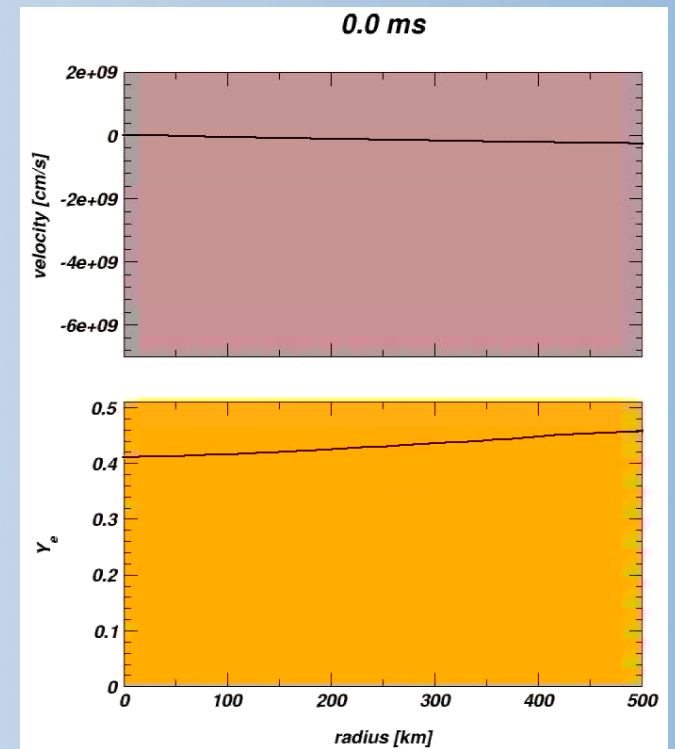
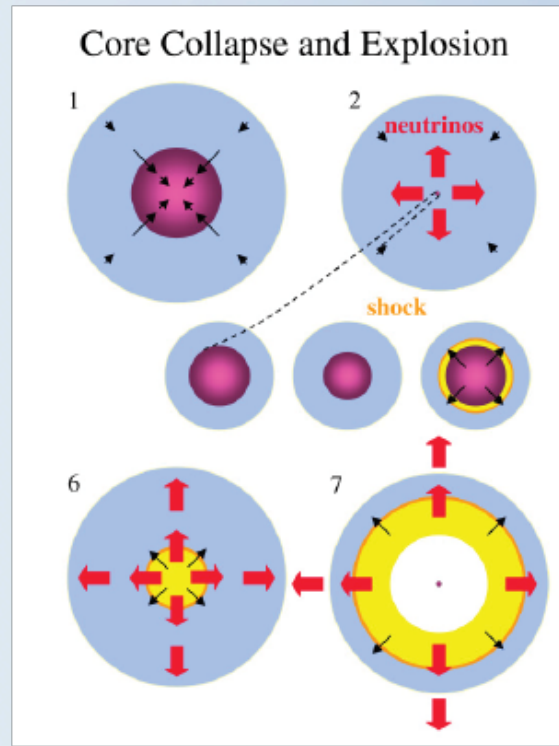
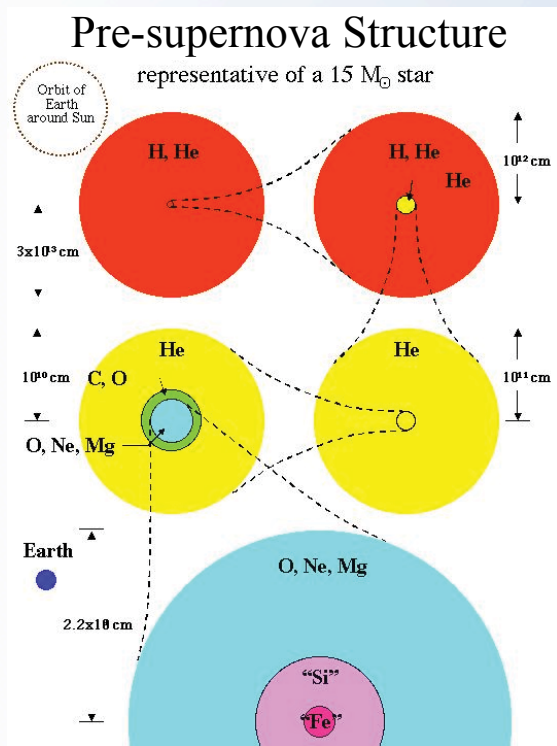


Lectures on Contemporary Core Collapse Supernova Theory

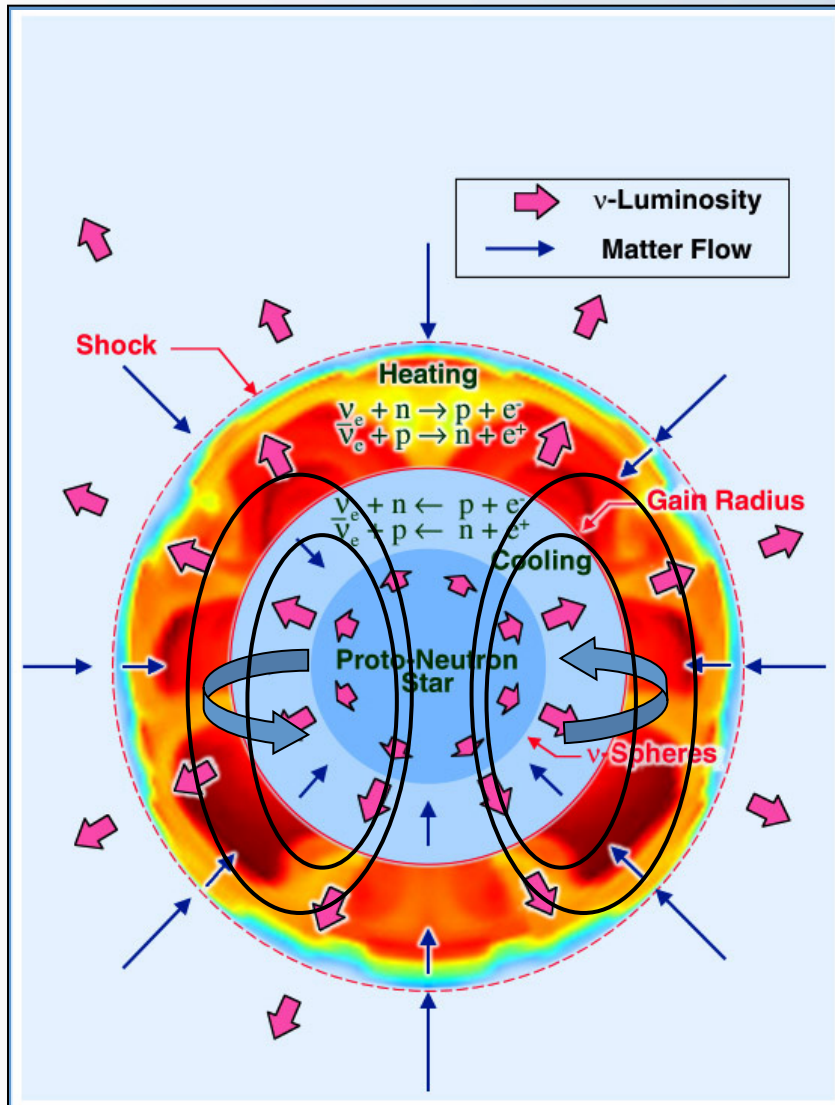
*Anthony Mezzacappa
Department of Physics and Astronomy
University of Tennessee
Joint Institute for Computational Sciences
Oak Ridge National Laboratory*

Core Collapse Supernova Paradigm and Problem Description



“Macrophysics”

How is the supernova shock wave revived?

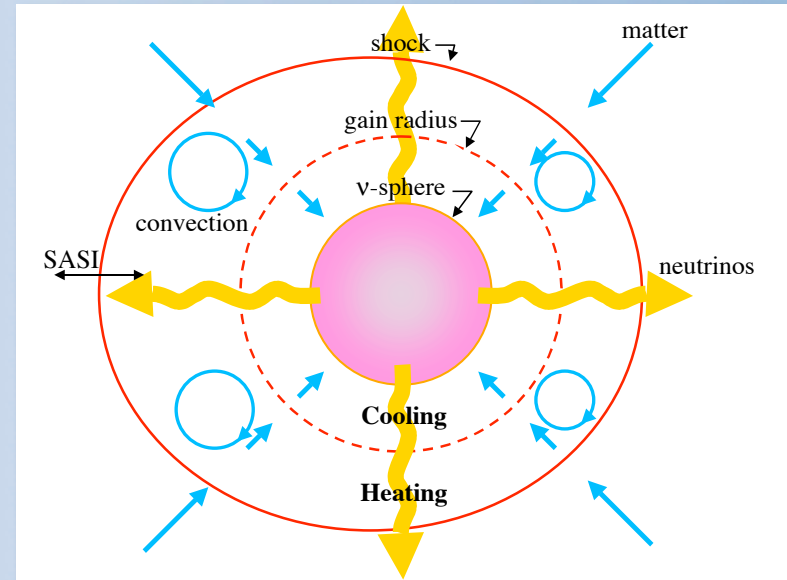
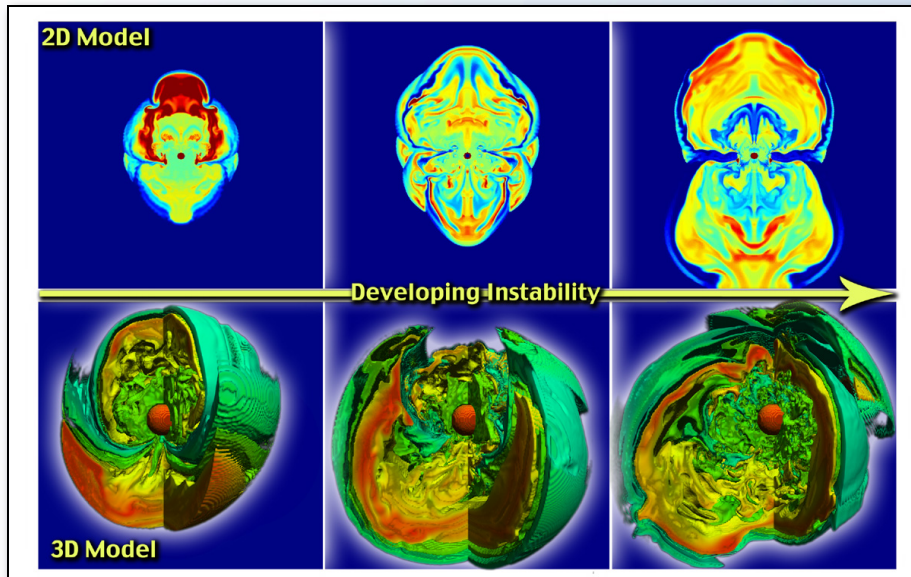


The most fundamental question in supernova theory

- Gravity
- Neutrino Heating
- Convection
- **Shock Instability**
- Nuclear Burning
- Rotation
- Magnetic Fields

**New Ingredient*

Stationary Accretion Shock Instability (SASI)



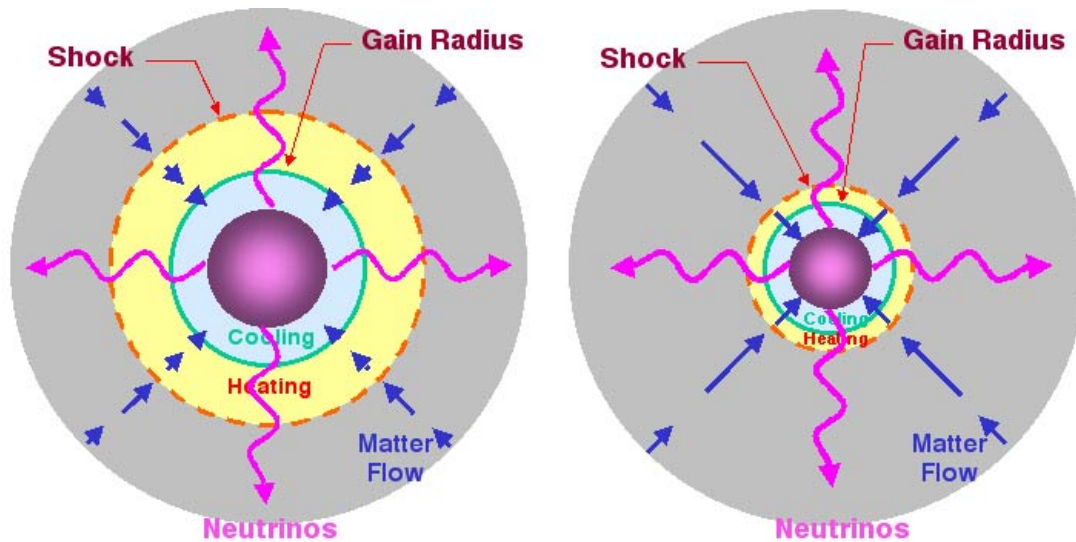
Blondin, Mezzacappa, & DeMarino, *Ap.J.* **584**, 971 (2003)

Shock wave unstable to non-radial perturbations.

SASI has *axisymmetric and nonaxisymmetric* modes that are both linearly unstable!

- Blondin and Mezzacappa, *Ap.J.* **642**, 401 (2006)
- Blondin and Shaw, *Ap.J.* **656**, 366 (2007)

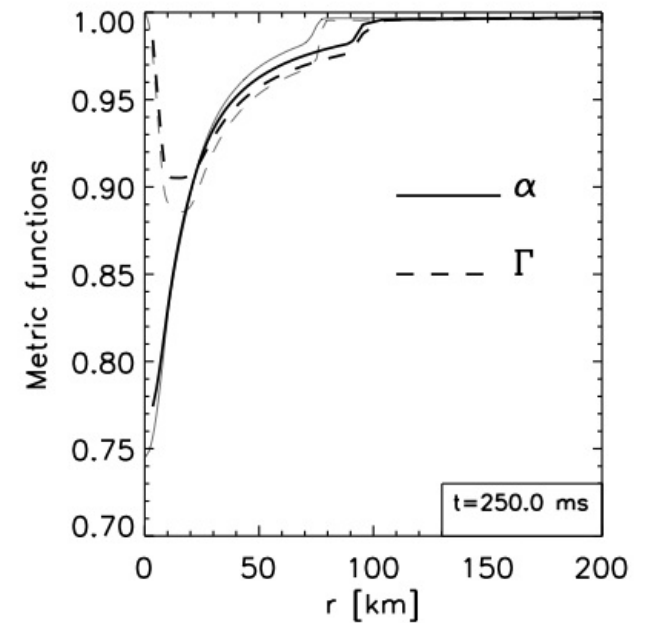
Newtonian versus GR



25 M Model

15 M Model

$$ds^2 = -\alpha^2 dt^2 + \left(\frac{r'}{\Gamma}\right)^2 da^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$



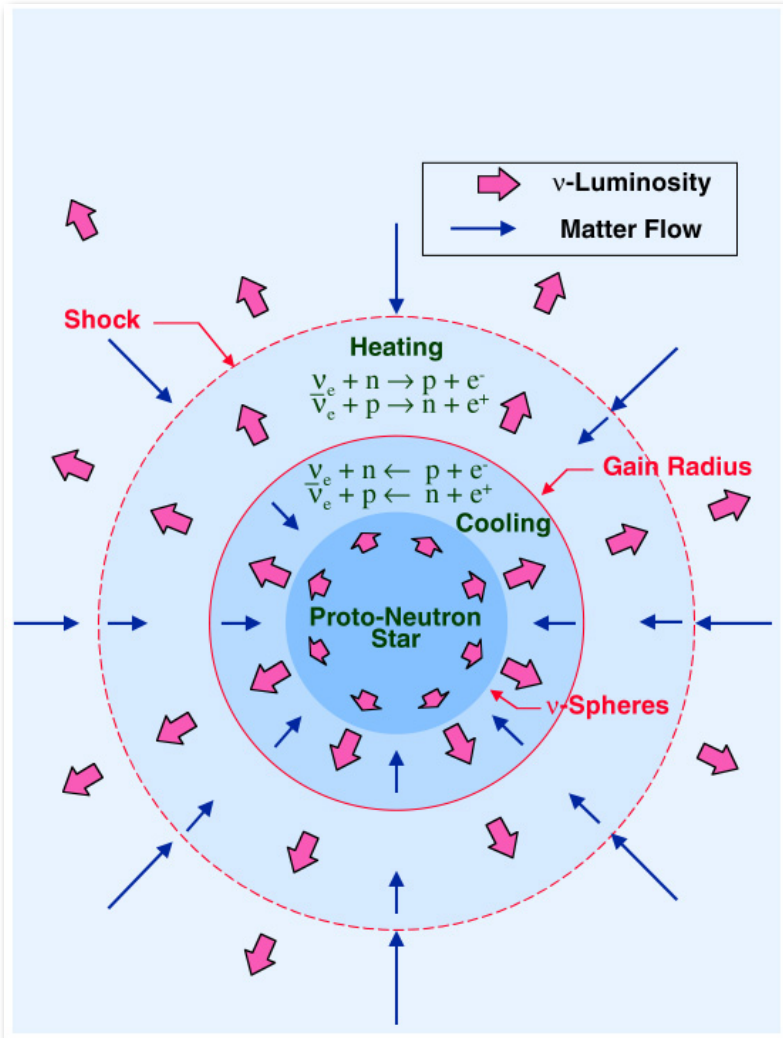
Bruenn, DeNisco, and Mezzacappa, *Ap.J.* **560**, 326 (2001)
 Liebendoerfer et al. *Ap.J.* **620**, 840 (2005)

SELECT MILESTONES IN CORE COLLAPSE SUPERNOVA THEORY

Publication	Milestone
Colgate and White (1966) Ap.J. 143 626	First to suggest core collapse supernovae are neutrino driven.
Wilson (1985), in Numerical Astrophysics, eds. Centrella, LeBlanc, and Bowers Bethe and Wilson (1985) Ap.J. 295 14	Discovery of delayed-shock mechanism. Framed contemporary core collapse supernova theory.
Herant, Benz, and Colgate (1992) Ap.J. 395 642 Herant et al. (1994) Ap.J. 435 339	First 2D models. Revolutionized core collapse supernova theory.
Blondin, Mezzacappa, and DeMarino (2003) Ap.J. 584 971	SASI discovery. Missing link.
Kitaura et al. (2006) A&A, 450 345 Buras et al. (2006) A&A 457 281 Marek and Janka (2009) Ap.J. 694 664 Suwa et al. (2010) PASJ 62 L49 Takiwaki, Kotake, and Suwa (2012) Ap.J. 749 98 Mueller, Janka, and Marek (2012) Ap.J. 756 84 Bruenn et al. (2013) Ap.J. 767 L6	Neutrino-driven explosions, most aided by convection and the SASI obtained across range of progenitors.



The Heart of the Matter



Neutrino heating depends on neutrino luminosities, spectra, and angular distributions.

$$\dot{\epsilon} = \frac{X_n}{\lambda_0^2} \frac{L_{\nu_e}}{4\pi r^2} \langle E_{\nu_e}^2 \rangle \langle \frac{1}{\mathcal{F}} \rangle + \frac{X_p}{\lambda_0^2} \frac{L_{\bar{\nu}_e}}{4\pi r^2} \langle E_{\bar{\nu}_e}^2 \rangle \langle \frac{1}{\bar{\mathcal{F}}} \rangle$$

⇒ Must compute neutrino distribution functions.

$$f(t, r, \theta, \phi, E, \theta_p, \phi_p)$$

Multifrequency
Multiangle

$$E_R(t, r, \theta, \phi, E) = \int d\theta_p d\phi_p f$$

$$F_R^i(t, r, \theta, \phi, E) = \int d\theta_p d\phi_p n^i f$$

Multifrequency
(solve for
lowest-order
multifrequency
angular moments:
energy and momentum
density/frequency)

Requires a closure prescription:

- MGFLD
- MGVEF/MGVET

Multigroup Flux-Limited Diffusion

The Boltzmann equation contains the same information as an infinite hierarchy of equations for the angular “moments” of the neutrino distribution function:

$$\int d\mu \left[\frac{\partial f}{\partial t} = L[f] \right] \Rightarrow \frac{\partial \psi^0}{\partial t} = \dots$$

$$\psi^0 \equiv \frac{1}{2} \int d\mu f$$

$$\int d\mu \mu \left[\frac{\partial f}{\partial t} = L[f] \right] \Rightarrow \frac{\partial \psi^1}{\partial t} = \dots$$

$$\psi^1 \equiv \frac{1}{2} \int d\mu \mu f$$

...

...

Approximation:

- Truncate hierarchy at the level of the “zerth” moment (neutrino energy density per frequency).
- **Closure:** Relate the first moment (momentum density per frequency) to the energy density per frequency so as to satisfy known limits:

$$\psi^1 = -\Lambda \left(\frac{\partial \psi^0}{\partial r} \right) \quad \Lambda \equiv \frac{1}{\frac{3}{\lambda} - \frac{1}{\psi^0} \frac{\partial \psi^0}{\partial r}}$$

Diffusion Limit: Fick's Law

$$\psi^1 \rightarrow -\frac{\lambda}{3} \frac{\partial \psi^0}{\partial r}$$

Free Streaming Limit

$$\psi^1 \rightarrow \psi^0$$

Multigroup Variable Eddington Factor/Tensor Method

The Boltzmann equation contains the same information as an infinite hierarchy of equations for the angular “moments” of the neutrino distribution function:

$$\int d\mu \left[\frac{\partial f}{\partial t} = L[f] \right] \Rightarrow \frac{\partial \psi^0}{\partial t} = \dots$$
$$\int d\mu \mu \left[\frac{\partial f}{\partial t} = L[f] \right] \Rightarrow \frac{\partial \psi^1}{\partial t} = \dots$$
$$\dots$$
$$\psi^0 \equiv \frac{1}{2} \int d\mu f$$
$$\psi^1 \equiv \frac{1}{2} \int d\mu \mu f$$
$$\dots$$

Approximation:

- Truncate hierarchy at level of “first” moment (neutrino momentum density per frequency).
- **Closure:** Relate the second and higher moments to the zeroth moment using “Eddington factors,” which are the ratio of these higher moments to the zeroth moment.

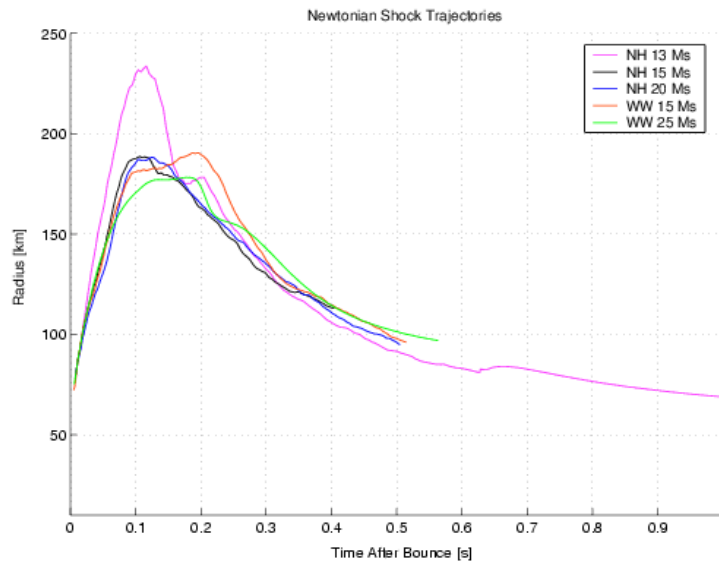
Eddington factors can be computed at different levels of approximation:

- “Prescribed” (analytic) closure (e.g., Maximum Entropy Closure in 1D).
- Computed closure (e.g., Maximum Entropy Closure in 2D/3D).
- Approximate Boltzmann solution.
- Exact Boltzmann solution.

1D

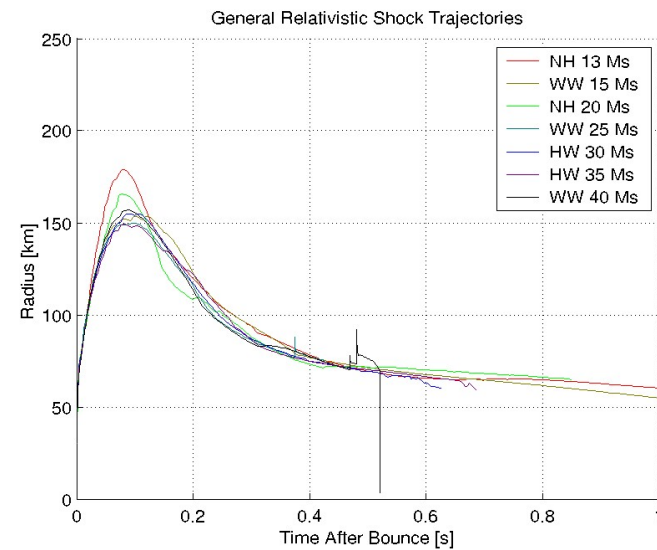
Completed: Spherical Models with Boltzmann Transport

Newtonian



Mezzacappa et al., PRL, **86**, 1935 (2001)

General Relativistic



Liebendoerfer et al., PRD, **63**, 103004 (2001)

The simulation of core collapse supernovae with fully general relativistic, multi-angle, multi-frequency, Boltzmann neutrino transport has been achieved for spherically symmetric cases.

⇒ What's missing?

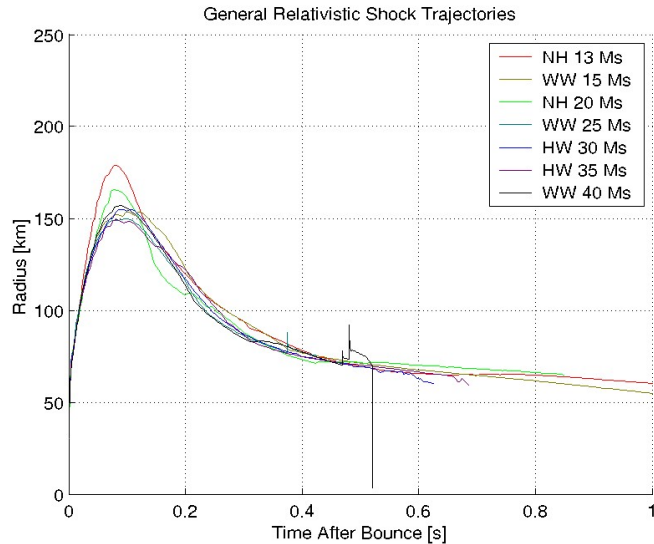
- Multi-D Effects
- ?

Agile-BOLTZTRAN

See also Lentz et al. 2012. *Ap.J.* **747**, 73.

2D

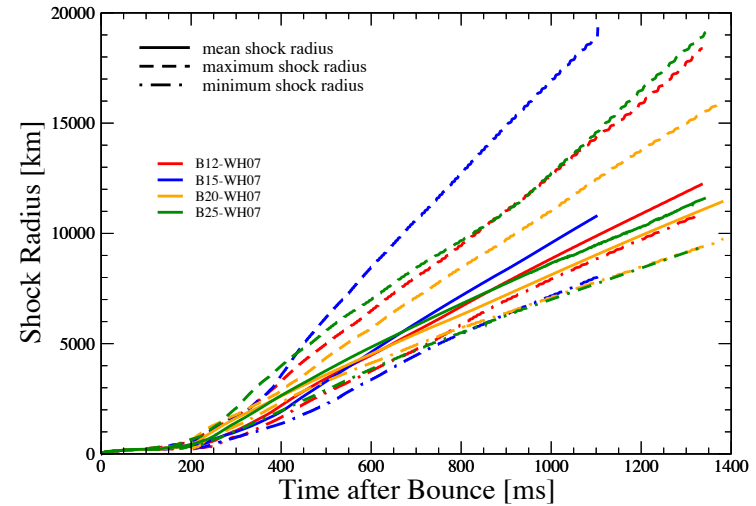
What a Difference a Dimension Makes



Liebendoerfer et al., PRD, **63**, 103004 (2001)

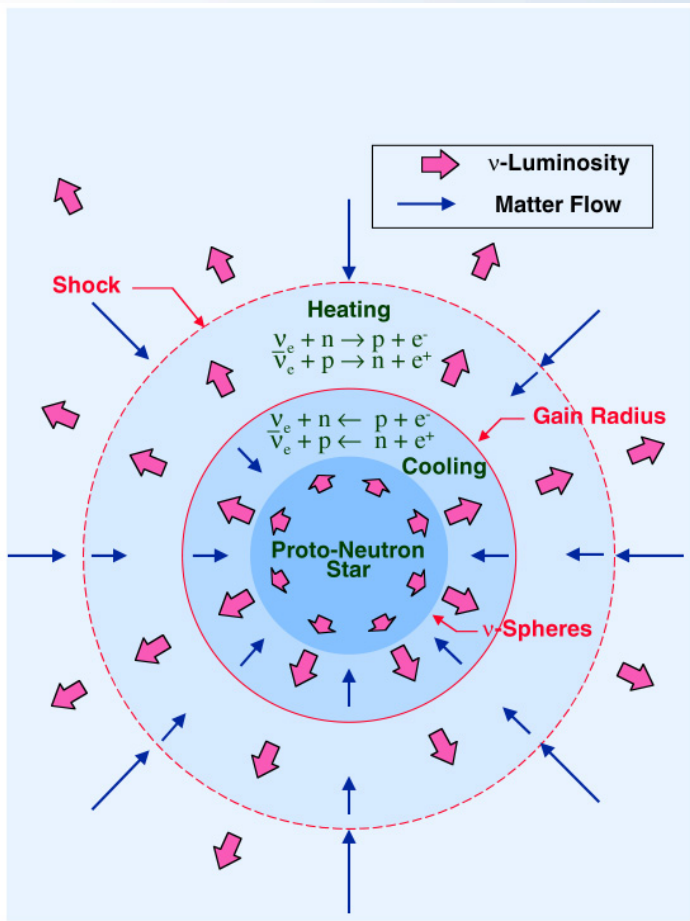
See also Lentz et al. 2012. *Ap.J.* **747**, 73.

Agile-BOLTZTRAN



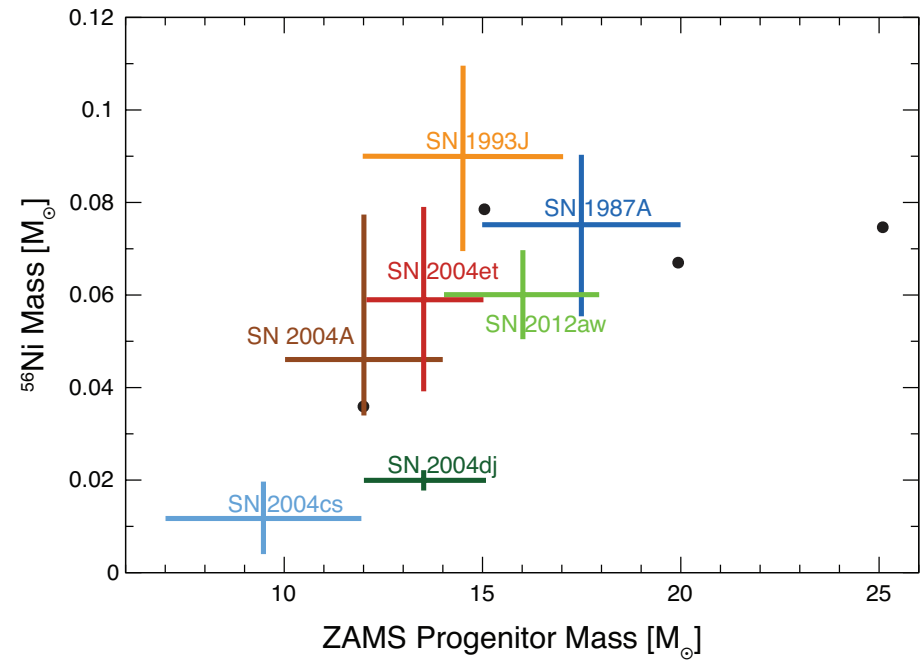
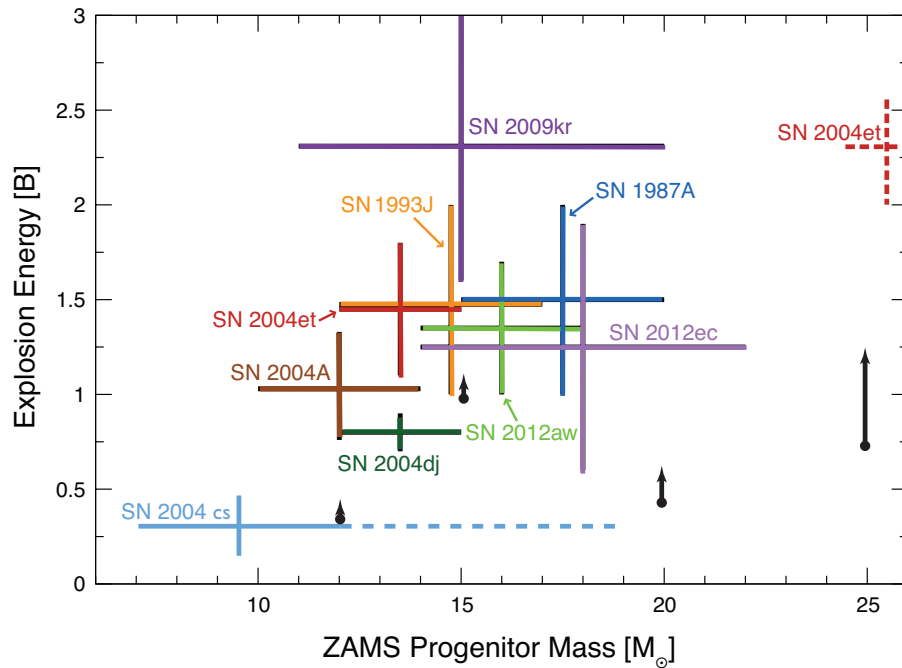
Bruenn et al. 2013. *Ap.J.* **767**, L6.
Bruenn et al. 2014. In preparation.

Complicating the Picture

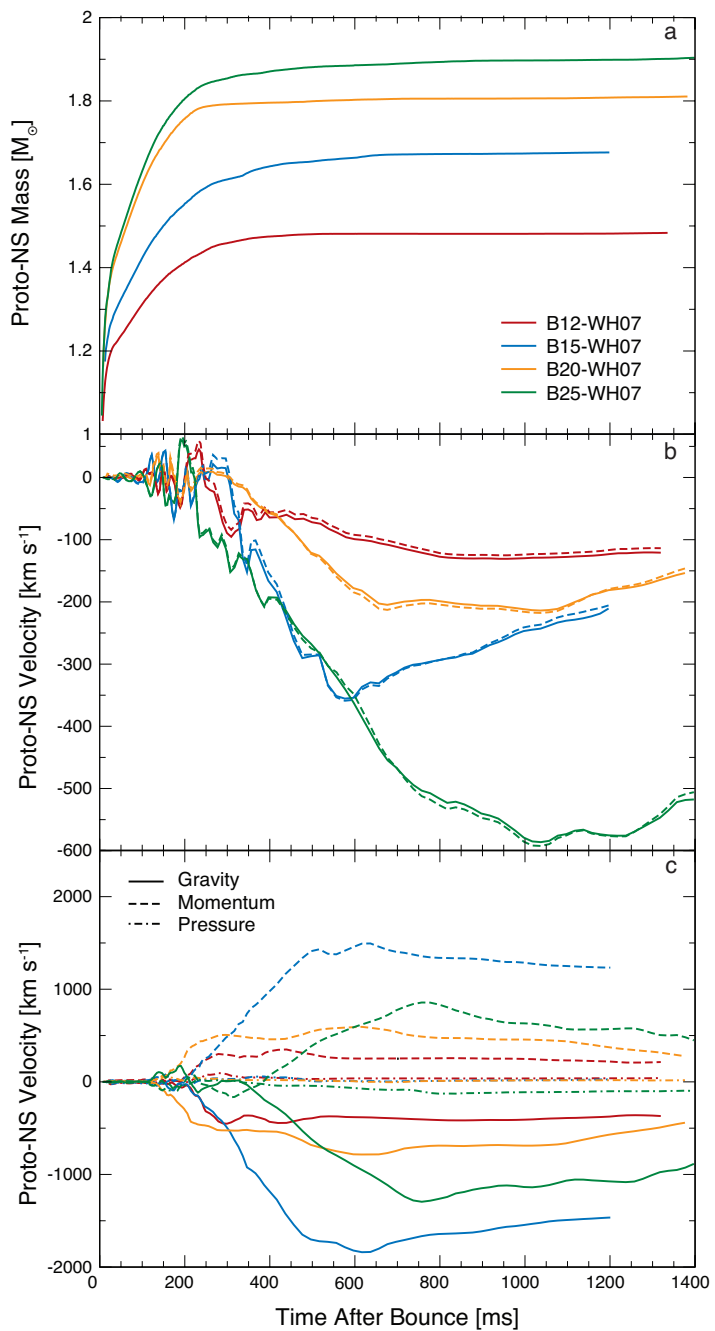




Comparison with Observations



Bruenn et al. 2014. In preparation.



7/23/14

Bruenn et al. 2014. In preparation.

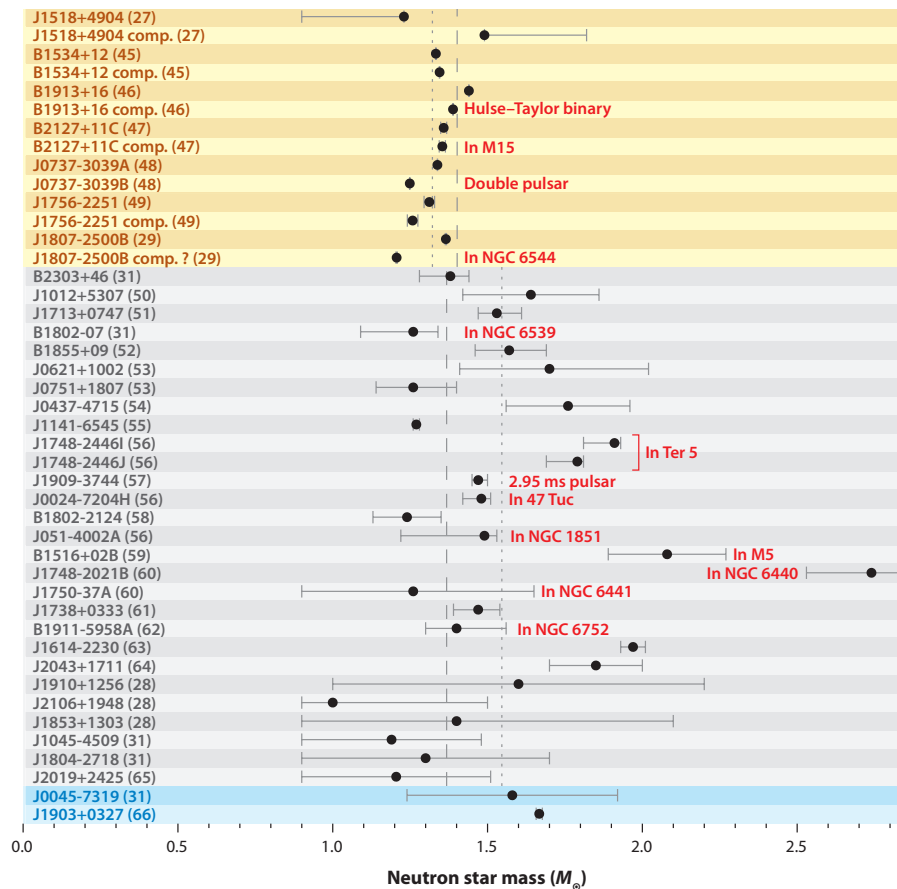


Figure 7

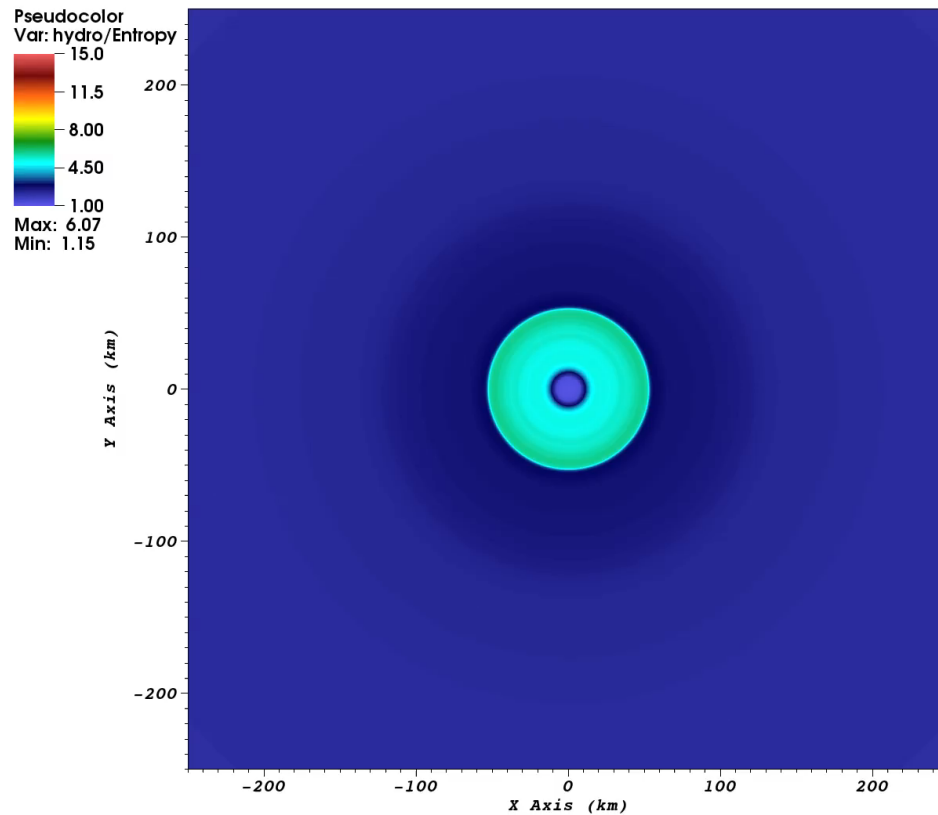
Measured neutron star masses with $1-\sigma$ errors. References in parentheses following source names are identifiers.

3D

15 M
LS (220)

3D Counterpart Models

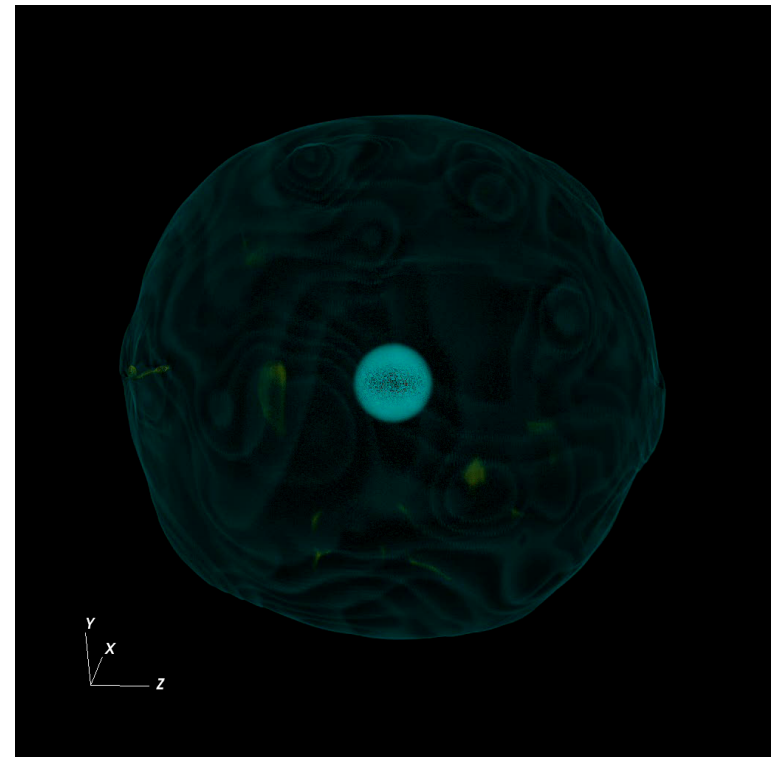
DB: C15-3D-1-000050000.silo
Cycle: 50000 Time: 4.92608



Simulation Stats

- 64,800 cores
- 35 weeks/postbounce second
- 100 M processor-hours/postbounce second

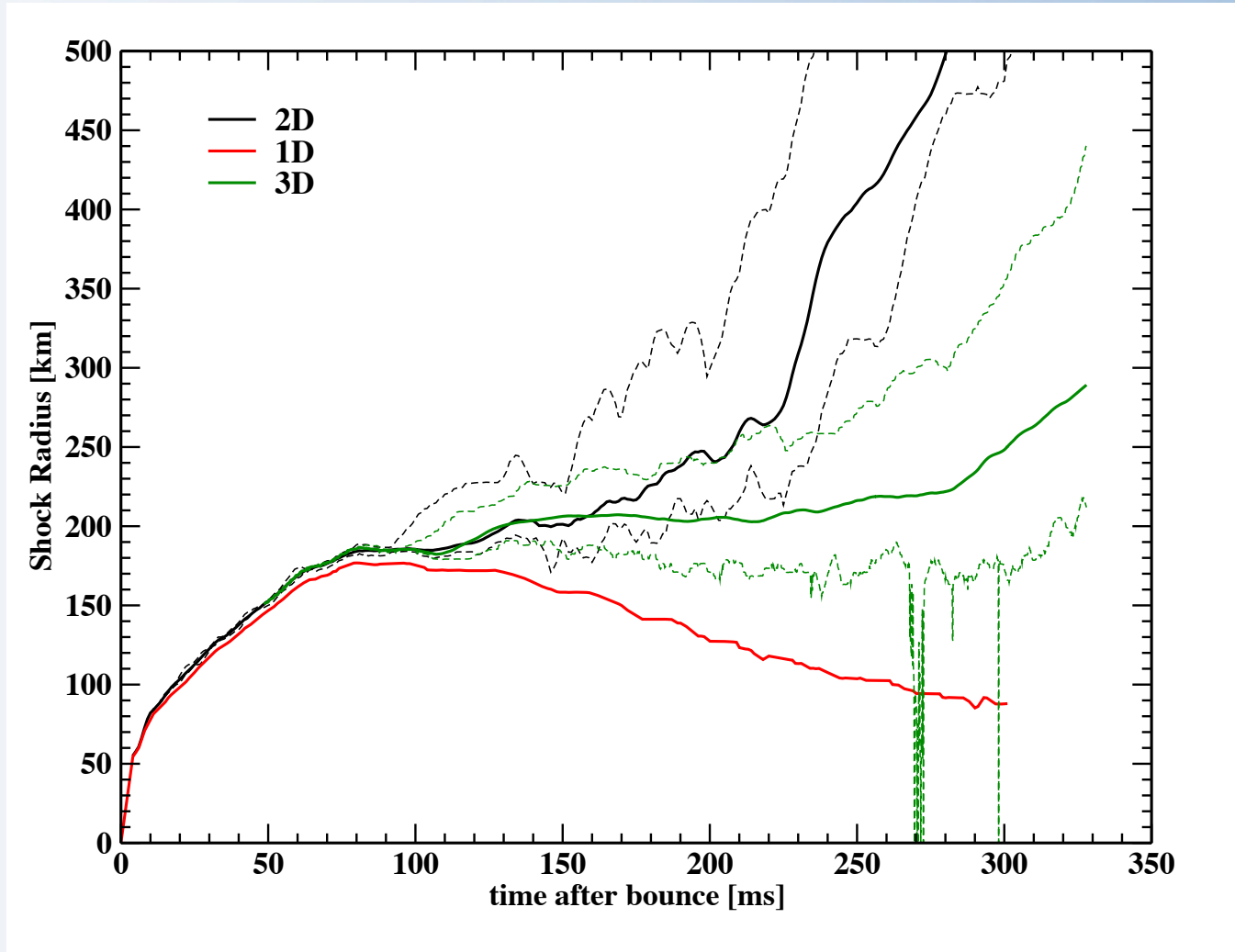
7/23/14



Lentz et al. 2014. In preparation.

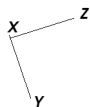
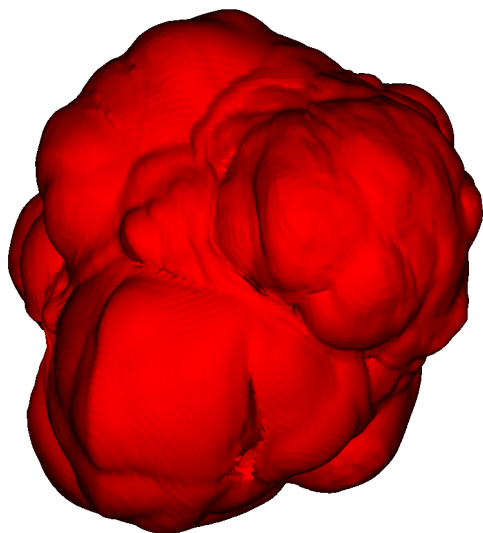
20

1D vs. 2D vs. 3D

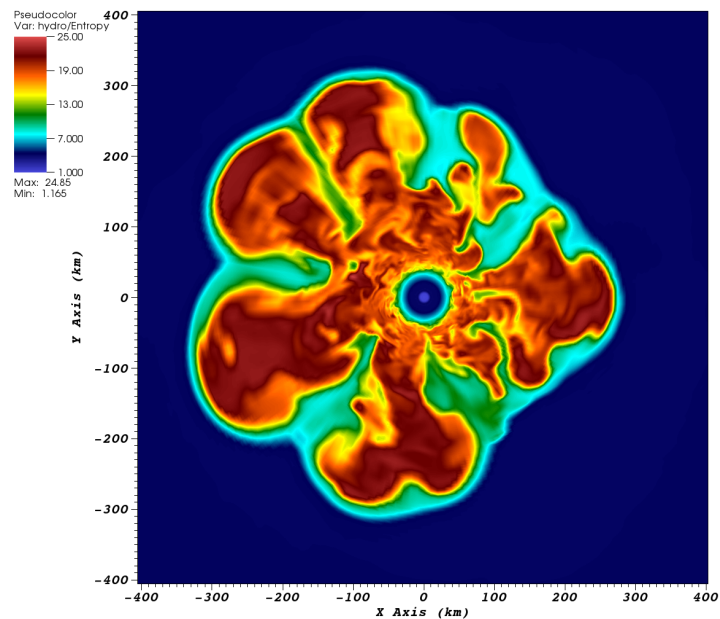


DB: C15-3D-1-002686000.silo
Cycle: 2686000 Time:329.681

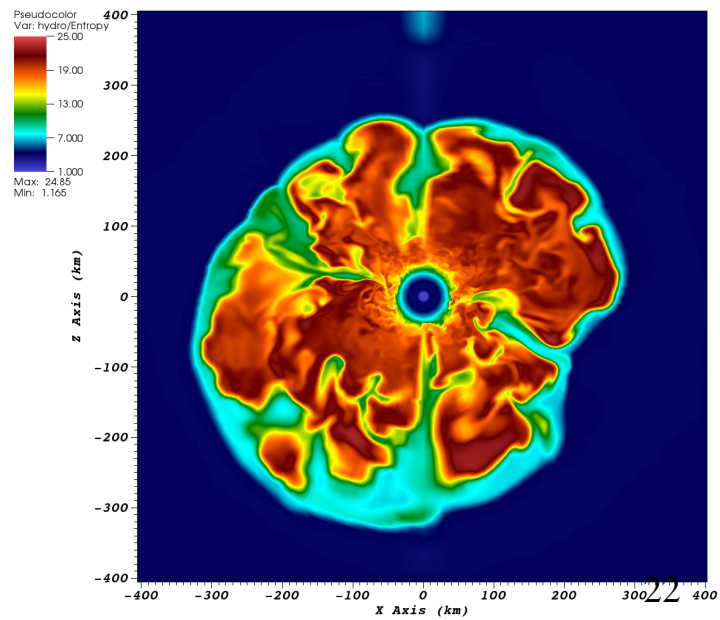
Contour
Var: hydro/Entropy
7
Max: 24.85
Min: 1.165

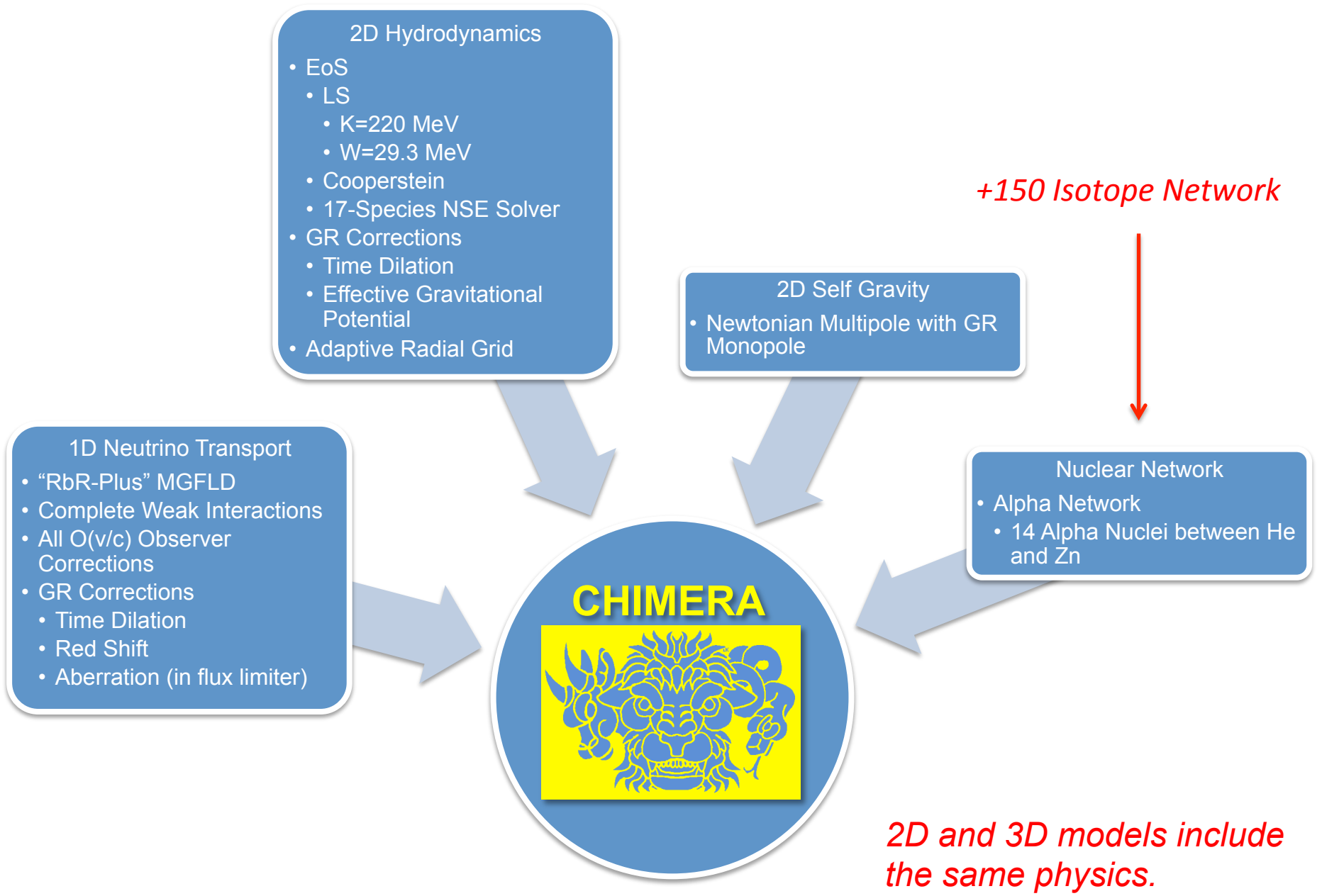


DB: C15-3D-1-002686000.silo
Cycle: 2686000 Time:329.681

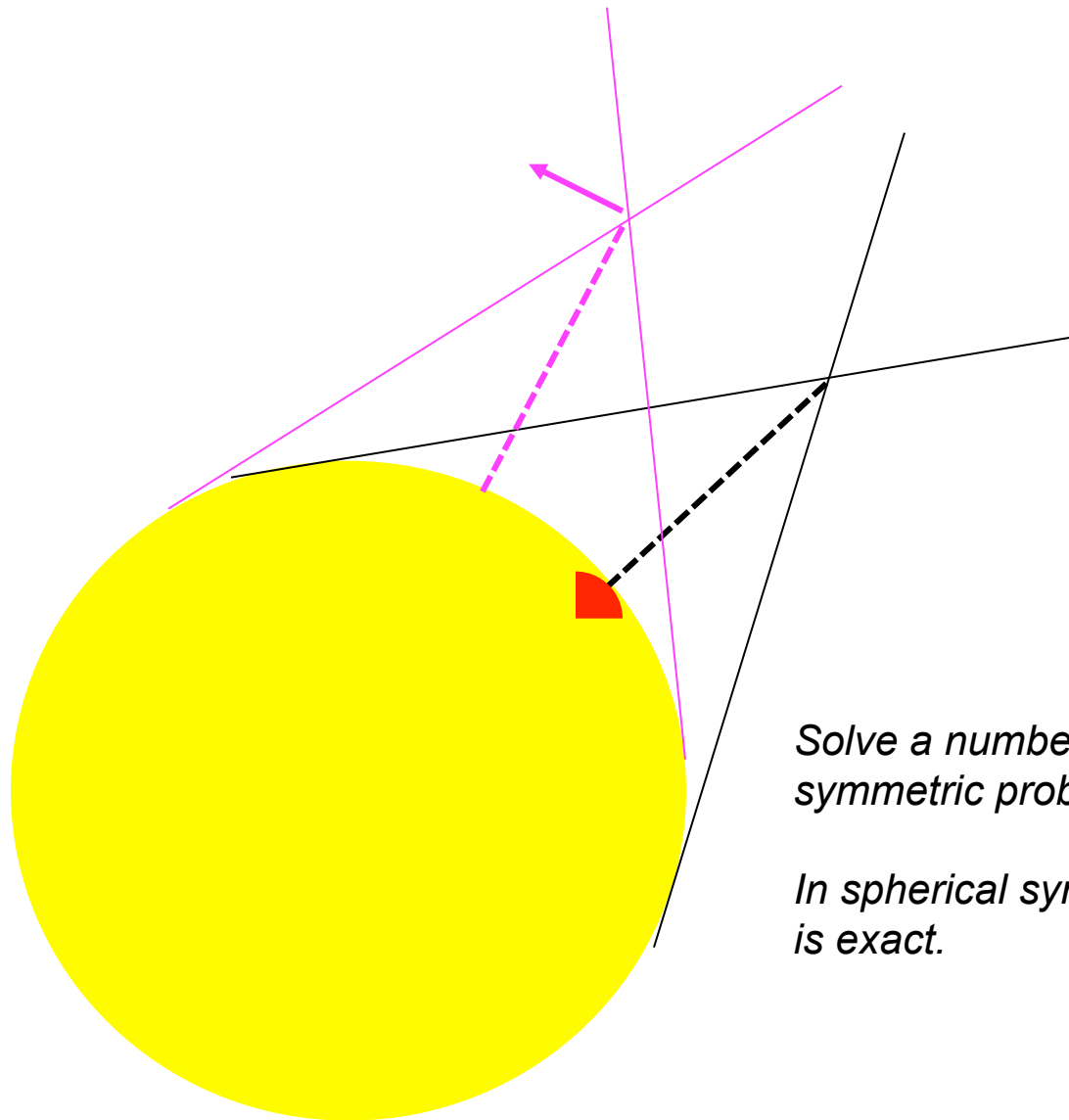


DB: C15-3D-1-002686000.silo
Cycle: 2686000 Time:329.681





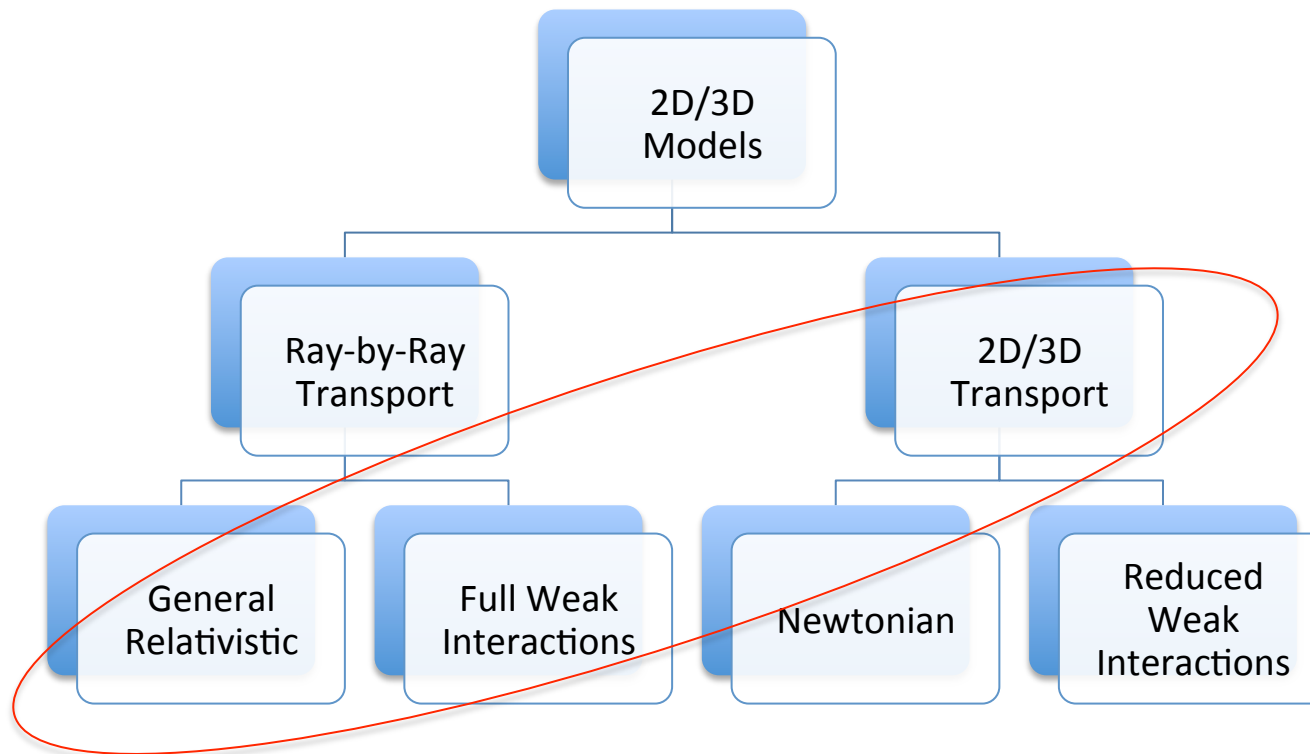
Ray-by-Ray Approximation



Solve a number of spherically symmetric problems.

In spherical symmetry, RbR is exact.

Parallel Lines of Investigation



Max Planck Group
Oak Ridge Group
Basel Group
Nippon Group

Princeton Group
Caltech Group
Max Planck Group

SELECT NEUTRINO TRANSPORT MILESTONES

Publication	Milestone
Bruenn (1975) Ann. N.Y. Acad. Sci. 262 80 Wilson (1975) Ann. N.Y. Acad. Sci. 262 54 Arnett (1977) Ap.J. 218 815	1D MGFLD Implementations
Wilson (1971) Ap.J. 163 209 Mezzacappa and Bruenn (1993) Ap.J. 410 740 Mezzacappa and Bruenn (1993) Ap.J. 405 669 Mezzacappa and Bruenn (1993) Ap.J. 405 637 Yamada, Janka, and Suzuki (1999) A&A 344 533 Burrows et al. (2000) Ap.J. 539 865 Liebendoerfer et al. (2004) Ap.J. Suppl. 150 263	1D Boltzmann Implementations
Kotake et al. (2006) AIP Conf. Proc. 847 421 Burrows et al. (2007) Ap.J. 655 416 Swesty and Myra (2009) Ap.J. Suppl. 181 1	2D MGFLD Implementations
Livne et al. (2004) Ap.J. 609 277 Ott et al. (2008) Ap.J. 685 1069	2D Boltzmann Implementations
Sumiyoshi and Yamada (2012) Ap.J. Suppl. 199 17	3D Boltzmann Implementation

SELECT CORE COLLAPSE SUPERNOVA MODELING MILESTONES

Publication	Milestone
Colgate and White (1966) Ap.J. 143, 626 Arnett (1966) Can. J. Phys. 44, 2553 Wilson (1971) Ap.J. 163, 209 Wilson (1974) PRL 32, 849	First numerical simulations.
Wilson (1985), in Numerical Astrophysics, eds. Centrella, LeBlanc, and Bowers Bethe and Wilson (1985) Ap.J. 295 14	First (1+1)D models – i.e., 1D models with multifrequency neutrino transport.
Herant, Benz, and Colgate (1992) Ap.J. 395 642 Herant et al. (1994) Ap.J. 435 339	First 2D models.
Mezzacappa et al. (2001) PRL 86 1935 Liebendoerfer et al. (2001) PRD 63 3004	First (1+2)D models – i.e., 1D models with Boltzmann neutrino transport.
Fryer and Warren (2004) Ap.J. 601 391	First 3D models.
Bruenn et al. (2006) Journ. Phys. Conf. Ser. 46 393 Buras et al. (2006) A&A 457 281 Burrows et al. (2007) Ap.J. 664 416	First (2+1)D models – i.e., 2D models with multifrequency neutrino transport. .
Hanke et al. (2013) Ap.J. 770 66 Mezzacappa et al. (2014), to appear in Proceedings of ASTRONUM 2013 (astro-ph 1405.7075v1)	First (3+1)D models – i.e., 3D models with multifrequency neutrino transport. .
Ott et al. (2008) Ap.J. 685 1069 Brandt et al. (2011) Ap.J. 728 8	First (2+3)D models – i.e., 2D models with Boltzmann neutrino transport

“Microphysics”

Weak Interactions

Important Neutrino Emissivities/Opacities

“Standard” Emissivities/Opacities

$$\star e^{-(+)} + p(n), A \leftrightarrow \nu_e(\bar{\nu}_e) + n(p), A'$$

$$e^+ + e^- \leftrightarrow \nu_{e,\mu,\tau} + \bar{\nu}_{e,\mu,\tau}$$

$$\star \nu + n, p, A \rightarrow \nu + n, p, A$$

$$\nu + e^-, e^+ \rightarrow \nu + e^-, e^+$$

Bruenn, *Ap.J. Suppl.* (1985)

- Nucleons in nucleus independent.
- No energy exchange in nucleonic scattering.

Langanke et al. PRL, **90**, 241102 (2003)

- Include correlations between nucleons in nuclei.

Reddy, Prakash, and Lattimer, PRD, **58**, 013009 (1998)

Burrows and Sawyer, PRC, **59**, 510 (1999)

- (Small) Energy is exchanged due to nucleon recoil.
- Many such scatterings.

$$\star N + N \leftrightarrow N + N + \nu_{e,\mu,\tau} + \bar{\nu}_{e,\mu,\tau}$$

$$\nu_e + \bar{\nu}_e \leftrightarrow \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau}$$

Hannestad and Raffelt, *Ap.J.* **507**, 339 (1998)

Hanhart, Phillips, and Reddy, *Phys. Lett. B*, **499**, 9 (2001)

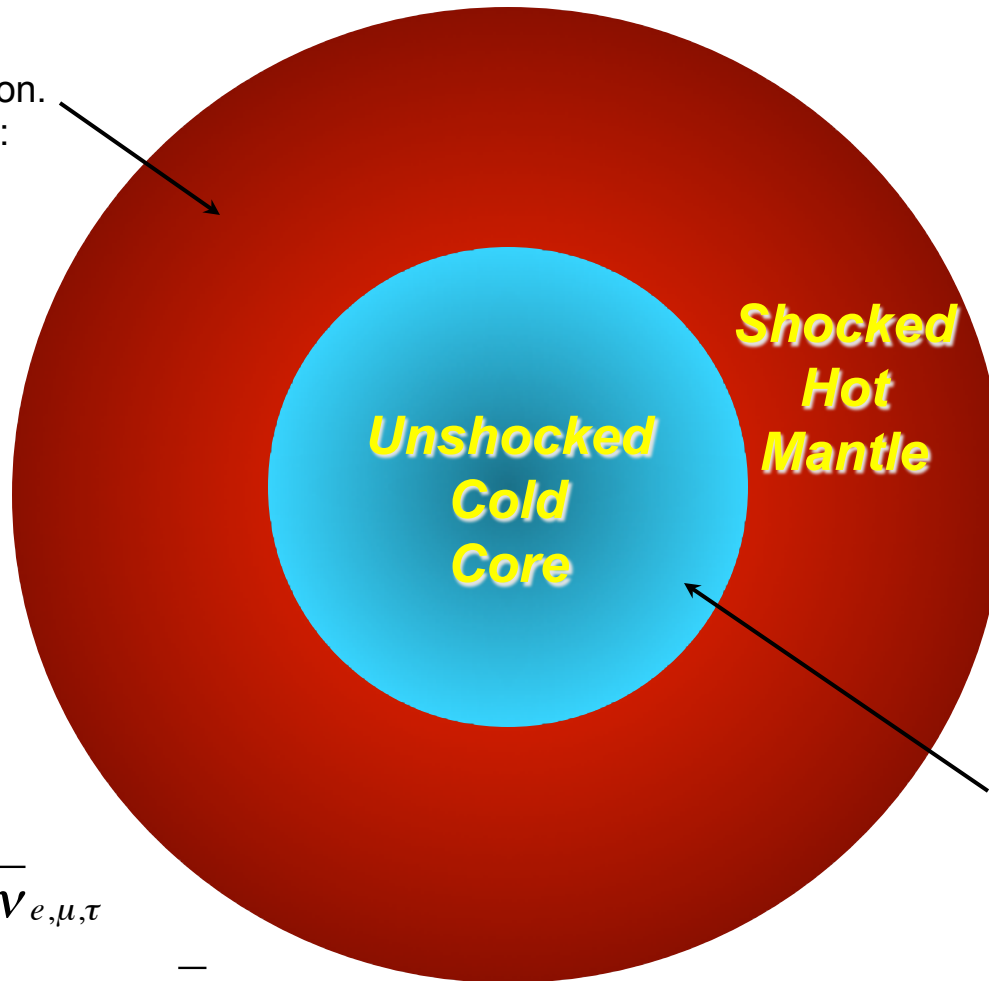
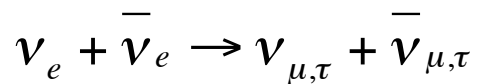
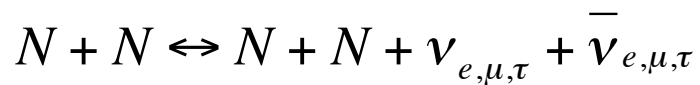
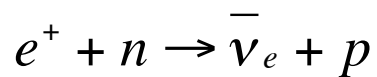
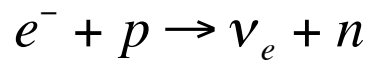
- New source of neutrino-antineutrino pairs.

Janka et al. PRL, **76**, 2621 (1996)

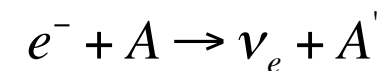
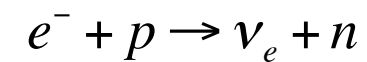
Buras et al. *Ap.J.*, **587**, 320 (2003)

Structure of Proto-Neutron Star

Large positron population.
Neutrino production via:



Low positron population.



Confluence of Electroweak and Supernova Theory

up to Baym, Pethick, and Sutherland's values of binding energy. To take into account the nonzero-temperature effects, the material is assumed to

$$P_1 = \frac{1}{1 + \exp\{-510/[T + 0.258(\rho z)^{1/3}]\}}$$

Thermonuclear burning of C, O, and S

from Negele and Vautherin's lowest de
up to Baym, Pethick, and Sutherland's
binding energy. To take into account t
temperature effects, the material is a

Confluence of Electroweak and Supernova Theory

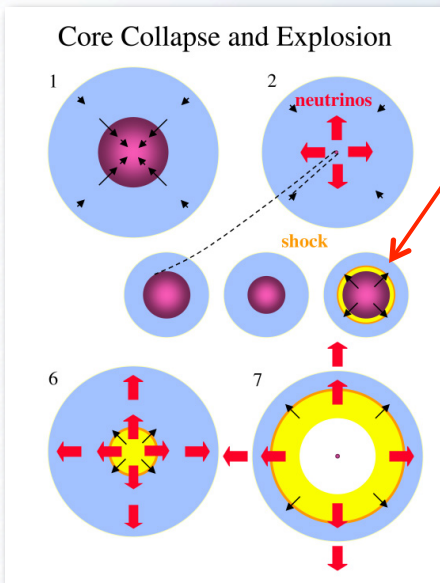
It should be stressed here that stars $\lesssim 7 M_{\odot}$ probably do not go to this state because they nondegenerately in their cores when and if they reach that stage of burning. The Pleiades cluster has masses $M \gtrsim 6 M_{\odot}$ at the main-sequence turnoff, contains a white dwarf (Arnett and suggests that stars $\lesssim 6 M_{\odot}$ may be able to lose their excess mass and settle down to become 1 without burning carbon. Those stars (if any) which burn carbon under degenerate conditions in detail elsewhere (cf. articles in Schramm and Arnett 1973; Arnett 1974). Recent work on the process on degenerate ignition and burning by $^{12}\text{C} + ^{12}\text{C}$ (Couch and Arnett 1975) suggests that

629

© American Astronomical Society • Provided by the NASA Astrophysics

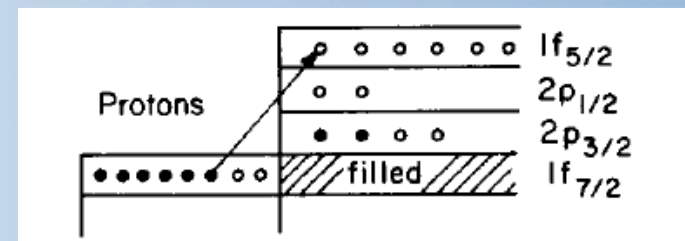
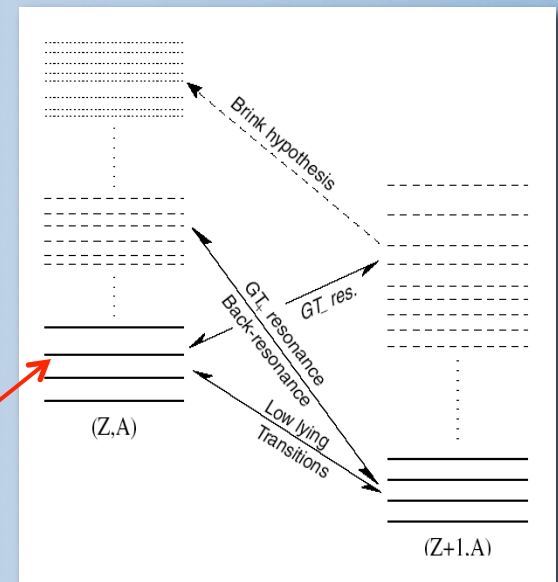
© American Astronomical Society • Provided by the NASA Astrophysics Data System

When Micro and Macro Worlds Collide



Initial shock location/strength depend on amount of electron capture on nuclei (and protons) during stellar core collapse.

Electron capture on stellar core nuclei depends on energy levels in the nucleus and how the nucleons populate them.



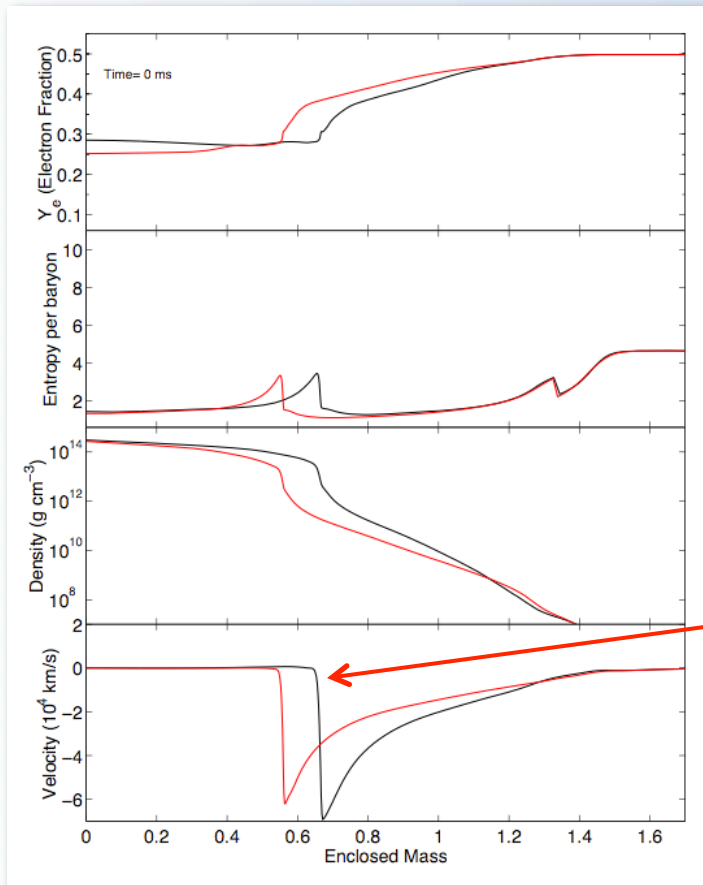
⇒ *Zeroth-order shell model (nucleons independent), electron capture on nuclei is blocked for $N > 40$.*

Fuller, Fowler, and Newman (1980) *Ap.J. Suppl.* **42** 447

Fuller, Fowler, and Newman (1982) *Ap.J.* **252** 715

Fuller, Fowler, and Newman (1982) *Ap.J. Suppl.* **48** 279

When Micro and Macro Worlds Collide



— No correlations.
— With correlations.

Significant change in shock formation mass.

Hix et al. *Phys. Rev. Lett.* 91 201102 (2003)

Shell Model Deployed: “Hybrid Model” [Langanke et al. *Phys. Rev. Lett.* 91, 241102 (2003)]

Interplay of Neutrino Opacities

(κ_l , lower left), net electron (or proton) fraction (Y_e , lower center, solid lines), net lepton fraction ($Y_L = Y_e + (n_{\nu_e} - n_{\bar{\nu}_e})/n_{\text{baryons}}$, lower center, dashed), pressure (lower right). All quantities are plotted relative to enclosed rest mass in M_\odot .

(A color version of this figure is available in the online journal.)

Table 2
Model Summary Table

Model	Bounce Properties				Post-bounce Peak	
	Core Mass (M_{sh}) (M_\odot)	Central ρ_c ($10^{14} \text{ g cm}^{-3}$)	Central Y_e	Central Y_L	Shock Radius (km)	ν_e -luminosity (Bethe s^{-1})
Base	0.430	3.234	0.2448	0.2804	161	408
Base-noNIS	0.431	3.234	0.2453	0.2811	150	478
Base-noNES	0.430	3.234	0.2450	0.2807	158	481
Base-ISnp	0.431	3.233	0.2451	0.2808	153	404
Base-noNPS	0.430	3.233	0.2448	0.2804	160	408
IPA	0.554	3.824	0.2843	0.3331	159	432
IPA-noNIS	0.618	4.239	0.3099	0.3712	148	449
IPA-noNES	0.608	4.162	0.3056	0.3647	159	454
IPA-ISnp	0.554	3.831	0.2849	0.3339	149	430
IPA-noNPS	0.551	3.825	0.2843	0.3331	159	432
Base-noEPpair	0.431	3.233	0.2448	0.2804	183	407
Base-noBrems	0.430	3.216	0.2443	0.2798	163	407
Base-noPair	0.435	3.216	0.2443	0.2798	185	410
Base-B85ea-np	0.431	3.239	0.2452	0.2808	159	393

3.1.1. NIS Comparisons Using LMSH EC Table

For this set of tests, we compare a model with our full opacity set (Base) to models without electron scattering (Base-

noNES), without positron scattering (Base-noNPS), the nucleon scattering of Reddy et al. (1998) w equivalent of Bruenn (1985) (Base-ISnp), and to a r all three of these changes (Base-noNIS). *At boun*

“Microphysics” Nuclear EOS

Comparison of Stellar Collapse with Different EOS

Different Calculations of Energy/Particle

- ⇒ Lattimer-Swesty (Compressible Liquid Droplet)
- ⇒ Shen et al. (Relativistic Mean Field)

Issues:

- ⇒ Energy/Particle
- ⇒ Inhomogeneous Matter
 - Nuclei and Nuclear Matter
 - 1D vs. 3D Nuclei
- ⇒ Other Constituents?

$$P = P(\rho, T, Y_e)$$

$$P = n^2 \left. \frac{\partial(F/n)}{\partial n} \right|_{T, Y_e}$$

$$F = E - TS$$

$$E = \langle \Phi | T + U | \Phi \rangle$$

Lattimer-Swesty EOS

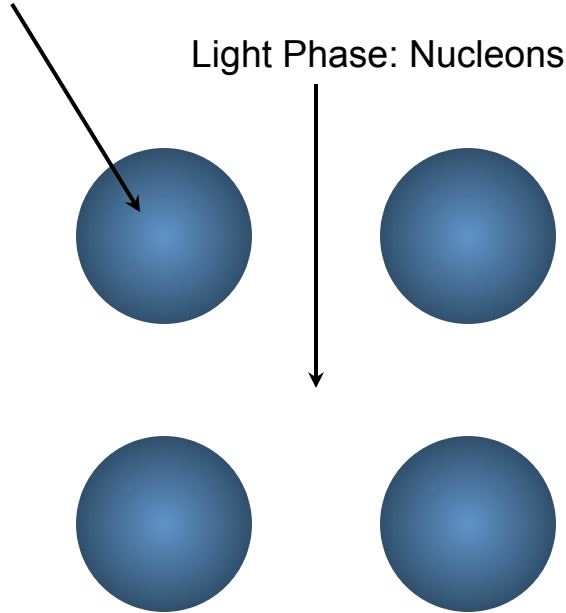
- *Nucl. Phys.* **A535**, 331 (1991)

Shen et al. EOS

- *Nucl. Phys.* **A637**, 435 (1998)

- ⇒ “Semi-Empirical”
 - Polynomial expansion about infinite symmetric nuclear matter at saturation.
- ⇒ “Phenomenological”
 - Nucleon interaction potential fit to nuclear matter and nuclei (e.g., Skyrme).
- ⇒ “Realistic”
 - Nucleon interaction potential fit to free nucleon scattering data.

Dense Phase: Nuclei



Compressible Liquid Drop Model

T=0

Nuclei arrange themselves in a b.c.c. lattice (minimizes free energy).

⇒ Components of electrically neutral “Wigner-Seitz” cells (more later).

Nuclear size determined by competition between surface and Coulomb forces.

- Surface effects favor larger nuclei.
 - Surface energy per nucleon smaller: lower percentage of nucleons on surface.
- Coulomb effects favor smaller nuclei.
 - Coulomb energy increases with A.

⇒ *As proton fraction decreases, surface effects win out.*

$$A \sim \frac{4\pi}{3} R^3$$

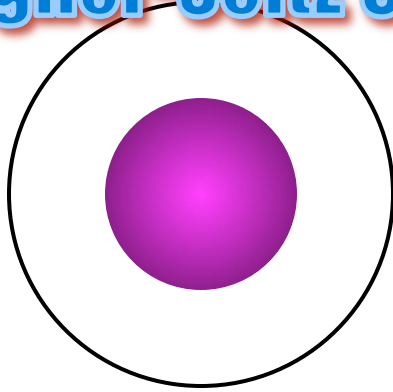
$$\Rightarrow 4\pi R^2 \sim A^{2/3}$$

$$\Rightarrow \frac{Z^2}{R} \sim \frac{A^2}{A^{1/3}} \sim A^{5/3}$$

$$E_{\text{Size}} = \frac{\omega_{\text{Surface}} A^{2/3} + \omega_{\text{Coulomb}} A^{5/3}}{A}$$

Wigner-Seitz Cell

(a)



$$E_{Coulomb} = \frac{1}{A} \frac{3}{5} \frac{Z^2 e^2}{R} g(u)$$

$$g(u) = 1 - \frac{3}{2} u^{1/3} + \frac{1}{2} u$$

$$u \equiv \frac{\rho}{\rho_0}$$

$\rho = \text{baryon density}$

$\rho_0 = \text{nuclear matter density}$

Wigner-Seitz Cell

(b)



Geometry Inversion

Energetically favored for

$$u \geq \frac{1}{2}$$

Coulomb energy minimized if:

$$u \rightarrow 1$$

Surface tension resists this in (a) but favors this in (b).

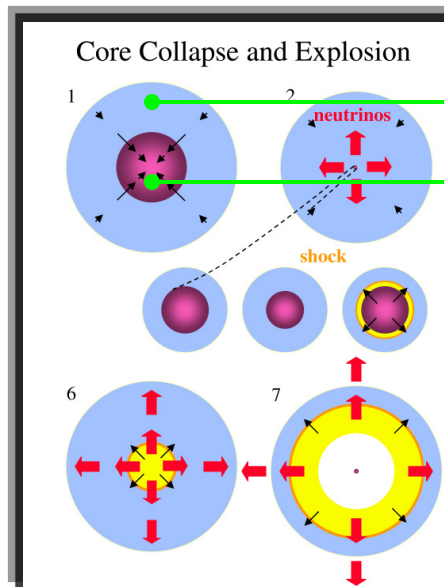
Possible Geometries Enroute to Bulk Nuclear Matter

Nuclear Pasta



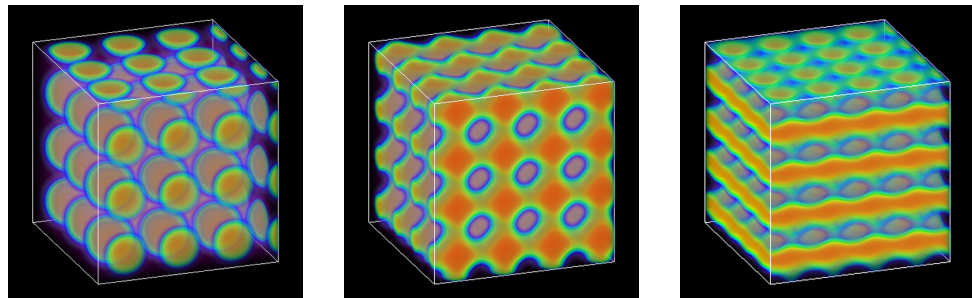
Ravenhall, Pethick, and Wilson (1983), *Phys. Rev. Lett.* **150**, 2006.

3D Inhomogeneous Matter



Transition from nuclei to nuclear matter occurs through a “pasta” phase.

Ravenhall, Pethick, and Wilson (1983), *Phys. Rev. Lett.* 150, 2006.



Newton et al., *Journ. Phys. Conf. Ser.*, **46**, 408 (2006)
 PRC 79 055801 (2009)

First look at stellar core (finite temperature) matter in 3D.

What impact will this have on the neutrino opacities?

– Horowitz et al., PRC 69 045804 (2004)

What impact will this have on stellar core collapse?

Industry Standard EOS

EOS	Reference
Lattimer-Swesty	Nucl. Phys. A 535 331 (1991)
H. Shen et al.	Nucl. Phys. A 637 435 (1998)
G. Shen et al.	PRC 83 035802 (2011)
Hempel et al.	Ap.J. 748 70 (2012)
Furusawa et al.	Ap.J. 772 95 (2013)
Meixner et al.	PRC submitted (2013)

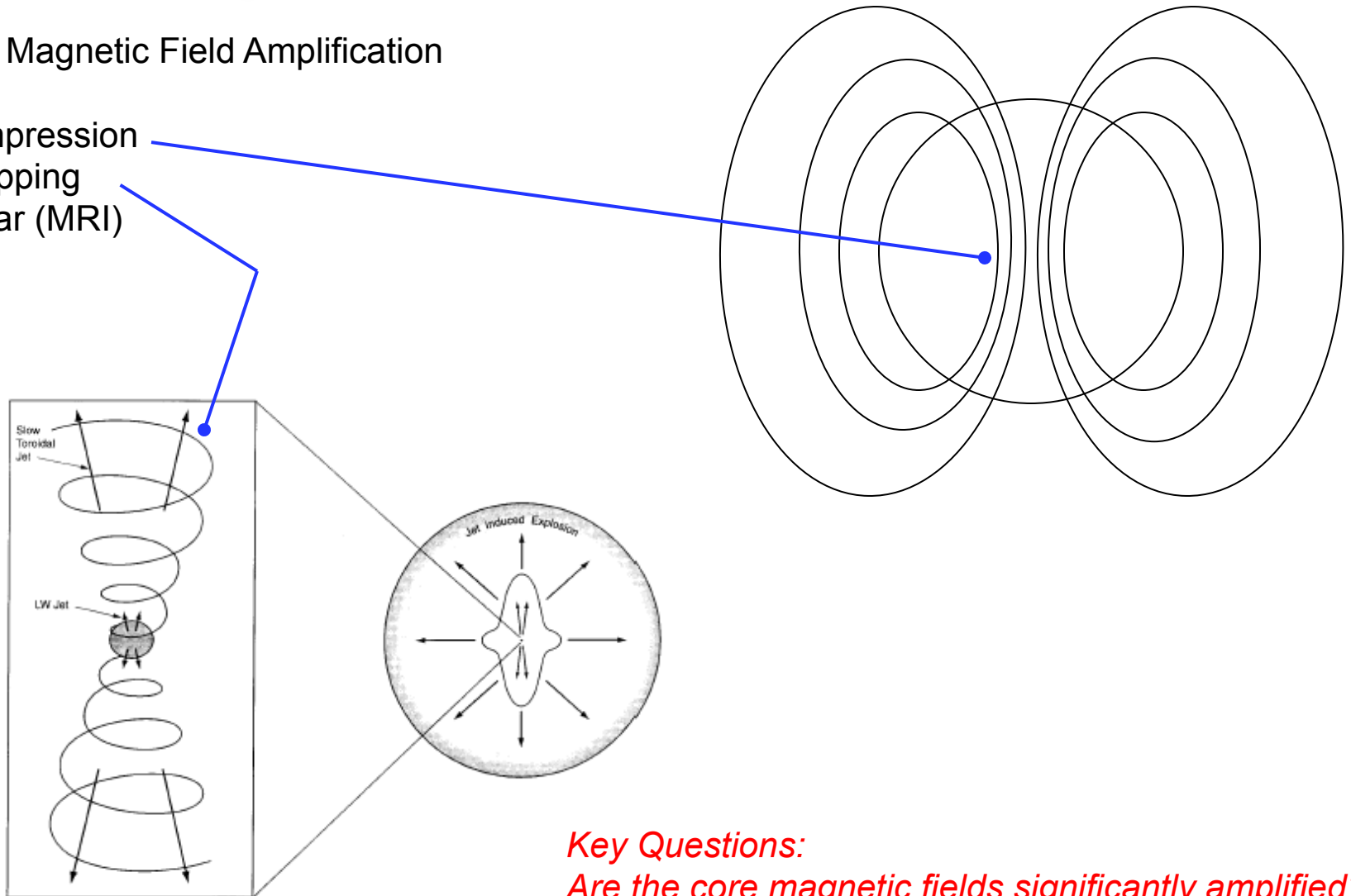
Other Physics

The Role of Magnetic Fields

Leblanc and Wilson, *Ap.J.* **161**, 541 (1970)
Symbalisty, *Ap.J.* **285**, 729 (1984)

Stellar Core Magnetic Field Amplification

- ⇒ Compression
- ⇒ Wrapping
- ⇒ Shear (MRI)



Key Questions:

*Are the core magnetic fields significantly amplified?
Will they collimate and drive outflows?*

Rapid Rotation

Burrows et al. *Ap.J.* **664**, 416 (2007)

2D modern multi-physics models with B fields.

For rapidly rotating progenitors, robust magnetically powered explosions obtained ($P_B > P$).

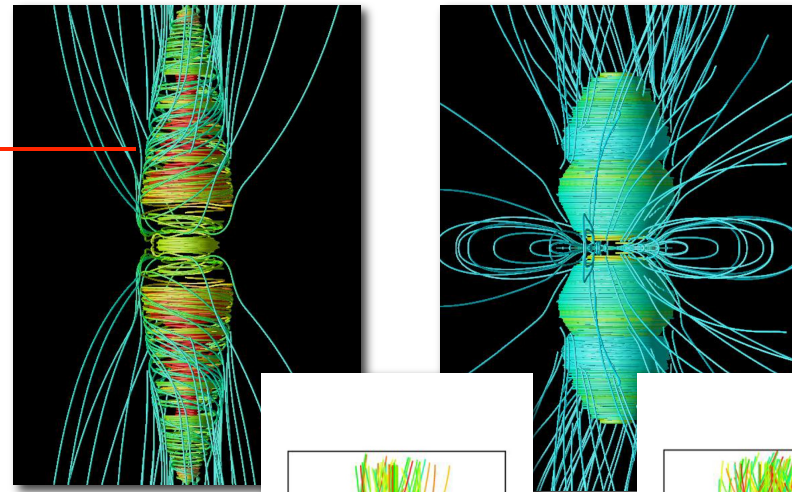
Energy available to B fields from rotation depends sensitively on the angular momentum distribution in the core after bounce.

“Magnetic Towers”

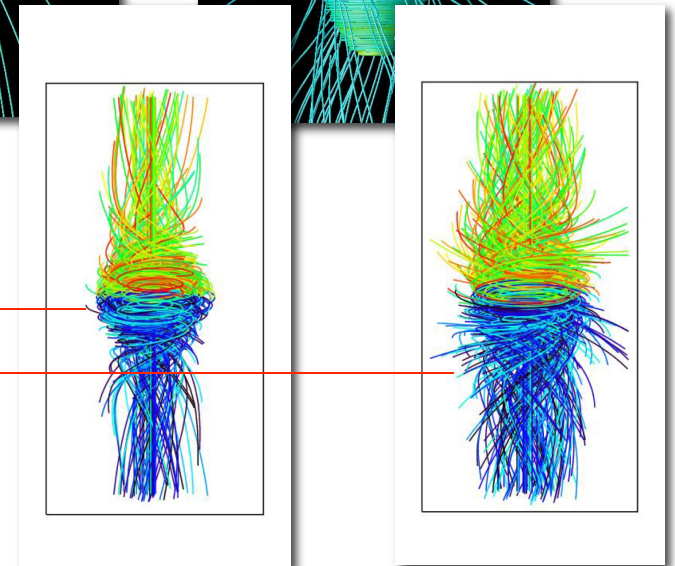
Uzbenksy and MacFadyen, *Ap.J.* **647**, 1192 (2006)

Uzbenksy and MacFadyen, *Ap.J.* **669**, 546 (2007)

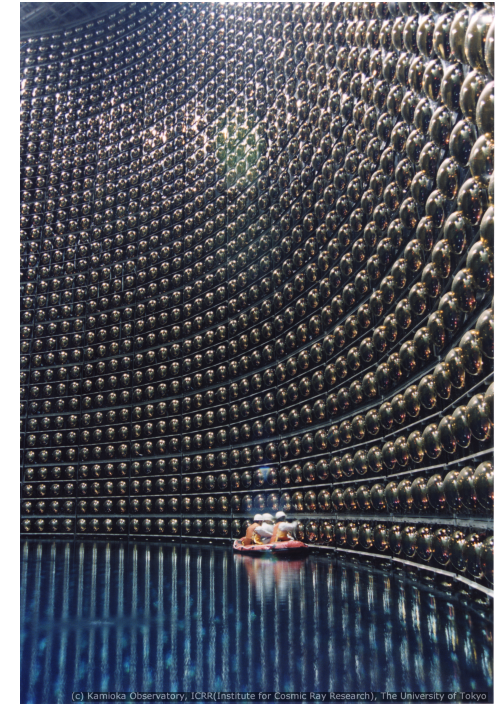
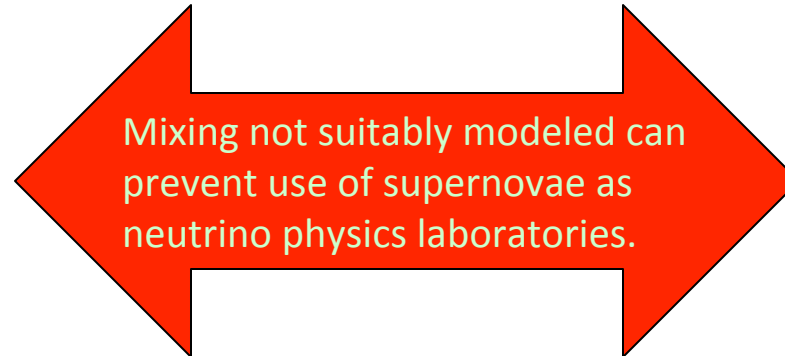
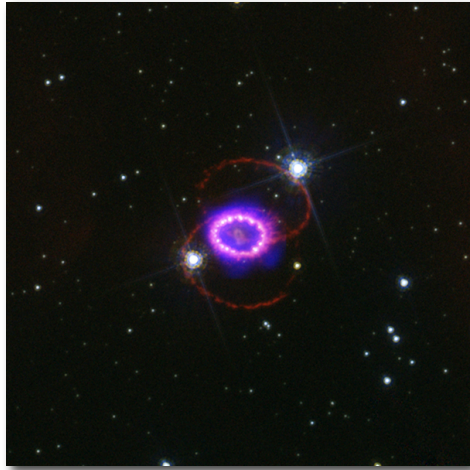
B Fields



Lagrangian Tracers



Neutrino Mass and Mixing: Implications



Neutrino Mixing Scenarios

- Mixing deep within the source (w/ neutrino-neutrino forward scattering).
- Mixing in the stellar envelope (MSW – i.e., w/ neutrino-electron forward scattering).
- Mixing between the supernova and the Earth (vacuum) and within the Earth.

Potential Impacts

- Explosion Mechanism
- Nucleosynthesis
- Terrestrial Neutrino Signatures

Outlook

