NEUTRINO AND NUCLEAR ASTROPHYSICS
The 2014 International Summer School on AstroComputing, UCSD, July 21 - August 1 2014

Neutrino
Quantum Kinetic Equations - I

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Based on 1309.2628, 1406.5558, 1406.6724, and references therein

Outline

Lectures

(I)

- Motivation: neutrinos and the cosmos
- Neutrinos in hot and dense media
- Structure of QKEs from quantum field theory

(II)

- Anatomy of the QKEs
  - Coherent evolution: flavor *and* spin
  - Inelastic collisions
- Comparison to other approaches & future challenges

Talk by A. Vlasenko

- Neutrino-antineutrino transformation in astrophysical environments

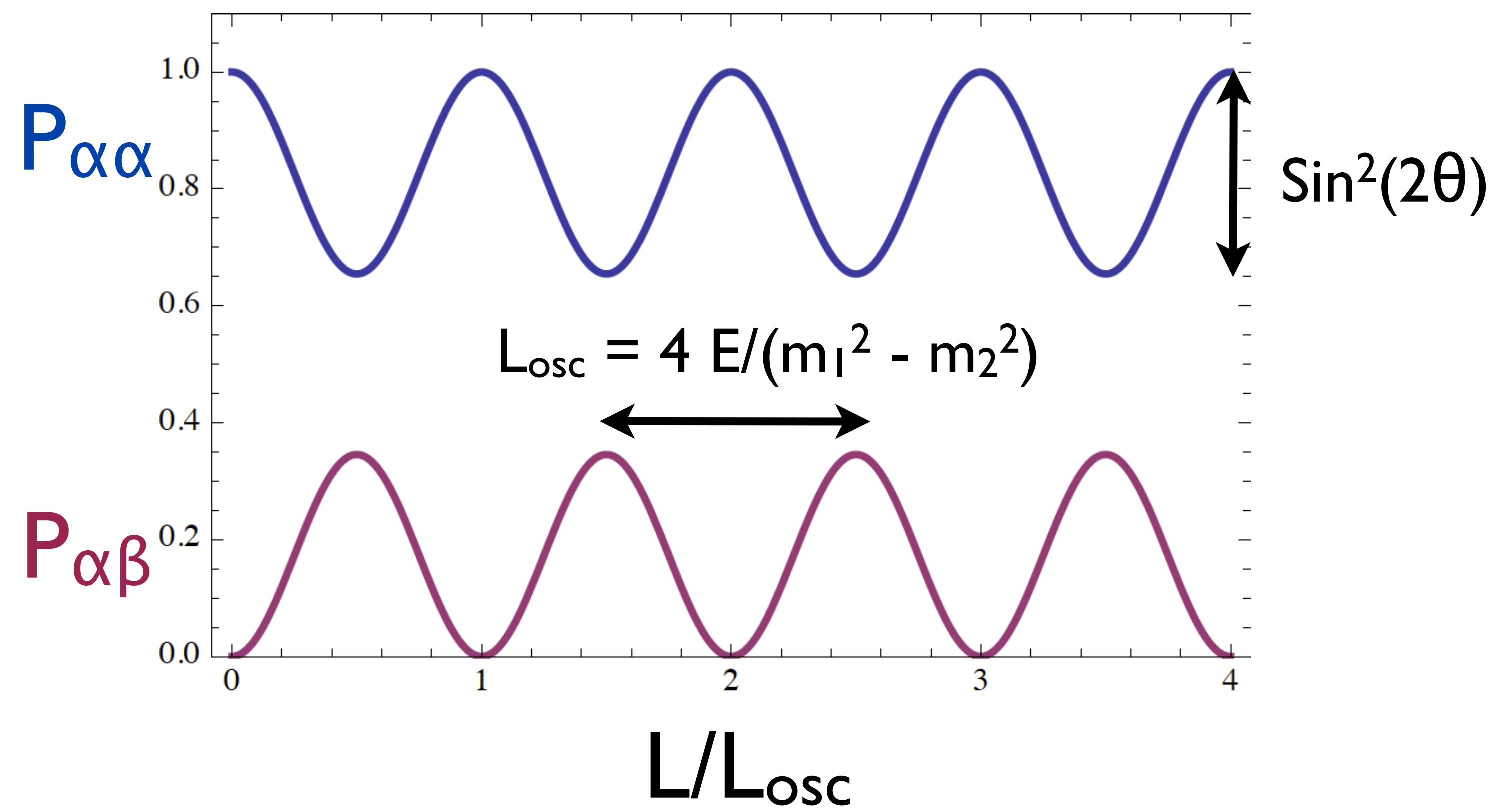
Neutrinos

- Elusive particles: lightest fermions, feel only the "weak" force

- Interaction ("flavor") states $\nu_{e,\mu,\tau}$ do not coincide with mass states $\nu_{1,2,3}$

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

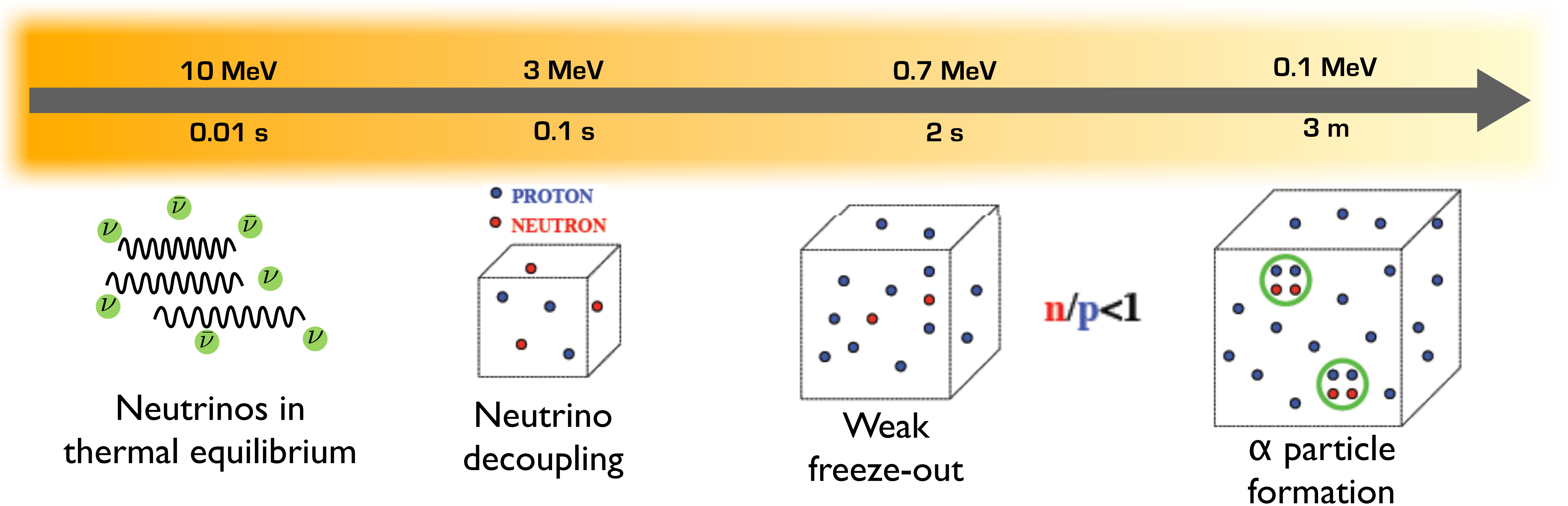
- A neutrino produced in a given flavor state can "oscillate" into another flavor state through QM interference effect!

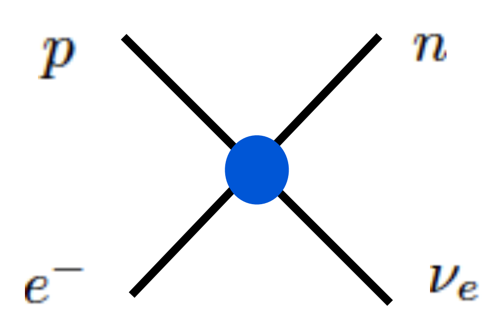


Despite elusive nature, ν's play a key role in cosmology / astrophysics

Neutrinos and the Cosmos (1)

1. What is the spectrum and flavor content of ν's when they decouple in the Early Universe? Far reaching implications for energy density, and n/p ratio → Big Bang Nucleosynthesis





$$e^- + p \leftrightarrow \nu_e + n, \; \bar{\nu}_e + p \leftrightarrow e^+ + n,$$
$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

Reaction rates depend strongly on E_ν

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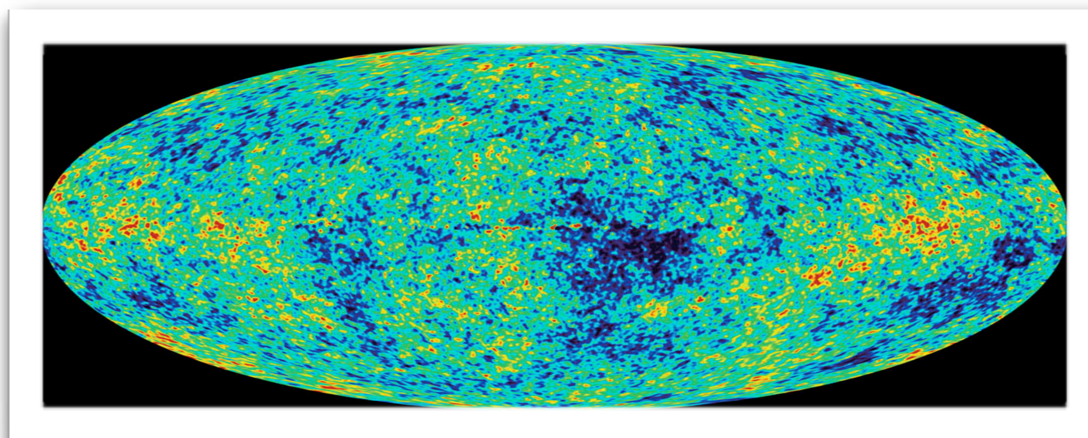
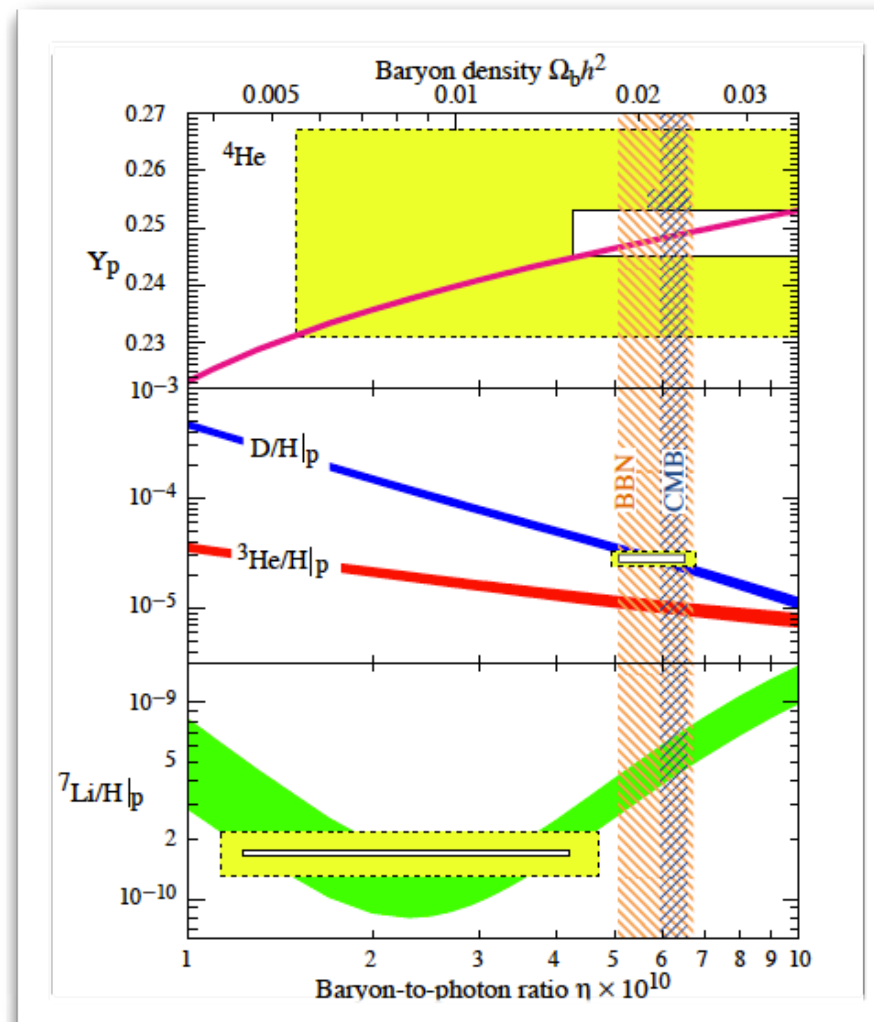
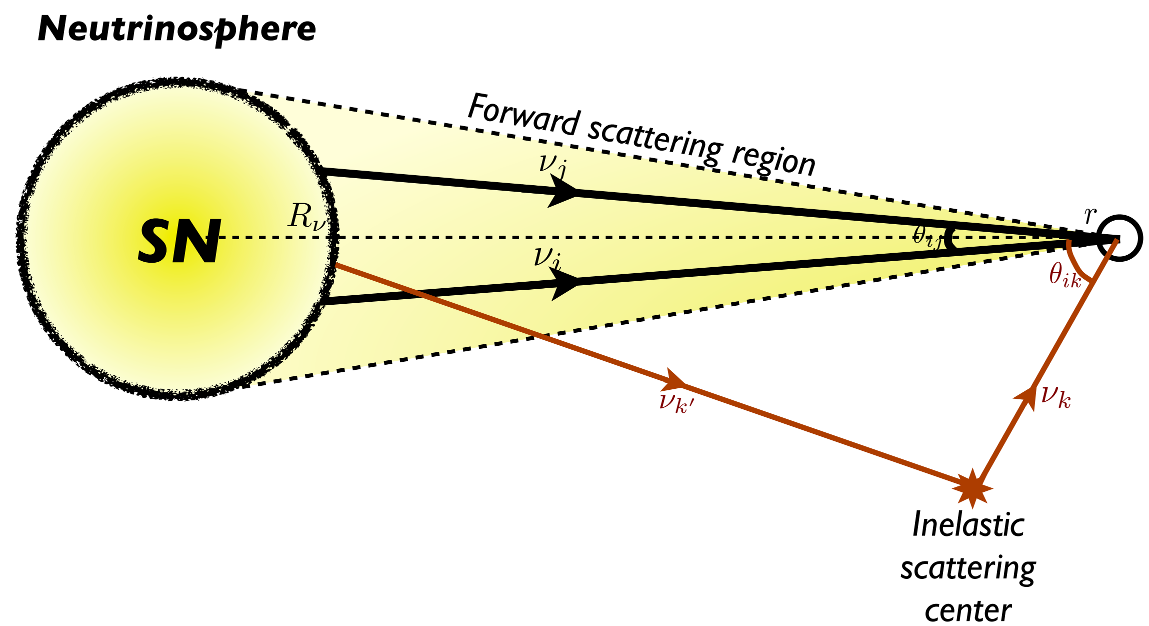


- Precise observations (η_B, N_{eff}, D, ^{4}He) + robust theory can turn BBN into a deep probe of physics beyond the Standard Model in the lepton sector (sterile ν's, non-zero L)

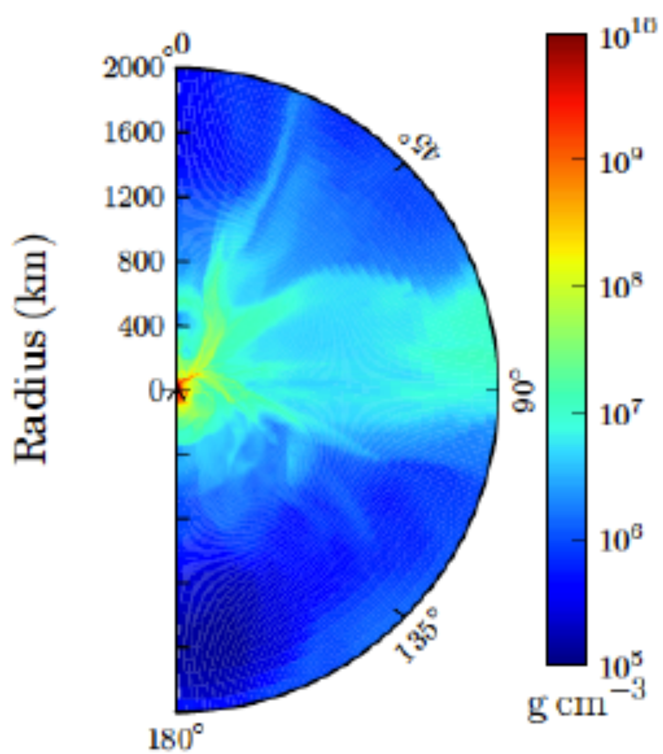


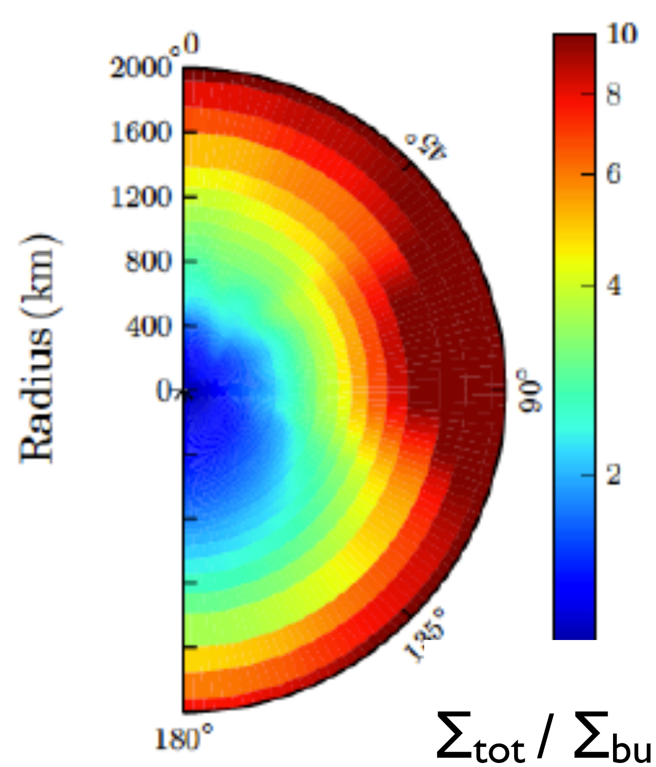
Neutrinos and the Cosmos (2)

2. What is the impact of inelastic collisions on ν propagation in the SN envelope? Implications for SN ν signal, nucleosynthesis in the neutrino-heated ejecta



Cherry-Carlson-Friedland-Fuller-Vlasenko 2012





$\Sigma_{tot} / \Sigma_{bulb}$

First studies indicate that < 1% of ν scatter, but there is a large effect on the neutrino potential Σ (angular dependence)

$\Sigma_{\nu\nu} \sim 1 - \cos\theta_{\hat{p}\hat{q}}$

The need for QKEs

To fully address the issues described above,
must set up the *analytic* and *computational* tools needed to describe neutrino kinetics in the EU and SN environments, simultaneously keeping track of the key quantum mechanical effect of coherent flavor oscillations AND de-cohering inelastic collisions with the medium

Neutrinos in hot / dense medium

- At a given time, ensemble of neutrinos described by incoherent mixture of states |k> with weight p_k ($\sum p_k = 1$)
- Physics controlled by density matrix

$$\boxed{\rho = \sum_k p_k \, |k\rangle\langle k|} \qquad i\frac{d\rho}{dt} = [H, \rho]$$

Example: in thermal equilibrium $\rho_{\rm eq} = \dfrac{e^{-H/(kT)}}{\mathrm{Tr}\left(e^{-H/(kT)}\right)}$

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$$\boxed{\rho = \sum_k p_k \, |k\rangle\langle k|} \qquad i\frac{d\rho}{dt} = [H, \rho]$$

- Ensemble average of any operator:

$$\boxed{\langle \hat{O} \rangle = \sum_k p_k \, \langle k|\hat{O}|k\rangle \; = \; \mathrm{Tr}(\rho \, \hat{O})}$$

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- In EU and SN we need densities and fluxes of ν_α, α=e,μ,τ,X ⇒ generalized number operator

$$\boxed{\hat{O} \quad \leftrightarrow \quad n_{ij} = a_j^\dagger a_i}$$

creation / annihilation operators:
i,j label one-particle states

$$|\nu(i)\rangle = a_i^\dagger \, |0\rangle$$

- 1-particle states associated with massive spin-1/2 field

$$|\nu(\vec{p}, i, h)\rangle = a^{\dagger}_{i,h}(\vec{p})|0\rangle \qquad |\bar{\nu}(\vec{p}, i, h)\rangle = b^{\dagger}_{i,h}(\vec{p})|0\rangle$$

creation operator for particle / antiparticle labeled by
3-momentum **p**, mass m_i, helicity h=L,R

- Dirac → 4 states: L- and R-handed neutrino *and* antineutrino

- Majorana → 2 states: L- and R-handed neutrino ($\Psi = \Psi^c \Rightarrow a_i = b_i$)

- Key dynamical objects are the "matrices of densities"

i = 1,2,3, ...   h,h' = L, R

neutrinos $$\langle a^{\dagger}_{j,h'}(\vec{p}') \, a_{i,h}(\vec{p}) \rangle \;=\; (2\pi)^3 \, 2 n_{ij}(\vec{p}) \, \delta^{(3)}(\vec{p} - \vec{p}') \, f^{ij}_{hh'}(\vec{p})$$

normalization (conventional)

$$n_{ij} = 2\omega_i \omega_j / (\omega_i + \omega_j)$$

creation / annihilation operators

$$\{a_{i,h}(\vec{p}), a^{\dagger}_{j,h'}(\vec{p}')\} = (2\pi)^3 \, 2\,\omega_i(\vec{p}) \, \delta_{hh'} \, \delta_{ij} \, \delta^{(3)}(\vec{p} - \vec{p}')$$

$$\omega_i(\vec{p}) = \sqrt{\vec{p}^2 + m_i^2}$$

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anti-neutrinos
$$\langle b^{\dagger}_{i,h'}(\vec{p}')\, b_{j,h}(\vec{p})\rangle = (2\pi)^3\, 2n_{ij}(\vec{p})\, \delta^{(3)}(\vec{p}-\vec{p}')\ \bar{f}^{ij}_{hh'}(\vec{p})$$

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- Physical content:

$f^{ii}_{hh}(\vec{p})$ Represents occupation number of neutrinos of mass m_i, helicity h, momentum p

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- Physical content:

$f^{ii}_{hh}(\vec{p})$ Represents occupation number of neutrinos of mass m_i, helicity h, momentum p

$f^{ij}_{hh}(\vec{p})$ Signals quantum coherence between states of same helicity and different mass

Non-zero if there are states in the ensemble that are coherent superpositions of states of same helicity and different mass, e.g., L-handed neutrino flavor states

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

$$\Rightarrow \quad f^{12}_{LL} \propto \sin\theta\, \cos\theta$$

- Key dynamical objects are the "matrices of densities"

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- Key dynamical objects are the "matrices of densities"

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$$\begin{array}{l|rcl} \text{neutrinos} & \langle a^\dagger_{j,h'}(\vec{p}\,')\, a_{i,h}(\vec{p})\rangle & = & (2\pi)^3\, 2n_{ij}(\vec{p})\, \delta^{(3)}(\vec{p}-\vec{p}\,')\; f^{ij}_{hh'}(\vec{p}) \\ \text{anti-neutrinos} & \langle b^\dagger_{i,h'}(\vec{p}\,')\, b_{j,h}(\vec{p})\rangle & = & (2\pi)^3\, 2n_{ij}(\vec{p})\, \delta^{(3)}(\vec{p}-\vec{p}\,')\; \bar{f}^{ij}_{hh'}(\vec{p}) \end{array}$$

- $2n_f$ x $2n_f$ matrix structure: Dirac case, need F and $\bar{F}$

$$F(\vec{p},x) = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\bar{F}(\vec{p},x) = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

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- $2n_f \times 2n_f$ matrix structure: Dirac case, need F and $\bar{F}$

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$n_f \times n_f$ blocks describing matrix of density for active states (L-handed neutrinos and R-handed antineutrinos)

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$n_f \times n_f$ blocks describing L-R (active-sterile) coherence

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- $2n_f \times 2n_f$ matrix structure: Majorana case

$$a_i(\vec{p},h) = b_i(\vec{p},h) \quad \longrightarrow \quad f \equiv f_{LL},\ \bar{f} \equiv \bar{f}_{RR} = f^T_{RR} \qquad \phi = f_{LR}$$

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$$F \to \mathcal{F} = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}$$

n_f x n_f blocks describing matrix of density for neutrinos and antineutrinos

- Key dynamical objects are the "matrices of densities"

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n_f x n_f blocks describing matrix of density for neutrinos and antineutrinos

n_f x n_f block describing L-R (neutrino-antineutrino) coherence

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- QKEs are nothing but the evolution equations for the f's

- We work in the flavor basis, related to the above by:

Matrix that puts neutrino propagator in diagonal form

$$\nu_\alpha = U_{\alpha i}\, \nu_i$$

$$\boxed{f_{\alpha\beta} = U_{\alpha i}\, f_{ij}\, U^*_{\beta j}}$$

flavor basis

mass basis

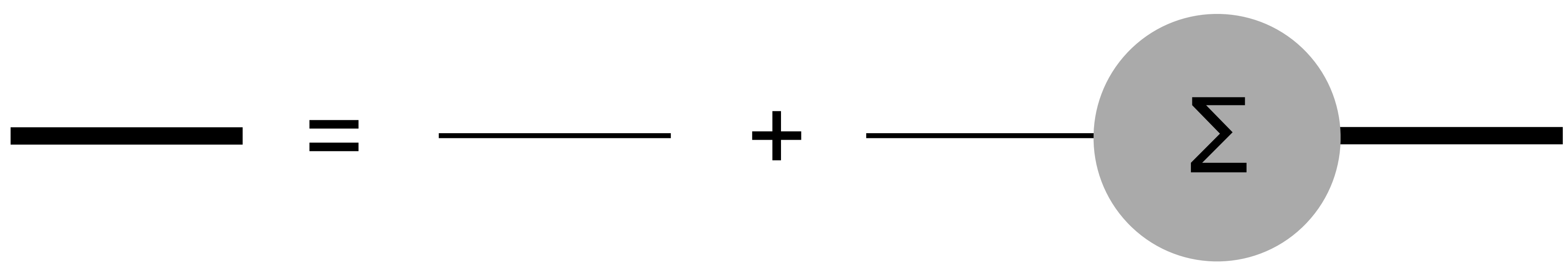
QKEs from Quantum Field Theory

Equations of motion for Green Functions

$$\langle \nu_\alpha^i(x) \bar{\nu}_\beta^j(y) \rangle$$

Kinetic equations for "matrix of densities" f(x,p)

$$f_{\alpha\beta}^{\lambda\lambda'}(x,p) \ \sim \ \langle \, a_\beta^\dagger(p,\lambda') \ a_\alpha(p,\lambda) \, \rangle$$



QKEs from Quantum Field Theory

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for Green Functions
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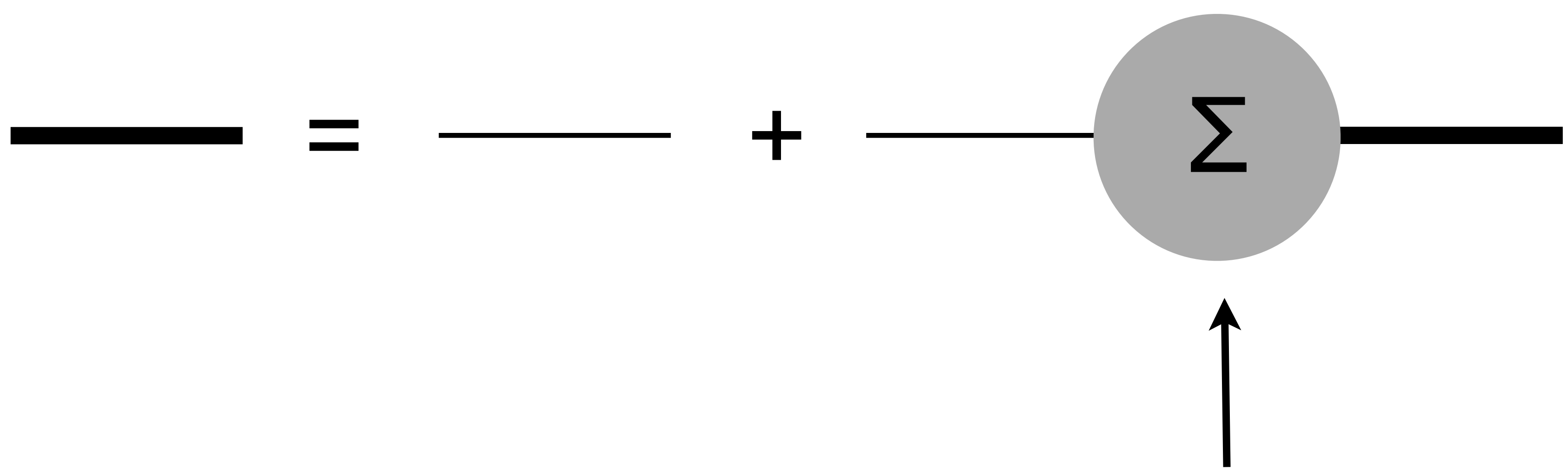

$\Sigma_{\text{forward}} \sim G_F\, n$

$\Sigma_{\text{inelastic}} \sim (G_F)^2\, T^5 \;;\; (G_F)^2\, n\, T^2$

QKEs from Quantum Field Theory

Equations of motion for Green Functions

$\langle \nu_\alpha^i(x)\bar{\nu}_\beta^j(y)\rangle$

$\rightarrow$

Kinetic equations for "matrix of densities" f(x,p)

$f_{\alpha\beta}^{\lambda\lambda'}(x,p) \sim \langle\, a_\beta^\dagger(p,\lambda')\; a_\alpha(p,\lambda)\,\rangle$

- Exploit hierarchy of scales. Work to 2nd order in small ratios (E~T):

$m_\nu/E \sim \Delta m_\nu/E \sim \Sigma_{forward}/E \sim \partial_X/E \sim O(\varepsilon)$ $\qquad$ $\Sigma_{inelastic}/E \sim O(\varepsilon^2)$

Small neutrino (Δ)masses

Comparable** potential induced by forward scattering on matter and other neutrinos

Slowly varying background

Weak interaction rates

QKEs from Quantum Field Theory

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- The physics: $L_{osc} \sim E/(\Delta m_\nu)^2$, L_{mfp}, $L_{gradients}$ >> $L_{deBroglie}$

QKEs from Quantum Field Theory

Equations of motion for Green Functions

$$\langle \nu^i_\alpha(x) \bar{\nu}^j_\beta(y) \rangle$$

$\Rightarrow$

Kinetic equations for "matrix of densities" f(x,p)

$$f^{\lambda\lambda'}_{\alpha\beta}(x,p) \;\sim\; \langle\, a^\dagger_\beta(p,\lambda')\; a_\alpha(p,\lambda)\,\rangle$$

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- Initial density matrix of the system [recall <O> = Tr (ρO)] → initial (or boundary) conditions for the QKEs

QKEs from Quantum Field Theory

Equations of motion for Green Functions
$\langle \nu^i_\alpha(x) \bar{\nu}^j_\beta(y) \rangle$

→

Kinetic equations for "matrix of densities" f(x,p)
$f^{\lambda\lambda'}_{\alpha\beta}(x,p) \ \sim \ \langle\, a^\dagger_\beta(p,\lambda')\ a_\alpha(p,\lambda)\,\rangle$

- Advantages of this approach (used already in other contexts, such as baryogenesis in the Early Universe):
  - First principles method, forced us to think about L-R coherence
  - No guesses or fudging: diagrammatic computations in non-eq QFT determine all terms of the QKEs
  - Systematic approximations (based on power counting in ε's)

Structure of the QKEs

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

$$\boxed{\begin{aligned} iDF &= [H, F] + iC \\ i\bar{D}\bar{F} &= [\bar{H}, \bar{F}] + i\bar{C} \end{aligned}}$$

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Derivative along ν world line: drift & force term

"Vlasov"

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Derivative along ν world line: drift & force term

"Vlasov"

Coherent evolution: vacuum mass & forward scattering (refractive potential)

"MSW"

Structure of the QKEs

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

$$iDF = [H, F] + iC$$
$$i\bar{D}\bar{F} = [\bar{H}, \bar{F}] + i\bar{C}$$

Derivative along ν world line: drift & force term

"Vlasov"

Coherent evolution: vacuum mass & forward scattering (refractive potential)

"MSW"

Inelastic collisions

"Boltzmann"

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- F, H, C: $2n_f \times 2n_f$ matrices, all components coupled in general
- D, H, C are functionals of F, $\bar{\text{F}}$: non-linear system

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Current state-of-the art:

- Early Universe: approximate treatment of inelastic collisions, inadequate in decoupling regime
- Supernovae:
  - no simultaneous treatment of forward AND inelastic collisions (separation of low- and high-density regimes)
  - no inclusion of spin degrees of freedom (n_f x n_f problem)

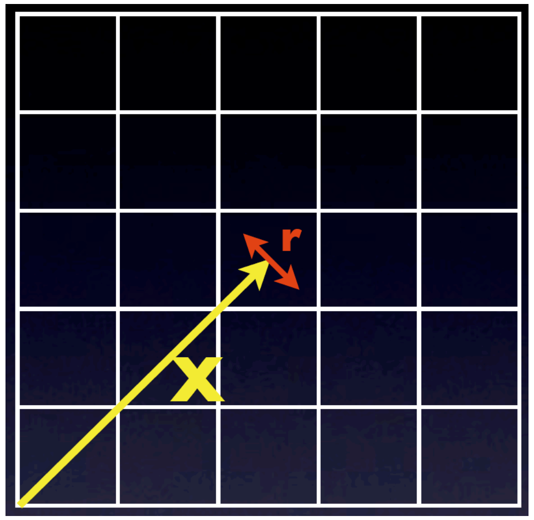
Backup

Green's function approach

- Dynamics contained in the two-point function

$$G^{ij}(x,r) \equiv \frac{1}{2}\langle\,[\nu^i(x+r/2),\bar{\nu}^j(x-r/2)]\,\rangle$$

Wigner transform $\big\downarrow$



$$G^{ij}(x,p) \equiv \int dr\, e^{ip\cdot x}\, G^{ij}(x,r)$$

- Take spinor projections (vector, tensor)

$$F_{L,R} \;=\; \frac{1}{4}\mathrm{Tr}\Big(\gamma_\mu P_{L,R}\, G(p,x)\Big)\, \bar{n}^\mu$$
$$\Phi^{(\dagger)} \;=\; \mp\frac{i}{16}\mathrm{Tr}\Big(\sigma_{\mu\nu} P_{L/R}\, G(p,x)\Big)(\bar{n}^\mu x_\pm^\nu - \bar{n}^\nu x_\pm^\mu)$$

$P_{L,R} \equiv (1 \mp \gamma_5)/2,\ x_\pm \equiv x_1 \pm i x_2,$

- Collet into $2n_f$ x $2n_f$ matrix $\quad \hat{F} = \begin{pmatrix} F_L & \Phi \\ \Phi^\dagger & F_R \end{pmatrix}$

- Take frequency projections

$$-2\int_0^\infty \frac{dp^0}{2\pi}\hat{F}(p,x) \;=\; F(\vec{p},x) \;=\; \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$
$$-2\int_{-\infty}^0 \frac{dp^0}{2\pi}\hat{F}(p,x) \;=\; \bar{F}(-\vec{p},x) = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{RL} \\ \bar{f}_{LR} & \bar{f}_{LL} \end{pmatrix}$$

In free theory coincide with definition in terms of creation and annihilation operators