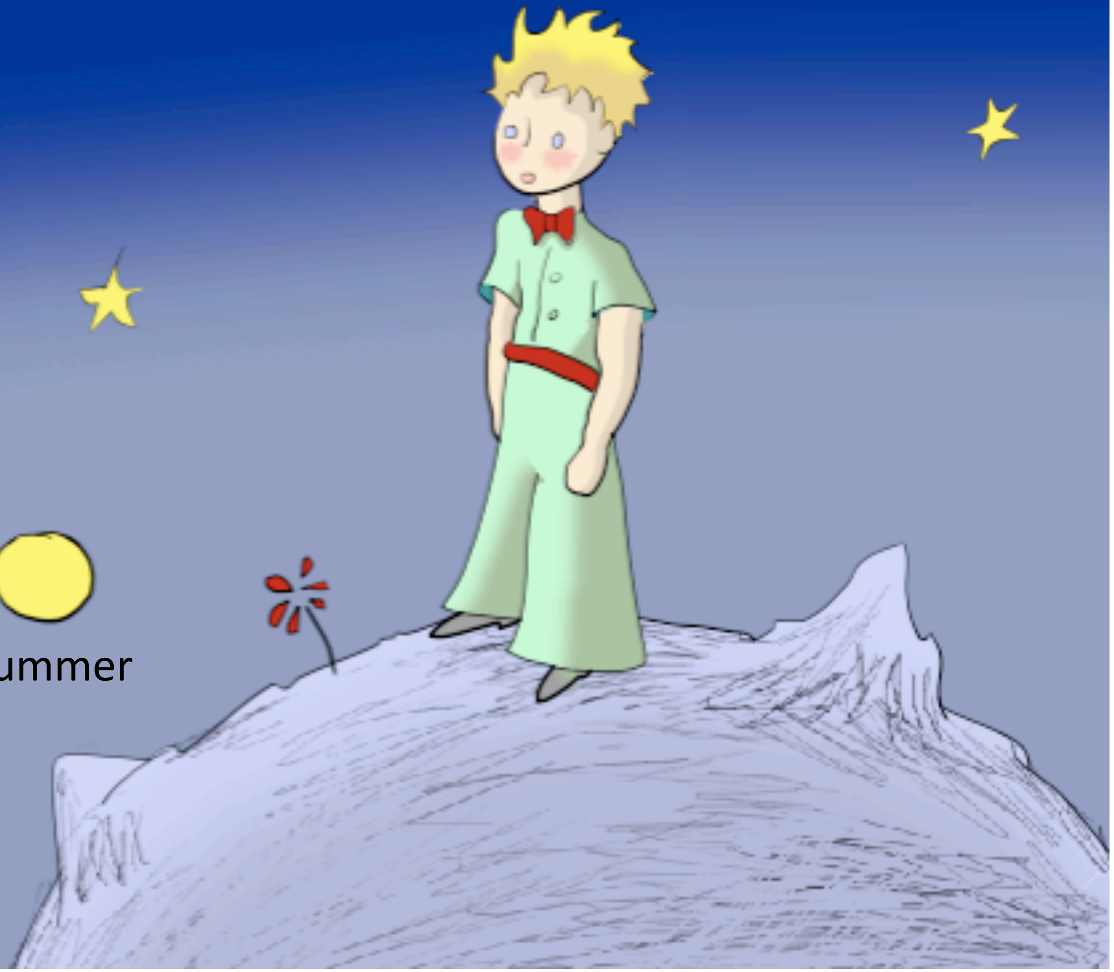
Variations on a theme of neutrinos

A.B. Balantekin

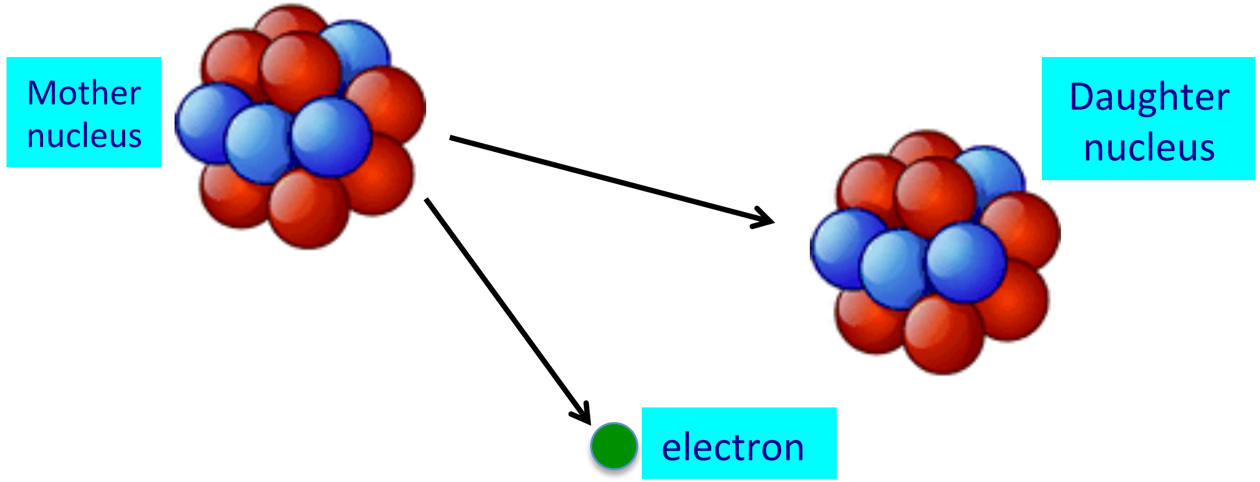


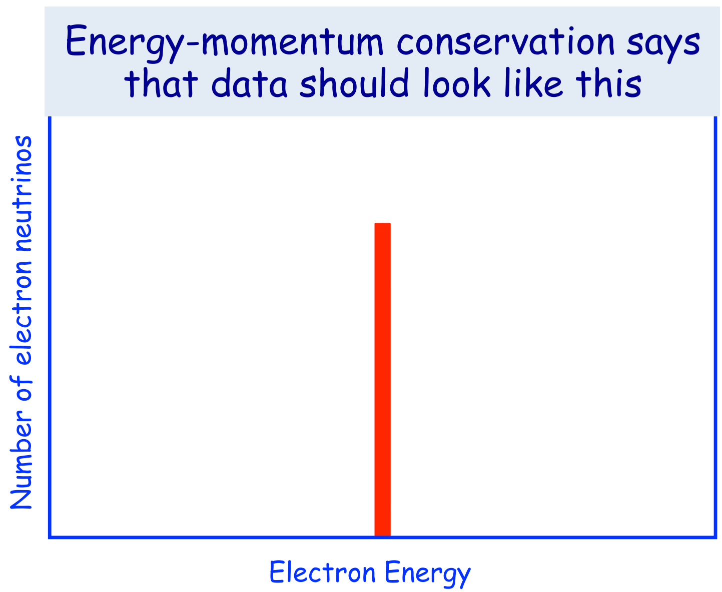
UC HIPECC 2014 International Summer School on Astrocomputing

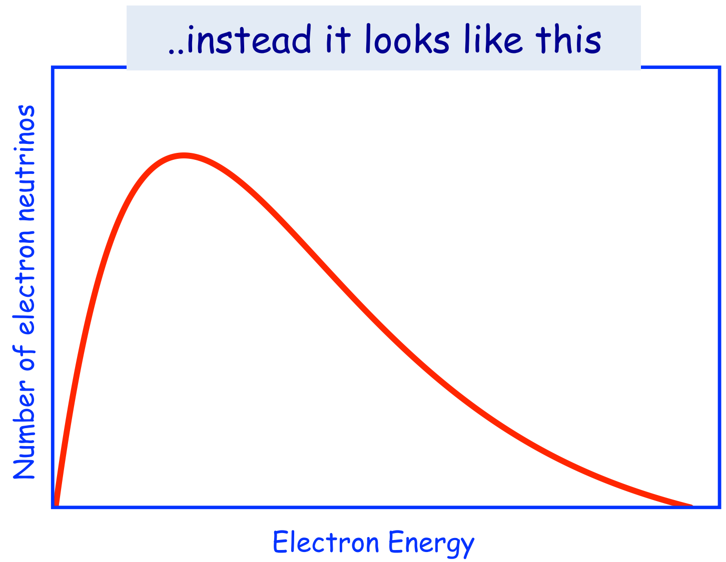
Neutrinos are fascinating particles as they are the only neutral fermions

- What is a neutrino?
- What is a particle?
- Why are they fascinating?

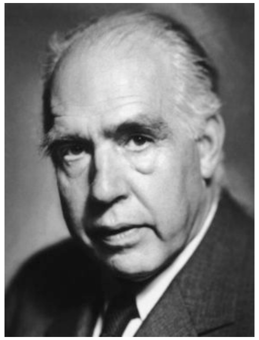
Neutrino came out of a puzzle about the radioactive decay in the early 1920's:



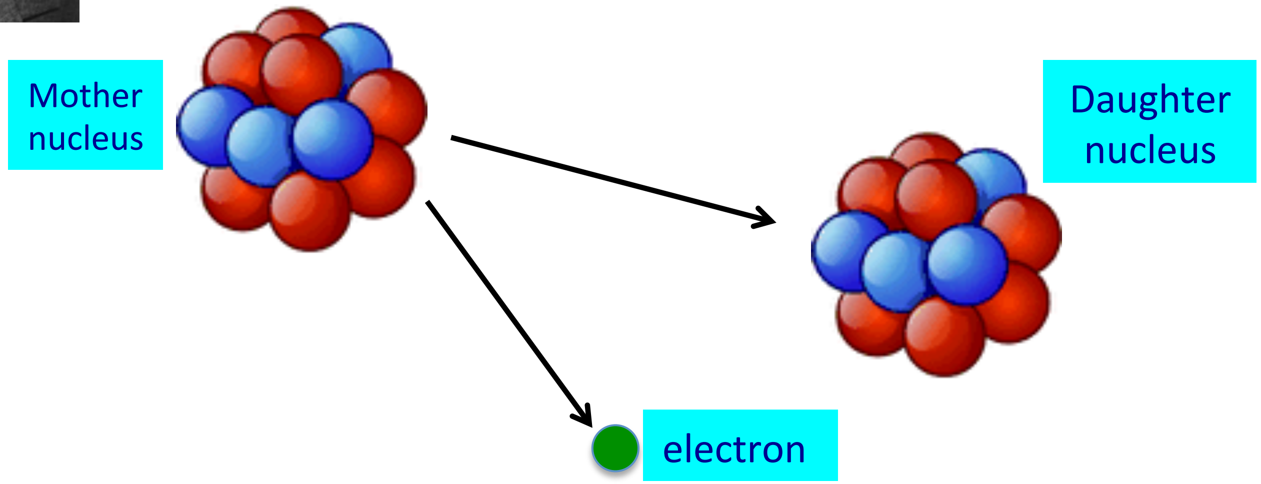




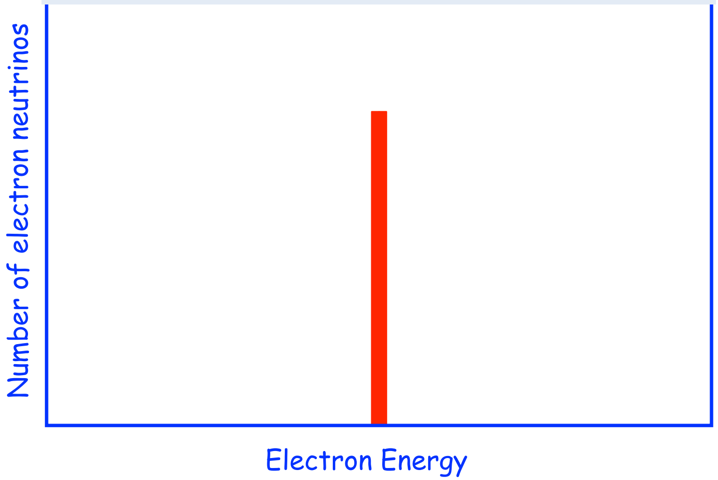
Niels Bohr



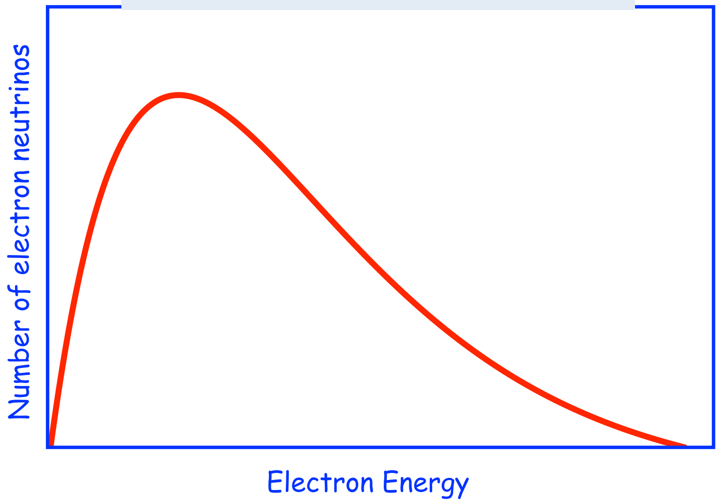
In radioactive decays energy-momentum conservation no longer holds!



Energy-momentum conservation says that data should look like this

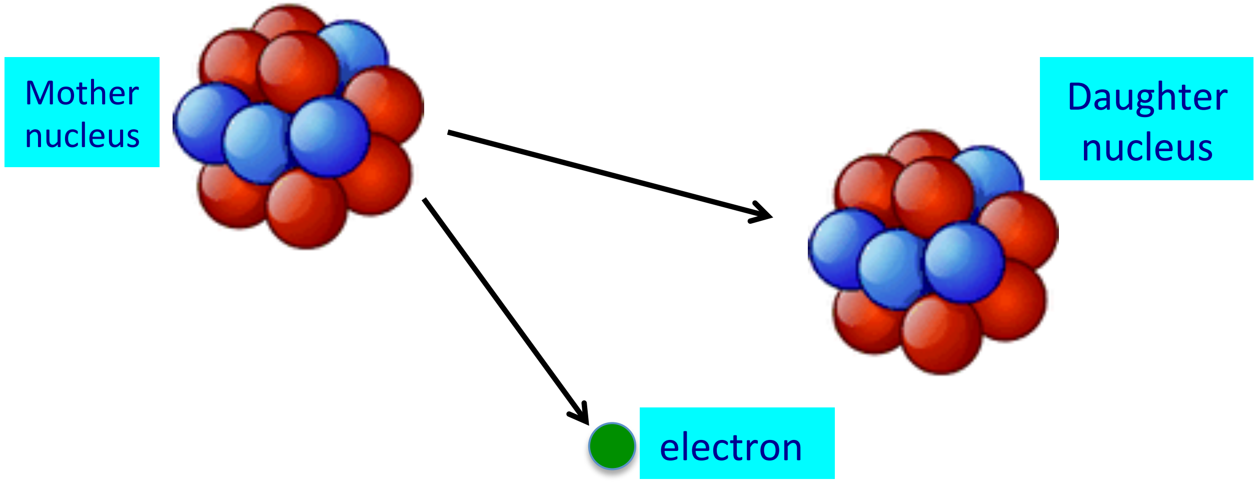


..instead it looks like this



Wolfgang Pauli

In this reaction there is a third particle produced that you cannot (yet) see!



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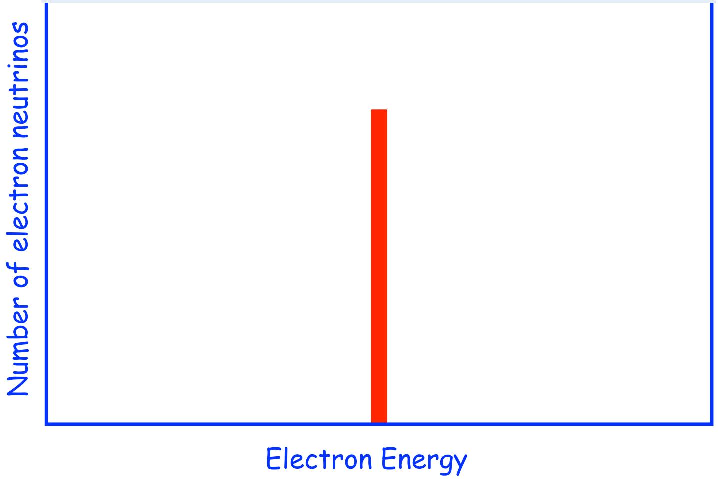

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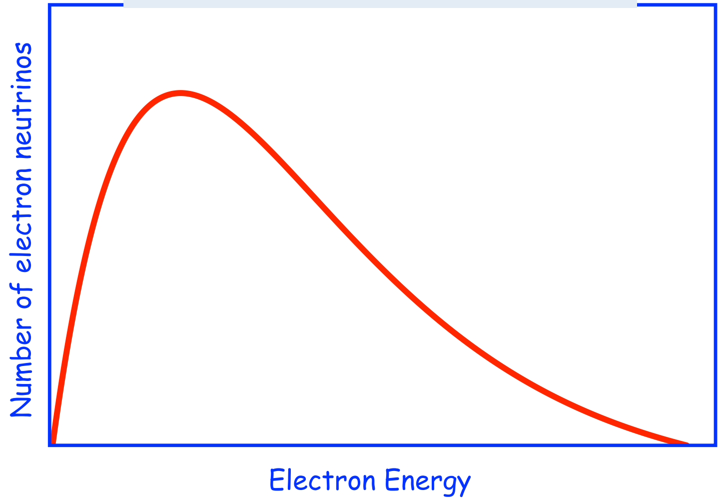

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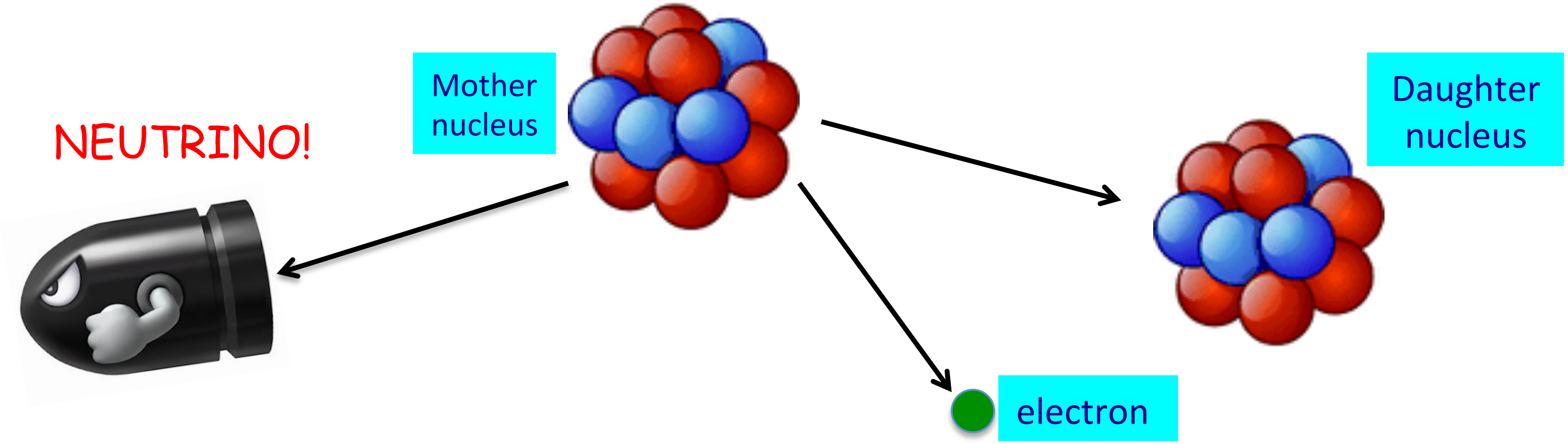

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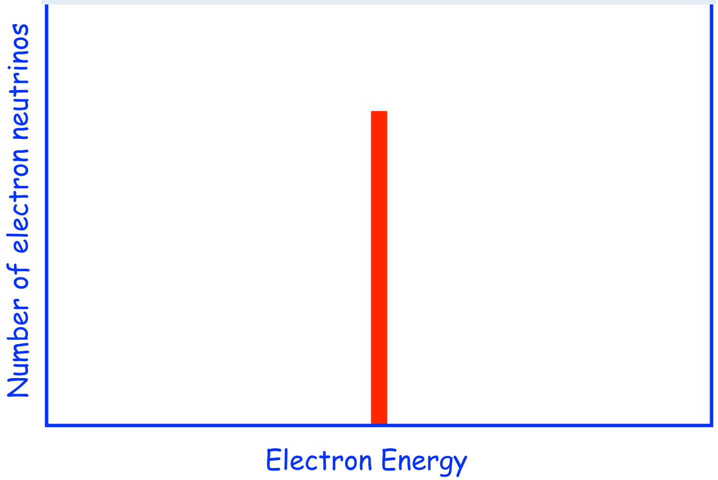

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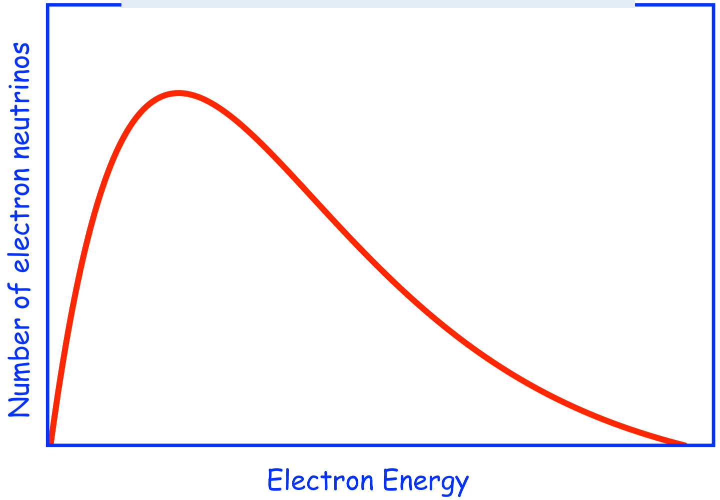


Wolfgang Pauli,
father of the neutrino
and Pauli exclusion
principle

Physicist goes to a ball

or

Mystery of Missing Energy

Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der
Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

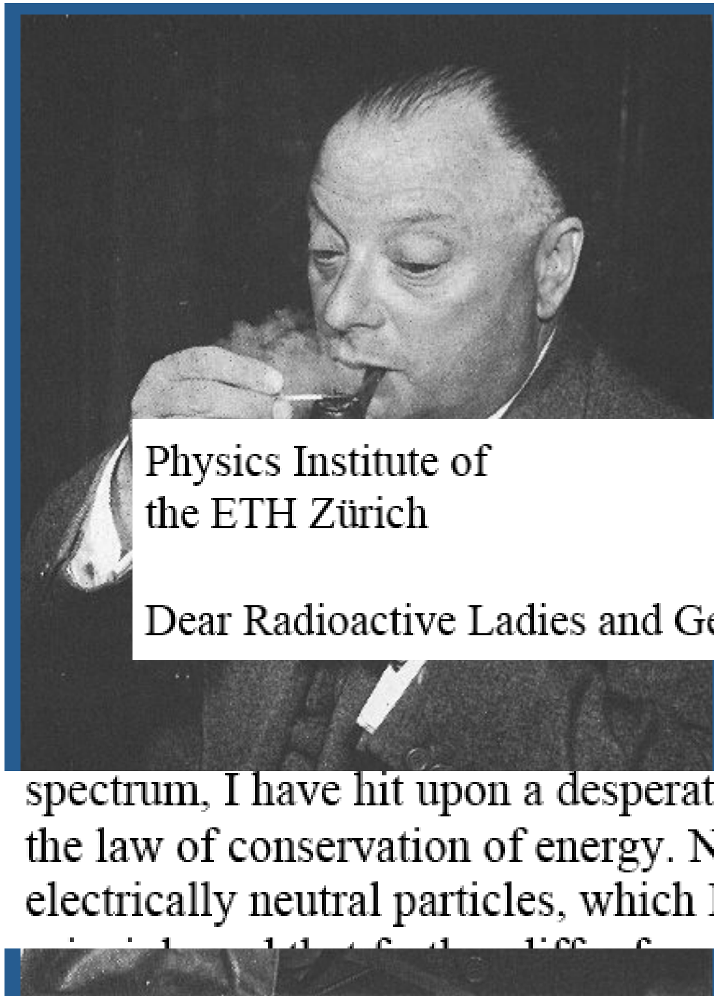
Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Physicist goes to a ball

or

ergy



Physics Institute of
the ETH Zürich   Zürich, Dec. 4, 1930

Dear Radioactive Ladies and Gentlemen,

spectrum, I have hit upon a desperate remedy to save the "exchange theorem" (1) of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei there could exist electrically neutral particles, which I will call neutrons, that have spin 1/2 and obey the exclusion

Wolfgang Pauli

way of rescue. Thus, dear radioactive people, scrutinize and judge. - Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7. With my best regards to you, and also to Mr. Back, your humble

principle

Pauli

Fermi

Majorana

Pontecorvo

Goeppert-Meyer

Neutrino Timeline

We will find out how neutrinos oscillate, why they play an important role in astrophysics/ cosmology and what they have to do with element production.

What is a particle?

The best answer follows from symmetry arguments!

What is a particle?

Wigner: A particle is an irreducible representation of the Poincare group.

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Recall the quantities Lorentz transformations leave invariant:

$$t^2 - \mathbf{r}^2 = \tau^2$$

$$E^2 - \mathbf{p}^2 = m^2$$

$$\mathbf{E}^2 - \mathbf{B}^2$$

$$\mathbf{E} \cdot \mathbf{B}$$

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Poincare group is the group including Lorentz boosts, translations and rotations.

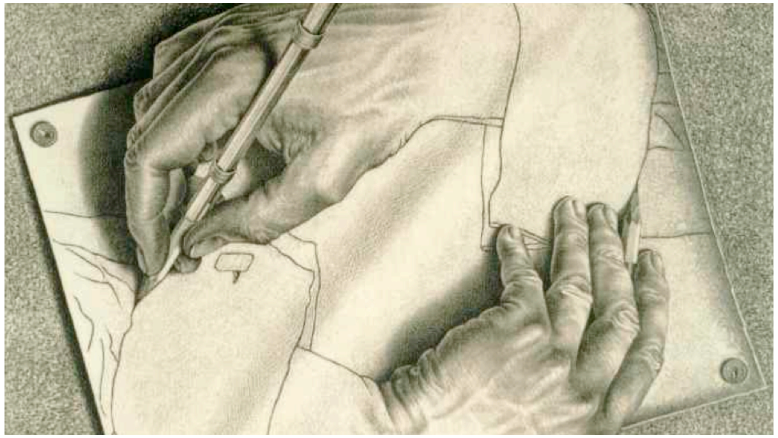
Next let us explore the concept of mass

What is mass?

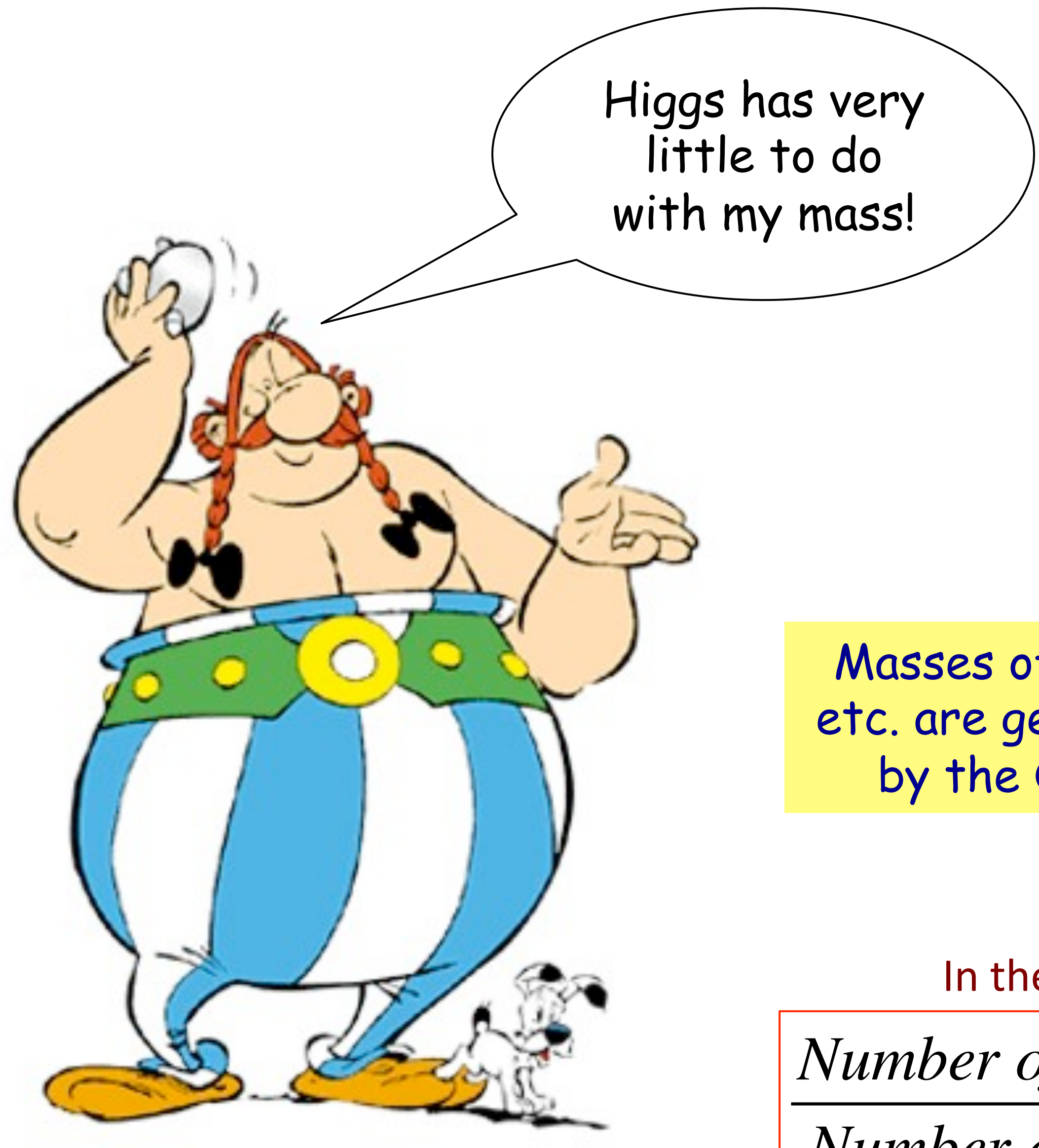
$$\Psi_L = \frac{1}{2}(1-\gamma_5)\Psi$$

$$\Psi_R = \frac{1}{2}(1+\gamma_5)\Psi$$

$$L = m\bar{\Psi}\Psi = m\left(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_l\right)$$



In the Standard Model all elementary masses possibly except those for neutrinos are generated by the Yukawa couplings of the Higgs.



Masses of protons, neutrons, etc. are generated dynamically by the QCD interactions!

In the Early Universe

$$\frac{Number\ of\ neutrons}{Number\ of\ protons} = \frac{e^{-m_n/T}}{e^{-m_p/T}}$$

Chiral representation of Dirac matrices

$$\gamma^0 = \beta = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \vec{\alpha} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & -\vec{\sigma} \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Helicity and chirality

Chirality: γ_5, Helicity: $\frac{\vec{\Sigma}\cdot\vec{k}}{|\vec{k}|}$, $\left[\gamma_5, \frac{\vec{\Sigma}\cdot\vec{k}}{|\vec{k}|}\right] = 0$

$$\gamma_5 |\lambda, \chi\rangle = \lambda |\lambda, \chi\rangle$$

$$\frac{\vec{\Sigma}\cdot\vec{k}}{|\vec{k}|} |\lambda, \chi\rangle = \chi |\lambda, \chi\rangle$$

$$\lambda = \pm 1, \quad \chi = \pm 1$$

These operators act on the fermion fields:

$$\Psi_s(\vec{r}) = \sum_{\vec{k}} \langle \vec{r} | \vec{k} \rangle a_s(\vec{k}), \quad s = 1,2,3,4$$

$$b^\dagger\left(\vec{k},\chi\right)=\sum_s a^\dagger\left(\vec{k}\right)\left\langle s\middle|\chi,\chi\right\rangle$$

$$d\left(-\vec{k},\chi\right)=\sum_s a^\dagger\left(\vec{k}\right)\left\langle s\middle|-\chi,\chi\right\rangle$$

Free, massless particle Hamiltonian:

$$H=\int d^3\vec{r}\ \Psi^\dagger(\vec{r})\vec{\alpha}\cdot\hat{p}\Psi(\vec{r})$$

$$=\sum_{\vec{k},\chi}\left|\vec{k}\right|\left[b^\dagger\left(\vec{k},\chi\right)b\left(\vec{k},\chi\right)-d\left(-\vec{k},\chi\right)d^\dagger\left(-\vec{k},\chi\right)\right]$$

"Dirac" mass term:

$$m_D\int d^3\vec{r}\ \Psi^\dagger(\vec{r})\beta\Psi(\vec{r})$$

$$=-\sum_{\vec{k},\chi}m_D\left[b^\dagger\left(\vec{k},\chi\right)d^\dagger\left(-\vec{k},-\chi\right)-d\left(-\vec{k},-\chi\right)b\left(\vec{k},\chi\right)\right]$$

Hence the total Hamiltonian is

$$H = \sum_{\vec{k},\chi} \left\{ \left|\vec{k}\right| \left[b^{\dagger}_{k\chi} b_{k\chi} - d_{-k\chi} d^{\dagger}_{-k\chi} \right] - m_D \left[b^{\dagger}_{k\chi} d^{\dagger}_{-k\chi} - d_{-k\chi} b_{k\chi} \right] \right\}$$

This Hamiltonian can be diagonalized by the transformation

$$\begin{pmatrix} B_{k\chi} \\ D^{\dagger}_{-k\chi} \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} b_{k\chi} \\ d^{\dagger}_{-k\chi} \end{pmatrix}$$

$$H = \sum_{k\chi} \sqrt{\vec{k}^{2} + m_D^2} \left(B^{\dagger}_{k\chi} B_{k\chi} - D_{-k\chi} D^{\dagger}_{-k\chi} \right)$$

$$\cos 2\vartheta = \frac{\left|\vec{k}\right|}{\sqrt{\vec{k}^{2} + m_D^2}} \qquad \sin 2\vartheta = \frac{m_D}{\sqrt{\vec{k}^{2} + m_D^2}}$$

Majorana mass term:

$$m_M \left(\left(\Psi_L \right)^C \right)^{\dagger} \beta \, \Psi_L$$

• Such a mass term violates lepton number conservation since it implies that neutrinos are their antiparticles.

Neutrino mass eigenstates are a combination of weak-interaction eigenstates: neutrinos mix!

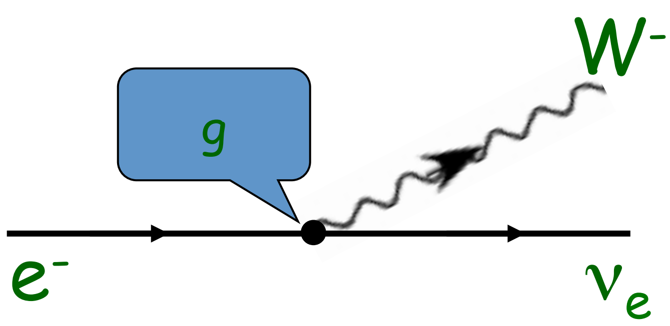
$$\nu_\alpha = \sum_i U_{\alpha i}\, \nu_i$$

$\alpha = e, \mu, \tau$

$i = 1,2,3,..$

U is unitary: $U^\dagger U = UU^\dagger = 1$

If the neutrino mass were zero this would be nothing more than a change of basis in the Standard Model:



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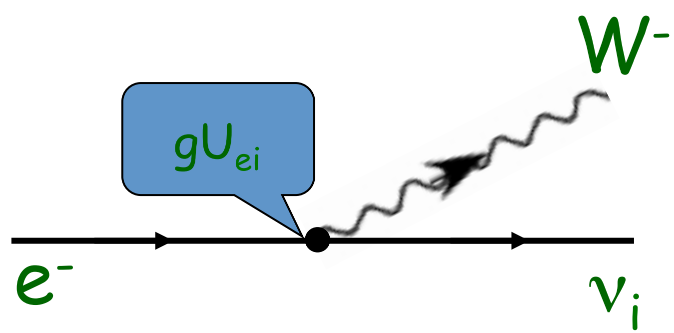

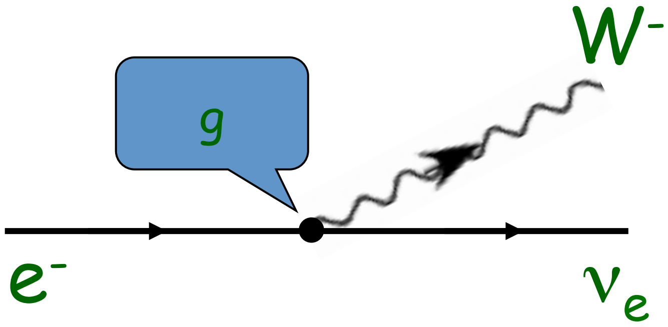

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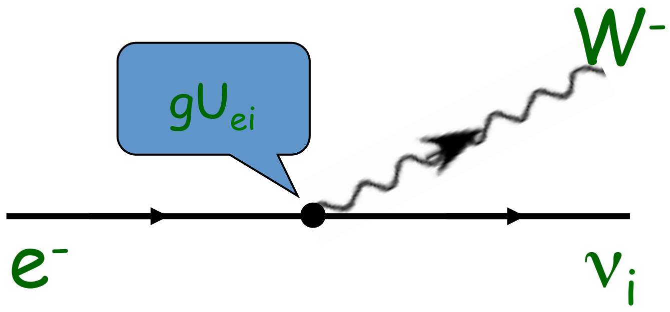


$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha}\gamma^{\lambda}\nu_{L\alpha}W_{\lambda}{}^{-} + \bar{\nu}_{L\alpha}\gamma^{\lambda}\ell_{L\alpha}W_{\lambda}{}^{+} \right)$$

$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha}\gamma^{\lambda}U_{\alpha i}\nu_{Li}W_{\lambda}{}^{-} + \bar{\nu}_{Li}\gamma^{\lambda}U_{\alpha i}{}^{*}\ell_{L\alpha}W_{\lambda}{}^{+} \right)$$

Left-handed

- Symmetries, in particular weak isospin invariance, define the Standard Model. The symmetry is $SU(2)_W x U(1)$.

- In the Standard Model, the left-handed and the right-handed components of the neutrino are treated differently: ν_L sits in an weak-isospin doublet (I_W =1/2) together with the left-handed component of the associated charged lepton, whereas ν_R is an weak-isospin singlet (I_W=0).

SU(2) × U(1) Standard Model

Weak isospin

$$SU(2)_W: \quad I^{(W)} = \frac{1}{2} \Rightarrow \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \begin{matrix} I_3^{(W)} = +\frac{1}{2} \\ I_3^{(W)} = -\frac{1}{2} \end{matrix}$$

Weak singlets

$$I^{(W)} = 0: \;\; \nu_R, \, e_R$$

Higgs Field sits in a weak doublet with $I_3^{(W)} = -\frac{1}{2}$.

• Symmetries, in particular weak isospin invariance, define the Standard Model. The symmetry is $SU(2)_W \times U(1)$.

• In the Standard Model, the left-handed and the right-handed components of the neutrino are treated differently: ν_L sits in an weak-isospin doublet ($I_W = 1/2$) together with the left-handed component of the associated charged lepton, whereas ν_R is an weak-isospin singlet ($I_W = 0$).

• A mass term connects left- and right-handed components. The usual Dirac mass term is $L = m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$. But such a neutrino mass term requires a right-handed neutrino, hence it is not in the Standard Model.

• The right-handed component of the neutrino carries no weak isospin quantum numbers. This permits *Majorana neutrino mass* in the Standard Model if one only uses right-handed neutrinos.

A very brief introduction to the effective field theories

A note on dimensional counting

- Lagrangian, L, has dimensions of energy (or mass).
- $L = \int d^3x\, \mathcal{L} \Rightarrow$ Lagrangian density, $\mathcal{L}$, has dimensions of energy/volume or M^4.
- Define the scaling dimension of x, [x] to be -1 $\Rightarrow$ scaling dimension of momentum (or mass) is [m] = +1 (recall that (p.x/h) is dimensionless and we take [h]=0).
- Clearly $[\mathcal{L}] = 4$. This should be true for any Lagrangian density of any theory.
- Consider the mass term for fermions, $\mathcal{L}_m = m\, \bar{\Psi}\Psi$. Then $[\bar{\Psi}\Psi] = 3$ or $[\Psi] = 3/2$.
- In the Standard Model the Higgs field vacuum expectation value gives the particle mass: $\mathcal{L} = H\, \bar{\Psi}\Psi$. Hence [H] = 1.

An example for the effective field theories

Euler-Heisenberg correction to the Q.E.D. Lagrangian

$$L = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \underbrace{\text{terms which are higher order in fields}}_{\text{Consistent with the symmetries of the system}}$$

Symmetries of the Electromagnetism

Lorentz Invariants: $\mathbf{E}^2 - \mathrm{B}^2$ and $\mathbf{E} \cdot \mathrm{B}$

$$\underbrace{\begin{aligned} \mathbf{E} &\rightarrow \mathbf{E} \\ \mathrm{B} &\rightarrow -\mathrm{B} \\ \mathbf{E}^2 - \mathrm{B}^2 &\rightarrow \mathbf{E}^2 - \mathrm{B}^2 \\ \mathbf{E} \cdot \mathrm{B} &\rightarrow -\mathbf{E} \cdot \mathrm{B} \end{aligned}}_{Under\ time-reversal}$$

Since we want the Lagrangian density to be *invariant* under *both* Lorentz *and* time-reversal transformations we pick $\mathbf{E}^2 - \mathrm{B}^2$.

An example for the effective field theories

Euler-Heisenberg correction to the Q.E.D. Lagrangian

$$L = \frac{1}{2}\,(\mathbf{E}^2 - \mathbf{B}^2) + \frac{2\alpha^2}{45 m_e^4}\left[(\mathbf{E}^2 - \mathbf{B}^2)^2 + 7\,(\mathbf{E}\cdot\mathbf{B})^2\right]$$

Using the Standard Model degrees of freedom one can parameterize the neutrino mass by a dimension 5 operator. (Recall that $I_3^W = 1/2$ for the ν_L and -1/2 for H_{SM}).

$$L = X_{\alpha\beta} H_{SM} H_{SM} \overline{\nu_{L\alpha}^{C}} \nu_{L\beta} / \Lambda$$

$$v^2 X_{\alpha\beta} / \Lambda = U m_\nu^{diagonal} U^T$$

This term is not renormalizable! It is the only dimension-five operator one can write using the Standard Model degrees of freedom. Hence the neutrino mass is the most accessible new physics beyond the Standard Model!

There are other ways to obtain neutrino mass:

$$L = H_{I=1} \overline{\nu_{L\alpha}{}^{c}} \, \nu_{L\beta}$$

Note: This Higgs is not in the Standard Model!

At lower energies, Beyond Standard Model physics is described by local operators

$$L = L_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \ldots$$

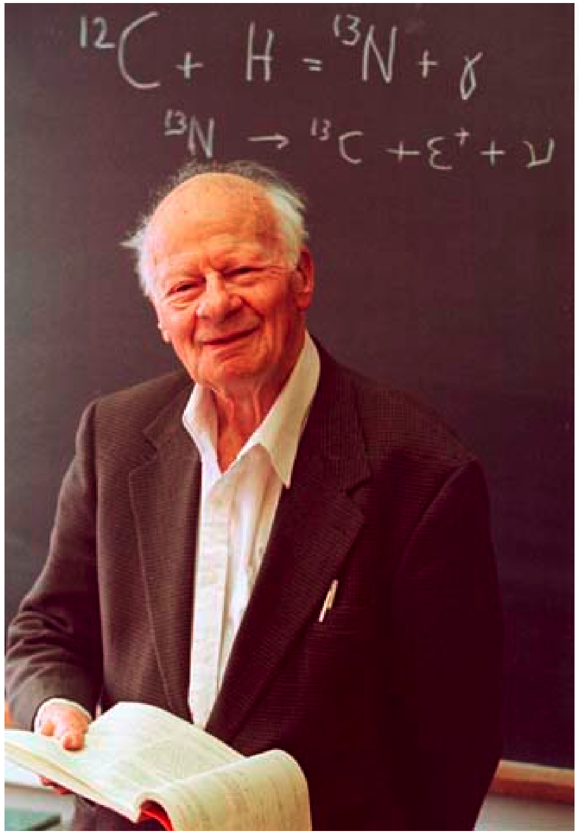
Majorana neutrino mass (unique)

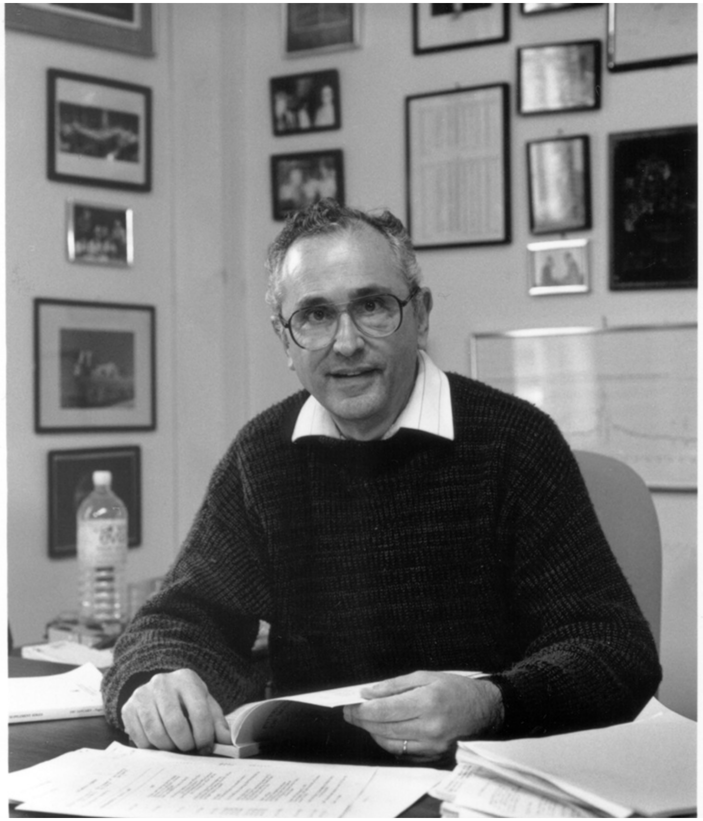
Includes Majorana neutrino magnetic moment

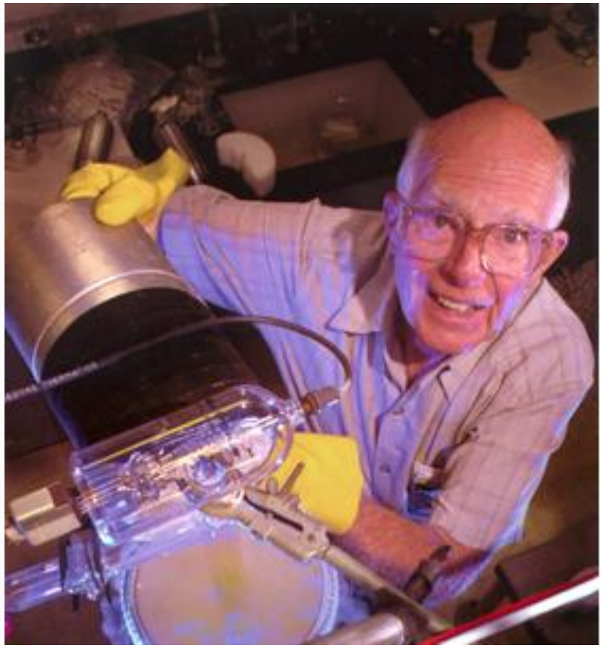
Majorana mass term:

$$m_M \left((\Psi_L)^C \right)^{\dagger} \beta \, \Psi_L$$

- Such a mass term violates lepton number conservation since it implies that neutrinos are their antiparticles.
- It is permitted by the weak-isospin invariance of the Standard Model.
- Neutrino mass terms are not included in the fundamental Lagrangian of the Standard Model. They arise from new physics. Of course it is possible to write down an *effective* Lagrangian for the neutrino mass in terms of only the Standard Model fields if you give up renormalizability.



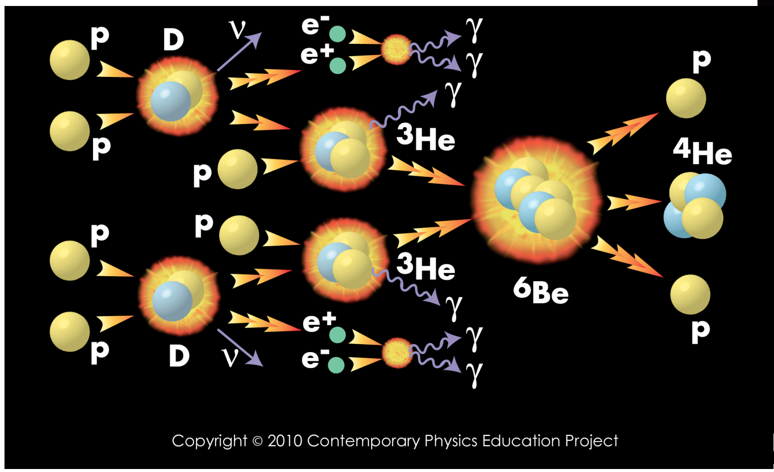


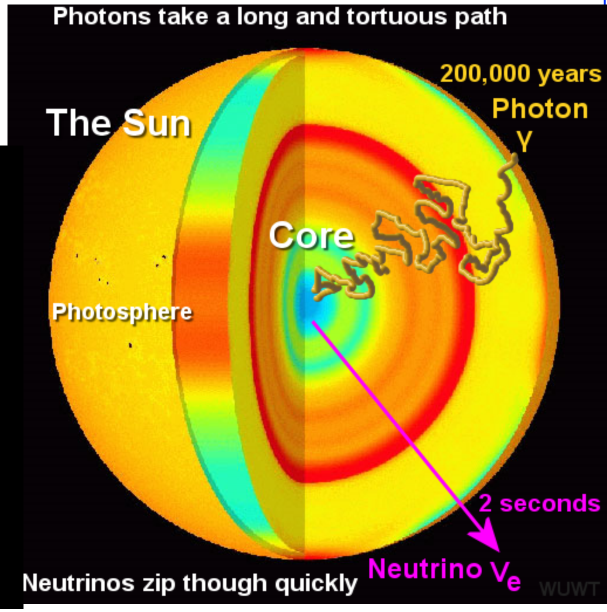


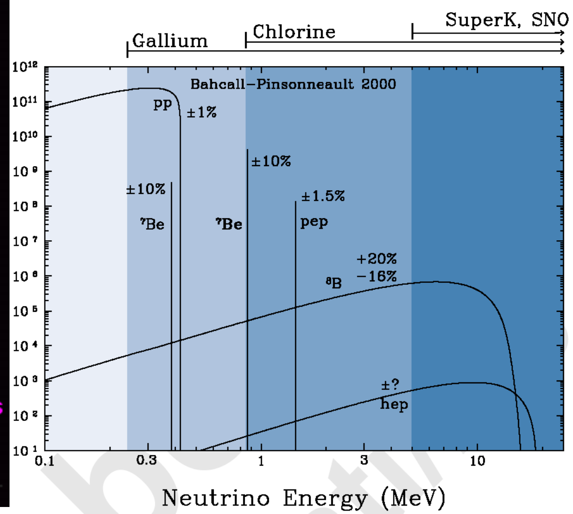
"...to see into the interior of a star and thus verify directly the hypothesis of nuclear energy generation.."

Bahcall and Davis, 1964

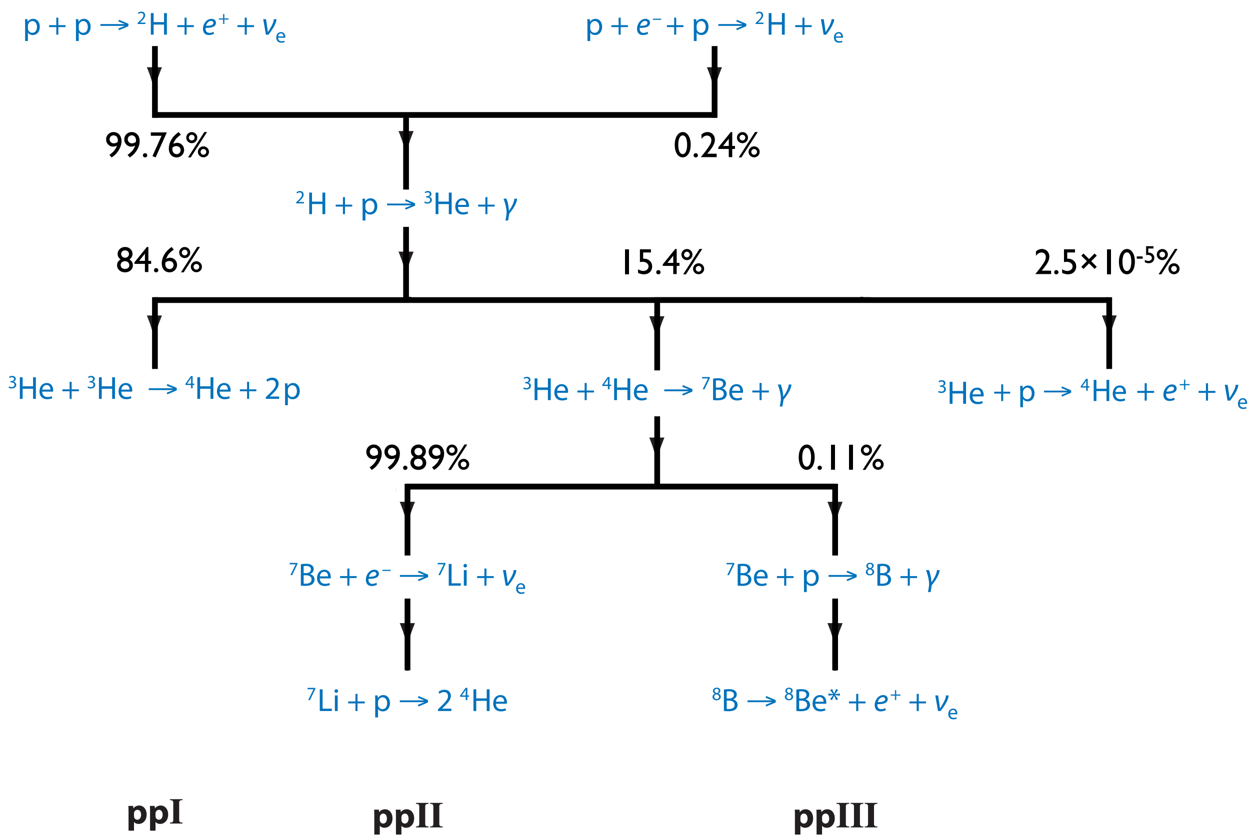
Solar Neutrinos

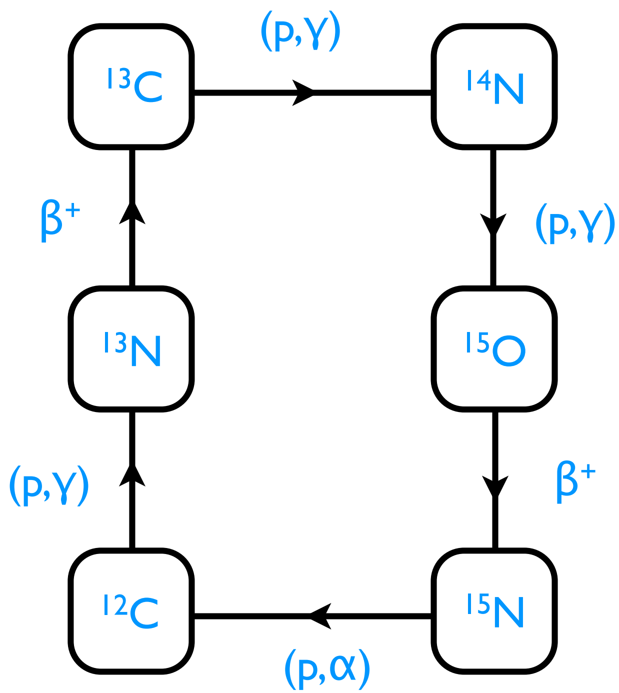






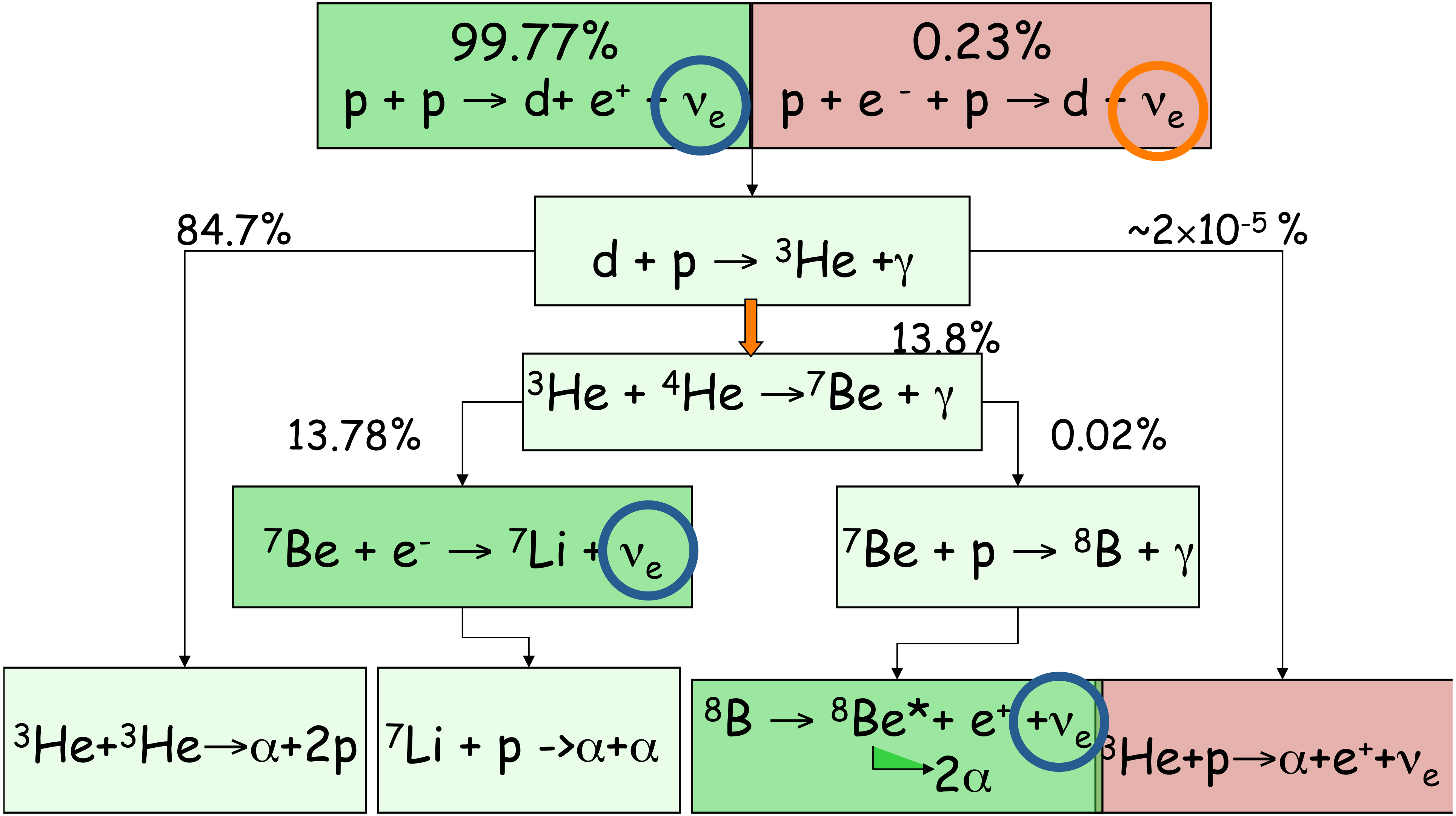
Neutrino producing reactions in the Sun





CN cycle

Nuclear reaction network in the Sun



Three paths leading to neutrinos are called pp-I, pp-II and pp-III chains, respectively.

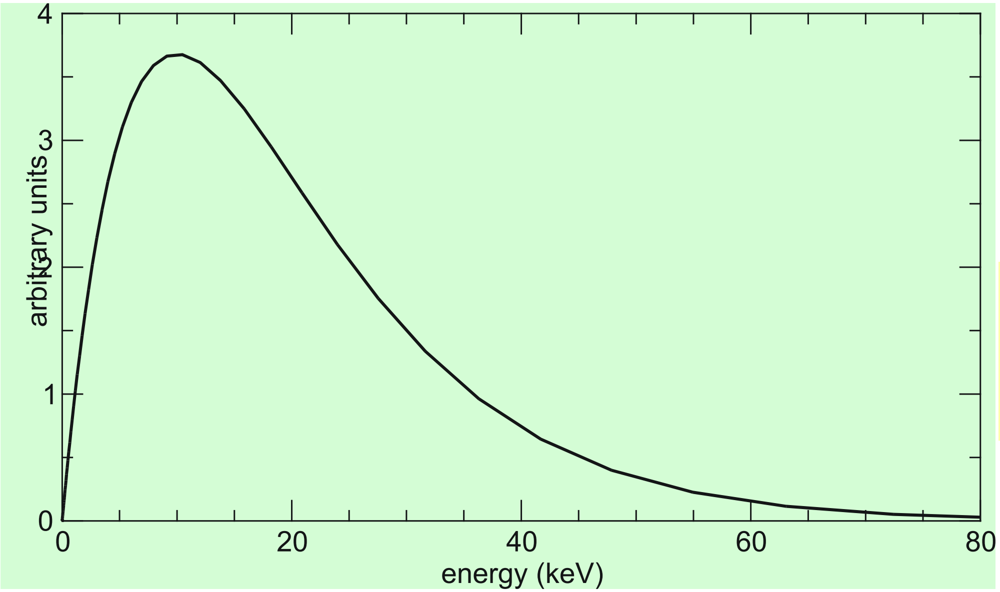
Reaction rates in astrophysical environments

Number of reactions per target nucleus per second: → $\lambda = \sigma n_p v$

Total number of reactions per second when the number of target nuclei per unit volume is n_T: → $R = \sigma n_p v n_t V$

Reaction rate per second and per unit volume: →

$$r = \frac{1}{1+\delta_{pt}} n_p n_t \sigma v$$



The probability Φ(v) to find a particle with a velocity between v and v+dv

$$\Phi(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 \, e^{-\frac{mv^2}{2kT}}, \int \Phi(v)\, dv = 1$$

Stellar reaction rates has to be averaged over this distribution

$$r = \frac{1}{1+\delta_{pt}} n_p n_t \int \sigma(v) \Phi(v) v \, dv = \frac{1}{1+\delta_{pt}} n_p n_t < \sigma v >$$

Initial Reaction Network for the pp Chain in the Sun

$$\frac{d[H]}{dt} = -2\lambda_{11}\frac{[H]^2}{2} - \lambda_{12}[H][D] + 2\lambda_{33}\frac{[He^3]^2}{2} - \lambda_{17}[H][Be^7] - \lambda'_{17}[H][Li^7]$$

$$\frac{d[D]}{dt} = \lambda_{11}\frac{[H]^2}{2} - \lambda_{12}[H][D]$$

$$\frac{d[He^3]}{dt} = \lambda_{12}[H][D] - 2\lambda_{33}\frac{[He^3]^2}{2} - \lambda_{34}[He^3][He^4]$$

$$\frac{d[He^4]}{dt} = \lambda_{33}\frac{[He^3]^2}{2} - \lambda_{34}[He^3][He^4] + 2\lambda_{17}[H][Be^7] + 2\lambda'_{17}[H][Li^7]$$

$$\frac{d[Be^7]}{dt} = \lambda_{34}[He^3][He^4] - \lambda_{17}[H][Be^7] - \lambda_{e7}[e][Be^7]$$

$$\frac{d[Li^7]}{dt} = \lambda_{e7}[e][Be^7] - \lambda'_{17}[H][Li^7]$$

Reaction Network After the Deuterium Equilibrium

$$
\begin{aligned}
\frac{d[H]}{dt} &= -3\lambda_{11}\frac{[H]^2}{2} + 2\lambda_{33}\frac{[He^3]^2}{2} - \lambda_{17}[H][Be^7] - \lambda'_{17}[H][Li^7] \\
\frac{d[He^3]}{dt} &= \lambda_{11}\frac{[H]^2}{2} - 2\lambda_{33}\frac{[He^3]^2}{2} - \lambda_{34}[He^3][He^4] \\
\frac{d[He^4]}{dt} &= \lambda_{33}\frac{[He^3]^2}{2} - \lambda_{34}[He^3][He^4] + 2\lambda_{17}[H][Be^7] + 2\lambda'_{17}[H][Li^7] \\
\frac{d[Be^7]}{dt} &= \lambda_{34}[He^3][He^4] - \lambda_{17}[H][Be^7] - \lambda_{e7}[e][Be^7] \\
\frac{d[Li^7]}{dt} &= \lambda_{e7}[e][Be^7] - \lambda'_{17}[H][Li^7]
\end{aligned}
$$

Reaction Network After the Li and Be Equilibrium

$$\frac{d[H]}{dt} = -3\lambda_{11}\frac{[H]^2}{2} + 2\lambda_{33}\frac{[He^3]^2}{2} - \lambda_{34}[He^3][He^4]$$

$$\frac{d[He^3]}{dt} = \lambda_{11}\frac{[H]^2}{2} - 2\lambda_{33}\frac{[He^3]^2}{2} - \lambda_{34}[He^3][He^4]$$

$$\frac{d[He^4]}{dt} = \lambda_{33}\frac{[He^3]^2}{2} + \lambda_{34}[He^3][He^4]$$

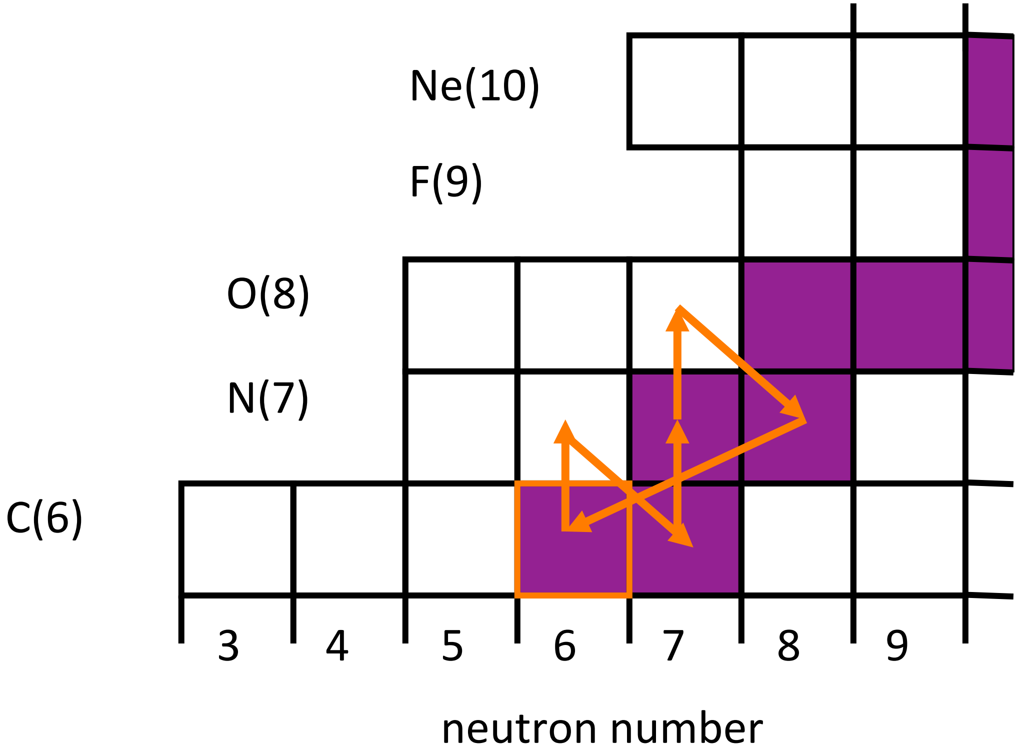
Reaction Network After the He^3 Equilibrium

$$\frac{d[H]}{dt} = -\lambda_{11}[H]^2 - 2\lambda_{34}[He^3]_{eq}[He^4]$$

$$\frac{d[He^4]}{dt} = \frac{1}{4}\lambda_{11}[H]^2 + \frac{1}{2}\lambda_{34}[He^3]_{eq}[He^4]$$

$$\frac{d[He^4]}{dt} = -\frac{1}{4}\frac{d[H]}{dt}$$

The CN cycle



$^{12}C(p,\gamma)^{13}N \quad (\beta^+)^{13}C \quad (p,\gamma)^{14}N \quad (p,\gamma)^{15}O \quad (\beta^+)^{15}N \; (p,\alpha)^{12}C$

Net effect: $4p \rightarrow \alpha + 2e^+ + 2\nu_e$

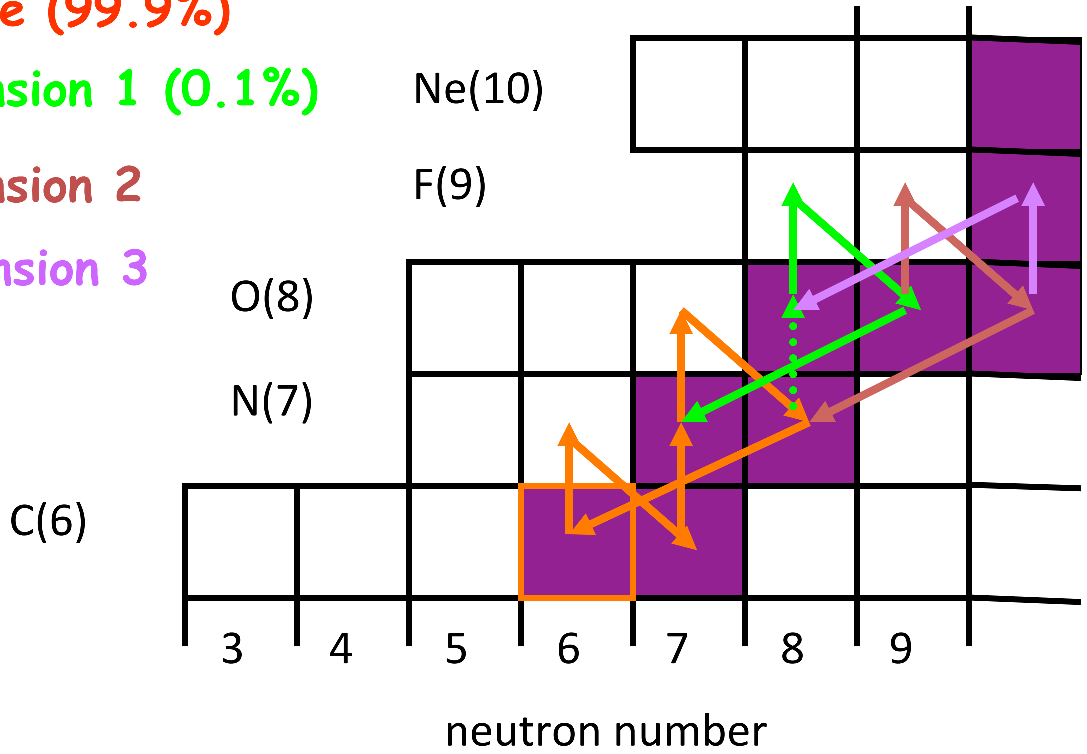
CNO cycle

CN cycle (99.9%)

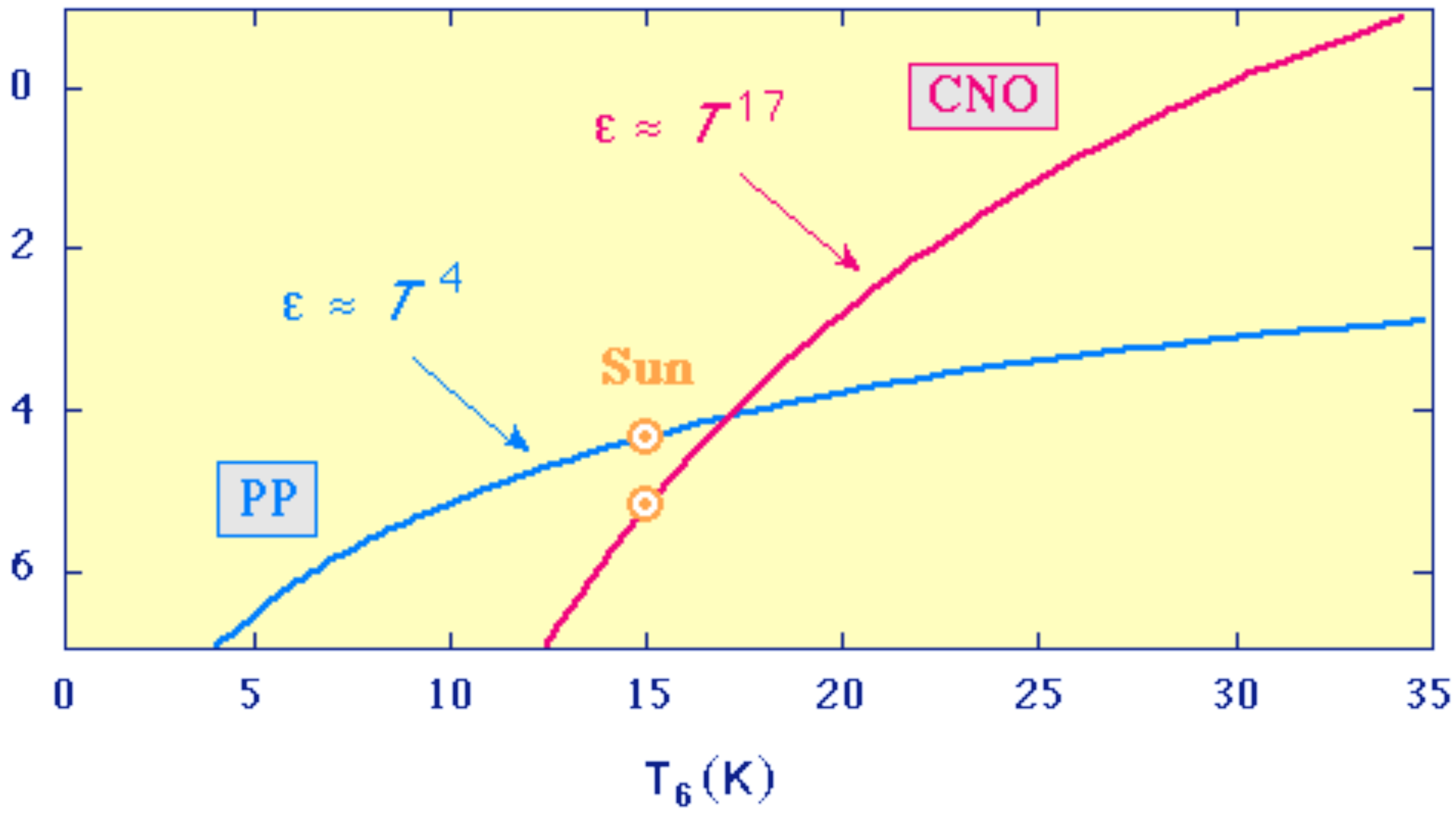
O Extension 1 (0.1%)

O Extension 2

O Extension 3



Competition between the p-p chain and the CNO Cycle



Helioseismology - Definitions of Characterizing Quantities

Static, stable star at spherically-symmetric equilibrium

- Pressure $p(r)$,
- Mass density $\rho(r)$,
- Gravitational potential $\phi(r)$,
- Rate of nuclear energy generation $\epsilon(r)$,
- Temperature $T(r)$,
- Energy flux $\mathbf{F}$,
- Entropy s.
- Adiabatic indices

$$\Gamma_1 = \left(\frac{\partial \log p}{\partial \log \rho}\right)_s \qquad \Gamma_3 - 1 = \left(\frac{\partial \log T}{\partial \log \rho}\right)_s,$$

- The total derivative $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$

A static star:

Equation of motion

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \rho \nabla \phi$$

Poisson's equation for gravitational attraction

$$\nabla^2 \phi = 4\pi G \rho$$

Equation of continuity

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0$$

Energy conservation

$$\frac{1}{p}\frac{Dp}{Dt} - \Gamma_1 \frac{1}{\rho}\frac{D\rho}{Dt} = \frac{\Gamma_3 - 1}{p}(\rho\epsilon - \nabla \cdot \mathbf{F})$$

Stellar Seismology

Eulerian perturbations

$$\rho(\mathbf{r},t) = \rho_0(\mathbf{r}) + \rho'(\mathbf{r},t), \quad \mathbf{v} = \frac{\partial}{\partial t}(\delta\mathbf{r})$$

Conservation of momentum

$$\rho\frac{\partial^2\delta\mathbf{r}}{\partial t^2} = -\nabla p' + \frac{\rho'}{\rho}\nabla p - \rho\nabla\phi'$$

Equation of continuity

$$\rho' + \nabla\cdot(\rho\delta\mathbf{r}) = 0$$

Poisson's equation

$$\nabla^2\phi' = 4\pi G\rho'$$

Energy conservation

$$\frac{\rho'}{\rho} + \frac{1}{\rho}\delta\mathbf{r}\cdot\nabla\rho = \frac{1}{\Gamma_1}\left(\frac{p'}{p} + \frac{1}{p}\delta\mathbf{r}\cdot\nabla p\right)$$

Normal modes

periodic time dependence

$$\rho'(\mathbf{r},t) \sim \rho'(r) Y_{\ell m}(\theta,\phi) \exp(-i\omega t)$$

Adiabatic sound speed

$$c^2 = \frac{\Gamma_1 p}{\rho}$$

Stellar oscillations

$$\frac{d^2\Psi}{dr^2} + \frac{1}{c^2}\left[\omega^2 - \omega_{\rm co}^2 - \frac{\ell(\ell+1)c^2}{r^2}\left(1 - \frac{N^2}{\omega^2}\right)\right]\Psi \simeq 0$$

$$\boxed{\Psi(r) = c^2 \rho^{1/2} \nabla \cdot \delta\mathbf{r}}$$

Acoustical cut-off freq.

$$\omega_{\rm co}^2 = \frac{c^2}{4H^2}\left(1 - 2\frac{dH}{dr}\right)$$

$$H = -(d \log \rho / dr)^{-1}$$

density scale height

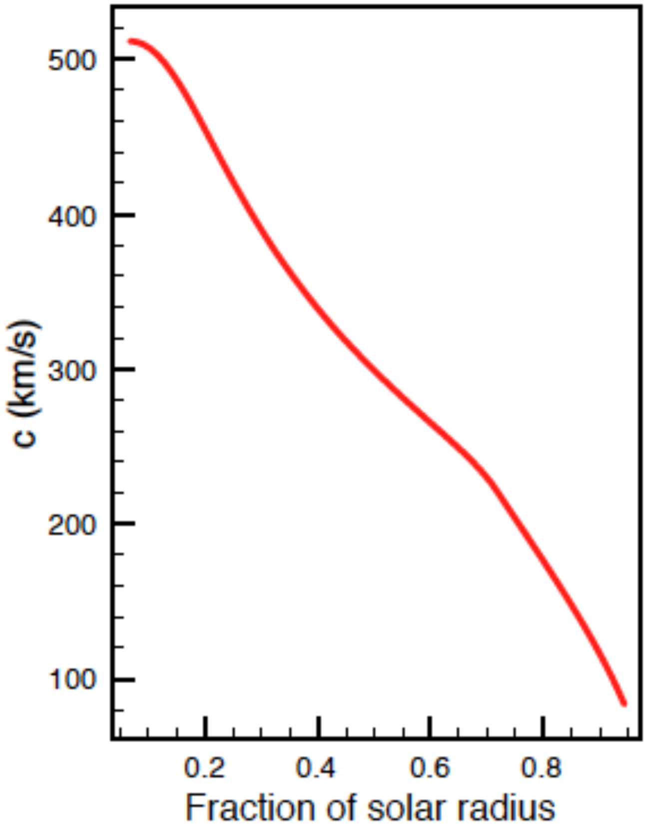
Buoyancy frequency

$$N^2 = \frac{Gm_0(r)}{r}\left(\frac{1}{\Gamma_1}\frac{d\log p}{dr} - \frac{d\log \rho}{dr}\right)$$

p- and g-modes

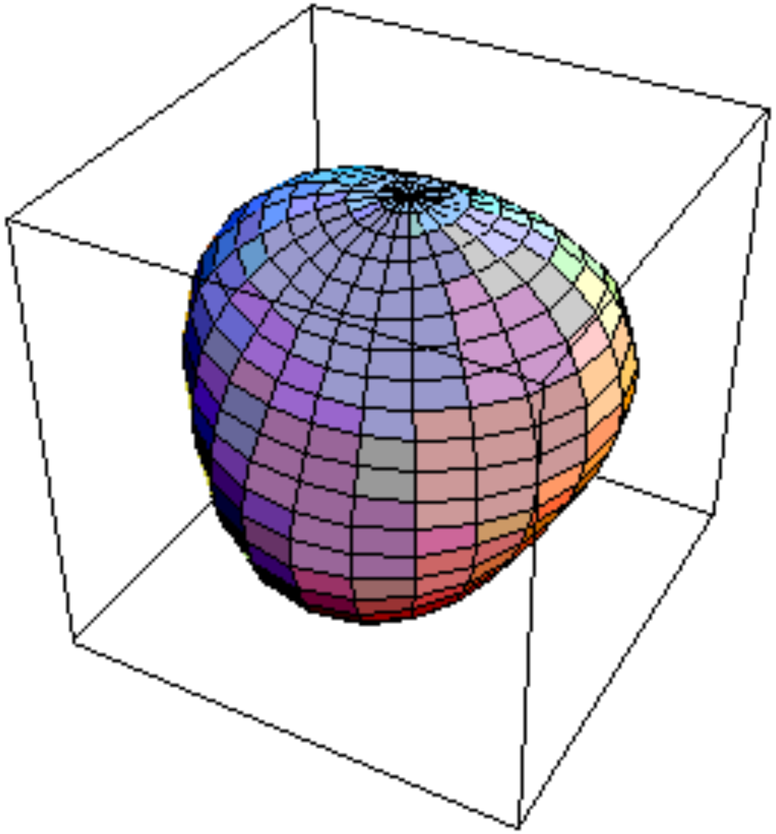
Stellar oscillations

$$\frac{d^2\Psi}{dr^2} + \frac{1}{c^2}\left[\omega^2 - \omega_{\rm co}^2 - \frac{\ell(\ell+1)c^2}{r^2}\left(1 - \frac{N^2}{\omega^2}\right)\right]\Psi \simeq 0$$



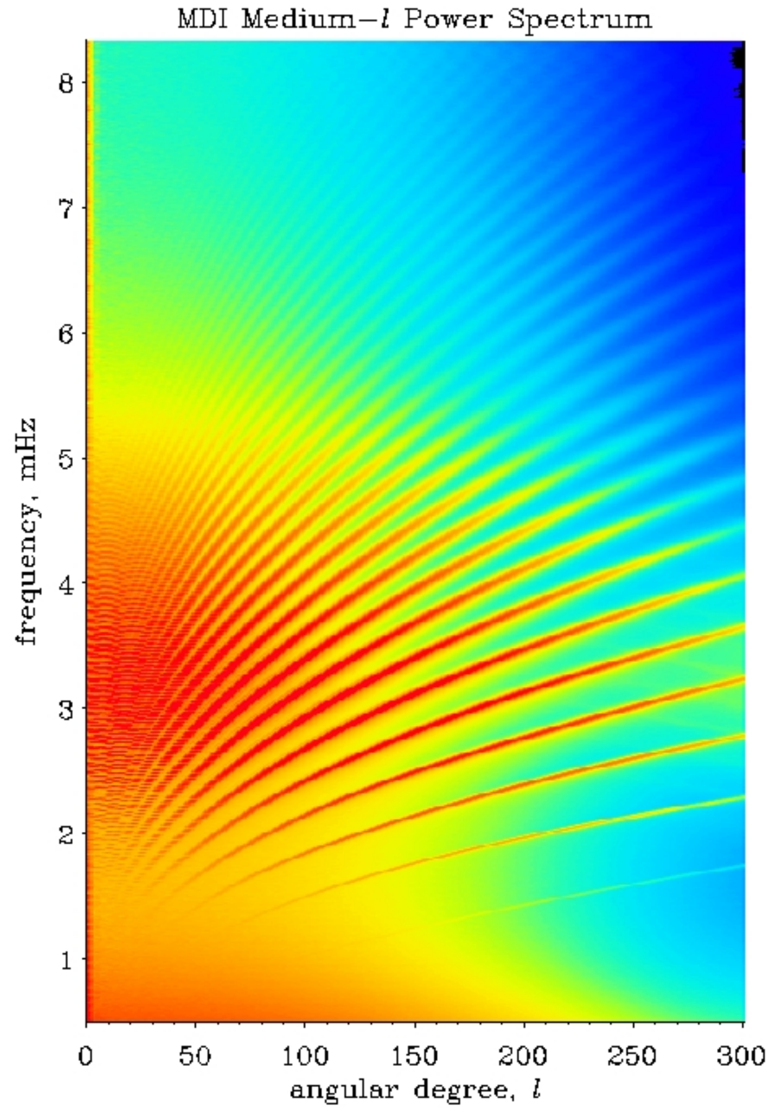
- N is $\sim$ constant in the radiative zone, but zero in the convective zone
- $\omega_{\rm co}$ is monotonically decreasing
- $N^2/\omega^2 \ll 1 \Rightarrow$ the oscillations die out in the radiative zone (p-modes)
- $\frac{\ell(\ell+1)c^2}{r^2\omega^2} \gg 1 \Rightarrow$ the oscillations die out in the convective zone (g-modes)

Example: L=3, m=2 p-mode

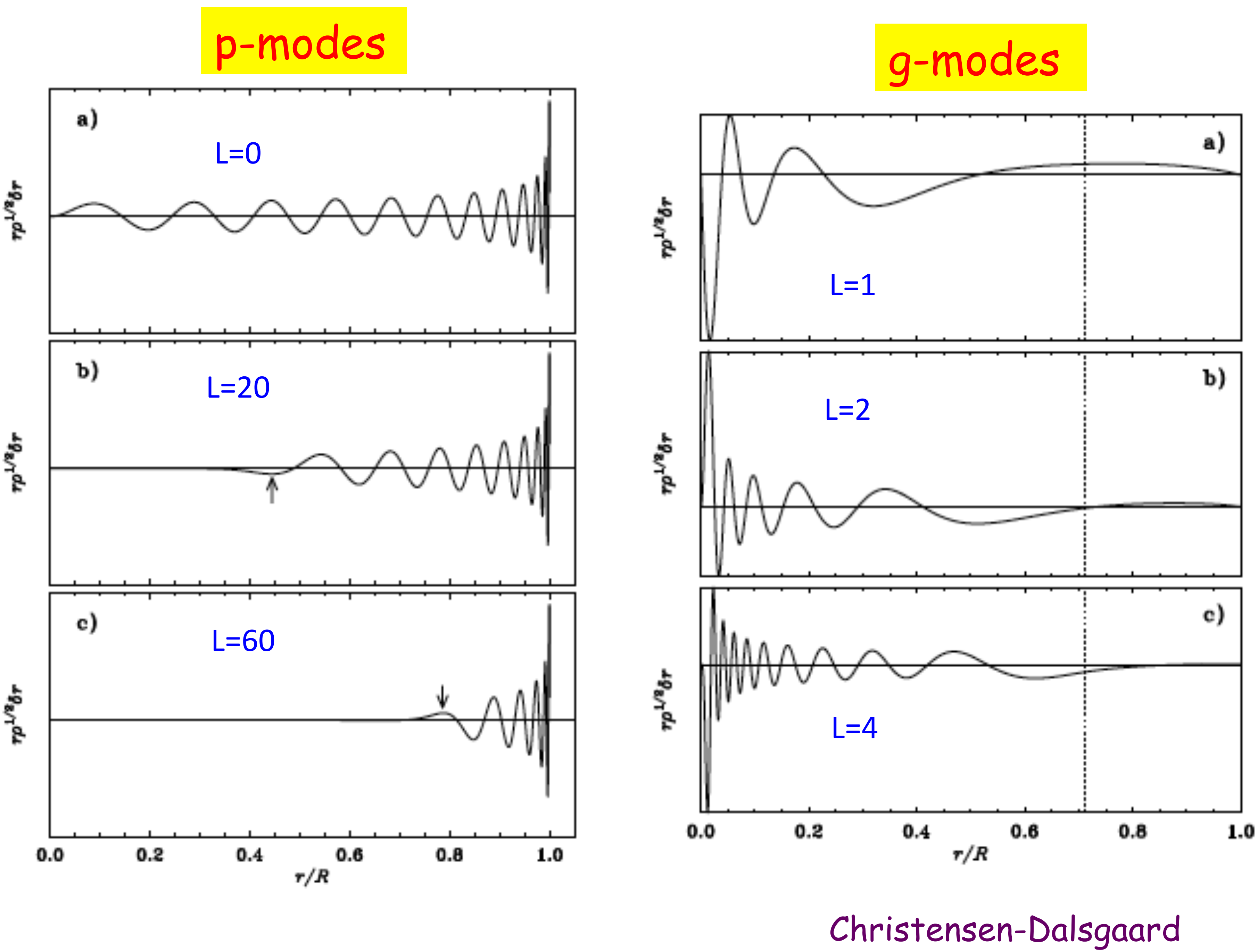


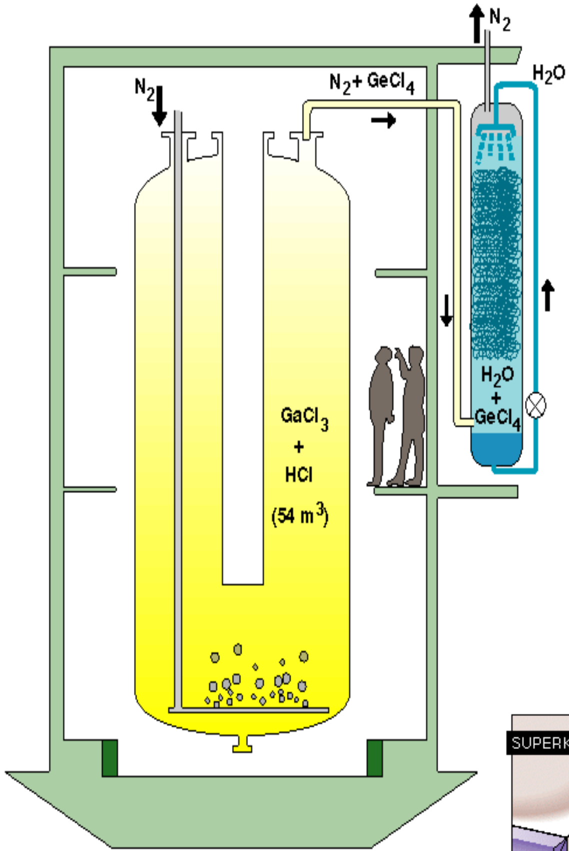
From D. Guenther

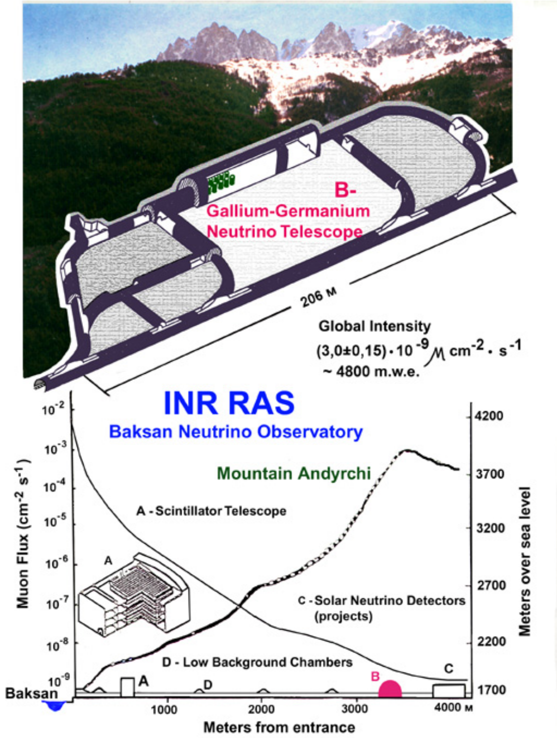

Single Dopplergram Minus 45 Images Average
(30-MAR-96 19:54:00)
-500. -400. -300. -200. -100. 0. 100. 200. 300. 400. 500.
Velocity (m/s)
SOI / MDI
Stanford Lockheed Institute for Space Research

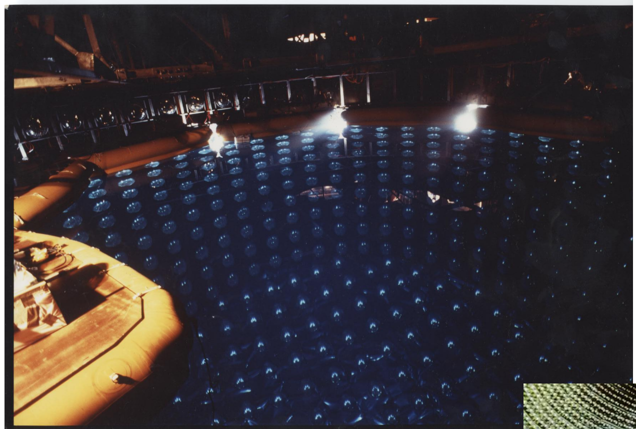

MDI Medium-l Power Spectrum
frequency, mHz
1
2
3
4
5
6
7
8
0 50 100 150 200 250 300
angular degree, l

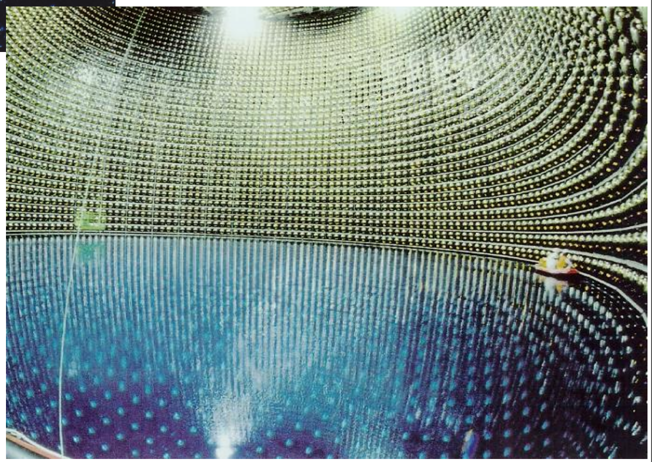
SOHO data

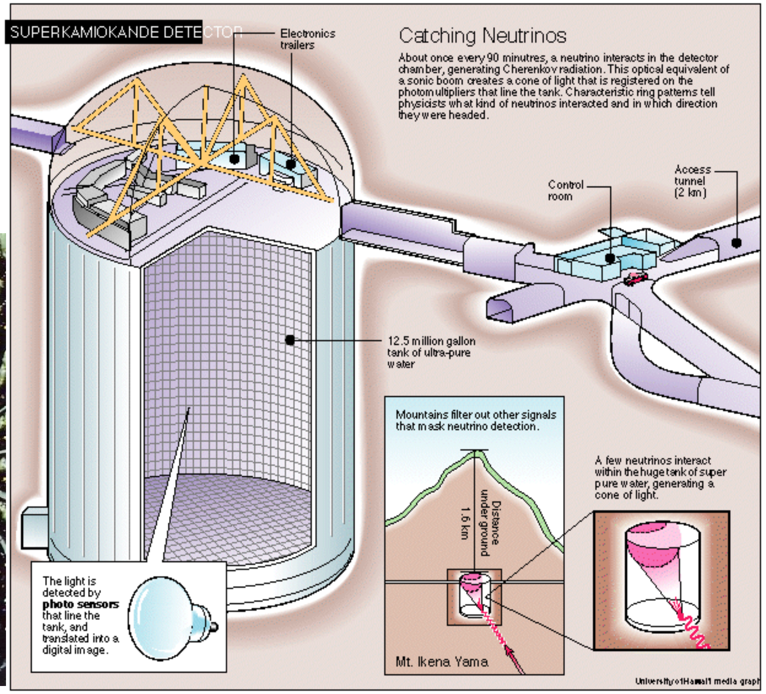

p-modes
a)
L=0
b)
L=20
c)
L=60
0.0
0.2
0.4
0.6
0.8
1.0
r/R
g-modes
a)
L=1
b)
L=2
c)
L=4
0.0
0.2
0.4
0.6
0.8
1.0
r/R
Christensen-Dalsgaard





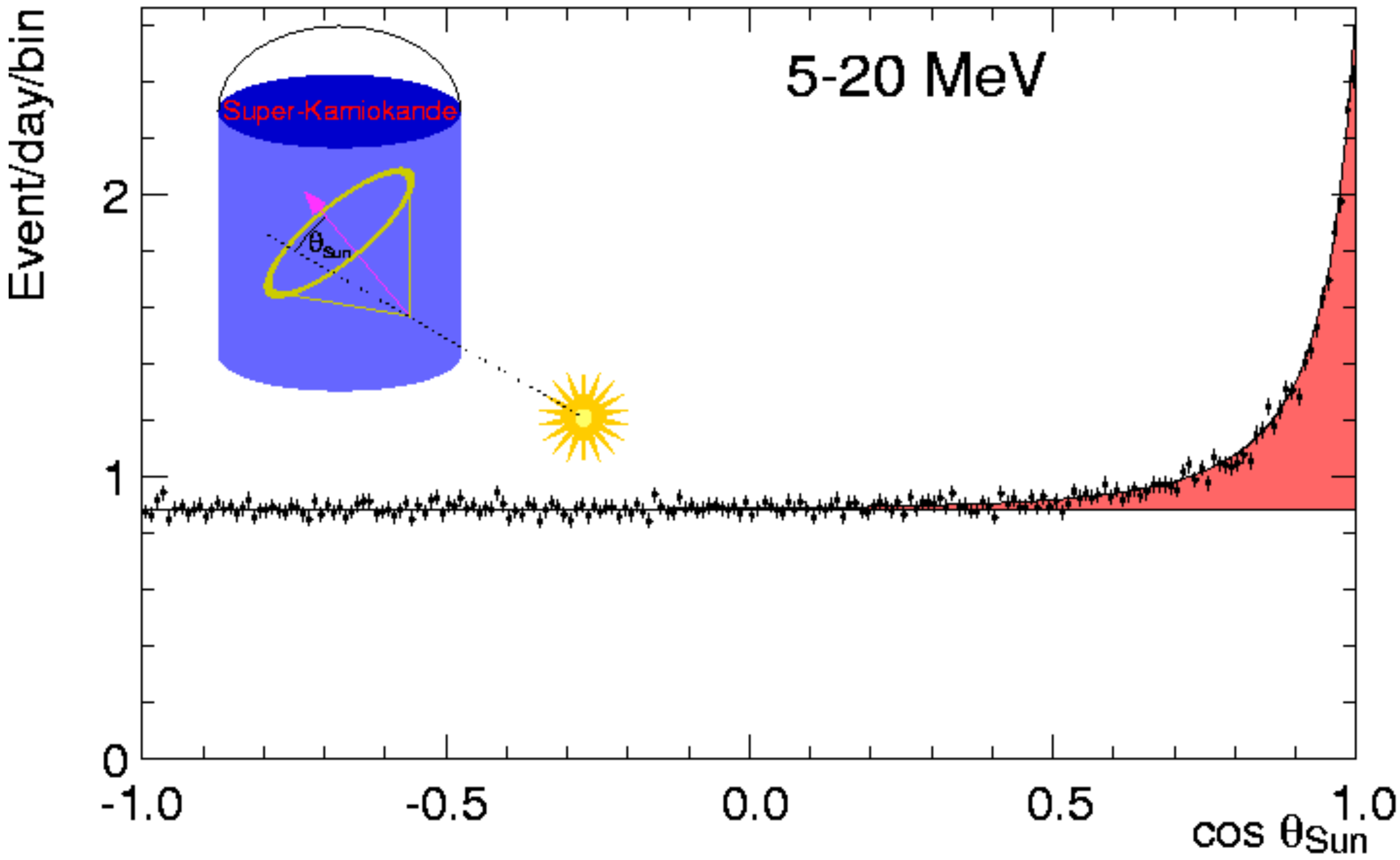




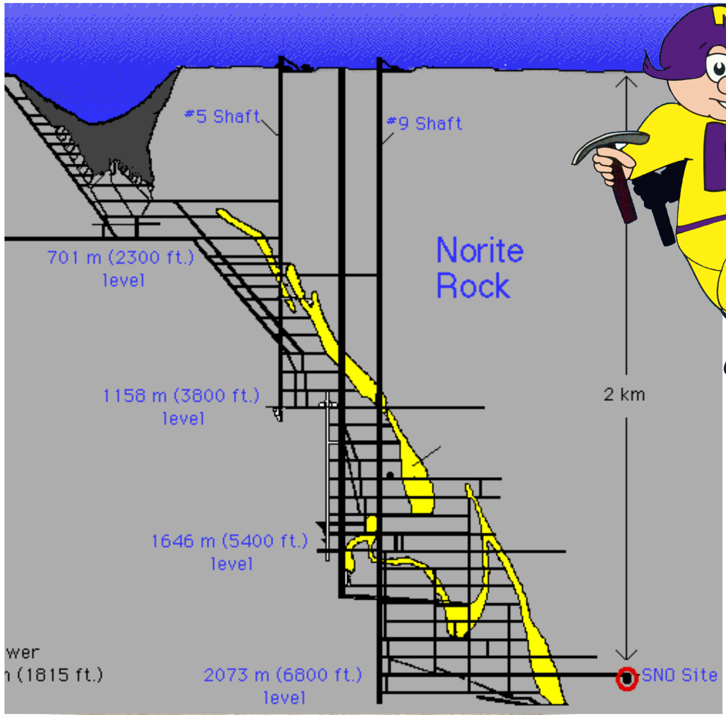


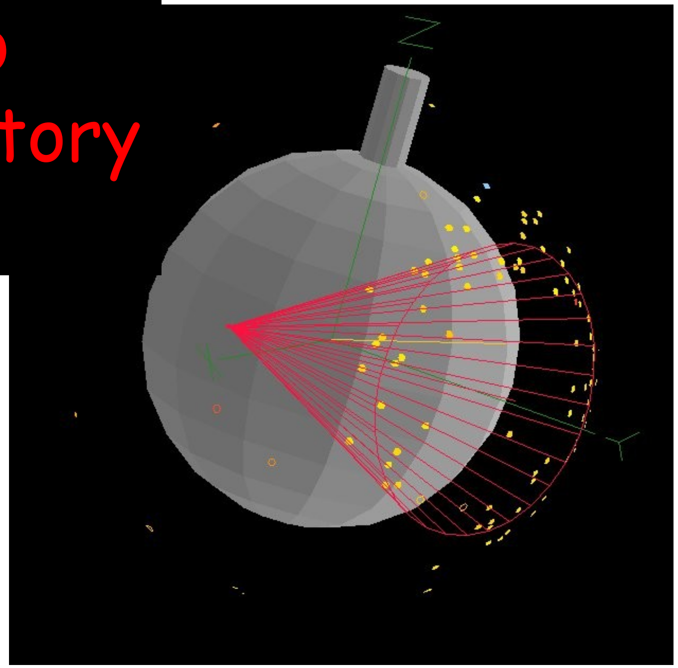
Solar neutrino experiments

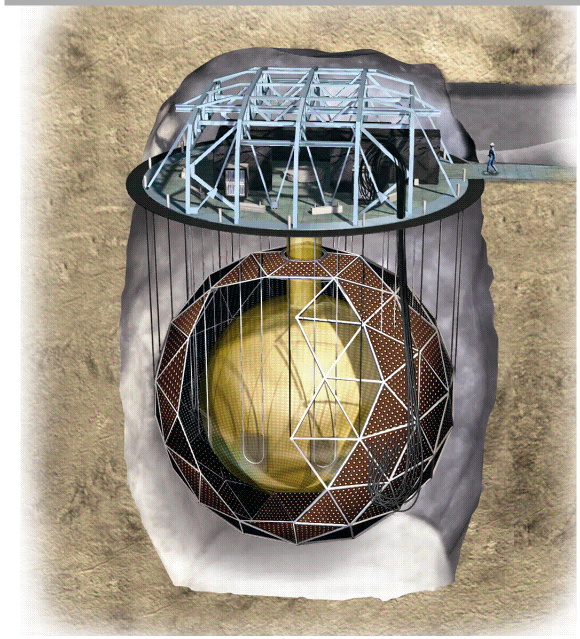
SuperKamiokande-I ^{8}B solar ν's

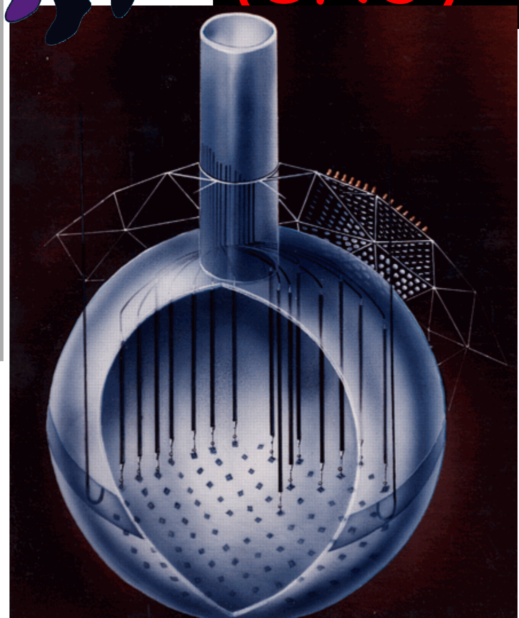


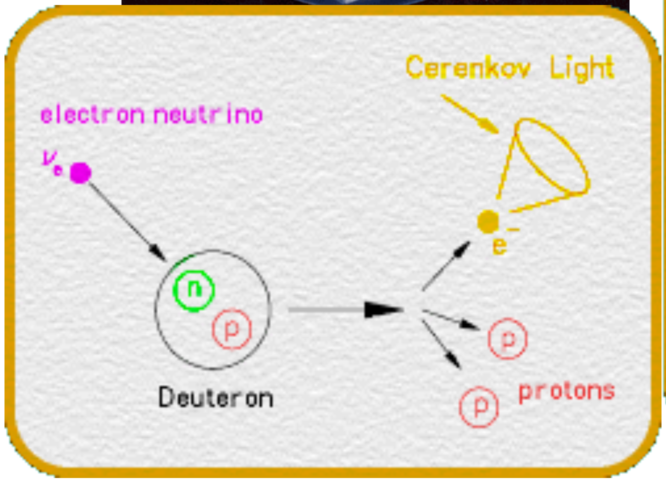
Sudbury Neutrino Observatory (SNO)

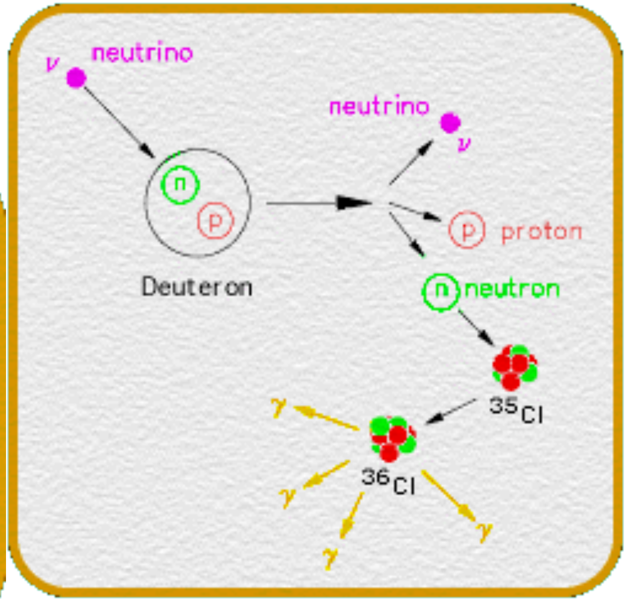






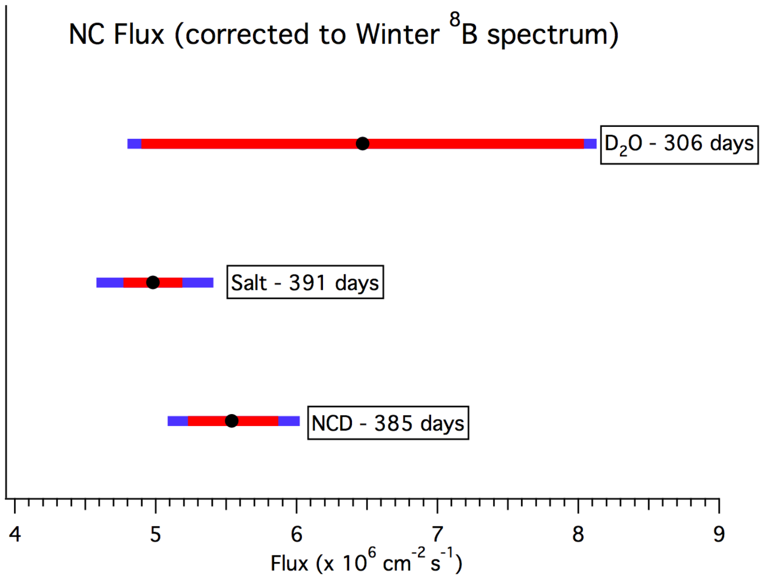


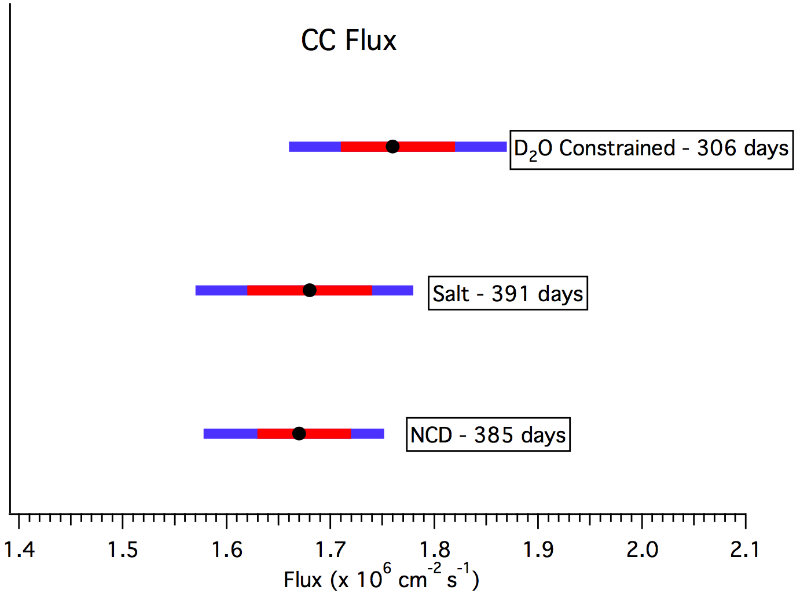


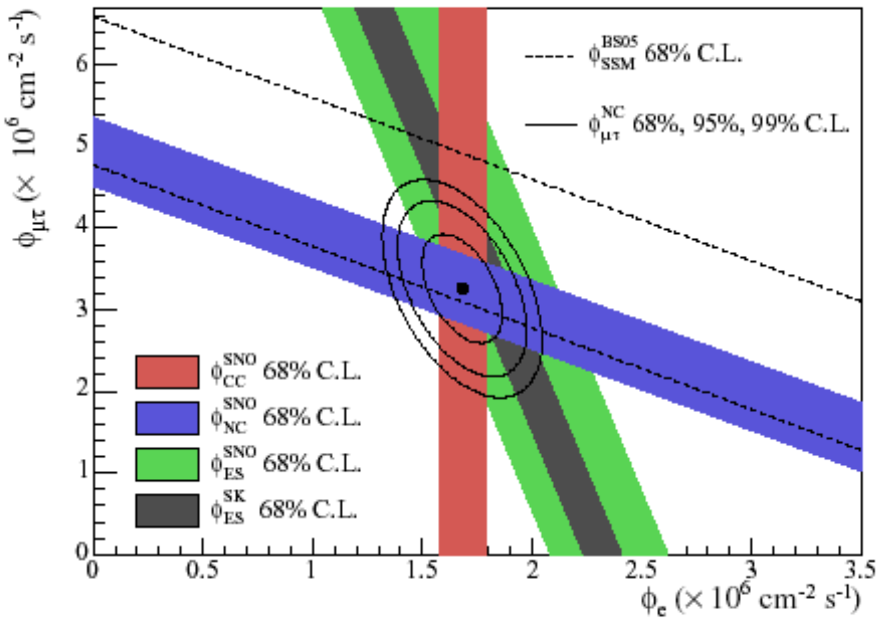


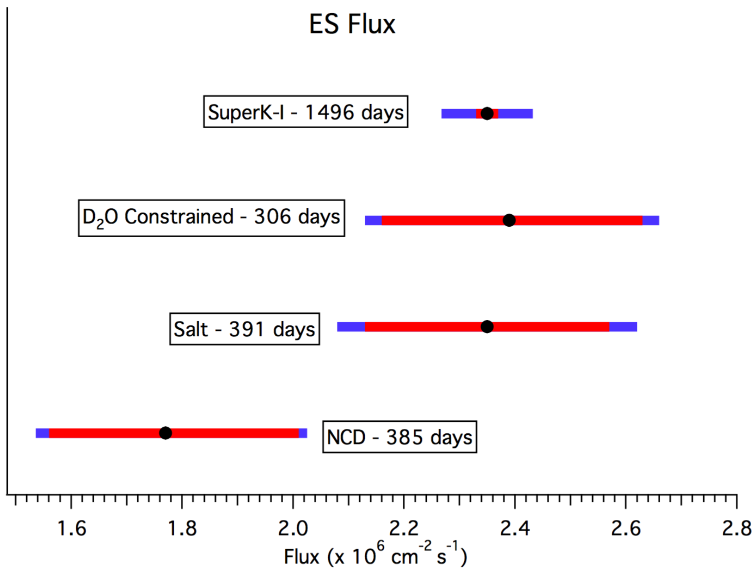
Three phases of SNO

stat — stat + syst

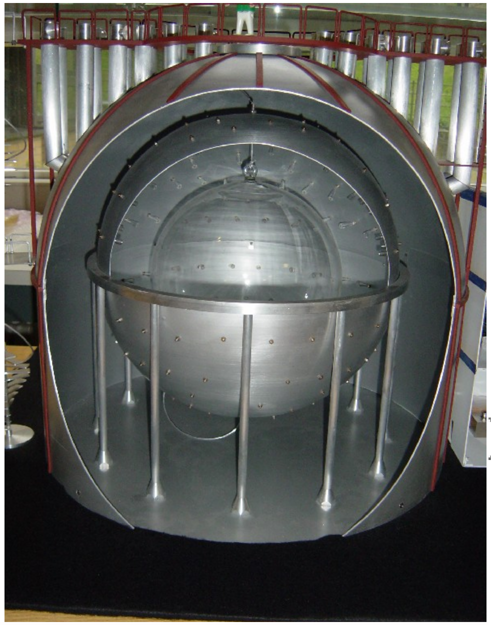


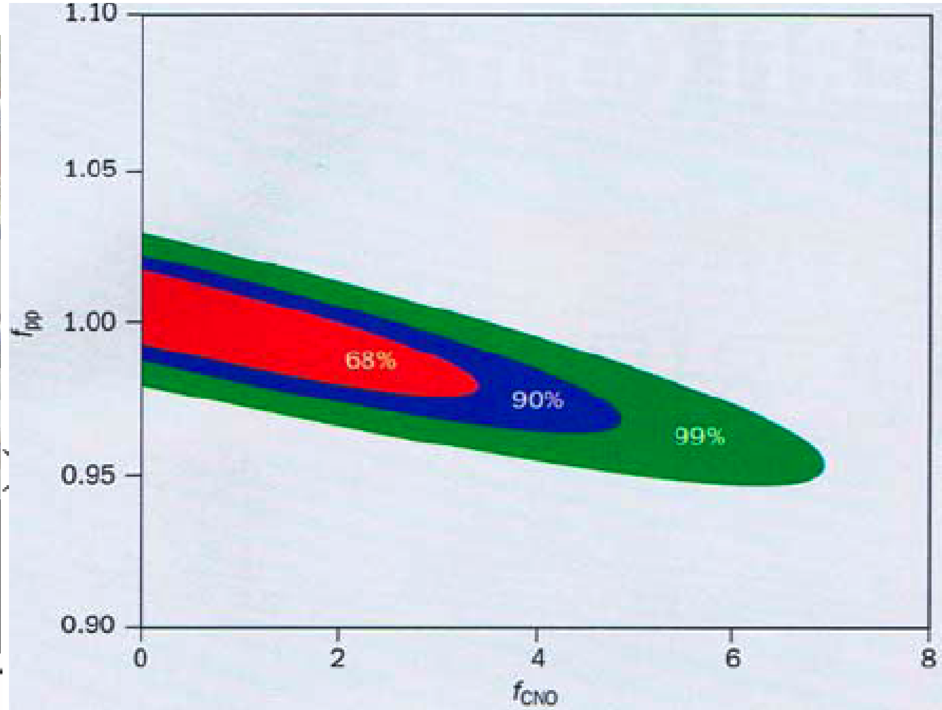


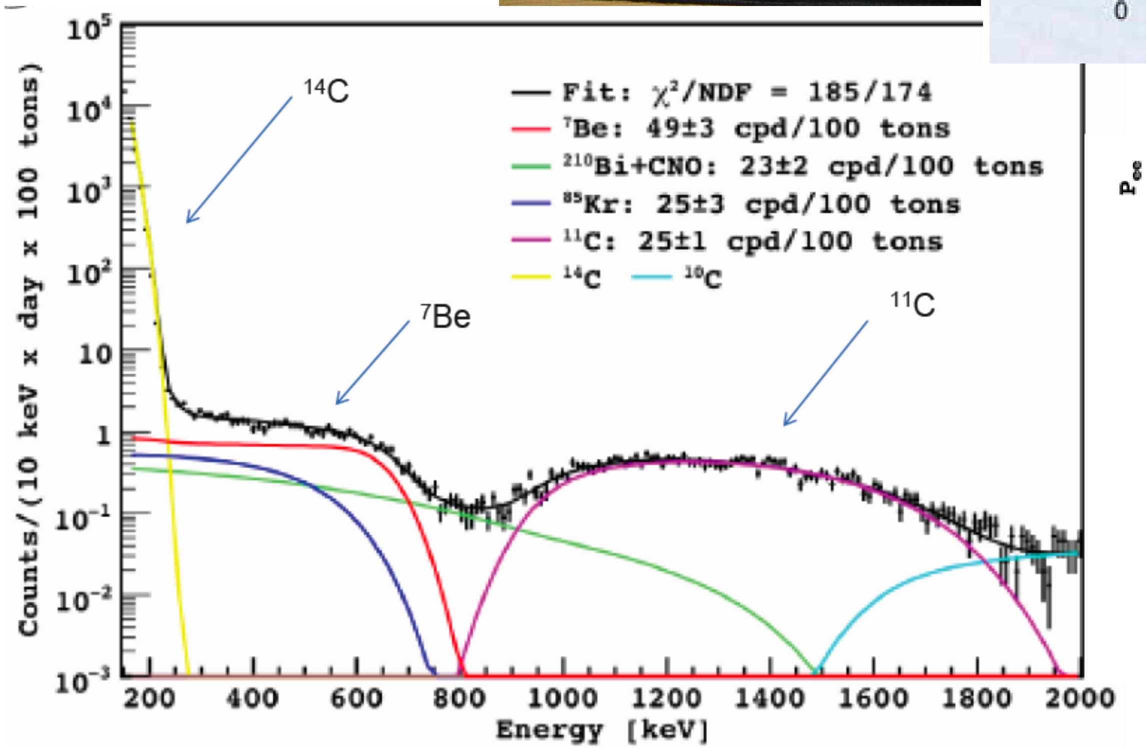


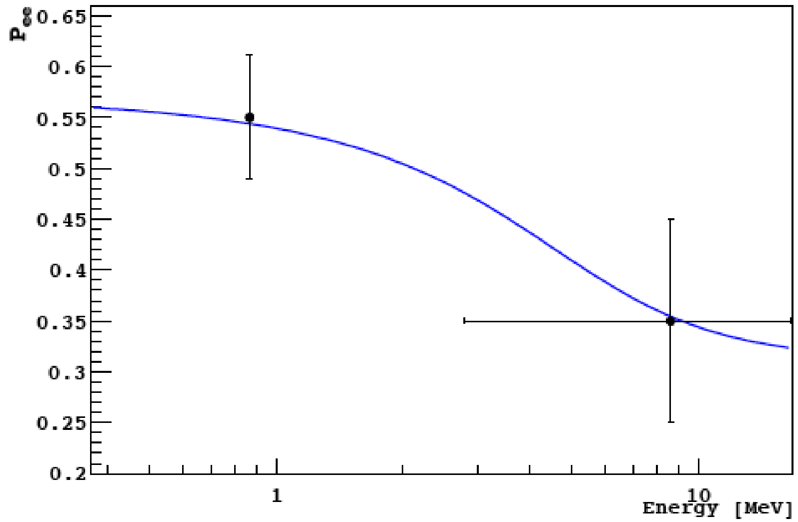


BOREXINO

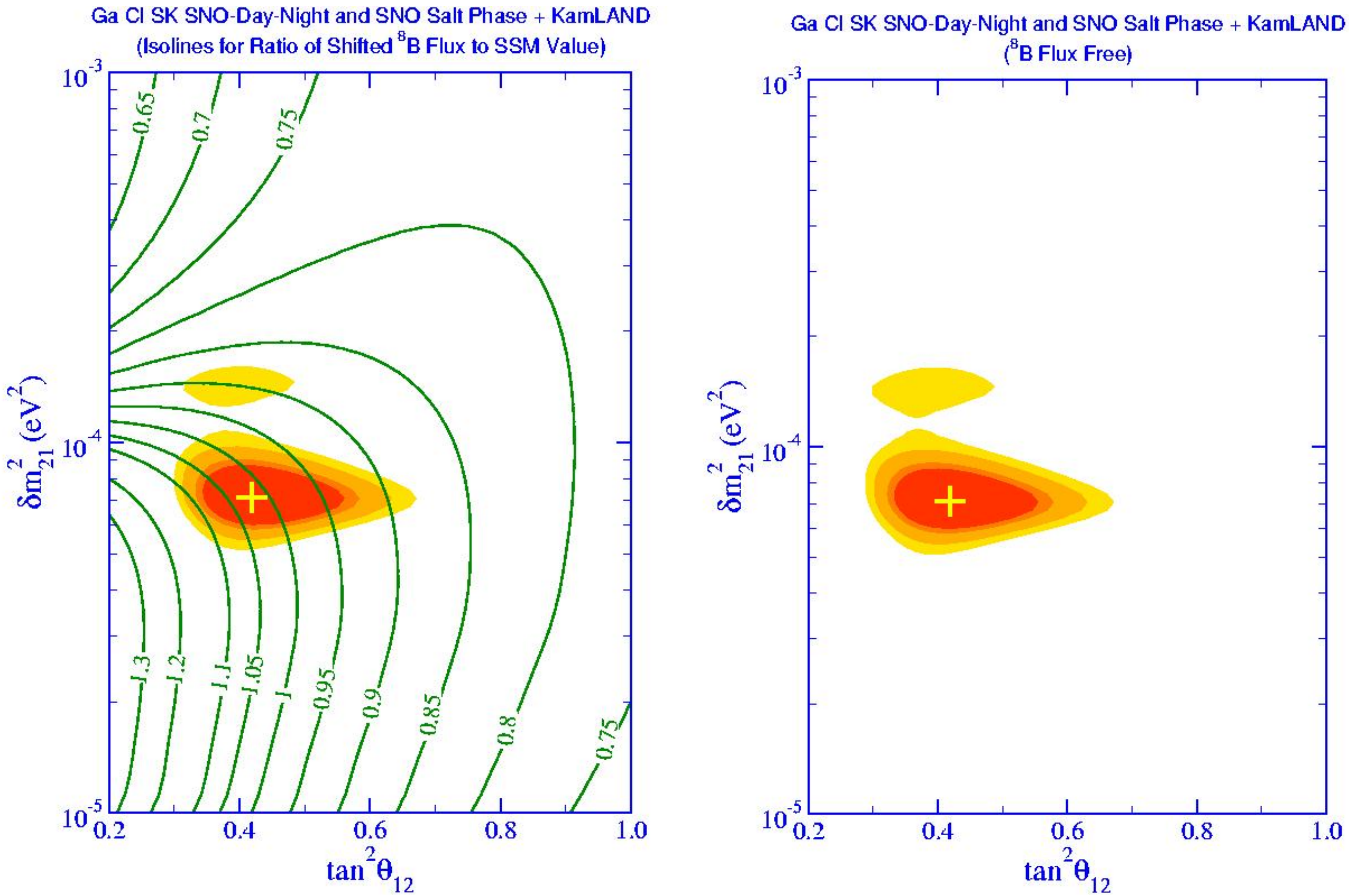



1.10
1.05
1.00
0.95
0.90
f_{pp}
68%
90%
99%
0
2
4
6
8
f_{CNO}

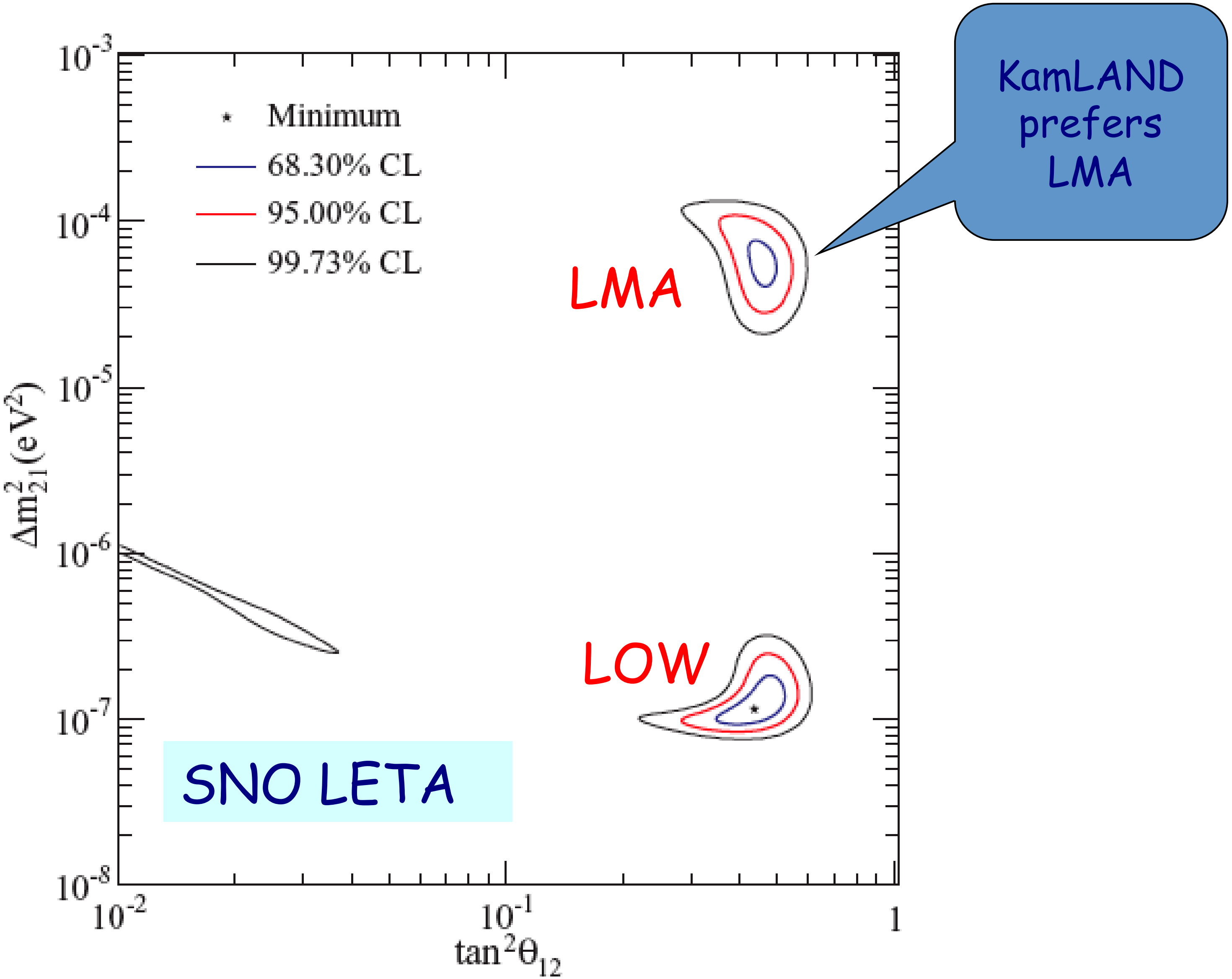

^{14}C
^{7}Be
^{11}C
Fit: χ^2/NDF = 185/174
^{7}Be: 49±3 cpd/100 tons
^{210}Bi+CNO: 23±2 cpd/100 tons
^{85}Kr: 25±3 cpd/100 tons
^{11}C: 25±1 cpd/100 tons
^{14}C
^{10}C
Counts/(10 keV x day x 100 tons)
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
200
400
600
800
1000
1200
1400
1600
1800
2000
Energy [keV]

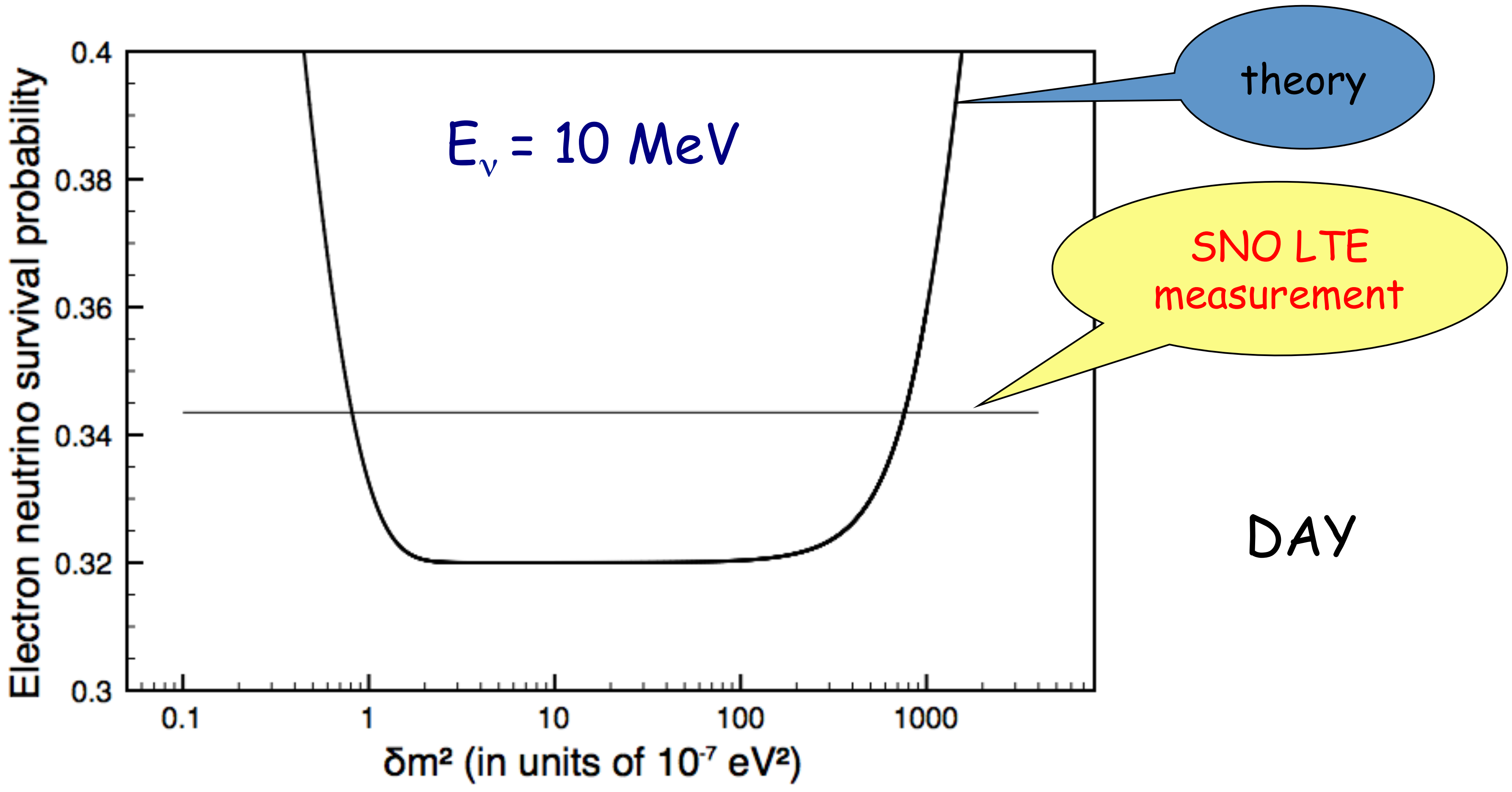

P_{ee}
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
1
10
Energy [MeV]

Already first SNO neutral current (salt) results could be analyzed without referring to the Standard Solar Model, A.B.B. & Yuksel, PRD 68, 113002 (2003)



Do antineutrinos mix the same way neutrinos do?





Experiments primarily sensitive to higher energy solar neutrinos cannot distinguish between LMA and LOW regions! It is desirable to pick the ***neutrino*** parameter region without KamLAND's ***antineutrinos***.

Neutrino mixing

$$\left|\nu_{flavor}\right\rangle = T\left|\nu_{mass}\right\rangle$$

$$T = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{atmospheric\ neutrinos} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{reactor\ neutrinos} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{solar\ neutrinos}$$

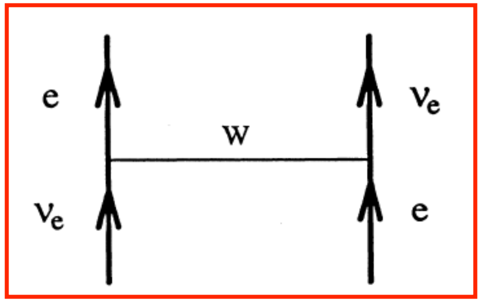
$$c_{ij} = \cos\theta_{ij} \qquad s_{ij} = \sin\theta_{ij}$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2(\Delta_{31}L) + \sin^2\theta_{12}\sin^2(\Delta_{32}L)\right] - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2(\Delta_{21}L)$$

$$\Delta_{ij} = \frac{\delta m_{ij}^2}{4E_\nu} = \frac{m_i^2 - m_j^2}{4E_\nu}, \qquad \Delta_{32} = \Delta_{31} - \Delta_{21}$$

The MSW Effect

The potential is provided by the coherent forward scattering of ν_e's off the electrons in dense matter



In vacuum: $E^2 = \mathbf{p}^2 + m^2$

In matter:

$(E - V)^2 = (\mathbf{p} - \mathbf{A})^2 + m^2$

$\Rightarrow E^2 = \mathbf{p}^2 + m_{\text{eff}}^2$

V $\propto$ background density

$\mathbf{A} \propto \mathbf{J}_{\text{background}}$ (currents) or

$\mathbf{A} \propto \mathbf{S}_{\text{background}}$ (spin)

In the limit of static, charge-neutral, and unpolarized background

V$\propto N_e$ and $\mathbf{A} = 0$

$\Rightarrow m_{\text{eff}}^2 = m^2 + 2EV + \mathcal{O}(V^2)$

There is a similar term with Z-exchange. But since it is the same for all neutrino flavors, it does not contribute to phase differences *unless* we invoke a sterile neutrino.

Note that matter effects induce an effective CP-violation since the matter in the Earth and the stars is not CP-symmetric!

Matter effects

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix} = \left[T\begin{pmatrix}E_1 & 0 & 0\\ 0 & E_2 & 0\\ 0 & 0 & E_3\end{pmatrix}T^\dagger + \begin{pmatrix}V_c+V_n & 0 & 0\\ 0 & V_n & 0\\ 0 & 0 & V_n\end{pmatrix}\right]\begin{pmatrix}\psi_e\\ \psi_\mu\\ \psi_\tau\end{pmatrix}$$

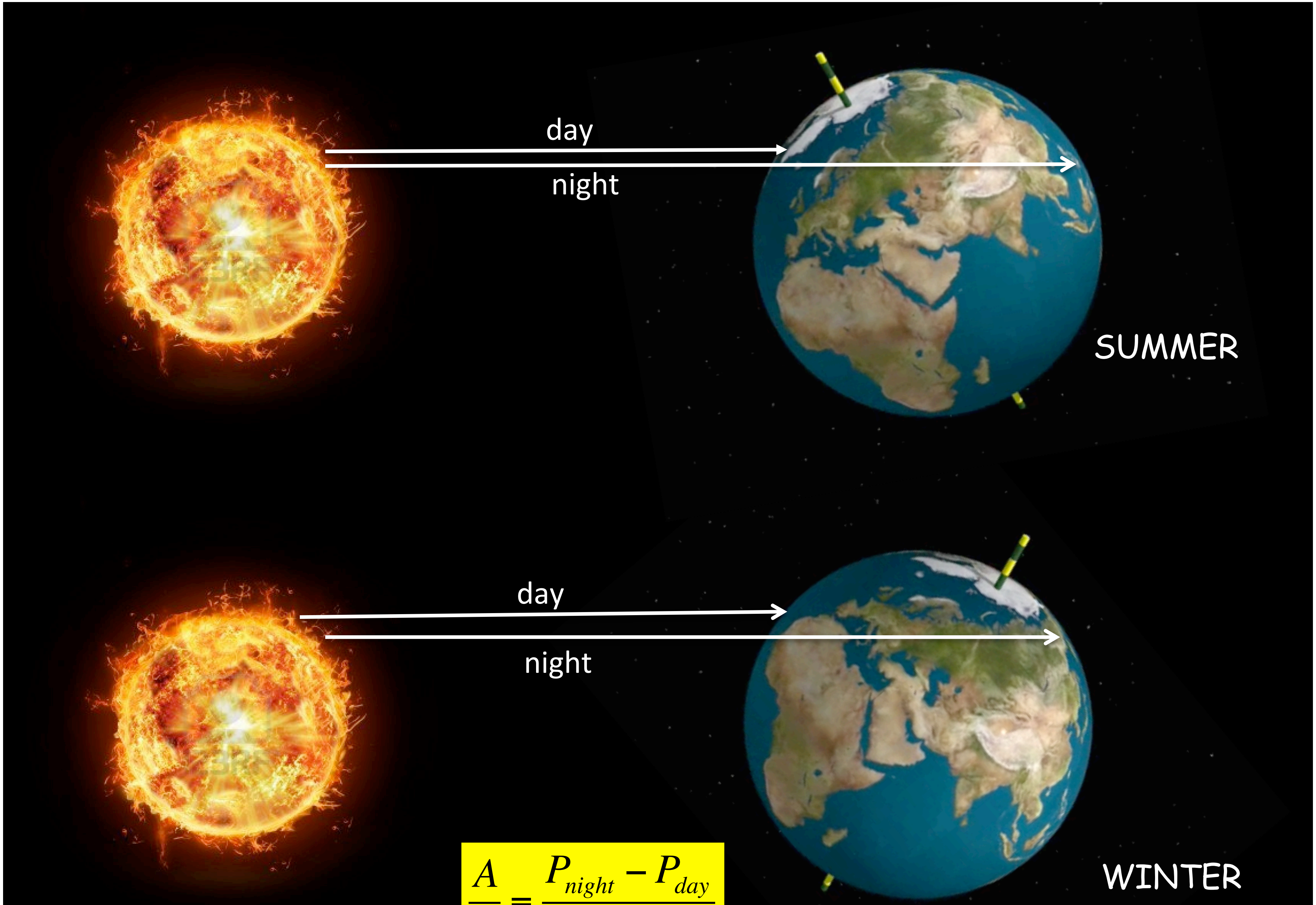
$$V_c = \sqrt{2}G_F N_e$$

$$V_n = -\frac{1}{\sqrt{2}}G_F N_n$$

Two-flavor limit

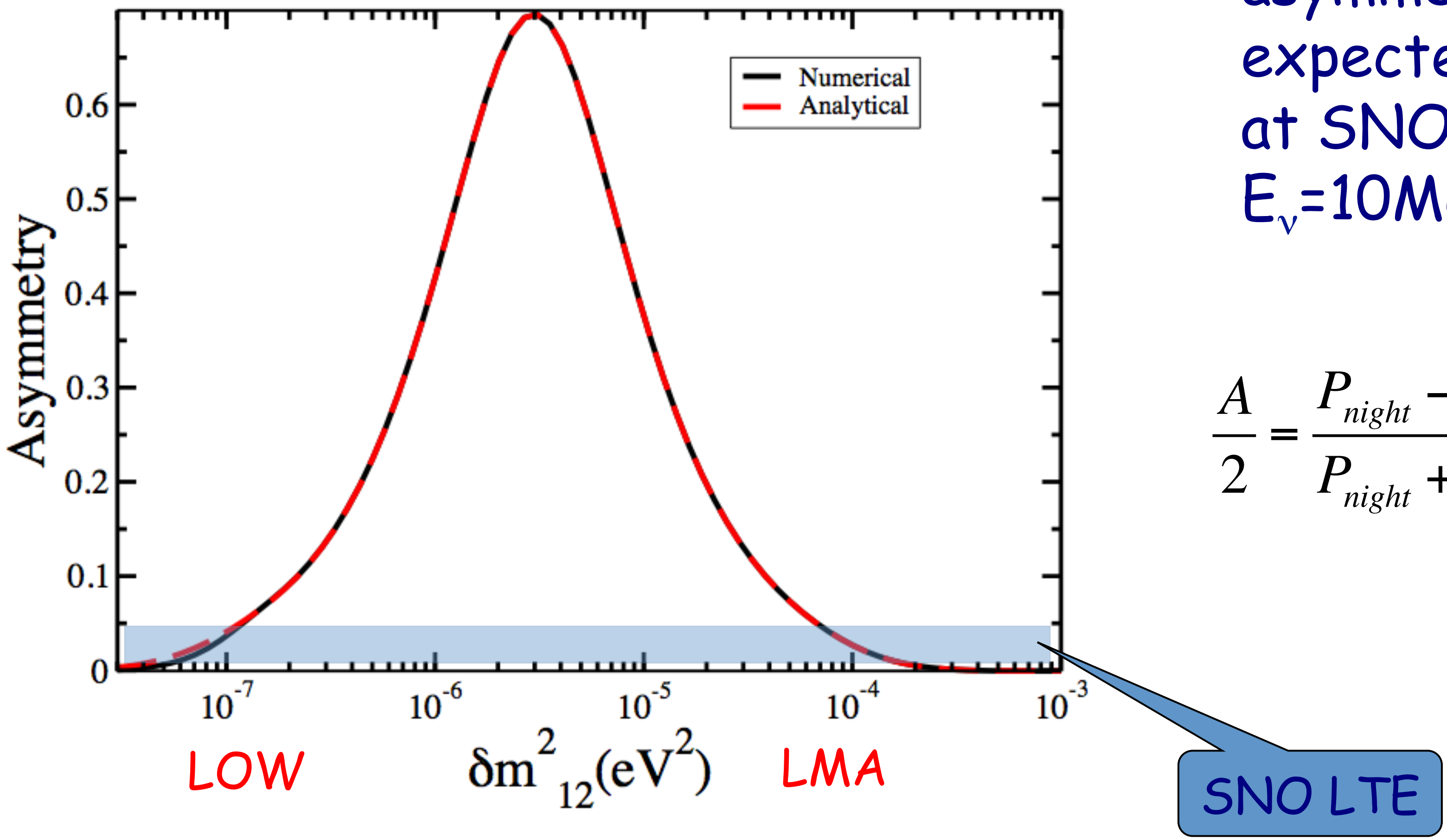
$$i\frac{\partial}{\partial t}\begin{pmatrix}|\nu_e\rangle\\ |\nu\mu\rangle\end{pmatrix} = \begin{pmatrix}\varphi & \frac{\delta m^2}{4E}\sin 2\theta\\ \frac{\delta m^2}{4E}\sin 2\theta & -\varphi\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\ |\nu\mu\rangle\end{pmatrix}$$

$$\varphi = -\frac{\delta m^2}{4E}\cos 2\theta + \frac{1}{\sqrt{2}}G_F N_e$$



Day-night asymmetry

$$\frac{A}{2} = \frac{P_{night} - P_{day}}{P_{night} + P_{day}}$$

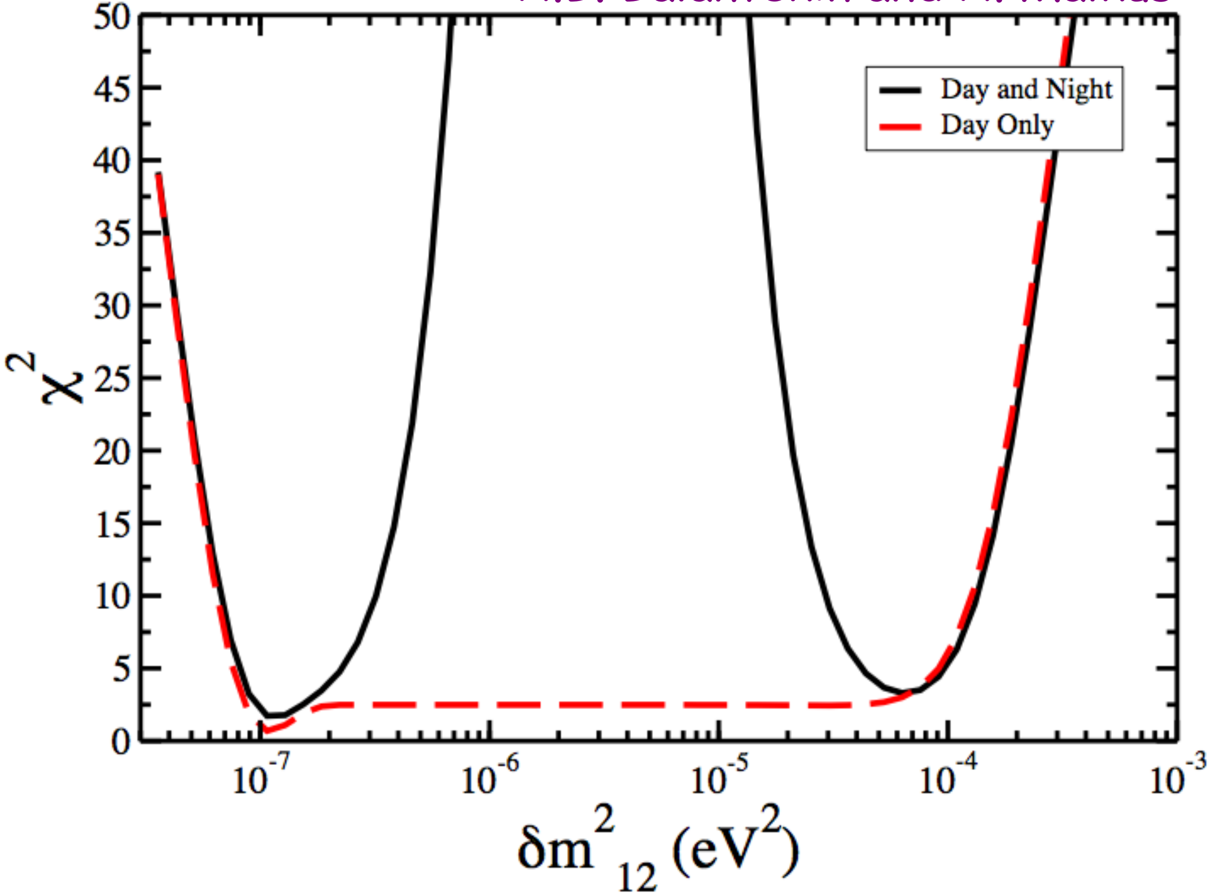


Day-night asymmetry expected at SNO for E_ν=10MeV

$$\frac{A}{2} = \frac{P_{night} - P_{day}}{P_{night} + P_{day}}$$

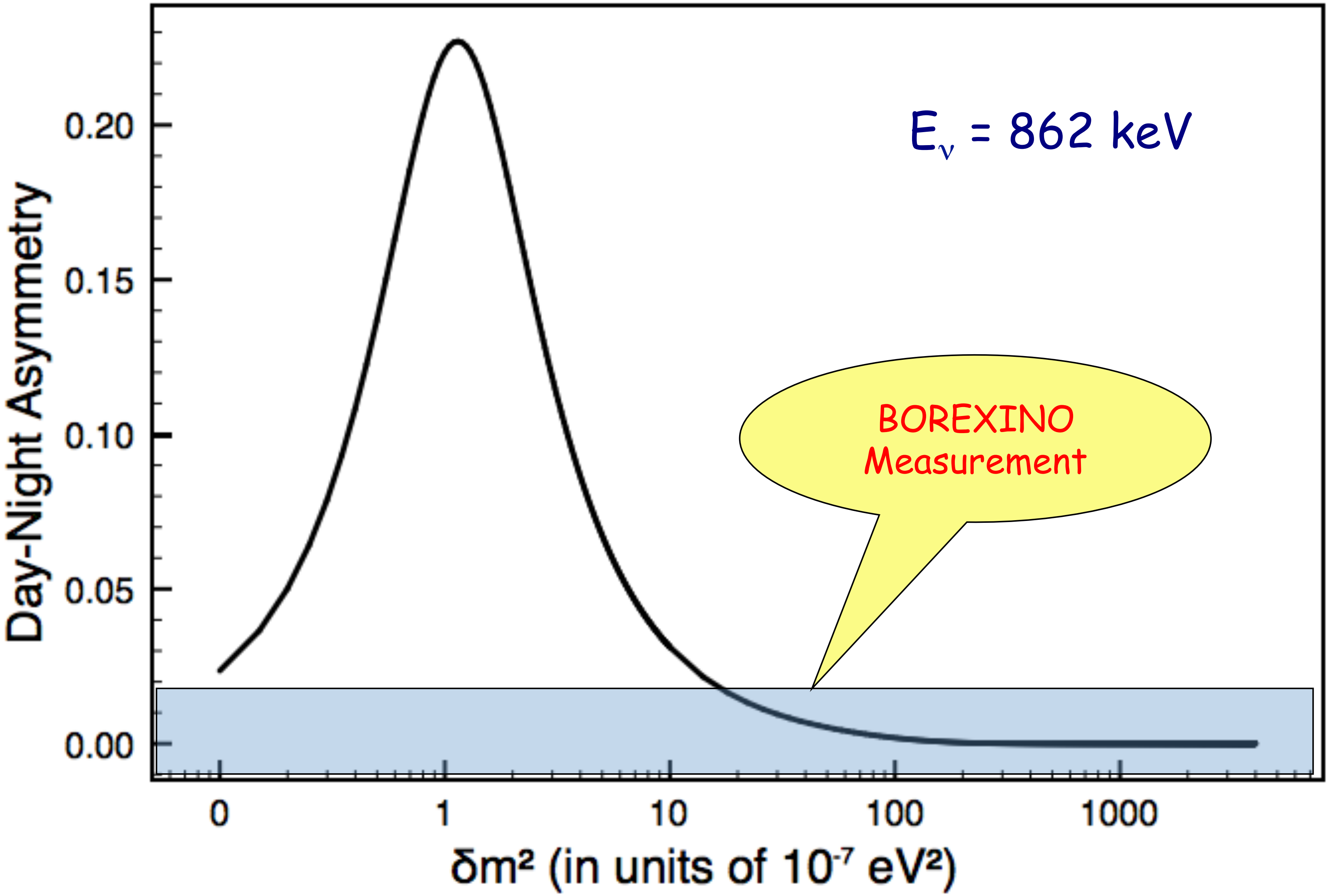
Experiments primarily sensitive to higher energy solar neutrinos cannot distinguish between LMA and LOW regions! It is desirable to pick the ***neutrino*** parameter region without KamLAND's ***antineutrinos***.

A.B. Balantekin and A. Malkus

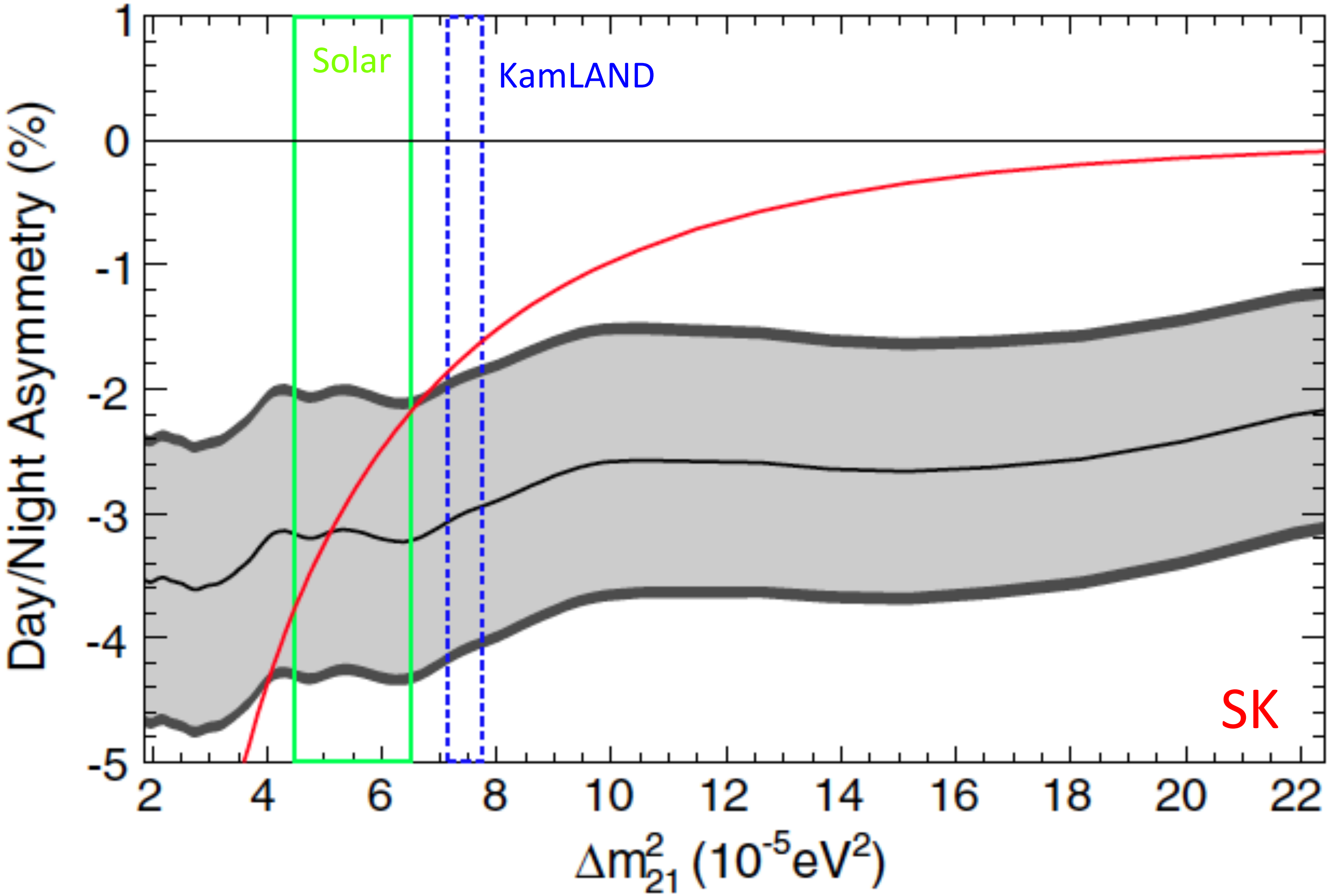


Fit to the SNO LTE data

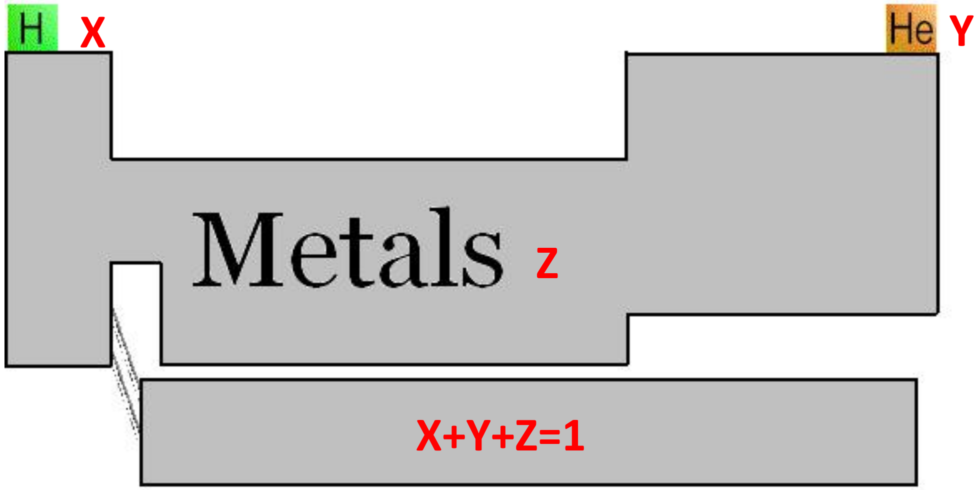
Experiments primarily sensitive to higher energy solar neutrinos cannot distinguish between LMA and LOW regions! It is desirable to pick the ***neutrino*** parameter region without KamLAND's ***antineutrinos***.



E_ν = 862 keV
BOREXINO
Measurement
Day-Night Asymmetry
0.20
0.15
0.10
0.05
0.00
0
1
10
100
1000
δm² (in units of 10⁻⁷ eV²)



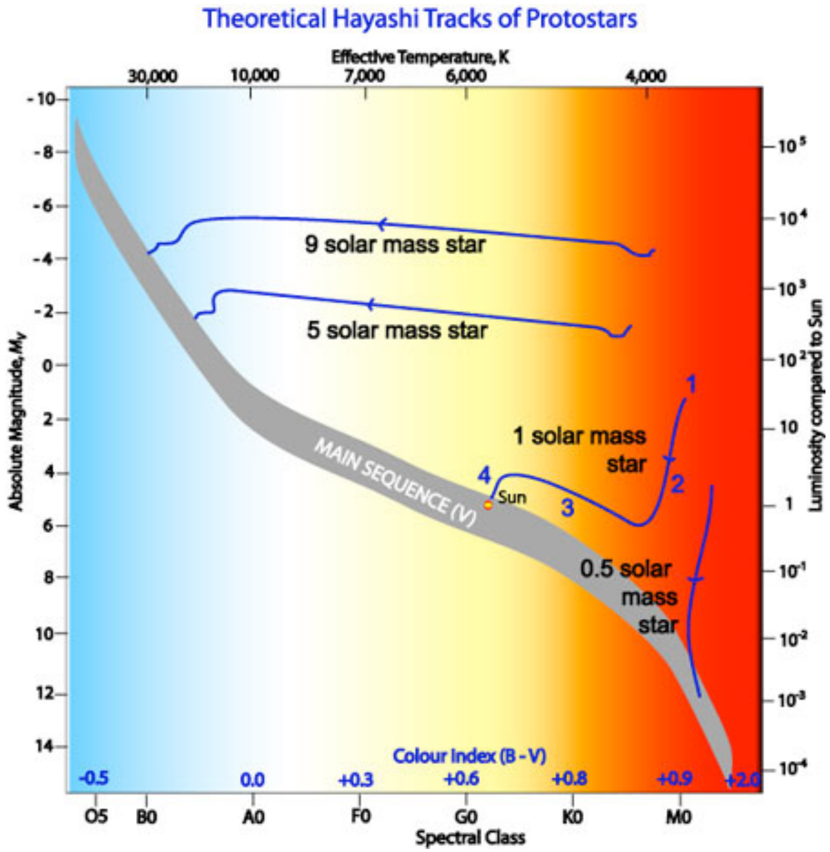
PRL **112,** 091805 (2014)



SSM assumption: The proto-Sun follows the convective Hayashi track ➔ zero-age Sun is homogeneous, i.e $Z_{initial} = Z_{surface\_today}$

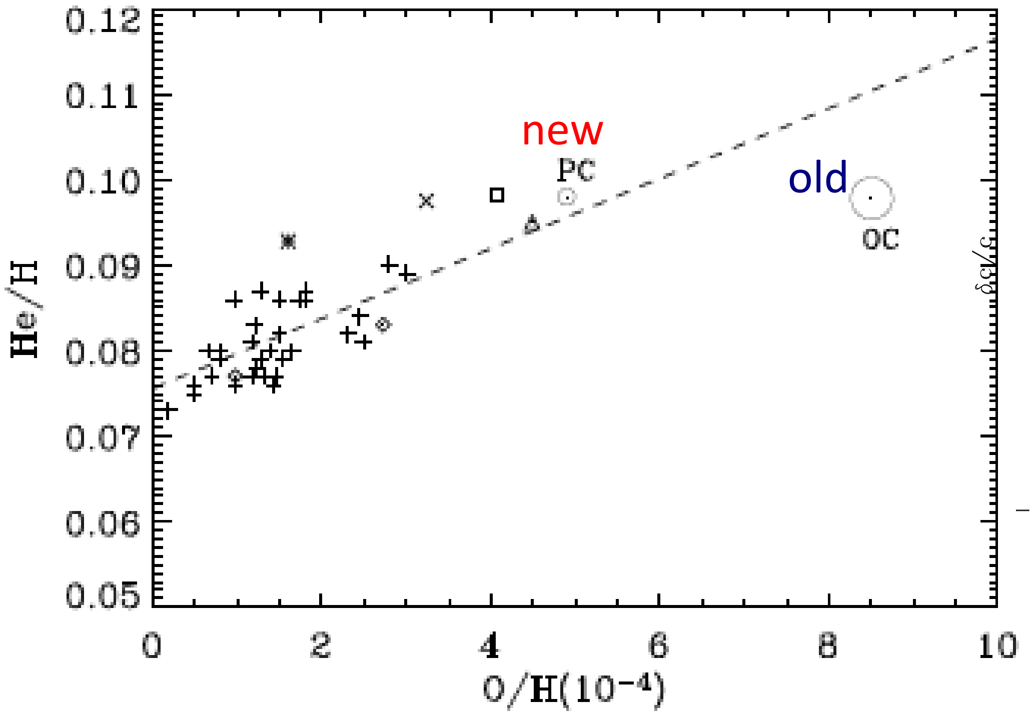
Initial parameters: $Y_{initial}$, $Z_{initial}$, solar mixing length

Evolve forward to today to reproduce present $R_{\odot}$, $L_{\odot}$, and $Y_{surface}$



$Z_{surface\_today}$ is deduced from photospheric absorption lines, which were recently evaluated using 3D methods. $Z_{surface\_today}$ obtained using improved methods does not match $Z_{initial}$ of the SSM!

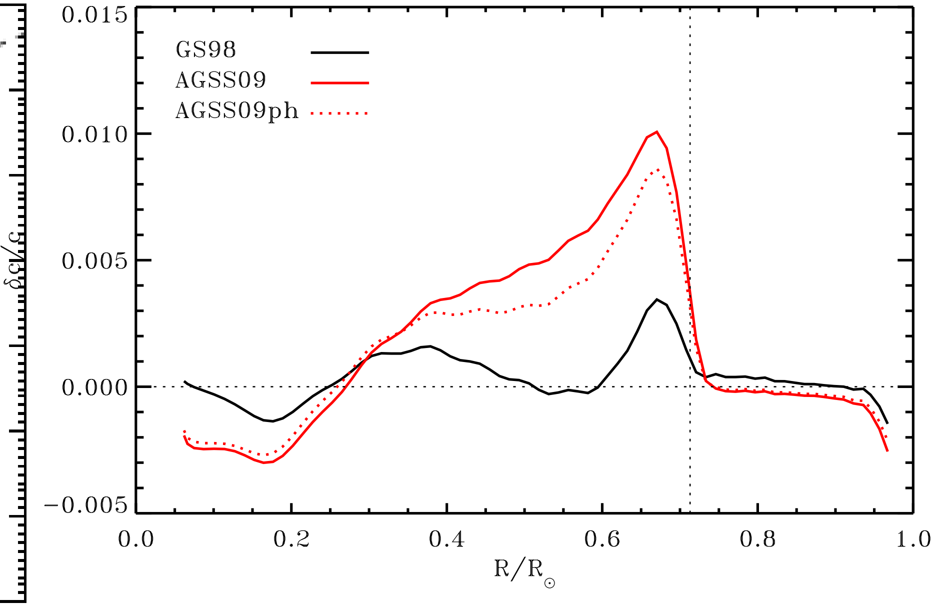
This fixes some old puzzles



Sun is no longer an “odd” star enriched in heavy elements!

Old ^{8}B neutrino flux = 4×10^{6} $cm^{-2}s^{-1}$
New ^{8}B neutrino flux = 5.31×10^{6} $cm^{-2}s^{-1}$

But creates new ones!



There is mismatch between the surface and the interior of the Sun!

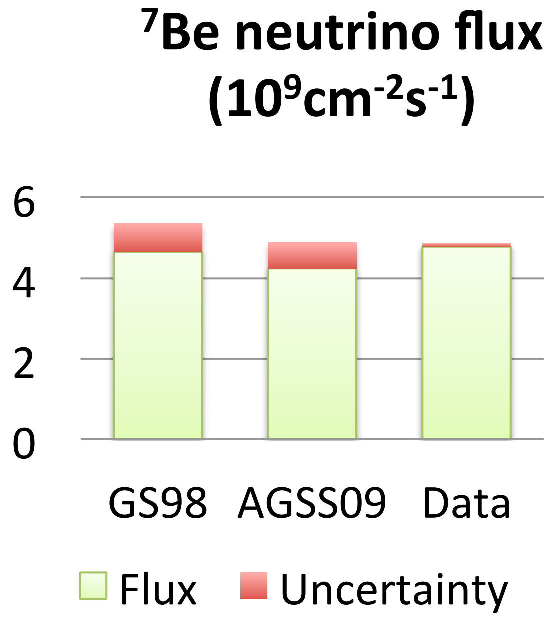
CNO Neutrinos are still not measured!

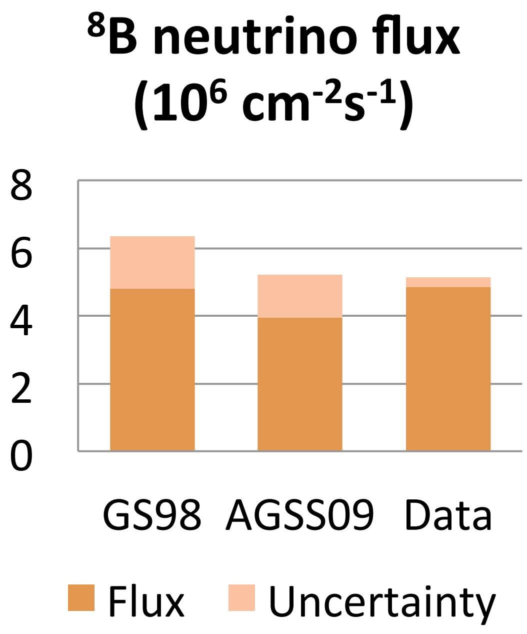
New Solar abundances:

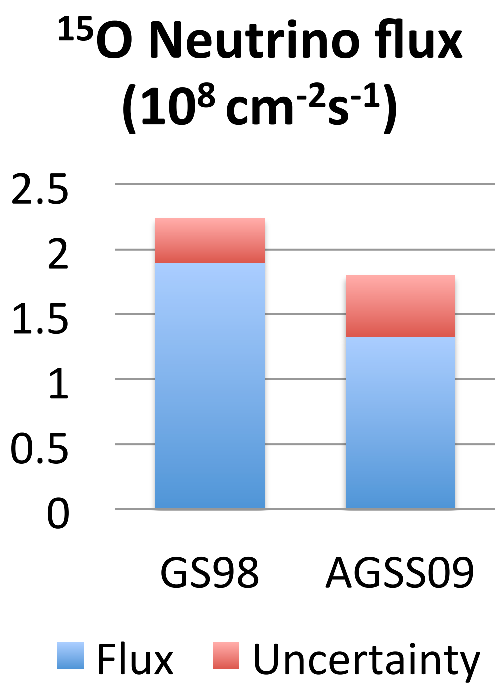
- Asplund *et al.* (AGSS09), $(Z/X)_{\odot}$=0.0178
- Grevesse and Sauvel (GS98), $(Z/X)_{\odot}$=0.0229

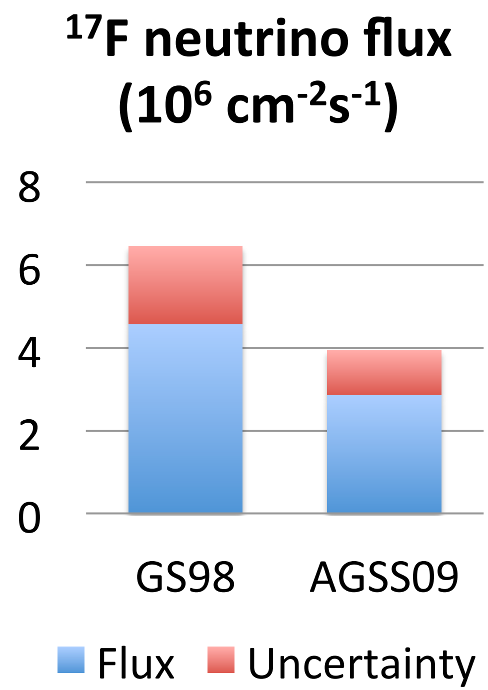
Drastically different!
Open problem in solar physics!

- New Evaluation of the nuclear reaction rates: Adelberger et al. (2011)
- New solar model calculations:Serenelli



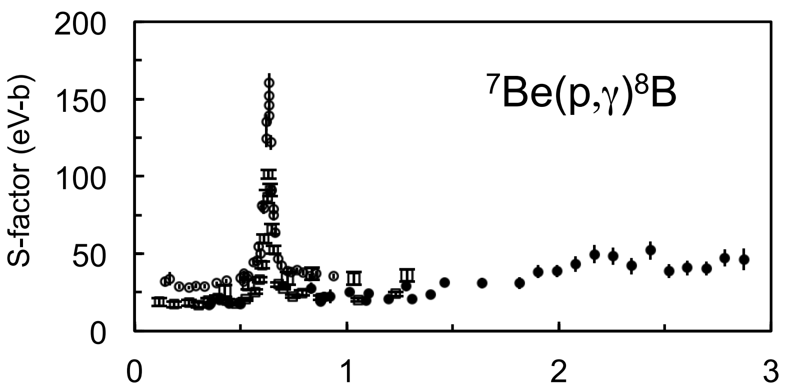


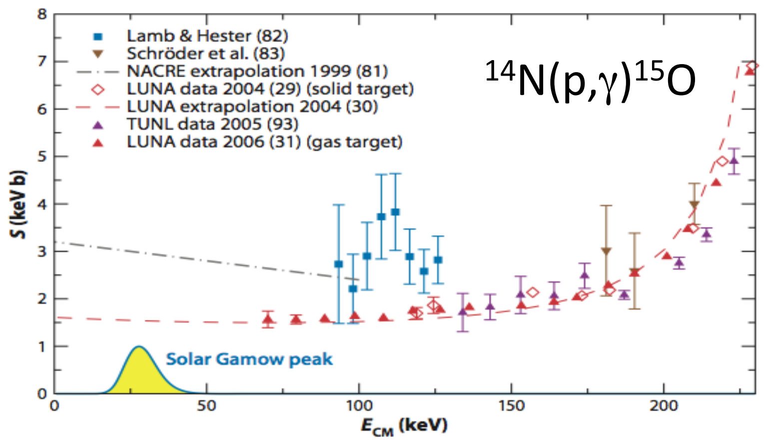




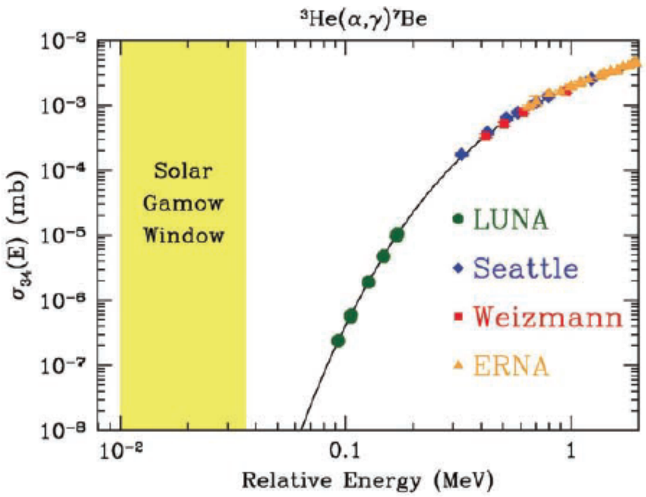
SSM Error Budget

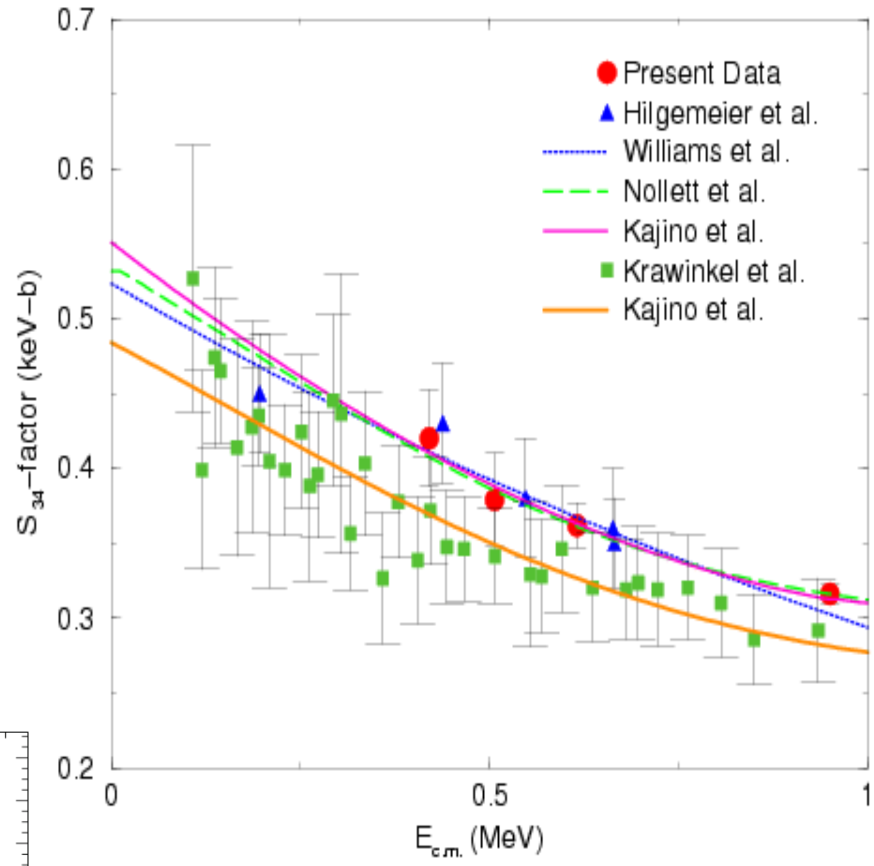
Source	Percentage Error
Diffusion coefficient of SSM	2.7%
Nuclear rates [mainly $^7Be(p,\gamma)^8B$ and $^{14}N(p,\gamma)^{15}O$]	9.9%
Neutrinos and weak interaction (mainly θ_{12})	3.2%
Other SSM input parameters	0.6%

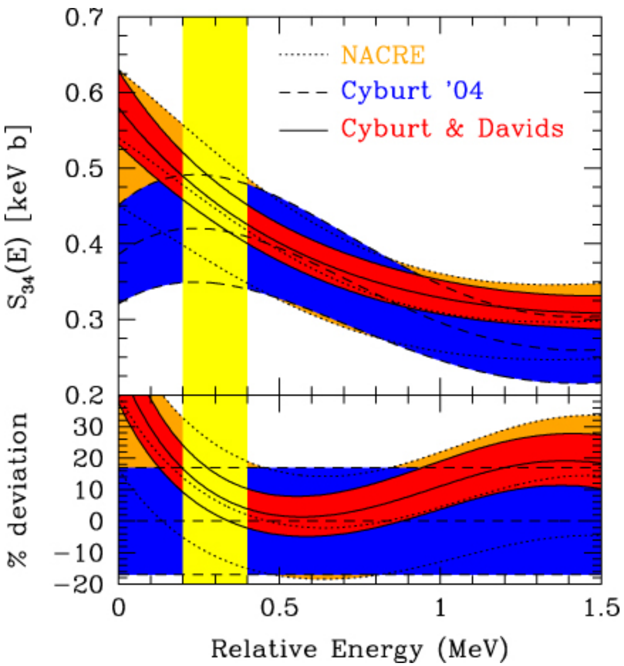


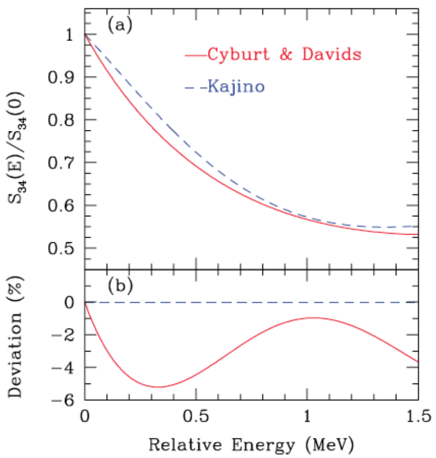


$^3He(\alpha,\gamma)^7Be$

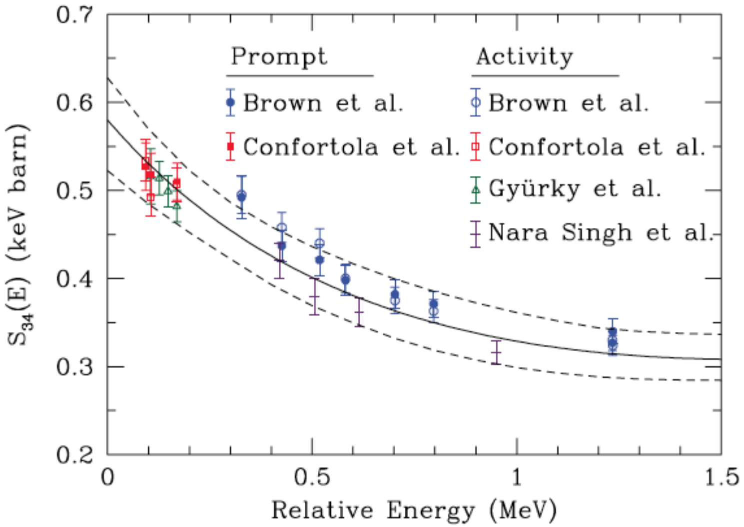




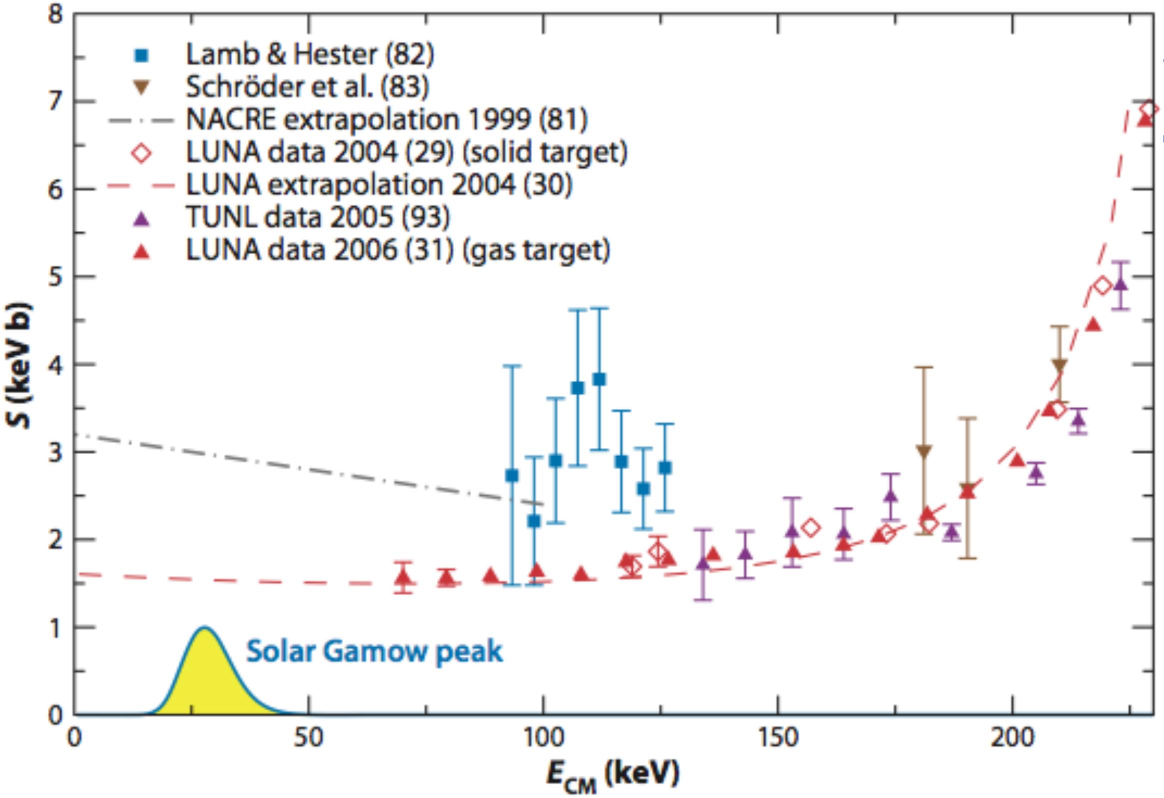


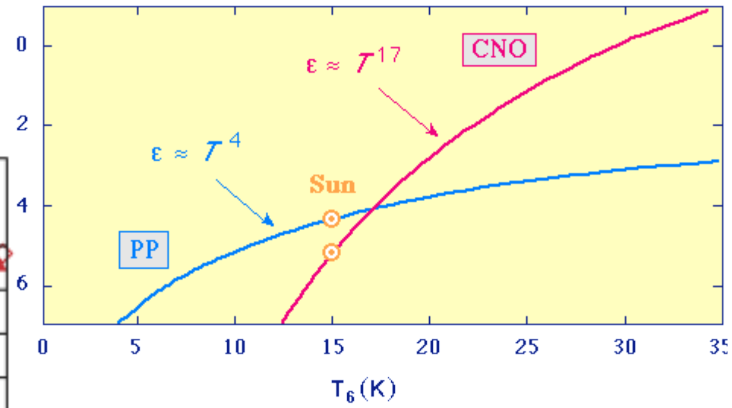


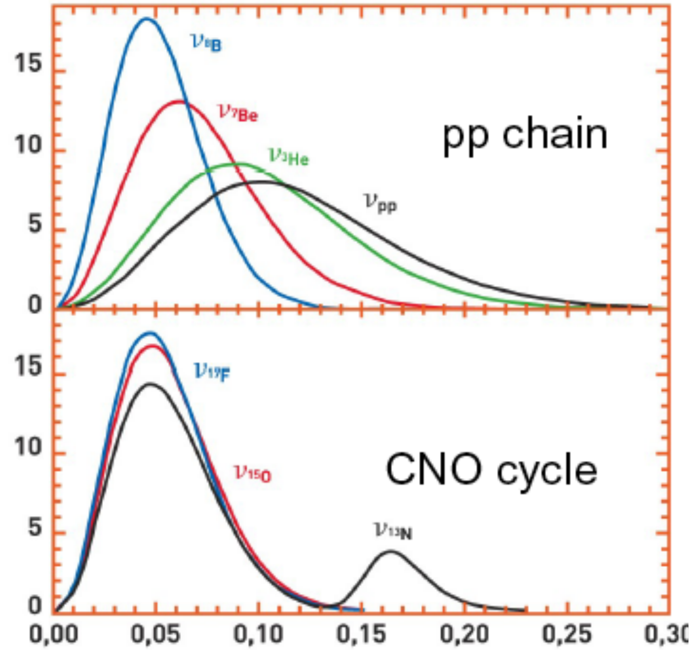
The main uncertainty for the Sun and Big-Bang nucleosynthesis



$^{14}N(p,\gamma)^{15}O$





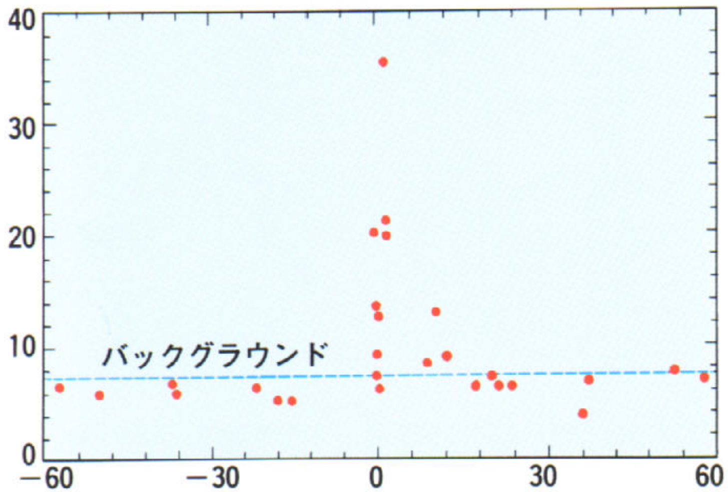


The determining reaction for the CNO burning

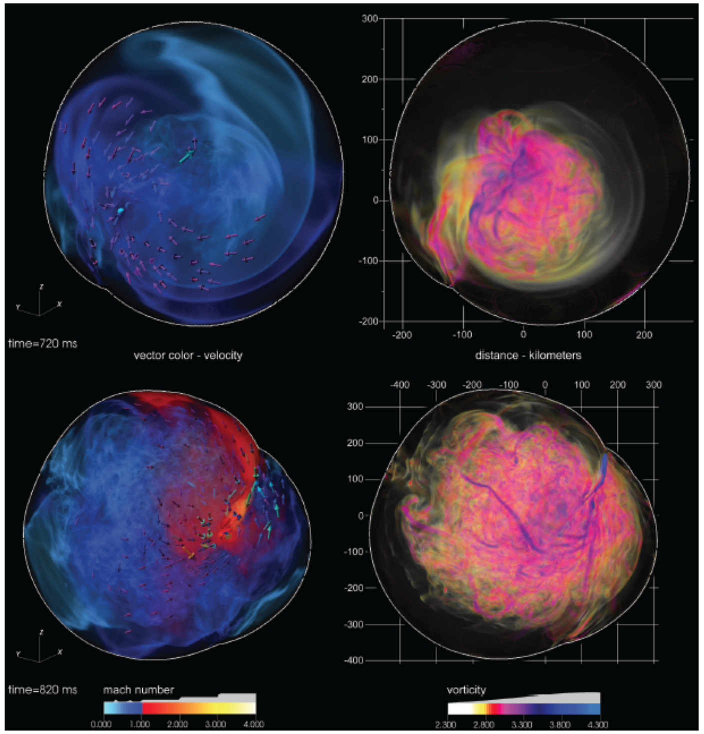
Neutrinos from core-collapse supernovae

- $M_{prog} \geq 8\ M_{Sun}$
- $\Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
- 99% of the energy is carried away by neutrinos and antineutrinos with

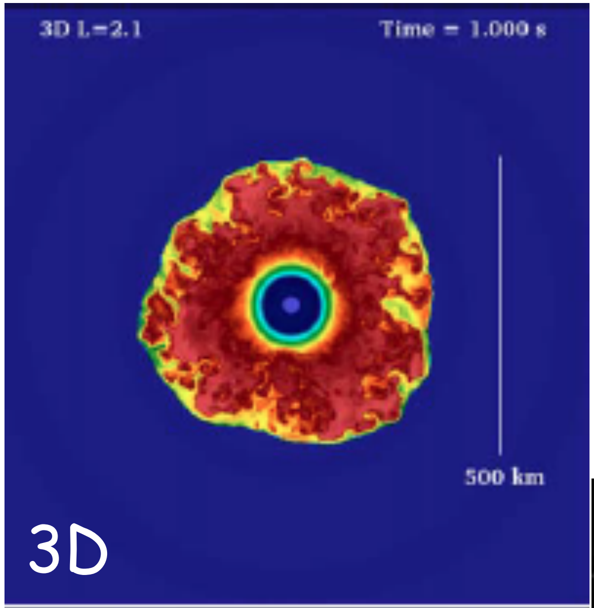
  $10 \leq E_\nu \leq 30$ MeV
- ~ 10^{58} Neutrinos!

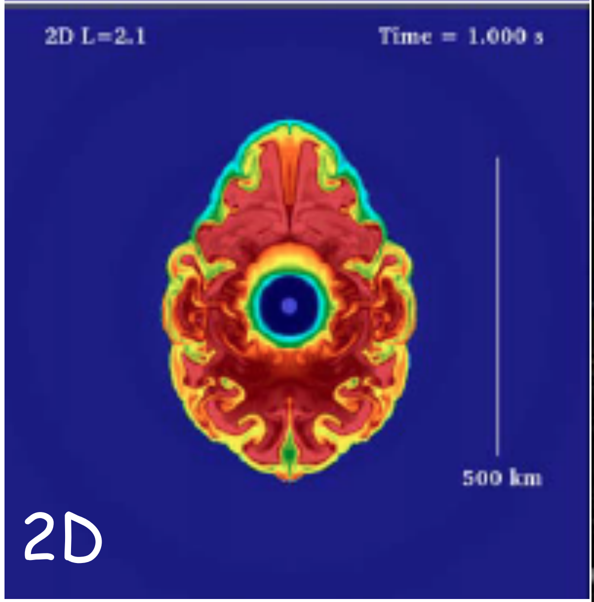


ORNL/UT

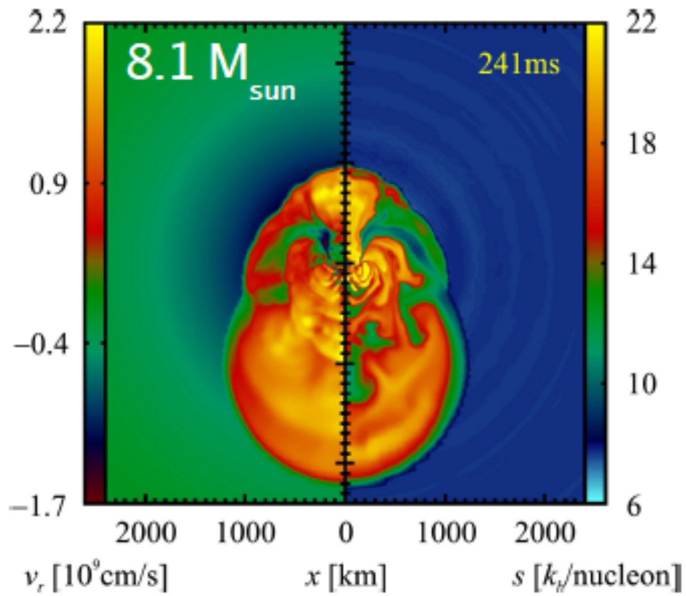


Development of 2D and 3D models for core-collapse supernovae: Complex interplay between turbulence, neutrino physics and thermonuclear reactions.





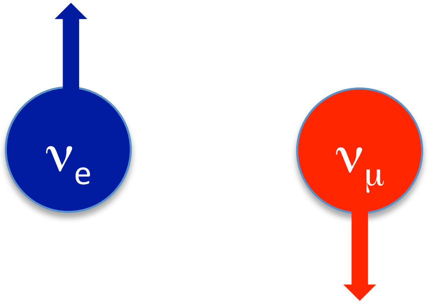
Princeton



Munich

If we want to catch a supernova with neutrinos we'd better know what neutrinos do inside a supernova.

Neutrino flavor isospin



$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

Free neutrinos (only mixing)

$$\hat{H} = \frac{m_1^2}{2E} a_1^\dagger a_1 + \frac{m_2^2}{2E} a_2^\dagger a_2 + (\cdot\cdot)\hat{1}$$

$$= \frac{\delta m^2}{4E}\cos 2\theta\left(a_\mu^\dagger a_\mu - a_e^\dagger a_e\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(a_e^\dagger a_\mu + a_\mu^\dagger a_e\right) + (\cdot\cdot)'\,\hat{1}$$

Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(a_\mu^\dagger a_\mu - a_e^\dagger a_e\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(a_e^\dagger a_\mu + a_\mu^\dagger a_e\right) + (\cdot\cdot)''\,\hat{1}$$

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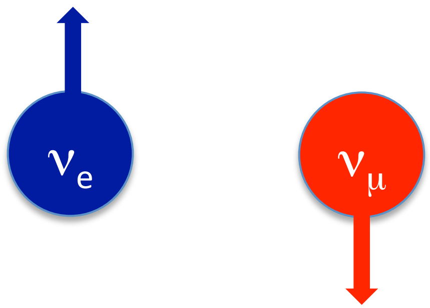

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$$a_e = \cos\theta\, a_1 + \sin\theta\, a_2$$

$$a_\mu = -\sin\theta\, a_1 + \cos\theta\, a_2$$

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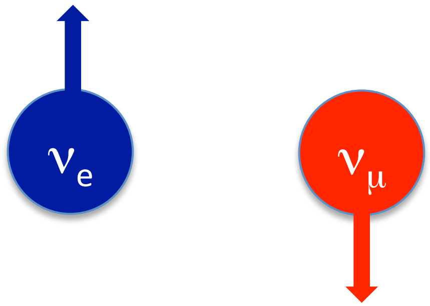

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$$= \frac{\delta m^2}{4E}\cos 2\theta\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)'\,\hat{1}$$

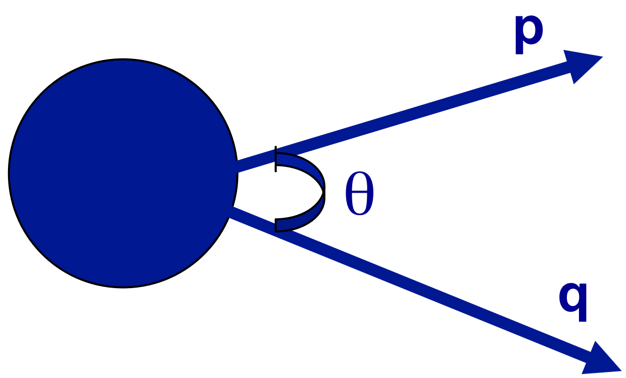
Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)''\,\hat{1}$$

Neutrino-Neutrino Interactions

Smirnov, Fuller and Qian, Pantaleone, McKellar, Friedland, Lunardini, Duan, Raffelt, Balantekin, Kajino, Pehlivan …

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V} \int dp\, dq \left(1 - \cos\theta_{pq}\right) \vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q$$



This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

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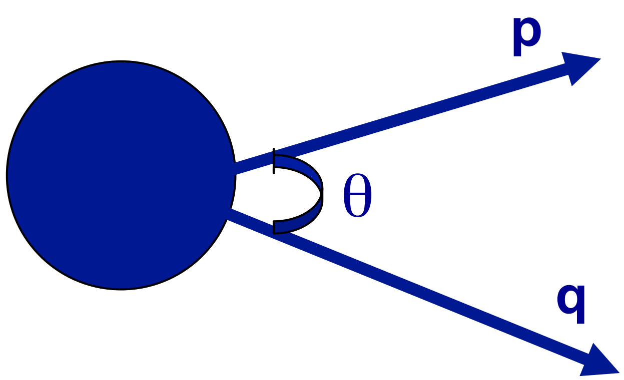


This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

$$\hat{H} = \int dp \left(\frac{\delta m^2}{2E}\vec{\mathbf{B}}\cdot\vec{\mathbf{J}}_p - \sqrt{2}G_F N_e \mathbf{J}_p^0\right) + \frac{\sqrt{2}G_F}{V}\int dp\, dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q$$

$$\vec{\mathbf{B}} = \left(\sin 2\theta,\ 0, -\cos 2\theta\right)$$

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral “swaps” or “splits”).

Including antineutrinos

$$H = H_\nu + H_{\bar{\nu}} + H_{\nu\nu} + H_{\bar{\nu}\bar{\nu}} + H_{\nu\bar{\nu}}$$

Requires introduction of a second set of SU(2) algebras!

Including three flavors

Requires introduction of SU(3) algebras.

Both extensions are straightforward, but tedious!
Balantekin and Pehlivan, J. Phys. G **34**, 1783 (2007).

Many neutrino system

This is the only many-body system driven by the weak interactions:

Table: Many-body systems

Nuclei	Strong	at most ∼250 particles
Condensed matter	E&M	at most N_A particles
ν**'s in SN**	Weak	$\sim 10^{58}$particles

Astrophysical extremes allow us to test physics that cannot be tested elsewhere!

Path Integral for the Evolution Operator

$$i\frac{\partial U}{\partial t} = (H_\nu + H_{\nu\nu})\, U$$

Use SU(2) coherent states to write the evolution operator as a path integral:

$$|z(t)\rangle = \exp\left(\int dpz(p,t) J_+(p)\right)|\phi\rangle$$

$$|\phi\rangle = \prod_p a_e^\dagger(p)|0\rangle$$

$$\langle z'(t_f)|U|z(t_i)\rangle = \int \mathcal{D}[z,z^*]\exp\left(iS[z,z^*]\right)$$

Stationary Phase Approximation

$$\langle z'(t_f)|U|z(t_i)\rangle = \int \mathcal{D}[z,z^*]\exp\left(iS[z,z^*]\right)$$

$$S(z,z^*) = \int_{t_i}^{t_f} dt \frac{\langle z(t)|i\frac{\partial}{\partial t} - H(t)|z(t)\rangle}{\langle z(t)|z(t)\rangle} + \log\langle z'(t_f)|z(t_f)\rangle$$

$$H = H_\nu + H_{\nu\nu}$$

$$\left(\frac{d}{dt}\frac{\partial}{\partial \dot{z}} - \frac{\partial}{\partial z}\right)L(z,z^*) = 0 \qquad \left(\frac{d}{dt}\frac{\partial}{\partial \dot{z}^*} - \frac{\partial}{\partial z^*}\right)L(z,z^*) = 0$$

Mean-field evolution equations

$$\Delta = \frac{\delta m^2}{2p}, \qquad A = \sqrt{2} G_F N_e$$

$$D = \sqrt{2} G_F \int dq (1 - \cos\theta_{pq}) \left[\left(|\psi_e(q,t)|^2 - |\psi_x(q,t)|^2 \right) \right]$$

$$D_{ex} = 2\sqrt{2} G_F \int dq (1 - \cos\theta_{pq}) \left(\psi_e(q,t) \psi_x^*(q,t) \right)$$

$$i \frac{\partial}{\partial t} \begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix} = \frac{1}{2} \begin{pmatrix} A + D - \Delta \cos 2\theta & D_{e\mu} + \Delta \sin 2\theta \\ D_{\mu e} + \Delta \sin 2\theta & -A - D + \Delta \cos 2\theta \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix}$$

The duality between $H_{\nu\nu}$ and BCS Hamiltonians

The ν-ν Hamiltonian

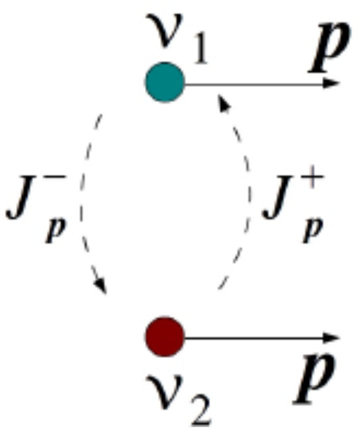
$$\hat{H} = \sum_p \frac{\delta m^2}{2p} \vec{\hat{B}} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \vec{J} \cdot \vec{J}$$

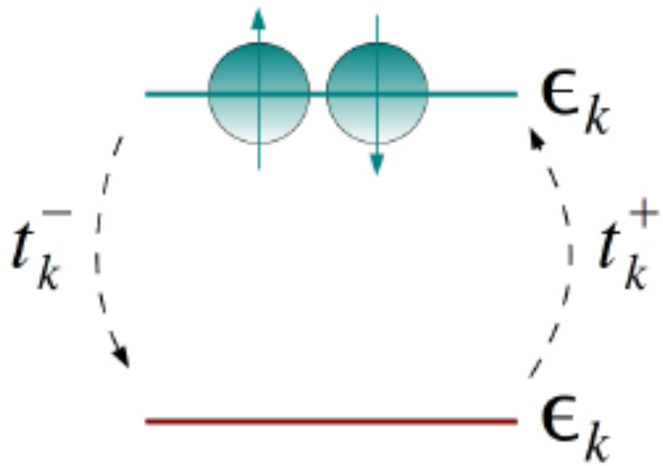
$\Longleftrightarrow$

The BCS Hamiltonian

$$\hat{H}_{\text{BCS}} = \sum_k 2\epsilon_k \hat{t}_k^0 - |G| \hat{T}^+ \hat{T}$$

Same symmetries leading to Analogous (dual) dynamics!
Pehlivan, Balantekin, Kajino, and Yoshida, Phys.Rev. D **84**, 065008 (2011)



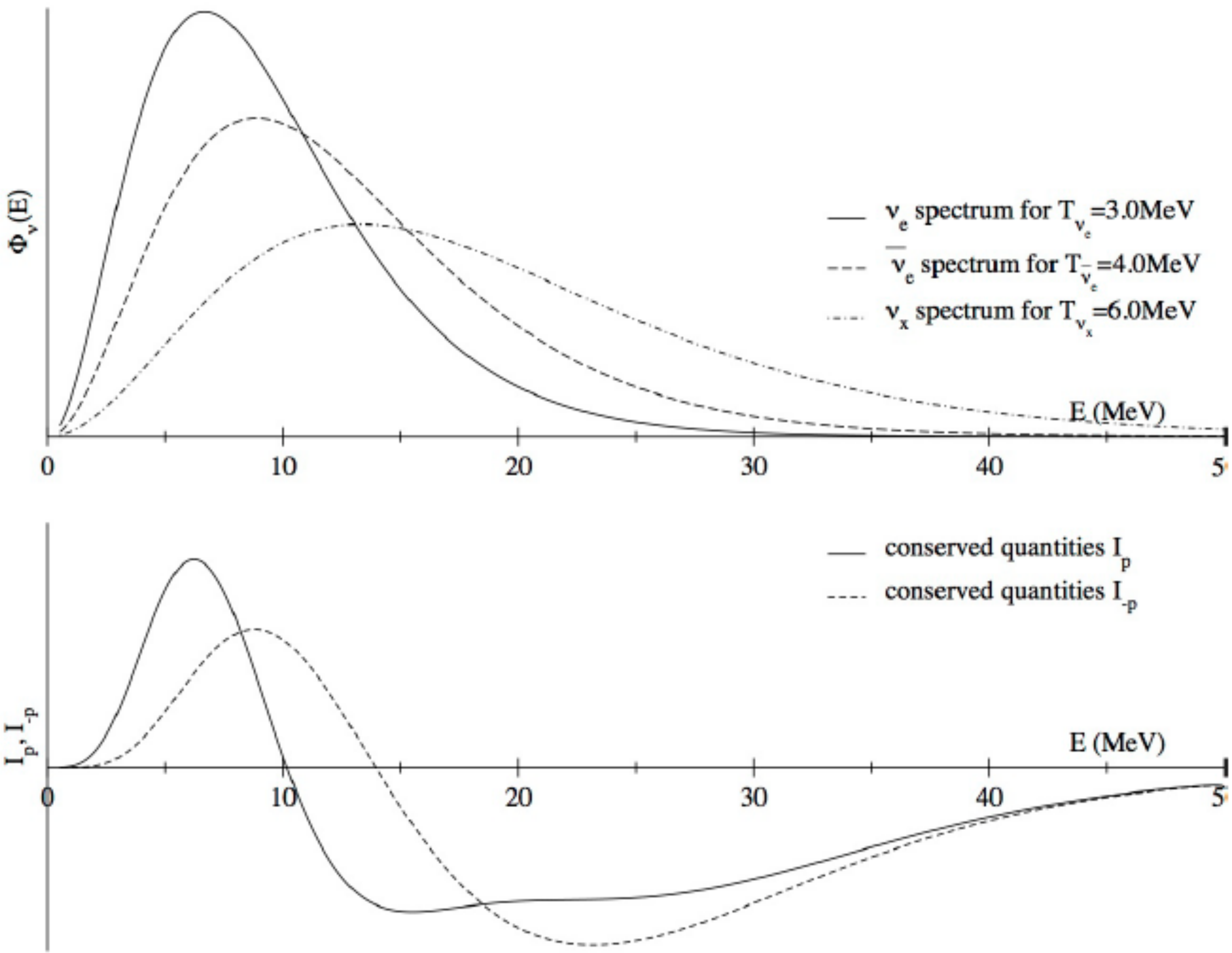


This symmetry naturally leads to splits in the neutrino energy spectra and was used to find conserved quantities in the single-angle case.

Conserved quantities of the collective motion

$$h_p = \vec{\mathbf{B}} \cdot \vec{\mathbf{J}}_p + \frac{4\sqrt{2}G_F}{\delta m^2 V} \sum_{p \neq q} qp \frac{\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q}{q - p}$$

- There is a second set of conserved quantities for antineutrinos.
- Note the presence of volume. In fact h_p/V are the conserved quantities for the neutrino densities.
- For three flavors a similar expression is written in terms of SU(3) operators.



$\Phi_\nu(E)$
ν_e spectrum for T_{ν_e}=3.0MeV
$\bar{\nu}_e$ spectrum for $T_{\bar{\nu}_e}$=4.0MeV
ν_x spectrum for T_{ν_x}=6.0MeV
E (MeV)
0
10
20
30
40
5
conserved quantities I_p
conserved quantities I_{-p}
I_p, I_{-p}
E (MeV)
0
10
20
30
40
5

The ν-ν Hamiltonian

$$\hat{H} = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \vec{J} \cdot \vec{J}$$

$\Longleftrightarrow$

The BCS Hamiltonian

$$\hat{H}_{\mathrm{BCS}} = \sum_k 2\epsilon_k \hat{t}^0_k - |G| \hat{T}^+ \hat{T}$$

Recall how we treat the BCS Hamiltonian. We diagonalize it in a quasiparticle basis. However that basis does not preserve particle number. We enforce the particle number conservation by introducing a Lagrange multiplier. This Lagrange multiplier turns out to be the chemical potential.

The ν-ν Hamiltonian

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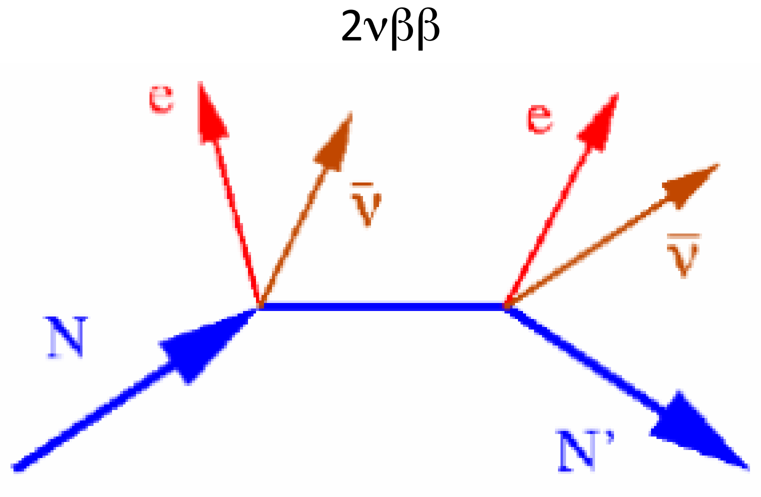
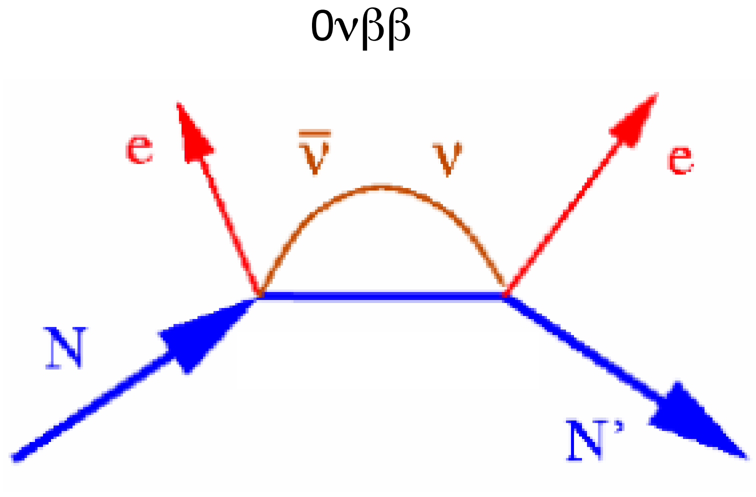
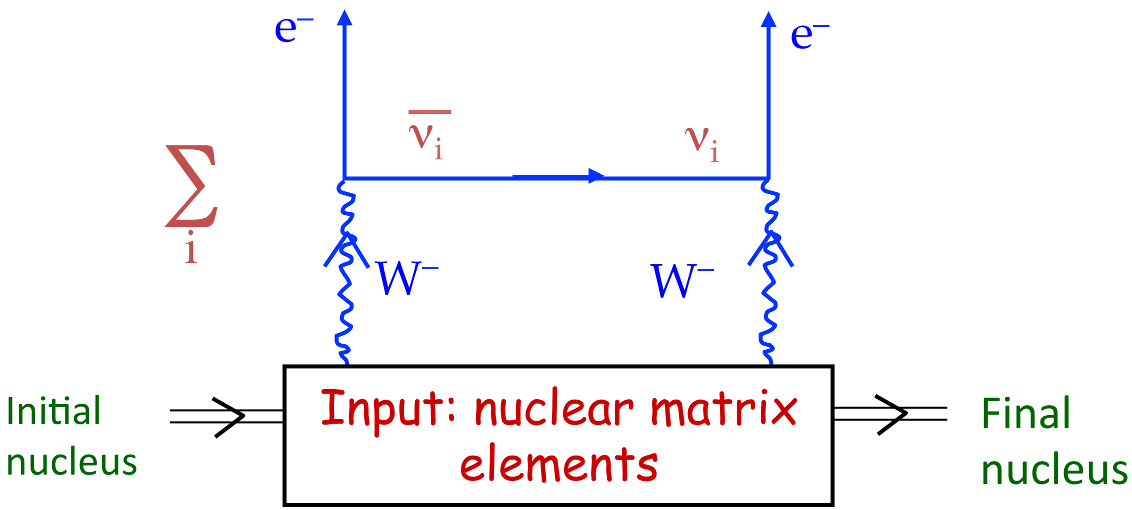
In the many neutrino case we can do the same. The Lagrange multiplier we have to introduce to preserve the total neutrino number shows up the the final neutrino energy spectra as a "split". This is the origin of the spectral splits (or swaps) numerically observed in many calculations.

Maria Goeppert *Mayer* was *awarded* the 1963 *Nobel* for the nuclear shell model, the San Diego Union headline read "*San Diego Housewife Wins Nobel Prize*".

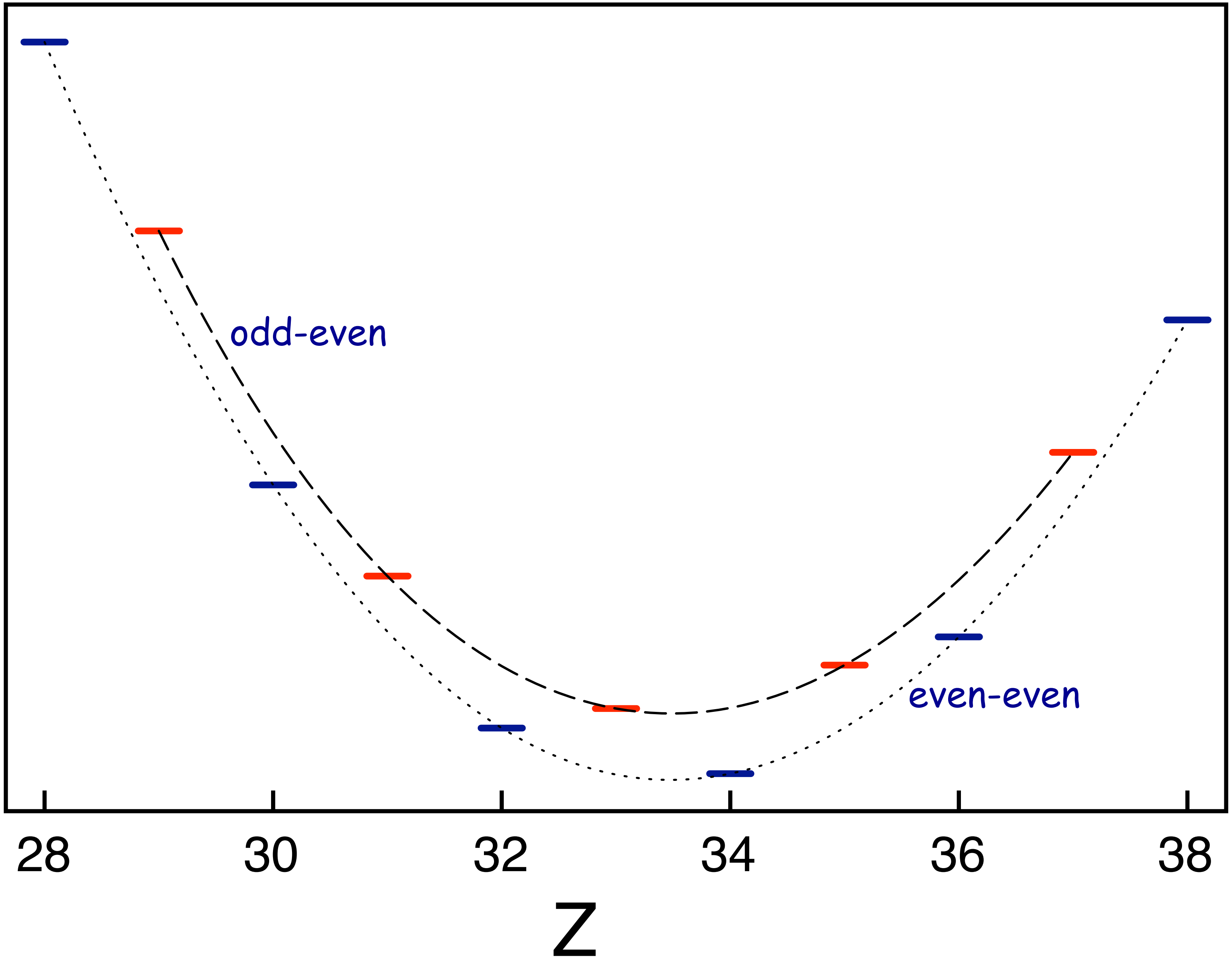


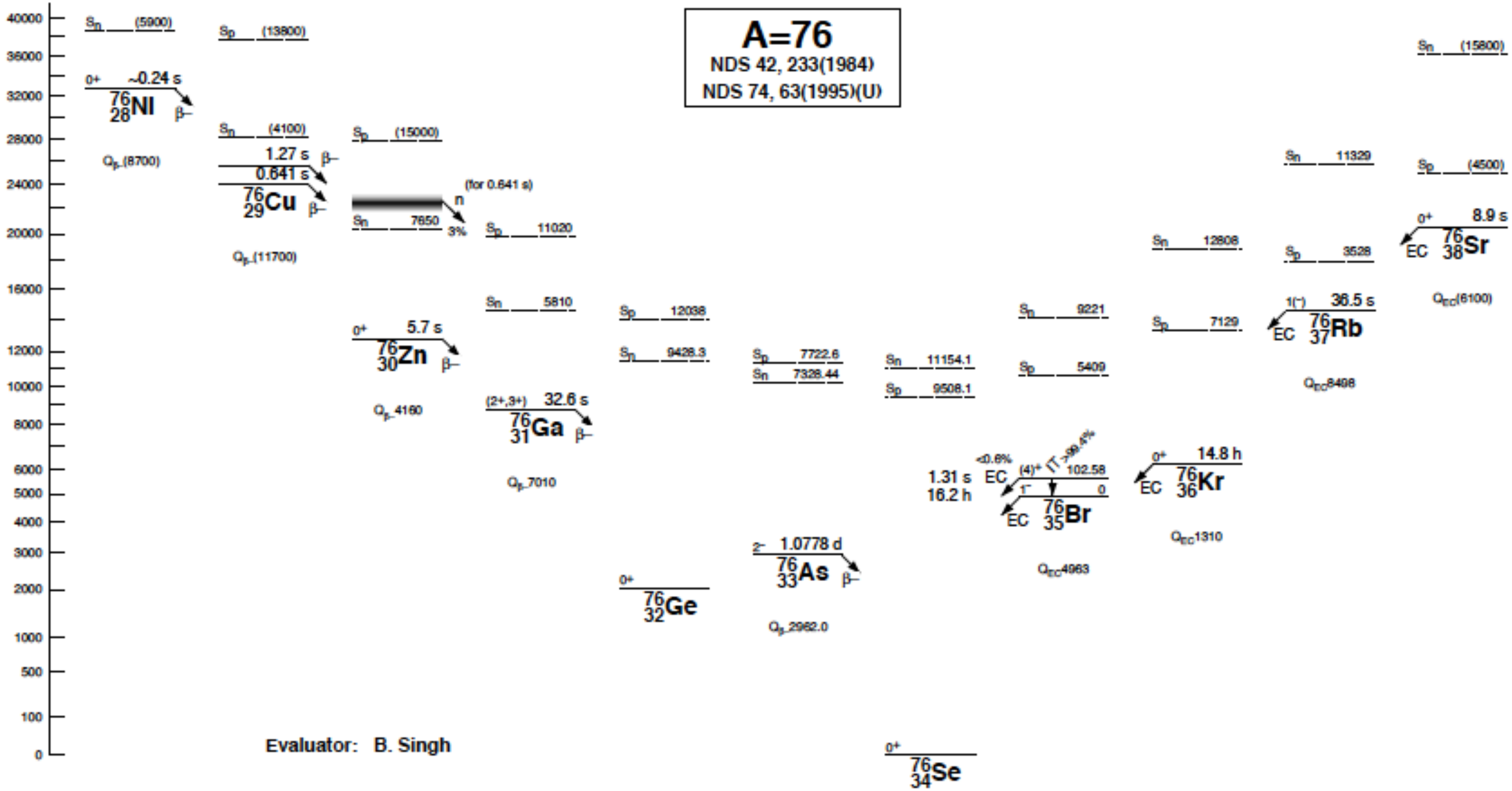


Majorana nature of the neutrinos permit neutrinoless double beta decay:

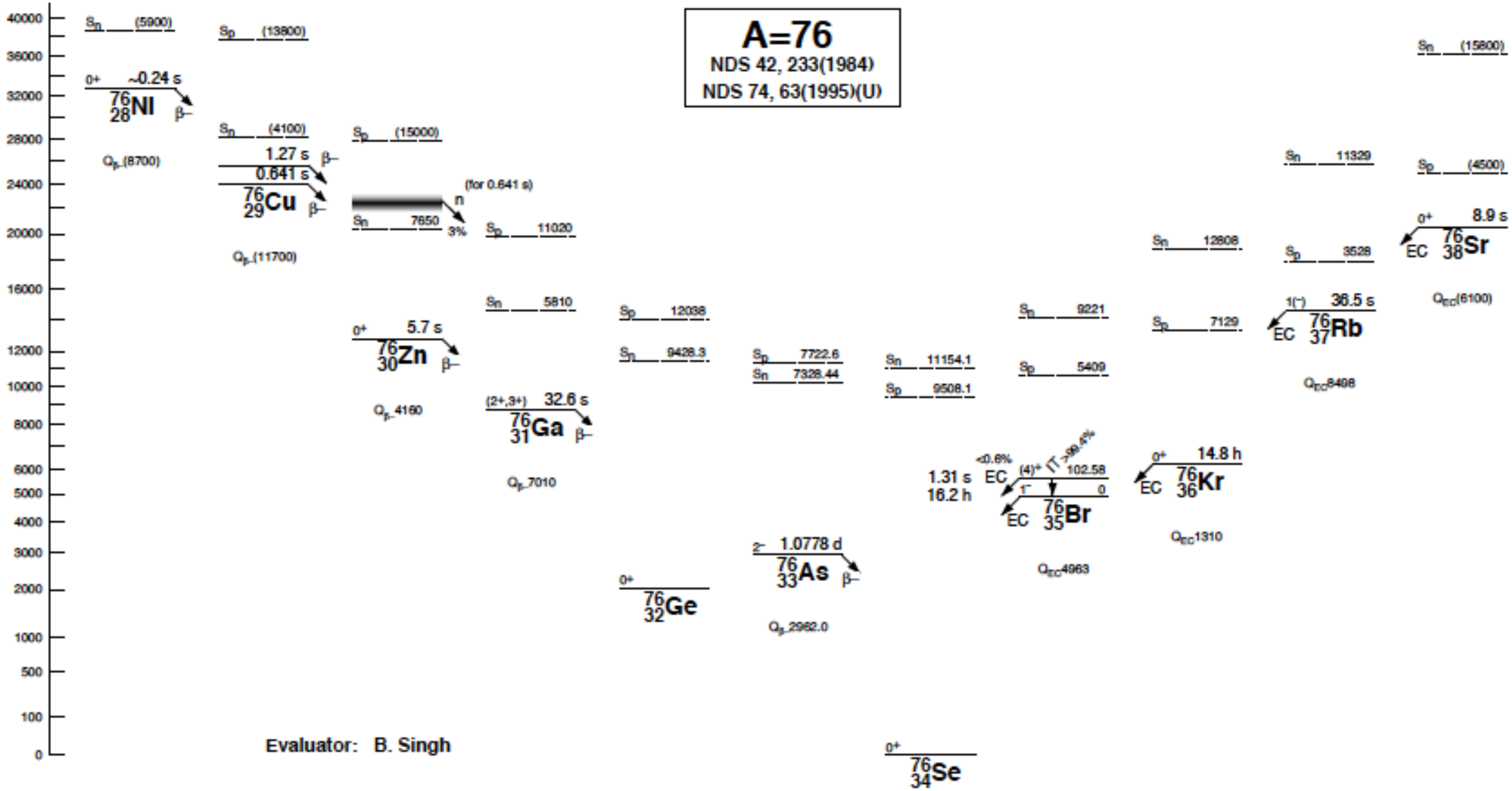


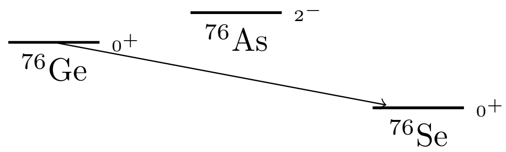
Pairing gives rise to double beta decay:



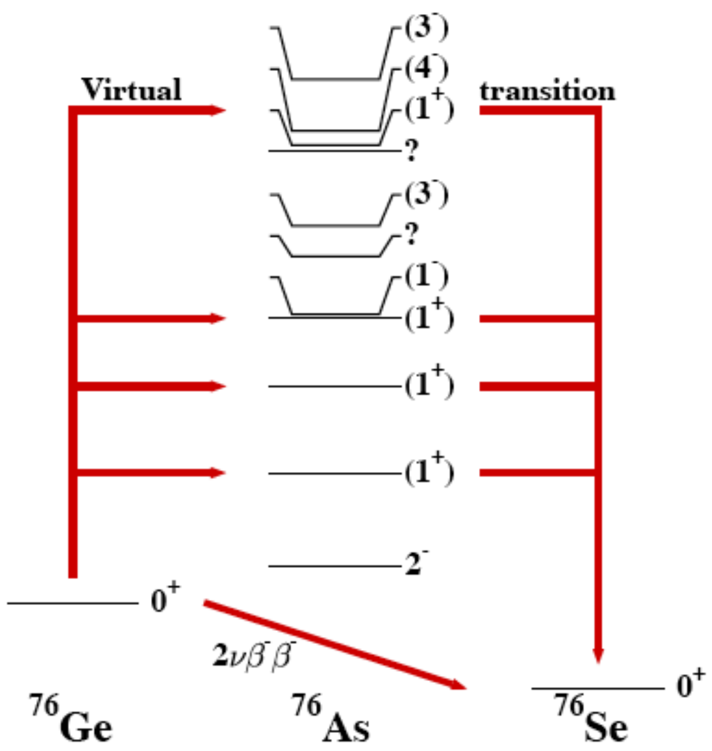


Evaluator: B. Singh


A=76
NDS 42, 233(1984)
NDS 74, 63(1995)(U)
Evaluator: B. Singh
76 28 Ni
0+ ~0.24 s β−
Sn (5900)
Qβ- (8700)
76 29 Cu
1.27 s β−
0.641 s β−
Sp (13800)
Sn (4100)
Qβ- (11700)
(for 0.641 s)
n
3%
76 30 Zn
0+ 5.7 s β−
Sp (15000)
Sn 7650
Qβ- 4160
76 31 Ga
(2+,3+) 32.6 s β−
Sp 11020
Sn 5810
Qβ- 7010
76 32 Ge
0+
Sp 12038
Sn 9428.3
76 33 As
2− 1.0778 d β−
Sp 7722.6
Sn 7328.44
Qβ- 2962.0
76 34 Se
0+
Sn 11154.1
Sp 9508.1
76 35 Br
1.31 s
16.2 h
(4)+ 102.58
1− 0
EC
<0.6%
IT >99.4%
Sn 9221
Sp 5409
QEC 4963
76 36 Kr
0+ 14.8 h EC
Sn 12808
Sp 7129
QEC 1310
76 37 Rb
1(−) 36.5 s EC
Sn 11329
Sp 3528
QEC 8498
76 38 Sr
0+ 8.9 s EC
Sn (15800)
Sp (4500)
QEC (6100)

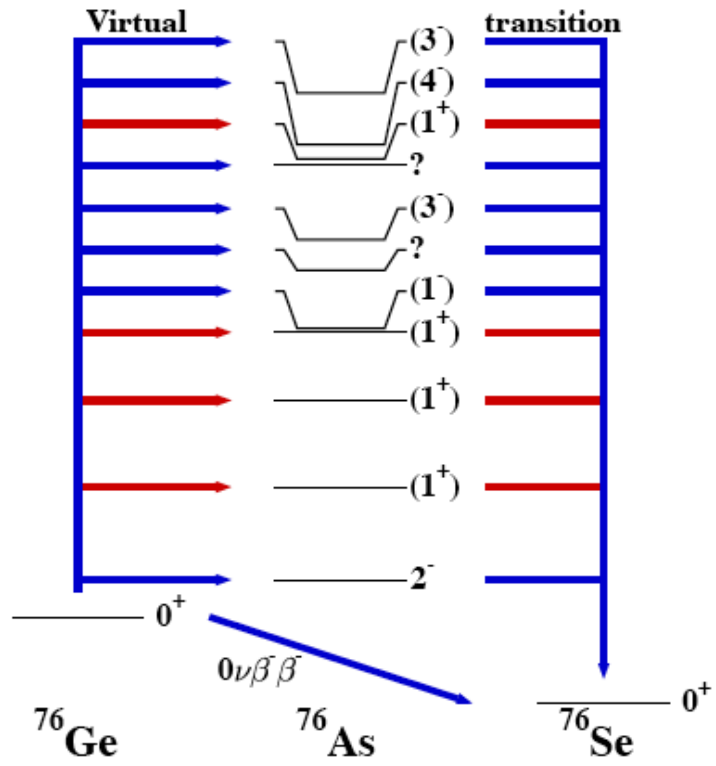

76Ge 0+
76As 2−
76Se 0+

Why are matrix elements of 0νββ and 2νββ different?



2νββ

Only intermediate 1+ states contribute (single-state dominance approximation?)



0νββ

All intermediate states contribute (closure approximation?)

Both approximations could be problematic!

Nuclear matrix elements for double beta decay

$$M^{2\nu} = \sum_n \frac{< f \| \vec{\sigma}\tau_+ \| n > \cdot < n \| \vec{\sigma}\tau_+ \| i >}{E_n - E_i + E_0}$$

Two-neutrino ββ decay

$$M^{0\nu} = M_{GT}^{0\nu} - \frac{M_F^{0\nu}}{g_A^2} + M_T^{0\nu}$$

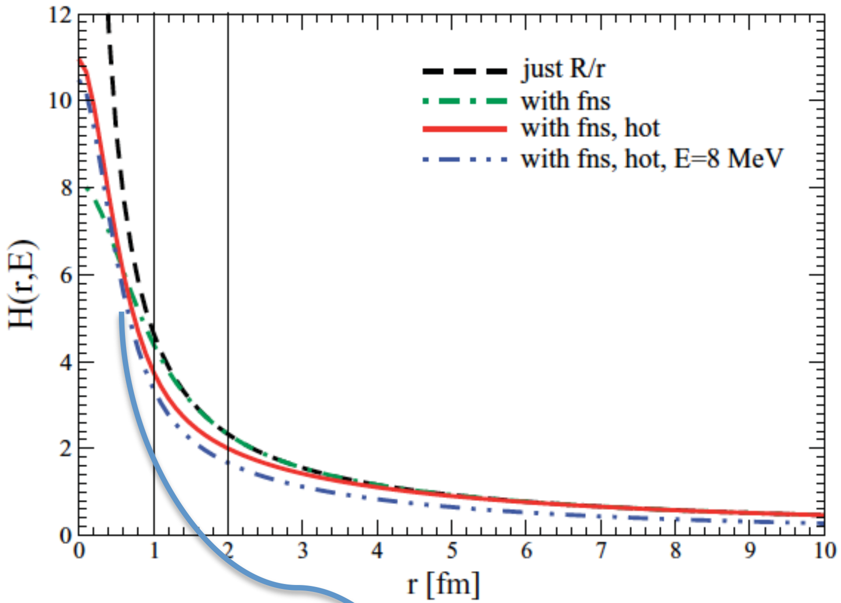
$$M_{GT}^{0\nu} \approx < f | \sum_{j,k} \frac{1}{r_{jk}} \vec{\sigma}(j) \cdot \vec{\sigma}(k)\, \tau_+(j) \tau_+(k) | f >$$

Neutrinoless ββ decay

Nuclear matrix elements for double beta decay

$$M^{2\nu} = \sum_n \frac{< f \| \vec{\sigma}\tau_+ \| n > \cdot < n \| \vec{\sigma}\tau_+ \| i >}{E_n - E_i + E_0}$$

Two-neutrino ββ decay



$$M^{0\nu} = M^{0\nu}_{GT} - \frac{M^{0\nu}_F}{g_A^2} + M^{0\nu}_T$$

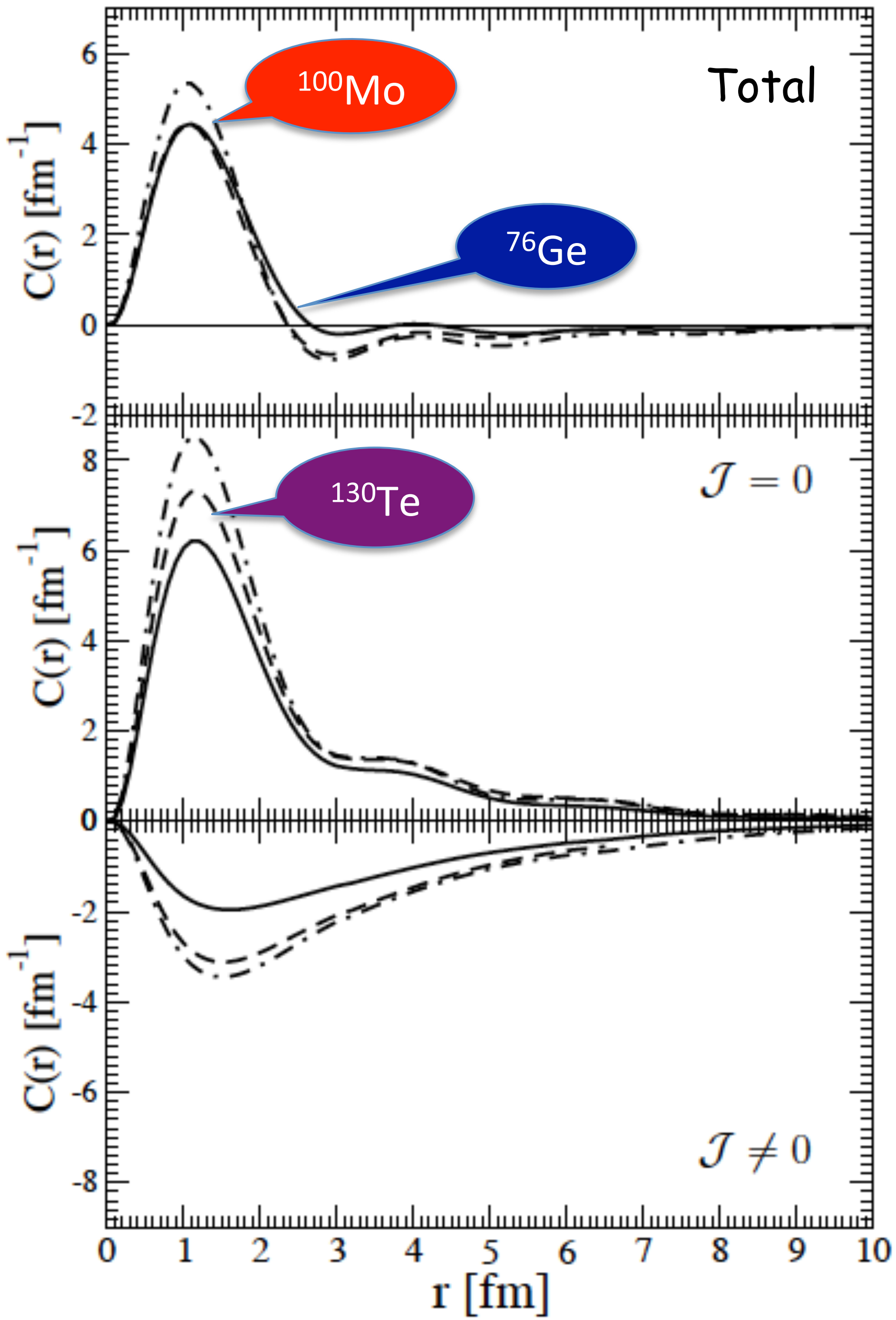
$$M^{0\nu}_{GT} \approx < f | \sum_{j,k} \frac{1}{r_{jk}} \vec{\sigma}(j)\cdot\vec{\sigma}(k)\, \tau_+(j)\tau_+(k) | f >$$

Neutrinoless ββ decay

Nuclear matrix elements

$$M_{GT}^{0\nu} = \int_0^\infty C_{GT}^{0\nu}(r)\,dr$$

Momentum of virtual neutrino, q ~ 1/r
r ~ 2 fm
q ~ 100 MeV



P. Vogel, J. Phys G **39**, 124002 (2012)

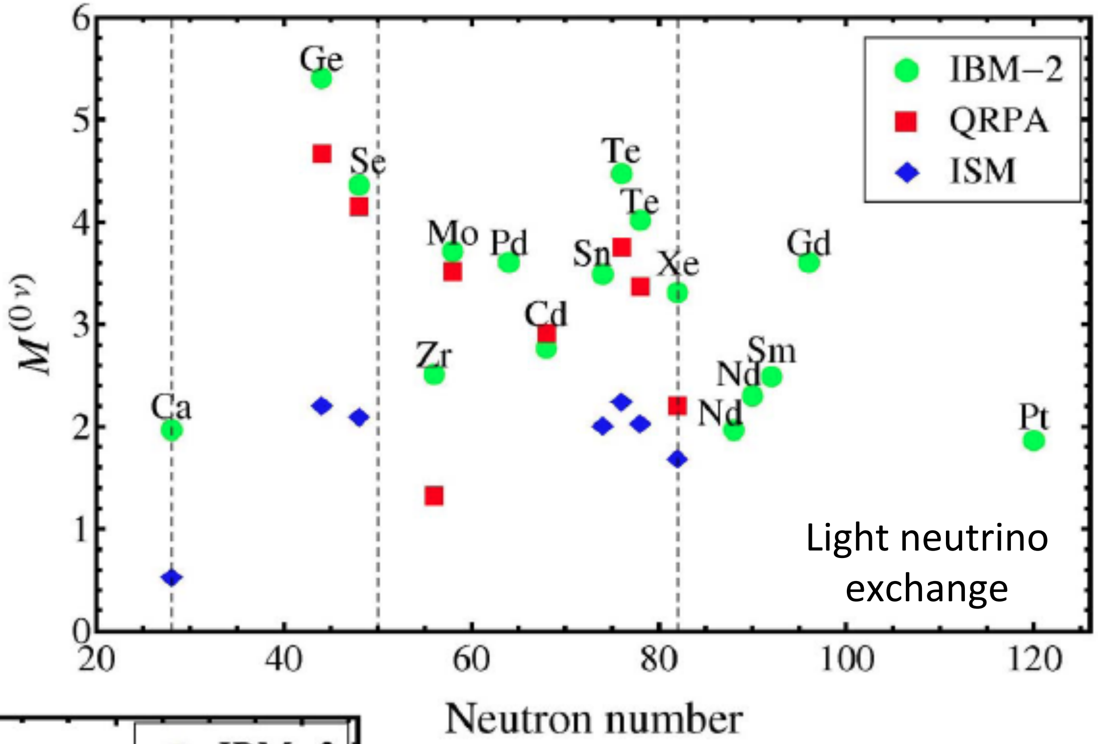
0ν double beta decay

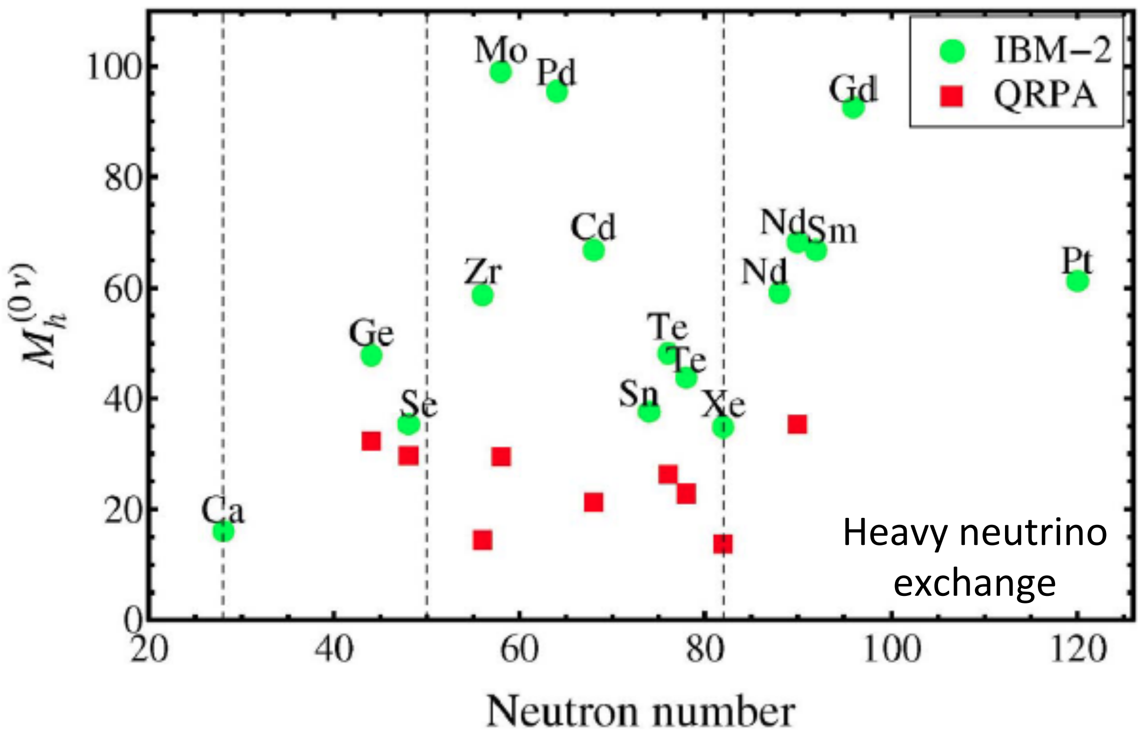
$(1/T_{1/2}) = G(E,Z)\, M^2 \langle m_{\beta\beta}\rangle^2$

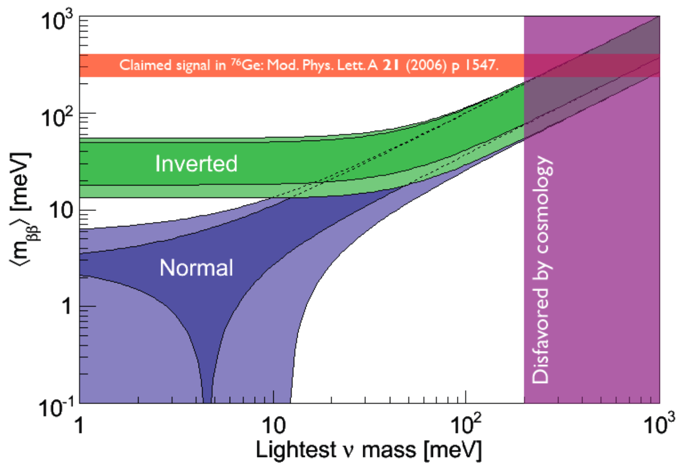
$G(E,Z)$: phase space

M : nuclear matrix element

$\langle m_{\beta\beta}\rangle = |\Sigma_j\, |U_{ej}|^2\, m_j\, e^{i\delta(j)}|$



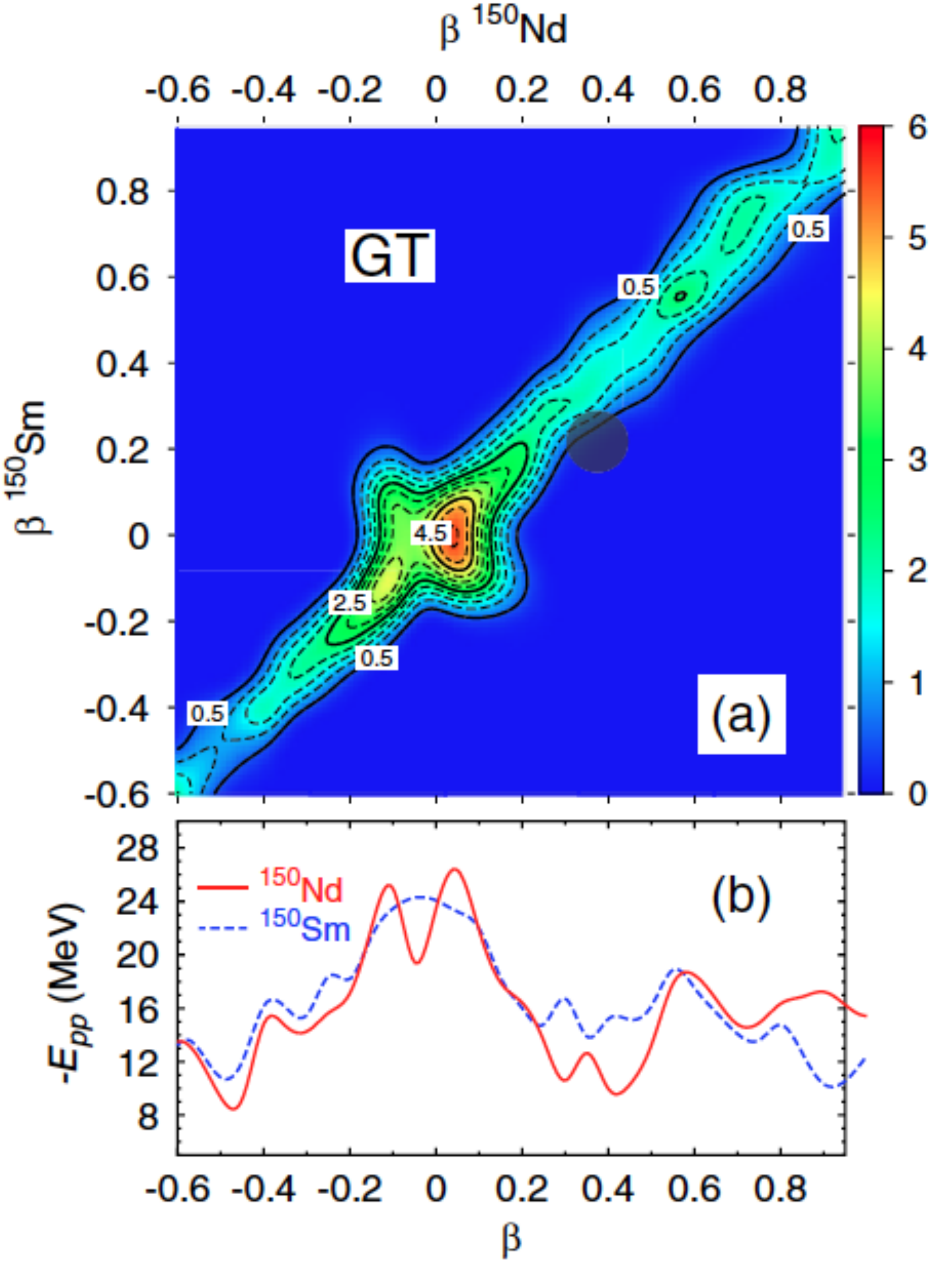




In neutrinoless double beta decay, the overlap between initial and final states should be not too small!

Example:

$^{150}Nd \rightarrow ^{150}Sm + ee$



Rodriguez & Martinez-Pinedo, PRL **105,** 252503 (2010)