NEUTRINO AND NUCLEAR ASTROPHYSICS

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# Neutrino Quantum Kinetic Equations - II

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Based on 1309.2628, 1406.5558, 1406.6724, and references therein

# Outline

Lectures

(I)

- Motivation: neutrinos and the cosmos
- Neutrinos in hot and dense media
- Structure of QKEs from quantum field theory

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(II)

- Anatomy of the QKEs
  - Coherent evolution: flavor *and* spin
  - Inelastic collisions
- Comparison to other approaches & future challenges

Talk by A. Vlasenko

- Neutrino-antineutrino transformation in astrophysical environments

# Structure of the QKEs

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

$$iDF = [H, F] + iC$$
$$i\bar{D}\bar{F} = [\bar{H}, \bar{F}] + i\bar{C}$$

Derivative along ν world line: drift & force term

"Vlasov"

Coherent evolution: vacuum mass & forward scattering (refractive potential)

"MSW"

Inelastic collisions

"Boltzmann"

- F, H, C: $2n_f \times 2n_f$ matrices, all components coupled in general
- D, H, C are functionals of F, $\bar{F}$: non-linear system

# Interlude on kinematics

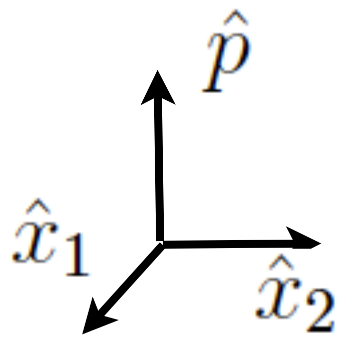
- For ultra-relativistic ν's of 3-momentum **p**, express all Lorentz tensors in terms of following basis:

$$\boxed{\begin{aligned} n^\mu(p) &= (1, \hat{p}) \\ \bar{n}^\mu(p) &= (1, -\hat{p}) \\ x_{1,2}&(p) \end{aligned}}$$

light-like

light-like

transverse




$$n \cdot n = \bar{n} \cdot \bar{n} = 0 \qquad n \cdot \bar{n} = 2$$

$$n \cdot x_i = \bar{n} \cdot x_i = 0 \qquad x_i \cdot x_j = -\delta_{ij}$$

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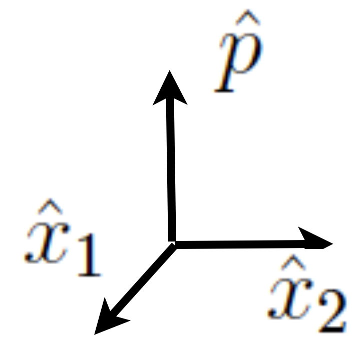


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$$n \cdot x_i = \bar{n} \cdot x_i = 0 \qquad x_i \cdot x_j = -\delta_{ij}$$

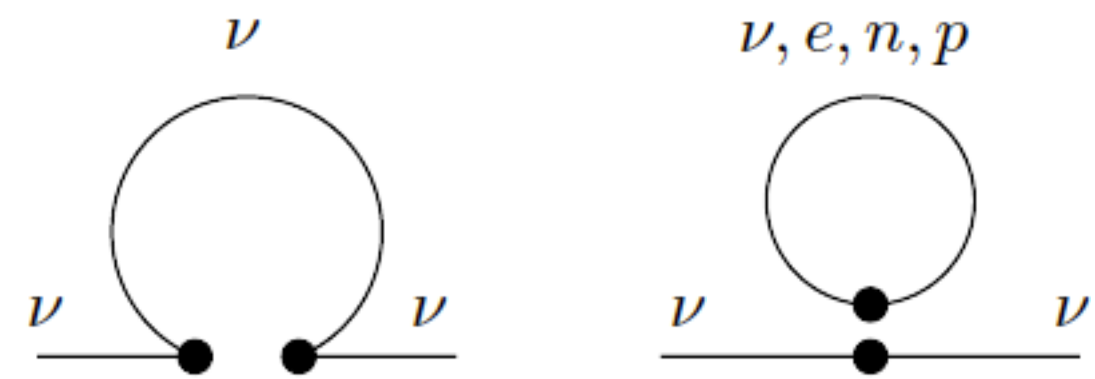
- Four-vector components along basis vectors:

$$V^\mu \quad \rightarrow \quad V^\kappa \equiv n \cdot V \qquad V^i \equiv x_i \cdot V$$

$$\partial^\kappa \equiv n \cdot \partial \qquad \partial^i \equiv x_i \cdot \partial$$

# Interlude on potentials

- Neutrino self-energy diagrams → in-medium *4-vector* potential (time- *and* space-like components in non-isotropic medium)



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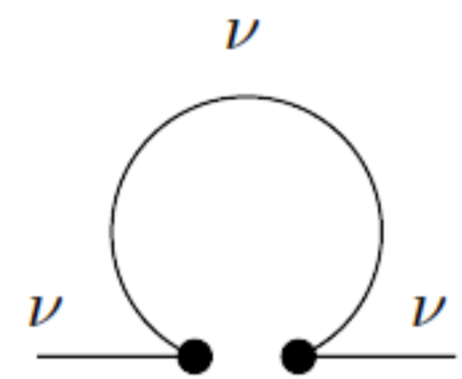


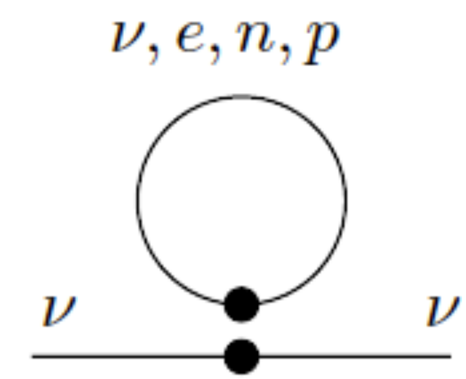


- Computed from neutrino interactions in the Standard Model. Ex: neutrino-matter interaction at low-energy can be put in the form

$$\boxed{\mathcal{L}_{\nu\psi} = -G_\psi \; \bar{\nu}\gamma_\mu P_L Y_\psi \nu \;\; \bar{\psi}\Gamma^\mu_\psi \psi}$$

$$G_\psi = n_\psi \times G_F \sim g^2/M_W^2$$
$$P_L = (1-\gamma_5)/2.$$
$$\Gamma^\mu_\psi = (g_V)_\psi \gamma^\mu - (g_A)_\psi \gamma^\mu \gamma^5.$$

$$\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \quad Y_\psi = \begin{pmatrix} Y_{e\psi} & 0 & 0 \\ 0 & Y_{\mu\psi} & 0 \\ 0 & 0 & Y_{\tau\psi} \end{pmatrix}$$

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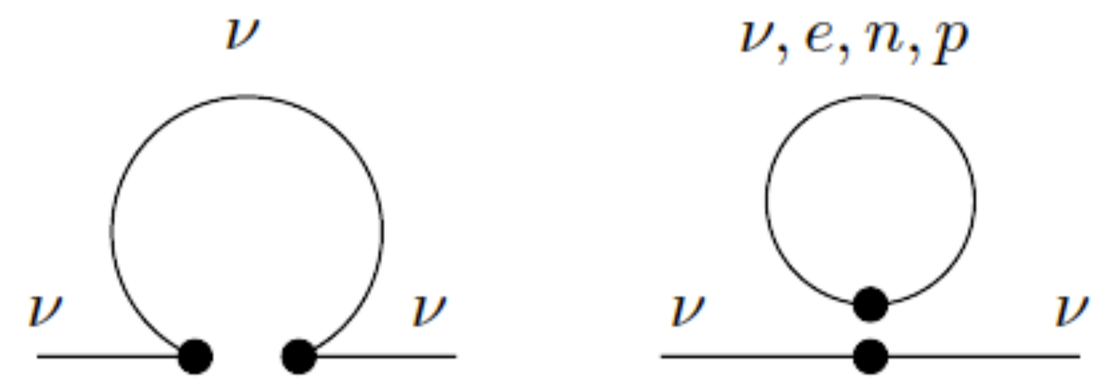


- $2n_f$ x $2n_f$ matrix structure:

$$\Sigma^{\mu}(x) = \begin{pmatrix} \Sigma_R^{\mu} & 0 \\ 0 & \Sigma_L^{\mu} \end{pmatrix}$$

- Induced interaction

$$\mathcal{L}_{\text{int}} = \quad -\bar{\nu}_L \not{\Sigma}_R \nu_L - \bar{\nu}_R \not{\Sigma}_L \nu_R + \text{h.c.}$$

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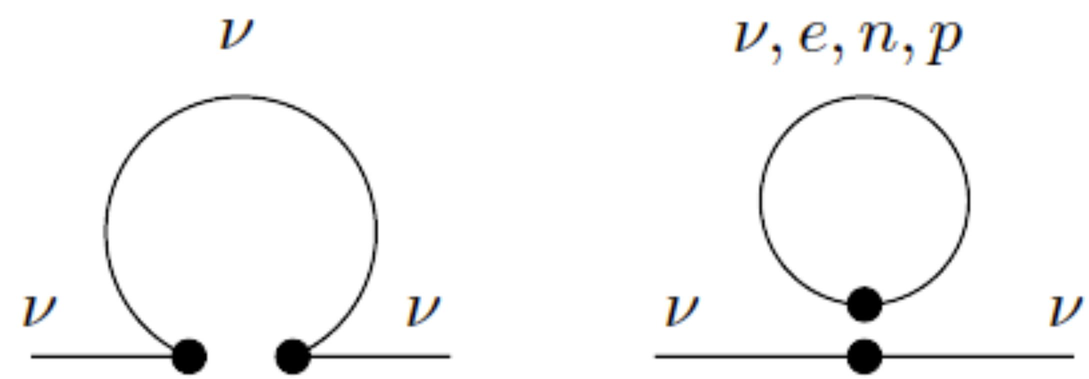


- $2n_f$ x $2n_f$ matrix structure:

Potential for L-handed ν's

$$\Sigma^{\mu}(x) = \begin{pmatrix} \Sigma^{\mu}_{R} & 0 \\ 0 & \Sigma^{\mu}_{L} \end{pmatrix}$$

- Induced interaction

$$\mathcal{L}_{\rm int} = \; -\bar{\nu}_L \not{\Sigma}_R \nu_L - \bar{\nu}_R \not{\Sigma}_L \nu_R + \text{h.c.}$$

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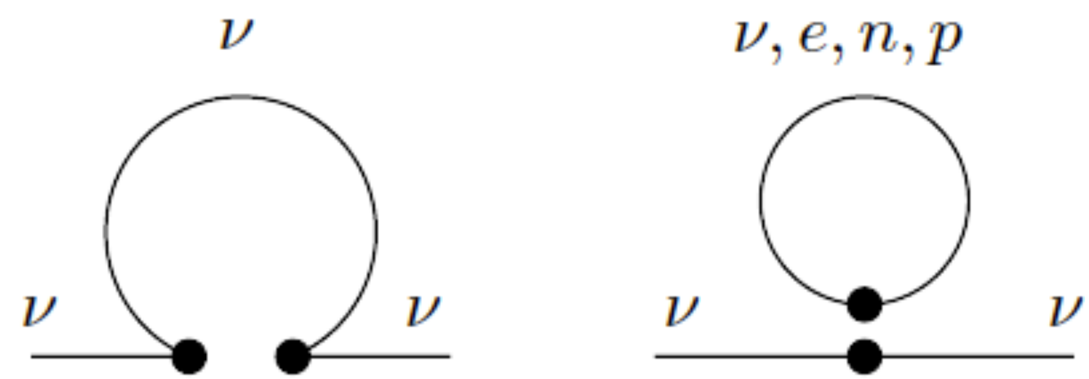


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$$\Sigma^{\mu}(x) = \begin{pmatrix} \Sigma_R^{\mu} & 0 \\ 0 & \Sigma_L^{\mu} \end{pmatrix}$$

Potential for L-handed ν's

Potential for R-handed ν's:

Dirac: $\Sigma_L \propto G_F m^2 \sim O(\epsilon^3)$

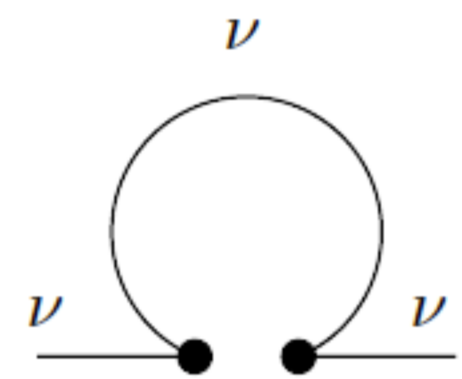
Majorana: $\Sigma_L = -\Sigma_R^T$

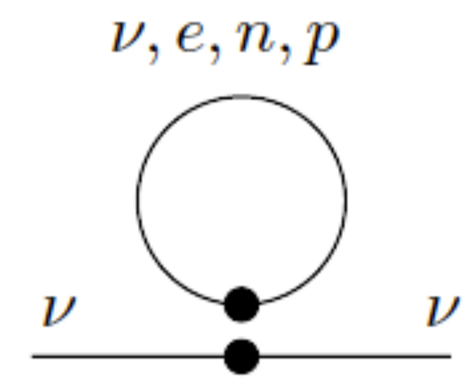
- Induced interaction

$$\mathcal{L}_{\rm int} = \ -\bar{\nu}_L \not{\Sigma}_R \nu_L - \bar{\nu}_R \not{\Sigma}_L \nu_R + {\rm h.c.}$$

# Interlude on potentials

- Neutrino self-energy diagrams → in-medium *4-vector* potential (time- *and* space-like components in non-isotropic medium)




- $2n_f$ x $2n_f$ matrix structure:

$$\Sigma^{\mu}(x) = \begin{pmatrix} \Sigma^{\mu}_{R} & 0 \\ 0 & \Sigma^{\mu}_{L} \end{pmatrix}$$

Potential for L-handed ν's

Potential for R-handed ν's:

Dirac: $\Sigma_L \propto G_F m^2 \sim O(\epsilon^3)$

Majorana: $\Sigma_L = -\Sigma_R^T$

- For a test ν of momentum **p**, get components

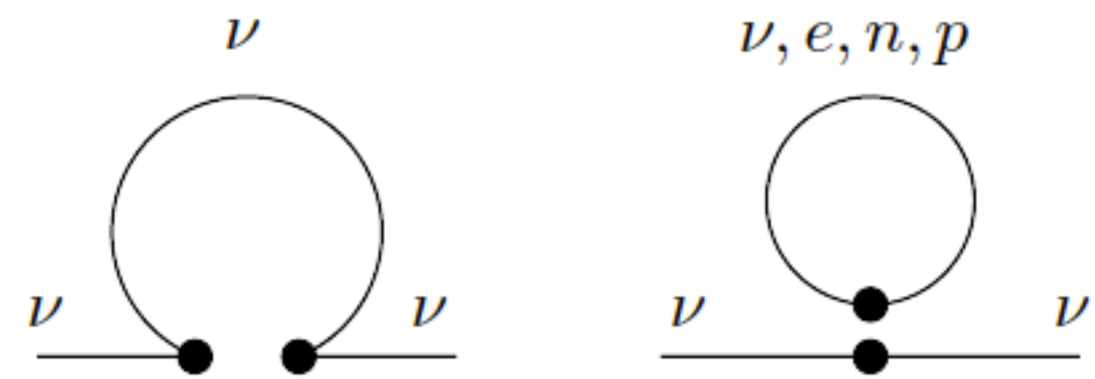
$$\boxed{\begin{aligned} \Sigma^{\kappa} &\equiv n(p) \cdot \Sigma \\ \Sigma^{i} &\equiv x^{i}(p) \cdot \Sigma \end{aligned}}$$

approximately along ν trajectory

approximately transverse to ν trajectory

# Interlude on potentials

- Neutrino self-energy diagrams → in-medium *4-vector* potential (time- *and* space-like components in non-isotropic medium)




- Explicit form of neutrino-induced $\Sigma_R$:

$$\Sigma^\mu_R\Big|_\nu = \sqrt{2} G_F \left( J^\mu_{(\nu)} + \mathbf{1} \left( \mathrm{tr}\ J^\mu_{(\nu)} \right) \right)$$

$$J^\mu_{(\nu)}(x) = \int \frac{d^3q}{(2\pi)^3} \, n^\mu(q) \, \left( f_{LL}(\vec{q}, x) - \bar{f}_{RR}(\vec{q}, x) \right)$$

$$n^\mu(q) = (1, \hat{q})$$

# Anatomy of the QKEs

# Drift & force terms

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

$$iDF = [H, F] + iC$$

$$i\bar{D}\bar{F} = [\bar{H}, \bar{F}] + i\bar{C}$$

Derivative along ν world line: drift & force term

“Vlasov”

# Drift & force terms

$$\boxed{\begin{aligned} DF &= \partial^\kappa F + \frac{1}{2|\vec{p}|}\{\Sigma^i, \partial^i F\} - \frac{1}{2}\left\{\frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial F}{\partial \vec{p}}\right\} \\ \bar{D}\bar{F} &= \partial^\kappa \bar{F} - \frac{1}{2|\vec{p}|}\{\Sigma^i, \partial^i \bar{F}\} + \frac{1}{2}\left\{\frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial \bar{F}}{\partial \vec{p}}\right\} \end{aligned}}$$

- Simple interpretation if one notes that ν(+) and ν̄(-) dispersion relations are:

$$\boxed{\omega_\pm = |\vec{p}| \pm \Sigma^\kappa}$$

$$\Sigma^\kappa \equiv n(p) \cdot \Sigma$$

# Drift & force terms

$$D F \;=\; \partial^\kappa F + \frac{1}{2|\vec{p}|}\left\{\Sigma^i, \partial^i F\right\} - \frac{1}{2}\left\{\frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial F}{\partial \vec{p}}\right\}$$

$$\bar{D} \bar{F} \;=\; \partial^\kappa \bar{F} - \frac{1}{2|\vec{p}|}\left\{\Sigma^i, \partial^i \bar{F}\right\} + \frac{1}{2}\left\{\frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial \bar{F}}{\partial \vec{p}}\right\}$$

- Simple interpretation if one notes that ν(+) and ν̄(-) dispersion relations are:

$$\omega_\pm = |\vec{p}| \pm \Sigma^\kappa$$

$$\Sigma^\kappa \equiv n(p) \cdot \Sigma$$

- Then one finds:

$$D = \partial_t + \frac{1}{2}\{\partial_{\vec{p}}\,\omega_+, \partial_{\vec{x}} \;\; \} - \frac{1}{2}\{\partial_{\vec{x}}\,\omega_+, \partial_{\vec{p}} \;\; \}$$

$$\bar{D} = \partial_t + \frac{1}{2}\{\partial_{\vec{p}}\,\omega_-, \partial_{\vec{x}} \;\; \} - \frac{1}{2}\{\partial_{\vec{x}}\,\omega_-, \partial_{\vec{p}} \;\; \}$$

- Generalization of familiar

$$d_t = \partial_t + \dot{\vec{x}}\;\partial_{\vec{x}} + \dot{\vec{p}}\;\partial_{\vec{p}}$$

$$\dot{\vec{x}} = \partial_{\vec{p}}\,\omega \qquad \dot{\vec{p}} = -\partial_{\vec{x}}\,\omega$$

# Coherent evolution

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

$$iDF = [H, F] + iC$$

$$i\bar{D}\bar{F} = [\bar{H}, \bar{F}] + i\bar{C}$$

Coherent evolution: vacuum mass & forward scattering (refractive potential)

"MSW"

# Coherent evolution

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

$$\boxed{\begin{aligned} iDF &= [H, F] + iC \\ i\bar{D}\bar{F} &= [\bar{H}, \bar{F}] + i\bar{C} \end{aligned}}$$

- Often written in the equivalent form of a Schrodinger-like equation for "ν flavor wave-function"

$$\boxed{i\partial_t \psi \;=\; H\psi}$$

$$\psi = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix}$$

- Mapping of the two approaches: off-diagonal entries in $f_{LL}$ encode information about relative QM phases

$$\boxed{f_{LL}^{\alpha\beta} = \psi_\alpha \psi_\beta^*}$$

- Not clear how to include inelastic collisions in wave-function approach

# Coherent evolution

- Controlled by $2n_f$ x $2n_f$ Hamiltonian-like operators

$$\boxed{H = \begin{pmatrix} H_R & H_m \\ H_m^\dagger & H_L \end{pmatrix} \qquad \bar{H} = \begin{pmatrix} \bar{H}_R & H_m \\ H_m^\dagger & \bar{H}_L \end{pmatrix}}$$

$$\begin{aligned} \overline{H}_R &= \Sigma_R^\kappa \pm \frac{1}{2|\vec{p}|}\left(m^\dagger m - \epsilon^{ij}\partial^i \Sigma_R^j + 4\Sigma_R^+ \Sigma_R^-\right) \\ \overline{H}_L &= \Sigma_L^\kappa \pm \frac{1}{2|\vec{p}|}\left(m m^\dagger + \epsilon^{ij}\partial^i \Sigma_L^j + 4\Sigma_L^- \Sigma_L^+\right) \end{aligned}$$

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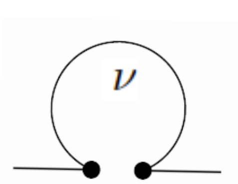
$$\overline{H}_R = \Sigma_R^\kappa \pm \frac{1}{2|\vec{p}|}\left(m^\dagger m - \epsilon^{ij}\partial^i\Sigma_R^j + 4\Sigma_R^+\Sigma_R^-\right)$$

$$\overline{H}_L = \Sigma_L^\kappa \pm \frac{1}{2|\vec{p}|}\left(m m^\dagger + \epsilon^{ij}\partial^i\Sigma_L^j + 4\Sigma_L^-\Sigma_L^+\right)$$

Standard vacuum mass
term + medium refraction
(included in all analyses)




$$\longrightarrow \quad \Sigma_R^\kappa(x) = \sqrt{2}G_F \int \frac{d^3q}{(2\pi)^3}\, n(p)\cdot n(q)\, \left(f_{LL}(\vec{q},x) - \bar{f}_{RR}(\vec{q},x)\right)$$

$1 - \cos\theta_{\hat{p}\hat{q}}$

# Coherent evolution

- Controlled by $2n_f \times 2n_f$ Hamiltonian-like operators

$$H = \begin{pmatrix} H_R & H_m \\ H_m^\dagger & H_L \end{pmatrix} \qquad \bar{H} = \begin{pmatrix} \bar{H}_R & H_m \\ H_m^\dagger & \bar{H}_L \end{pmatrix}$$

$$\begin{aligned} \overline{H}_R &= \Sigma_R^\kappa \pm \frac{1}{2|\vec{p}|}\left(m^\dagger m - \epsilon^{ij}\partial^i\Sigma_R^j + 4\Sigma_R^+\Sigma_R^-\right) \\ \overline{H}_L &= \Sigma_L^\kappa \pm \frac{1}{2|\vec{p}|}\left(m m^\dagger + \epsilon^{ij}\partial^i\Sigma_L^j + 4\Sigma_L^-\Sigma_L^+\right) \end{aligned}$$

Standard vacuum mass term + medium refraction (included in all analyses)

Additional $O(\varepsilon^2)$ terms if potential has space-like components

$$\Sigma_{L,R}^\pm \equiv 1/2\,(\Sigma_{L,R}^1 \pm i\Sigma_{L,R}^2)$$

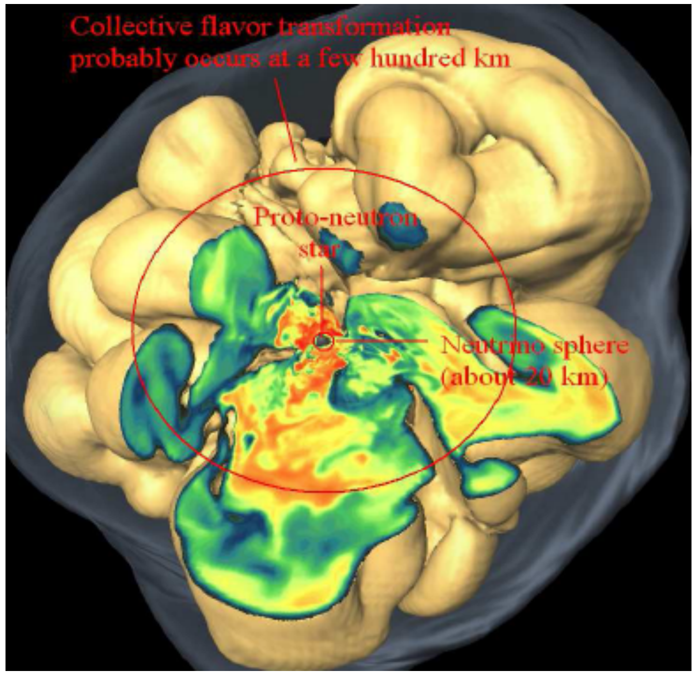
# Coherent evolution

- Controlled by $2n_f$ x $2n_f$ Hamiltonian-like operators

$$H = \begin{pmatrix} H_R & \boxed{H_m} \\ H_m^\dagger & H_L \end{pmatrix} \qquad \bar{H} = \begin{pmatrix} \bar{H}_R & H_m \\ H_m^\dagger & \bar{H}_L \end{pmatrix}$$

$$\boxed{H_m = -\frac{1}{|\vec{p}|}\left(\Sigma_R^+ m^\dagger - m^\dagger \Sigma_L^+\right)}$$

- Qualitatively new O($\varepsilon^2$) effect: coherent conversion of LH ↔ RH ν's
  - Need anisotropic environment (transverse component of Σ)
  - Need axial components, coupling to spin [ 1-flavor $H_m \sim m_\nu/p\ (\Sigma_R - \Sigma_L)^+$ ]
  - Potentially big impact: Dirac (active-sterile) vs Majorana ($\nu$-$\bar{\nu}$)



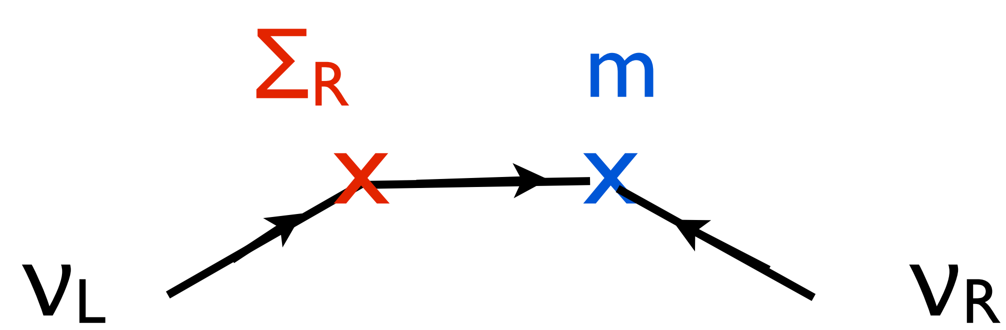
# More on spin-mixing term

- Effect can be derived using effective hamiltonian approach

$$\langle i, \vec{p}', h' \,|\, j, \vec{p}, h \rangle \equiv -i(2\pi)^2\, 2|\vec{p}|\, \delta^{(4)}(p - p') \boxed{\mathcal{H}^{ij}_{h'h}(p)}$$

- Use medium-modified neutrino Lagrangian in perturbation theory

$$\boxed{\mathcal{L}_{\rm int} = -\bar{\nu}_L\, m\, \nu_R - \bar{\nu}_L \not{\Sigma}_R \nu_L - \bar{\nu}_R \not{\Sigma}_L \nu_R + \text{h.c.}}$$



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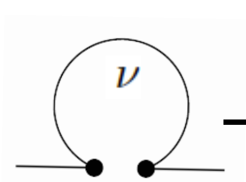
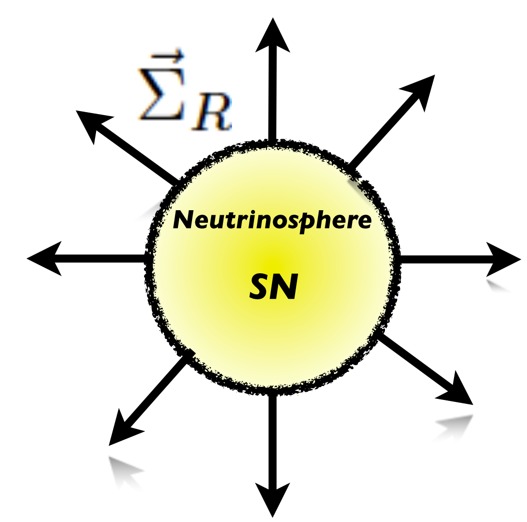
* $\Sigma_{L,R}$: medium-induced vector potentials




$$\longrightarrow \quad \Sigma^{\mu}_{R}(x) = \sqrt{2}G_F \int \frac{d^3q}{(2\pi)^3} n^{\mu}(q) \Big( f_{LL}(\vec{q},x) - \bar{f}_{RR}(\vec{q},x) \Big)$$

* Even in simple "bulb" model for SN: $\vec{\Sigma}_R(x) \neq 0$



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- Use medium-modified neutrino Lagrangian in perturbation theory

$$\boxed{\mathcal{L}_{\rm int} = -\bar{\nu}_L\, m\, \nu_R - \bar{\nu}_L \not{\Sigma}_R \nu_L - \bar{\nu}_R \not{\Sigma}_L \nu_R + \text{h.c.}}$$

- 1-flavor result

$$\boxed{\begin{aligned} \mathcal{H}_{LL}(p) &= \Sigma^0_R - \vec{\Sigma}_R \cdot \hat{p} \\ \mathcal{H}_{RR}(p) &= \Sigma^0_L - \vec{\Sigma}_L \cdot \hat{p} \\ \mathcal{H}_{LR}(p) &= -\frac{m}{2|\vec{p}|}\, \vec{\Sigma}_A \cdot \vec{x}_+(p) \end{aligned}}$$

Axial potential

$$\Sigma^\mu_A \equiv \Sigma^\mu_L - \Sigma^\mu_R$$

$$\Downarrow$$

medium birefringence
+
mixing (transverse part)

# More on spin-mixing term

- Effect can be derived using effective hamiltonian approach

$$\langle i, \vec{p}', h' \,|\, j, \vec{p}, h \rangle \equiv -i(2\pi)^2\, 2|\vec{p}|\, \delta^{(4)}(p - p') \boxed{\mathcal{H}^{ij}_{h'h}(p)}$$

- Similar mixing is induced by magnetic moment (Dirac for simplicity)

$$\boxed{\Delta\mathcal{L} = (\mu_\nu/2)\ \bar{\nu}_R \sigma_{\mu\nu} F^{\mu\nu} \nu_L + \text{h.c.}}$$

- 1-flavor result

$$\boxed{\mathcal{H}_{LR}(p) = \mu_\nu\, \vec{B} \cdot \vec{x}_+(p)}$$

transverse component
of the magnetic field

See de Gouvea & Shalgar for impact on SN neutrino collective oscillations

# Inelastic collisions

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

$$iDF = [H, F] + iC$$

$$i\bar{D}\bar{F} = [\bar{H}, \bar{F}] + i\bar{C}$$

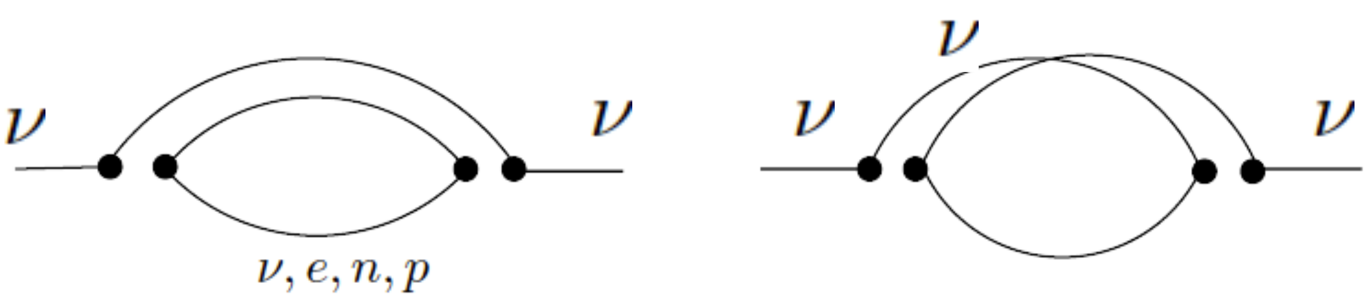
Inelastic collisions

"Boltzmann"

# Inelastic collisions

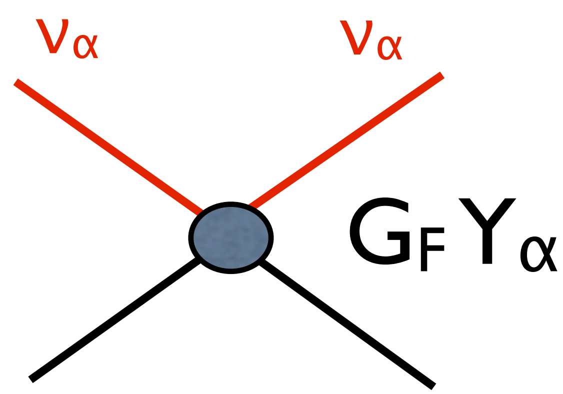
- Controlled by $2n_f \times 2n_f$ gain and loss potentials $\Pi^{\pm}[F, \bar{F}, f_{e,n,p,..}]$

$$
\boxed{\begin{aligned}
C &= \frac{1}{2}\{\Pi^+, F\} - \frac{1}{2}\{\Pi^-, I - F\} \\
\bar{C} &= \frac{1}{2}\{\bar{\Pi}^+, \bar{F}\} - \frac{1}{2}\{\bar{\Pi}^-, I - \bar{F}\}
\end{aligned}}
$$




- $\Pi^{\pm}$ are non-diagonal in both flavor and spin ($\rightarrow$ decoherence)

- Example: $C_{LL}$ (upper $n_f$ x $n_f$ block) induced by neutrino scattering off medium particles (e,p,n,...) in isotropic environment

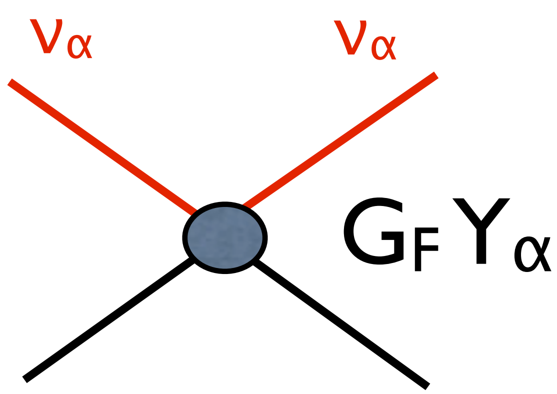



$$Y = \begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \end{pmatrix}$$

Medium response function (knows about medium particle distributions and their interactions)

$$C_{LL}(\vec{p}) = -\int d^3p'\, R(\vec{p},\vec{p}')\left\{Y\left(1-f_{LL}(\vec{p}')\right)Y\,,\, f_{LL}(\vec{p})\right\} + \text{gain}$$

- Example: $C_{LL}$ (upper $n_f$ x $n_f$ block) induced by neutrino scattering off medium particles (e,p,n,...) in isotropic environment




$$Y = \begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \end{pmatrix}$$

Medium response function (knows about medium particle distributions and their interactions)

$$\boxed{C_{LL}(\vec{p}) = -\int d^3p' \, R(\vec{p}, \vec{p}') \left\{ Y \left( 1 - f_{LL}(\vec{p}') \right) Y \, , \, f_{LL}(\vec{p}) \right\} + \text{gain}}$$

$$\neq \begin{pmatrix} C_e & 0 & 0 \\ 0 & C_\mu & 0 \\ 0 & 0 & C_\tau \end{pmatrix}$$

# Comparison with other QKEs

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\boxed{iDF = [H, F] + iC}$$

NPB 406, 423 (1993)

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1310.2164
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1302.2347

- Quite different from Volpe et al., who include "abnormal densities" (correlations of $\nu$ and $\overline{\nu}$ of opposite momentum) and discuss their coherent evolution coupled to "normal densities". We do not include this, based on $L_{\text{gradients}} \gg L_{\text{deBroglie}}$

# Summary & future challenges

- Neutrino QKEs can be formulated from QFT + power counting in ratio of length scales ($L_{osc}$, $L_{mfp}$, $L_{gradients}$ >> $L_{deBroglie}$)
- Many expected features, some surprising ones (spin oscillations). See A. Vlasenko's talk for first applications to astrophysics
- Challenges:
  - Explicit form of the collision term (in progress)
  - Computational implementation

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$$\begin{pmatrix} f & \cancel{\phi} \\ \cancel{\phi^\dagger} & \bar{f}^T \end{pmatrix}$$

Early Universe: $F(\mathbf{x},\mathbf{p}) \xrightarrow[\text{no L-R coherence}]{\text{isotropy:}} F(t,|\mathbf{p}|) \xrightarrow[\text{binning}]{} F_{|\mathbf{p}|}(t)$

$2 * (n_f)^2 * n_{|\mathbf{p}|}$ coupled ODEs, initial value problem

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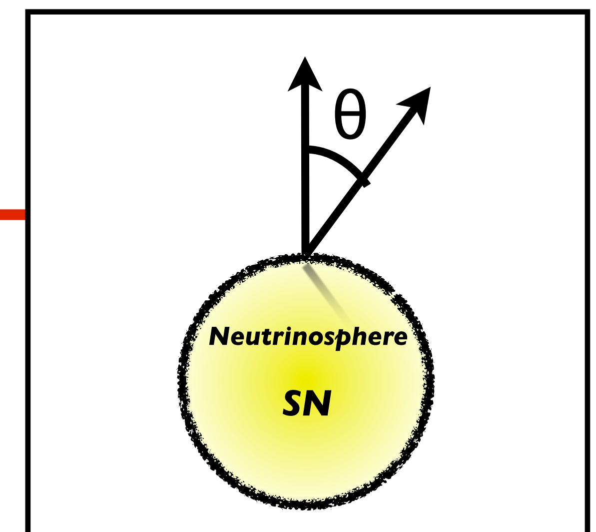


$$\begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}$$

Supernovae with spherical symmetry:

$$F(\mathbf{x},\mathbf{p}) \xrightarrow[\text{geometry}]{} F(r,|\mathbf{p}|,\theta) \xrightarrow[\substack{\text{binning} \\ \text{all } \theta \text{ contribute}}]{} F_{|\mathbf{p}|,\theta}\,(r)$$

$4 * (n_f)^2 * n_{|\mathbf{p}|} * n_\theta$ coupled ODEs, boundary value problem