

Collective Neutrino Oscillations

Huaiyu Duan



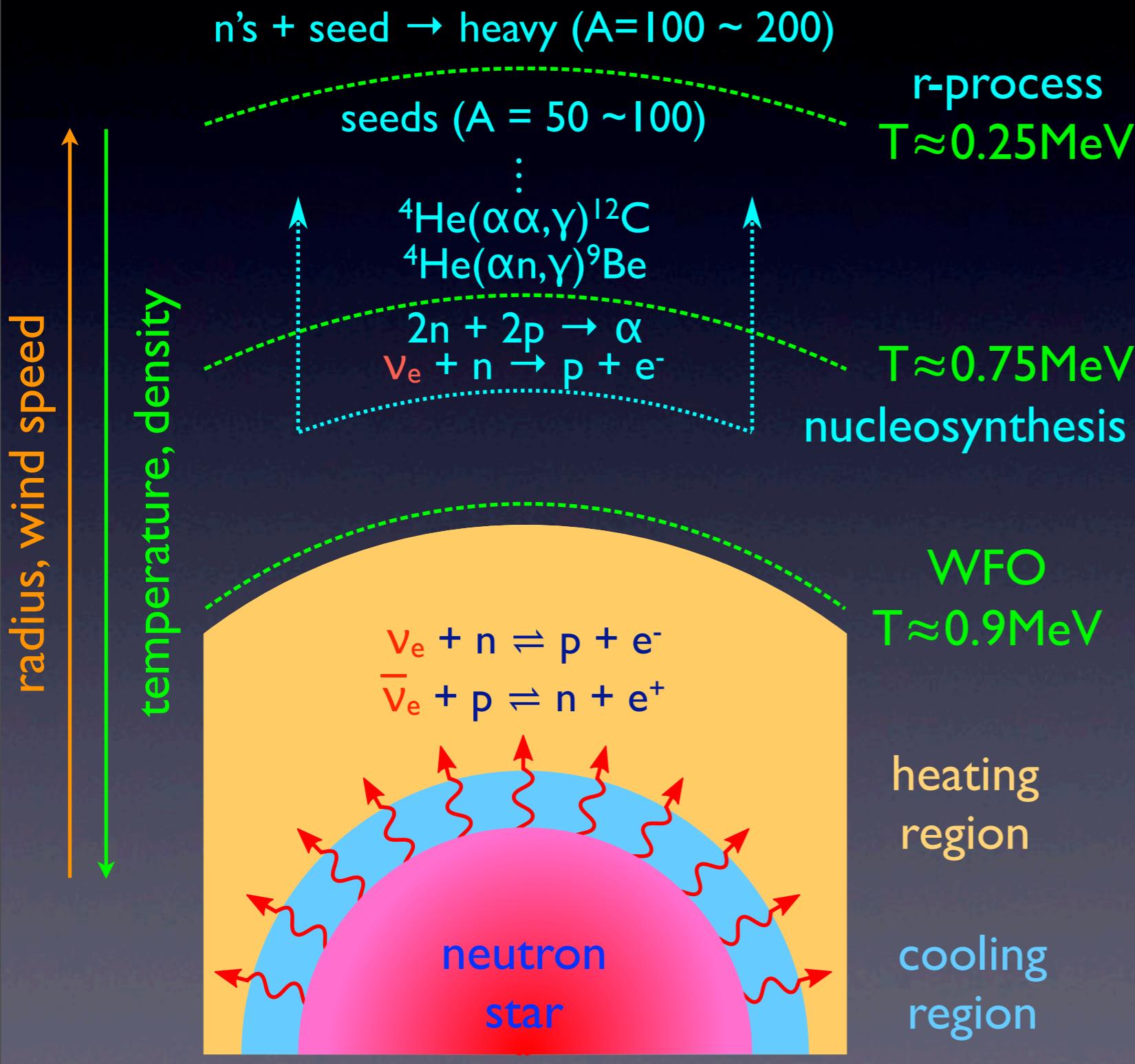
THE UNIVERSITY *of*
NEW MEXICO

*International Summer School on AstroComputing 2014
Neutrino & Nuclear Astrophysics*

Outline

- ◆ Introduction & overview
- ◆ Understandings & insights
- ◆ New developments & challenges

Neutrinos in Supernovae

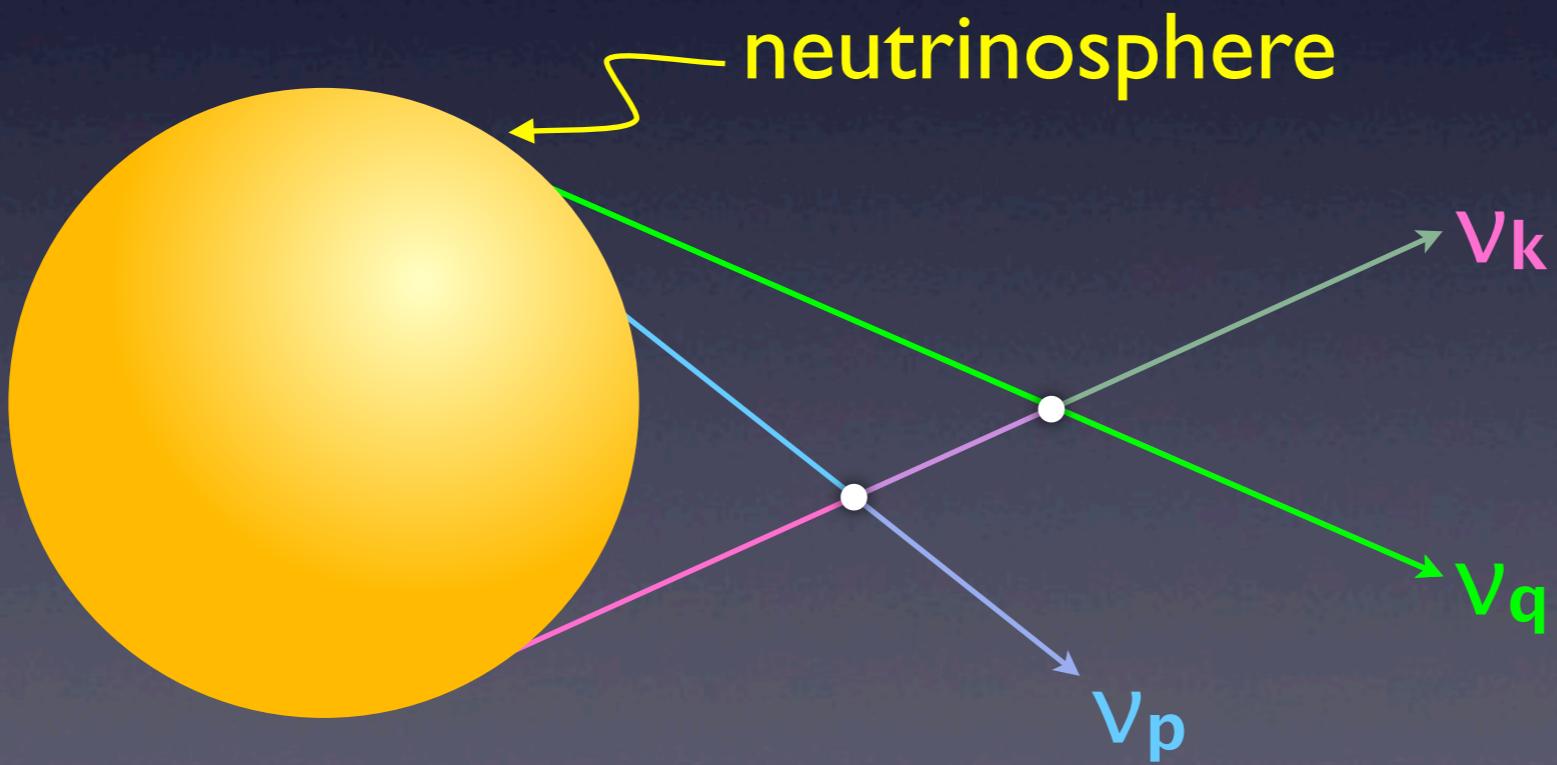


- $\sim 10^{53}$ ergs, 10^{58} neutrinos in ~ 10 seconds
- All neutrino species, $10 \sim 30$ MeV
- Dominate energetics
- Influence nucleosynthesis
- Probe into SNe

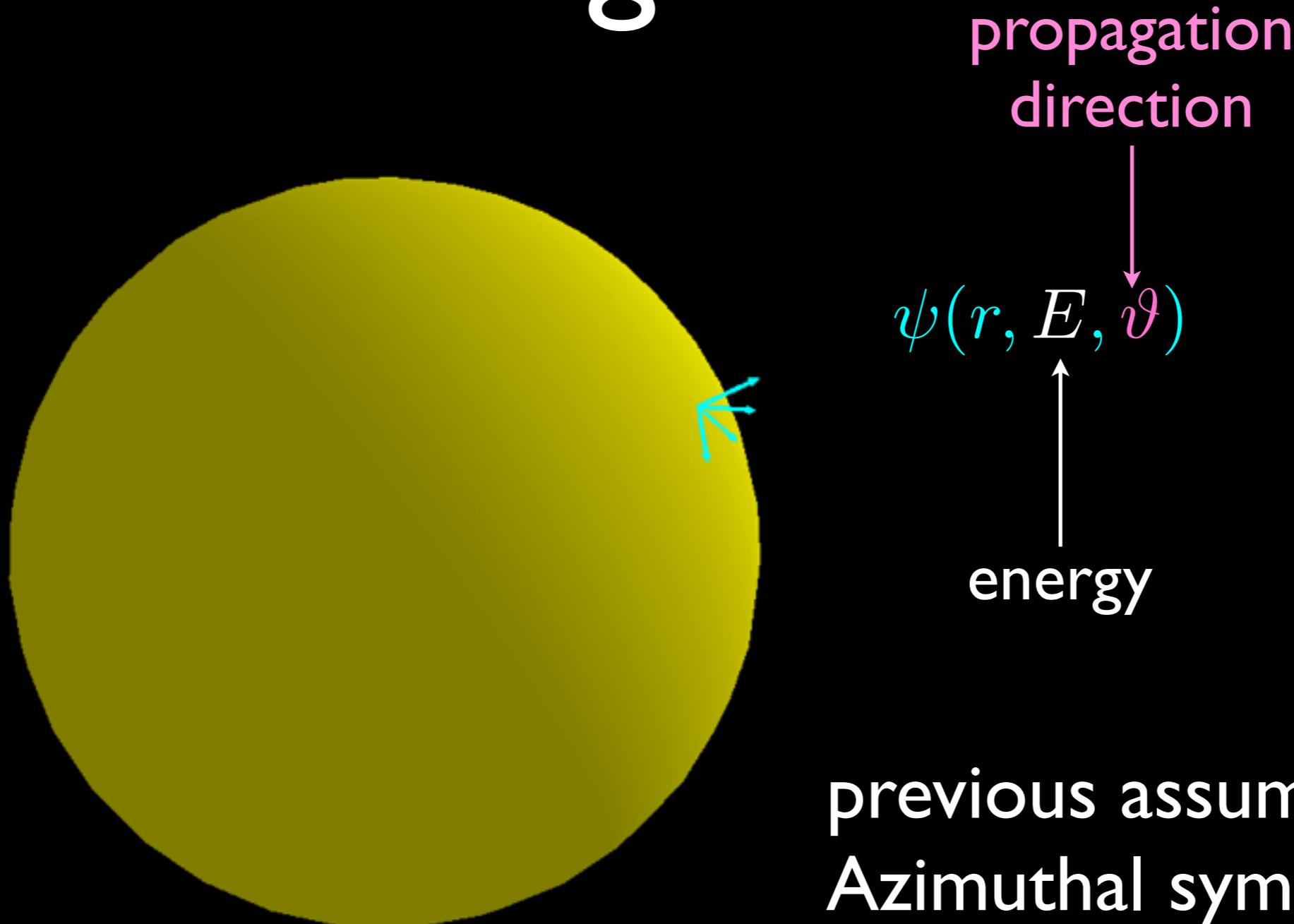
ν oscillations in SN

$$i \frac{d}{d\lambda} |\psi_{\nu, \mathbf{p}}\rangle = \hat{H} |\psi_{\nu, \mathbf{p}}\rangle$$

$$\mathcal{H} = \frac{\mathbf{M}^2}{2E} + \sqrt{2}G_F \text{diag}[n_e, 0, 0] + \mathcal{H}_{\nu\nu}$$



(I+2)D Multi-Angle/Bulb Model

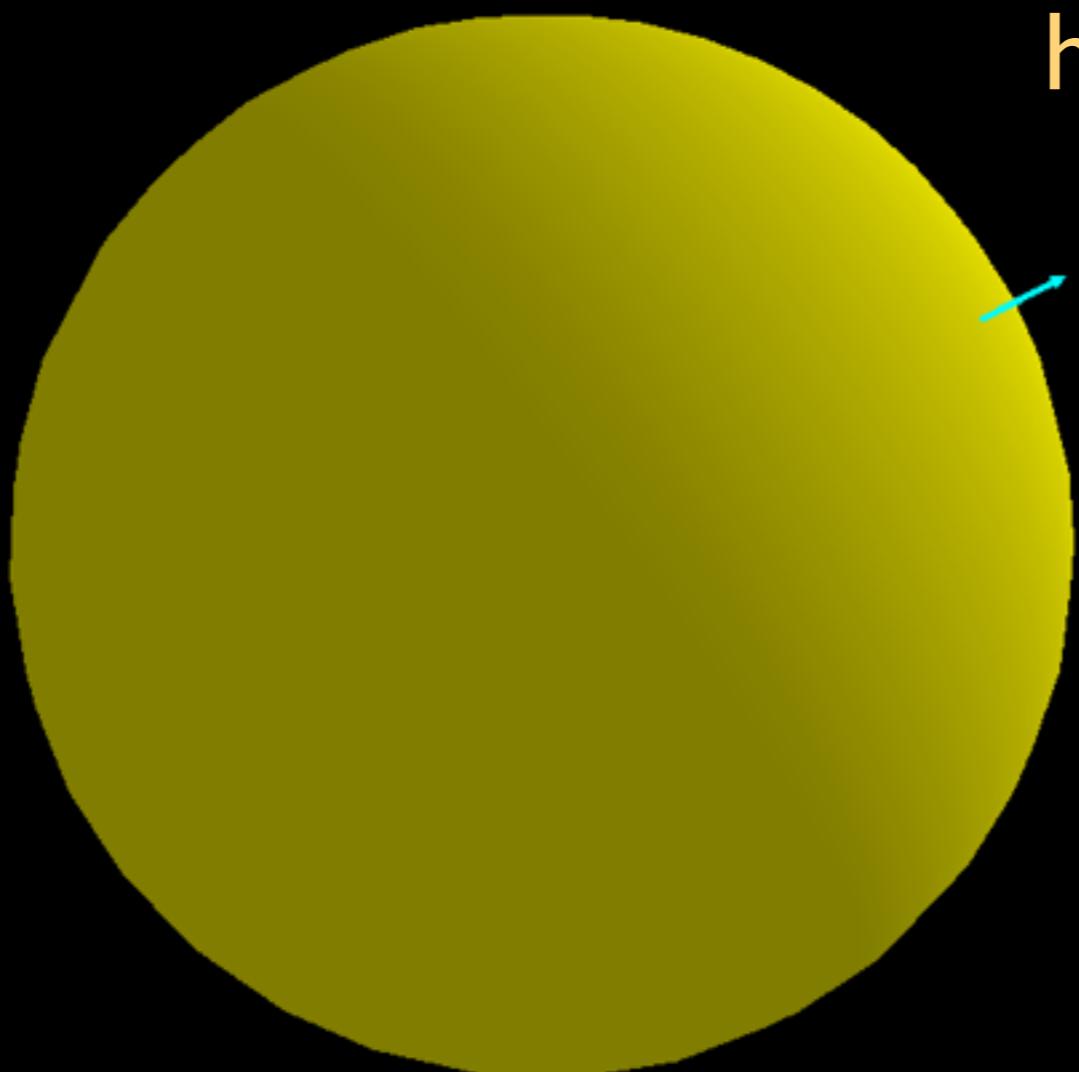


previous assumptions +
Azimuthal symmetry around
any radial direction

$(I+I)D$

Single-Angle

Equivalent to an expanding homogeneous neutrino gas

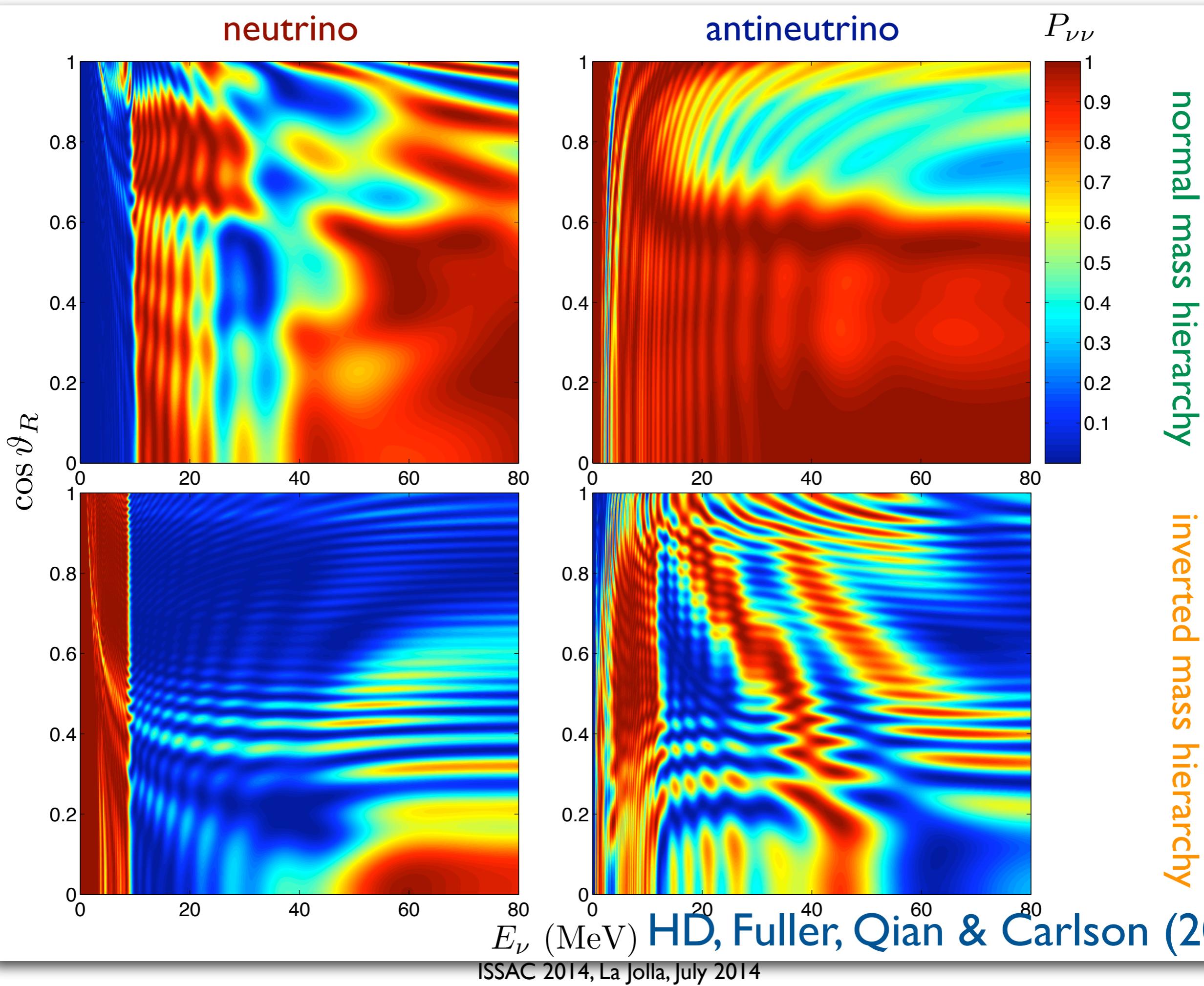


$$\psi(r, E)$$

↑
energy

A diagram showing a vertical arrow pointing upwards from the center of the yellow sphere towards the text "energy". To the right of the arrow, the mathematical expression $\psi(r, E)$ is written in blue.

previous assumptions +
Trajectory independent
neutrino flavor evolution

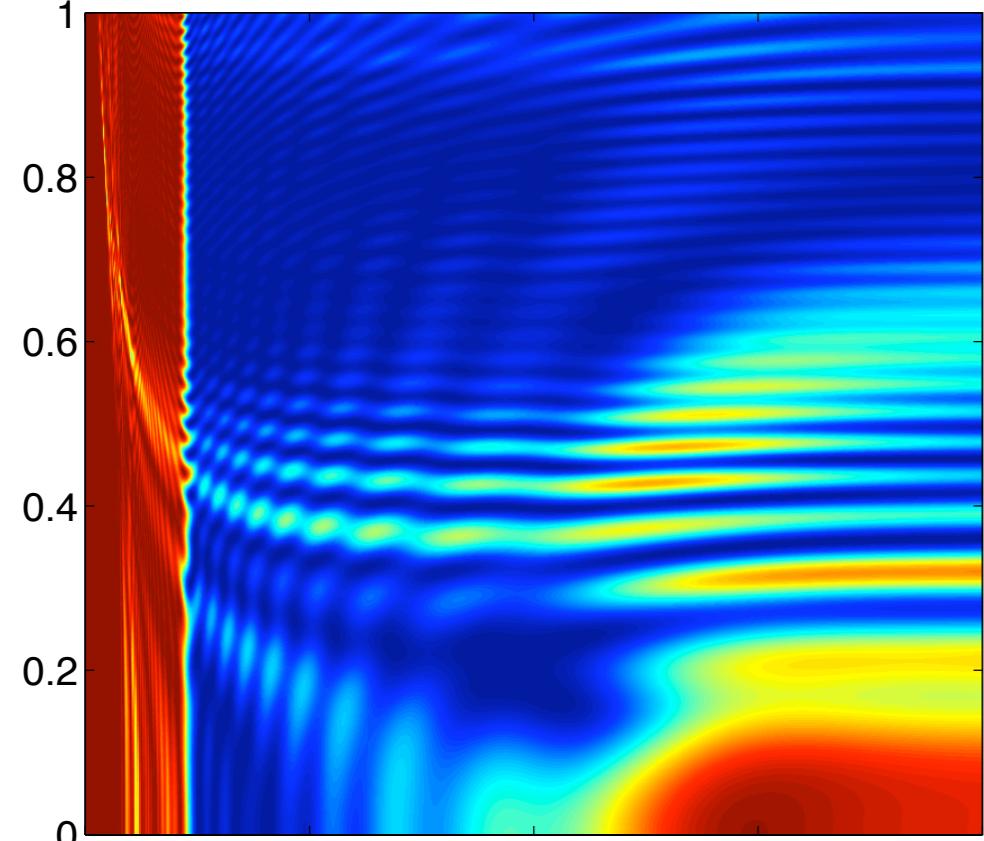
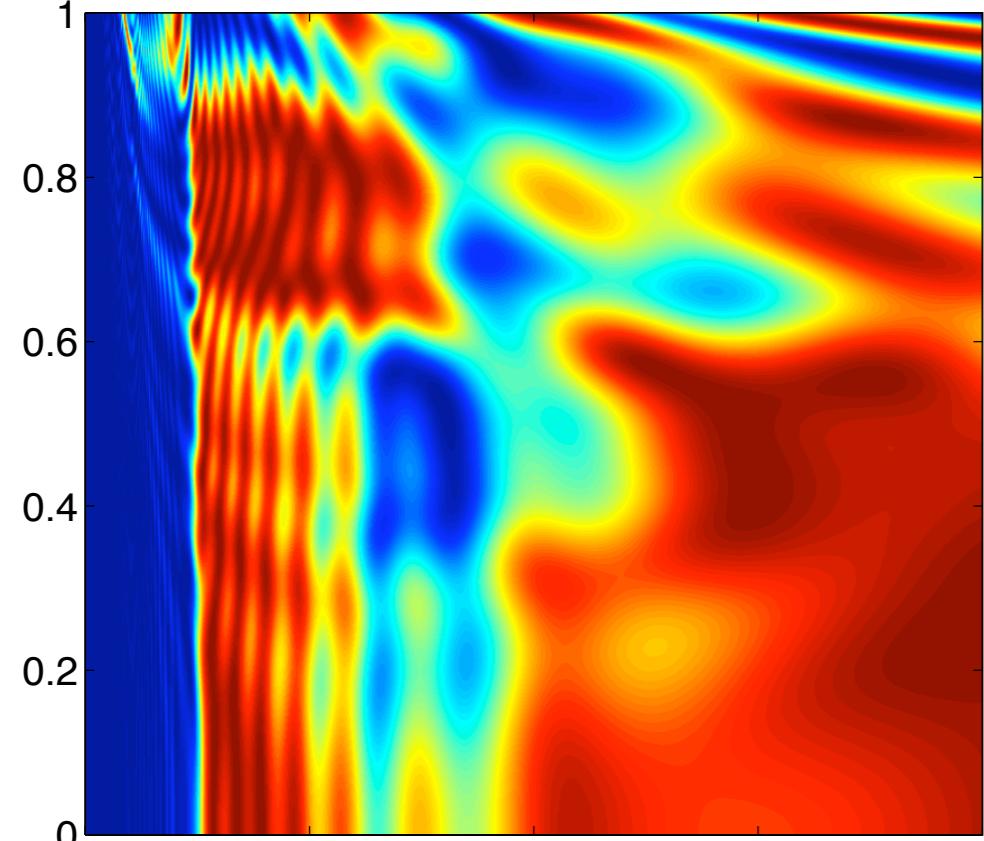


normal mass hierarchy

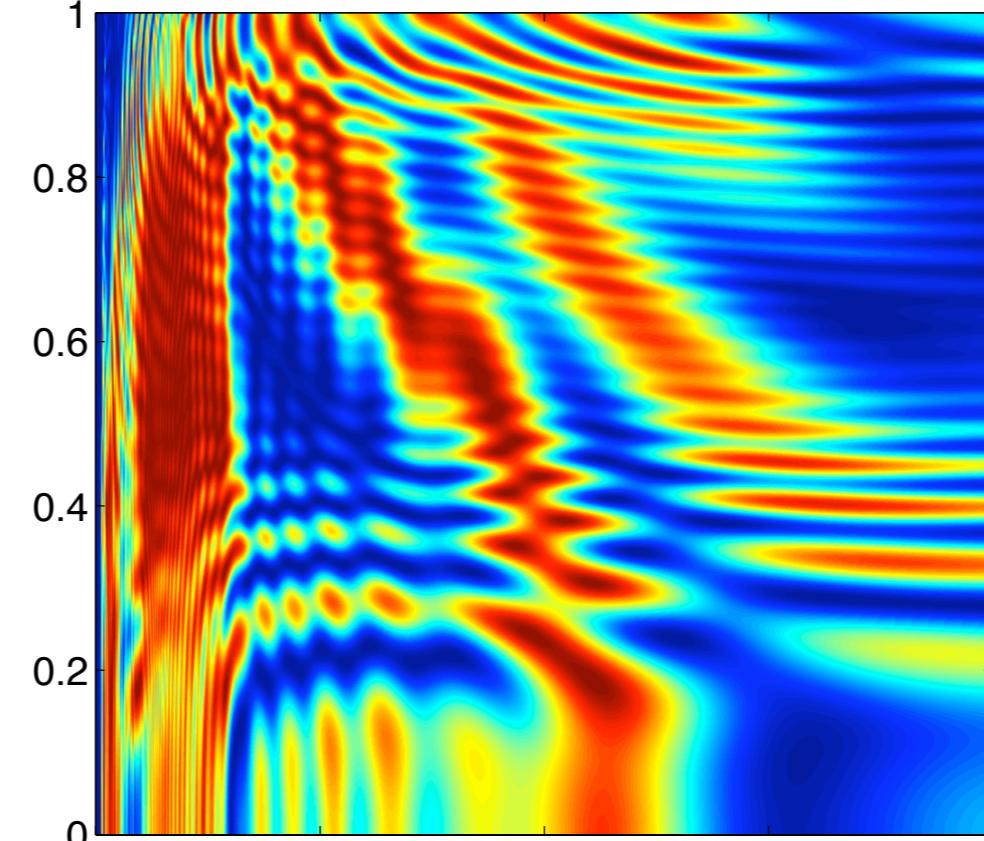
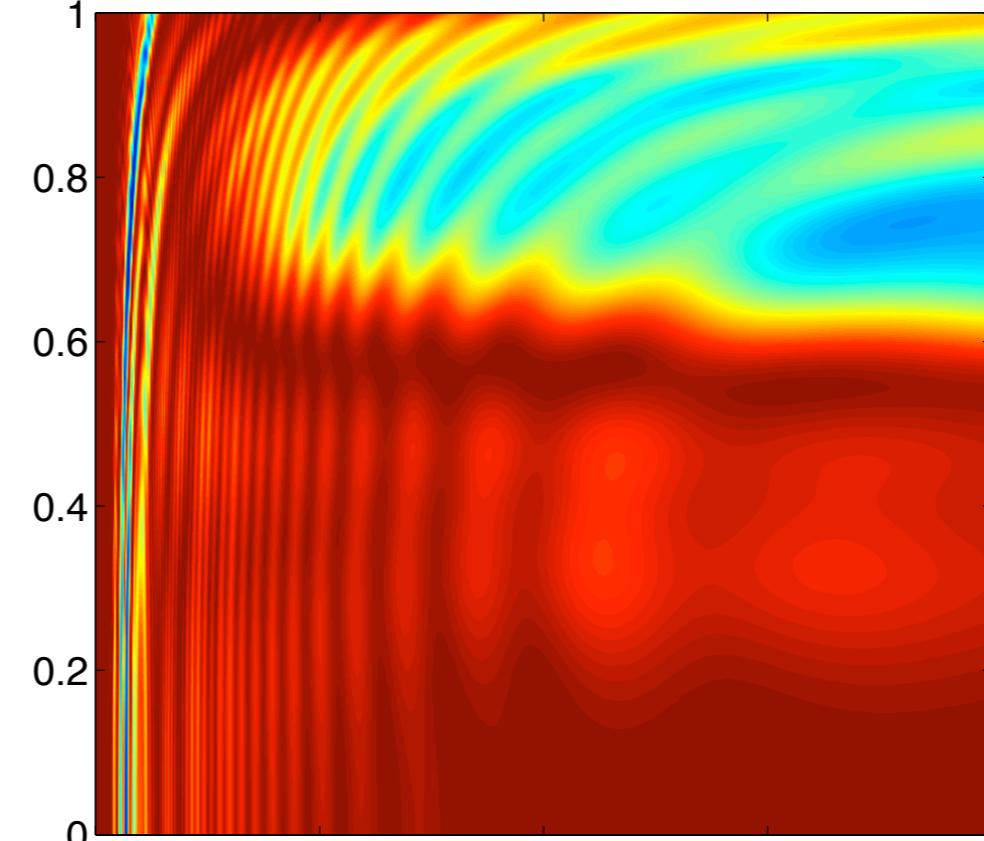
inverted mass hierarchy

$P_{\nu\nu}$

$\cos \vartheta_R$



antineutrino

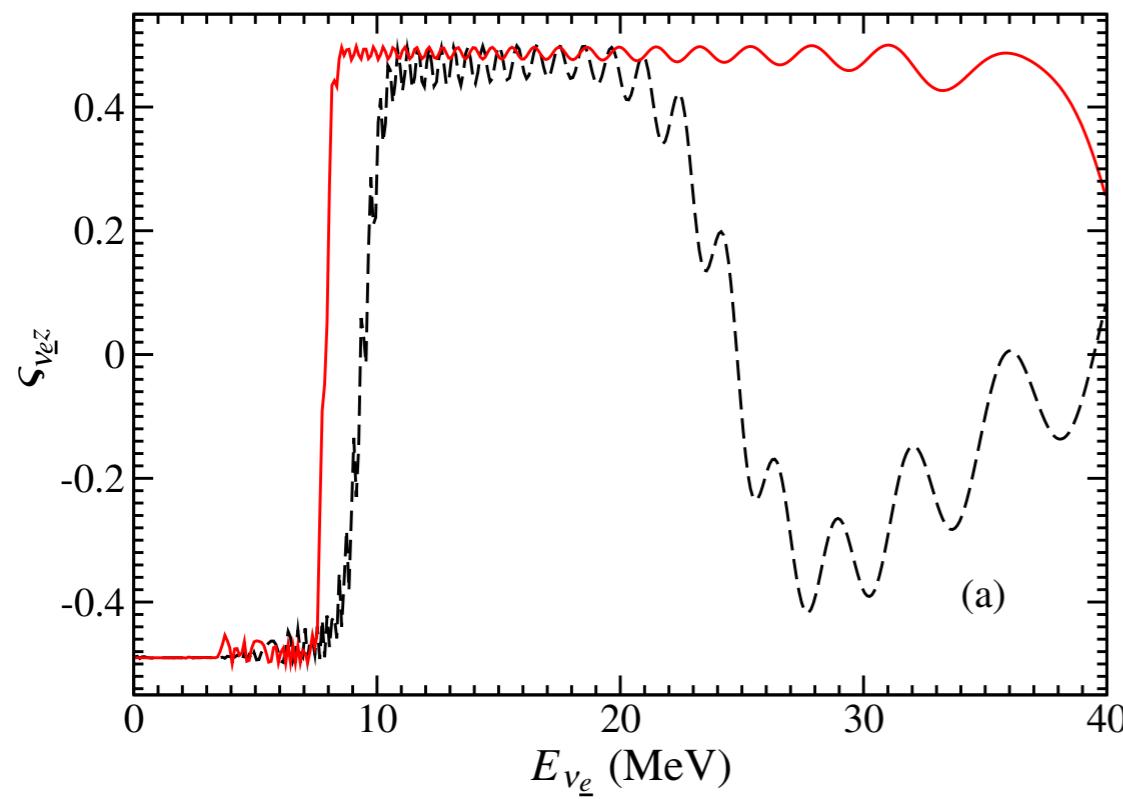


neutrino

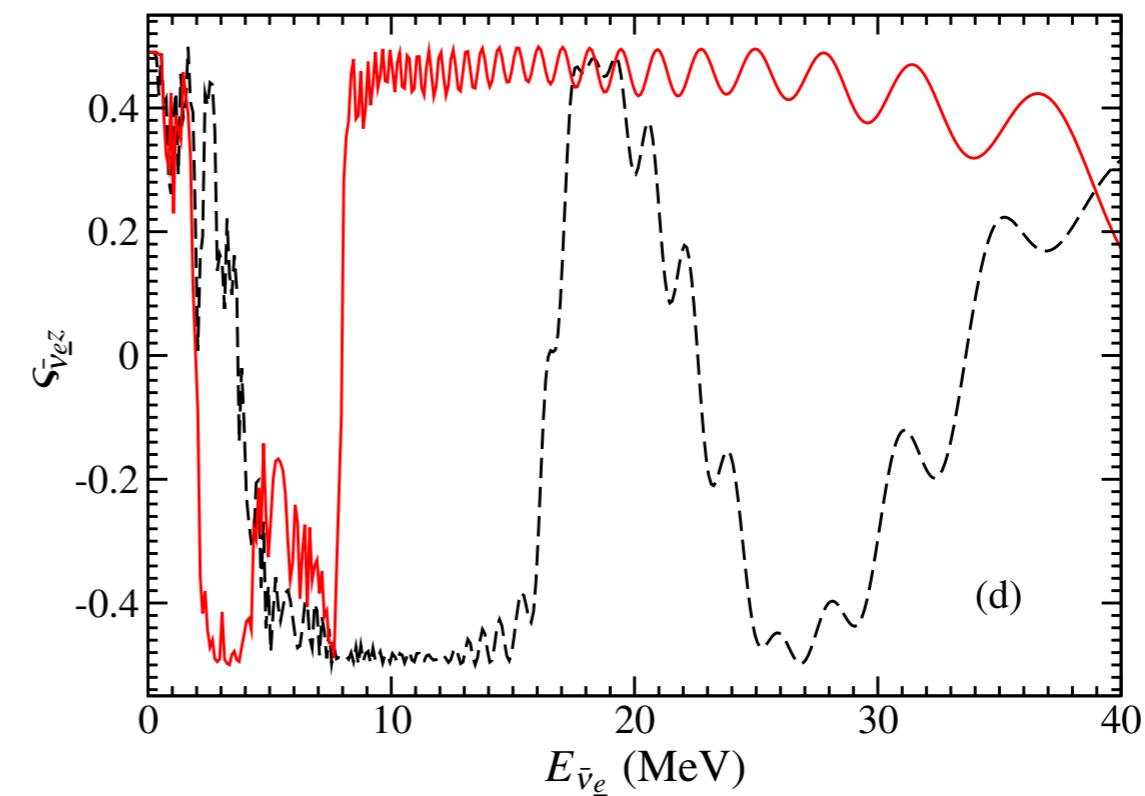
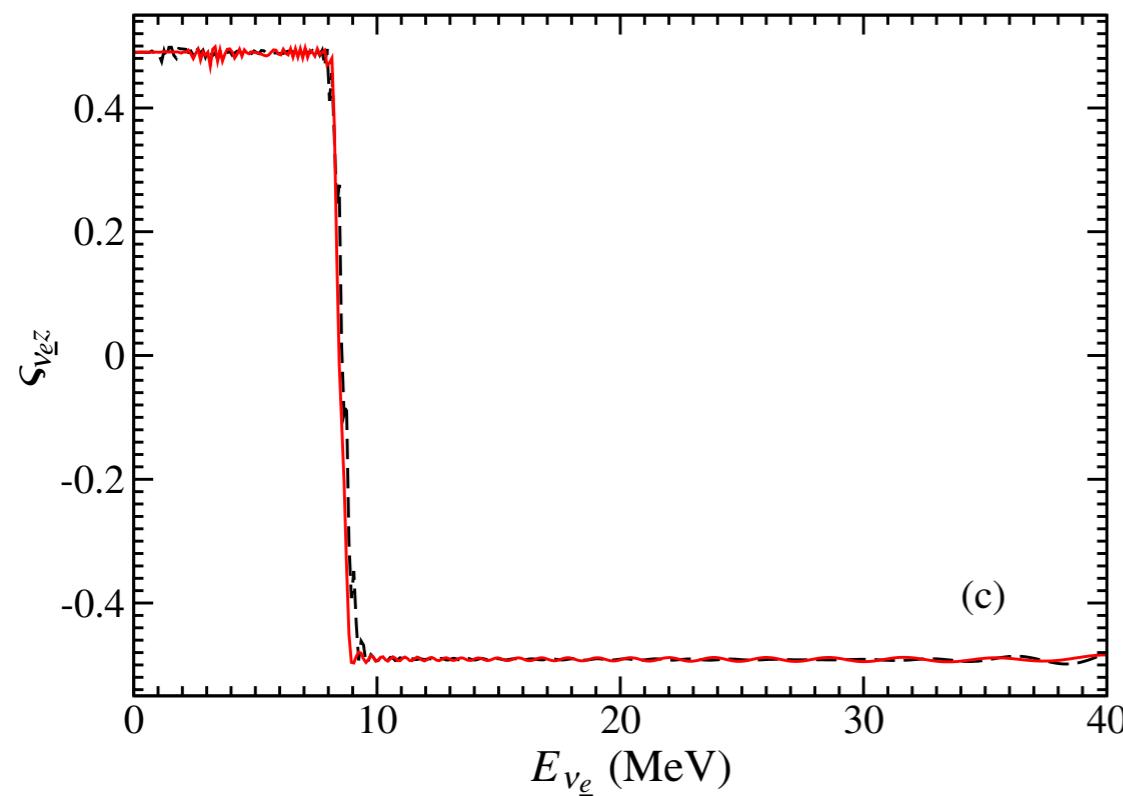
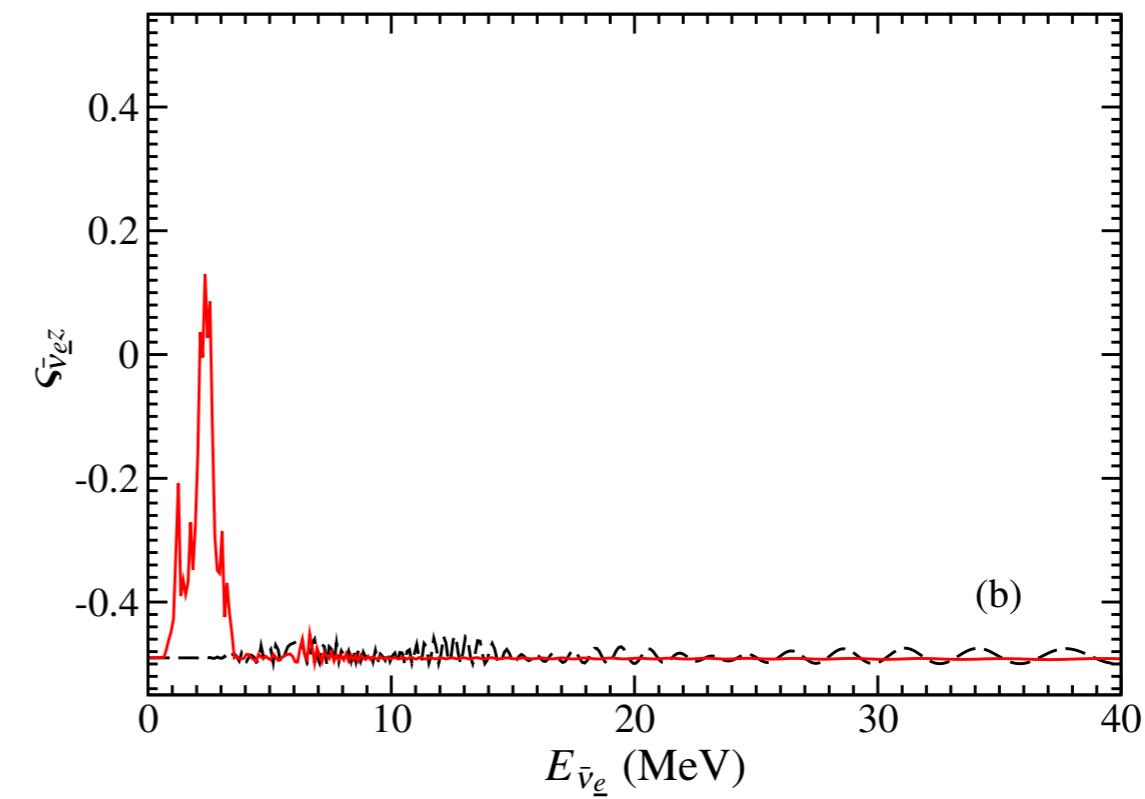
HD, Fuller, Qian & Carlson (2006)

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neutrino



antineutrino



normal mass hierarchy

inverted mass hierarchy

HD, Fuller, Qian & Carlson (2006)

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Friday, July 25, 14

Neutrino Self-Coupling

$$i \frac{d}{d\lambda} |\psi_{\nu, \mathbf{p}}\rangle = \hat{H} |\psi_{\nu, \mathbf{p}}\rangle$$

mass squared
matrix \mathbf{M}^2 

neutrino energy $\frac{\mathbf{p}^2}{2E}$ 

electron density n_e 

$H = \frac{\mathbf{p}^2}{2E} + \sqrt{2}G_F \text{diag}[n_e, 0, 0] + H_{\nu\nu}$

$H_{\nu\nu} = \sqrt{2}G_F \int d\mathbf{p}' (1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') (\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'}^*)$ 

v-v forward scattering
(self-coupling)

Tools & Toy Models

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Vacuum Oscillations

neutrinos are generated/detected in flavor states

neutrino mass eigenstates \neq neutrino flavor states

$$|\nu_1\rangle = \cos \theta_v |\nu_e\rangle + \sin \theta_v |\nu_\mu\rangle \quad \text{with mass } m_1$$

$$|\nu_2\rangle = -\sin \theta_v |\nu_e\rangle + \cos \theta_v |\nu_\mu\rangle \quad \text{with mass } m_2$$

vacuum mixing angle

$$i \frac{d}{dx} \begin{bmatrix} \langle \nu_e | \psi_\nu \rangle \\ \langle \nu_\mu | \psi_\nu \rangle \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega \cos 2\theta_v & \omega \sin 2\theta_v \\ \omega \sin 2\theta_v & \omega \cos 2\theta_v \end{bmatrix} \begin{bmatrix} \langle \nu_e | \psi_\nu \rangle \\ \langle \nu_\mu | \psi_\nu \rangle \end{bmatrix}$$

vac. osc. freq. $\omega = \frac{\delta m^2}{2E_\nu}$

$$\delta m^2 = m_2^2 - m_1^2$$

Neutrino Flavor Isospin

Two-component system



spin-1/2

$$2 \times 2 \text{ Hermitian matrix } \mathbf{H} = H_0 \mathbb{1} + \mathbf{H} \cdot \boldsymbol{\sigma}$$

Neutrino Flavor Isospin

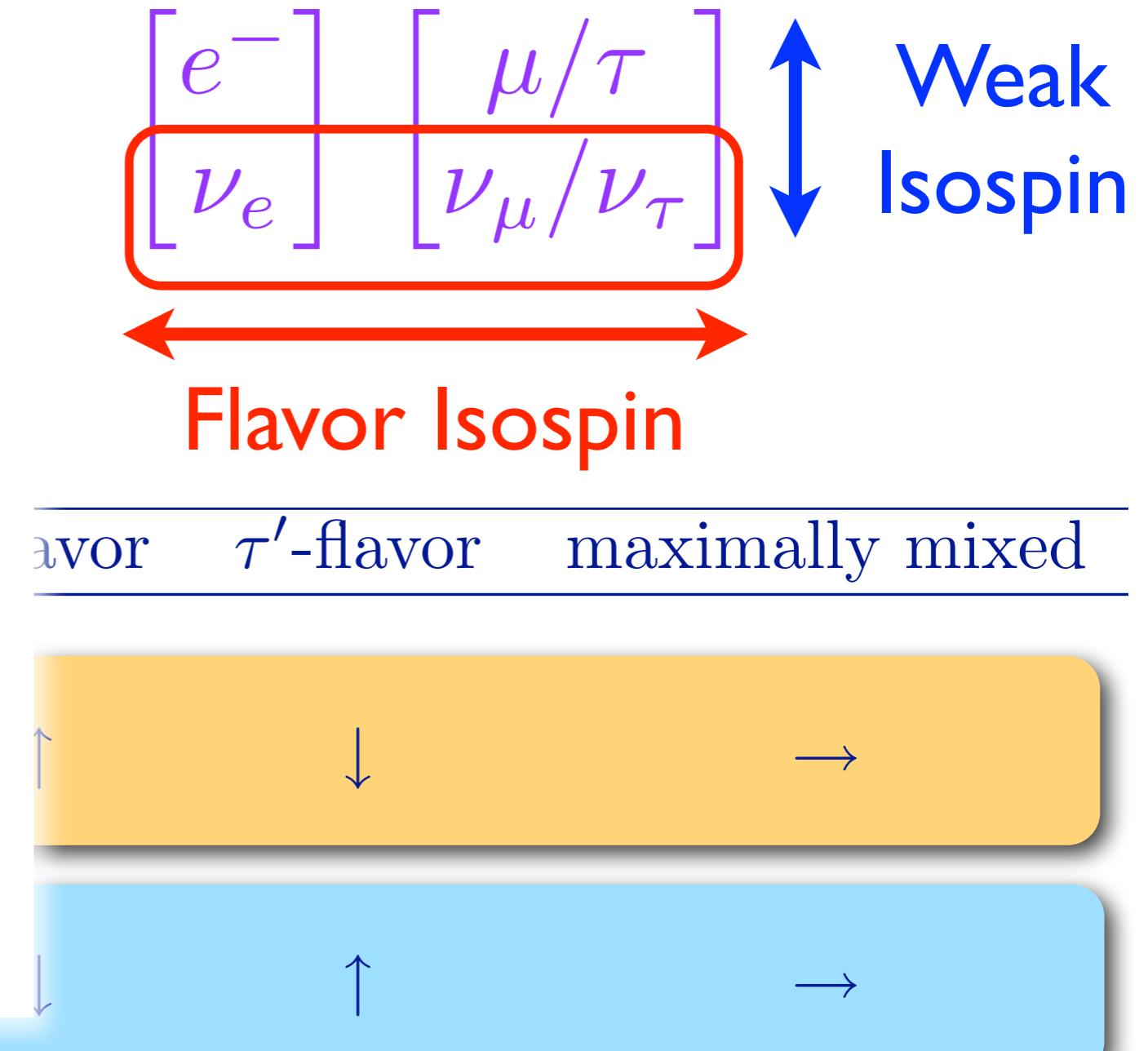
	d	c	t	γ
mass	2.4 MeV	1.27 GeV	171.2 GeV	0
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name	u up	c charm	t top	photon

	d	s	b	g
mass	4.8 MeV	104 MeV	4.2 GeV	0
charge	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name	d down	s strange	b bottom	gluon

	ν_e	ν_μ	ν_τ	Z ⁰
mass	<2.2 eV	<0.17 MeV	<15.5 MeV	91.2 GeV
charge	0	0	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	weak force

	e	μ	τ	W ⁺
mass	0.511 MeV	105.7 MeV	1.777 GeV	80.4 GeV
charge	-1	-1	-1	± 1
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name	electron	muon	tau	weak force

Wikimedia: Standard Model of Elementary Particles



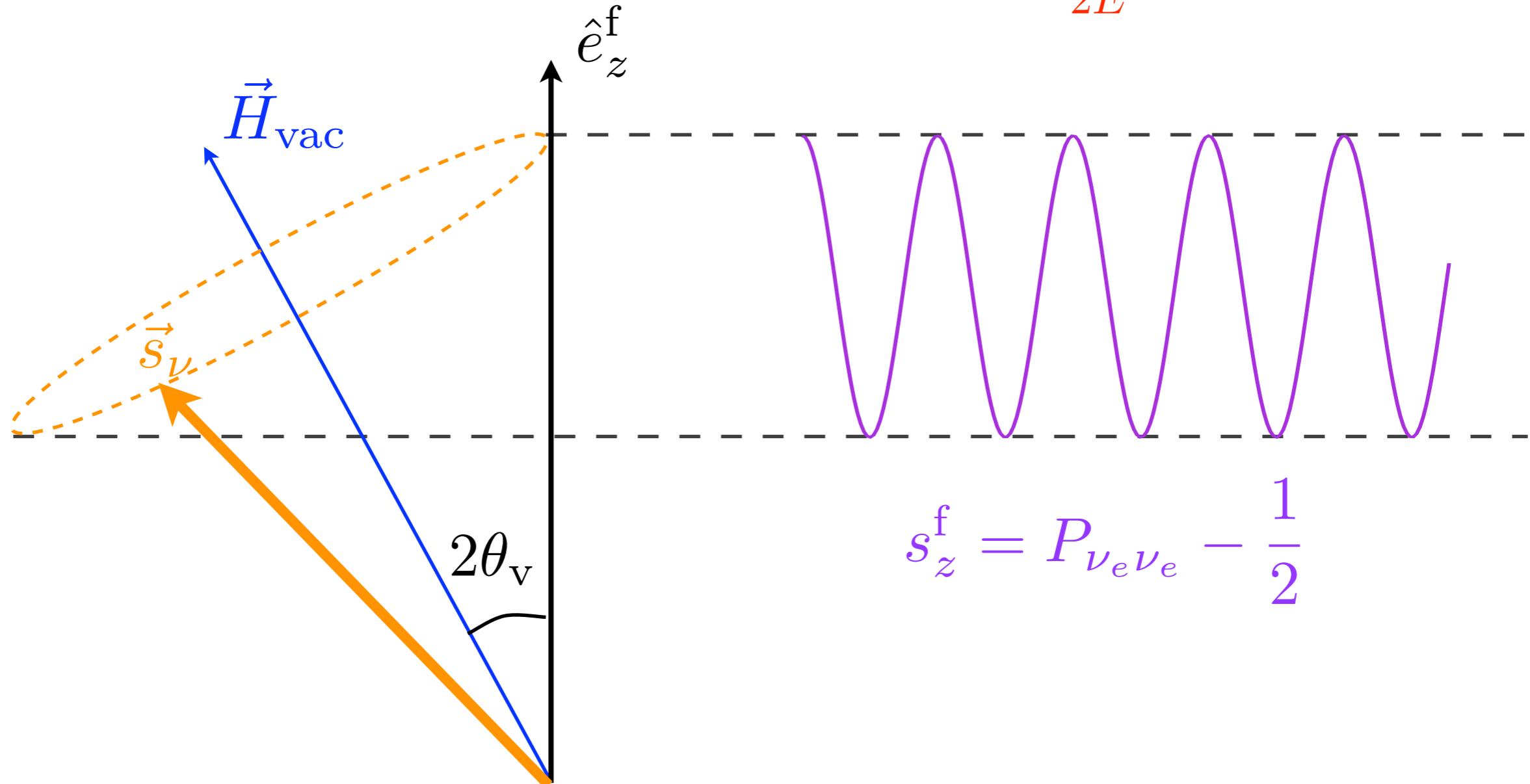
Vacuum Oscillations

Again

$$\vec{H} = \omega \vec{H}_{\text{vac}}$$

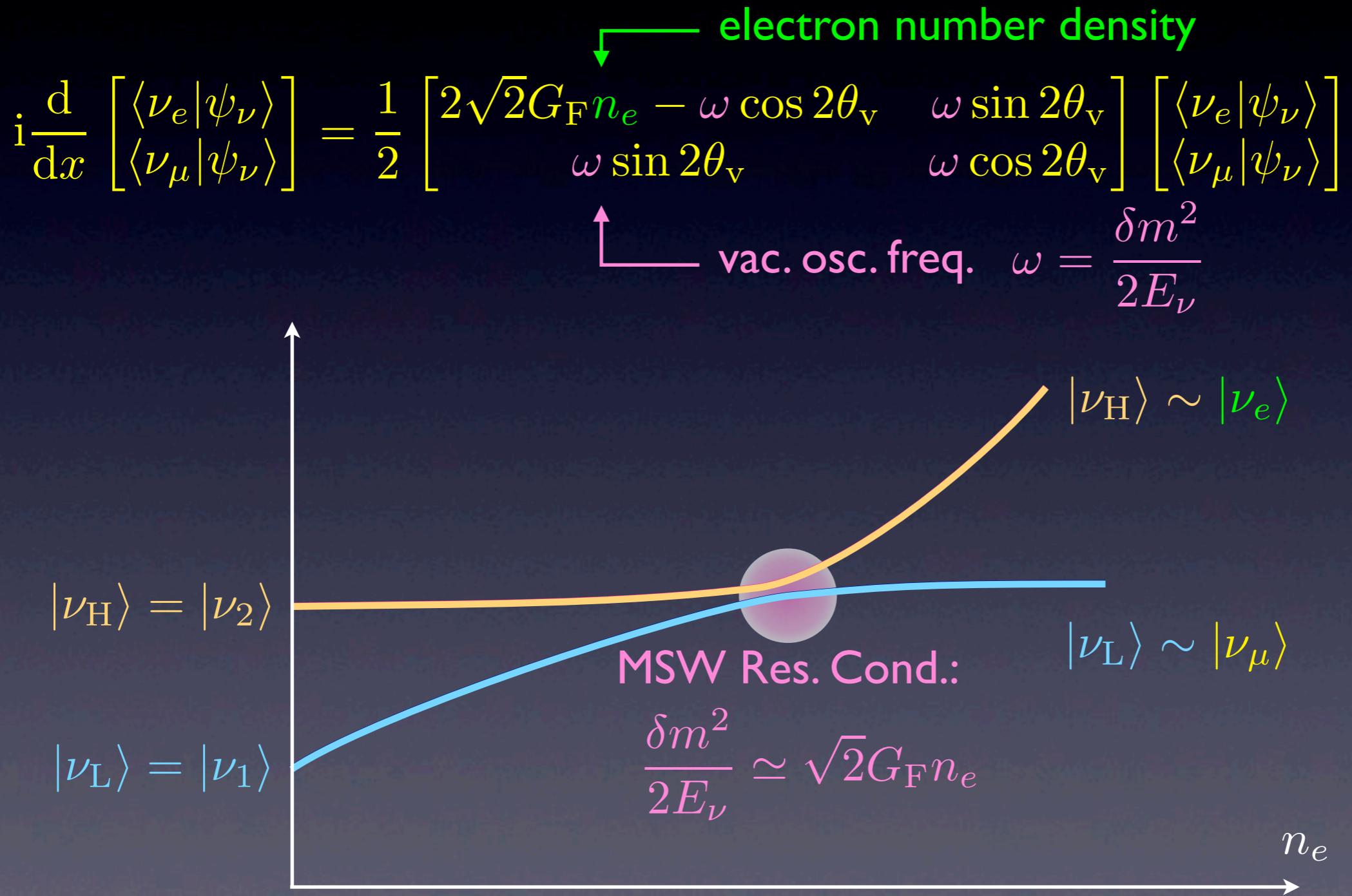
$$\vec{H}_{\text{vac}} \equiv -\hat{e}_x^{\text{f}} \sin 2\theta_{\nu} + \hat{e}_z^{\text{f}} \cos 2\theta_{\nu}$$

$$\omega \equiv \pm \frac{\delta m^2}{2E}$$



$$s_z^{\text{f}} = P_{\nu_e \nu_e} - \frac{1}{2}$$

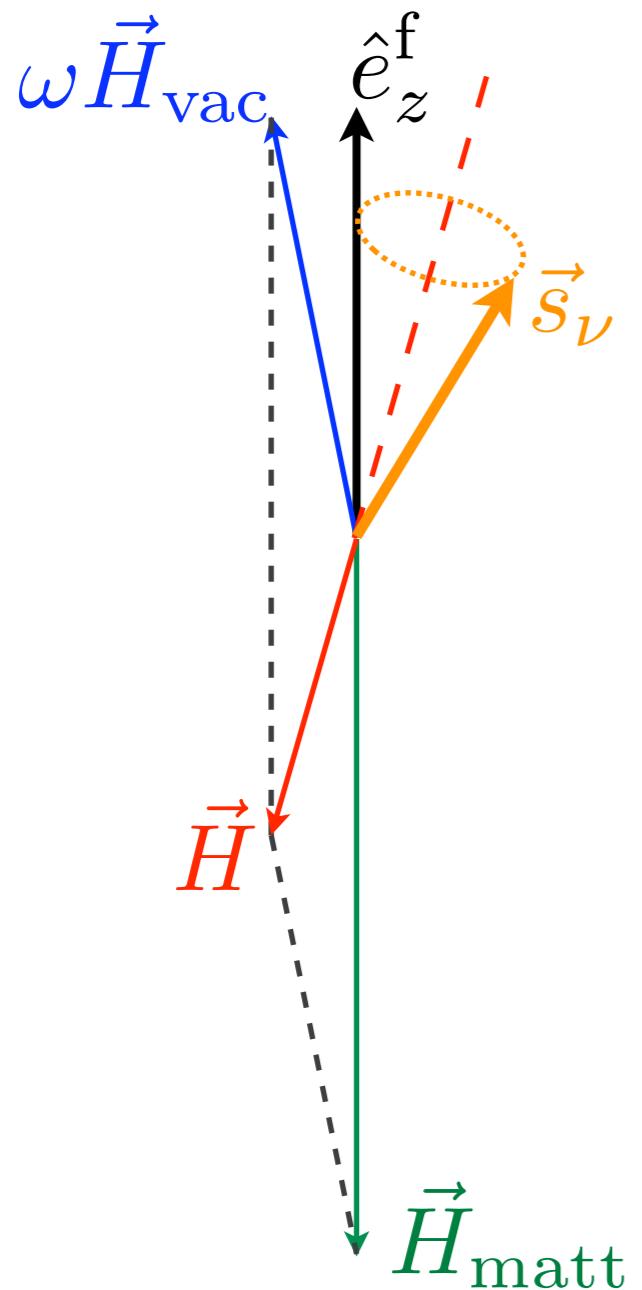
MSW Effect



MSW Again

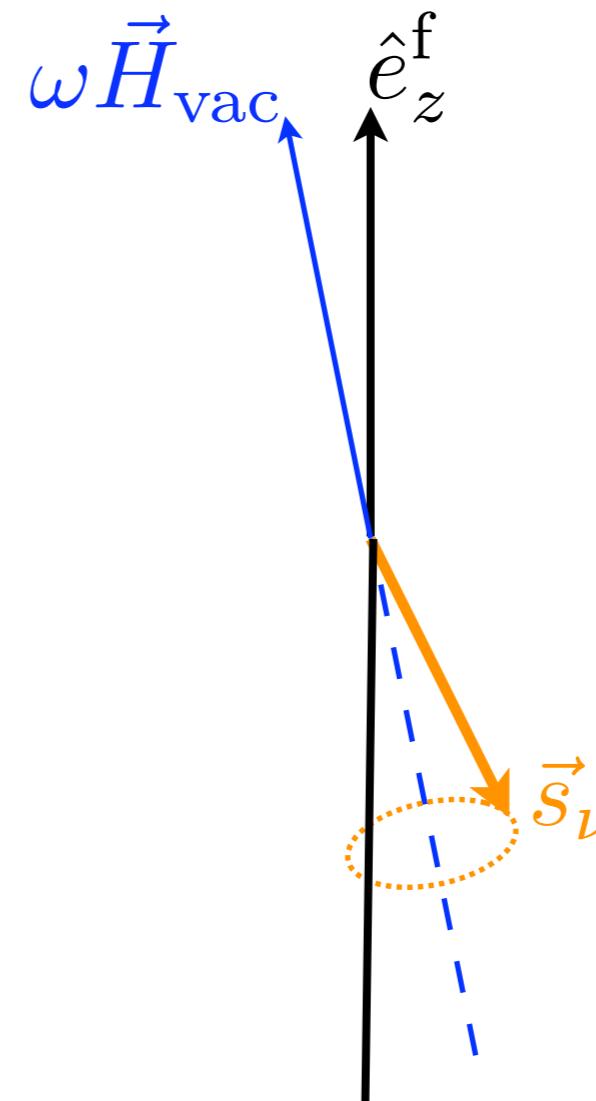
$$\vec{H} = \omega \vec{H}_{\text{vac}} + \vec{H}_{\text{matt}}$$

$$\vec{H}_{\text{matt}} \equiv -\hat{e}_z^{\text{f}} \sqrt{2} G_F n_e$$

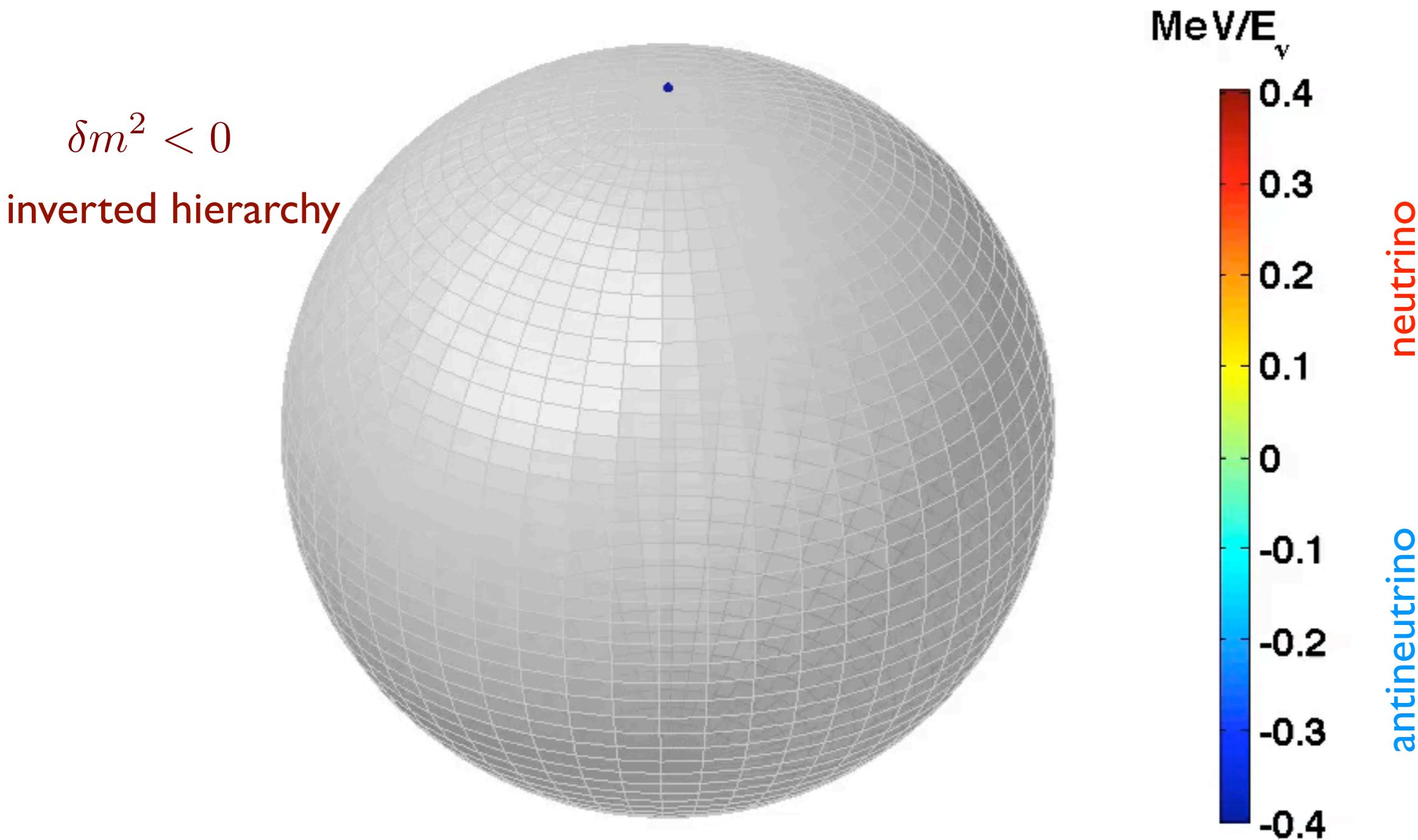


$$\omega = \frac{\delta m^2}{2E}, \delta m^2 > 0$$

neutrino, normal hierarchy



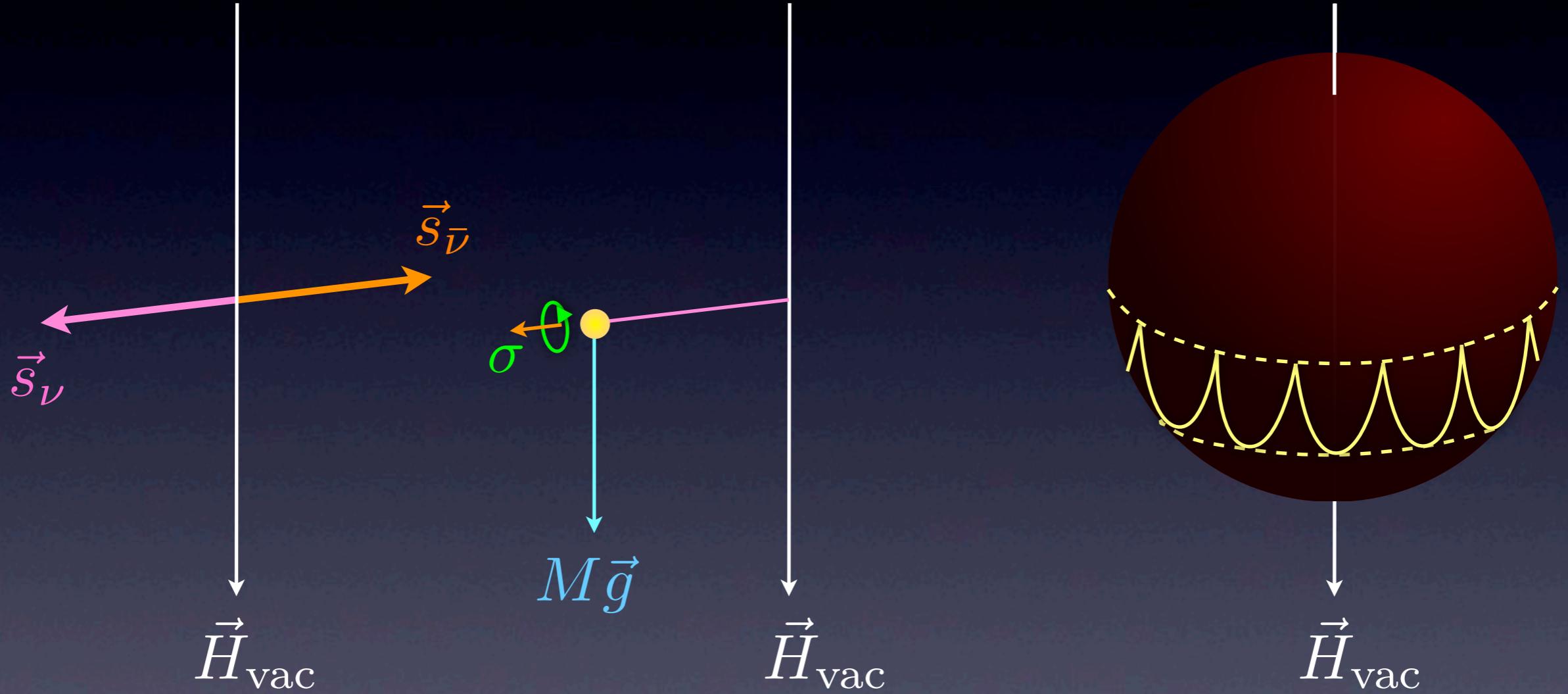
MSW Mechanism



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Bipolar System

Mono-energetic ν - $\bar{\nu}$ gas



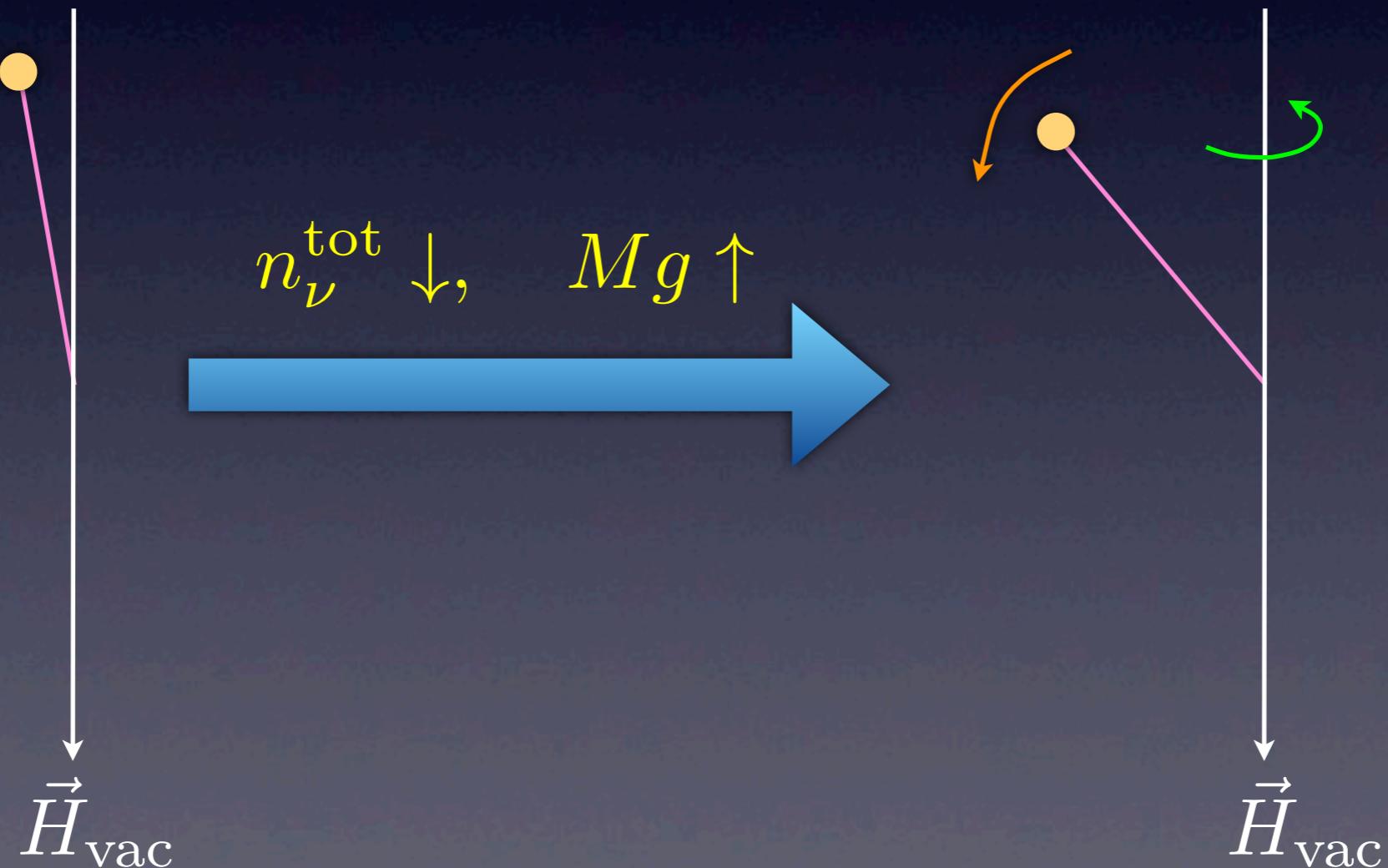
(Hannestad et al, 2006;
HD et al, 2007)

$$\sigma \sim \frac{n_\nu - n_{\bar{\nu}}}{n_\nu + n_{\bar{\nu}}} \quad M\vec{g} \sim \frac{\vec{H}_{\text{vac}}}{n_\nu + n_{\bar{\nu}}}$$

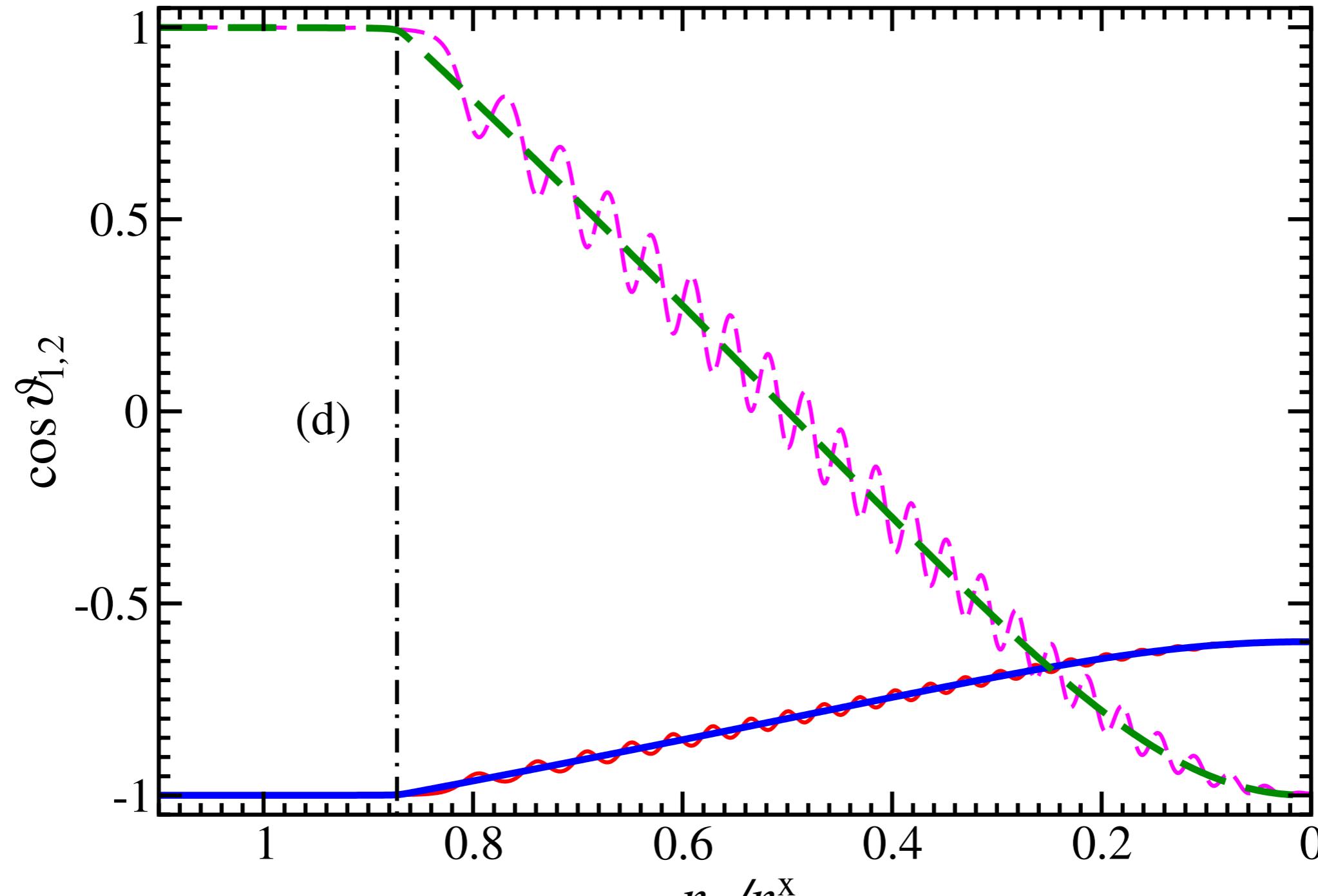
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Bipolar System

Inverted Mass Hierarchy



Bipolar System

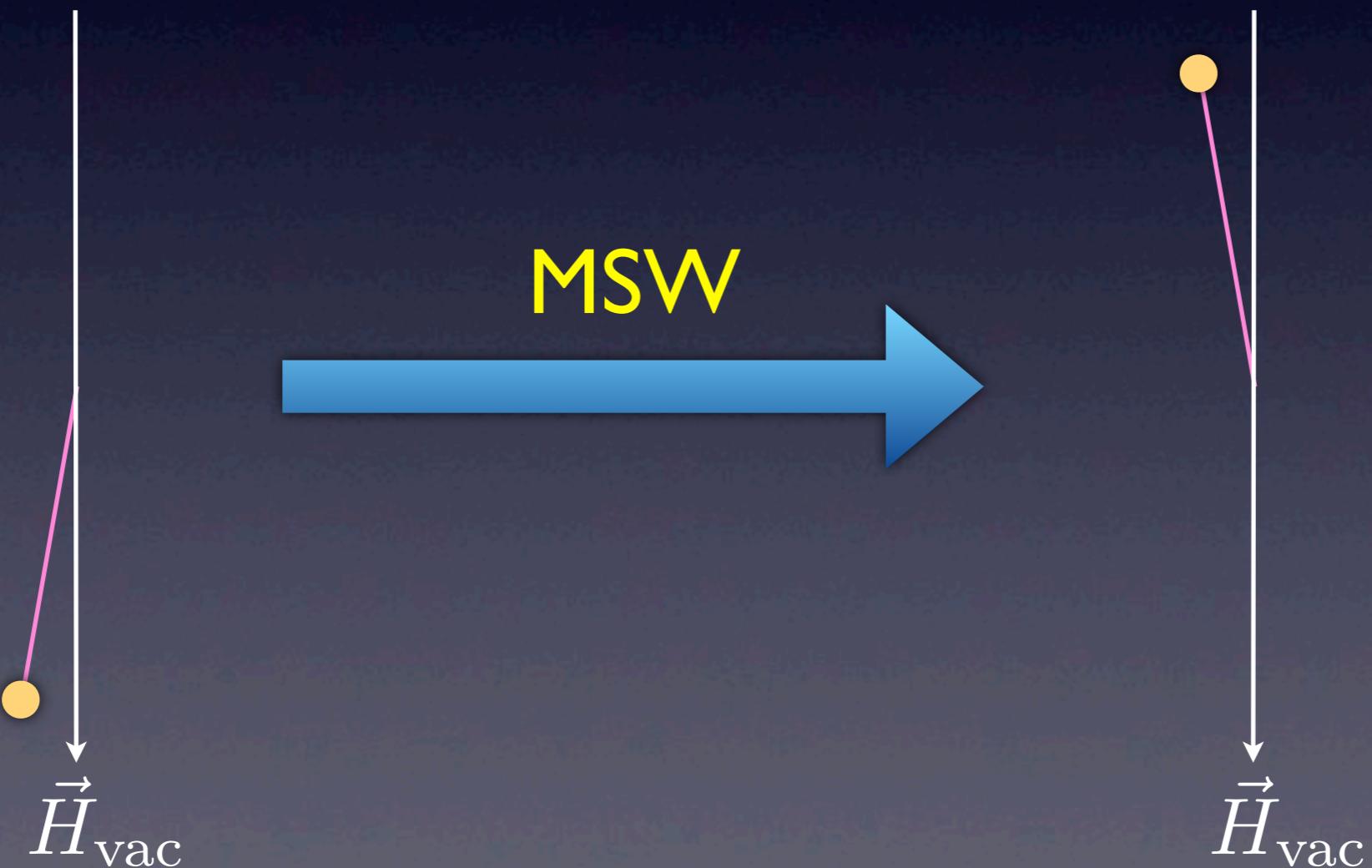


(HD et al, 2007)

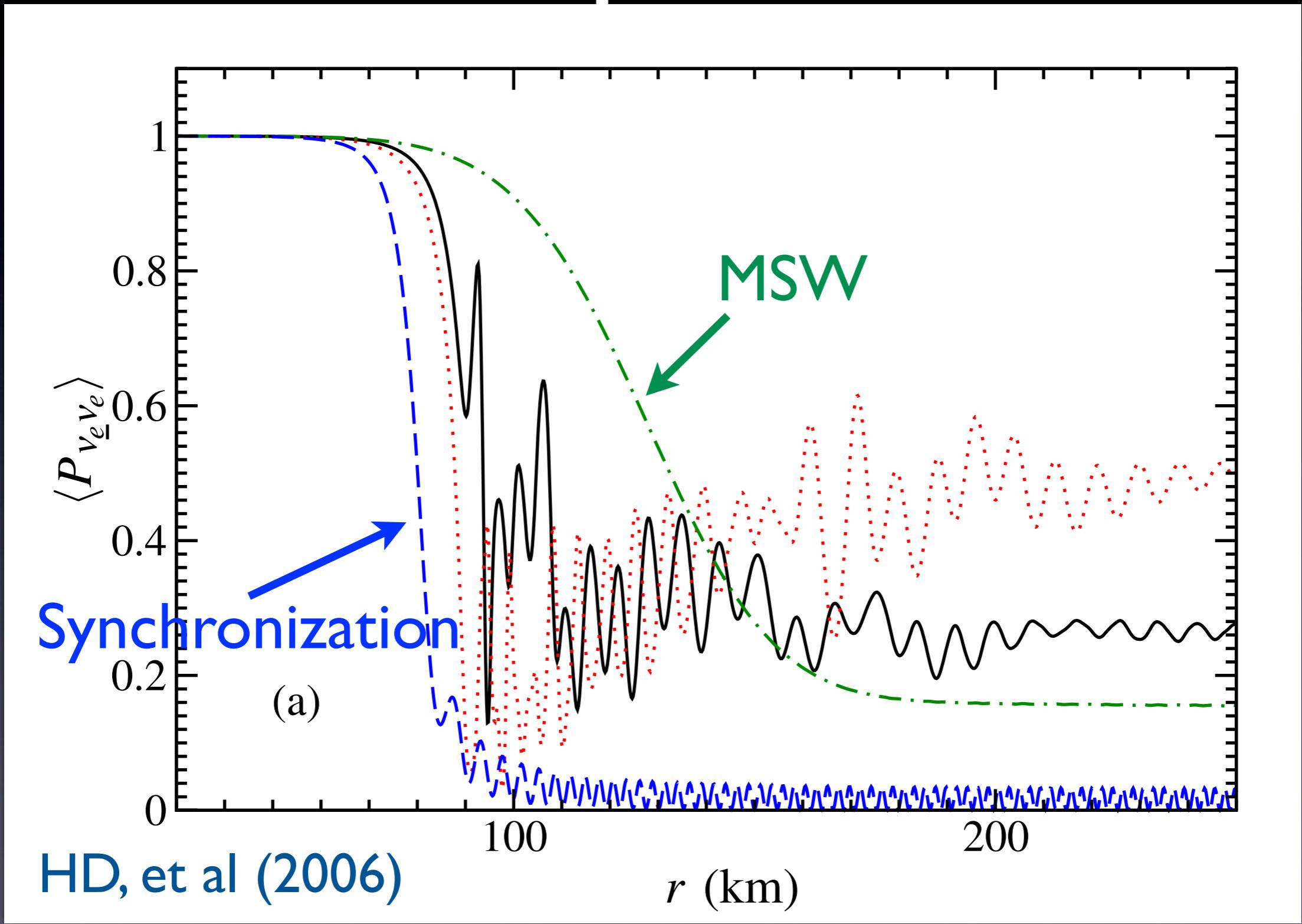
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Bipolar System

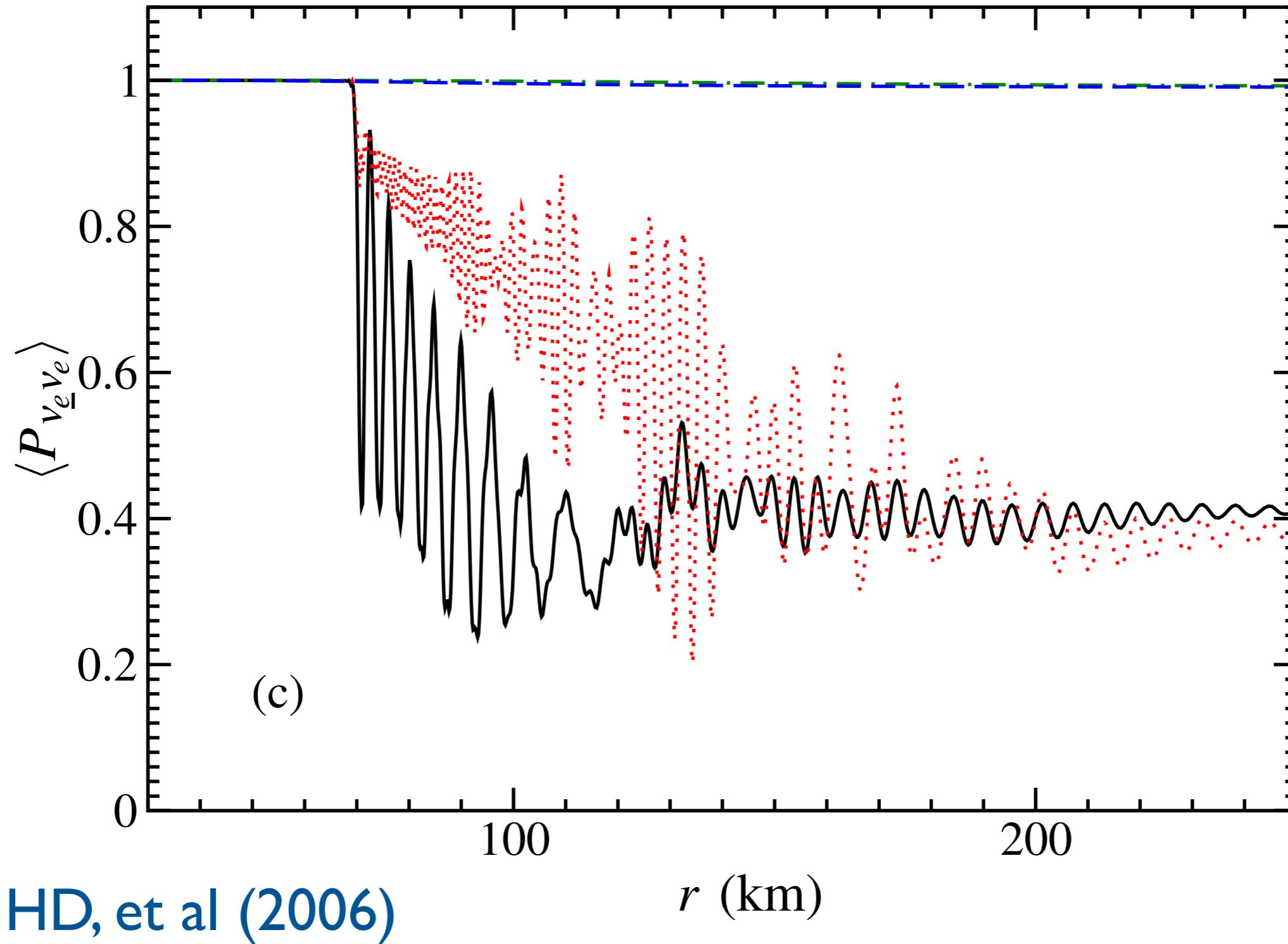
Normal Mass Hierarchy



Comparison



Comparison



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Homogeneous Gas

$$\frac{d}{dr} \vec{s}_\omega = \vec{s}_\omega \times \vec{H}_\omega$$

$$\vec{H}_\omega = \vec{H}_{\text{vac}} + \cancel{\vec{H}_{\text{matt}}} + \vec{H}_{\nu\nu}$$

$$\vec{H}_{\text{vac}} = \omega \hat{e}_z^v$$

Depend on neutrino energy;
disrupt collective oscillations

$$\vec{H}_{\text{matt}} = -\sqrt{2}G_F n_e \hat{e}_z^f$$

Independent of neutrino energy;
“Ignored” for collective oscillations

$$\vec{H}_{\nu\nu} = -2\sqrt{2}G_F n_\nu^{\text{tot}} \int_{-\infty}^{\infty} d\omega' f_{\omega'} \vec{s}_{\omega'} = -\mu \langle \vec{s} \rangle$$

avg NFIS
coupling strength
anti-ferromagnetic

NFIS distribution

Independent of neutrino energy;
Drive collective oscillations

Collective Oscillations

rotational symmetry of EoM



collective precession of flavor isospins



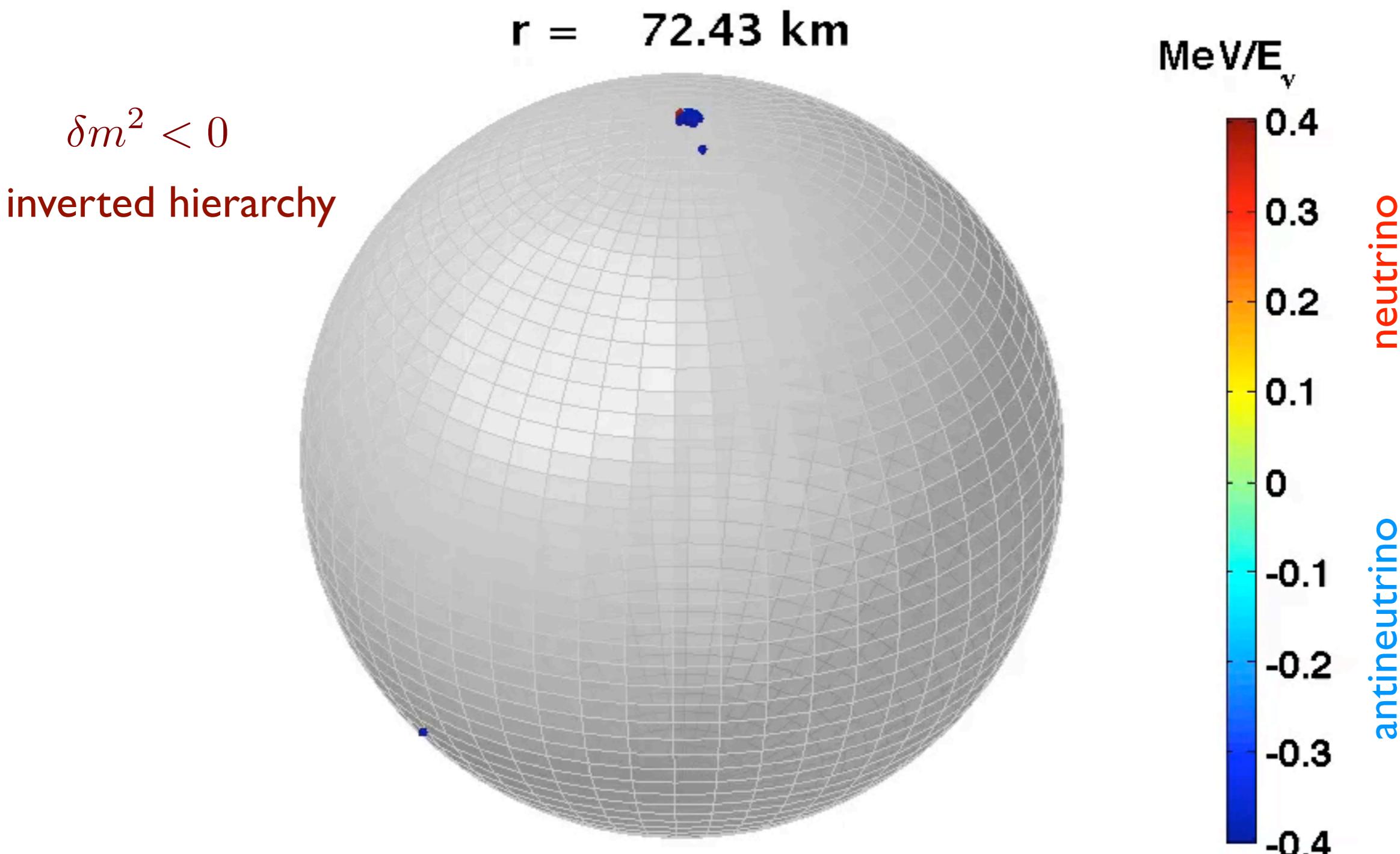
rotating “magnetic field”



magnetic spin resonance

new flavor transformation mechanism

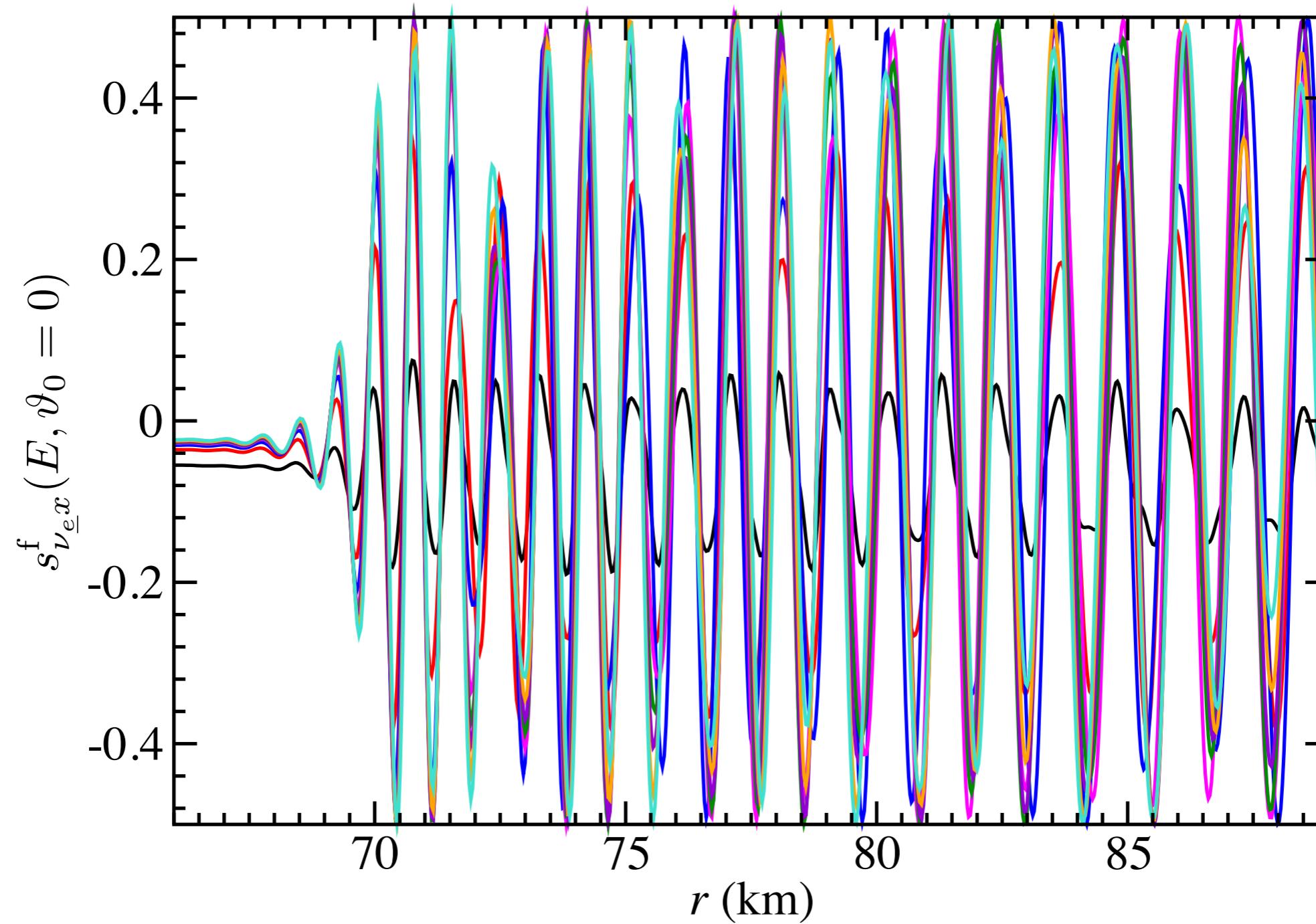
Collective Oscillations



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Multi-angle calculation

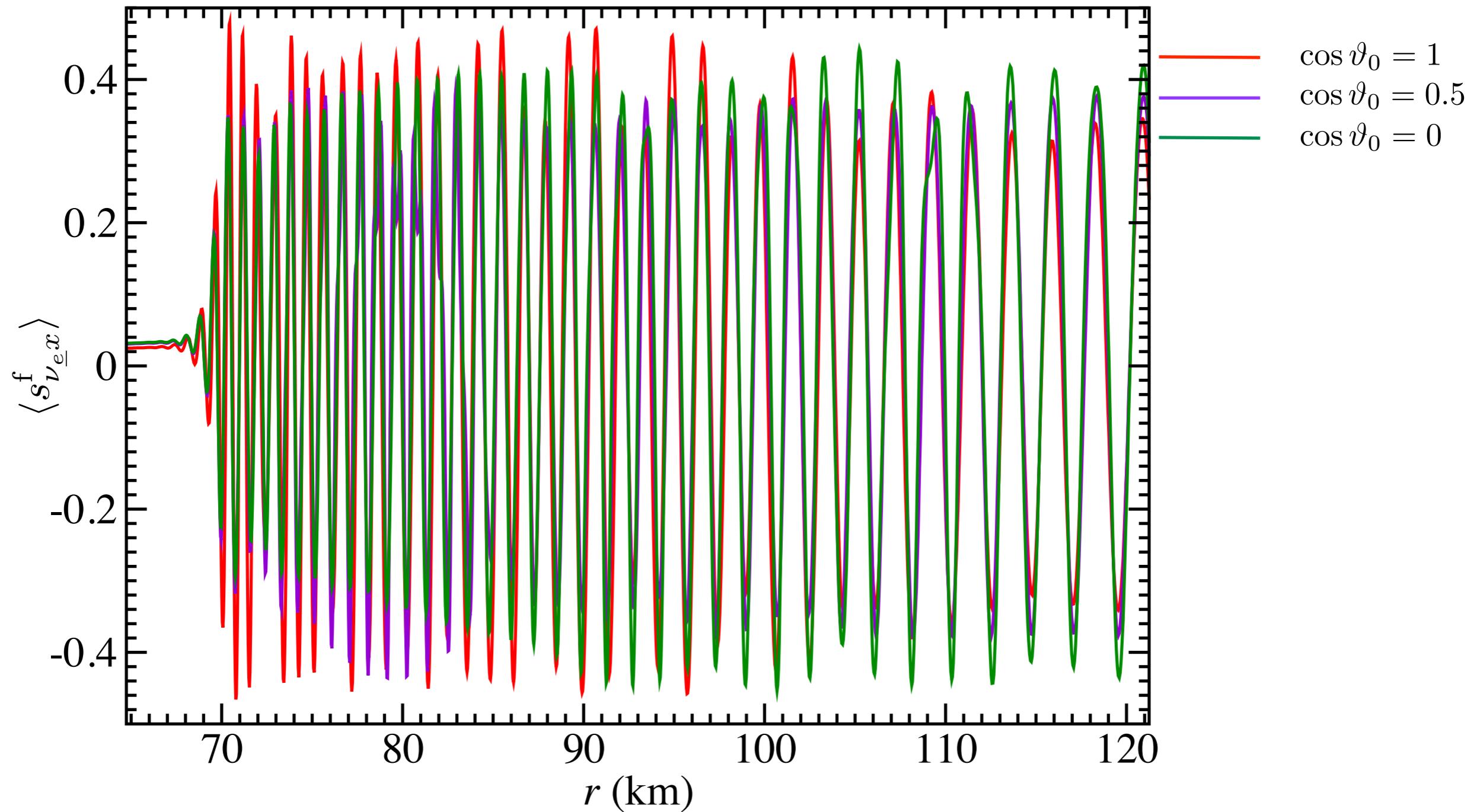
$$\delta m^2 = -3 \times 10^{-3} \text{ eV}^2 \simeq \delta m_{\text{atm}}^2, \quad \theta_v = 0.1$$



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Multi-angle calculation

$$\delta m^2 = -3 \times 10^{-3} \text{ eV}^2 \simeq \delta m_{\text{atm}}^2, \quad \theta_v = 0.1$$



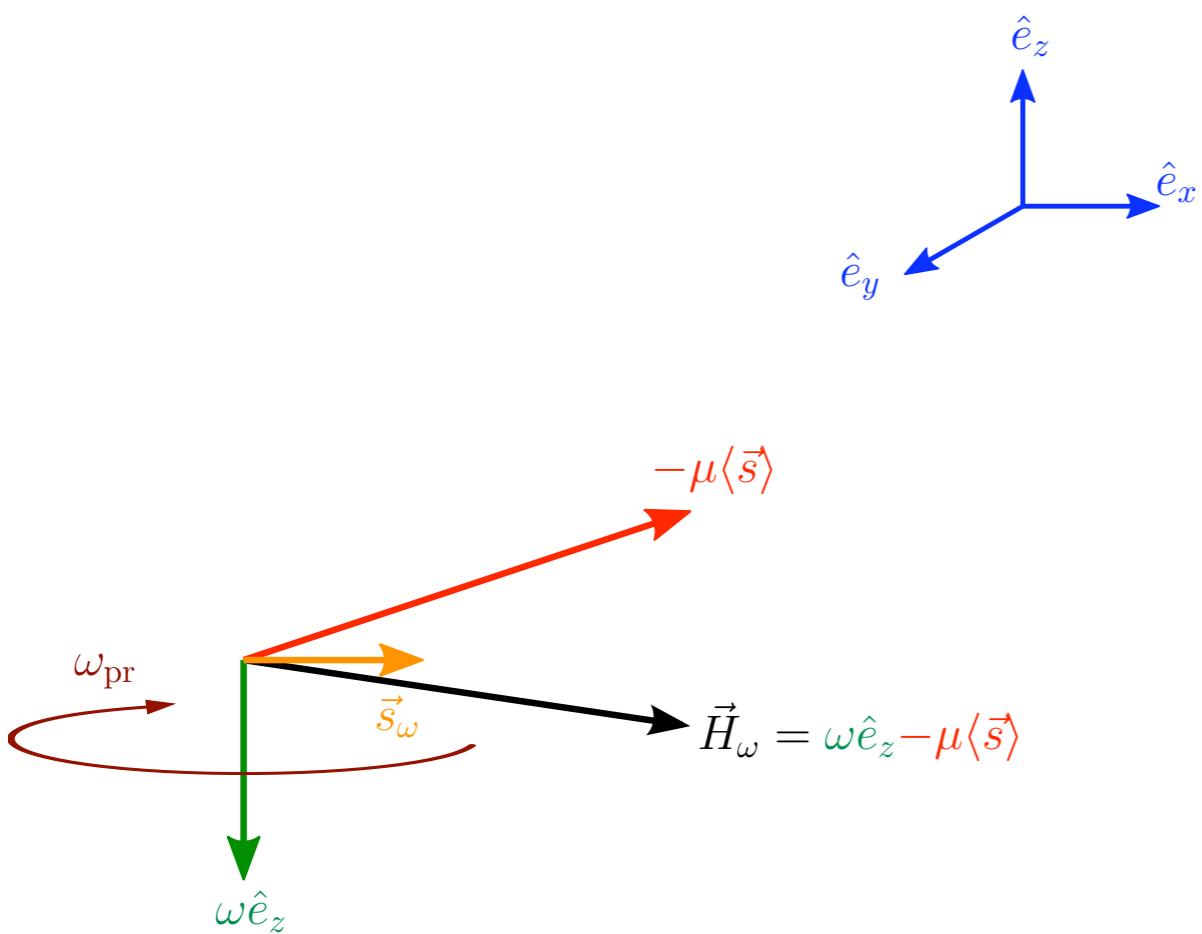
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Precession Mode

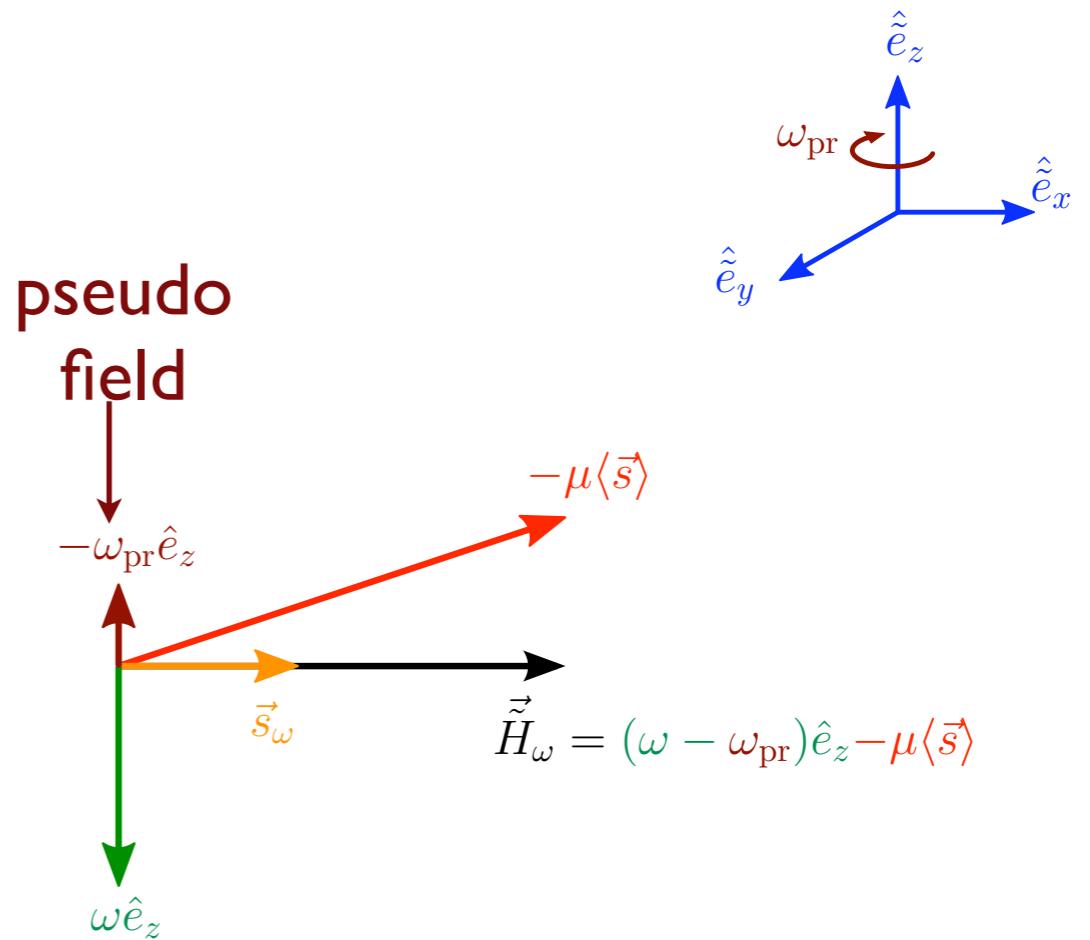
precession
ansatz

all \vec{s}_ω precess about \hat{e}_z with a common angular speed ω_{pr}

static frame

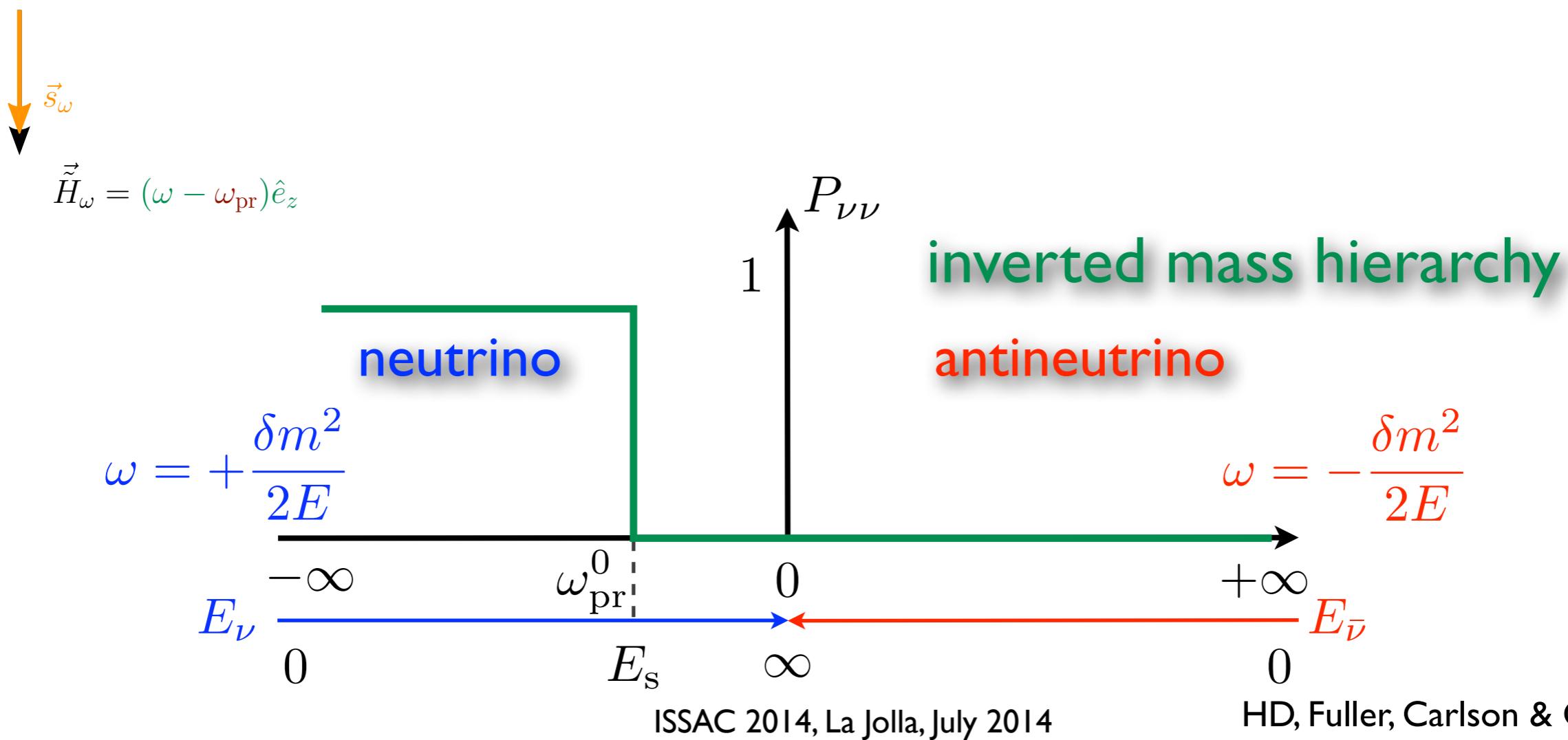
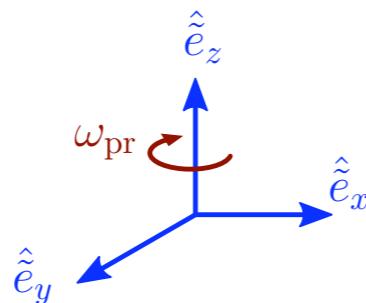


corotating frame

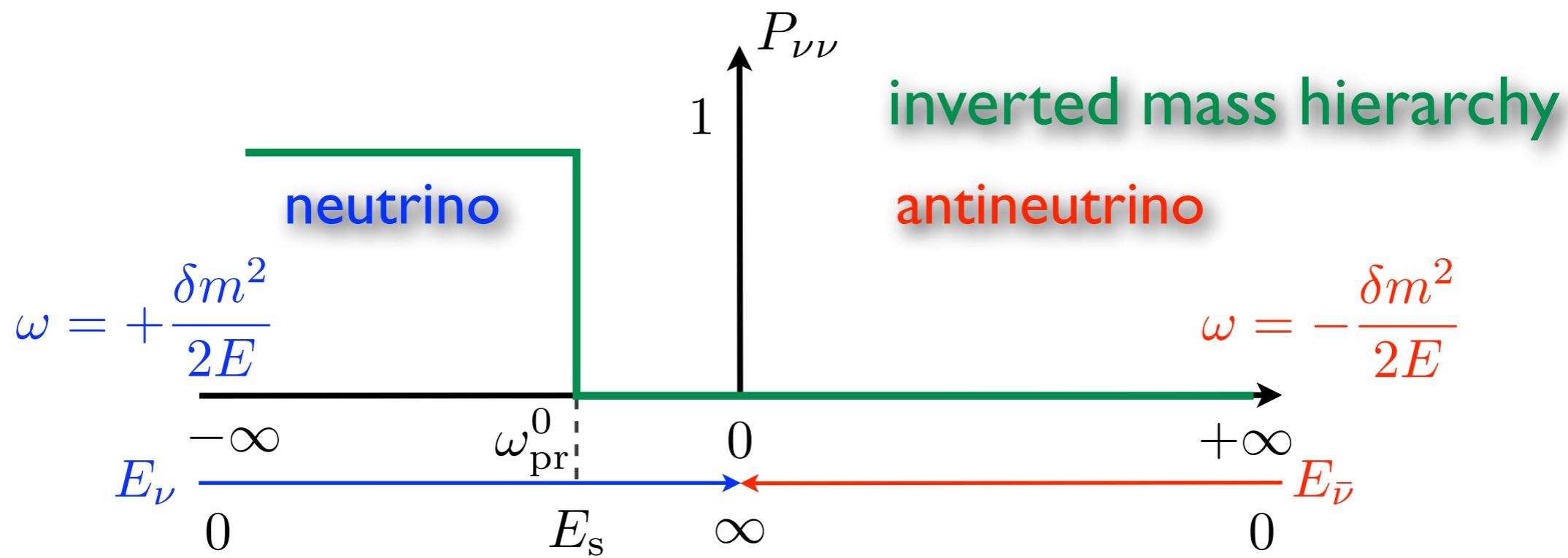
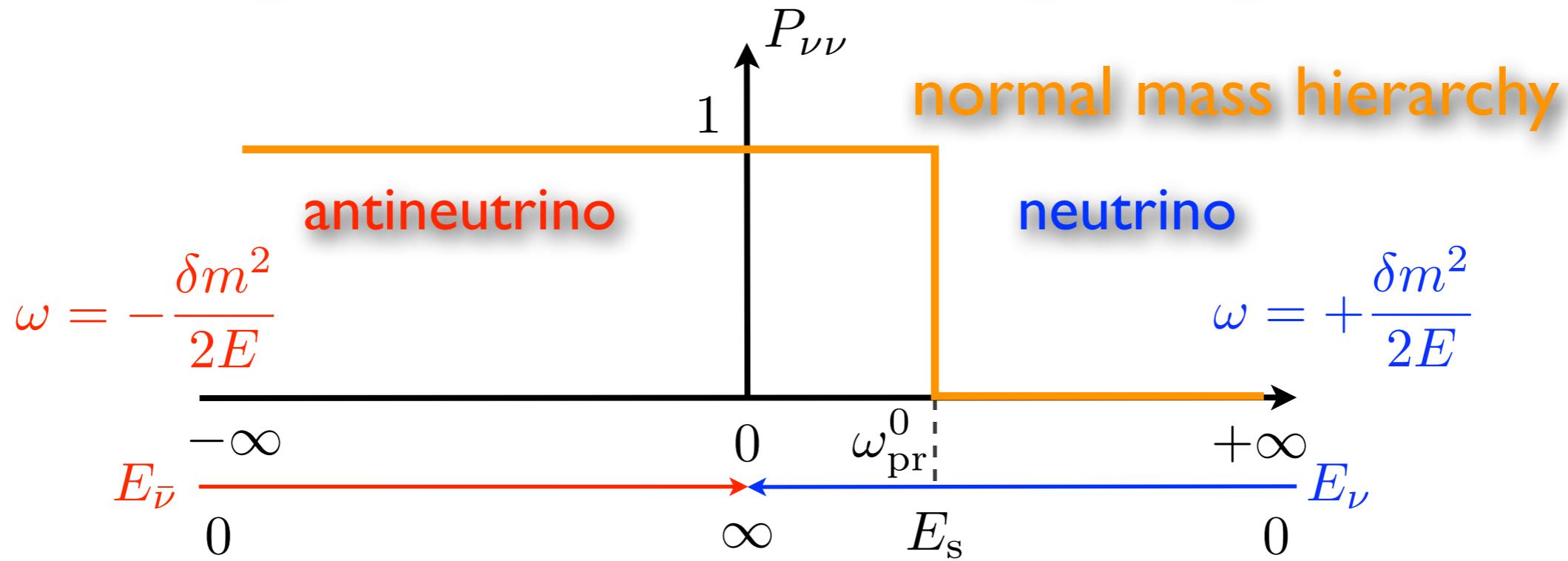


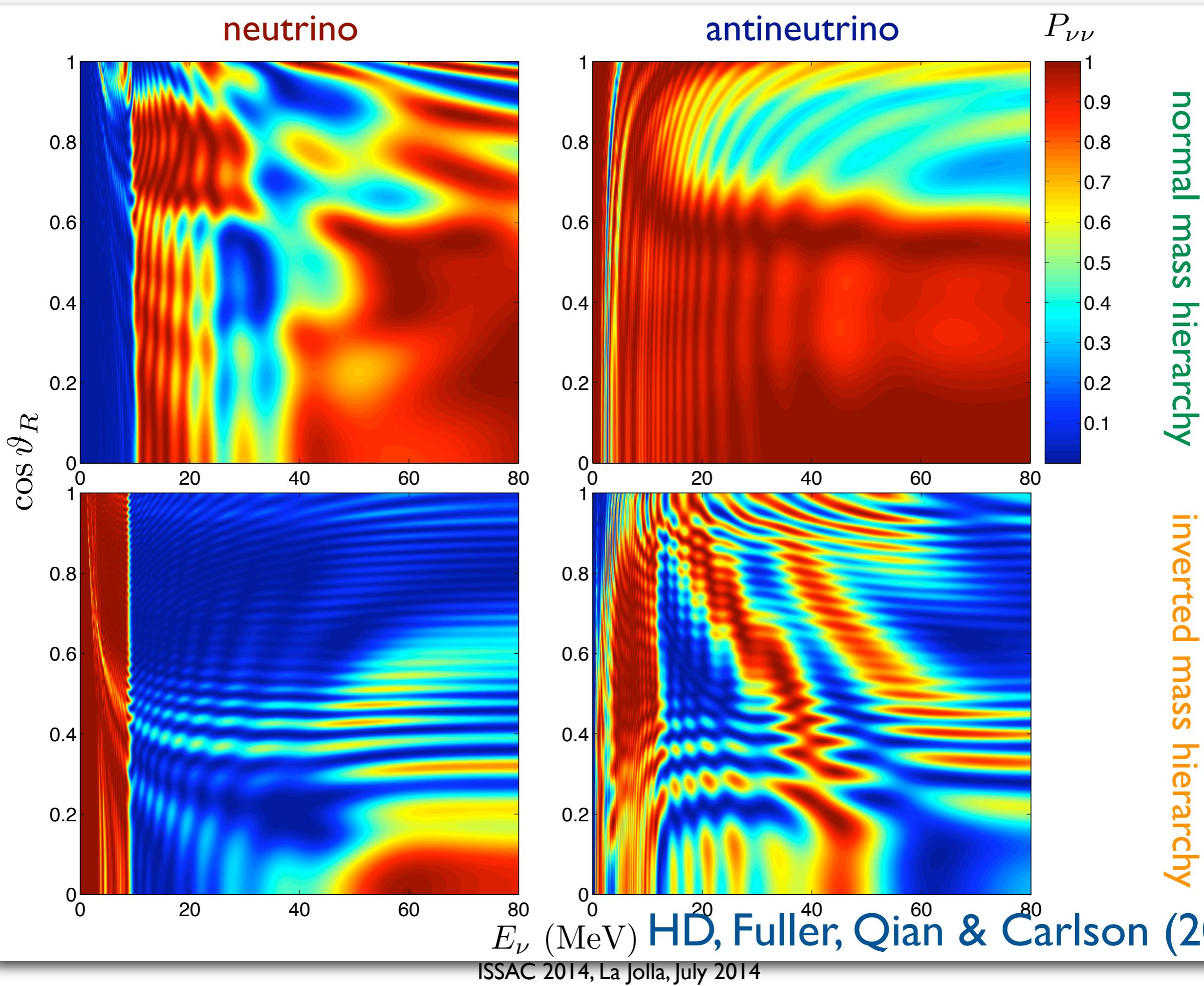
Adiabatic Process

$$n_\nu^{\text{tot}} \rightarrow 0$$



Spectral Swap/Split





normal mass hierarchy

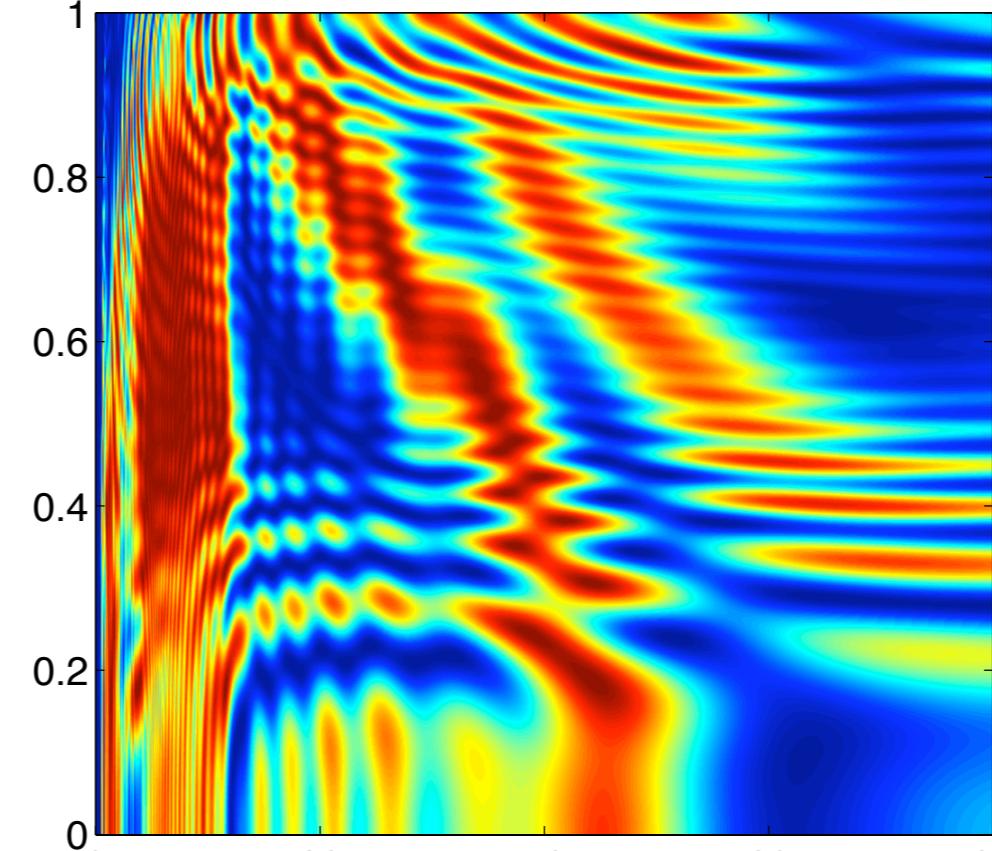
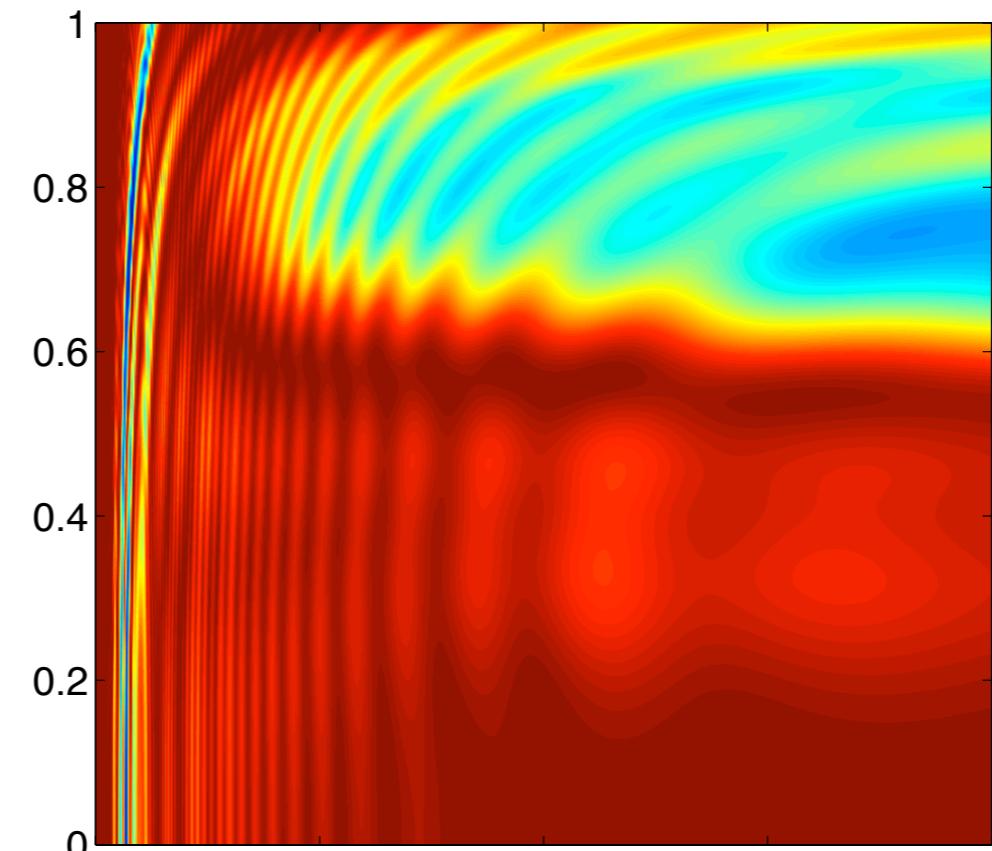
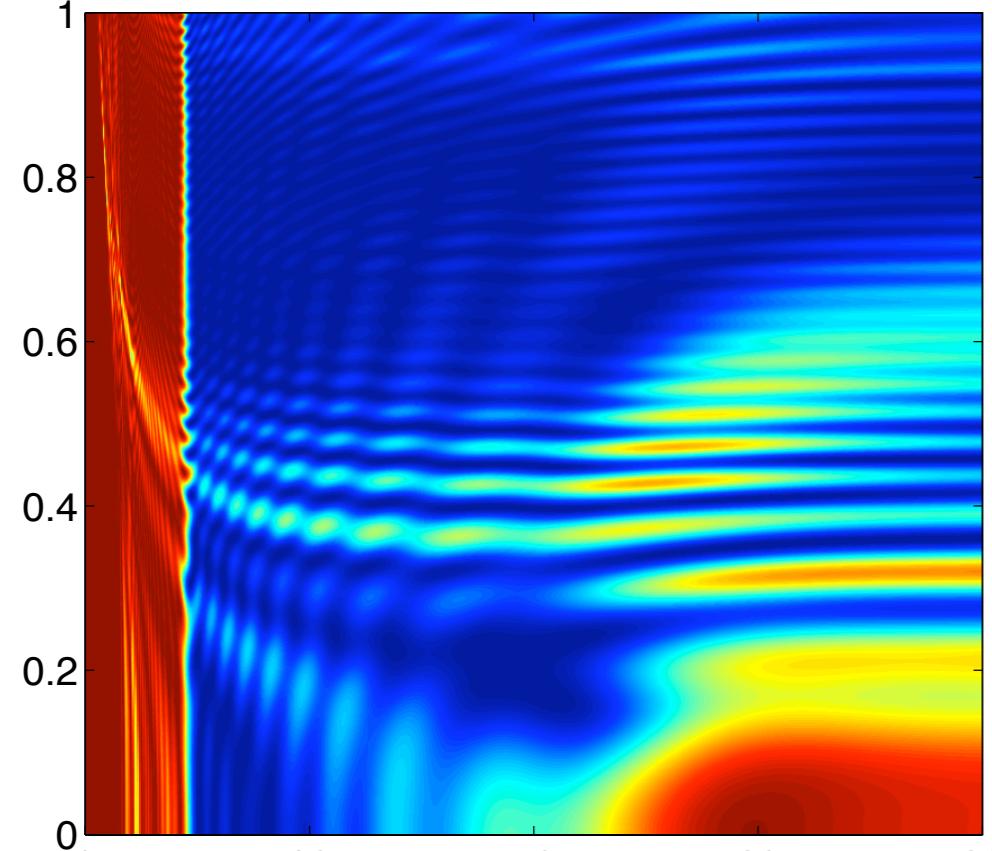
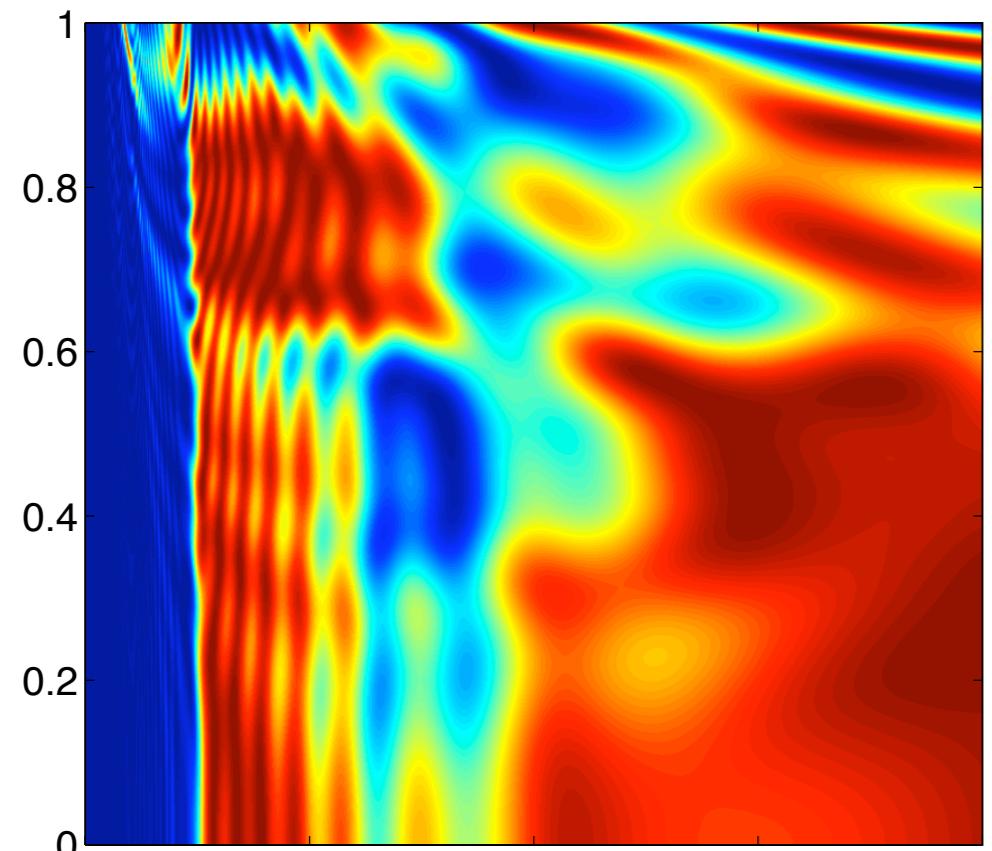
inverted mass hierarchy

$P_{\nu\nu}$

$\cos \vartheta_R$

neutrino

antineutrino



Linear Stability Analysis

$$\vec{s}_\omega \rightarrow \rho_\omega = \begin{bmatrix} s_z & s_x - i s_y \\ s_x + i s_y & -s_z \end{bmatrix}$$

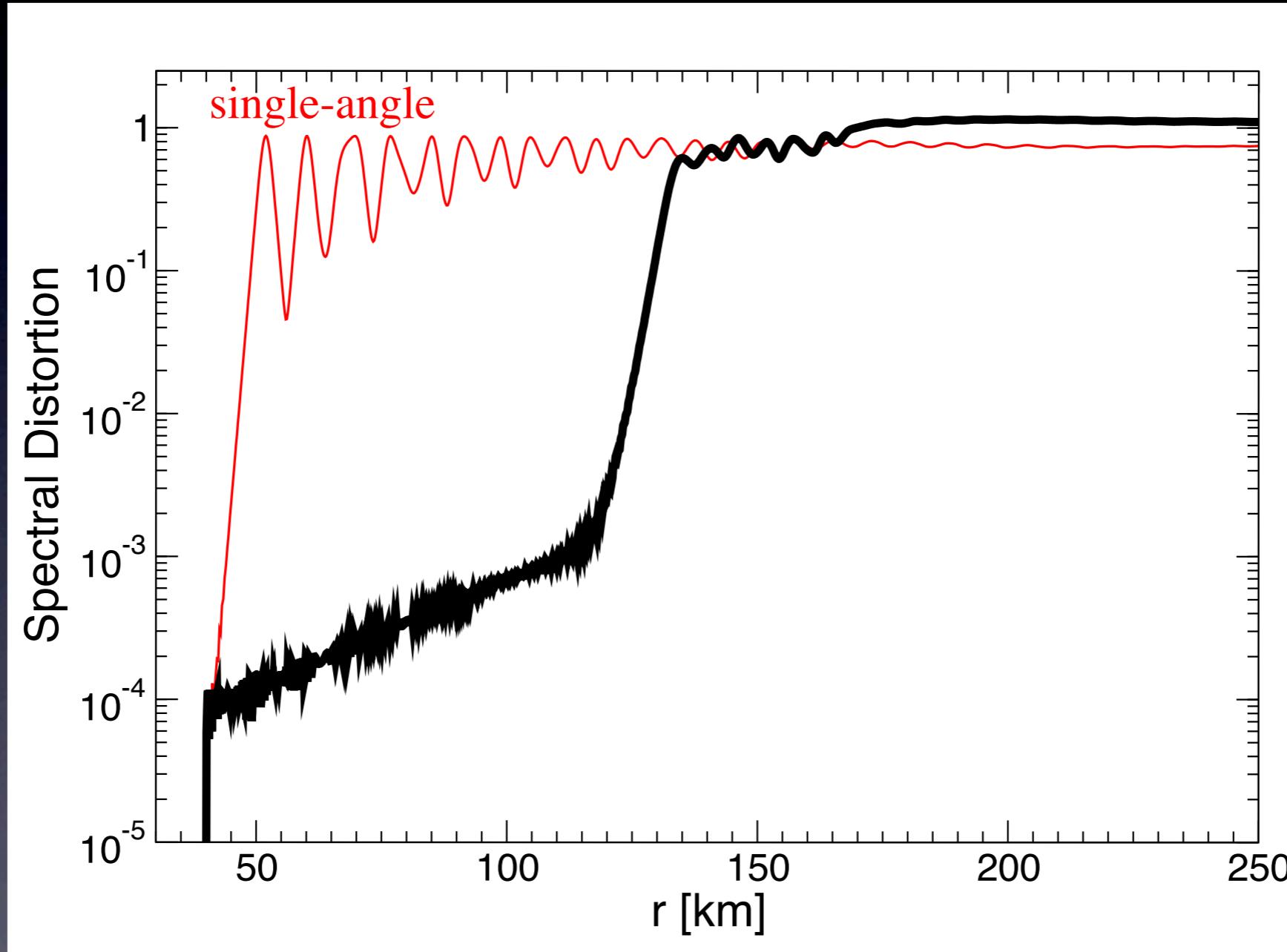
$|s_z| \approx 1, |s_x| \sim |s_y| \ll 1 \Rightarrow$ Keep linear terms of $S = s_x - i s_y$

$$i \dot{S}_\omega \approx \omega S_\omega - \mu \int f_{\omega'} S_{\omega'} d\omega'$$

Pure precession $\Rightarrow S_\omega \propto e^{-i\omega_{\text{pr}} t}$

Imaginary ω_{pr} ($= \gamma + i\kappa$) \Rightarrow flavor instability
(Banerjee et al, 2011)

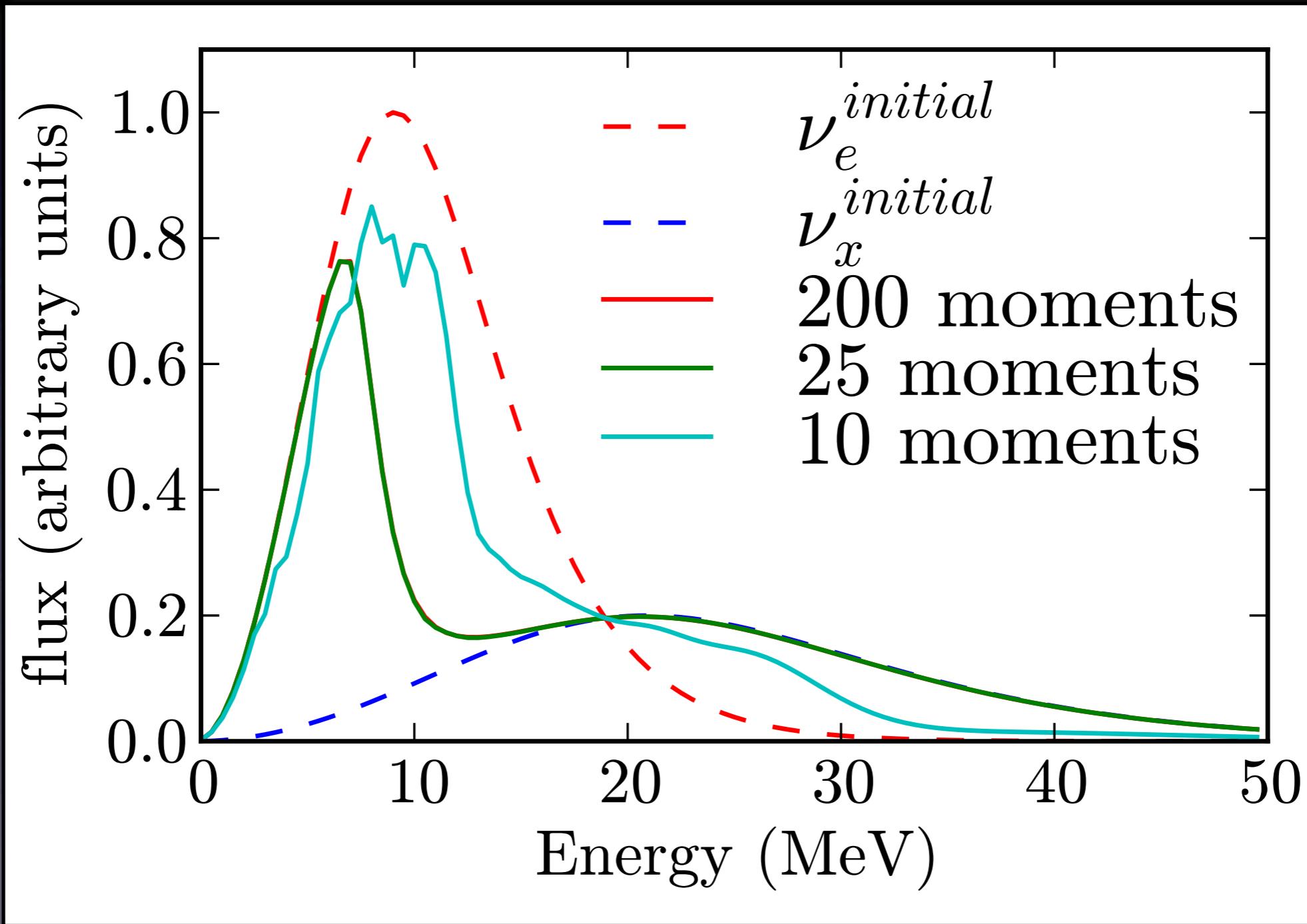
Multiangle Suppression



New Developments and Challenges

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Moment Method



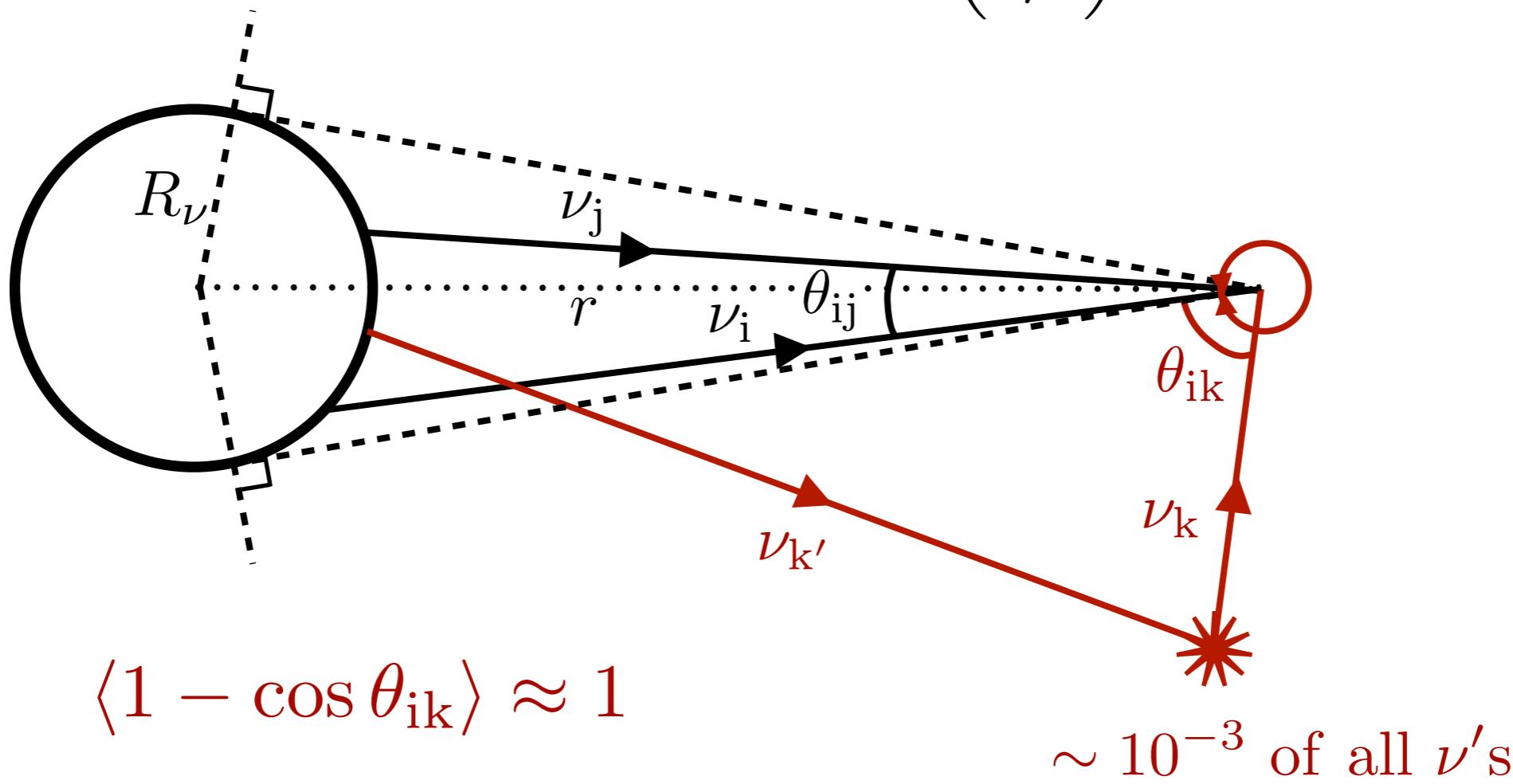
Shalgar & HD (in preparation)

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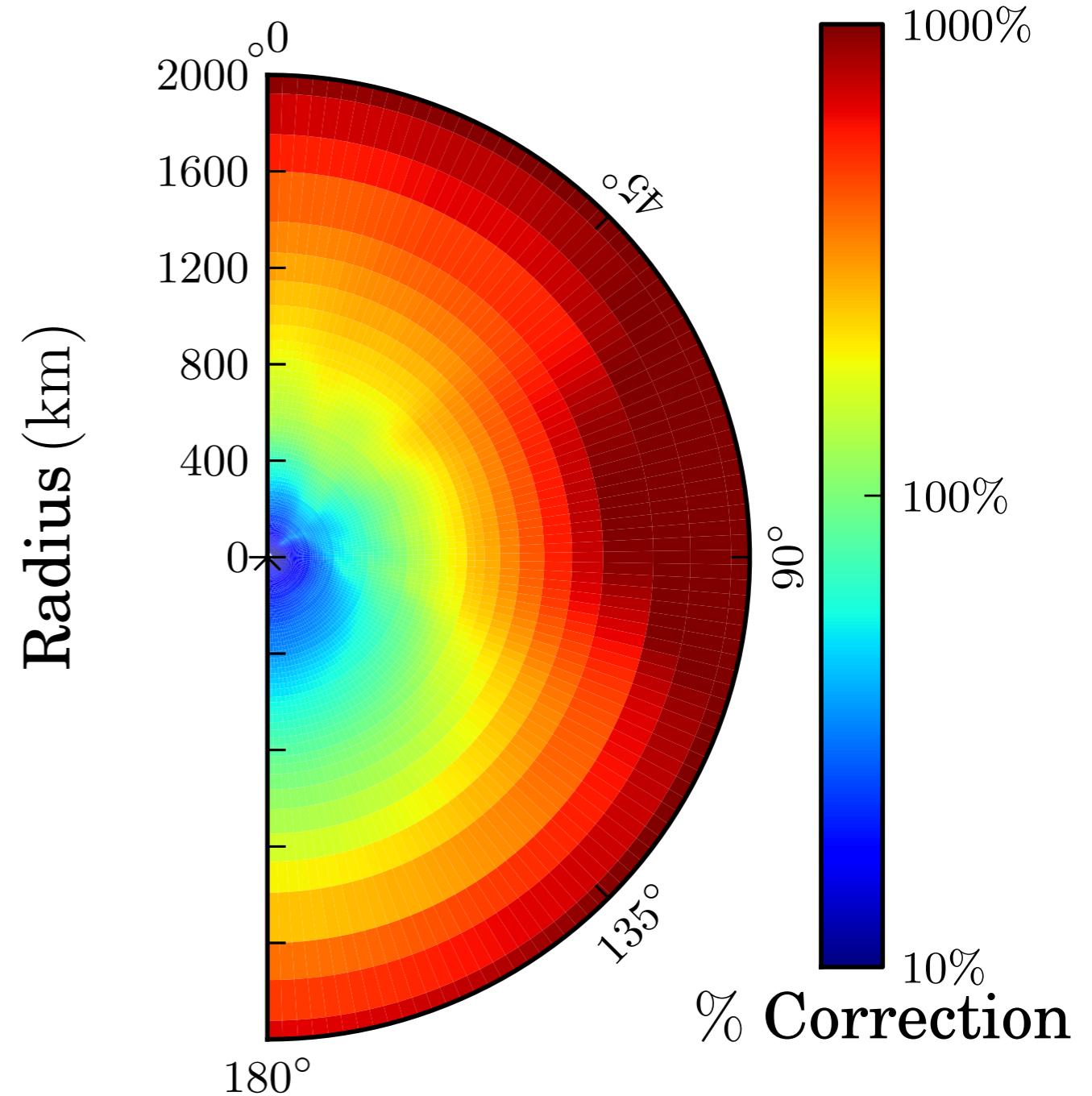
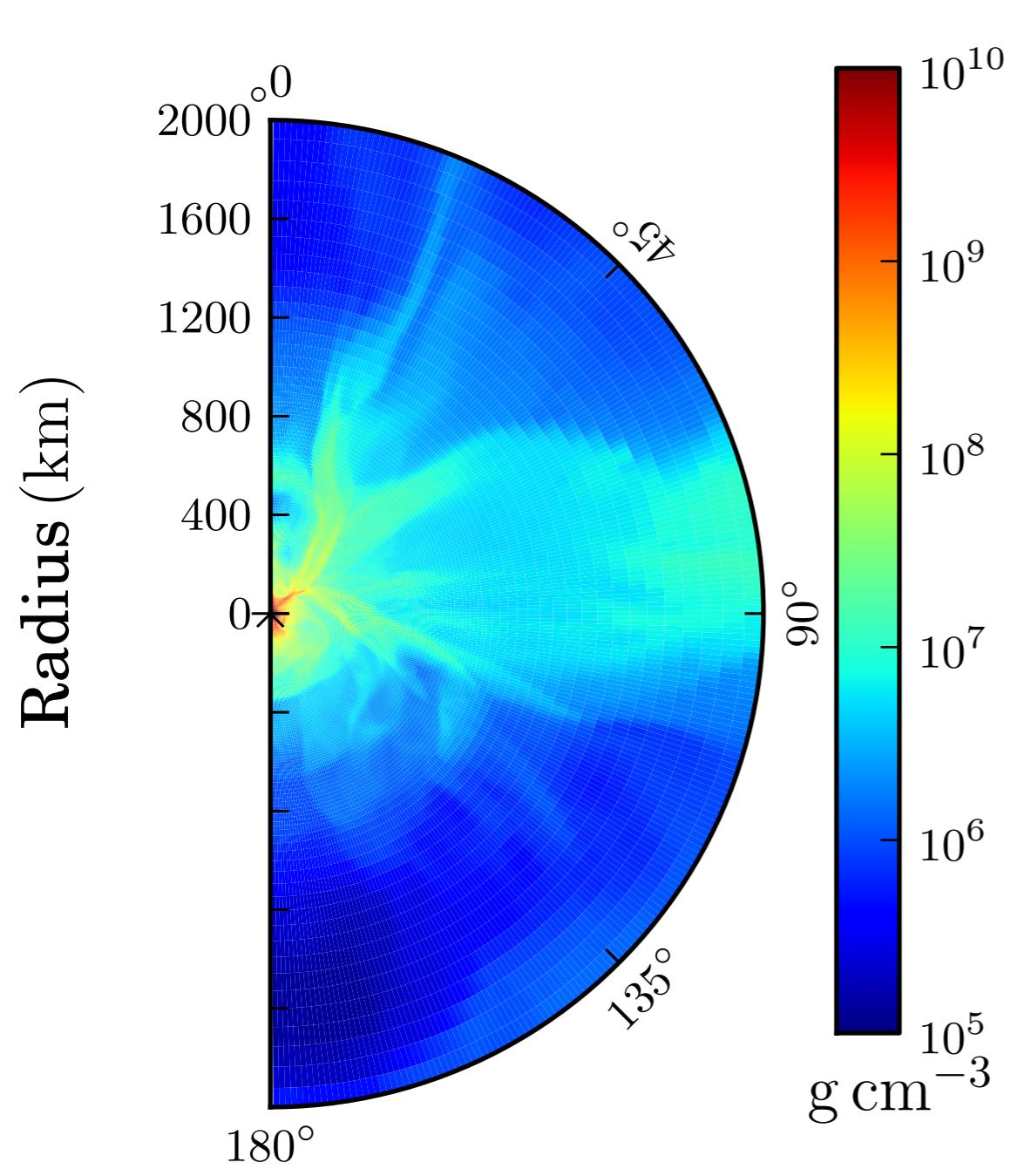
Neutrino Halo

(Cherry et al, 2012)

$$r \gg R_\nu \Rightarrow \langle 1 - \cos \theta_{ij} \rangle \propto \left(\frac{R_\nu}{r} \right)^2$$



Neutrino Halo



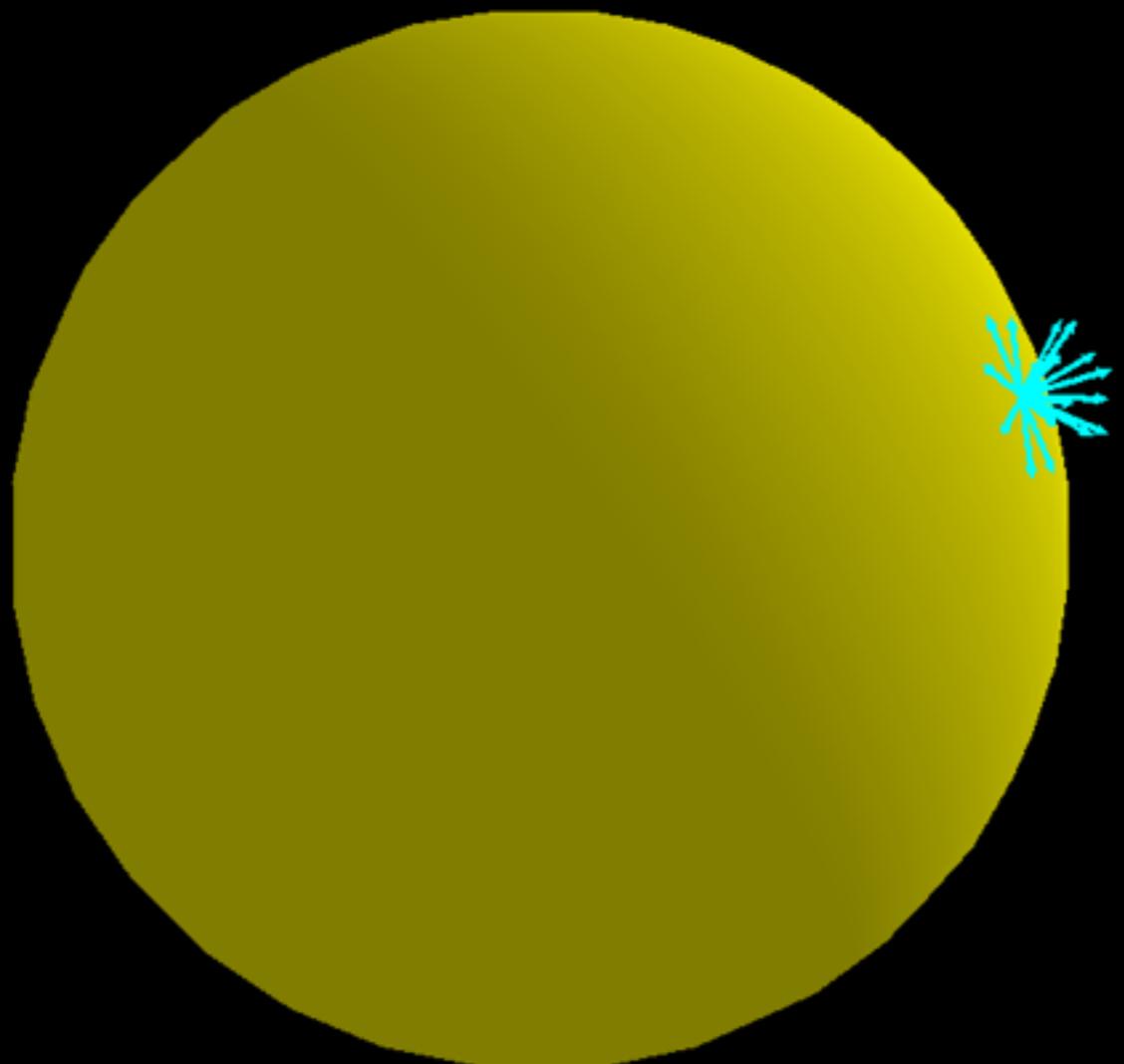
(Cherry et al, 2012)

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Spontaneous Symmetry Breaking?

- A symmetry in the EoM does not guarantee that its solution(s) will also be symmetric.
- Even if the system may be approximately symmetric initially, a non-symmetric mode may quickly dominate if it is unstable.
- Numerical calculations suggest that supernova neutrino oscillations may not be axially symmetric even in the (1+2)D model. [Raffelt et al, 2013; Mirizzi, 2013]

(l+3)D



propagation
direction

$\psi(r, E, \vartheta, \varphi)$

↑
energy

A diagram on the right side of the slide. It features a yellow sphere on the left. From its top-right surface, a cyan starburst-like pattern extends upwards and to the right. To the right of the sphere, the text "propagation direction" is written in purple, with a purple arrow pointing downwards towards the starburst. Below the sphere, the text " $\psi(r, E, \vartheta, \varphi)$ " is written in cyan. A white double-headed vertical arrow is positioned between the sphere and the text, with the word "energy" written in white below it.

previous assumptions +
Spherical symmetry about
the center (Consistency?)

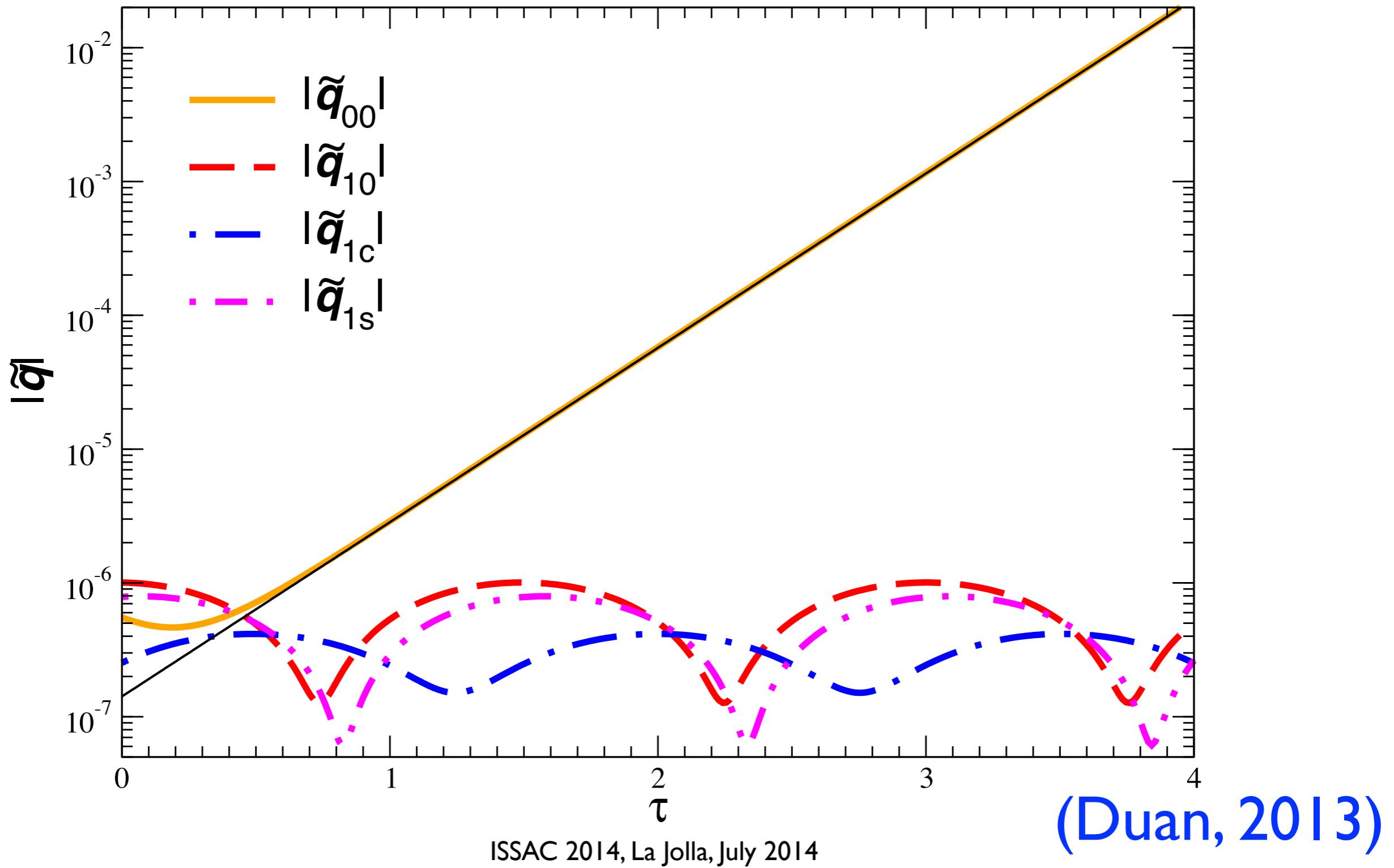
Homogeneous Gas Again

$$1 - \mathbf{p} \cdot \mathbf{p}' = 4\pi \left[Y_{0,0}(\mathbf{p})Y_{0,0}^*(\mathbf{p}') - \frac{1}{3} \sum_{m=0,\pm 1} Y_{1,m}(\mathbf{p})Y_{1,m}^*(\mathbf{p}') \right]$$

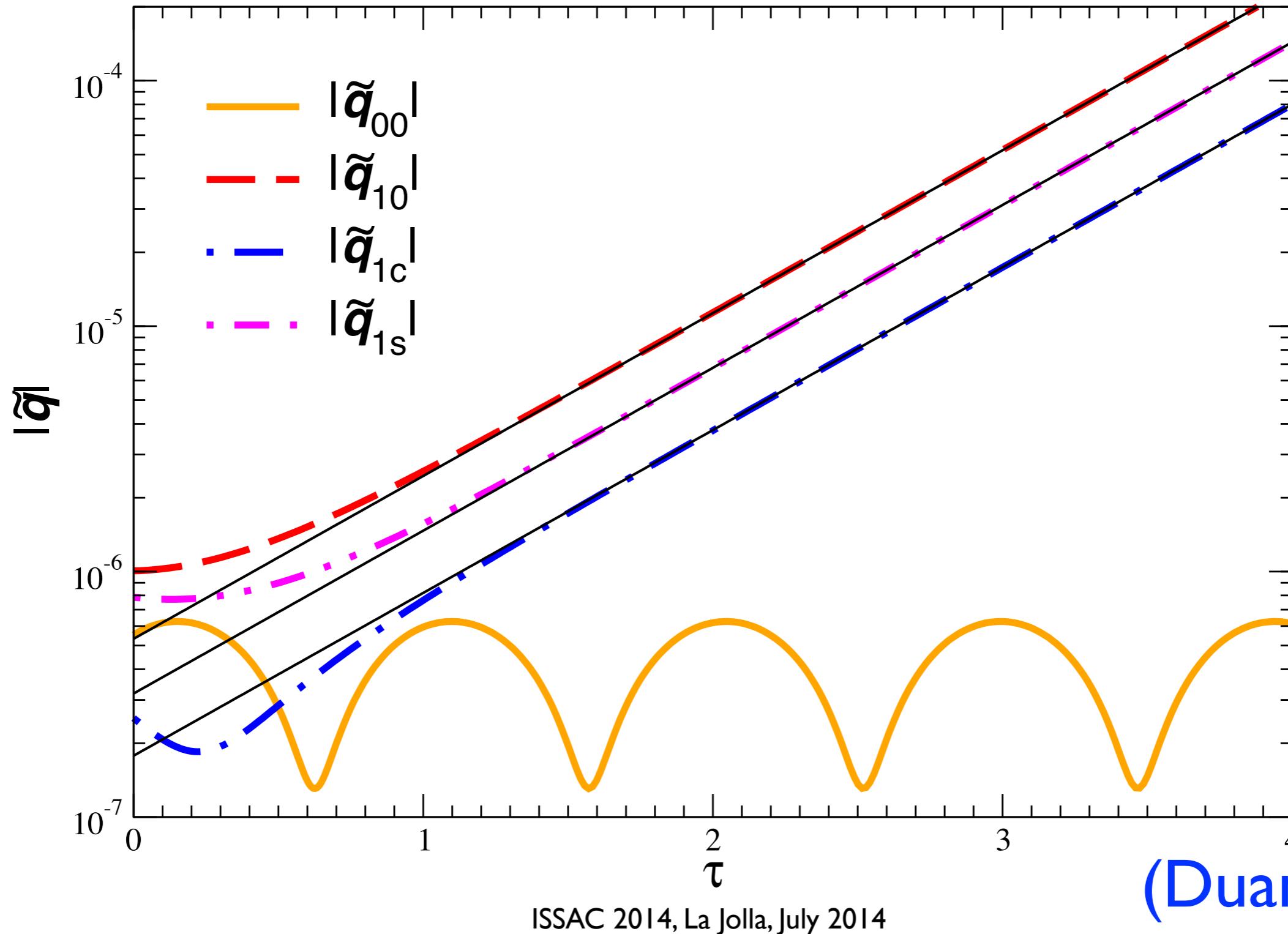
- Multipole modes are decoupled in the linear Regime
- $l=0$: $\mu_{\text{eff}} = \mu$, unstable in IH
- $l=1$: $\mu_{\text{eff}} = -\mu/3$ unstable in NH
- $l > 1$: $\mu_{\text{eff}} = 0$, always stable

(Duan, 2013)

Inverted Hierarchy



Normal Hierarchy



Implications for SN ν

- Collective oscillations can occur in either mass hierarchy.
- Oscillations can occur deeper in the NH case than the IH case.
- The angle-dependent modes break the axial symmetry and the spherical symmetry -- new computing paradigm is needed.

Summary

- Neutrinos offer a unique and direct probe into the center of stars, including supernovae.
- Neutrinos are essential to supernova dynamics and nucleosynthesis.
- Collective neutrino oscillations — a collective quantum phenomenon on the scale of $10 \sim 100$ km?

How do you want do your calculations?