FLASH Code Tutorial

part II Gravity modules

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Motivation

- dynamics of stratified atmospheres
- N-body dynamics in gaseous media (e.g. stellar cluster, planets)
- collapse of gas cores (e.g. star formation)
- formation of molecular clouds



collapse of a cloud core



molecular cloud formation

• coupling to hydro-dynamics / MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$
$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla P = \rho \mathbf{g}$$
$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + P) \mathbf{v}] = \rho \mathbf{v} \cdot \mathbf{g}$$

g : gravitational acceleration

- \Rightarrow via source term
- \Rightarrow no explicit total energy conservation using gravitational potential

- Gravity modules
 - source/physics/Gravity
 - → **default**: constant gravitational acceleration
 - source/Grid/GridSolvers
 - \Rightarrow for self-gravity
 - Multipole
 - Multigrid
 - BHTree

- time-independent external fields:
 - source/physics/Gravity/GravityMain
 - Constant

 \Rightarrow **g** = const in gdirec = x,y or z direction

- PlanPar \Rightarrow g = G_N M / h² h = x,y or z-direction
- PointMass

 \implies **g** = **G**_N *M* / r^2 **r**/r

 \implies similarly for sink particles

- Self-gravity:
 - \rightarrow source/Grid/GridSolvers

provides gravitational potential $\Phi(\mathbf{x})$ via

- multi-pole solver
- multi-grid solver (Paul Ricker 2008)
- tree-based solver (Richard Wunsch)

$$\Rightarrow \nabla^2 \Phi(\mathbf{x}) = 4\pi G_N \rho(\mathbf{x}) \Rightarrow \mathbf{g}(\mathbf{x}) = -\nabla \Phi(\mathbf{x})$$

$$g_{x;ijk} = \frac{1}{2\Delta x} (\phi_{i-1,j,k} - \phi_{i+1,j,k})$$

$$\Rightarrow g_{y;ijk} = \frac{1}{2\Delta y} (\phi_{i,j-1,k} - \phi_{i,j+1,k})$$

$$g_{z;ijk} = \frac{1}{2\Delta z} (\phi_{i,j,k-1} - \phi_{i,j,k+1})$$

implementation via finite difference scheme

- setup with self-gravity:
 - in Config file

or

mathrmale ==> REQUIRES physics/Gravity/GravityMain/Poisson/Multigrid

or

⇒ REQUIRES physics/Gravity/GravityMain/Poisson/BHTree

• setup with self-gravity:

boundary conditions

bcTypes	Type of boundary condition
0	Isolated boundaries
1	Periodic boundaries
2	Dirichlet boundaries
3	Neumann boundaries
4	Given-value boundaries

• Multipole solver

formal solution of $\nabla^2 \Phi(\mathbf{x}) = \alpha \rho(\mathbf{x})$; $\alpha = 4\pi G_N$

$$\implies \qquad \phi(\mathbf{x}) = -\frac{\alpha}{4\pi} \int d^3 \mathbf{x}' \, \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}$$

with Green's function

$$\implies \qquad \frac{1}{|\mathbf{x} - \mathbf{x}'|} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{1}{2\ell+1} \frac{r_{<}^{\ell}}{r_{>}^{\ell+1}} Y_{\ell m}^{*}(\theta', \varphi') Y_{\ell m}(\theta, \varphi)$$

where $Y_{lm}(\theta, \varphi)$ are the spherical harmonic functions

 \Rightarrow origin at center of mass (CM): $\mathbf{x}_{cm} = \frac{\int \mathbf{x} \rho(\mathbf{x}) d\mathbf{x}}{\int \rho(\mathbf{x}) d\mathbf{x}}$

• Multipole solver

 $\Rightarrow \Phi(\mathbf{x}) \rightarrow \Phi(r, \mu(r))$

with source moments $\mu(r)$

e.g.
$$\mu_{\ell m}^{\mathrm{ei}}(r) \equiv \frac{(\ell-m)!}{(\ell+m)!} \int_{r>r'} d^3 \mathbf{x}' r'^{\ell} \rho(\mathbf{x}') P_{\ell m}(\cos\theta') \cos m\varphi'$$

and μ^{oi} , μ^{eo} , μ^{oo} , (even/odd, inner/outer)

- \Rightarrow compute μ 's up to l_{\max}
- \rightarrow **note**: scales at least as $2 \times N_{\text{grid}} \times l_{\text{max}}^2$
- \Rightarrow only useful for low l_{max} , i.e. nearly spherical problems

- Multipole solver
 - main runtime parameters

Variable	Type	Default	Description		
${\tt mpole_lmax}$	integer	10	Maximum multipole moment		
quadrant	logical	.false.	Use symmetry to solve a single quadrant in 2D ax- isymmetric cylindrical (r, z) coordinates, instead of a half domain.		

- supported grid geometries:
 - \rightarrow ID, spherical
 - \Rightarrow 2D, cylindrical, spherical
 - \rightarrow 3D, Cartesian, axi-symmetric



- Multipole solver
 - possible test suit:
 - unitTest/Multipole
 - ⇒ only multipole solver without gravity based on MacLaurin spheroid analytic solution

- Multipole solver
 - improved Multipole solver

source/Grid/GridSolvers/Multipole_new

- improvements:
 - efficient memory layout
 - elimination of over- and underflow errors when using astrophysical (dimensions $\approx 10^9$) domains
 - elimination of subroutine call overhead (I call per cell)
 - •

- Multigrid solver
 - based on improved Huang-Greengard (2000) method by P. Ricker (ApJS, 2008)
 - iterative method
 - uses fine-coarse structure of AMR grid
 - \Rightarrow restrict & interpolate solution to different refinement levels \Rightarrow V-cycle
 - main functions in

source/Grid/GridSolvers/Multigrid/
⇒ gr_hg*.F90

• Multigrid solver: V-cycle

```
DESCRIPTION
   This is the main Poisson solve routine for the Huang & Greengard
!!
!!
   (2000, SIAM J. Sci. Comput., 21, 1551) algorithm. This routine
!!
   defines the multigrid cycle as expressed in the article.
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  coarse ^
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                                0---->0
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       tl
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                                               o---->...until |r| < rtol
                         o---->o gr_iSls
   src
                       take residual_____
                                                  _take new residual__>
  fine L
!!
```

• Multigrid solver: more details

• residual
$$R(\mathbf{x}) \equiv 4\pi G \rho(\mathbf{x}) - \nabla^2 \tilde{\phi}(\mathbf{x})$$

• finite difference operator:

$$\mathcal{D}_{\ell}\tilde{\phi}_{ijk}^{b\ell} \equiv \frac{1}{\Delta x_{\ell}^2} \left(\tilde{\phi}_{i+1,jk}^{b\ell} - 2\tilde{\phi}_{ijk}^{b\ell} + \tilde{\phi}_{i-1,jk}^{b\ell} \right) + \dots$$

• restrict operator: fine \rightarrow coarse

$$\left(\mathcal{R}_{\ell}\tilde{\phi}\right)_{ijk}^{\mathcal{P}(c),\ell} \equiv \frac{1}{2^d} \sum_{i'j'k'} \tilde{\phi}_{i'j'k'}^{c,\ell+1}$$

• interpolation operator at block edges: coarse \rightarrow fine

$$(\mathcal{I}_{\ell}\tilde{\phi})_{i'j'k'}^{c,\ell+1} \equiv \sum_{p,q,r=-2}^{2} \alpha_{i'j'k'pqr} \tilde{\phi}_{i+p,j+q,k+r}^{\mathcal{P}(c),\ell}$$

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• Multigrid solver: Interpolation operator



• Multigrid solver: more details

- 1. Restrict the source function $4\pi G\rho$ to all levels. Subtract the global average for the **periodic** case.
- 2. Interpolation step: For ℓ from 1 to ℓ_{\max} ,
 - (a) Reset the grid so that ℓ is the maximum refinement level
 - (b) Solve $\mathcal{D}_{\ell} \tilde{\phi}_{ijk}^{b\ell} = 4\pi G \rho_{ijk}^{b\ell}$ for all blocks b on level ℓ . \rightarrow sin-type Green's function or FFT
 - (c) Compute the residual $R_{ijk}^{b\ell} = 4\pi G \rho_{ijk}^{b\ell} \mathcal{D}_{\ell} \tilde{\phi}_{ijk}^{b\ell}$
 - (d) For each block b on level ℓ that has children, prolong face values for $\tilde{\phi}_{ijk}^{b\ell}$ onto each child block.
- 3. Residual propagation step: Restrict the residual $R_{ijk}^{b\ell}$ to all levels.
- 4. Correction step: Compute the discrete L_2 norm of the residual over all leaf-node blocks and divide it by the discrete L_2 norm of the source over the same blocks. If the result is greater than a preset threshold value, proceed with a correction step: for each level ℓ from 1 to ℓ_{max} ,
 - (a) Reset the grid so that ℓ is the maximum refinement level
 - (b) Solve $\mathcal{D}_{\ell}C_{ijk}^{b\ell} = R_{ijk}^{b\ell}$ for all blocks b on level ℓ . \rightarrow sin-type Green's function or FFT
 - (c) Overwrite $R_{ijk}^{b\ell}$ with the new residual $R_{ijk}^{b\ell} \mathcal{D}_{\ell} C_{ijk}^{b\ell}$ for all blocks b on level ℓ .
 - (d) Correct the solution on all leaf-node blocks b on level $\ell: \tilde{\phi}_{ijk}^{b\ell} \to \tilde{\phi}_{ijk}^{b\ell} + C_{ijk}^{b\ell}$.
 - (e) For each block b on level ℓ that has children, interpolate face boundary values of $C_{ijk}^{b\ell}$ for each child.
- 5. If a correction step was performed, return to the residual propagation step.

• Multigrid solver: more details

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 - (e) For each block b on level ℓ that has children, interpolate face boundary values of $C_{ijk}^{b\ell}$ for each child.
- 5. If a correction step was performed, return to the residual propagation step.

```
do m = 1, gr_hgMeshRefineMax
    call gr_hgSetMaxLevel(m) ! This is a new FLASH3 routine
    ! call gr_hgNorm(0, 1, gr_iSource, norm_tmp, MG_NODES_LEAF_ONLY)
    call gr_hgSetZeroBoundary(m, gr_iSoln) !oK
    call gr_hgSolveLevel(m, gr_iSource, gr_iSoln, SolveBlock, MG_NODES_ALL_NODES) ! Working here.....
    call gr_hgResidual(m, gr_iSource, gr_iSoln, gr_iSls) !oK
    call gr_hgProlongBndries(m, gr_iSoln, gr_iSoln, 0) ! LBR doesn't wanna know. But we looked at it...
 enddo
 ! Correction step. Restrict residuals from finer levels to coarser levels.
 ! Solve for correction on these levels and interpolate boundary conditions to
 ! finer levels. Solve for corrections there and apply. Repeat. Repeat these
 ! correction steps until the desired residual norm is achieved.
 do n = 0, gr_hgMaxCorrections
    call gr_hgNorm(0, 2, gr_iSls, norm_lhs, MG_NODES_LEAF_ONLY)
    if (norm_lhs/norm_rhs <= gr_hgMaxResidualNorm) exit
    do m = gr_hgMeshRefineMax-1, 1, -1
       call gr_hgRestrict(m+1, gr_iSls, gr_iSls)
    enddo
    call gr_hgSetMaxLevel(1)
    call gr_hgSetZeroBoundary(1, gr_iCorr)
    call gr_hgSolveLevel(1, gr_iSls, gr_iCorr, SolveBlock, MG_NODES_ALL_NODES, dt, chi)
    call gr_hgLevelAdd(1, gr_iSoln, gr_iCorr, MG_NODES_LEAF_ONLY) !oK
    call gr_hgProlongBndries(1, gr_iCorr, gr_iCorr, 0) !infamous
    do m = 2, gr_hgMeshRefineMax
       call gr_hgSetMaxLevel(m)
       call gr_hgSetZeroBoundary(m, gr_iCorr)
       call gr_hgSolveLevel(m, gr_iSls, gr_iCorr, SolveBlock, MG_NODES_ALL_NODES)
       call gr_hgResidual(m, gr_iSls, gr_iCorr, gr_iSls) ! not called on top level
       call gr_hgLevelAdd(m, gr_iSoln, gr_iCorr, MG_NODES_LEAF_ONLY)
       call gr_hgProlongBndries(m, gr_iCorr, gr_iCorr, 0)
    enddo
```

- Multigrid solver: Additional fields needed:
 - 'gpot' key GPOT_VAR: actual gravitational potential
 - 'gpol' key GPOT_VAR: gravitational potential from previous time step => needed to start solution iteration
 - 'isls' key ISLS_VAR: residual potential variable
 - 'icor' key ICOR_VAR: correction potential variable
 - 'imgm' key IMGM_VAR: image mass for periodic BCs
 - 'imgp' key IMGP_VAR: image potential for periodic BCs

- Multigrid solver:
 - supported geometries

 \rightarrow only Cartesian geometry in 1, 2 and 3 D

- supported boundary conditions
 - \rightarrow isolated
 - \rightarrow perodic
 - \Rightarrow Dirichlet (Φ given at the boundaries)
 - \rightarrow Neumann, not yet implemented

 Multigrid solver: main files source/Grid/GridSolvers/Multigrid

> _hgBndry.F90 haData.F90 hgFinalize.F90 haGuardCell.F90 haInit.F90 haInitSource.F90 hgLevelAdd.F90 hgLevelAddScalar.F90 hgLevelMultiplyScalar.F90 hgLevelZero.F90 r_hgMapBcType.F90 r_hgNorm.F90 hgPoissonSolve1D.F90 hqPoissonSolve2D.F90 hgPoissonSolve3D.F90 hgPoissonSolveBlock.F90 hgProlongBndries.F90 hgRecordNodeTypeState.F90 haResidual.F90 hgRestoreNodeTypes.F90 _hgRestrict.F90 r_hgSetExtBoundary.F90 r_hgSetMaxLevel.F90 r_hgSetZeroBoundary.F90 hgSolve.F90 hgSolveLevel.F90 hg_amr_1blk_bcset_work.F90

- Multigrid solver: Solution convergence \Rightarrow UG: truncation error: $O(\Delta x^2)$
 - \Rightarrow weaker for AMR (here: 5% of vol. on highest refinement level)



- Barns & Hut Tree Solver (*Barns & Hut*, Nature 1986)
 - implemented by Richard Wünsch (Prague)
 - based on an oct-tree with monopole moments
 matches oct-tree AMR structure of PARAMESH
 - basic properties
 - scaling: $N \log(N)$: N number of grid/mass cells instead N^2 of for direct summation
 - works only for NBX = NBY = NBZ
 - NBX = 2^n (not yet tested for NBX $\neq 8$)

- Tree Solver: steps to calculate the gravitational potential:
 - build the tree:
 - I. build oct-tree in block (block-tree)
 - $\Rightarrow log_2(nbx)$ levels
 - \rightarrow stored in gr_bhTreeArray
 - 2. masses and CMs of top nodes are distributed to all processors via gr_bhTreeParentTree (4,MAXBLOCKS,NPROCS)
 - 3. calculation of masses and CMs of parent blocks on all CPUs
 - \Rightarrow stored also in gr_bhTreeParentTree

 \Rightarrow tree down to leaf block information on all CPUs



block-tree structure in memory for
 blocks with 8×8×8 grid cells
 ⇒ linear array in memory: size = 8³ + 4×(8²+8¹+8⁰)= 804



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- Tree Solver: steps to calculate the gravitational potential:
 - communication of the tree:
 - I. determine which block-trees to be communicated:

$$\frac{S_{\rm node}(l)}{D_{\rm min}} < {\tt gr_bhTreeLimAngle}$$

 $\Rightarrow S(l) : \text{size of the local block-tree at level } l = 0...3$ $\Rightarrow D_{\min} : \text{minimum distance to all remote blocks}$ all eight corners of both, the local and the remote blocks, are checked

 \Rightarrow typical values for gr_bhTreeLimAngle: 0.5 ... 1.0

- Tree Solver: steps to calculate the gravitational potential:
 - communication of the tree:
 - I. determine which block-tree levels to be communicated:

 $\frac{S_{\rm node}(l)}{D_{\rm min}} < {\tt gr_bhTreeLimAngle}$

- 2. communication of block-tree levels
 - \Rightarrow to allocate memory on remote CPUs
 - \Rightarrow all values for a given CPU in a single message
- 3. selected block-tree nodes are send to remote CPUs
 ⇒ all block-tree nodes are packed into a single message



- Tree Solver: steps to calculate the gravitational potential:
 - calculation of the potential \Rightarrow the "tree walk"
 - \Rightarrow most time consuming part (~ 90 % of tree solver)
 - \Rightarrow fully parallel, no more communication needed
 - walk the tree for local block A
 ⇒ start with refinement level one
 B is the currently processed tree node
 - if **B** is a parent block

 $\Rightarrow S_{B}/D_{AB} < \theta_{\lim} \ (S_{B}: \text{size of } B, D_{AB}: \text{min distance to } CM_{B})$ $\Rightarrow \Delta \Phi_{A,\text{cell}} = -G M_{B}/D_{\text{cell},B}$

else continue with child-blocks of B

- Tree Solver: steps to calculate the gravitational potential:
 - if *B* is a leaf block:
 - \Rightarrow block tree of B is walked separately for each level \Rightarrow block tree consists of nodes N

$$\implies$$
 S_N/D_{cell,N} < θ_{lim} (S_N : size of node N)

$$\Rightarrow \Delta \Phi_{A,\text{cell}} = - G M_N / D_{\text{cell},N}$$

else continue with children of ${\cal N}$

• Tree Solver: Test with Bonnor-Ebert sphere

 $\log_{10}|(\Phi-\Phi_{anl})/\Phi_{anl}(0)|$



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- Tree Solver:
 - supported geometry
 - \rightarrow so far, only 3D cartesian geometry
 - boundary conditions
 - \rightarrow isolated
 - \rightarrow periodic \rightarrow generation of 'Ewald' field values necessary

	PARAMETER	gr_bhTreeLimAngle	REAL	0.5	[0]	
	PARAMETER	gr_bhIlist	INTEGER	0	[0,1]	
 Parameters 	PARAMETER	gr_bhEwaldFieldNx	INTEGER	64	[1]	
	PARAMETER	gr_bhEwaldFieldNy	INTEGER	64	[1]	
	PARAMETER	gr_bhEwaldFieldNz	INTEGER	64	[1]	
	PARAMETER	gr_bhEwaldSeriesN	INTEGER	10		
	PARAMETER	<pre>gr_bhEwaldAlwaysGenerate</pre>	BOOLEAN	TRUE		
	PARAMETER	gr_bhEwaldFName	STRING	"ewald_f	ald_field"	
	PARAMETER	gr_bhUseEwaldDecomp	BOOLEAN	TRUE		
	PARAMETER	gr_bhEwaldIsoFac	INTEGER	1000		

Tree Solver: the source files in source/Grid/GridSolvers/BHTree/Wunsch

Grid_solvePoisson.F90 gr_bhBlockRelationship.F90 gr_bhBuildTree.F90 gr_bhBuildTreeBlock.F90 gr_bhComBlkProperties.F90 gr_bhComParentTree.F90 gr_bhData.F90 gr_bhDestroyTree.F90 gr_bhErfc.F90 gr_bhEwald.F90 gr_bhEwaldField.F90 gr_bhExchangeTrees.F90 gr_bhFinalize.F90 gr_bhFindNeighbours.F90 gr_bhGetTreePos.F90 gr_bhGetTreeSize.F90 gr_bhILContrib.F90 gr_bhInit.F90 gr_bhInitTemplates.F90 gr_bhLeafContrib.F90 gr_bhLocalInterface.F90 gr_bhParentContrib.F90 _bhPotential.F90 _bhPotentialBlock.F90

• MacLaurin test problem: collapse of a spheroid

source/Simulation/SimulationMain/MacLaurin



spheroid with ellipticity e = 0.9

FLASH code: Sinks

tasks

- try MacLauren setup with
 - multipole
 - multigrid and
 - BHTree

Poisson solvers

 set up and run a collapse Bonnor-Ebert problem: source/Simulation/SimulationMain/BonnorEbert

replace Config with /pfs/banerjee/Config_BE