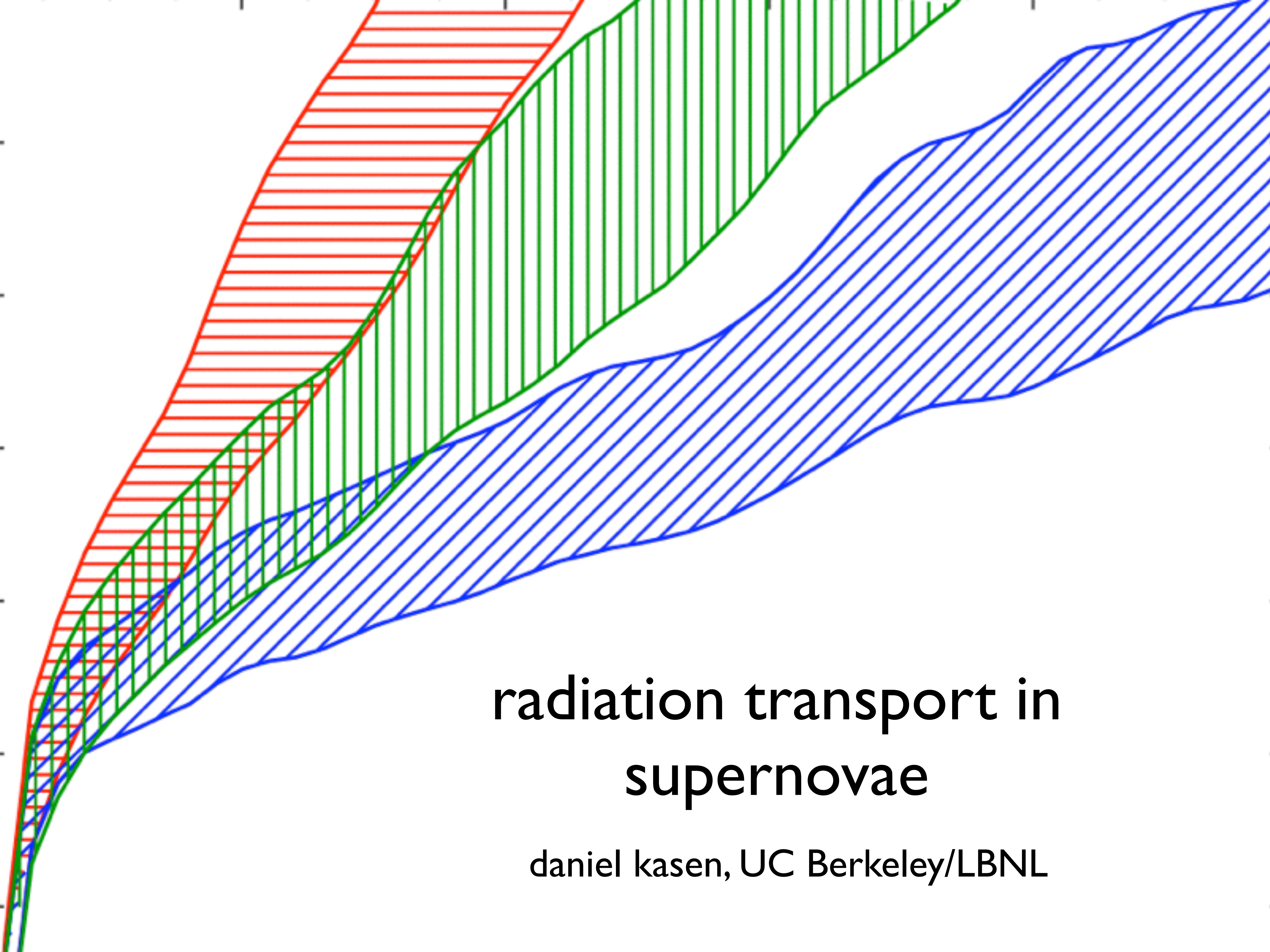
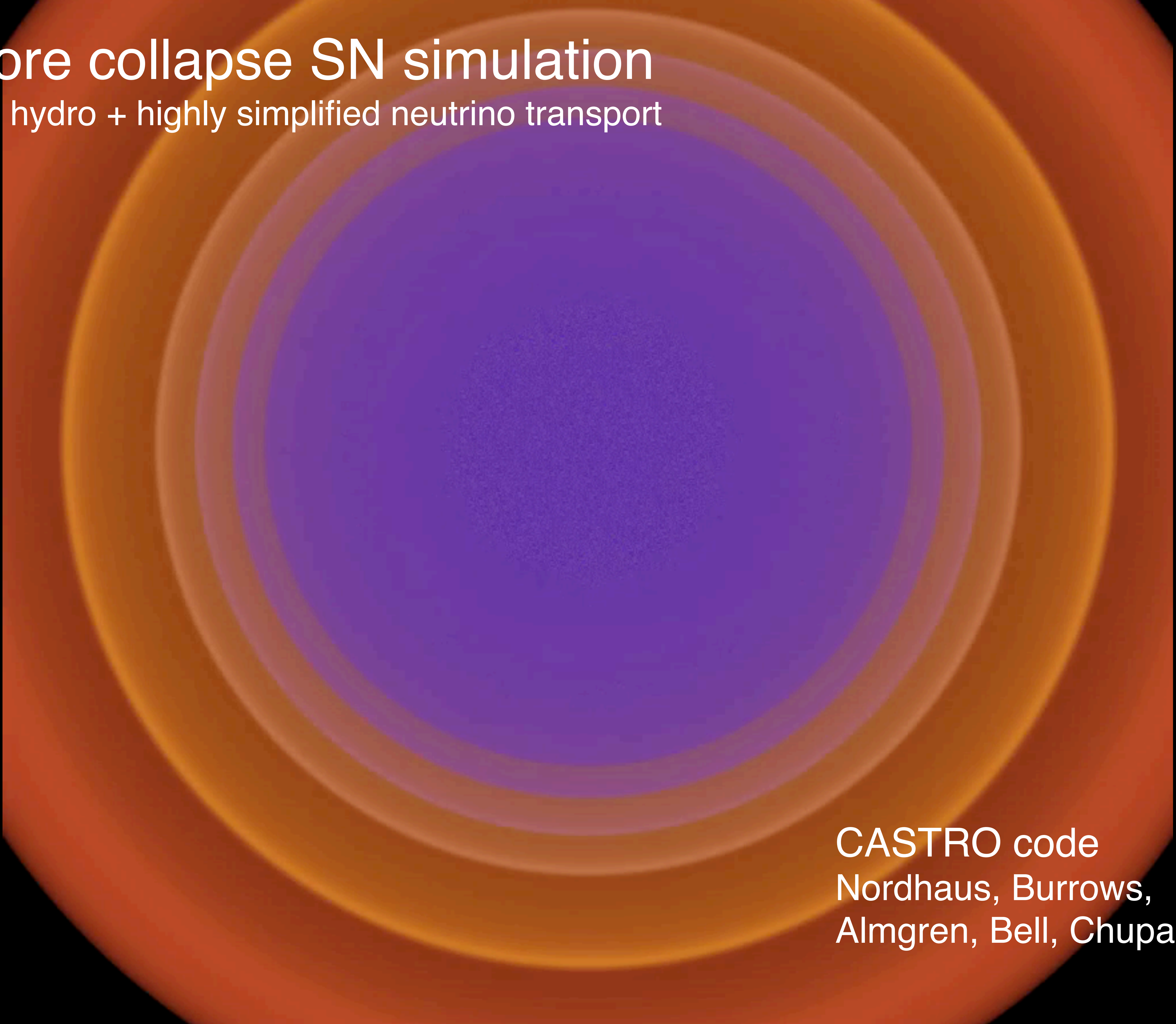


radiation transport in supernovae

daniel kasen, UC Berkeley/LBNL

why do radiation transport?

- That’s what we see! (photon and neutrino light curves and spectra)
- Radiation can be dynamically important (transports energy and momentum)
- Radiation can alter the composition (neutrinos can exchange protons and neutrons)

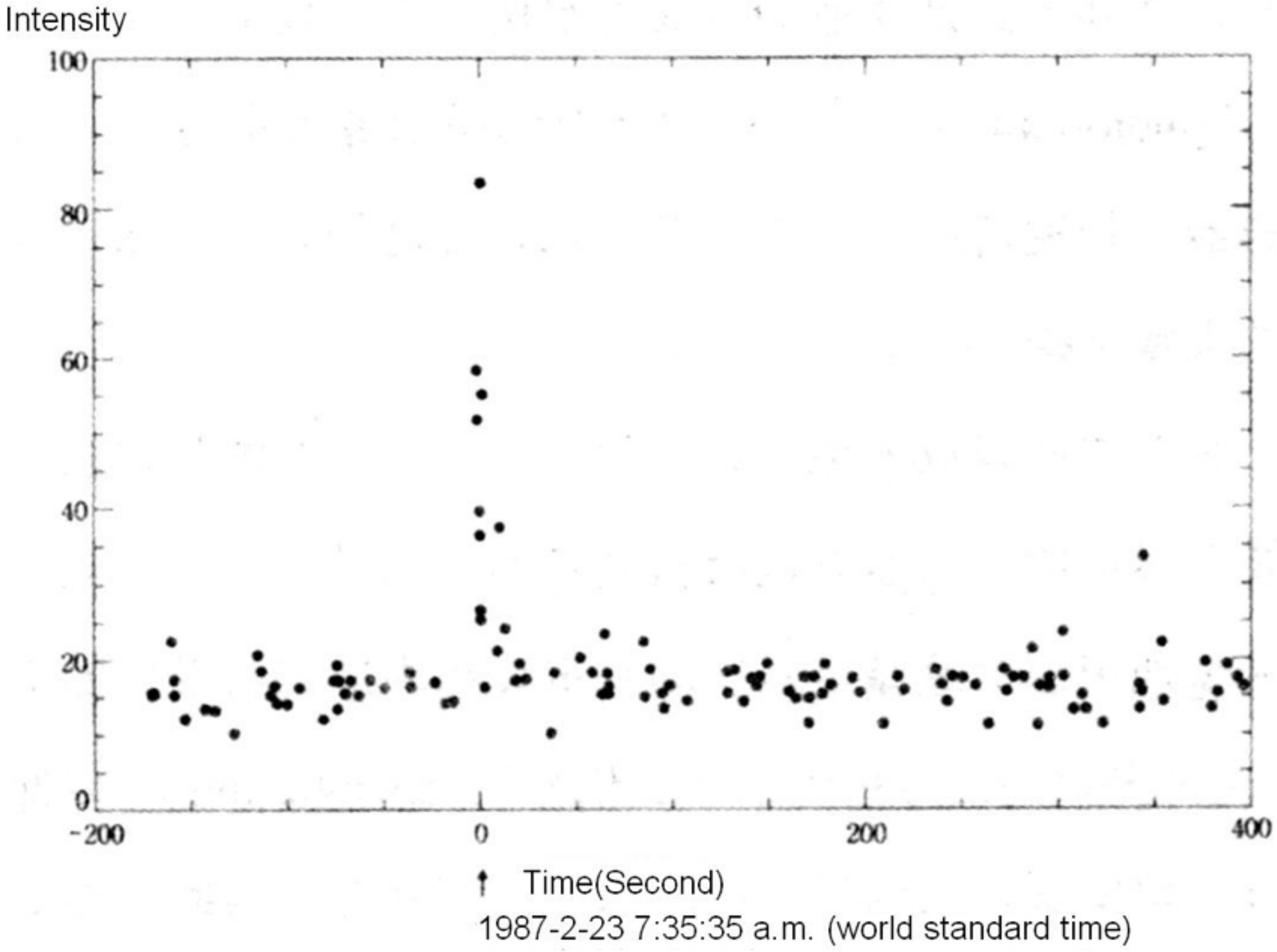


core collapse SN simulation

3D hydro + highly simplified neutrino transport

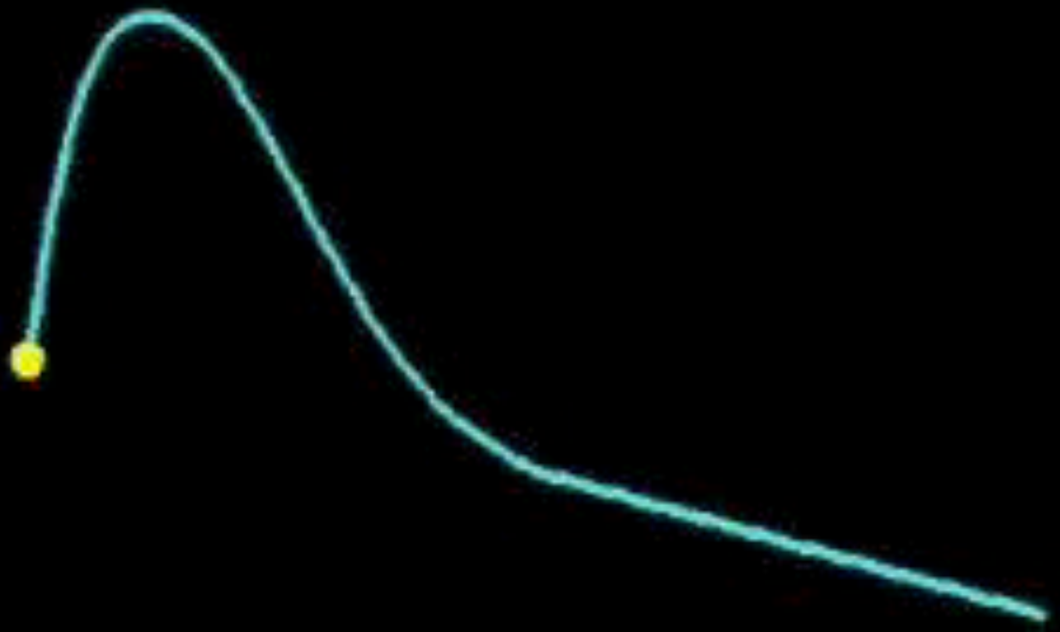
CASTRO code
Nordhaus, Burrows,
Almgren, Bell, Chupa

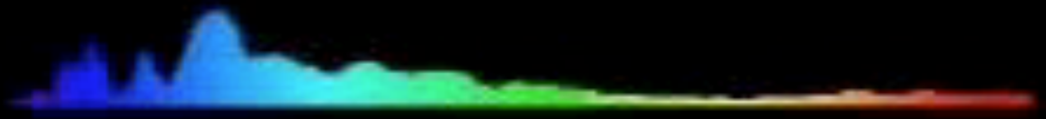
neutrinos from SN1987A



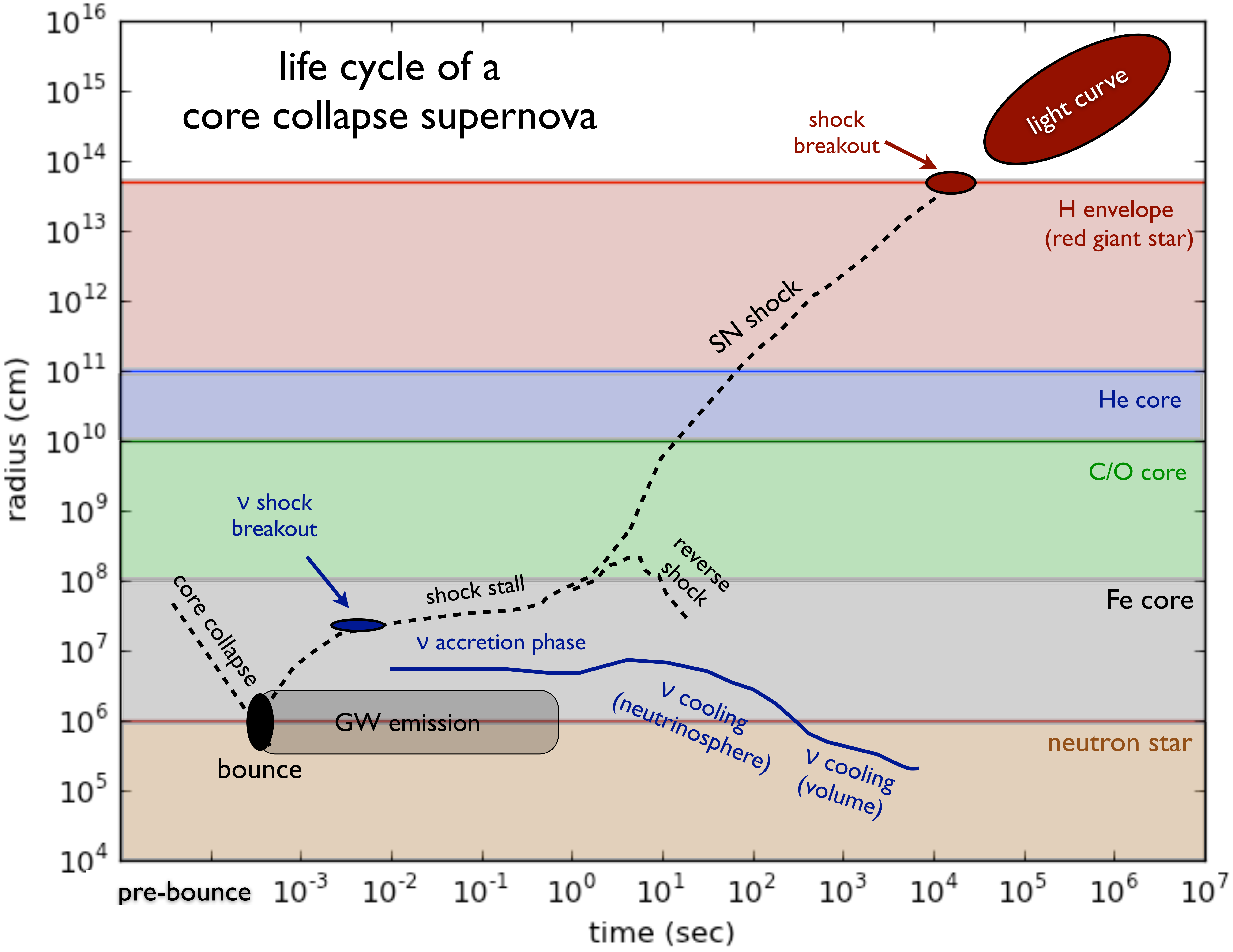
SN location
in galaxy

optical light curve
(time to peak ~ 20 days)



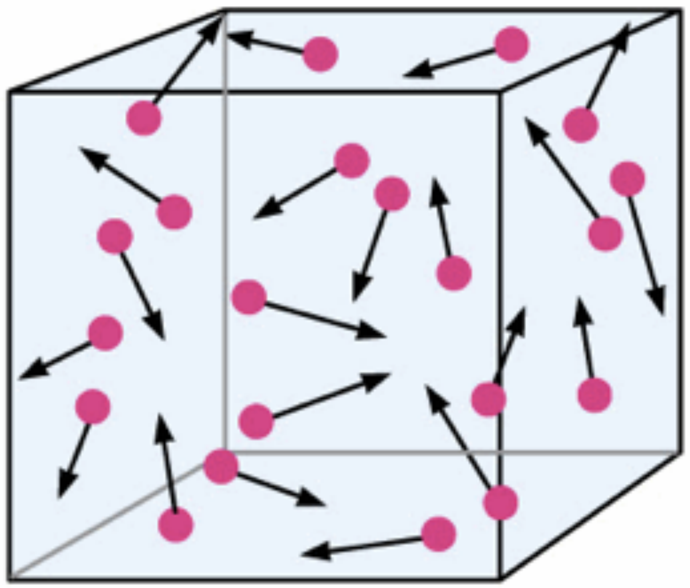


optical spectrum



life cycle of a core collapse supernova

what counts as radiation?



some typical interaction cross-sections

photon-electron scattering

$$\sigma_{\mathrm{t}} \sim 10^{-24}\ \mathrm{cm}^2$$

neutrino-nucleon scattering

$$\sigma_\nu \sim G_F^2 E_\nu^2 \sim 10^{-44} \left(\frac{E_e}{1\ \mathrm{MeV}}\right)^2\ \mathrm{cm}^2$$

electron-electron (coulomb) scattering

$$\sigma_e \sim \sigma_{\mathrm{t}} \left(\frac{E_e}{m_e c^2}\right)^{-2} \sim 10^{-13} \left(\frac{E_e}{1\ \mathrm{eV}}\right)^{-2}\ \mathrm{cm}^2$$

Photons and neutrinos move around much more easily!
they are not necessarily in equilibrium/isotropic

when do we need transport?

compare transport timescale to dynamical

optical depth: $\tau = \sigma n R$

free-streaming time (τ < 1) $t_{\rm fs} = R/c$

diffusion time (τ > 1) $t_d = \tau(R/c)$

neutrinos near neutron star surface (r ~ 10 km, σ ~ 10	τ **~ several** **t ~ ms**
photons in a solar-type star (r ~ 10	τ **~ 10^{11}** **t ~ 10,000 yrs**
photons in an expanded SN remnant (r ~ 10	τ **~ 100** **t ~ month**

neutrinos for the explosion, photons for the aftermath

how to describe radiation

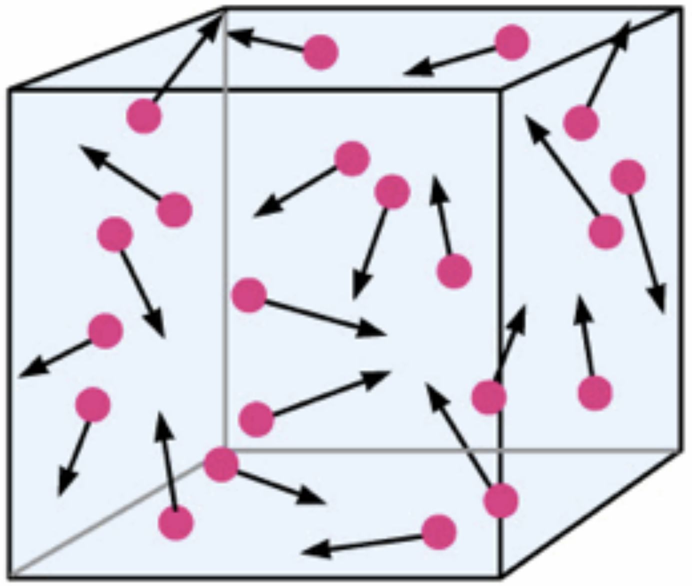
the field is fully described by a ***distribution function***

$$f = f(x, y, z, t, E, \theta, \phi)$$

# of particles at (x,y,z,t) with energy E moving in direction (θ,ϕ)

or, equivalently: $f = f(\vec{x}, \vec{p}, t)$

distribution of particles in phase space



often we use the ***specific intensity***

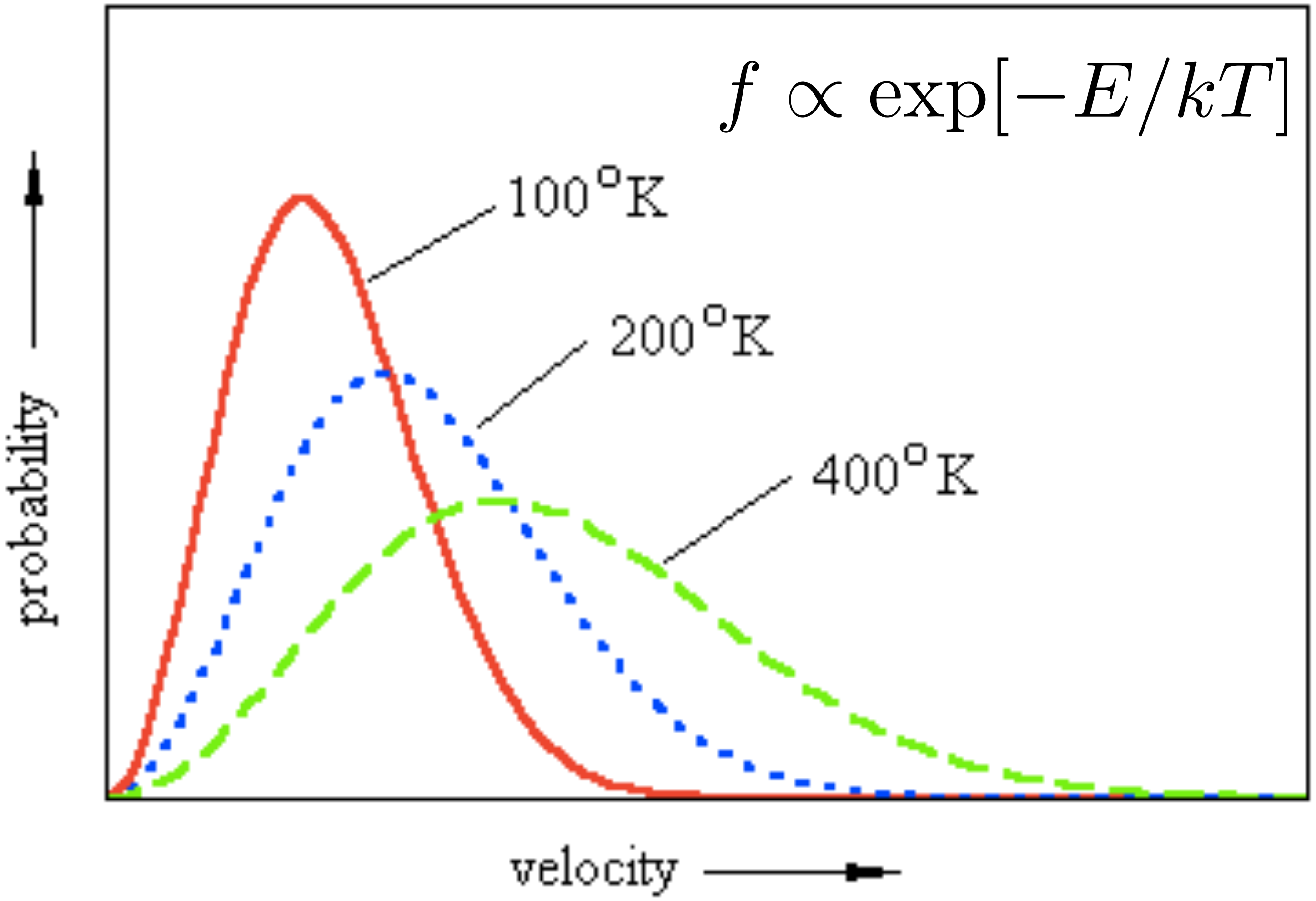
$$I_\nu = h\nu c \times f$$

however you do it, we have a function of 7 variables!

thermodynamic equilibrium

if collisions are frequent we reach equilibrium.
f becomes isotropic (no (θ,ϕ) dependence) and
the E dependence is a known function of temperature

e.g., maxwell-boltzmann distribution



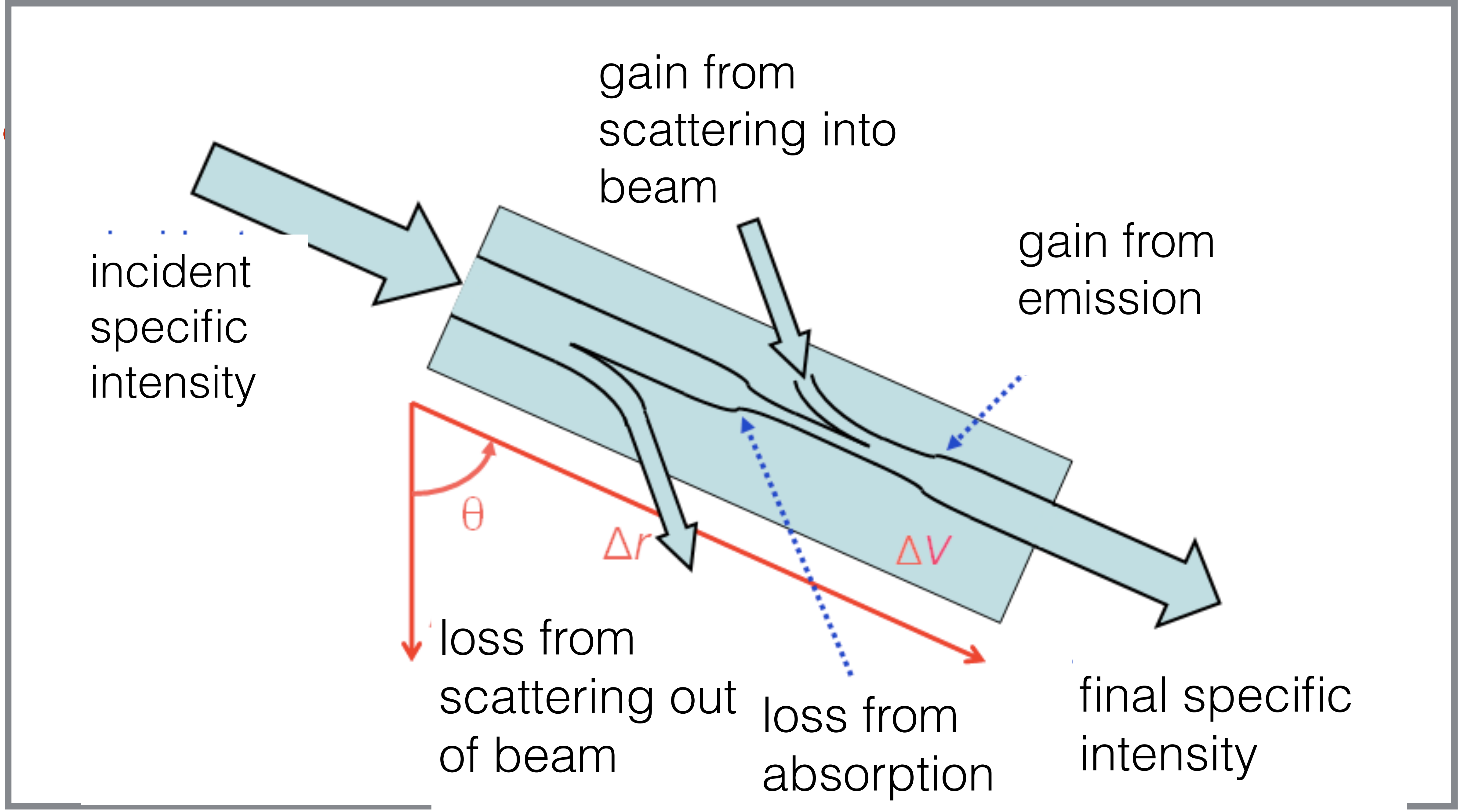
for gas, we can usually make this assumption and do hydrodynamics (neglect θ,ϕ,E dependence)

but radiation will not be in equilibrium unless the optical depths are very large

the radiative transfer equation

a.k.a the Boltzmann equation

$$\frac{1}{c}\frac{dI_\nu}{dt} + \frac{dI_\nu}{ds} = -\chi_{\text{abs}} I_\nu + \eta + \frac{\chi_{\text{sc}}}{4\pi} \oint I_\nu d\Omega'$$



incident specific intensity

gain from scattering into beam

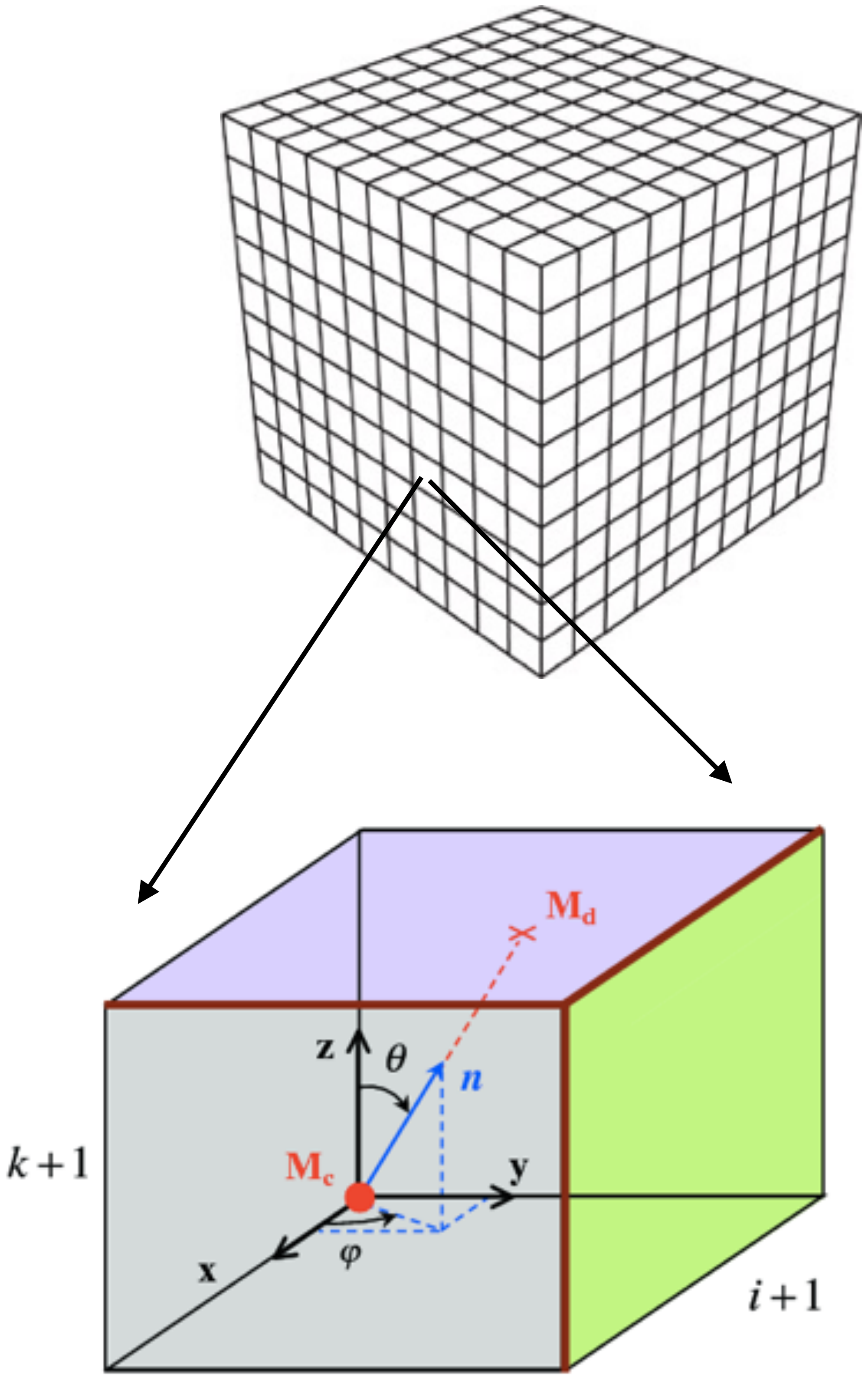
gain from emission

loss from scattering out of beam

loss from absorption

final specific intensity

The full 7-D transport problem is ***hard***



example discretization

dimension	# of points
spatial (x,y,z)	256x256x256
angular (θ,ϕ)	30x30
frequency (v)	30
total	**4.5 x 10**

very memory intensive (~1 Tb in this example) and computationally expensive to solve at every time step

a variety of radiation transport methods

leakage scheme
flux limited diffusion
M1 methods
ray-by-ray
variable eddington tensor (VET)
monte carlo

various levels of approximation (reduction of dimensions)

“approximate” transport

"no transport" transport

ways to capture effects of heating and cooling

optically thin, $\tau \ll 1$ (no attenuation)

local emission (cooling) = η (all radiation escapes)

impinging radiation field (heating)
$F = L/4 \pi r^2$ (if a spherical source, "light-bulb")

leakage scheme, $\tau > 1$ (include attenuation)

local emission (cooling) = $\eta e^{-\tau}$ (not all radiation escapes)

impinging radiation field (heating)
$F = L/4 \pi r^2 e^{-\tau}$ (spherical source radiation is attenuated)

need to integrate to determine τ and do some appropriate average

most approximate transport methods attempt to reduce the dimensionality of the 7-D problem

dimension	approximations	proper
spatial (x,y,z)	1D (spherical symmetry) or 2D (axial symmetry) ray-by-ray methods	3-D
frequency (ν)	grey transport	multi-group
angular (θ,ϕ)	diffusion approximation M1, moment methods	Boltzmann transport

grey approximation

neglect frequency dependence

$$\int_0^\infty [\text{RT Eq}] d\nu$$

integrate out the frequency dependence

$$\frac{1}{c}\frac{dI}{dt} + \frac{dI}{ds} = -\chi_{\rm p} I + \eta + \frac{\chi_{\rm J}}{4\pi}\oint I d\Omega'$$

with the mean exctinction coefficients

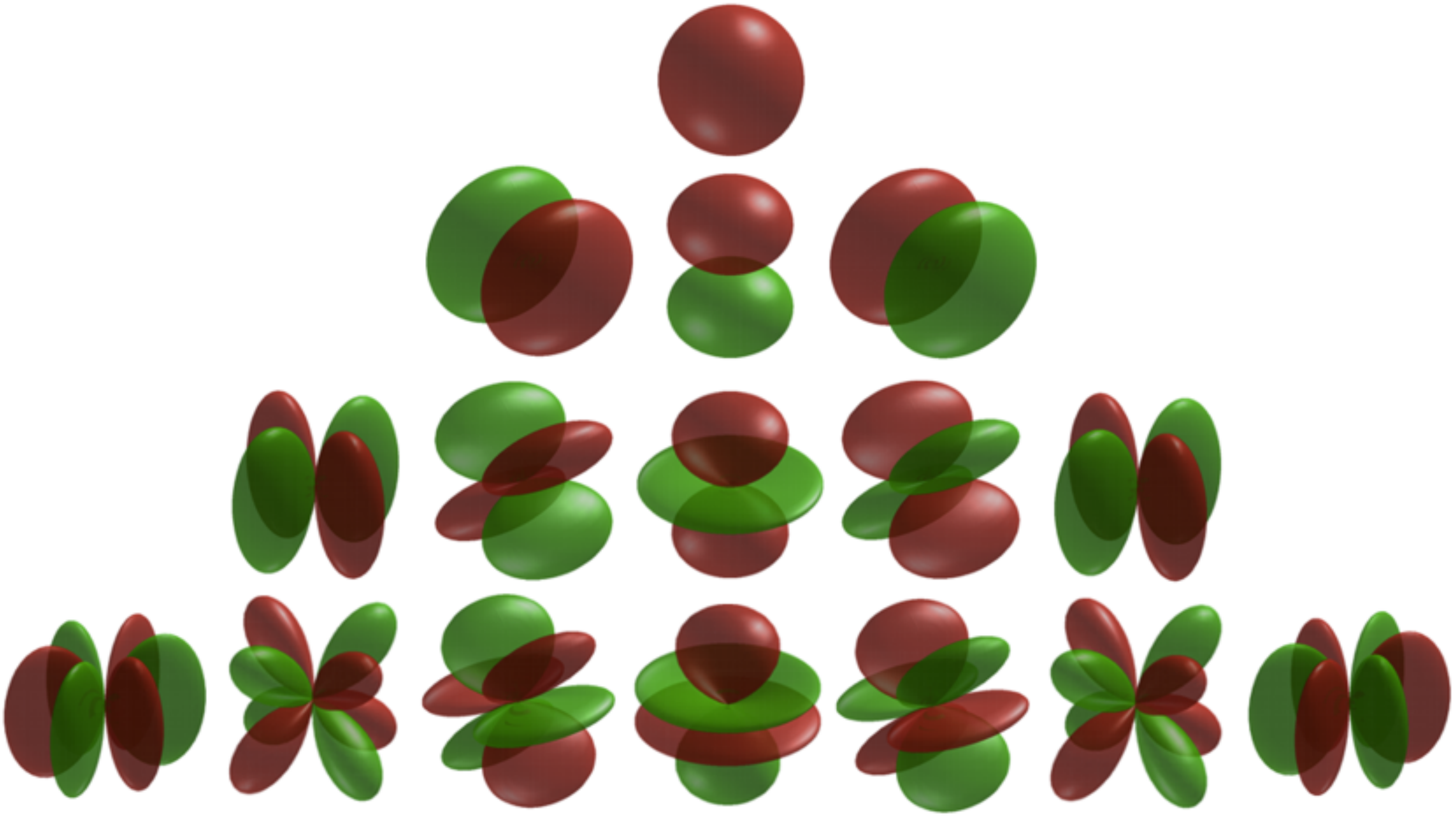
$$\chi_p = \int_0^\infty \chi(\nu) B_\nu(T) d\nu \Big/ \int_0^\infty B_\nu(T) d\nu$$

$$\chi_j = \oint \int_0^\infty \chi(\nu) I_\nu d\nu d\Omega \Big/ \oint \int_0^\infty I_\nu d\nu d\Omega$$

not great for supernova neutrinos, since many cross-sections depend on frequency, $\sigma \sim E^2$

simplifying the angular dependence with moments of the radiation field

a decomposition of the angle dependence
not unlike spherical harmonics



moments of the radiation transport equation

integrate out the angle dependence

0th moment: $$\oint [\text{RT Eq}]\, d\Omega$$

$$\boxed{\frac{dE_\nu}{dt} + \vec{\nabla} \cdot \vec{F}_\nu = -\chi_{\text{abs}} c E_\nu + 4\pi\eta}$$

escaping flux absorbed emitted

$$E_\nu = \frac{1}{c}\oint I_\nu d\Omega \qquad \vec{F}_\nu = \oint I_\nu \hat{n} d\Omega$$

radiation energy density radiation flux

expression of radiation energy conservation!

but one equation, and four unknowns (E_ν $\vec{F}_\nu$)

moments of the radiation transport equation

integrate out the angle dependence

1$^{\text{th}}$ moment: $\oint [\text{RT Eq}]\ \vec{n}\ d\Omega$

$$\boxed{\frac{1}{c}\frac{d\vec{F}_\nu}{dt} + c\vec{\nabla}\cdot\mathbf{P}_\nu = -\chi_{\text{abs}}\vec{F}_\nu}$$

$$\mathbf{P}_\nu = \oint I_\nu \hat{n}\hat{n} d\Omega$$ radiation pressure tensor

expression of radiation momentum conservation!

3 new equations, but added more unknowns in ***P***

we could just keep going…need to *close* the system

diffusion approximation

use **only** the 0th moment

$$\boxed{\frac{dE_\nu}{dt} + \vec{\nabla} \cdot \vec{F}_\nu = -\chi_{\rm abs} c E_\nu + 4\pi\eta}$$

and close with the law diffusion

$$\boxed{\vec{F}_\nu = -\frac{c}{3\chi} \vec{\nabla} E_\nu}$$

i.e., radiation “flows down the energy gradient” with a diffusion constant c/3χ

numerical solution of diffusion equation

basic case of 1D diffusion

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2}$$

diffusion equation (parabolic)

discretize the equation (implicit approach)

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \frac{a}{2(\Delta x)^2}\left((u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}) + (u_{i+1}^n - 2u_i^n + u_{i-1}^n)\right)$$

$$\begin{bmatrix} b_1 & c_1 & & & 0 \\ a_2 & b_2 & c_2 & & \\ & a_3 & b_3 & \ddots & \\ & & \ddots & \ddots & c_{n-1} \\ 0 & & & a_n & b_n \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_n \end{bmatrix}.$$

need to solve a
linear system of equations
a tridiagonal matrix

use some numerical method to solve linear equations (e.g., conjugate gradient, multi-grid)

flux-limited diffusion

One problem with the diffusion approximations that the flux can become infinitely large when the material is optically thin —> faster than light energy transport

fix it up with a fudge factor D(r)

$$\vec{F_\nu} = -D(R)\frac{c}{\chi}\vec{\nabla}E_\nu$$

where a common choice is (Levermore and Pomraning 1981)

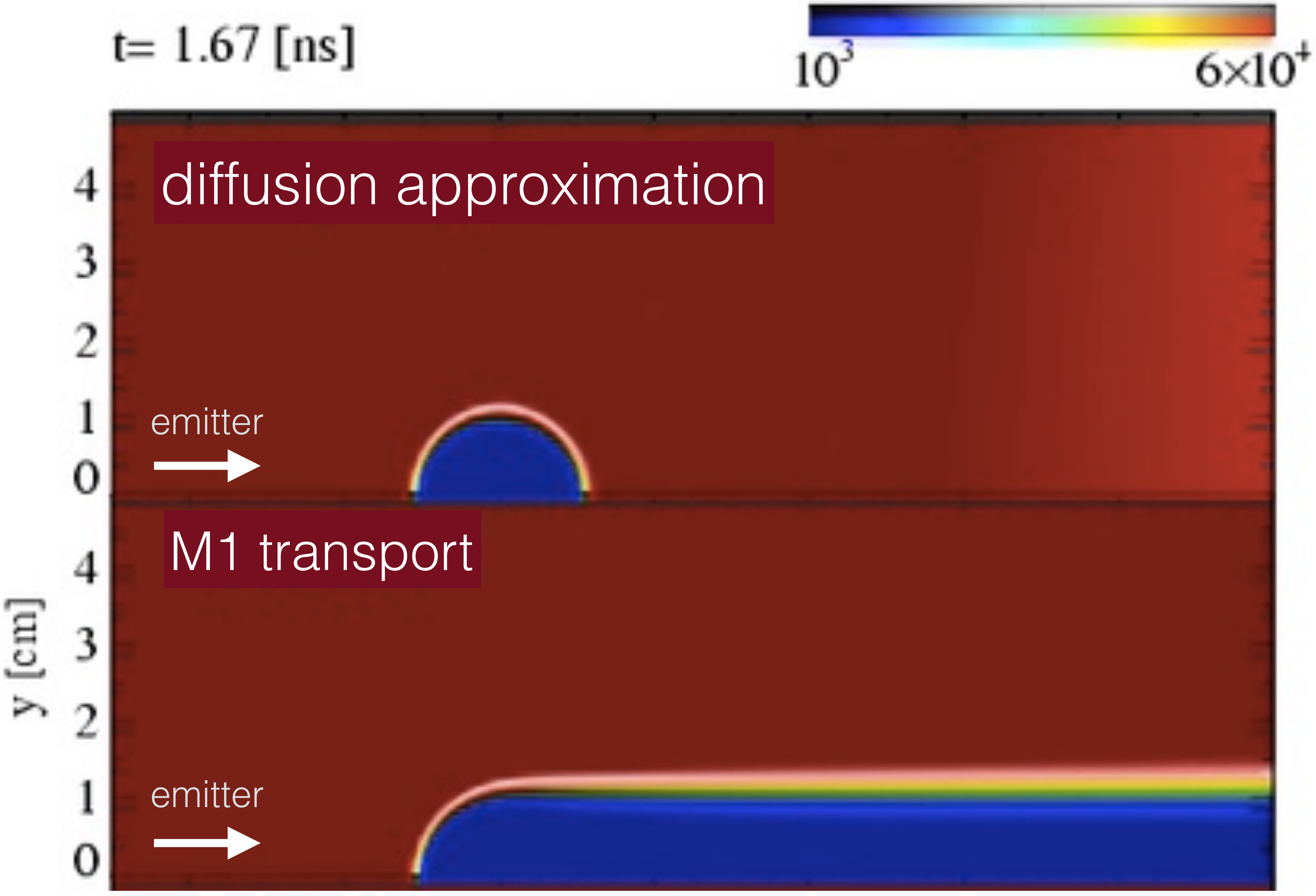
$$D(R) = \frac{2+R}{6+3R+R^2} \quad \text{where } R = \frac{\vec{\nabla}E_\nu}{\chi E}$$

now when $\chi \to 0$ $F_\nu \sim cE_\nu$ optically thin limit

$\chi \to \infty$ $F_\nu \sim \vec{\nabla}E_\nu$ optically thick limit

limitations of diffusion approximation

shadow problem



M1 transport

use **both** the 0th and 1st moment equations

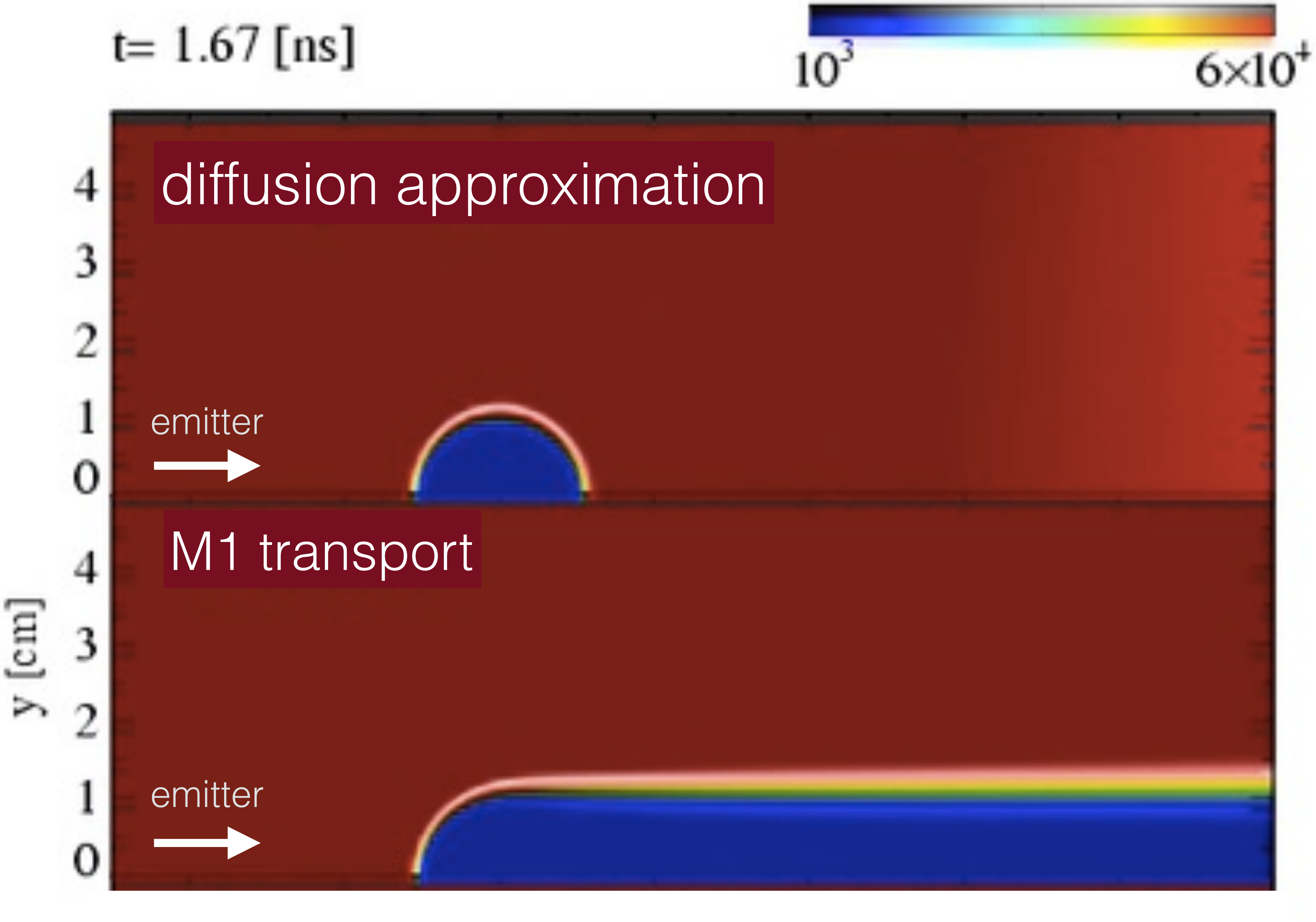
$$\boxed{\frac{dE_\nu}{dt} + \vec{\nabla} \cdot \vec{F}_\nu = -\chi_{\mathrm{abs}} c E_\nu + 4\pi\eta}$$

$$\boxed{\frac{1}{c}\frac{d\vec{F}_\nu}{dt} + c\vec{\nabla} \cdot \mathbf{P}_\nu = -\chi_{\mathrm{abs}} \vec{F}_\nu}$$

use an analytic closure relation that relates **P** to F using local info (e.g., entropy considerations) (somewhat like is done with a flux limiter)

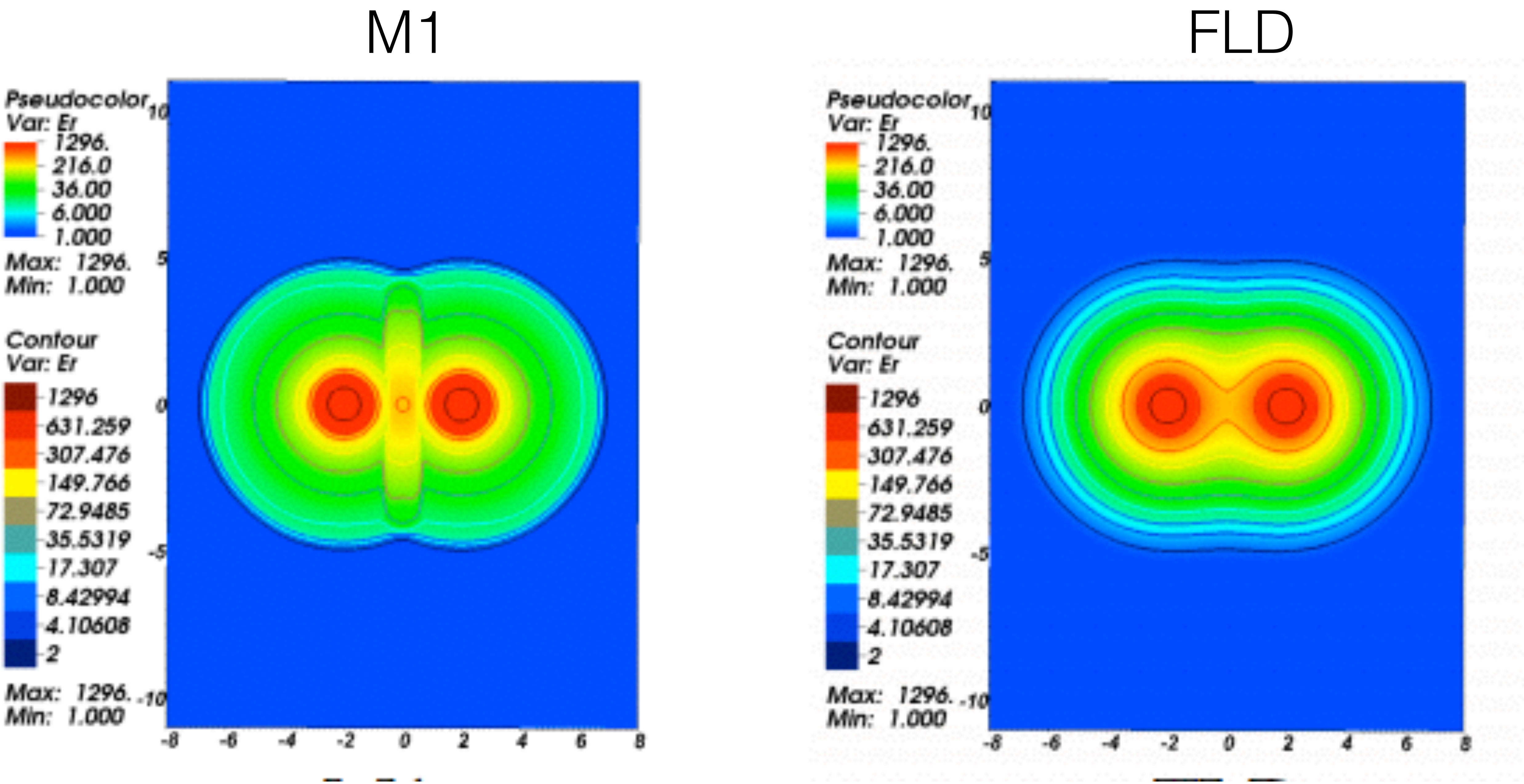
limitations of diffusion approximation

shadow problem



limitations of M1 transport

“collision” of radiation fronts



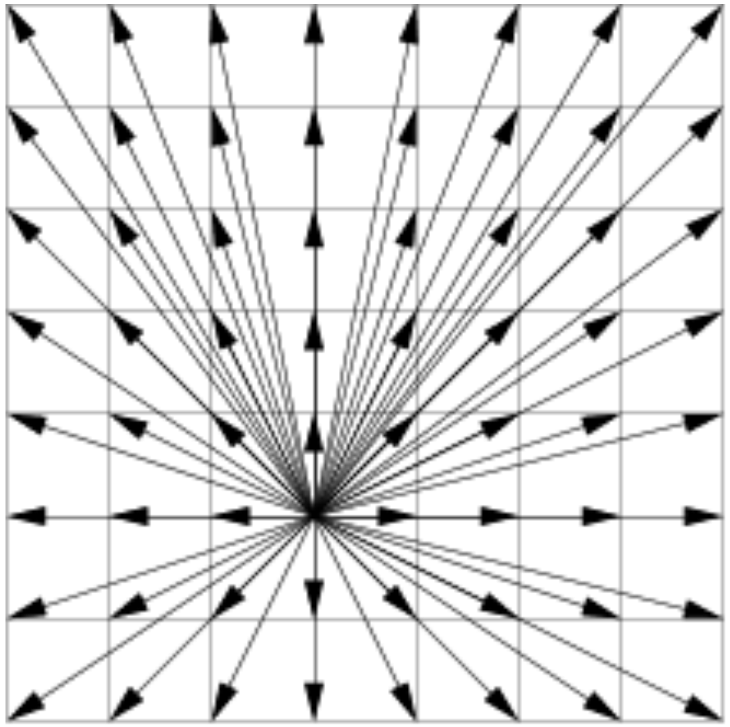
from Jim Stone’s HIPACC lecture

http://hipacc.ucsc.edu/LectureSlides/22/333/130801_1_Stone.pdf

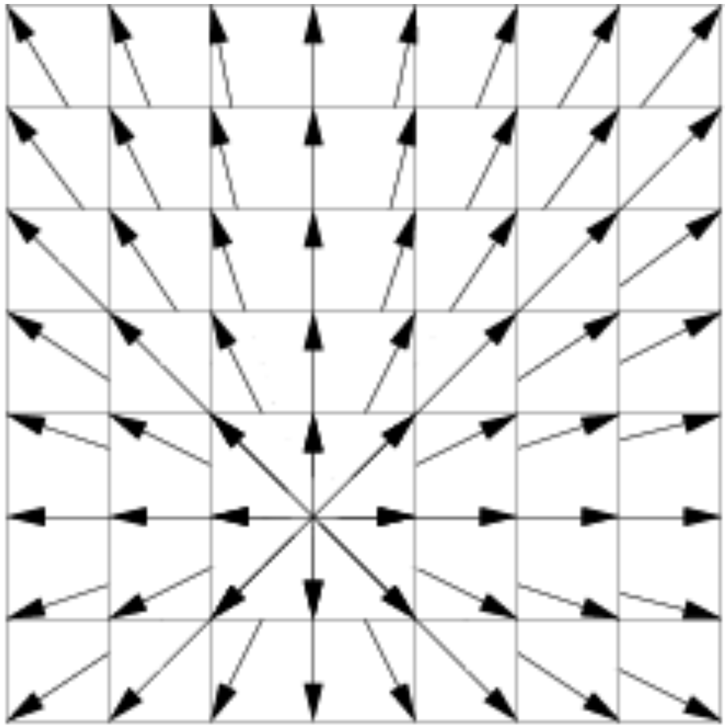
“full” (Boltzmann) transport

formal integration of the transfer equation

$$\boxed{\frac{1}{c}\frac{dI_\nu}{dt} + \frac{dI_\nu}{ds} = -\chi_{\mathrm{abs}} I_\nu + \eta + \frac{\chi_{\mathrm{sc}}}{4\pi}\oint I_\nu d\Omega'}$$



long characteristics



short characteristics

guess I, η, χ and integrate the equations, then iterate
need to apply acceleration techniques to speed convergence (e.g., accelerated lambda iteration)

variable eddington tensor (VET)

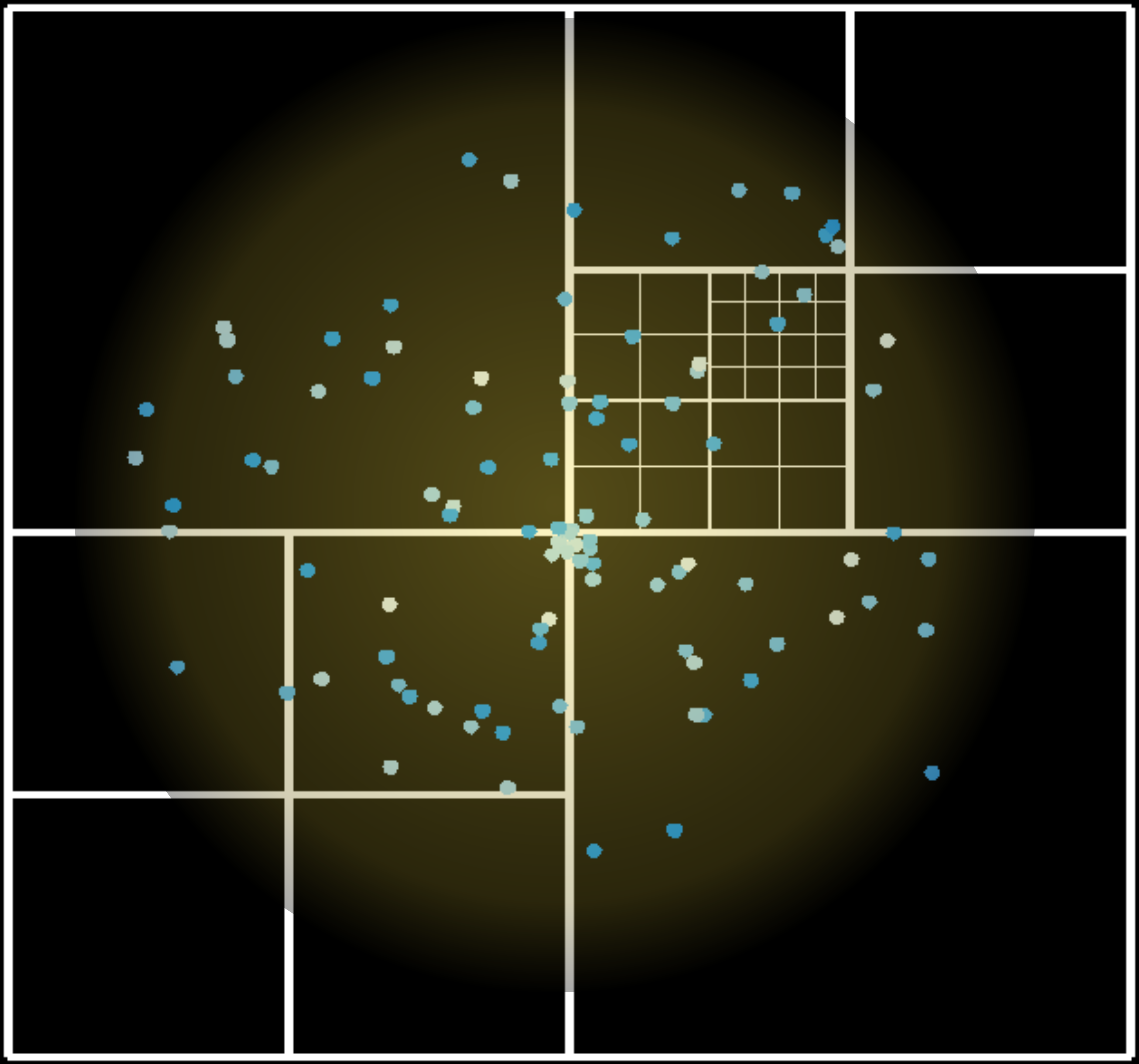
use both the 0th and 1st moment equations

$$\boxed{\frac{dE_\nu}{dt} + \vec{\nabla} \cdot \vec{F}_\nu = -\chi_{\rm abs} c E_\nu + 4\pi\eta}$$

$$\boxed{\frac{1}{c}\frac{d\vec{F}_\nu}{dt} + c\vec{\nabla} \cdot \mathbf{P}_\nu = -\chi_{\rm abs}\vec{F}_\nu}$$

close using the Eddington Tensor: $\mathbf{f} = \mathbf{P}_\nu / E_\nu$

to get **f**, solve the full Boltzmann equation,
but don’t need to do this at every time step



monte carlo transport

radiation field represented by discrete particles that randomly interact

monte carlo transport

each packet represents a number of photons with a position vector (x,y,z), a direction vector (D_x, D_y, D_z), a frequency, and a total packet energy.

probability of traveling a distance x before scattering

$$P = \exp(-\tau) = \exp(-\kappa\rho x) = \mathcal{R}$$

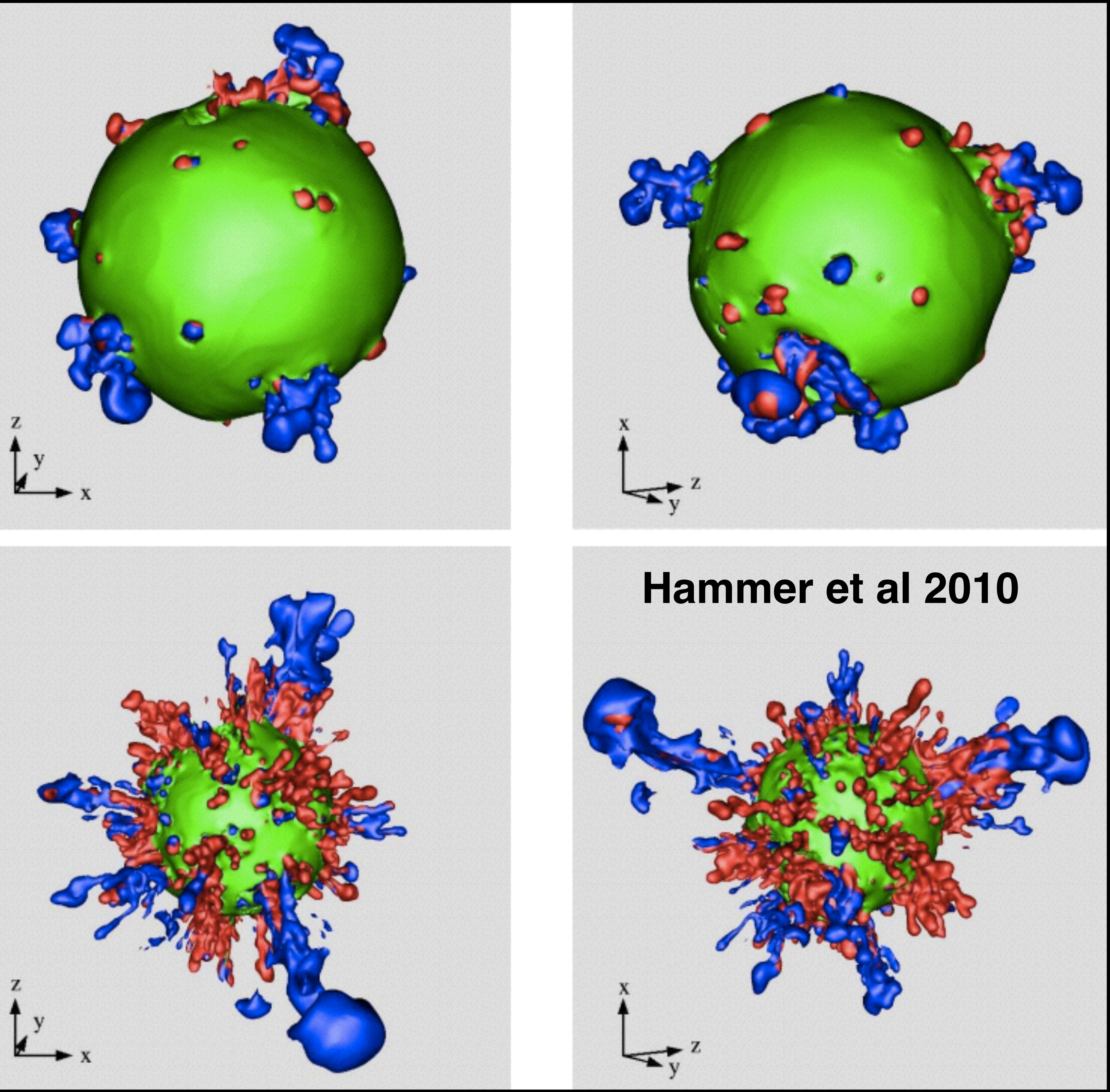
R is a random number sampled uniformly between (0, 1]

solve for x (distance traveled before scattering)

$$x = -(\kappa\rho)^{-1}\log(\mathcal{R})$$

supernova
light curves
and spectra

3D core collapse hydro simulation

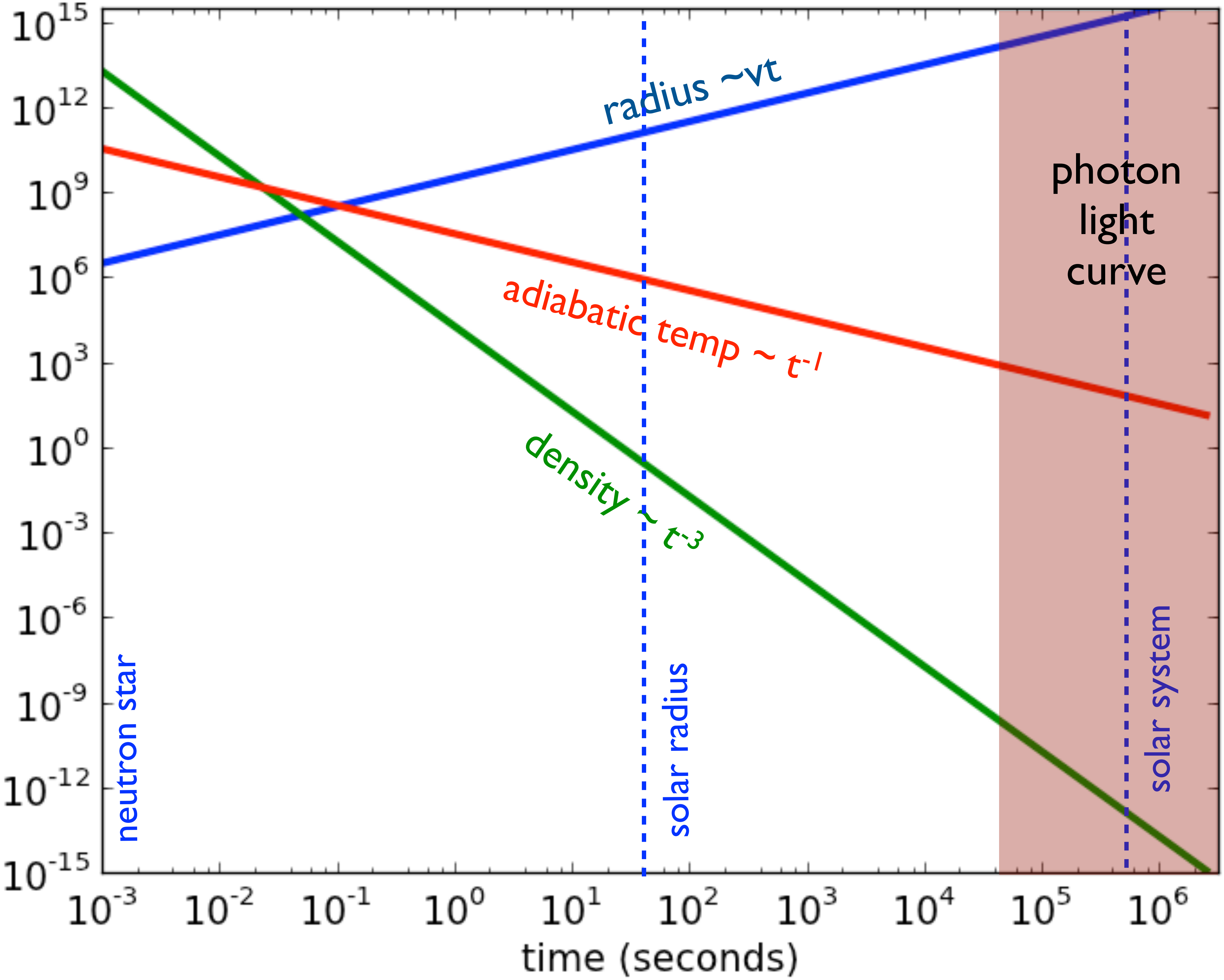


Hammer et al 2010

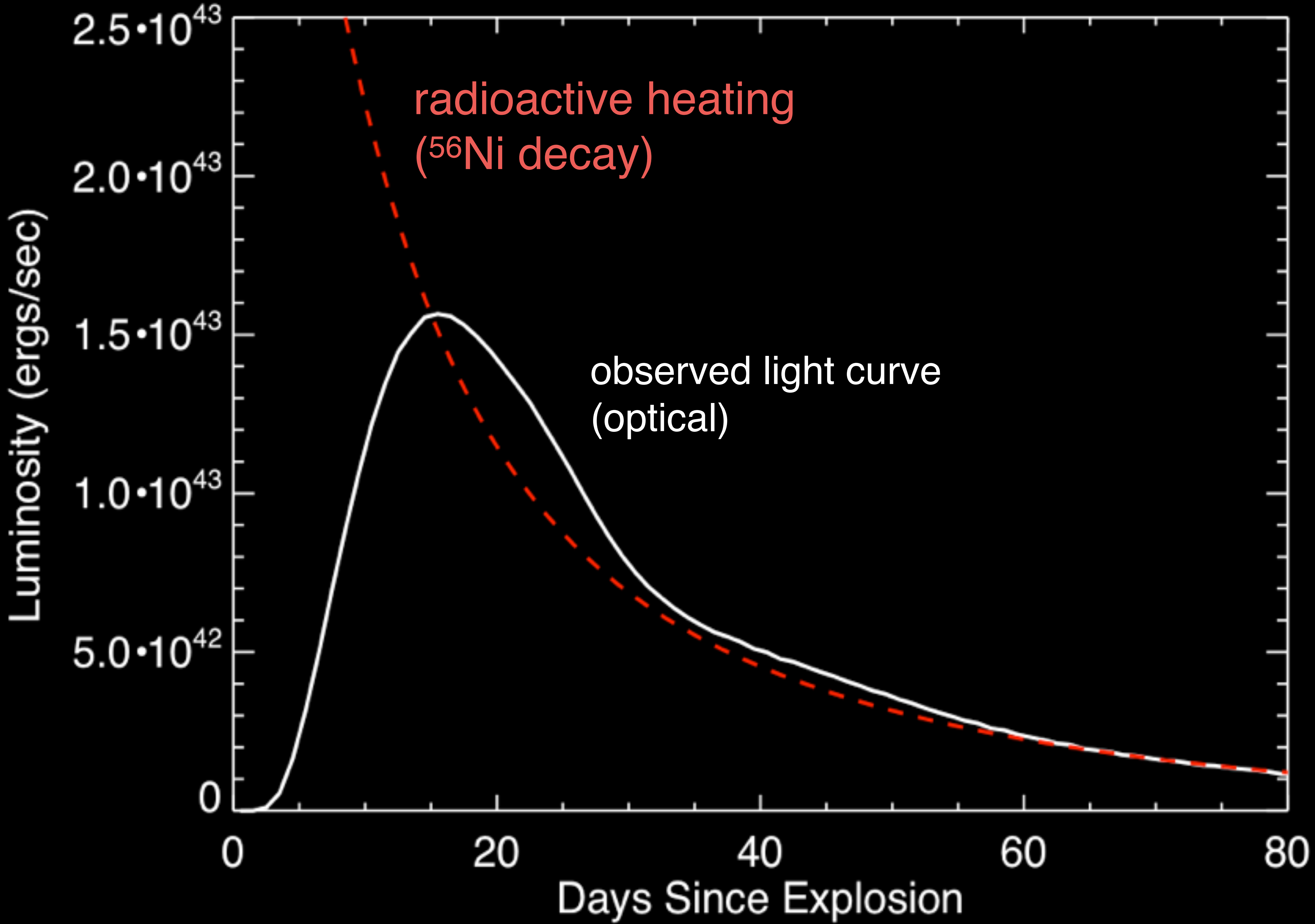
🟥 ^{16}O
🟩 ^{12}C
🟦 ^{56}Ni

SN ejecta and explosive nucleosynthesis

expansion of the ejecta



supernova light curves



what sets the light curve duration?

the *diffusion time* of photons through the optically thick remnant

$$t_\mathrm{d} = \tau \left[\frac{R}{c} \right]$$

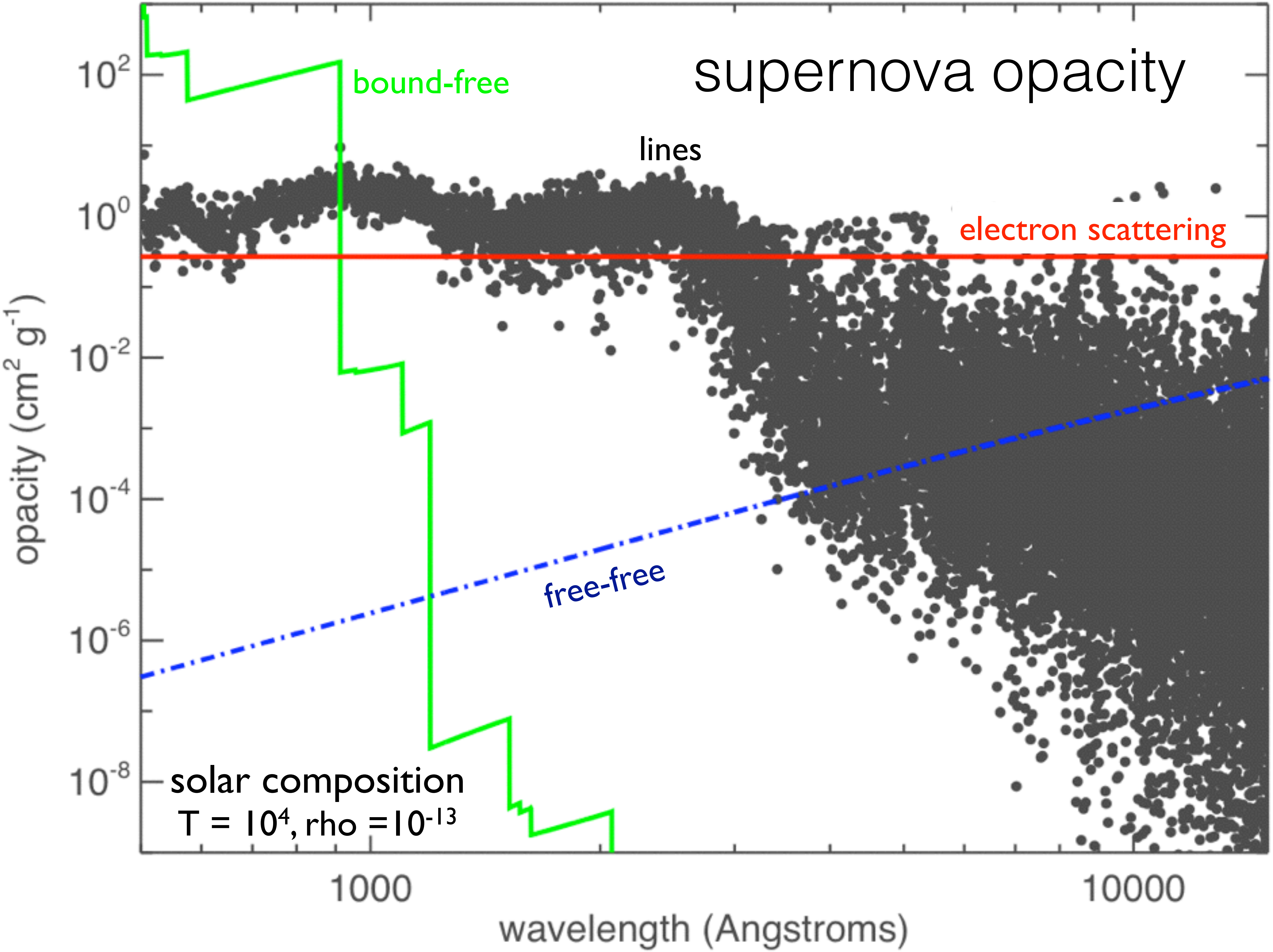
but since the remnant is expanding, R = vt

$$t_\mathrm{d} \sim \frac{M\kappa}{(vt)c}$$

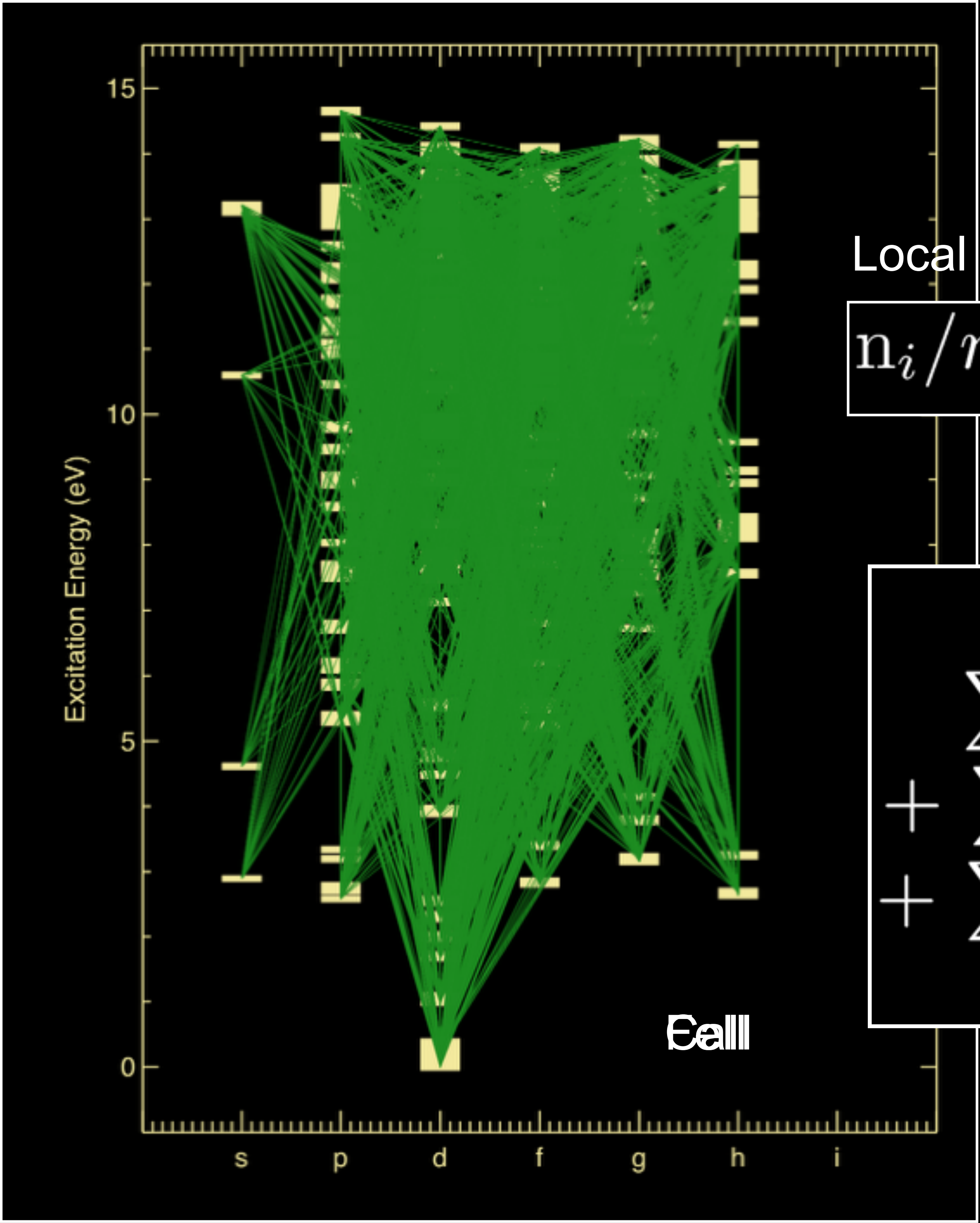
solving for time (i.e., diffusion time ~ elapsed time)

$$\boxed{t_\mathrm{d} \sim \left[\frac{M\kappa}{vc} \right]^{1/2}}$$

e.g., arnett (1979)


supernova opacity
bound-free
lines
electron scattering
free-free
solar composition
T = 10⁴, rho =10⁻¹³
opacity (cm² g⁻¹)
wavelength (Angstroms)
10²
10⁰
10⁻²
10⁻⁴
10⁻⁶
10⁻⁸
1000
10000

line interactions



~1/2 GB atomic data

Local Thermodynamic Equilibrium (LTE)

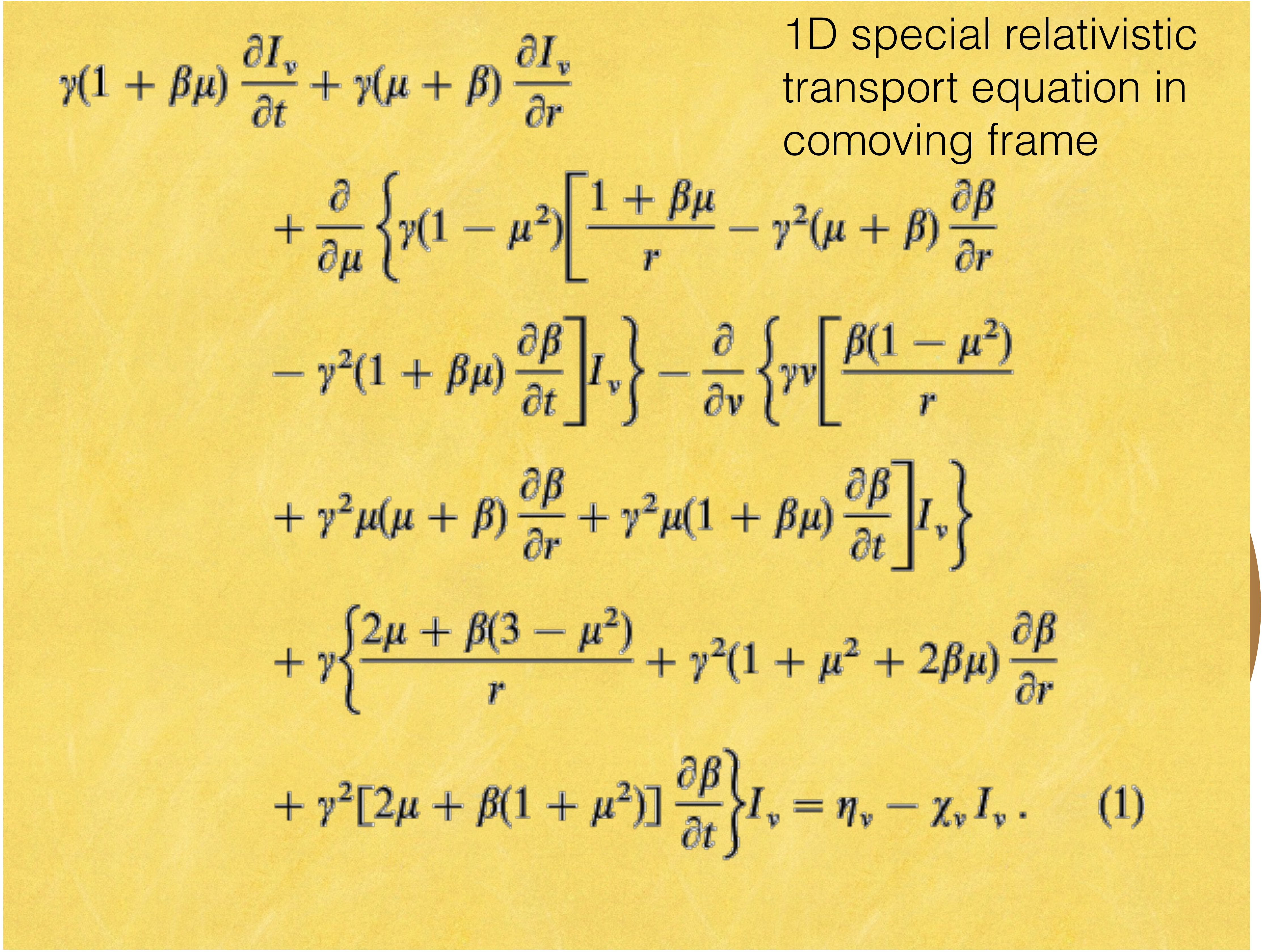
$$\boxed{\mathrm{n}_i/n_j = \frac{g_i}{g_j}\exp(-\Delta E/kT)}$$

non-equilibrium (NLTE)

$$\boxed{\begin{aligned}\partial n_i/\partial t = \\ \textstyle\sum_{j\neq i}(n_j R_{ji} - n_i R_{ij}) \\ + \textstyle\sum_{j\neq i}(n_j C_{ji} - n_i C_{ij}) \\ + \textstyle\sum_{j\neq i}(n_j G_{ji} - n_i G_{ij}) \\ = 0\end{aligned}}$$

nxn matrix, where n = number of atomic levels (sparsity depends on number of transitions included)

transport in moving media

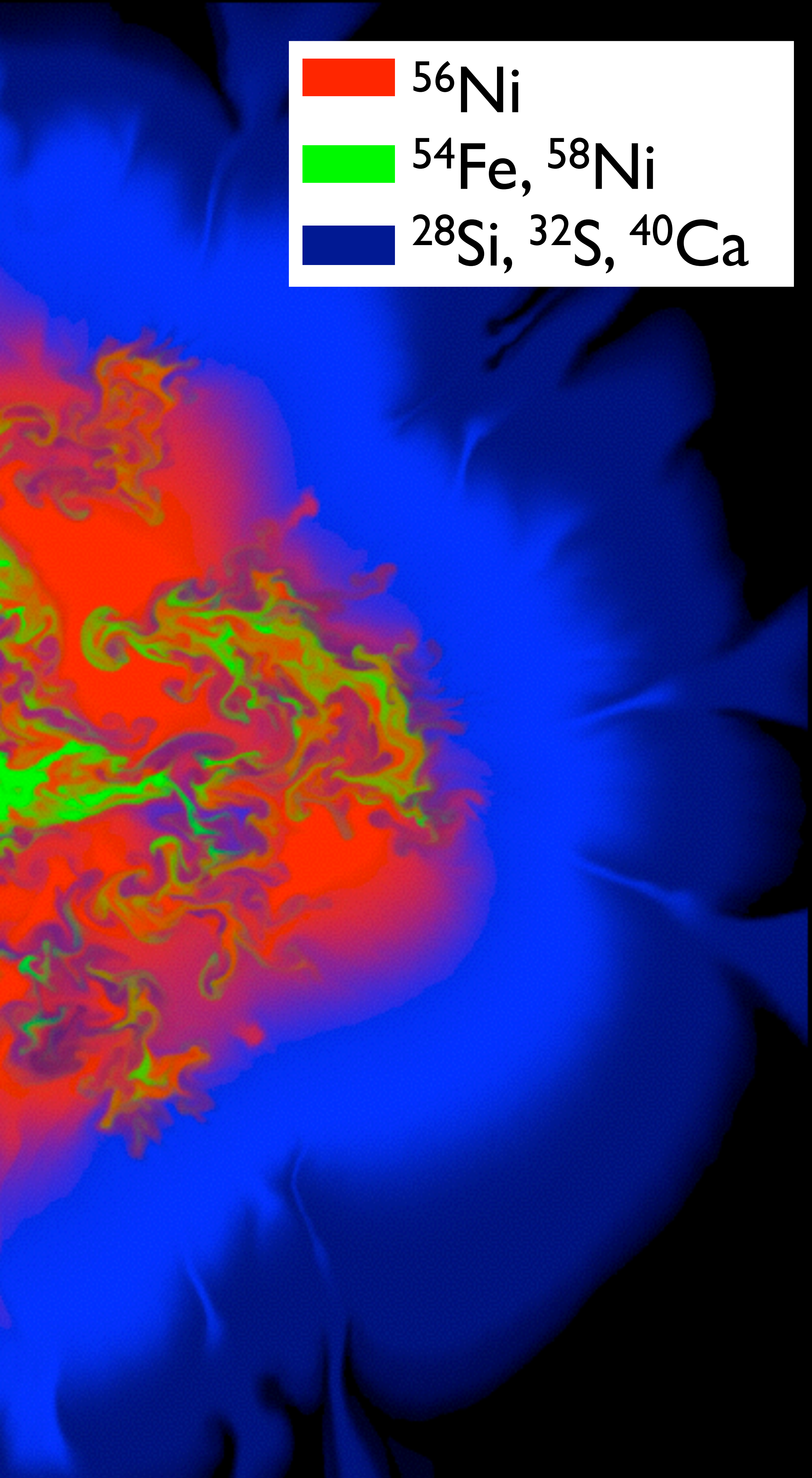


$$
\begin{aligned}
&\gamma(1+\beta\mu)\frac{\partial I_\nu}{\partial t}+\gamma(\mu+\beta)\frac{\partial I_\nu}{\partial r}\\
&\quad+\frac{\partial}{\partial\mu}\left\{\gamma(1-\mu^2)\left[\frac{1+\beta\mu}{r}-\gamma^2(\mu+\beta)\frac{\partial\beta}{\partial r}\right.\right.\\
&\quad\left.\left.-\gamma^2(1+\beta\mu)\frac{\partial\beta}{\partial t}\right]I_\nu\right\}-\frac{\partial}{\partial\nu}\left\{\gamma\nu\left[\frac{\beta(1-\mu^2)}{r}\right.\right.\\
&\quad\left.\left.+\gamma^2\mu(\mu+\beta)\frac{\partial\beta}{\partial r}+\gamma^2\mu(1+\beta\mu)\frac{\partial\beta}{\partial t}\right]I_\nu\right\}\\
&\quad+\gamma\left\{\frac{2\mu+\beta(3-\mu^2)}{r}+\gamma^2(1+\mu^2+2\beta\mu)\frac{\partial\beta}{\partial r}\right.\\
&\quad\left.+\gamma^2[2\mu+\beta(1+\mu^2)]\frac{\partial\beta}{\partial t}\right\}I_\nu=\eta_\nu-\chi_\nu I_\nu\,. \qquad (1)
\end{aligned}
$$

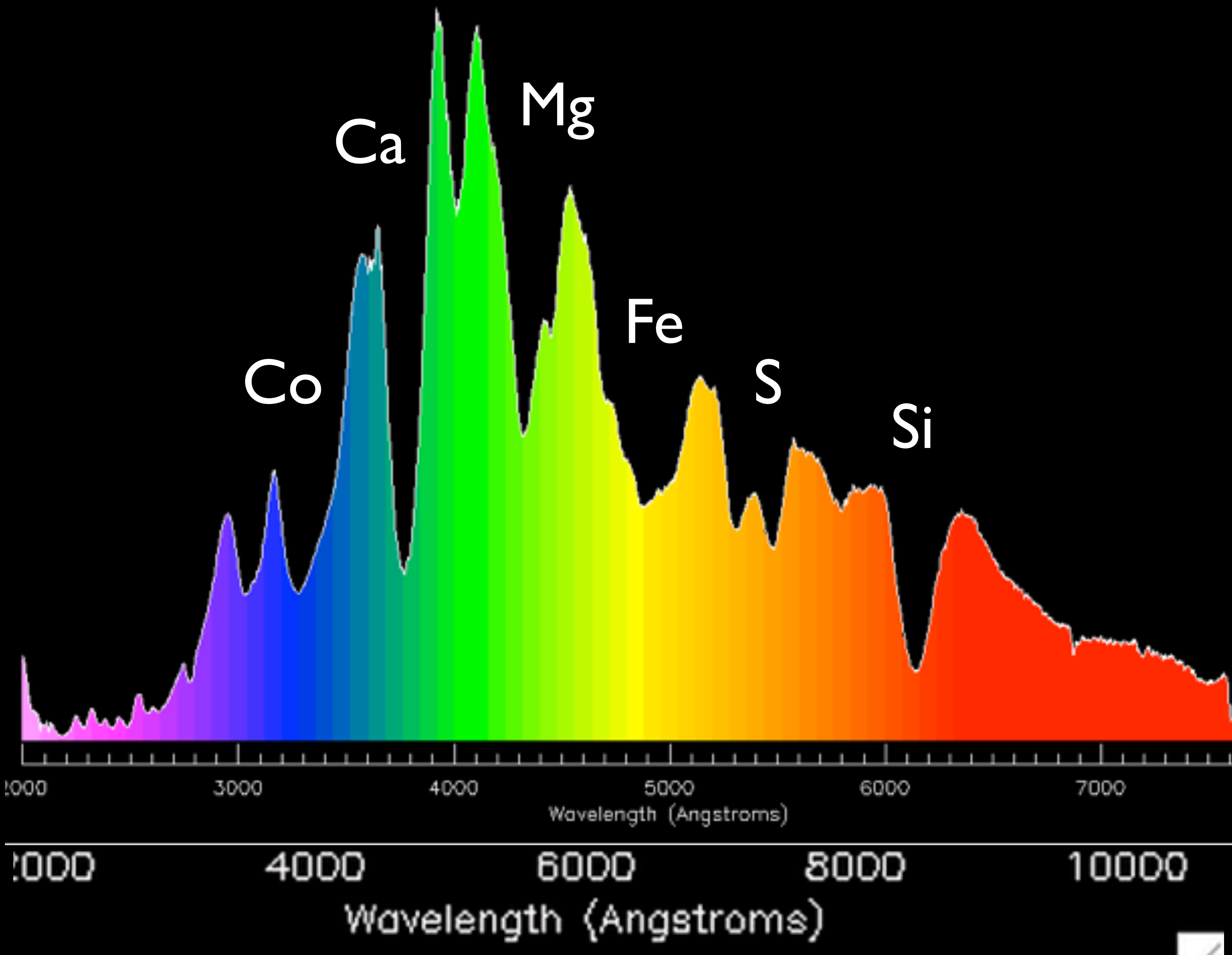
1D special relativistic transport equation in comoving frame

thermal optical spectrum

sedona radiation transport calculation



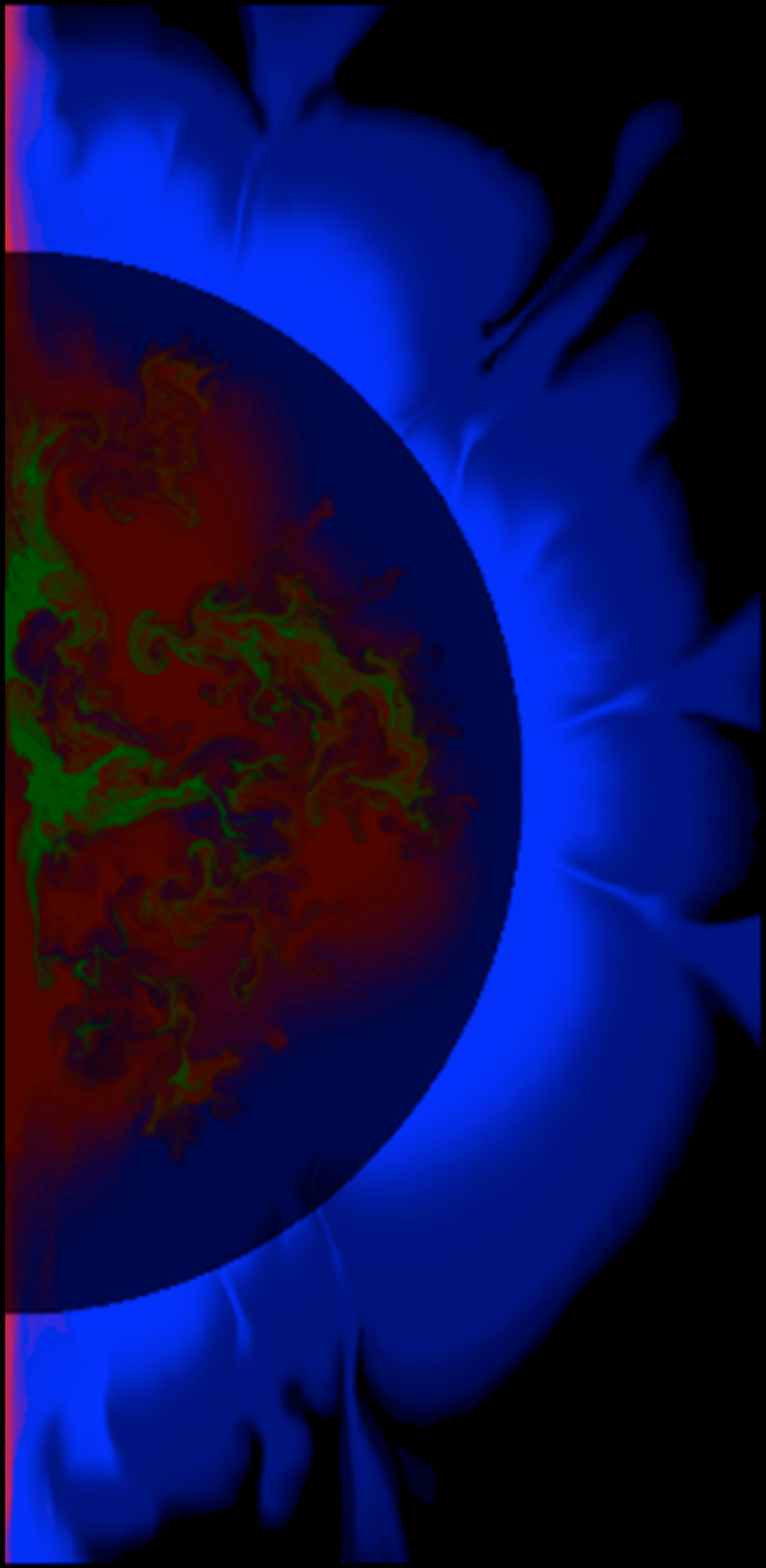
T ~ 10,000 K blackbody

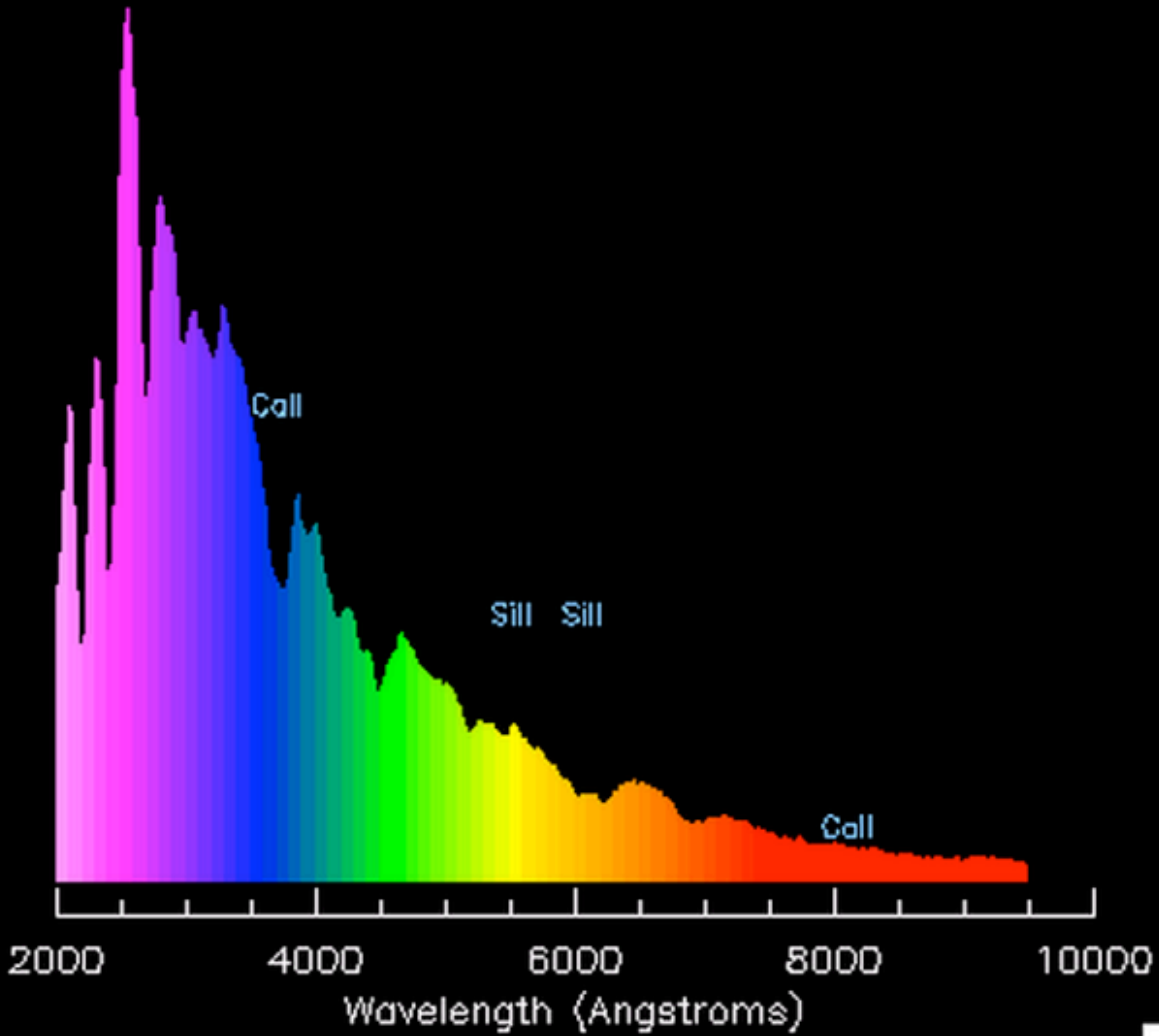


model predicted spectral evolution

t = 6.0 days

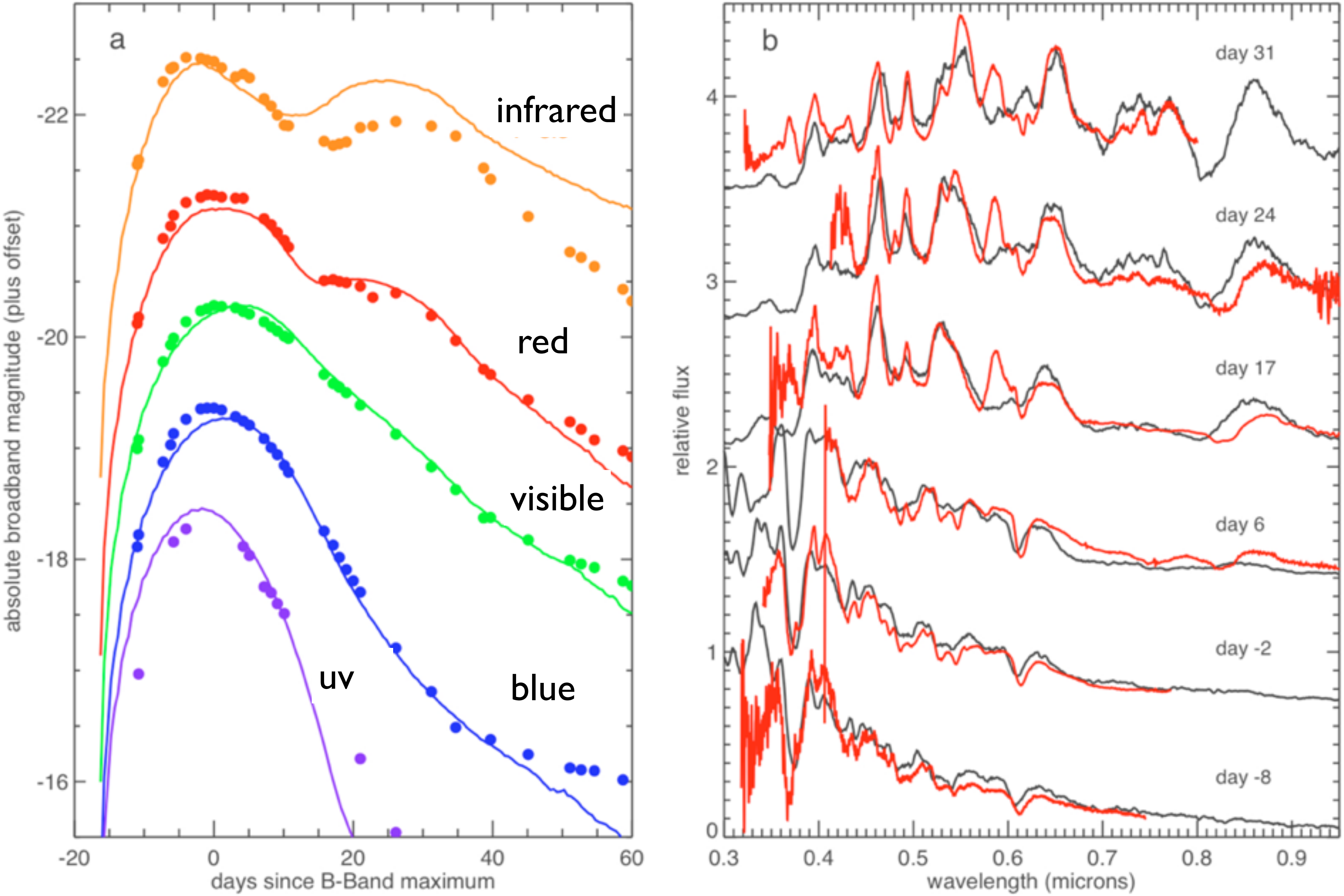
sedona radiation transport calculation





comparing models to optical observations

Type Ia supernova



tomorrow: transport in neutron star mergers