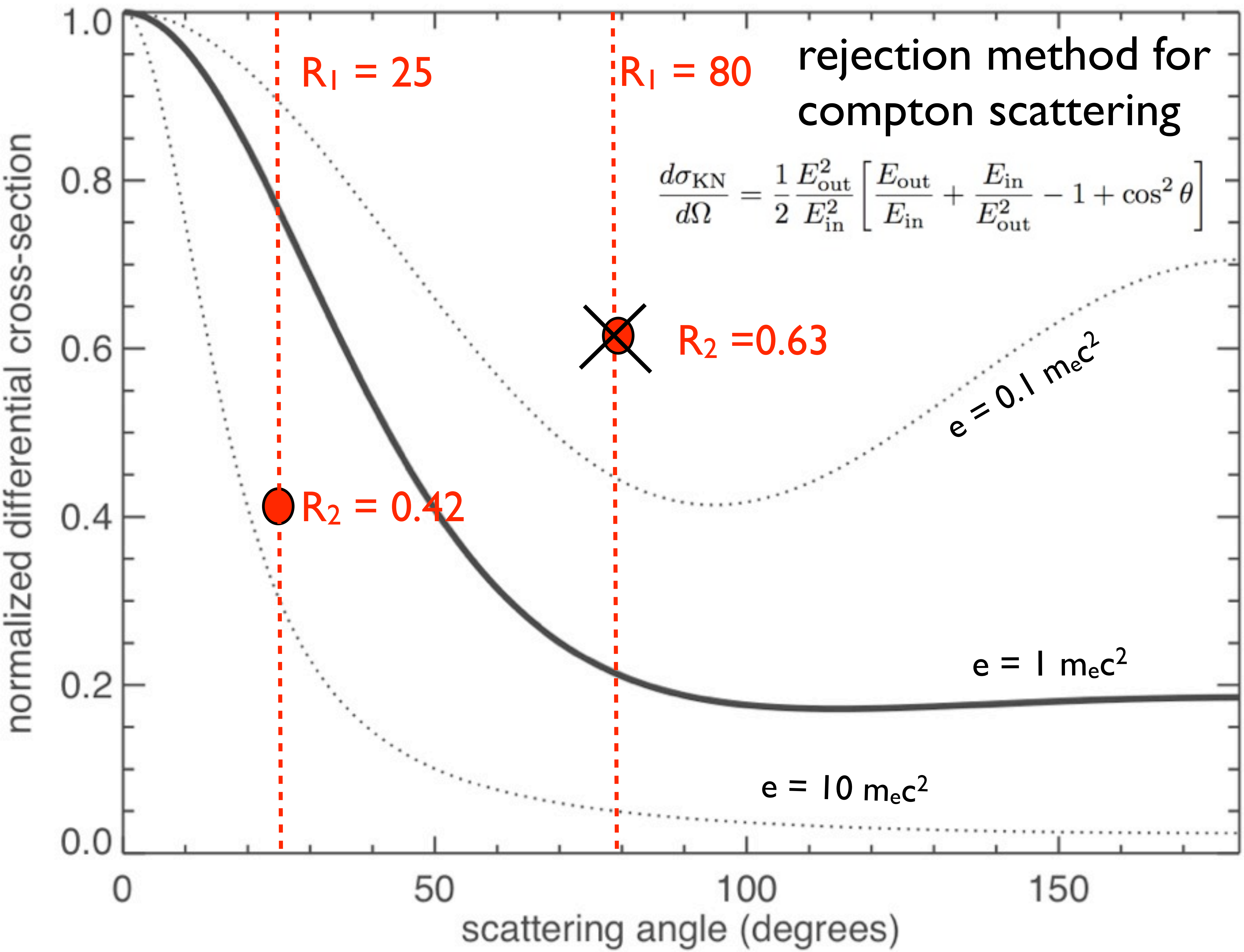
rejection method for
compton scattering
$$\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2}\frac{E_{out}^2}{E_{in}^2}\left[\frac{E_{out}}{E_{in}} + \frac{E_{in}}{E_{out}^2} - 1 + \cos^2\theta\right]$$
R_1 = 80
R_2 =0.63
e = 0.1 m_ec^2
e = 1 m_ec^2
e = 10 m_ec^2
normalized differential cross-section
scattering angle (degrees)
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150



rejection method for compton scattering
$$\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2}\frac{E_{out}^2}{E_{in}^2}\left[\frac{E_{out}}{E_{in}} + \frac{E_{in}}{E_{out}^2} - 1 + \cos^2\theta\right]$$
$R_1 = 80$
$R_2 = 0.63$
$e = 0.1\ m_ec^2$
$e = 1\ m_ec^2$
$e = 10\ m_ec^2$
normalized differential cross-section
scattering angle (degrees)
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150



rejection method for
compton scattering
$$\frac{d\sigma_{\mathrm{KN}}}{d\Omega} = \frac{1}{2}\frac{E_{\mathrm{out}}^2}{E_{\mathrm{in}}^2}\left[\frac{E_{\mathrm{out}}}{E_{\mathrm{in}}} + \frac{E_{\mathrm{in}}}{E_{\mathrm{out}}^2} - 1 + \cos^2\theta\right]$$
R_1 = 25
R_1 = 80
R_2 =0.63
e = 0.1 m_ec^2
e = 1 m_ec^2
e = 10 m_ec^2
normalized differential cross-section
scattering angle (degrees)
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150



rejection method for compton scattering
$$\frac{d\sigma_{\rm KN}}{d\Omega} = \frac{1}{2}\frac{E_{\rm out}^2}{E_{\rm in}^2}\left[\frac{E_{\rm out}}{E_{\rm in}} + \frac{E_{\rm in}}{E_{\rm out}^2} - 1 + \cos^2\theta\right]$$
R_1 = 25
R_1 = 80
R_2 =0.63
R_2 = 0.42
e = 0.1 m_ec^2
e = 1 m_ec^2
e = 10 m_ec^2
normalized differential cross-section
scattering angle (degrees)
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150

special relativistic transport in 1-D radiating flows

e.g., mihalas&mihalas

$$\frac{dI}{ds} = -\chi I + \eta + \oint d\Omega \, \mathbf{R}(\hat{\mathbf{n}}, \hat{\mathbf{n}}')I$$

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$$\gamma(1+\beta\mu)\frac{\partial I_\nu}{\partial t}+\gamma(\mu+\beta)\frac{\partial I_\nu}{\partial r}$$

$$+\frac{\partial}{\partial\mu}\left\{\gamma(1-\mu^2)\left[\frac{1+\beta\mu}{r}-\gamma^2(\mu+\beta)\frac{\partial\beta}{\partial r}\right.\right.$$

$$\left.\left.-\gamma^2(1+\beta\mu)\frac{\partial\beta}{\partial t}\right]I_\nu\right\}-\frac{\partial}{\partial\nu}\left\{\gamma\nu\left[\frac{\beta(1-\mu^2)}{r}\right.\right.$$

$$\left.\left.+\gamma^2\mu(\mu+\beta)\frac{\partial\beta}{\partial r}+\gamma^2\mu(1+\beta\mu)\frac{\partial\beta}{\partial t}\right]I_\nu\right\}$$

$$+\gamma\left\{\frac{2\mu+\beta(3-\mu^2)}{r}+\gamma^2(1+\mu^2+2\beta\mu)\frac{\partial\beta}{\partial r}\right.$$

$$\left.+\gamma^2[2\mu+\beta(1+\mu^2)]\frac{\partial\beta}{\partial t}\right\}I_\nu=\eta_\nu-\chi_\nu I_\nu\,. \quad (1)$$

comoving frame spherical special relativistic transport eq.

mixed frame monte carlo transport

opacities/emissivities calculated in the comoving frame
monte carlo particles propagated in the observer frame
lorentz transformation photon four vector at scattering events

$$\nu_0 = \gamma\nu(1 - \mathbf{d}\cdot\mathbf{v}/c)$$

$$\chi = \gamma\chi_0(1 - \mathbf{d}\cdot\mathbf{v}/c)$$

$$\mathbf{d}_0 = \left(\mathbf{d} - \frac{\gamma\mathbf{v}}{c}\left[1 - \frac{\gamma\mathbf{d}\cdot\mathbf{v}/c}{\gamma+1}\right]\right)\left[\gamma(1 - \mathbf{d}\cdot\mathbf{v}/c)\right]^{-1}$$

lorentz transformations

automatically accounts for all aberration, advection, doppler shifts, and adiabatic loses to all orders of v/c

general relativistic effects (geodesic tracking) can also be included
e.g., Dolence et al., (2009), Dexter et al., (2009)

implicit monte carlo methods

fleck and cummings 1971

$$G^0_0 = \left[\frac{1}{V\Delta t}\sum_i \epsilon_0 l_i \chi_0(\nu_0)\right] - \chi_0 a T_g^4$$

$$G^i_0 = \frac{1}{cV\Delta t}\sum_i \epsilon_0 l_i \chi_0(\nu_0) d^i_0$$

momentum four-force vector (i.e., radiative heating/cooling, radiative acceleration)

timescale for matter/radiation coupling

$$t_{\rm RM} \sim \frac{l_p}{c}\frac{nkT}{aT^4} \ll t_{\rm dyn}, t_{\rm diff}$$

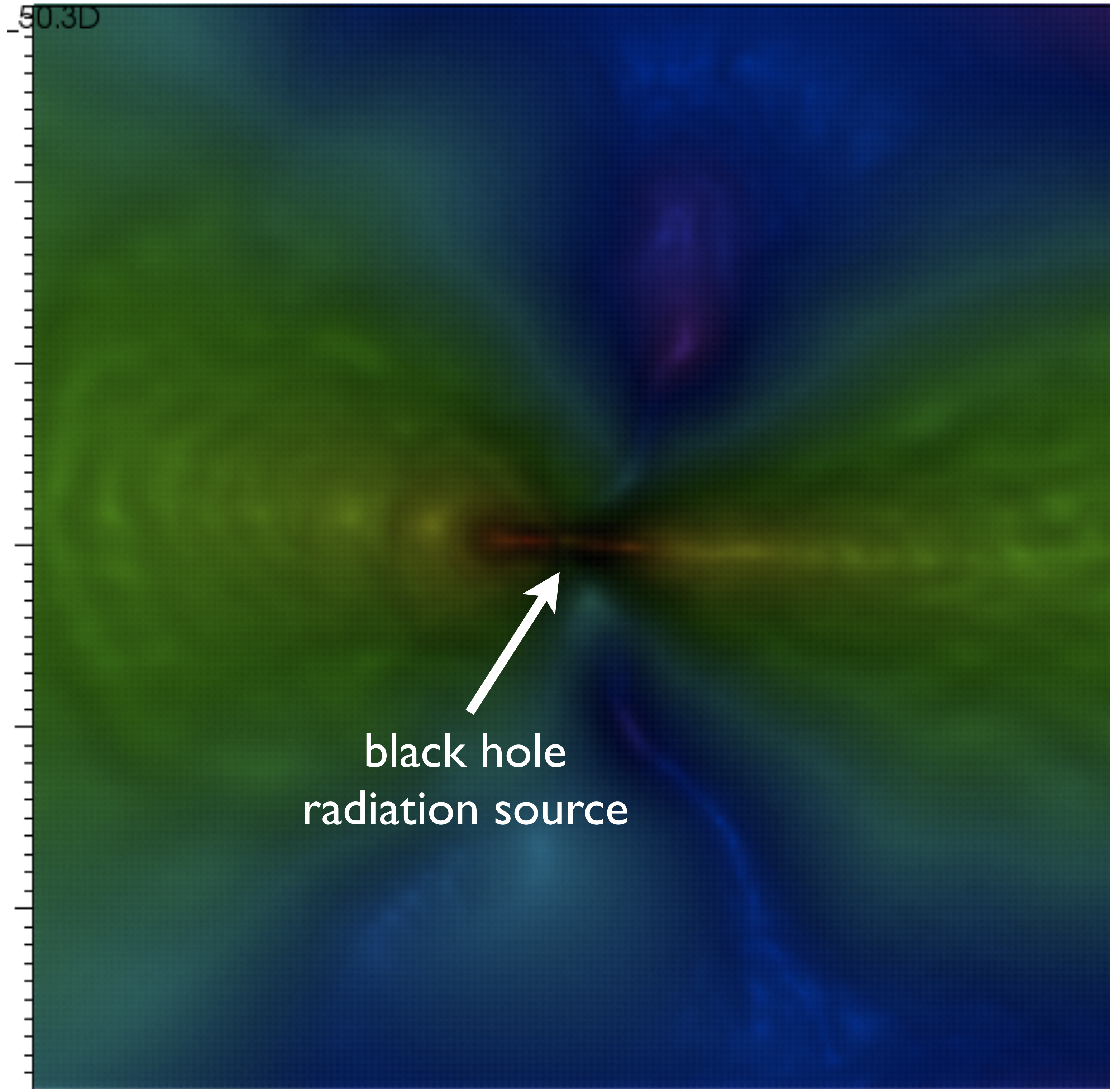
implicit methods: particle absorption/re-emission (i.e., creation/destruction) is replaced by "effective scattering"

population control and load balancing

For highly asymmetric 3D radiative flows, some zones may be under (over)-sampled by monte carlo particles

strategies

pressure tensor methods
russian roulette
particle splitting/killing
directionally biased emission
replicate heavily loaded zones



black hole accretion disk
(Nathan Roth, UCB)

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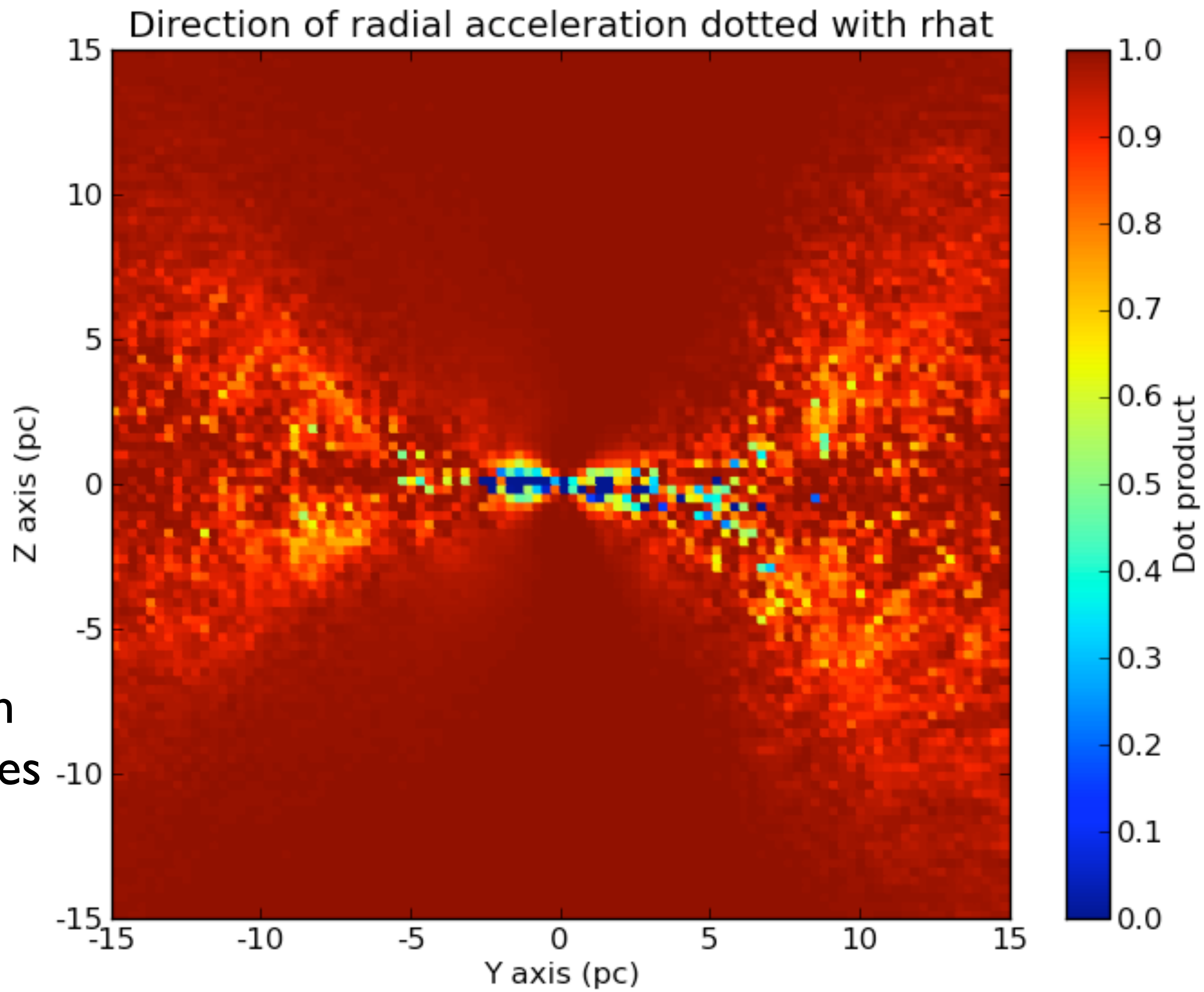


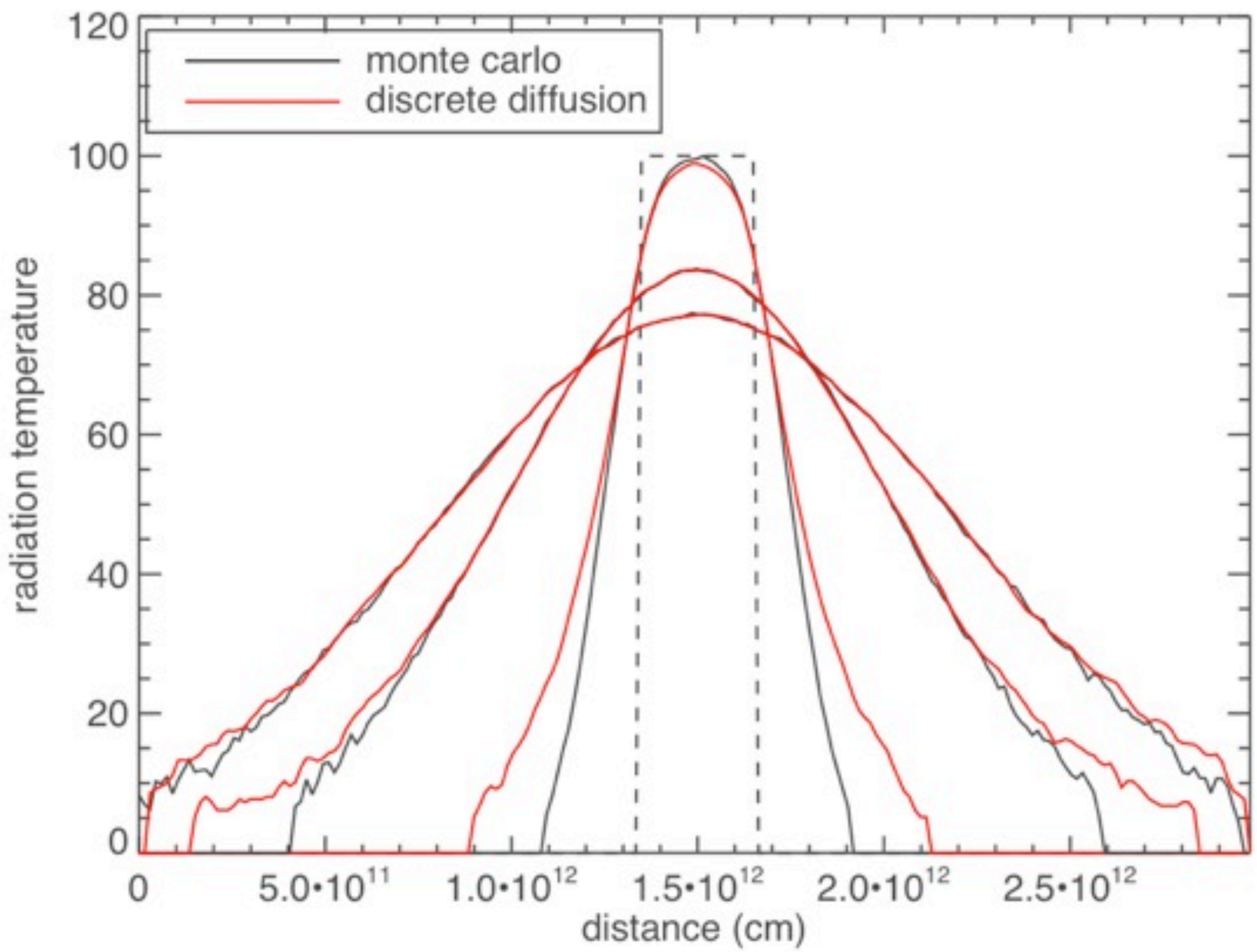
black hole accretion disk
(Nathan Roth, UCB)

discrete diffusion monte carlo

gentile 2001, densmore et al 2007

For regions of high opacity, monte carlo is very inefficient.
Instead, sample from the diffusion approximation:

$$F = -\frac{c}{3\chi}\nabla E_{\rm rad}$$



jump probabilities

$$P_L = \frac{c}{3\chi_L \Delta x}\frac{\Delta t}{\Delta x}$$

$$P_R = \frac{c}{3\chi_R \Delta x}\frac{\Delta t}{\Delta x}$$

$$P_{\rm abs} = c\chi_{\rm abs}\Delta t$$

$$P_{\rm stay} = 1$$

$$\text{norm} = [1 + P_R + P_L + P_{\rm abs}]^{-1}$$

monte carlo parallelization strategies

using hybrid MPI/open MP, run on 10,000-100,000 cores
using Cray XE6 (Hopper @ NERSC),
Cray XT5 (Jaguar @ ORNL) Blue Gene/P (Intrepid @ ALCF)

full replication

each core holds entire model and propagates particles independently;
MPI all reduce of radiation/matter coupling terms after each time step.
Memory limited (2D, low resolution 3D).

domain decomposed

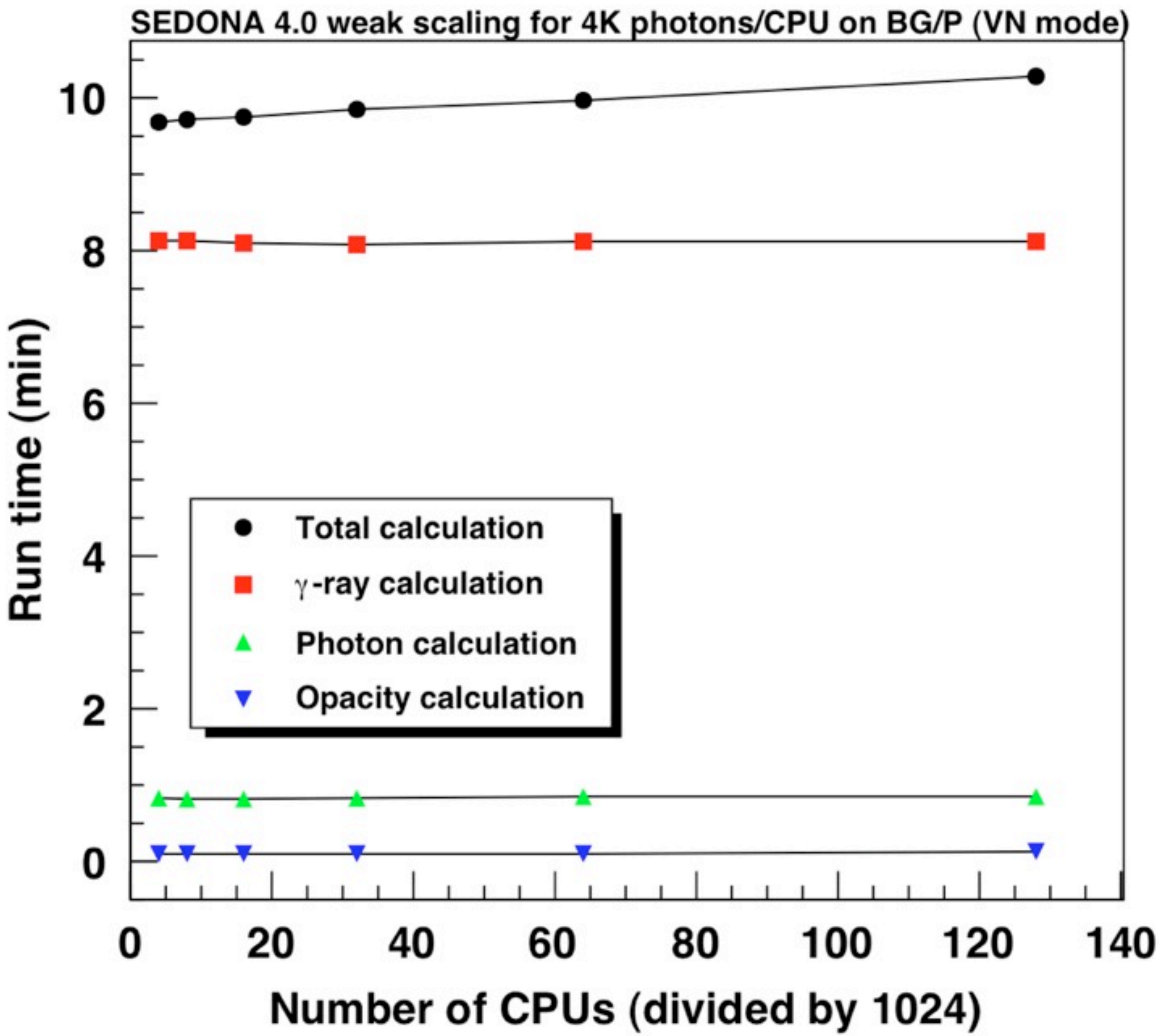
spatial grid partitioned over cores
particles leaving local domain communicated via MPI to neighbors

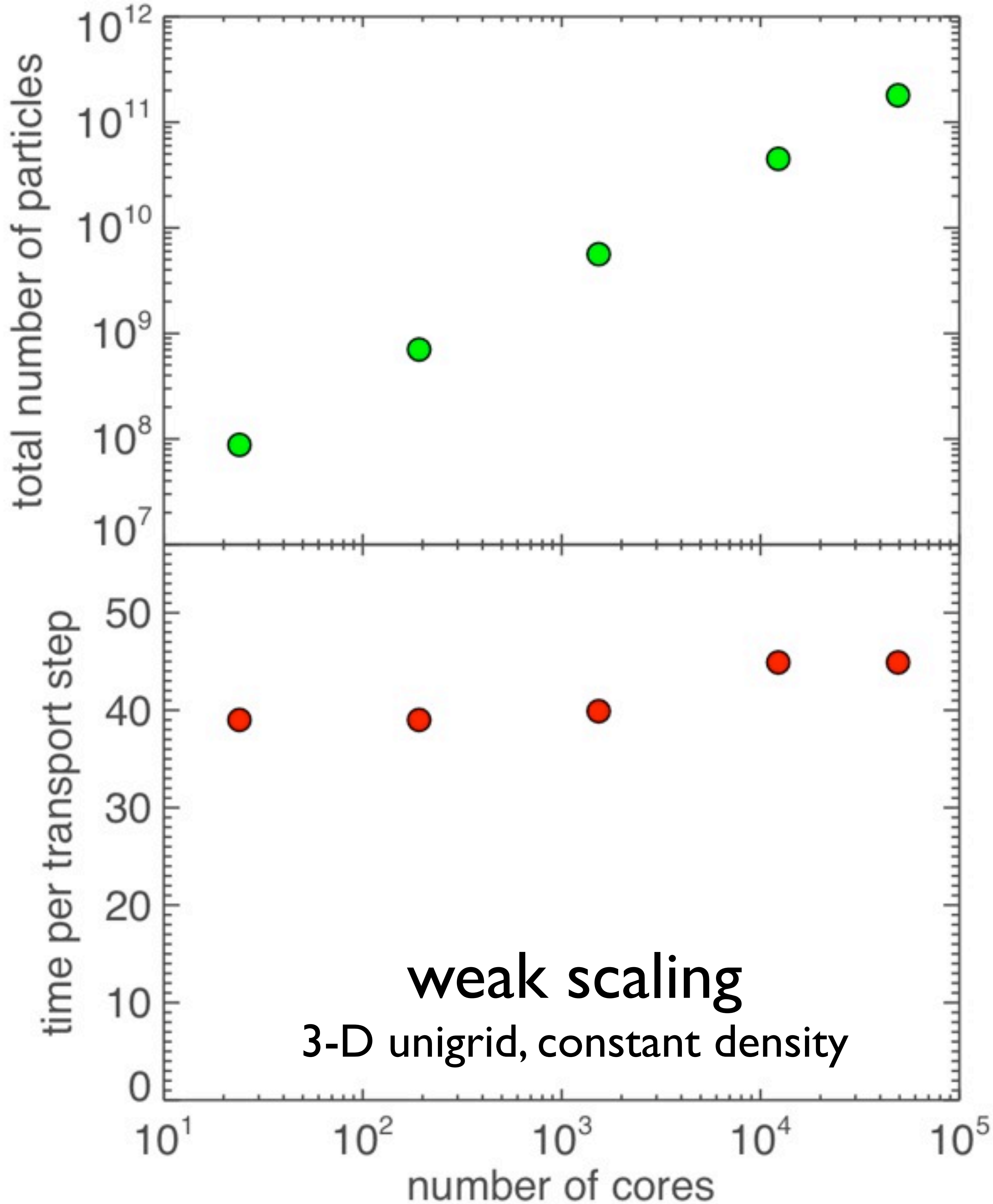
hybrid

use openMP threading to do additional particles on shared memory node,
can fully replicate certain domains on additional nodes to extend scaling
and manage load balancing.

weak scaling: 2D transport calculation

full replication -- embarrassingly paralell





domain decomposed monte carlo transport

hybrid MPI/open MP
BoxLib AMR framework

on *Hopper* XE6 (NERSC)
2 twelve-core AMD "Mangy-Cours"
(4 NUMA "nodes" of 6 cores)
2.1 GHz processors per node

<u>@ 49,152 cores (2048 nodes)</u>

total particles	= 1.8×10^{11}
total cells	= 4.5×10^{7}
wavelength points	= 10,000
total memory	= 65 TB


CAPITOL FULL DIMENSIONAL STEREO
Tex Williams
SMOKE SMOKE SMOKE
Capitol
HIGH FIDELITY
RECORDING
SMOKE
SMOKE
SMOKE