

Neutron Stars

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Computational Explosive Astrophysics Summer School
LBL

- ▶ **Neutron Stars**
 - ▶ Observed Properties of Neutron Stars
 - ▶ Structure of Neutron Stars
 - ▶ Formation and Evolution of Neutron Stars
- ▶ **Mass Measurements and Implications**
 - ▶ How Masses of Neutron Stars Are Measured
 - ▶ Implications of a Large Maximum Mass
 - ▶ Neutron Star Radii, the Maximum Mass, and the EOS
 - ▶ Nuclear Physics Constraints

Observed Properties of Neutron Stars

- ▶ Over 1800 known as pulsars
- ▶ A few dozen accreting or quiescent sources in binary systems
- ▶ Less than a dozen isolated neutron stars

Pulsars: The Early History

1932 - Chadwick discovers neutron.

1934 - W. Baade and F. Zwicky predict existence of neutron stars as end products of supernovae.

1939 - Oppenheimer and Volkoff predict mass limit of neutron stars.

1966 - Colgate and White simulate supernovae forming neutron stars.

1966 - Wheeler predicts Crab nebula powered by rotating neutron star.

1967 - C. Schisler discovers pulsing radio sources, including the Crab, with military radar.

1967 - Hewish, Bell, Pilkington, Scott and Collins discover the pulsar PSR 1919+21, Aug 6. Only Hewish awarded Nobel Prize (1974).

1968 - Crab pulsar discovered.

1968 - T. Gold identifies pulsars with magnetized, rotating neutron stars.

1968 - The term “pulsar” first appears in print, in the *Daily Telegraph*.

1969 - “Glitches” provide evidence for superfluidity in neutron star.

1971 - Accretion powered X-ray pulsar discovered by Uhuru (*not* Lt.).

1974 - Binary pulsar PSR 1913+16 discovered by Hulse and Taylor with orbital decay due to gravitational radiation. Nobel prize 1993.

1982 - First millisecond pulsar, PSR B1509-58, discovered by Backer et al.

1992 - Discovery of planets orbiting PSR B1509-58, Wolszczan and Frail.

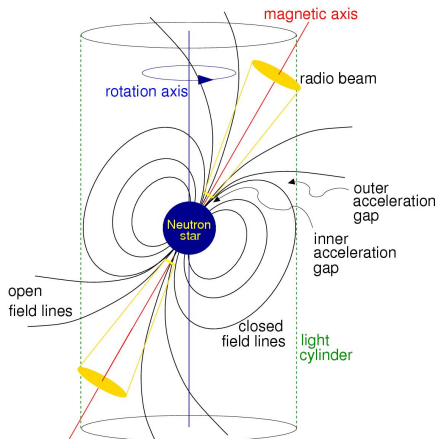
1992 - Prediction of magnetars by Duncan & Thompson.

Amazing Facts About Neutron Stars

- ▶ **Densest objects this side of an event horizon:** $10^{15} \text{ g cm}^{-3}$
Four teaspoons on the Earth would weigh as much as the Moon.
- ▶ **Largest surface gravity:** $10^{14} \text{ cm s}^{-2}$, about $10^{11}g$
- ▶ **Fastest spinning massive objects known**
PSR J1748-2446ad, located in the globular cluster Terzan 5 28,000 light years away, spins at 716 Hz. (33 pulsars have been found in this cluster.) The velocity at this star's equator is $c/4$.
- ▶ **Largest known magnetic field strengths:** $B = 10^{15} \text{ G}$, Sun = 1 G.
- ▶ **Highest temperature superconductor:** $T_c = 10 \text{ billion K}$
The record superconductor on the Earth is mercury thallium barium calcium copper oxide ($\text{Hg}_{12}\text{Tl}_3\text{Ba}_{30}\text{Ca}_{30}\text{Cu}_{45}\text{O}_{125}$), at 138 K.
- ▶ **Highest temperature since Big Bang:** $T = 700 \text{ billion K}$
- ▶ **Fastest velocity of a massive object in the Galaxy:** $> 1083 \text{ km/s}$
- ▶ **Largest burst of energy in our Galaxy since SN 1604**
A burst from magnetar SGR 1806-20 was brighter than the full moon in gamma rays and released more energy in 0.1 s than Sun emits in 100,000 years. It ionized ionosphere to daytime levels.
- ▶ **The only place in the universe except for the Big Bang where neutrinos become *trapped*.**

Pulsars: Why do they pulse?

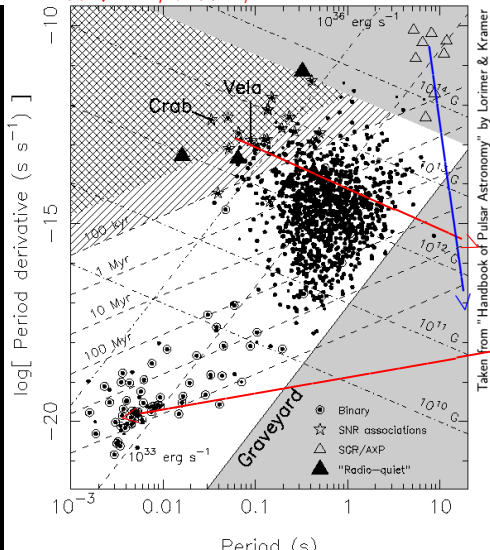
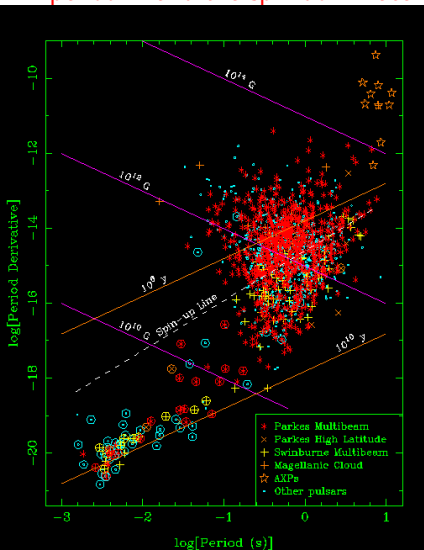
- ▶ All models involve the lighthouse effect, in which particles and light are emitted from magnetic poles that are misaligned with the orbital poles (magnetic dipole model). The beam widths are measured in some cases to be several degrees, so we are fortunate to see any given pulsar.
- ▶ It is known that spinning magnetic dipoles can emit energy.
- ▶ Nobody understands in detail how the beaming is accomplished.
- ▶ For the magnetic dipole model,
$$dE_{rot}/dt = \dot{E}_{rot} \propto B^2 R^6 P^{-4}$$
$$E_{rot} \propto MR^2 P^{-2}$$
$$\dot{E}_{rot} \propto MR^2 P^{-3} \dot{P}$$
$$B \propto R^{-2} \sqrt{MP\dot{P}} > 10^{19} \sqrt{\frac{P\dot{P}}{s}} \text{ G}$$
Characteristic age $\tau = P/(2\dot{P})$
- ▶ Neutron stars with too small B or ν can not pulse.



From *Handbook of Pulsar Astronomy* by Lorimer and Kramer

The $P - \dot{P}$ Diagram – The H-R Diagram for Pulsars

The magnetic field strength and age can be expressed in terms of the period P and the spin-down rate \dot{P} : $B \propto \sqrt{P\dot{P}}$, $\tau \propto P/\dot{P}$.

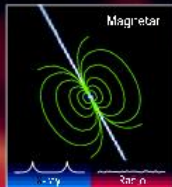


Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

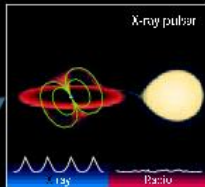


The Lives of Pulsars

If pulsar has a high-mass companion star



If pulsar has a high-mass companion star

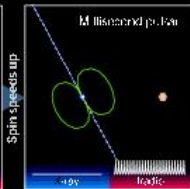
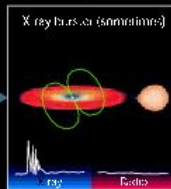


very pulsar is born in a supernova, but different ones go on to live out different life stories. The usual progression is for a pulsar simply to spin down, losing first its X-ray and eventually its radio pulses (bottom side of dia-

Supernova explosion

A Pulsar's Life Cycle

Low-mass companion star

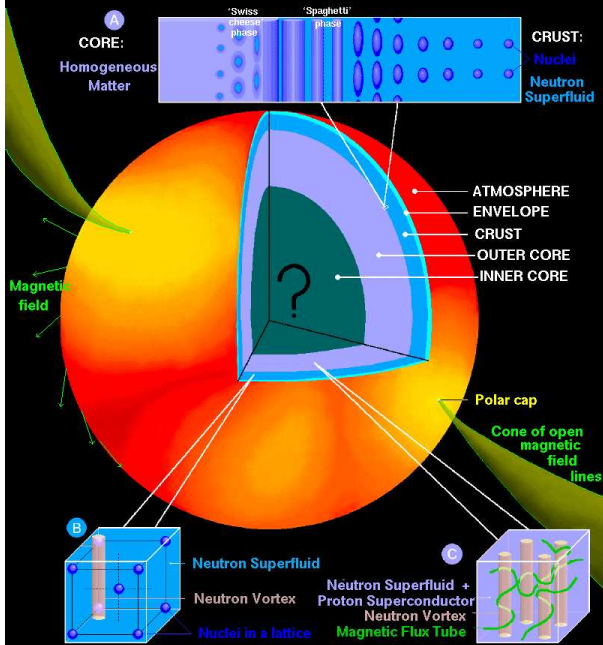


Spin slows down



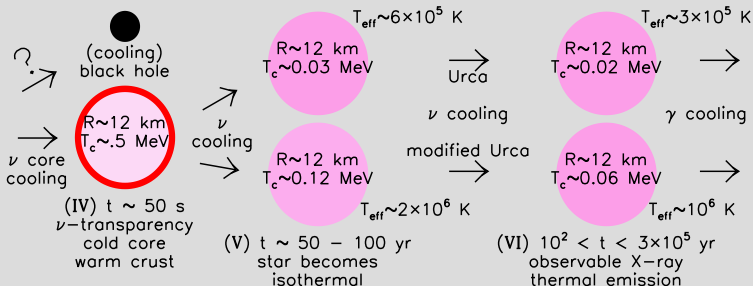
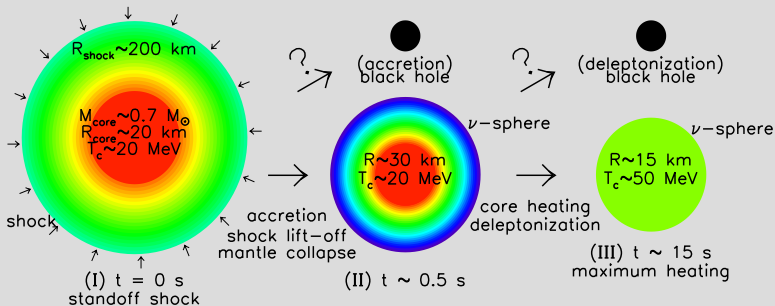
gram). But if it is born with an exceptionally intense magnetic field it becomes a "magnetar" with special properties of its own. If it has a close companion star, it may be spun up and reinvigorated late in life to become a millisecond pulsar.

A NEUTRON STAR: SURFACE AND INTERIOR

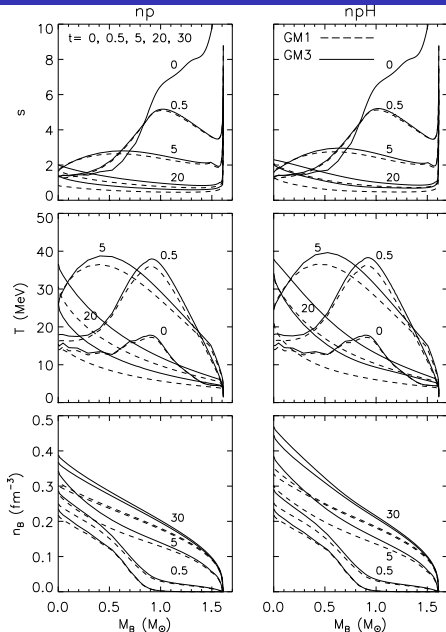


Credit: Dany Page, UNAM

Proto-Neutron Stars and Neutron Star Evolution

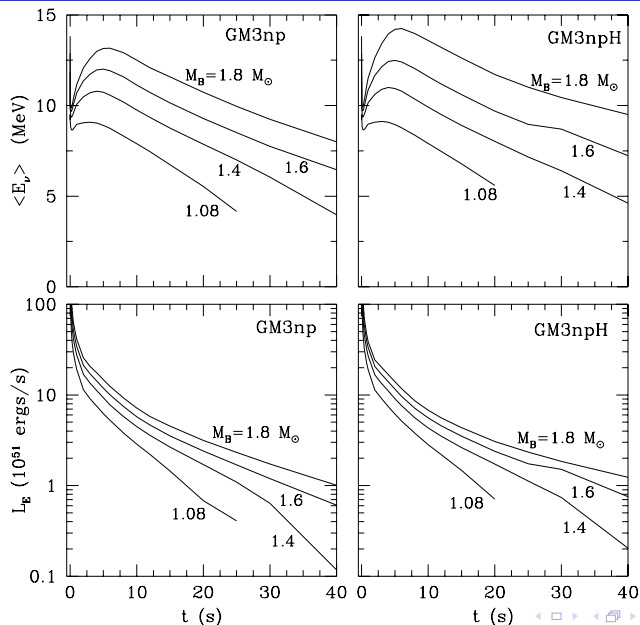


Model Simulations of Proto-Neutron Stars

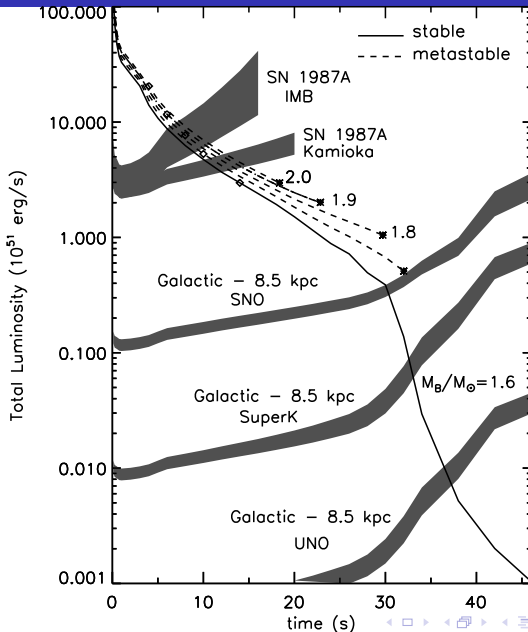


Pons et al. 2000

Model Simulations of Proto-Neutron Stars



Model Signal From Proto-Neutron Stars

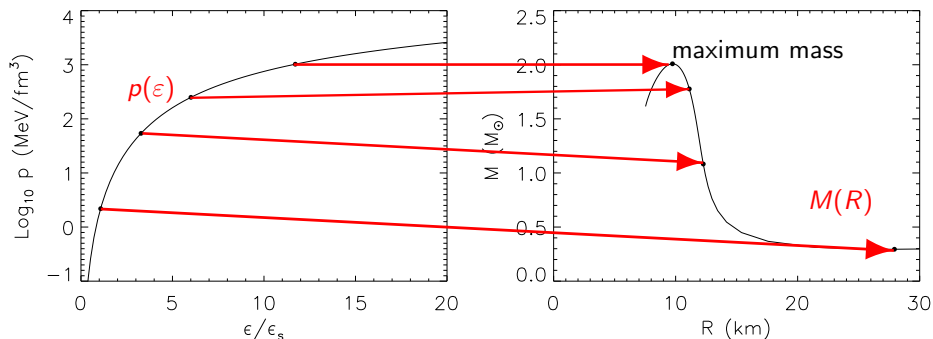


Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^2} \frac{(m + 4\pi pr^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi \frac{\epsilon}{c^2} r^2$$



Schematic Nucleonic Energy Density

n : number density; x : proton fraction; T : temperature

$n_s \simeq 0.16 \pm 0.01 \text{ fm}^{-3}$: nuclear saturation density

$B \simeq -16 \pm 1 \text{ MeV}$: saturation binding energy

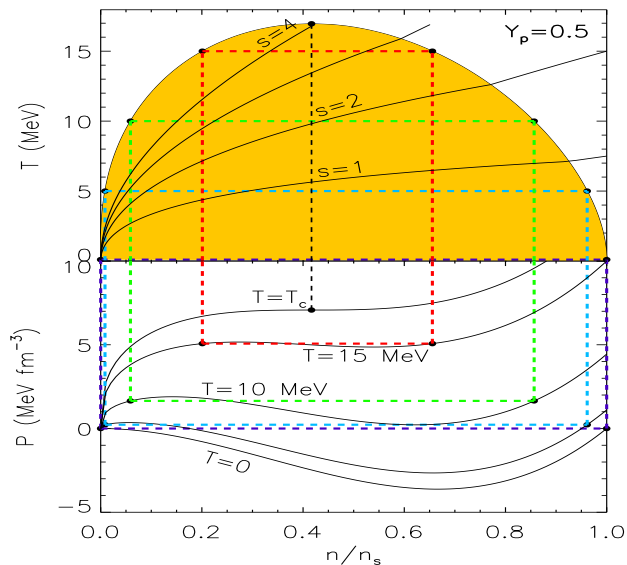
$K \simeq 220 \pm 15 \text{ MeV}$: incompressibility parameter

$S_v \simeq 30 \pm 6 \text{ MeV}$: bulk symmetry parameter

$a \simeq 0.065 \pm 0.010 \text{ MeV}^{-1}$: bulk level density parameter

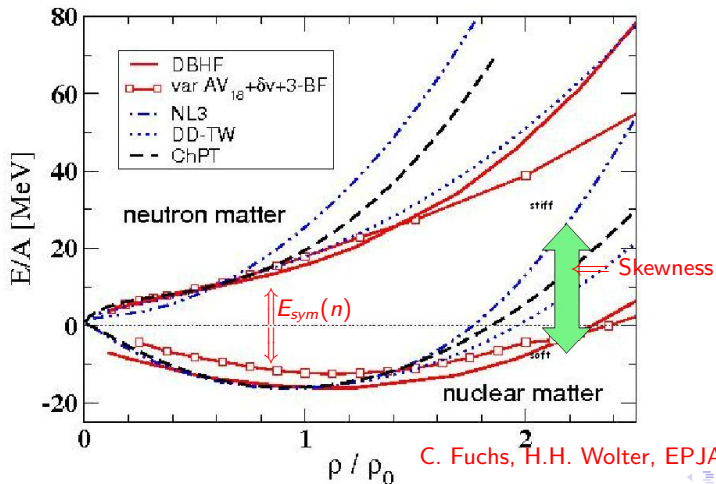
$$\begin{aligned}\epsilon(n, x, T) &= n \left[B + \frac{K}{18} \left(1 - \frac{n}{n_s} \right)^2 + S_v \frac{n}{n_s} (1 - 2x)^2 + a \left(\frac{n_s}{n} \right)^{2/3} T^2 \right] \\ P &= n^2 \frac{\partial(\epsilon/n)}{\partial n} = \frac{n^2}{n_s} \left[\frac{K}{9} \left(\frac{n}{n_s} - 1 \right) + S_v (1 - 2x)^2 \right] + \frac{2an}{3} \left(\frac{n_s}{n} \right)^{2/3} T^2 \\ \mu_n &= \frac{\partial \epsilon}{\partial n} - \frac{x}{n} \frac{\partial \epsilon}{\partial x} \\ &= B + \frac{K}{18} \left(1 - \frac{n}{n_s} \right) \left(1 - 3 \frac{n}{n_s} \right) + 2S_v \frac{n}{n_s} (1 - 4x^2) - \frac{a}{3} \left(\frac{n_s}{n} \right)^{2/3} T^2 \\ \hat{\mu} &= -\frac{1}{n} \frac{\partial \epsilon}{\partial x} = \mu_n - \mu_p = 4S_v \frac{n}{n_s} (1 - 2x) \\ s &= \frac{1}{n} \frac{\partial \epsilon}{\partial T} = 2a \left(\frac{n_s}{n} \right)^{2/3} T\end{aligned}$$

Phase Instabilities



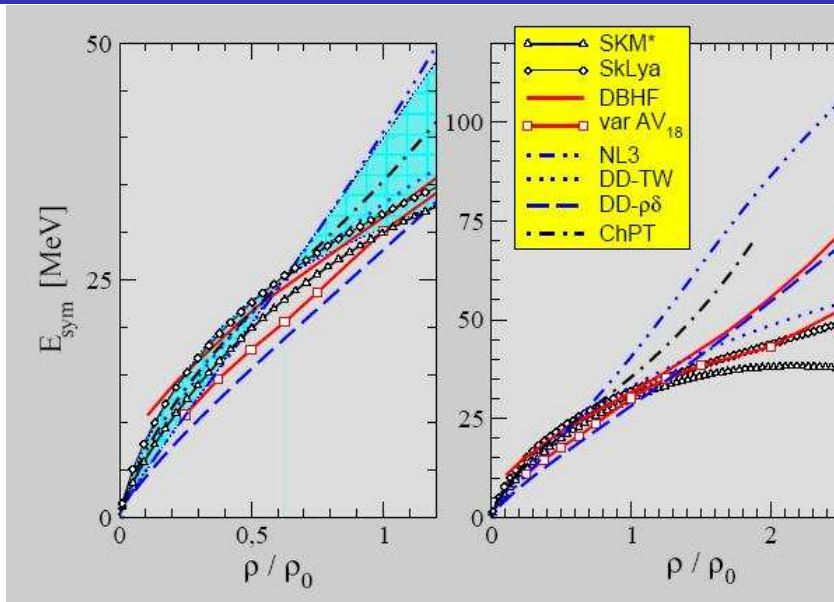
The Uncertain Nuclear Force

The density dependence of $E_{sym}(n) = E_{neutrons}(n) - E_{symmetric}(n)$ is crucial but poorly constrained. The skewness, $\partial^3 E / \partial n^3$, is also uncertain.



C. Fuchs, H.H. Wolter, EPJA 30(2006) 5

The Uncertain $E_{\text{sym}}(n)$

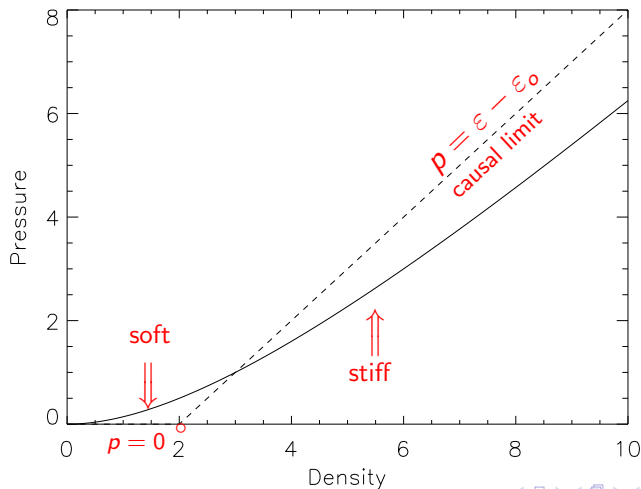


C. Fuchs, H.H. Wolter, EPJA 30(2006) 5



Extreme Properties of Neutron Stars

- ▶ The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ϵ_0 is the only
EOS parameter

The TOV
solutions scale
with ϵ_0

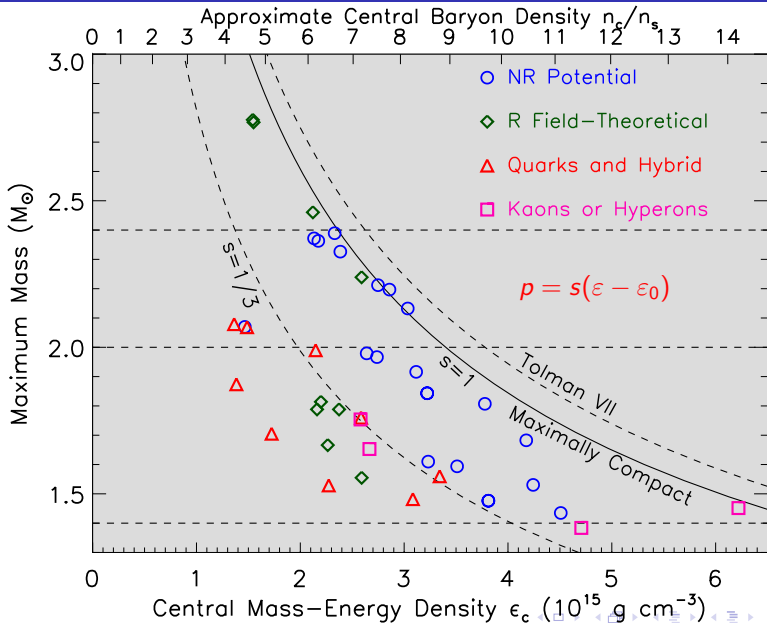
Extreme Properties of Neutron Stars

- ▶ $M_{max} = 4.1 (\epsilon_s/\epsilon_0)^{1/2} M_\odot$ (Rhoades & Ruffini 1974)
- ▶ $M_{B,max} = 5.41 (m_B c^2/\mu_o)(\epsilon_s/\epsilon_0)^{1/2} M_\odot$
- ▶ $R_{min} = 2.82 GM/c^2 = 4.3 (M/M_\odot) \text{ km}$
- ▶ $\mu_{B,max} = 2.09 \text{ GeV}$
- ▶ $\epsilon_{c,max} = 3.034 \epsilon_0 \simeq 51 (M_\odot/M_{largest})^2 \epsilon_s$
- ▶ $p_{c,max} = 2.034 \epsilon_0 \simeq 34 (M_\odot/M_{largest})^2 \epsilon_s$
- ▶ $n_{B,max} \simeq 38 (M_\odot/M_{largest})^2 n_s$
- ▶ $BE_{max} = 0.34 M$
- ▶ $P_{min} = 0.74 (M_\odot/M_{sph})^{1/2} (R_{sph}/10 \text{ km})^{3/2} \text{ ms}$
 $= 0.20 (M_{sph,max}/M_\odot) \text{ ms}$

A phenomenological limit for hadronic matter (Lattimer & Prakash 2004)

- ▶ $P_{min} \simeq 1.00 (M_\odot/M_{sph})^{1/2} (R_{sph}/10 \text{ km})^{3/2} \text{ ms}$
 $= 0.27 (M_{sph,max}/M_\odot) \text{ ms}$

Maximum Energy Density in Neutron Stars



Mass-Radius Diagram and Theoretical Constraints

GR:

$$R > 2GM/c^2$$

$P < \infty$:

$$R > (9/4)GM/c^2$$

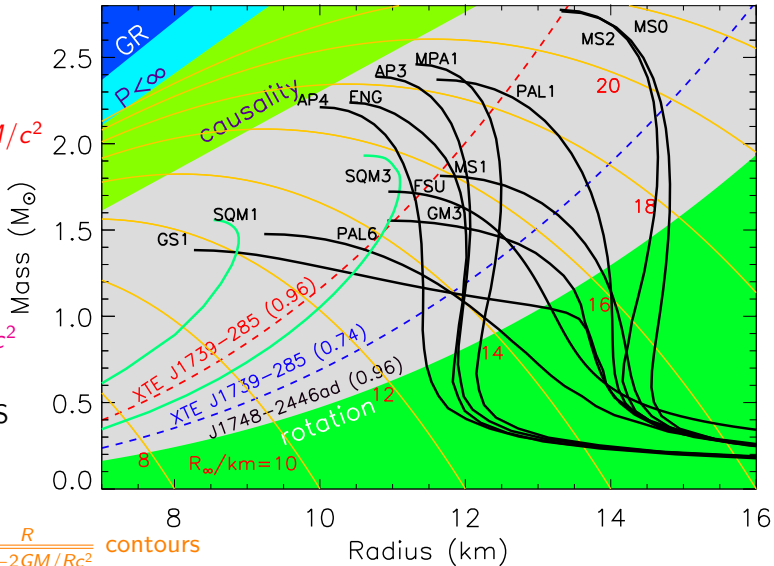
causality:

$$R \gtrsim 2.9GM/c^2$$

— normal NS

— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours



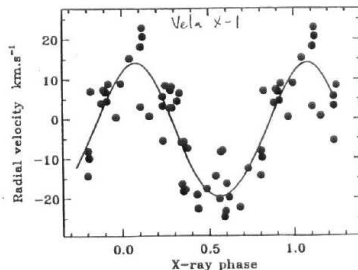
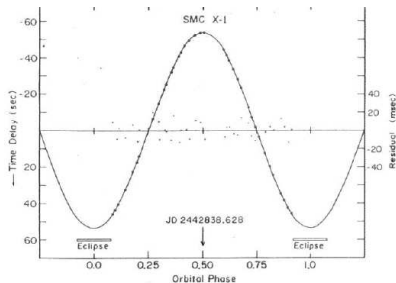
Mass Measurements In X-Ray Binaries

Mass function

$$\begin{aligned}f(M_1) &= \frac{P(v_2 \sin i)^3}{2\pi G} \\ &= \frac{(M_1 \sin i)^3}{(M_1 + M_2)^2} \\ &> M_1\end{aligned}$$

$$\begin{aligned}f(M_2) &= \frac{P(v_1 \sin i)^3}{2\pi G} \\ &= \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2} \\ &> M_2\end{aligned}$$

In an X-ray binary, v_{optical} has the largest uncertainties. In some cases, $\sin i \sim 1$ if eclipses are observed. If eclipses are not observed, limits to i can be made based on the estimated radius of the optical star.



Pulsar Mass Measurements

Mass function for pulsar precisely obtained.

It is also possible in some cases to obtain the rate of periastron advance and the Einstein gravitational redshift + time dilation term:

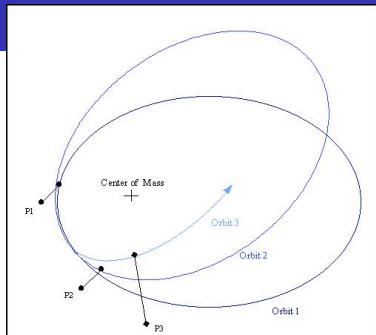
$$\dot{\omega} = 3(2\pi/P)^{5/3} (GM/c^2)^{2/3} / (1 - e^2)$$

$$\gamma = (P/2\pi)^{1/3} e M_2 (2M_2 + M_1) (G/M^2 c^2)^{2/3}$$

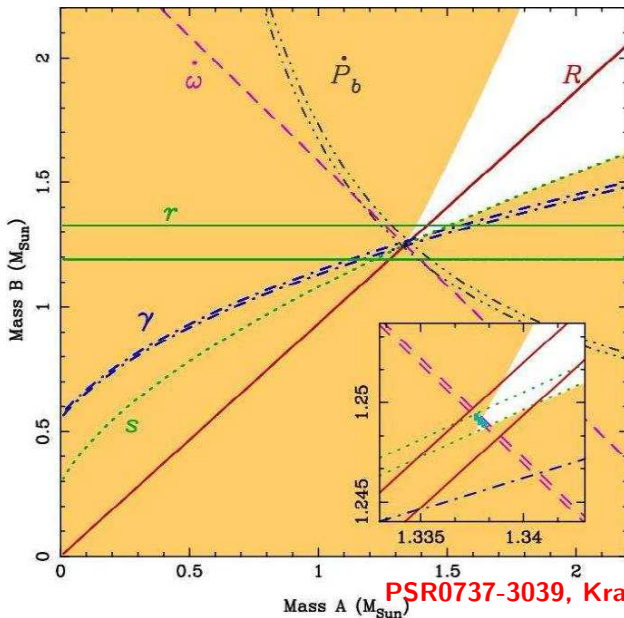
Gravitational radiation leads to orbit decay:

$$\dot{P} = -\frac{192\pi}{5c^5} \left(\frac{2\pi G}{P}\right)^{5/3} (1 - e^2)^{-7/2} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \frac{M_1 M_2}{M^{1/2}}$$

In some cases, can constrain Shapiro time delay, r is magnitude and $s = \sin i$ is shape parameter.

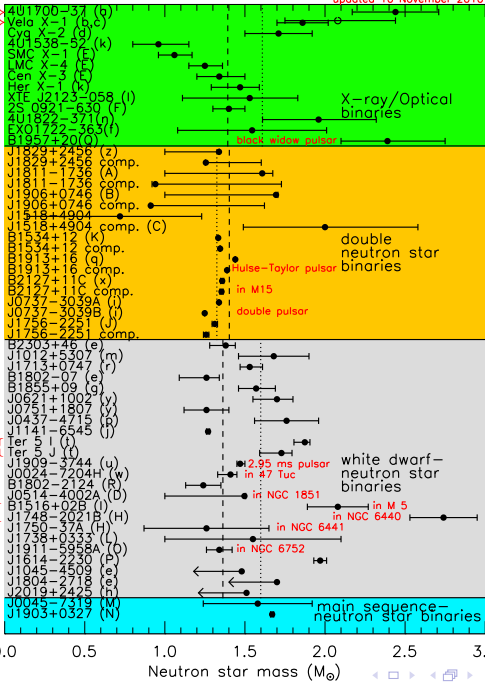


The Double Pulsar Binary



PSR0737-3039, Kramer et al. (2007)

Black hole? \Rightarrow
Firm lower mass limit? \Rightarrow



$M > 1.68 M_{\odot}$ {
95% confidence

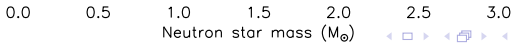
Freire et al. 2007 {

Although simple average mass of w.d. companions is $0.27 M_{\odot}$ larger, weighted average is $0.08 M_{\odot}$ smaller

w.d. companion? statistics?

Demorest et al. 2010

Champion et al. 2008



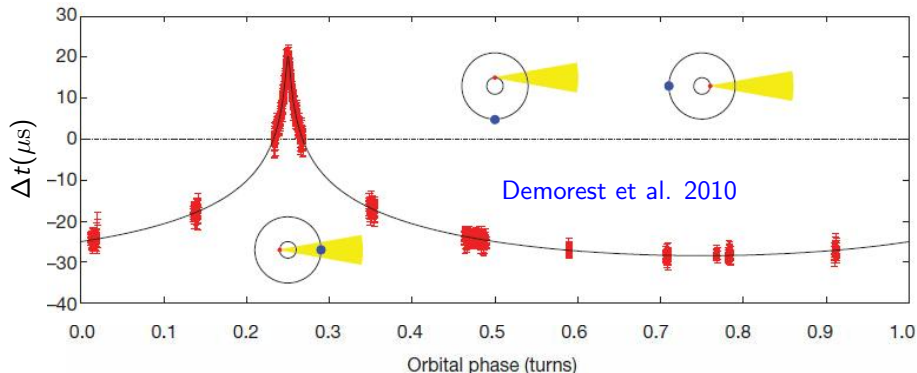
PSR J1614-2230

A 3.15 ms pulsar in an 8.69d orbit with an $0.5 M_{\odot}$ white dwarf companion.

Shapiro delay yields edge-on inclination: $\sin i = 0.99984$

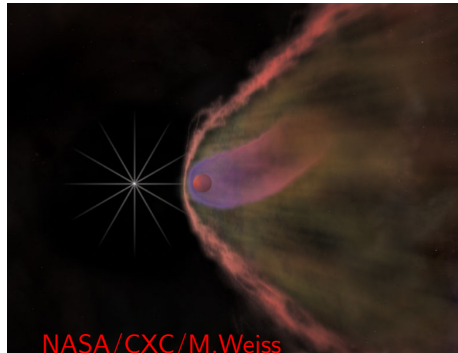
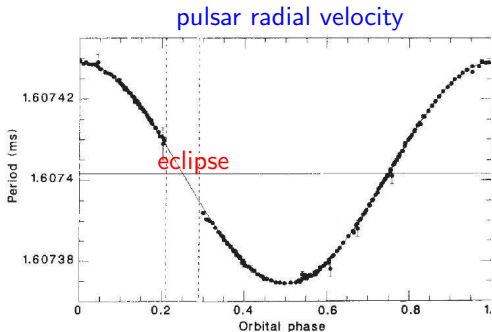
Pulsar mass is $1.97 \pm 0.04 M_{\odot}$

Distance > 1 kpc, $B \simeq \times 10^8$ G



Black Widow Pulsar PSR B1957+20

1.6ms pulsar in circular 9.17h orbit with a $M_c \sim 0.03 M_\odot$ companion. Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the companion or its Roche lobe. It is believed the companion is ablated by the pulsar leading to mass loss and an eclipsing plasma cloud. Companion nearly fills its Roche lobe. Ablation by pulsar leads to eventual disappearance of companion. The optical light curve does not represent the center of mass of the companion, but the motion of its irradiated hot spot.



Implications of Maximum Masses

$$M_{max} > 2 M_{\odot}$$

- ▶ Upper limits to energy density, pressure and baryon density:
 - ▶ $\epsilon < 13.1\epsilon_s$
 - ▶ $p < 8.8\epsilon_s$
 - ▶ $n_B < 9.8n_s$
- ▶ Lower limit to spin period:
 $P > 0.56$ ms
- ▶ Lower limit to neutron star radius:
 $R > 8.5$ km
- ▶ Upper limits to energy density, pressure and baryon density in the case of a quark matter core:
 - ▶ $\epsilon < 7.7\epsilon_s$
 - ▶ $p < 2.0\epsilon_s$
 - ▶ $n_B < 6.9n_s$

$$M_{max} > 2.4 M_{\odot}$$

- ▶ Upper limits to energy density, pressure, baryon density:
 - ▶ $\epsilon < 8.9\epsilon_s$
 - ▶ $p < 5.9\epsilon_s$
 - ▶ $n_B < 6.6n_s$
- ▶ Lower limit to spin period:
 $P > 0.68$ ms
- ▶ Lower limit to neutron star radius:
 $R > 10.4$ km
- ▶ Upper limits to energy density, pressure, baryon density in the case of a quark matter core:
 - ▶ $\epsilon < 5.2\epsilon_s$
 - ▶ $p < 1.4\epsilon_s$
 - ▶ $n_B < 4.6n_s$

Neutron Star Matter Pressure and the Radius

$$p \approx Kn^\gamma$$

$$\gamma = d \ln p / d \ln n \sim 2$$

$$R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)}$$

$$R \propto p_f^{1/2} n_f^{-1} M^0$$

$$(1 < n_f/n_s < 2)$$

Wide variation:

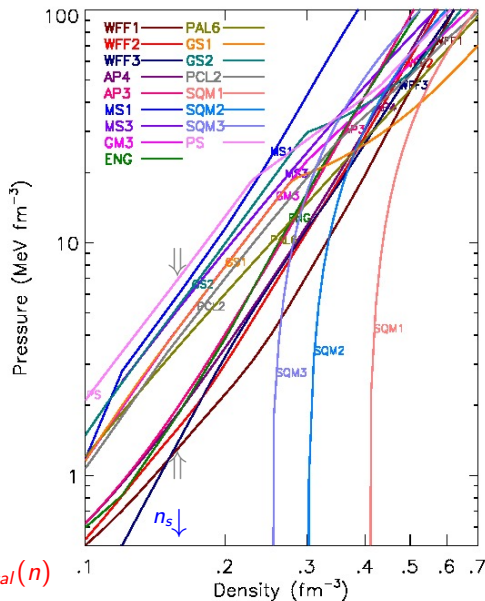
$$1.2 < \frac{p(n_s)}{\text{MeV fm}^{-3}} < 7$$

GR phenomenological result
(Lattimer & Prakash 2001)

$$R \propto p_f^{1/4} n_f^{-1/2}$$

$$p_f = n^2 dE_{\text{sym}}/dn$$

$$E_{\text{sym}}(n) = E_{\text{neutron}}(n) - E_{\text{symmetrical}}(n)$$

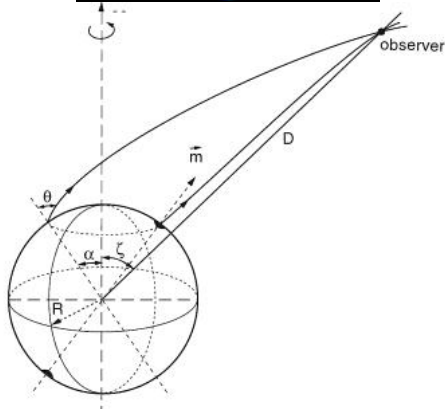


Radiation Radius

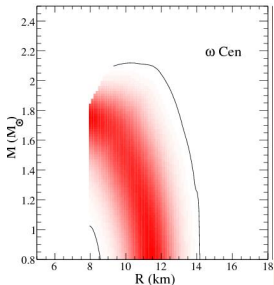
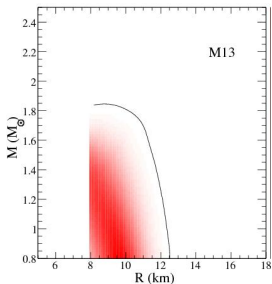
- ▶ The measurement of flux and temperature yields an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{d} = \frac{R}{d} \frac{1}{\sqrt{1 - 2GM/Rc^2}}$$

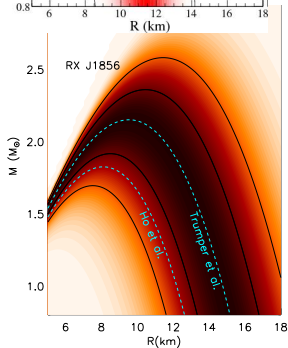
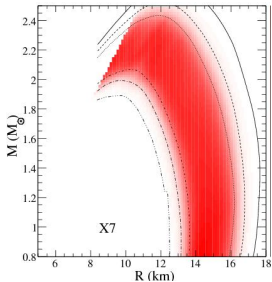
- ▶ Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- ▶ Best chances for accurate radii:
 - ▶ Nearby isolated neutron stars (parallax measurable)
 - ▶ Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmospheres)



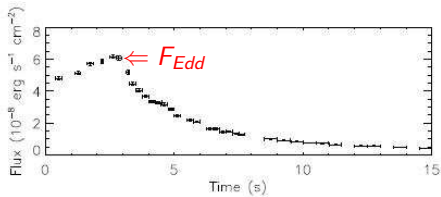
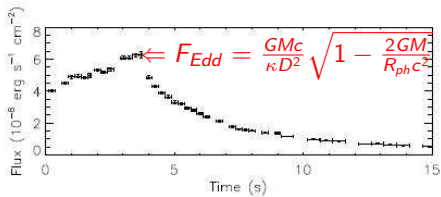
Inferred M-R Probability Estimates from Thermal Sources



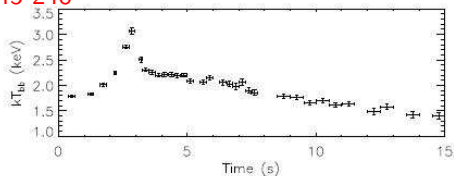
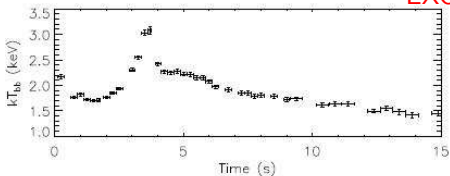
Steiner, Lattimer & Brown 2010



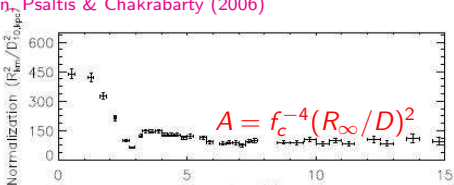
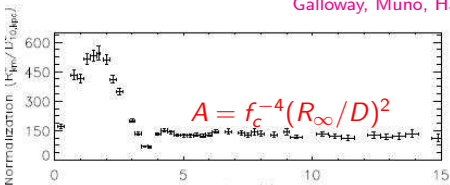
Photospheric Radius Expansion X-Ray Bursts



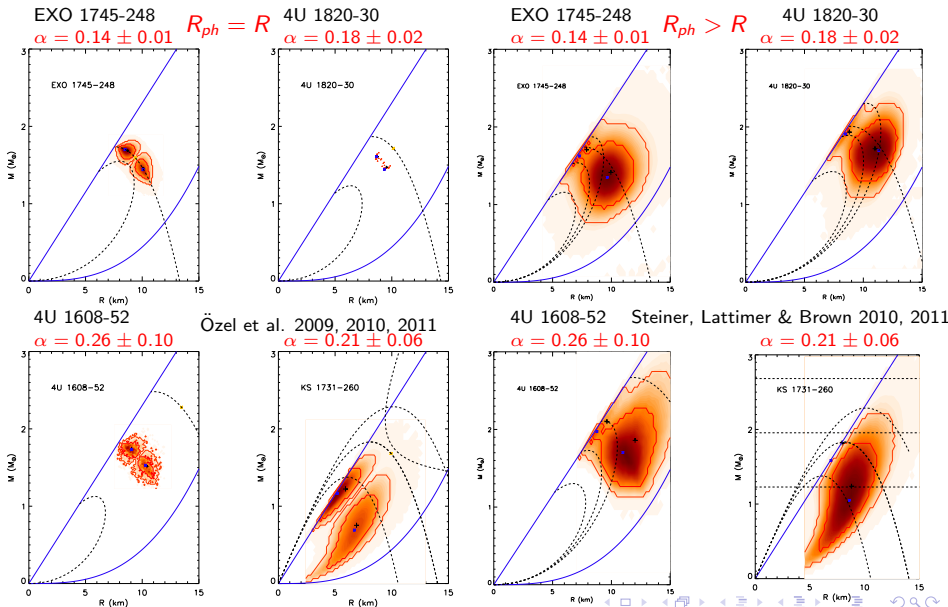
EXO 1745-248



Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)

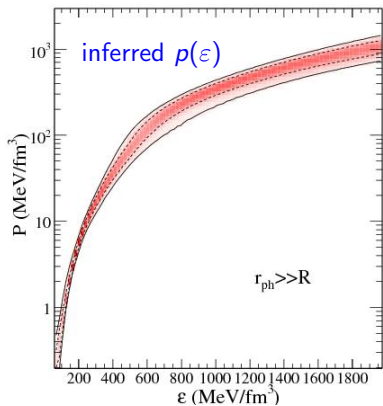


$M - R$ Probability Estimates from PRE Bursts

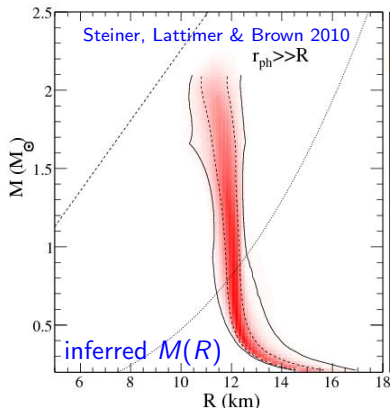


Bayesian TOV Inversion

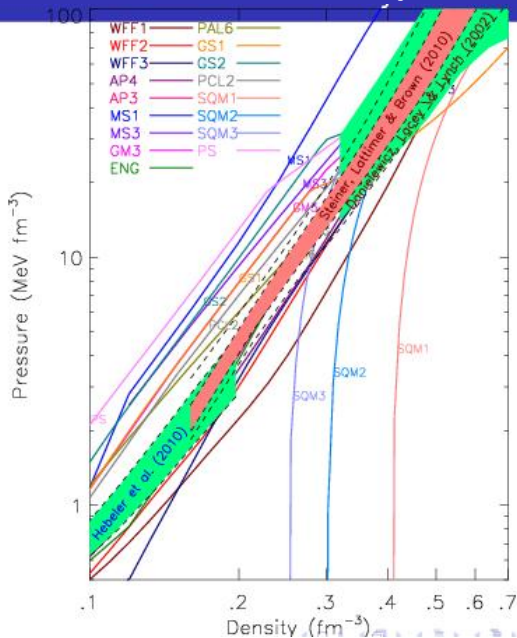
- ▶ $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- ▶ $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- ▶ $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1 ; $\varepsilon > \varepsilon_2$: Polytropic EOS with n_2



- ▶ EOS parameters ($K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$) uniformly distributed
- ▶ M and R probability distributions for 7 neutron stars treated equally.



Consistency with Neutron Matter and Heavy-Ion Collisions



Urca Processes

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.

$$n \rightarrow p + e^- + \nu_e,$$

$$p \rightarrow n + e^+ + \bar{\nu}_e$$

Energy conservation guaranteed by beta equilibrium

$$\mu_n - \mu_p = \mu_e$$

Momentum conservation requires

$$|k_{Fn}| \leq |k_{Fp}| + |k_{Fe}|.$$

Charge neutrality requires $k_{Fp} = k_{Fe}$, therefore $|k_{Fp}| \geq 2|k_{Fn}|$.

Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \geq x_{DU} = 1/9$.

With muons

$$(n > 2n_s), x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$$

If $x < x_{DU}$, bystander nucleons needed: modified Urca process.

$$(n, p) + n \rightarrow (n, p) + p + e^- + \nu_e,$$

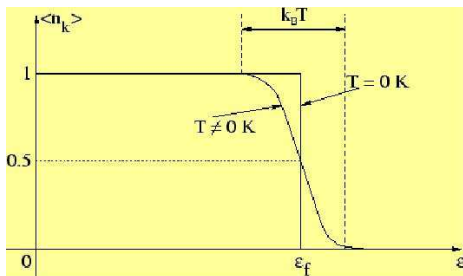
$$(n, p) + p \rightarrow (n, p) + n + e^+ + \bar{\nu}_e$$

Neutrino emissivities:

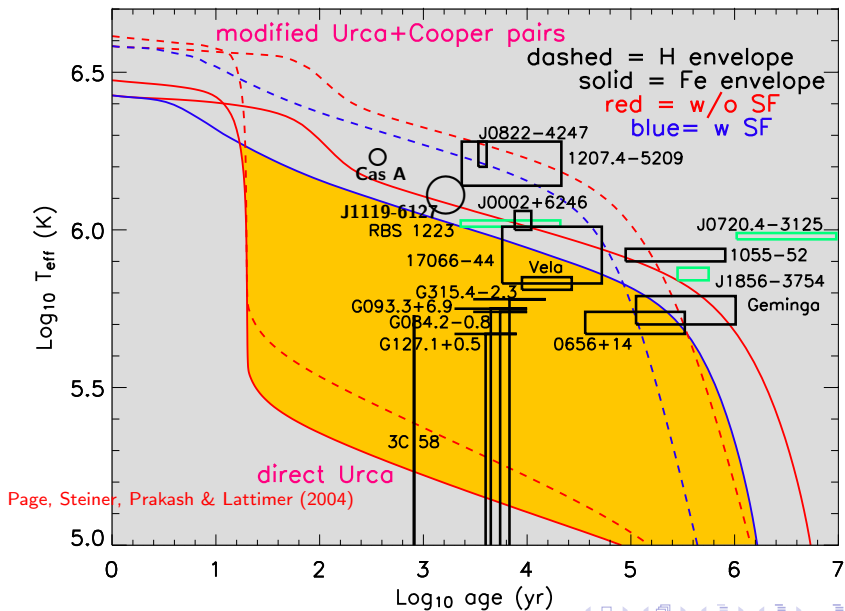
$$\dot{\epsilon}_{MU} \simeq (T/\mu_n)^2 \dot{\epsilon}_{DU} \sim 10^{-6} \dot{\epsilon}_{DU}.$$

Beta equilibrium composition:

$$\begin{aligned} x_\beta &\simeq (3\pi^2 n)^{-1} (4E_{sym}/\hbar c)^3 \\ &\simeq 0.04 (n/n_s)^{0.5-2}. \end{aligned}$$



Neutron Star Cooling



Transitory Rapid Cooling

MU emissivity: $\dot{\epsilon}_{MU} \propto T^8$

PBF emissivity ($f \sim 10$):

$\dot{\epsilon}_{PBF} \propto F(T) T^7 \propto T^8 \simeq f \dot{\epsilon}_{MU}$

Specific heat: $C_V \propto T$

Neutrino dominated cooling:

$C_V dT/dt = -L_\nu$

$\Rightarrow T \propto (t/\tau)^{-1/6}$

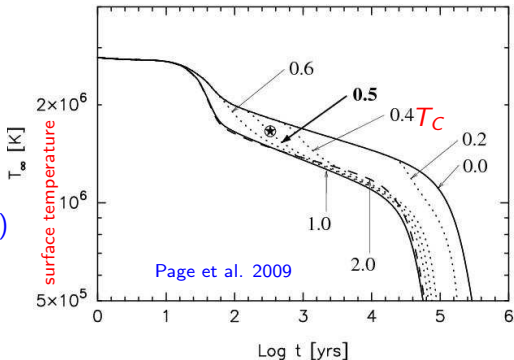
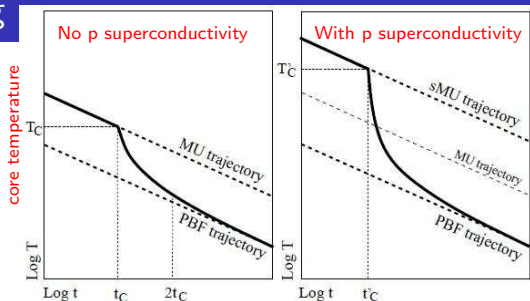
$\tau_{PBF} = \tau_{MU}/f$

$(d \ln T / d \ln t)_{transitory}$

$\simeq (1 - 10)(d \ln T / d \ln t)_{MU}$

$\simeq (1 - 25)(d \ln T / d \ln t)_{MU}$ (p SC)

Very sensitive to n^1S_0 critical temperature (T_C) and existence of proton superconductivity



Cas A

Remnant of Type IIb
(gravitational collapse,
no H envelope) SN in
1680 (Flamsteed).

3.4 kpc distance

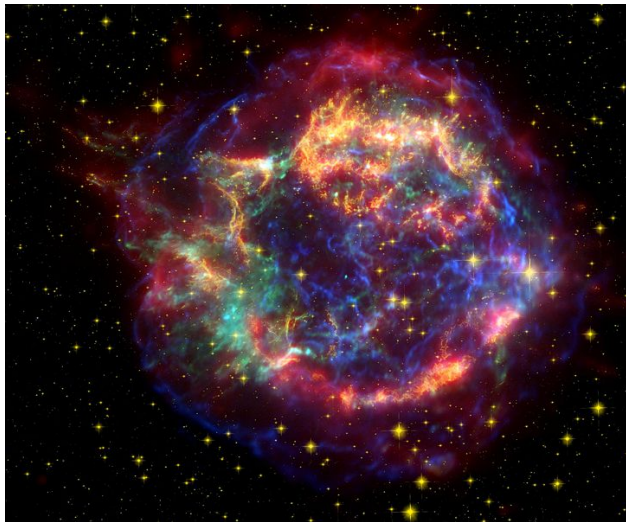
3.1 pc diameter

Strongest radio source
outside solar system,
discovered in 1947.

X-ray source detected
(Aerobee flight, 1965)

X-ray point source
detected
(Chandra, 1999)

1 of 2 known CO-rich
SNR (massive
progenitor and neutron star?)



Spitzer, Hubble, Chandra



Cas A Superfluidity

X-ray spectrum indicates thin C atmosphere,
 $T_e \sim 1.7 \times 10^8$ K
(Ho & Heinke 2009)

10 years of X-ray data show cooling at the rate
 $\frac{d \ln T_e}{d \ln t} = -1.23 \pm 0.14$
(Heinke & Ho 2010)

Modified Urca:
 $\left(\frac{d \ln T_e}{d \ln t}\right)_{MU} \simeq -0.08$

We infer that
 $T_C \simeq 5 \pm 1 \times 10^8$ K
 $T_C \propto (t_c L / C_V)^{-1/6}$

