# *Nuclear reactions in the early universe I*

*Mark Paris – Los Alamos Nat'l Lab*
*Theoretical Division*
*ISSAC 2014 UCSD*

# Acknowledgments


- IGPPS University Collaborative subcontract #257842
  - "Towards a Unitary and Self-Consistent Treatment of Big Bang Nucleosynthesis"
  - Started FY2014
  - $45k of LDRD-DR for first year (before -9%)
- Supporting grant: LANL Institutional Computing
  - Project Name: w14_bigbangnucleosynthesis
  - Project duration: 2 years (commenced April '14)
  - Year1: 1M, Year2: 1M CPU-hours
- LANL Collaborators
  - T-2: Gerry Hale, Anna Hayes & Gerry Jungman

# Supporting activities 2013—2014

- Paris – T-2 staff member [Jan. 2012 hire]
  - International conferences (2 invited, 1 contributed), seminars, workshops
  - 4 peer-review publications on light nuclear reactions
  - LANL Institutional Computing 2 year grant
  - **LDRD-ER (FY15): BBN proposal oral review 14 May '14 (yesterday)**
- Fuller – Director CASS, UCSD
  - Conferences, colloquia, workshops (many)
  - Publications (many)
  - NSF Grant No. PHY- 09-70064 at UCSD
- Grohs – Graduate Program UCSD – ABD
  - 15 Feb 2013-Sterile Neutrinos: Dark Matter, Neff, and BBN Implications-CASS Journal Club-UCSD; 10 Sep 2013-Nucleosynthesis, Neff, and Neutrino Mass Implications from Dark Radiation-NUPAC Seminar-UNM; 13 Jan 2014-Nucleosynthesis, $N_{eff}$, and Neutrino Mass Implications from Dark Radiation-HEP Seminar-Caltech; 14 Feb 2014-Evidence (to the trained eye) for Sterile Neutrino Dark Matter-CASS Journal Club-UCSD; 28 Mar 2014-Photon Diffusion in the Early Universe-PCGM30 (Pacific Coast Gravity Meeting)-UCSD; 18 Apr 2014-Neutrinos in Cosmology I-CASS Journal Club-UCSD
  - Dissertation targeted Spring 2015

# Organization

Nuclear reactions in the early universe

- Lectures (Paris/E. Grohs)
  I. Overview of cosmology/Kinetic theory/Big bang nucleosynthesis (BBN)
  II. Scattering & reaction formalism/Neutrino energy transport
- Workshop sessions (E. Grohs/Paris)
  I. BBN exercises: compute Nuclear Statistical Equilibrium/electron fraction
  II. Compute primordial abundances vs $\Omega_b h^2$: code parallelization
- Lecture notes
  - Will be available online (URL TBA)



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# Possibly useful references

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# Outline

Lecture I

- Overview
- Cosmological dynamics in GR
- Big bang nucleosynthesis (BBN)
- Boltzmann equation
  - Flat & curved spacetime

Lecture II

- Unitary reaction network (URN) of light nuclei
- Neutrino energy transport
- Evan Grohs: observations of primordial abundances

# Big Bang nucleosynthesis (BBN) in CSM context

- Cosmological Standard Model – $\Lambda$CDM
- Formation of $^4$He, deuterium (D), $^3$H, $^3$He, $^7$Be/Li, … in the primordial 'fireball'
- Epochs (Hot/dense > cool/rarified)
  - Planck > GUT/Inflation > EWPT > QHT > BBN > RC > GF/LSS
- Time of BBN: ~1sec → ~$10^2$ sec; $T_{BBN}$: ~1 MeV → ~10 keV
- Relevant physics: cooling thermonuclear reactor
  - work of expansion cools radiation & matter
  - weak (neutrino) & strong nuclear interactions (& ???)
  - Boltzmann transport, non-equilibrium phenomena
- Comparison to observations
  - stunning successes: CMB, helium, deuterium
  - perplexing anomalies: dark matter/energy, lithium problem

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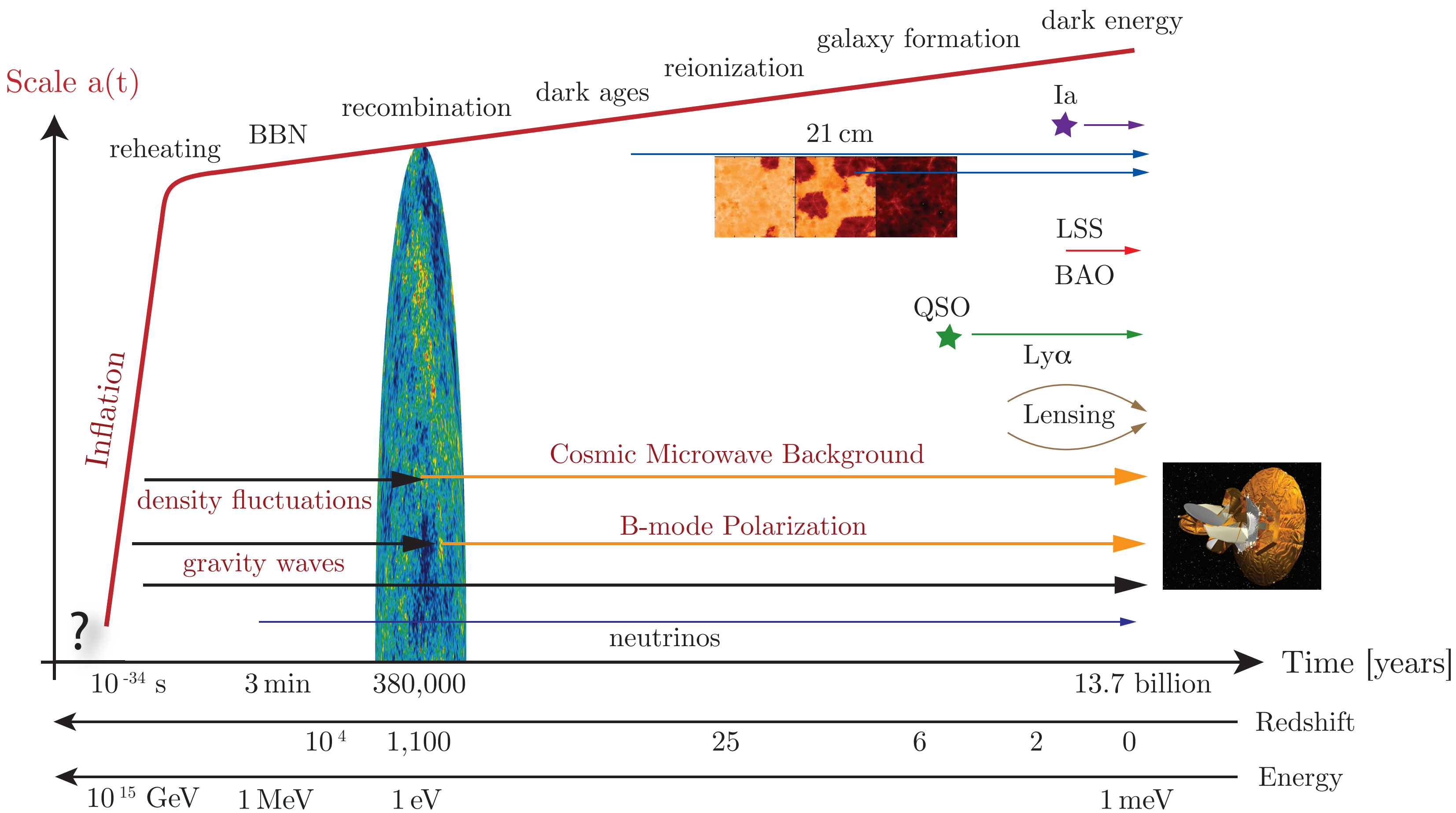

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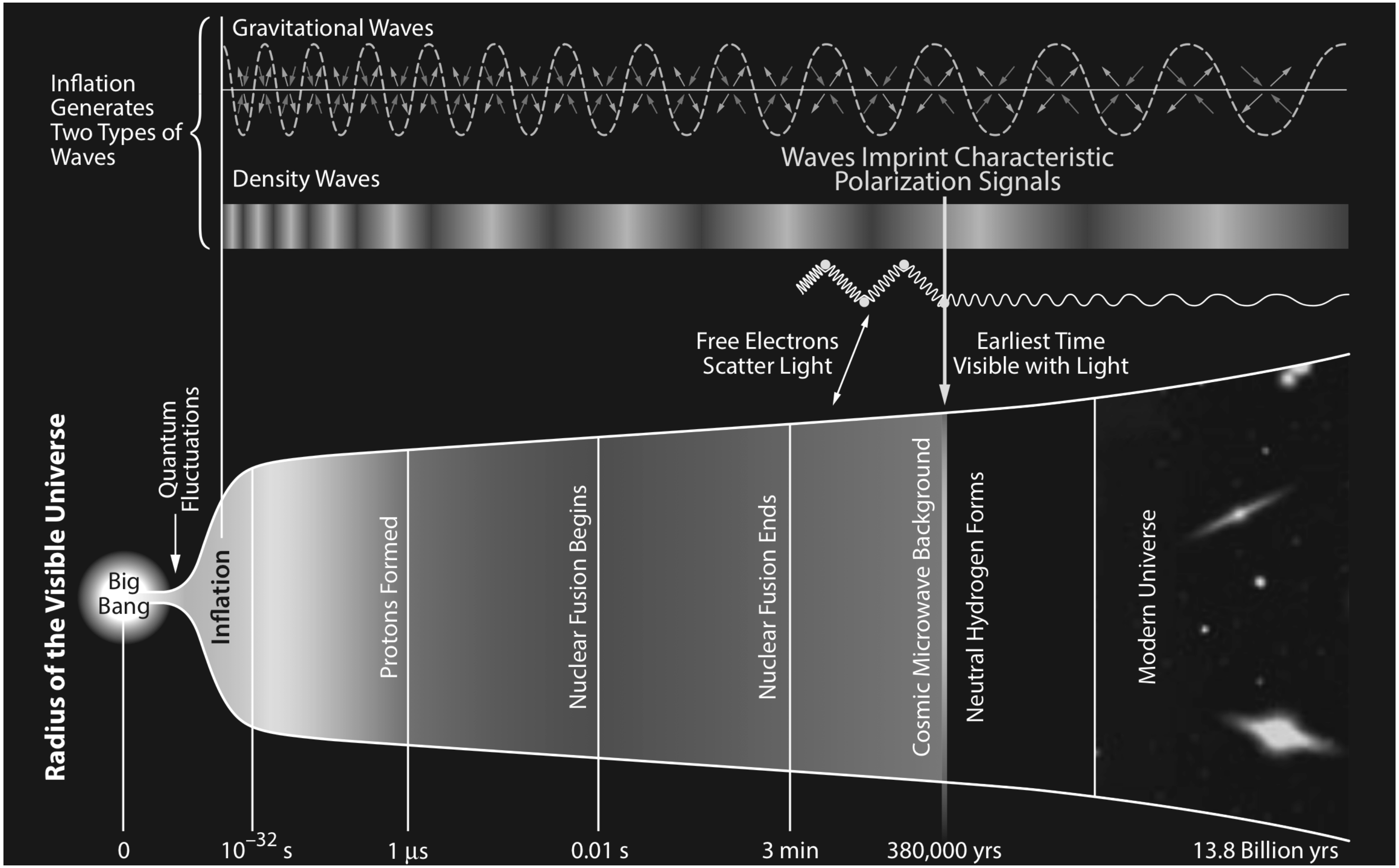




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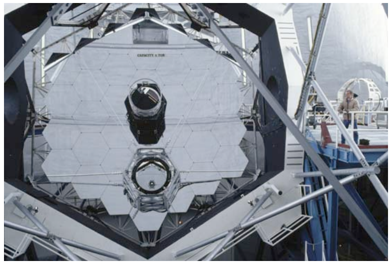
# Observations [more from Evan G. tomorrow]

- Observational astronomy
  - existing 10m-class telescopes: Keck, …
    - Gold-plated: 2% D meas. Pettini & Cooke '13
  - adaptive optics
  - space- & ground-based observatories
  - planned $30^{+}$m-class telescopes: ELT, TMT, …
- Cosmic microwave background
  - Planck, WMAP, PolarBear, APT, SPT, CMBPol, …
- Implications
  - test physics beyond SM; lab tests difficult/impossible
  - precision constraints expected to test ***nuclear*** physics
- ***Unprecedented precision for primordial nuclear abundances***



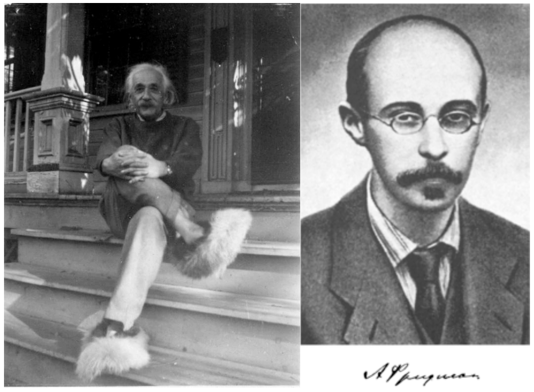
# Standard FLRW Cosmology

- Robertson, Walker show homogen., isotropic > Friedmann, Lemaître solution to GR unique:

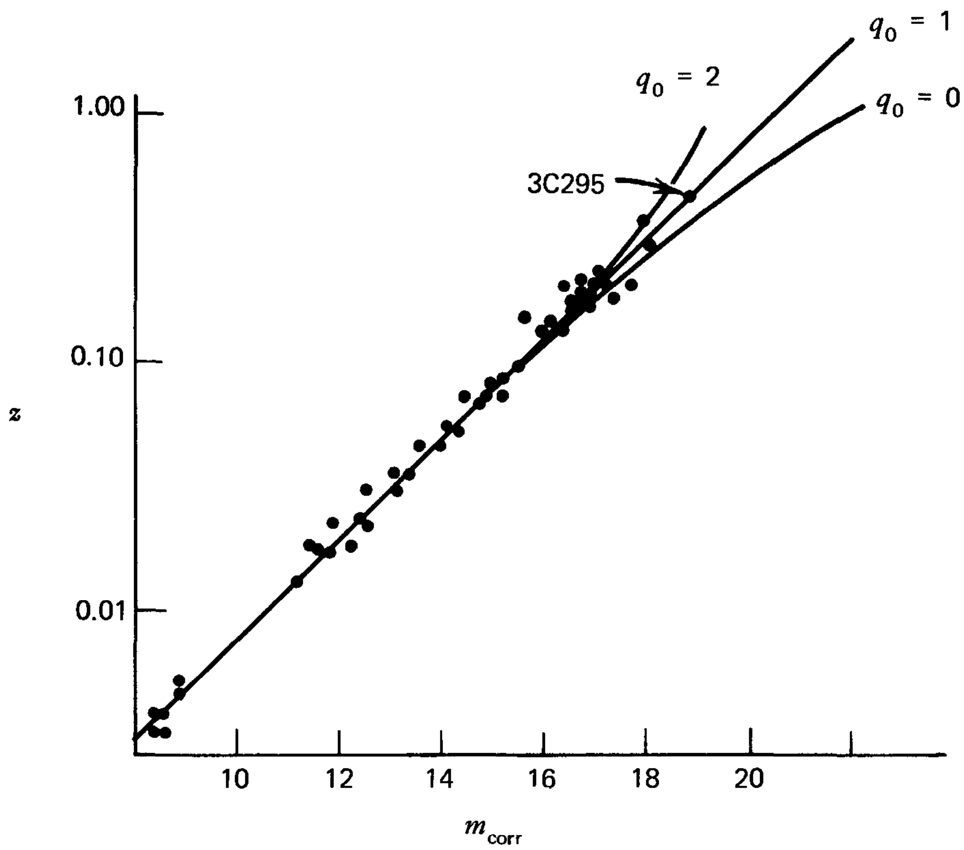
$$G_{00} = 8\pi T_{00}; \quad g_{00} = 1, \quad g_{ij} = -a^2(t), \quad K_{space} \equiv 0; \quad \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho(t)$$

- The ‘Old’, Big Three observations
  - expansion: Hubble “constant,” $H_0$= 67.1 km/s/Mpc (Planck)
  - CMB: T = 2.73 K
  - BBN: concordance at baryon/photon ratio
- HIF universe ⇨ may *only* tune RHS of Einstein-Friedmann Eqn
  - radiation: photons, neutrinos, dark radiation
  - matter: baryonic, dark
  - ΛCDM model: set of assumptions to confront data
    - Wayne Hu (Uchicago): “alive and well” but issues with growth of density fluctuations

# Einstein-Friedmann equations (0)



- An enduring legacy…



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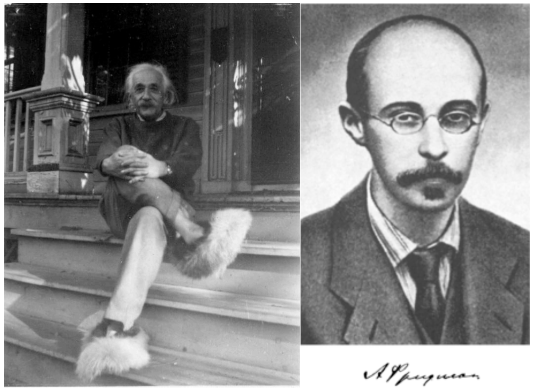

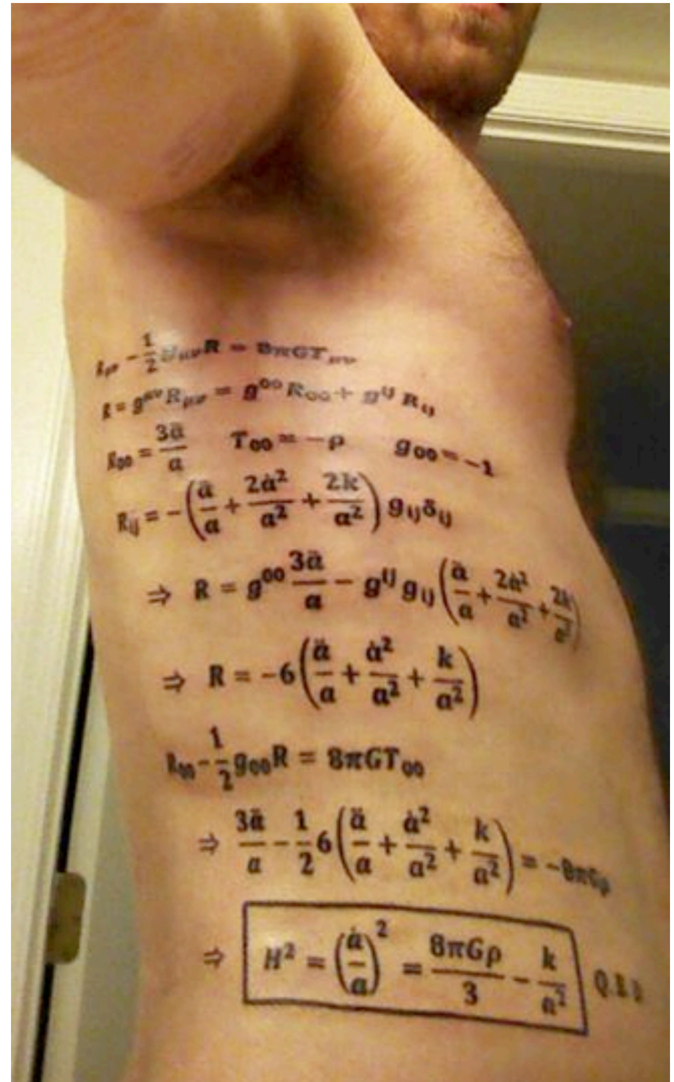


- An enduring legacy…



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# Einstein-Friedmann equations (I)

- Universe dynamics from GR ⟺ energy-momentum density

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = -p g_{\mu\nu} + (p+\rho) u_\mu u_\nu$$

- Einstein/Ricci/Curv Scalar

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R$$

$$R_{\mu\nu} = g^{\alpha\beta} R_{\alpha\mu\beta\nu} = R^{\beta}{}_{\mu\beta\nu} = R_{\alpha\mu}{}^{\alpha}{}_{\nu}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$R^{\mu}{}_{\nu\rho\sigma} = \frac{\partial \Gamma^{\mu}{}_{\nu\sigma}}{\partial x^{\rho}} - \frac{\partial \Gamma^{\mu}{}_{\nu\rho}}{\partial x^{\sigma}} + \Gamma^{\tau}{}_{\nu\sigma}\Gamma^{\mu}{}_{\rho\tau} - \Gamma^{\tau}{}_{\nu\rho}\Gamma^{\mu}{}_{\sigma\tau}$$

- Metric/connection

$$g_{00} = 1, \qquad g_{ij} = a^2(t)\tilde{g}_{ij}$$

$$\Gamma^{0}{}_{00} = \tfrac{1}{2} g^{0\alpha}(2 g_{\alpha 0,0} - g_{00,\alpha}) = 0$$
$$\Gamma^{0}{}_{i0} = \tfrac{1}{2} g^{0\alpha}(g_{\alpha i,0} + g_{\alpha 0,i} - g_{i0,\alpha}) = 0$$
$$\Gamma^{0}{}_{ij} = \tfrac{1}{2} g^{0\alpha}(g_{\alpha i,j} + g_{\alpha j,i} - g_{ij,\alpha}) = -\tfrac{1}{2} g_{ij,0} = -\dot{a} a \tilde{g}_{ij}$$
$$\Gamma^{i}{}_{00} = \tfrac{1}{2} g^{i\alpha}(2 g_{\alpha 0,0} - g_{00,\alpha}) = 0$$
$$\Gamma^{i}{}_{j0} = \tfrac{1}{2} g^{i\alpha} g_{\alpha j,0} = \tfrac{1}{2}\frac{1}{a^2}\tilde{g}^{ik}\frac{\partial[a^2 \tilde{g}_{kj}]}{\partial x^0} = \frac{\dot{a}}{a}\delta^{i}{}_{j} = \Gamma^{i}{}_{0j}$$
$$\Gamma^{i}{}_{jk} = \tfrac{1}{2}\tilde{g}^{il}(\tilde{g}_{lj,k} + \tilde{g}_{lk,j} - \tilde{g}_{jk,l}) \equiv \tilde{\Gamma}^{i}{}_{jk}.$$

# Einstein-Friedmann equations (II)

- Knowing energy density ( $\rho$ ) and pressure (p)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho,$$

$$-\frac{\ddot{a}}{a} = \frac{4\pi}{3}(\rho + 3p)$$

- Covariantly conserved energy-momentum (not indep. eqn.)

$$T^{\mu\nu}{}_{;\nu} = 0 \qquad \dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$$

- Two equations for three unknowns: $a(t), \rho(t), p(t)$
  - Equation of state: $p = w\rho^x$



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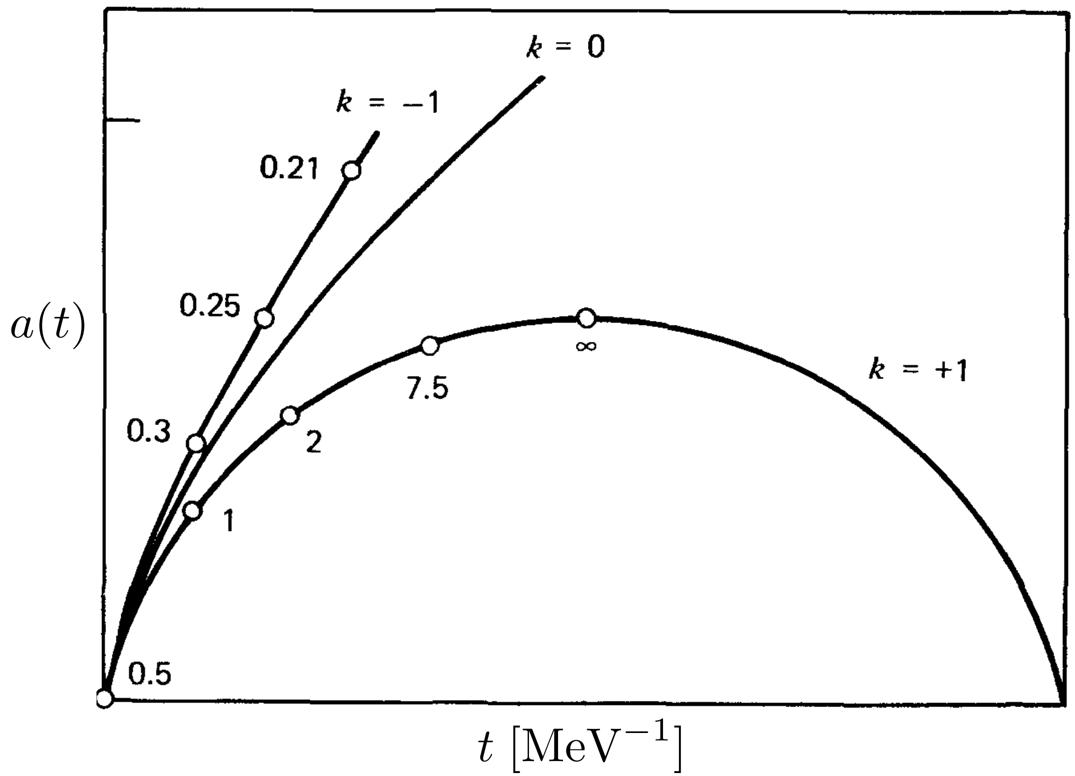
# Einstein-Friedmann equations (III)

- Solution classes




$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho,$$
$$-\frac{\ddot a}{a} = \frac{4\pi}{3}(\rho + 3p)$$

$p = w\rho^x$

- $w > 0 \implies \ddot a < 0$ a(t) negative curvature
- $w < 0 \implies \ddot a > 0$ a(t) positive curvature; inflation

- Acceleration parameter
$$q(t) = -\frac{\ddot a / \dot a}{\dot a / a}$$
- Hubble constant
$$H_0 = \frac{\dot a(t_0)}{a(t_0)} > 0$$
- Redshift
$$1 + z = \frac{a(t_0)}{a(t_1)}, \quad t_0 > t_1$$
- Current critical density
$$\rho_{c,0} = \frac{3}{8\pi} H_0^2 m_{Pl}^2 \approx 5 \frac{\text{protons}}{\text{m}^3}$$



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# Einstein-Friedmann equations (IV)

- Maximally symmetric subspace
  - Consequence of homogeneity & isotropy
  - 'Maximal' number L.I. Killing vector fields N(N+1)/2 (dim N)
  - Flows of Killing vector fields generate isometries of manifold
  - Friedman universe has MS spacelike hypersurfaces
- Tensors in MS spaces
  - scalar:

$$\partial_\mu S(x) = 0$$

  - vector:

$$A^i(x) \equiv 0 \qquad (A^0(x) \neq 0)$$

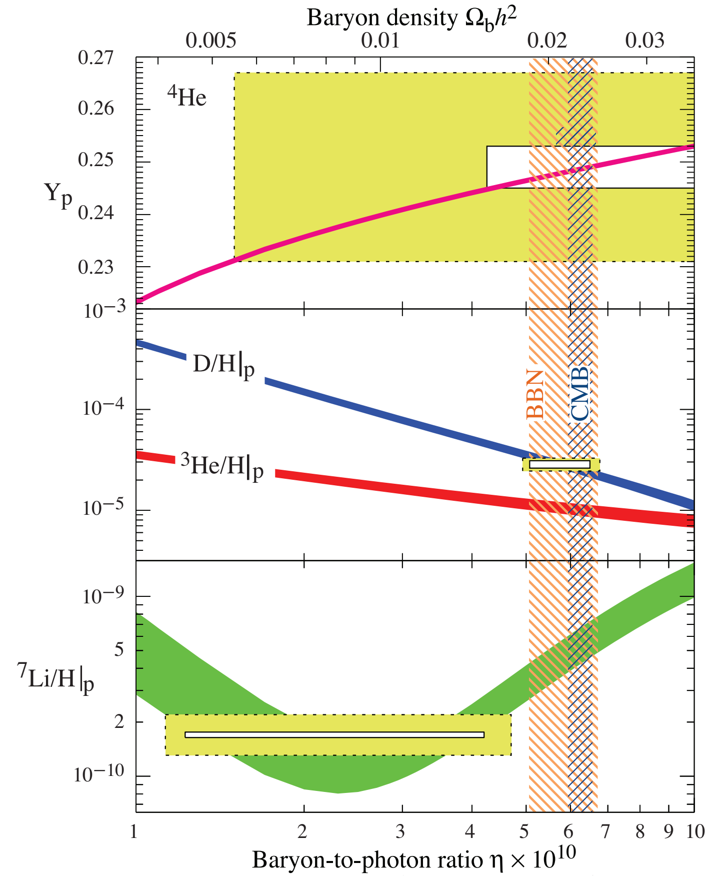
  - rank-2 tensor:

$$B_{ij} = B_{ji} = C g_{ij} \qquad C \neq C(x)$$

# Standard BBN – $^7$Li anomaly

- n/p ratio
  - exquisite sensitivity to neutrino *distribution*
  - ~1:5
- Helium
  - exquisite sensitivity to neutrons
  - mass fraction $Y_p$~1/4 (p:primordial)
- Deuterium
  - ~1:$10^5$
  - Pettini & Cooke obs. better by fact 5
- Lithium
  - mass A=7
    - 3—5 $\sigma$ discrepancy > Li anomaly

h=$H_0$/(100 km/s/Mpc)




*Review Particle Properties*

*Workshop II: generate 'Schramm plot'*



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# The New, 'Big Five' observations

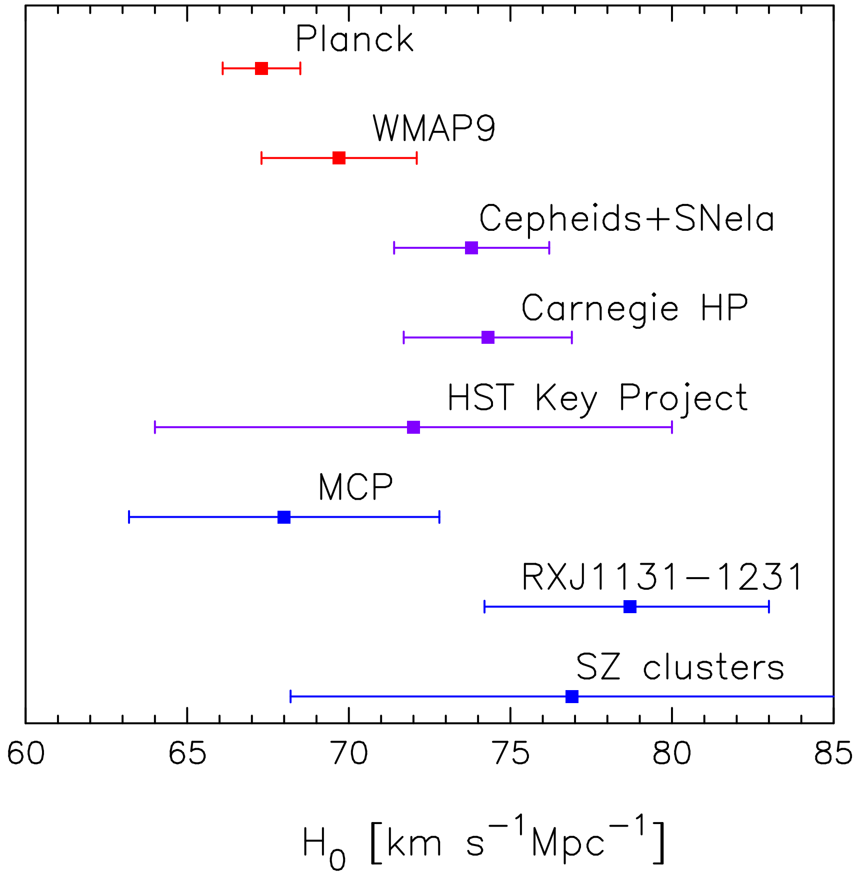
GF: "***VERY EXCITING* situation developing** . . . because of the advent of . . ."

- comprehensive cosmic microwave background (CMB) observations (WMAP, Planck, ACT, SPT, PolarBear, CMBPol,...)
  - $N_{eff}$ : "effective number" of relativistic species; $Y_p$ : $^4$He mass fraction (relative to proton); $\eta$ ($\Omega_b$): baryon-to-photon number fraction; Primordial deuterium abundance $(D/H)_p$; $\Sigma m_\nu$
- 10/30-meter class telescopes, adaptive optics, and orbiting observatories
  - e.g., precision determinations of deuterium abundance dark energy/ matter content, structure history etc.
- Laboratory neutrino mass/mixing measurements
  - mini/micro-BooNE, EXO, LBNE

GF: "is setting up a nearly over-determined situation where ***new*** Beyond Standard Model **neutrino physics** likely ***must*** show itself!"

# Λ CDM: Possible discrepancies (I)

- Hubble expansion




- Planck XVI (2013)
  - "tension between the CMB-based estimates and the astrophysical measurements of $H_0$ is intriguing and merits further discussion"
  - "*highly model dependent*"
  - Λ CDM extraction
    - requires assumptions about relativistic energy density (RED)
    - extra RED could explain discrepancy

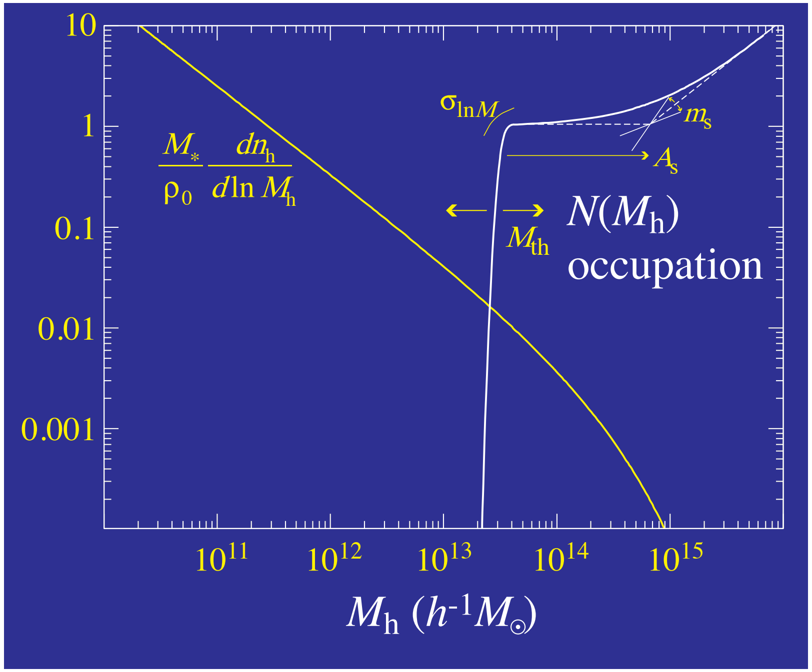
# ΛCDM: Possible discrepancies (II)

- Clustering
  - Abundance of rare massive DM halos exponentially sensitive to growth of structure
  - rms fluct. total mass 8 $h^{-1}$ Mpc spheres with variance

$$\sigma_R^2 = \int \frac{dk}{k} \mathcal{P}_m(k) \left[ \frac{3 j_1(kR)}{kR} \right]^2$$

  - Discrepancy b/w CMB & lensing
  - extra RED can reconcile CMB-inferred $\sigma_8$ with direct observational determinations

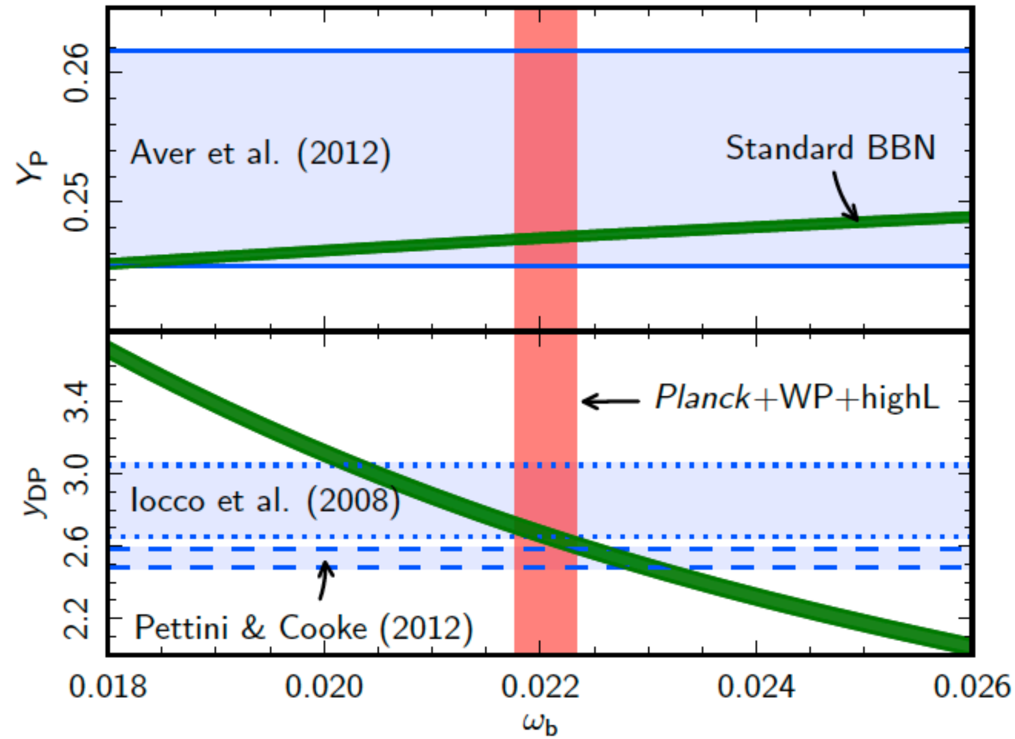
- Dark matter & structure formation




Wayne Hu/2013 October

# $\Lambda$CDM: Possible discrepancies (III)

- Big Bang nucleosynthesis
  - Lithium anomaly
  - $Y_P$,$Y_{DP}$ exquisite sensitivity to active neutrino spectrum:
    - Most neutrons → $^4$He
    - $Y_p \leftarrow n/p \leftarrow f_\nu(p,T)$
  - Thermal effects
    - Hotter later: less neutrons
    - Non-equilibrium $\nu$: less neutrons
  - Probe neutrino sector by studying constraints on various scenarios imposed by precision BBN




Planck XVI (2013)



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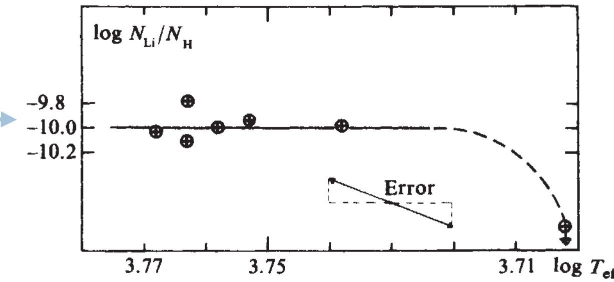
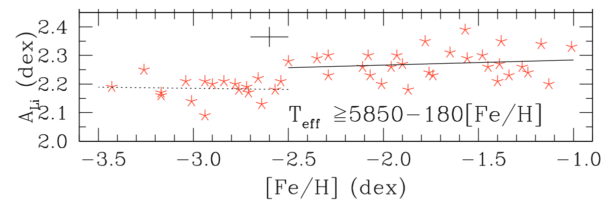
# Possible solutions to lithium anomaly in BBN

- Astronomical explanation
  - Spite-Spite plateau
  - Robust? Melendez et.al.(2010) ‘broken’
- Nuclear physics
  - resonant destruction of mass 7 nuclides
    - $^9$B compound system example (below)
  - Unitarity: fundamental, neglected property of QM
- Physics beyond the standard model (BSM)
  - new particles’ effect on thermal history, etc.




**Even if the lithium anomaly is not nuclear or BSM in origin, precision cosmology forces better treatment of nuclear and astroparticle physics**

# Possible refinements to BBN

- Physics beyond the standard model
  - Increasing observational precision requires "sharpening the tool"
    - improve on existing BBN codes from late 60's
    - replace equilibrium thermal history > full neutrino transport
  - BBN can be used to ***test*** BSM & nuclear physics
- Fundamental principle of nuclear physics: Unitarity
  - Existing codes' nuclear reaction networks don't observe unitarity
  - LANL-developed unitary reaction network (URN) for thermonuclear boost & burn
  - Two objectives from nuclear physics perspective
    - Test LANL URN in similar (but different, high-entropy) environment
    - Address fundamental problem in cosmology
- NB: ***without correct URN, req'd. by QM, BSM physics uncertain***

# BBN project: introduce a new theoretical tool

- Outline for the rest of talk
  - $1^{st}$ refinement: neutrino sector
  - $2^{nd}$ refinement: nuclear physics

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- Outline for the rest of talk
  - 1st refinement: neutrino sector
  - 2nd refinement: nuclear physics



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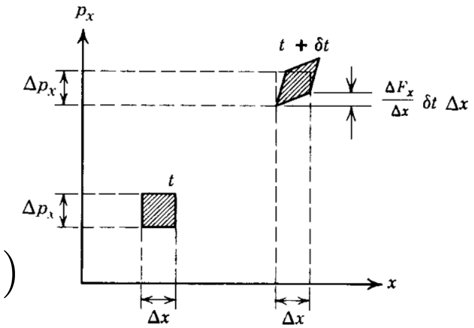
# Unitary, self-consistent primordial nucleosynthesis

- BBN as a tool for precision cosmology
  - incorporate unitarity into strong & electroweak interactions
  - couple unitary reaction network (URN) to full Boltzmann transport code
    - neutrino energy distribution function evolution/transport code
    - fully coupled to nuclear reaction network
    - calculate light primordial element abundance for non-standard BBN
      - active-sterile neutrino mixing
      - massive particle *out-of-equilibrium* decays→energetic active SM particles
  - New tools/codes for nuc-astro-particle community:
    - test new physics w/BBN
    - existing codes are based on Wagoner's (1969) code
      - we will improve this situation dramatically

# Kinetic theory: flat spacetime

- distribution function




$$dN(\mathbf{r},\mathbf{p},t) = d^3r\, dn(\mathbf{r},\mathbf{p},t) = d^3r\, \frac{d^3p}{(2\pi)^3}\, f(\mathbf{r},\mathbf{p},t)$$

$$\frac{d}{dt} f(\mathbf{r},\mathbf{p},t) = \frac{\partial}{\partial t} f(\mathbf{r},\mathbf{p},t) + \frac{\partial \mathbf{r}}{\partial t}\cdot\frac{\partial}{\partial \mathbf{r}} f(\mathbf{r},\mathbf{p},t) + \frac{\partial \mathbf{p}}{\partial t}\cdot\frac{\partial}{\partial \mathbf{p}} f(\mathbf{r},\mathbf{p},t)$$

- Boltzmann equation $\frac{d}{dt} f(\mathbf{r},\mathbf{p},t) = \left(\frac{\partial f}{\partial t}\right)_c$
  - collisionless: $\left(\frac{\partial f}{\partial t}\right)_c = 0$
  - collisional:

$$\left(\frac{\partial f}{\partial t}\right)_c = R_i - R_o,$$
$$= \int \frac{d^3p_2}{2E_2(2\pi)^3}\frac{d^3p_1'}{2E_{1'}(2\pi)^3}\frac{d^3p_2'}{2E_{2'}(2\pi)^3}$$
$$\times\, (2\pi)^4\delta^{(4)}\left(p_1' + p_2' - (p_1 + p_2)\right)|\mathcal{M}_{fi}|^2 \left(F_{1'2'} - F_{12}\right)$$

$$d\sigma = \frac{1}{2E_1 2E_2|v_1 - v_2|}\frac{d^3p_1'}{2E_{1'}(2\pi)^3}\frac{d^3p_2'}{2E_{2'}(2\pi)^3}$$
$$\times\, (2\pi)^4\delta^{(4)}\left(p_1' + p_2' - (p_1 + p_2)\right)|\mathcal{M}_{fi}|^2,$$

$$\left(\frac{\partial f}{\partial t}\right)_c = \int \frac{d^3p_2}{(2\pi)^3}\, d\sigma|\mathbf{v}_1 - \mathbf{v}_2|(F_{1'2'} - F_{12})$$

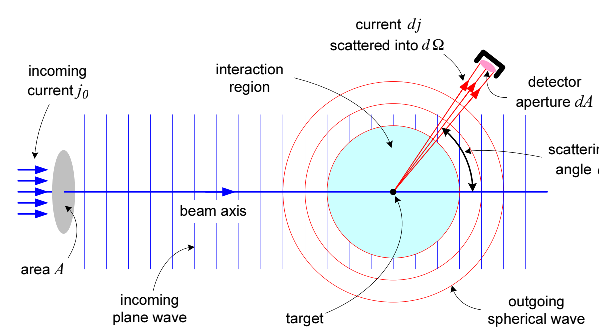
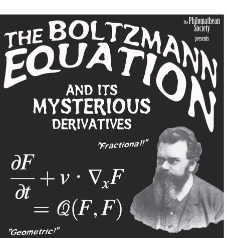



Fig. H. Haberzettl



# Kinetic theory: curved spacetime

- Liouville operator

$$\frac{d}{d\lambda} f(x^\mu(\lambda), p^\mu(\lambda)) = \frac{\partial f}{\partial x^\mu}\frac{dx^\mu}{d\lambda} + \frac{\partial f}{\partial p^\mu}\frac{dp^\mu}{d\lambda}$$

$$L_F(f(p,t)) = \left[\frac{\partial}{\partial t} - \frac{\dot{a}}{a} p \frac{\partial}{\partial p}\right] f(p,t) = \frac{1}{E} C(E)$$

$$p^0 = E = (p^2 + m^2)^{1/2}$$

Geodesic equation

$$p^\mu \equiv \frac{dx^\mu}{d\lambda}$$

$$\frac{dp^\mu}{d\lambda} + \Gamma^\mu{}_{\nu\rho} p^\nu p^\rho = 0$$

- Relativistic Boltzmann equation

$$n(t) = \int \frac{d^3p}{(2\pi)^3} f(p,t)$$

$$\dot{n}(t) + 3H(t)n(t) = a^{-3}\frac{d}{dt}(a^3 n(t)) = \int \frac{d^3p}{(2\pi)^3}\frac{C(E)}{E}$$

$$H(t) = \frac{\dot{a}}{a}$$

Hubble expansion

$$\dot{\xi} + 3H\xi = 0$$

$$\dot{n} + 3Hn = \mathcal{C}[n]$$

$$\frac{\partial}{\partial t}\left(\frac{n}{\xi}\right) = \frac{1}{\xi}\mathcal{C}[n]$$

# Entropy production

- Boltzmann H-theorem
  - Entropy current

$$S^\mu = -\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p^0}\left[f\log f \mp (1\pm f)\log(1\pm f)\right]$$

$$S^\mu{}_{;\mu} = -\int \frac{d^3p}{(2\pi)^3}\log f C(E) \geq 0$$

$$\text{Equilibrium} \implies S^\mu{}_{;\mu} \equiv 0$$

- Equivalence relations
  - Collision integral is zero; proper entropy is constant; equilibrium distributions
  - Collision integral non-zero; proper entropy generation; non-equilibrium

# Equilibrium distributions

- Fermi-Dirac

$$\mathscr{Z}_{FD} = \sum_{N=0}^{1} \left(e^{-\beta(\epsilon-\mu)}\right)^N = 1 + e^{-\beta(\epsilon-\mu)}.$$

$$f_{FD} = \langle N \rangle_{FD} = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$$

- Bose-Einstein

$$\mathscr{Z}_{BE} = \sum_{N=0}^{\infty} \left(e^{-\beta(\epsilon-\mu)}\right)^N = \frac{1}{1 - e^{-\beta(\epsilon-\mu)}},$$

$$f_{BE} = \langle N \rangle_{BE} = \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$$

- Maxwell-Boltzmann

$$f_{BE} = f_{FD} \approx e^{-\beta(\epsilon-\mu)} = f_{MB}$$

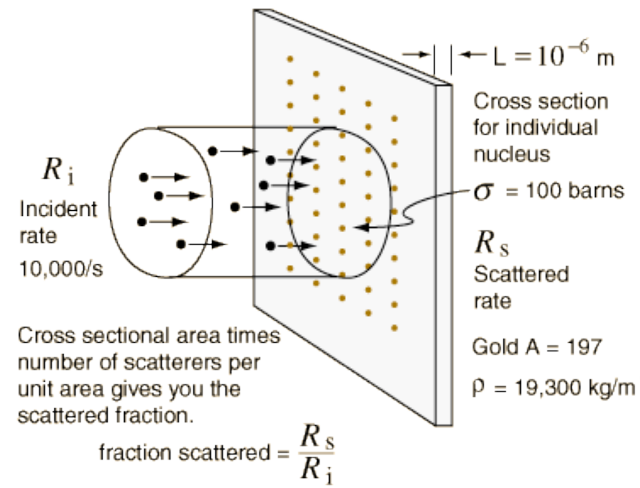
The equilibrium distributions satisfy the condition that the collision integral is zero. But here we derive them from the grand canonical ensemble.

# Kinetic regimes

- Equilibrium $\Gamma \gg H(t)$
  - Hubble exp. negligible for kinetics
  - Forward/Reverse rates *detail balance*
  - Reaction rate sufficiently fast to explore much phase space
  - Caveat: FLRW no timelike Killing field
- Kinetic $\Gamma \simeq H(t)$
  - Hubble exp. and reactions compete
  - Non-zero net=F-R rate
  - Boltzmann H-theorem: dS/dt>0 but ~ 0
    - However, assume adiabatic
- Decoupled $\Gamma \ll H(t)$
  - e.g. Relativistic: $T \sim a^{-1}$
  - Free-streaming; distribution frozen

Reaction rate

$$d\Gamma_{34,12} = dn_2 \langle \sigma_{34,12} v_{12,rel} \rangle$$

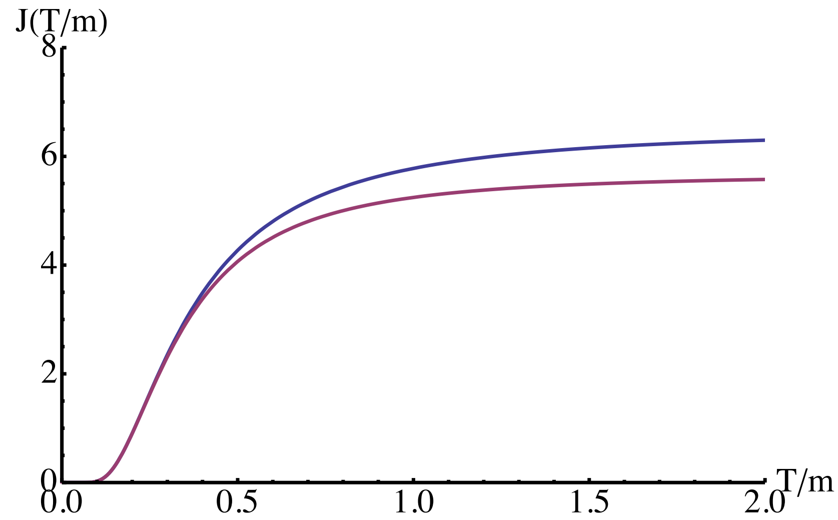


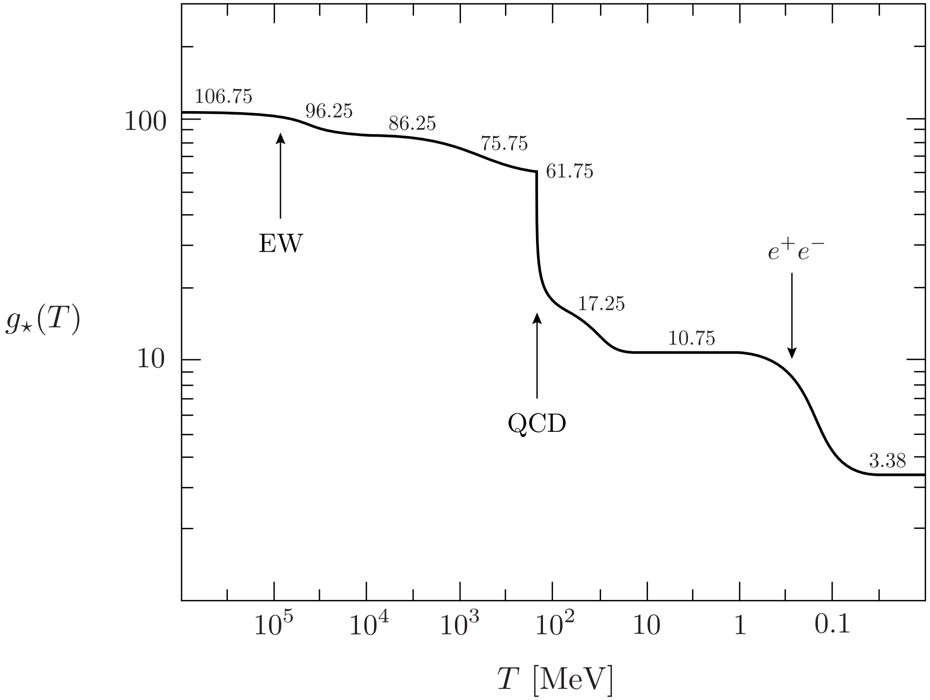
# Cosmological transitions (*Caveat Emptor*)

$$\rho(T) = \sum_{i=\gamma,\nu_j,\ell^\pm,\ldots} \int \frac{d^3p}{(2\pi)^3} f_i(p)\sqrt{p^2+m_i^2}$$

$$= \sum_i g_i \frac{T_i^4}{2\pi^2} J_{\pi_i}(x_i)$$

$$J_{\pi_i}(x_i) = \int_0^\infty d\xi\, \xi^2 \frac{\sqrt{\xi^2+x_i^2}}{e^{\sqrt{\xi^2+x_i^2}}+\pi_i}, \quad x_i = m_i/T$$




$$\rho(T) = g_\star(T)\frac{\pi^2}{30}T_\gamma^4$$

$$g_\star = \sum_i g_i \left(\frac{T_i}{T_\gamma}\right)^4 \frac{J_{\pi_i}(x_i)}{J_\gamma(0)}$$

NB: $T_i \doteq T_\gamma$

$$J_i(x_i) \to \theta\left(T - \frac{m_i}{6}\right) J(x_i) \to \text{ arbitrary!}$$

# Reaction network reduction of Boltzmann eqn

- Reaction network reduction of BEq. (classical, non-degenerate)

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_{\alpha_1}) = \sum_{\alpha_2 \beta} \int_{\substack{p_{\beta_1} p_{\beta_2} \\ p_{\alpha_1} p_{\alpha_2}}} (2\pi)^4 \delta^{(4)}(p_{\beta_1} + p_{\beta_2} - (p_{\alpha_1} + p_{\alpha_2}))$$

$$\times \left|\mathcal{M}_{\beta\alpha}\right|^2 (f_{\beta_1} f_{\beta_2} - f_{\alpha_1} f_{\alpha_2})$$

$$= -\sum_{\alpha_2 \beta} n^{(0)}_{\alpha_1} n^{(0)}_{\alpha_2} \langle \sigma_{\beta\alpha} v_\alpha \rangle \left[ \frac{n_{\alpha_1} n_{\alpha_2}}{n^{(0)}_{\alpha_1} n^{(0)}_{\alpha_2}} - \frac{n_{\beta_1} n_{\beta_2}}{n^{(0)}_{\beta_1} n^{(0)}_{\beta_2}} \right]$$

$$\langle \sigma_{\beta\alpha} v_\alpha \rangle \equiv \frac{1}{\mathcal{N}} \int \frac{d^3 p_{\beta_1}}{(2\pi)^3} \int \frac{d^3 p_{\beta_2}}{(2\pi)^3} |\mathbf{v}_1 - \mathbf{v}_2| d\sigma_{\beta\alpha} f_{\alpha_1} f_{\alpha_2}$$

$$\mathcal{N} = \int \frac{d^3 p_{\beta_1}}{(2\pi)^3} \int \frac{d^3 p_{\beta_2}}{(2\pi)^3} f_{\alpha_1} f_{\alpha_2} \equiv n^{(0)}_{\alpha_1} n^{(0)}_{\alpha_1}$$

$$n_i^{(0)} = g_i \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T} \approx g_i \left( \frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T}$$

# n-p weak equilibrium [Workshop exercise]

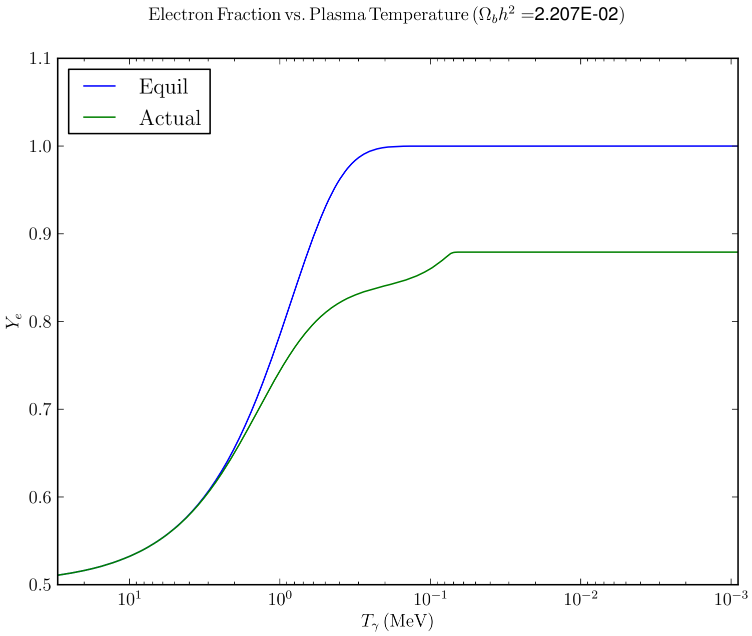
- At high T ~ 10's MeV $X_n \sim X_p \sim 1/2$
- At 10 MeV > T > 1 MeV (ignore nucleons)

$$n\nu_e \leftrightarrow pe^-, \qquad ne^+ \leftrightarrow p\bar{\nu}_e, \qquad n \leftrightarrow pe^-\bar{\nu}_e.$$

- Equilibrium condition

$$\mu_p = \mu_n \implies \frac{n_n^{(0)}}{n_p^{(0)}} = e^{Q/T}$$

$$Q = m_n - m_p \simeq 1.293 \text{ MeV}$$




$$\frac{dX_n}{dt} = -\lambda(n \to p)X_n + \lambda(p \to n)(1 - X_n)$$

$$\lambda(i \to j) = n_\ell^{(0)}\langle\sigma_{ji}v_i\rangle$$

$$X_n = \frac{n_n}{n_b} \qquad n_b = n_n + n_p$$

$$X_p \approx X_{e^-}$$

# Big bang nucleosynthesis [Workshop exercise]

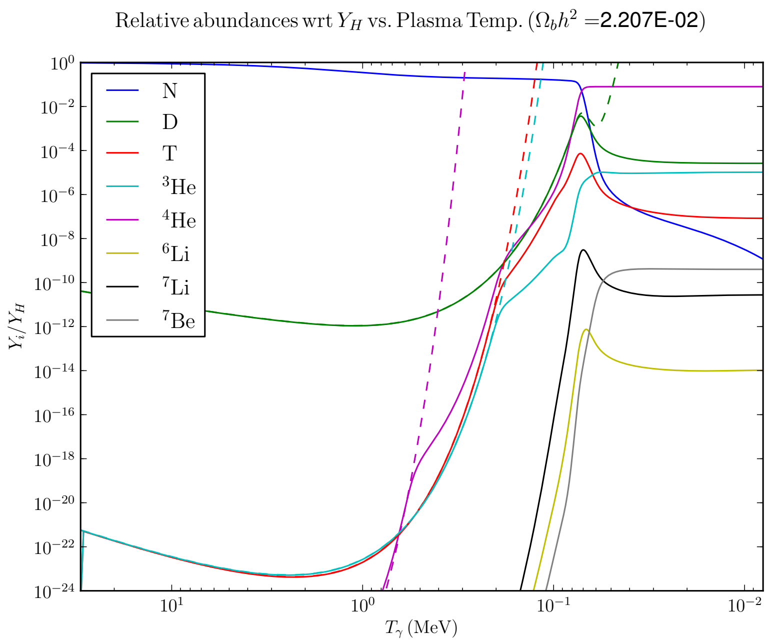
- Full reaction network [*NB: should be unitary*]

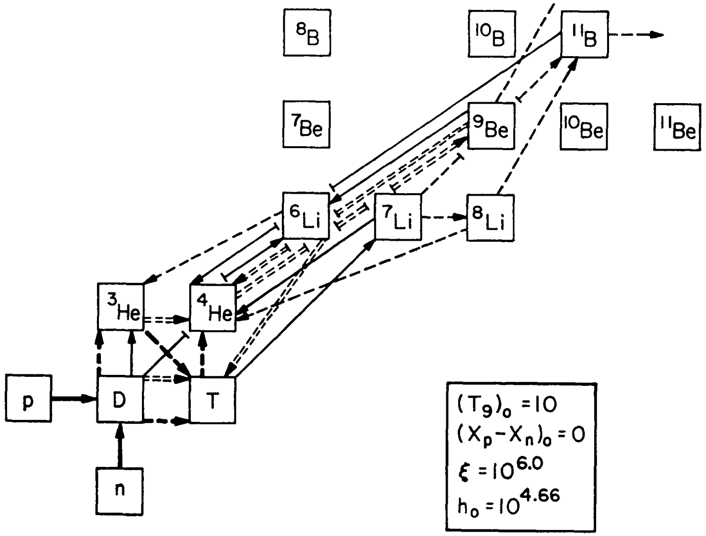
$$Y_i = \frac{n_i}{n_b}$$

$$\frac{dY_{\alpha_1}}{dt} = \sum_{\alpha_2 \beta} \left[ -n_b \langle \sigma_{\beta\alpha} \rangle Y_{\alpha_1} Y_{\alpha_2} + n_b \langle \sigma_{\alpha\beta} \rangle Y_{\beta_1} Y_{\beta_2} \right]$$

- Nuclear statistical equilibrium

$$\mu_A = Z\mu_p + N\mu_n$$



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# End Lecture I