SPH extras

(things that have been mentioned in passing)

Resources

- There are a number of excellent papers/reviews on SPH that discuss many of the problems with SPH...

Reviews:

Rosswog S., 2009, New Astron. Rev., 53, 78

Price D.J., 2012, J. Comp. Phys., 231, 759

Papers:

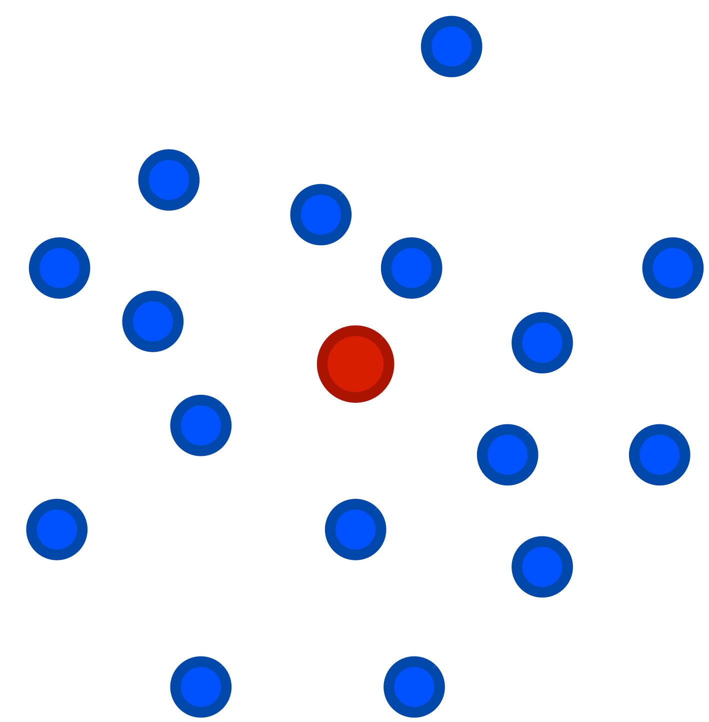
Cullen L. & Dehnen W., 2010, 408, 669

Read J.I., Hayfield T., 2012, MNRAS, 422, 3037

Hopkins P., 2013, MNRAS, 428, 2840

Number of neighbours

- SPH approximates an integral over local properties with a sum over the neighbours.
- Compact support for the smoothing kernel.
- We try to fix the number of neighbours (say ~ 50 in 3D).
- Naïvely, we expect by increasing the neighbour number, we should better approximate the integral



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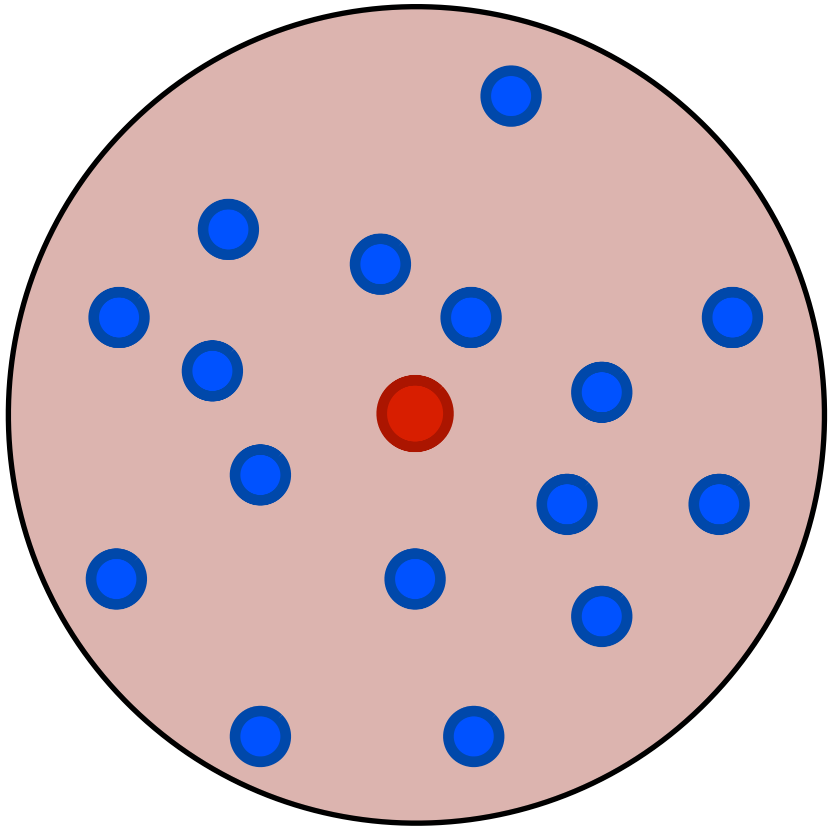

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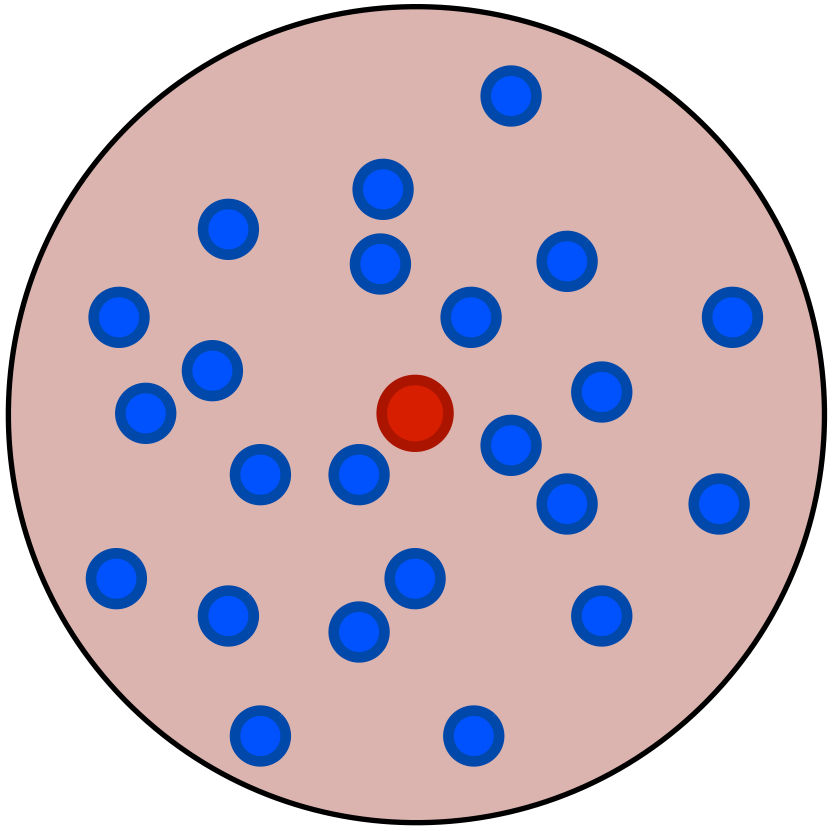

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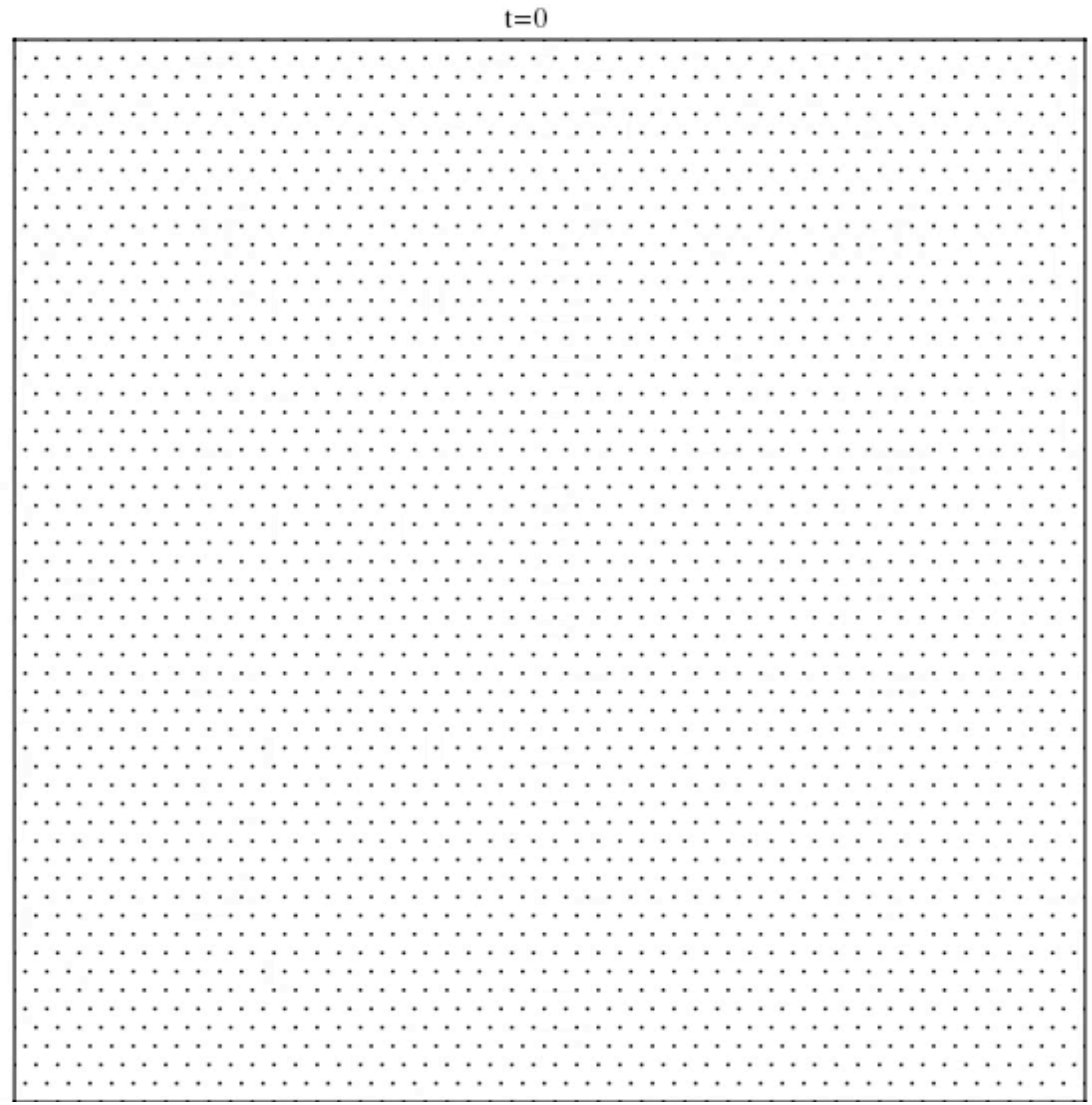
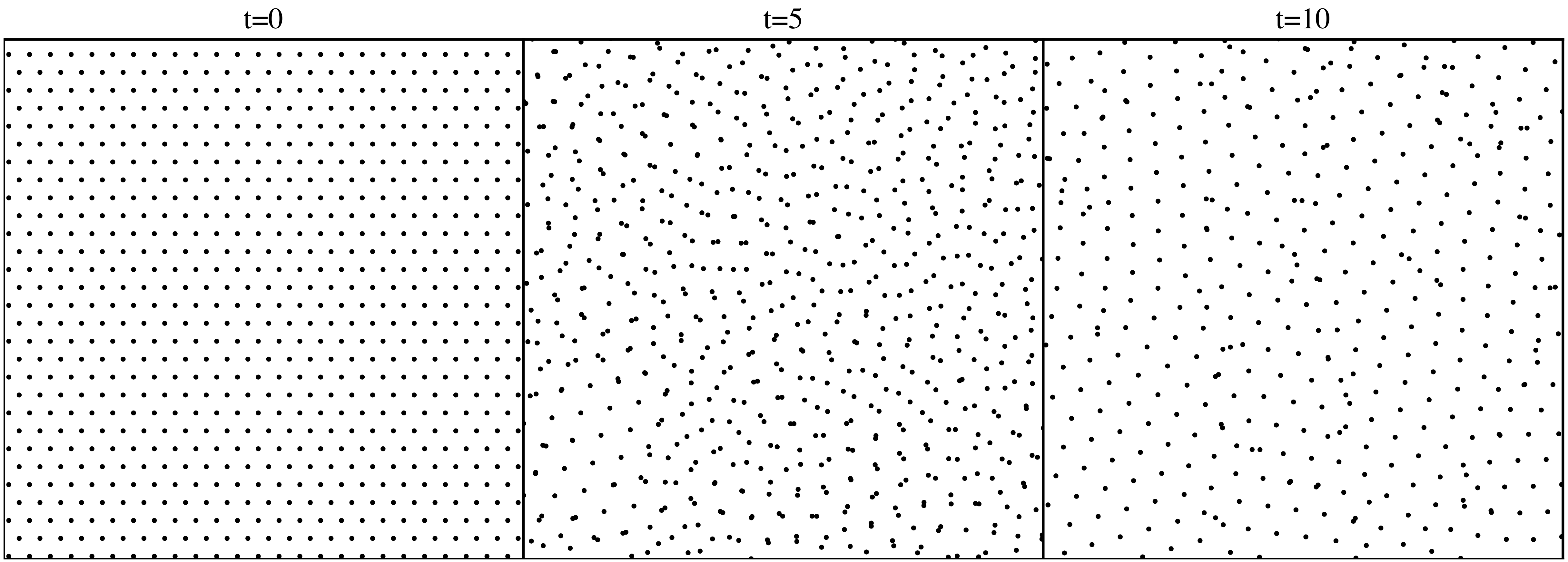


The smoothing kernel

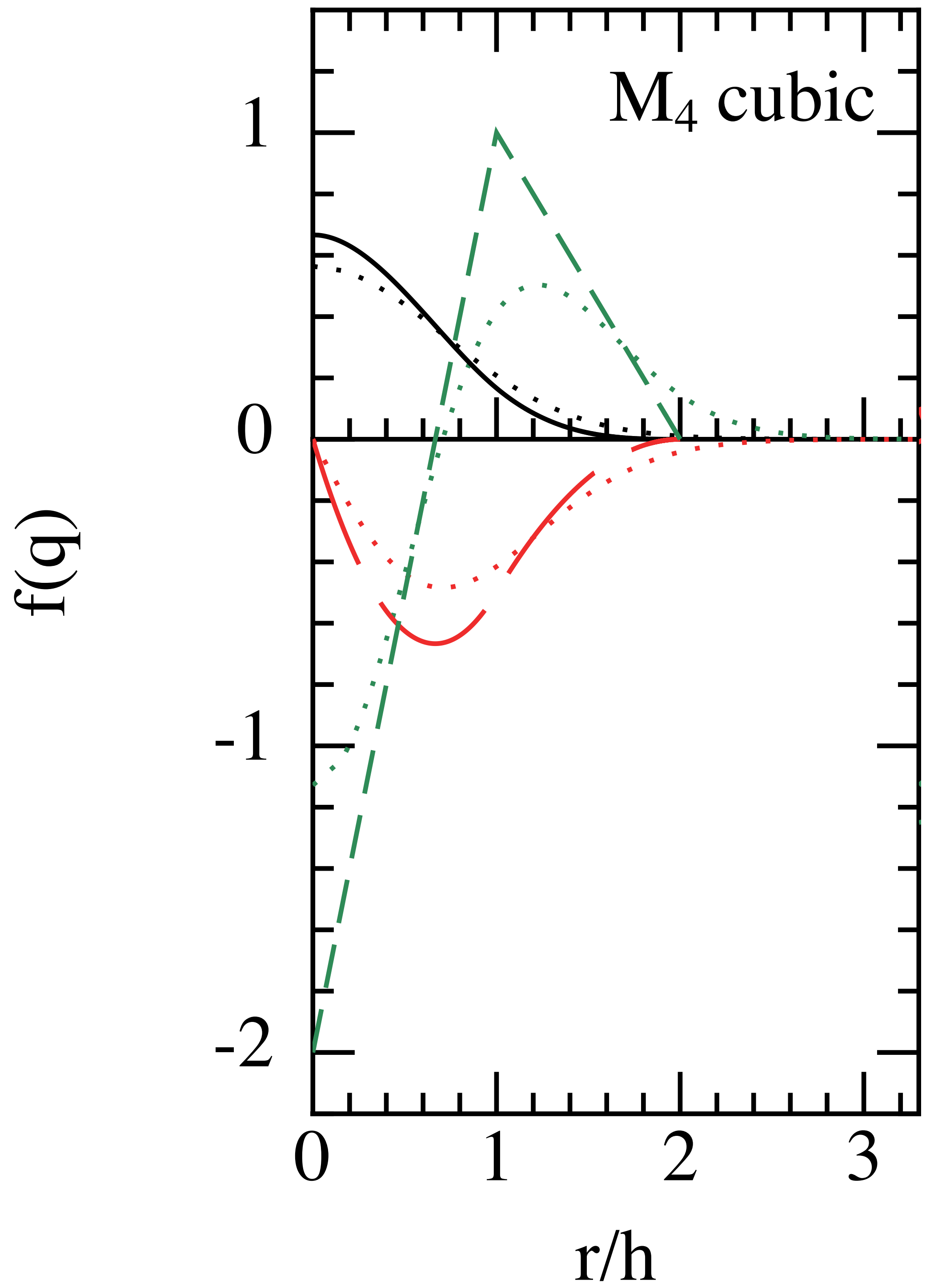


Close packed lattice with N_{neigh} ~100

Particles start to pair up.

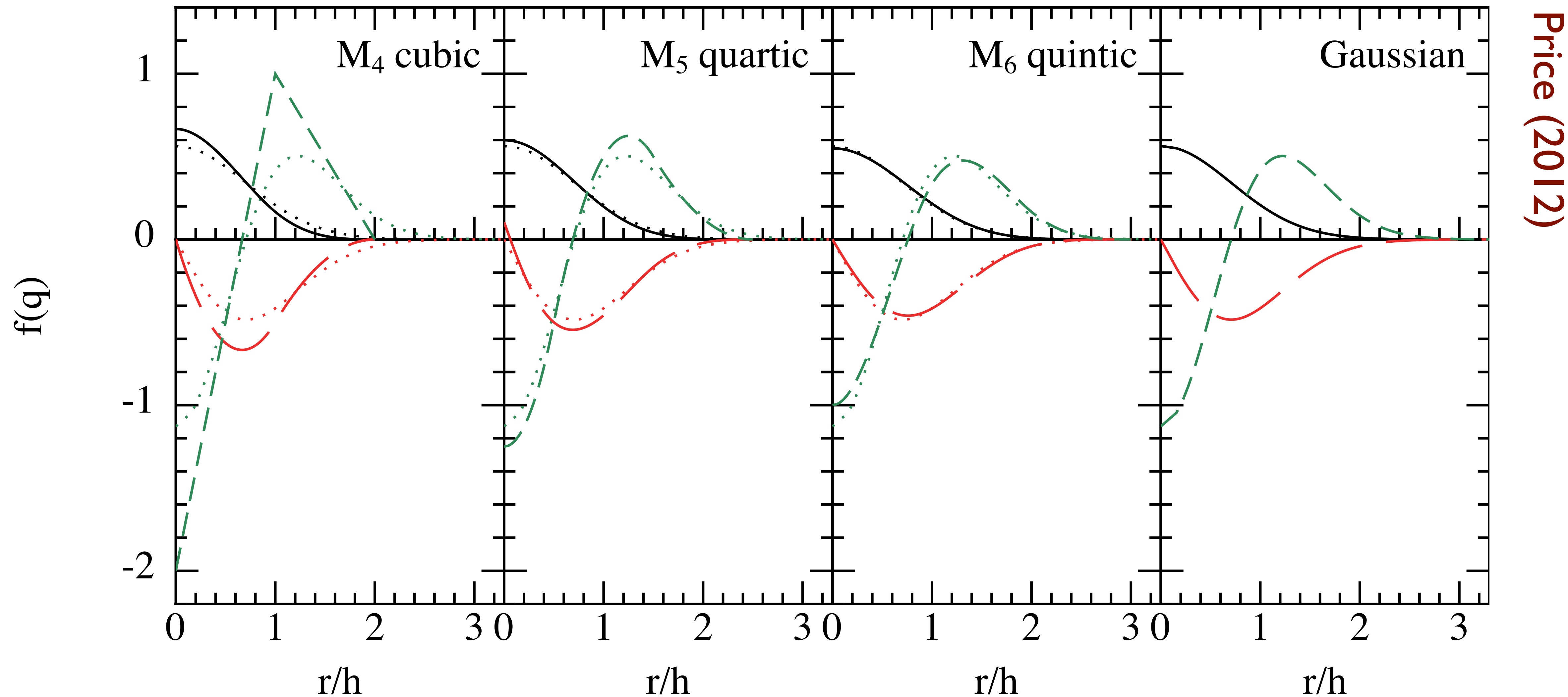
End up with about half the resolution you where aiming for...

The smoothing kernel



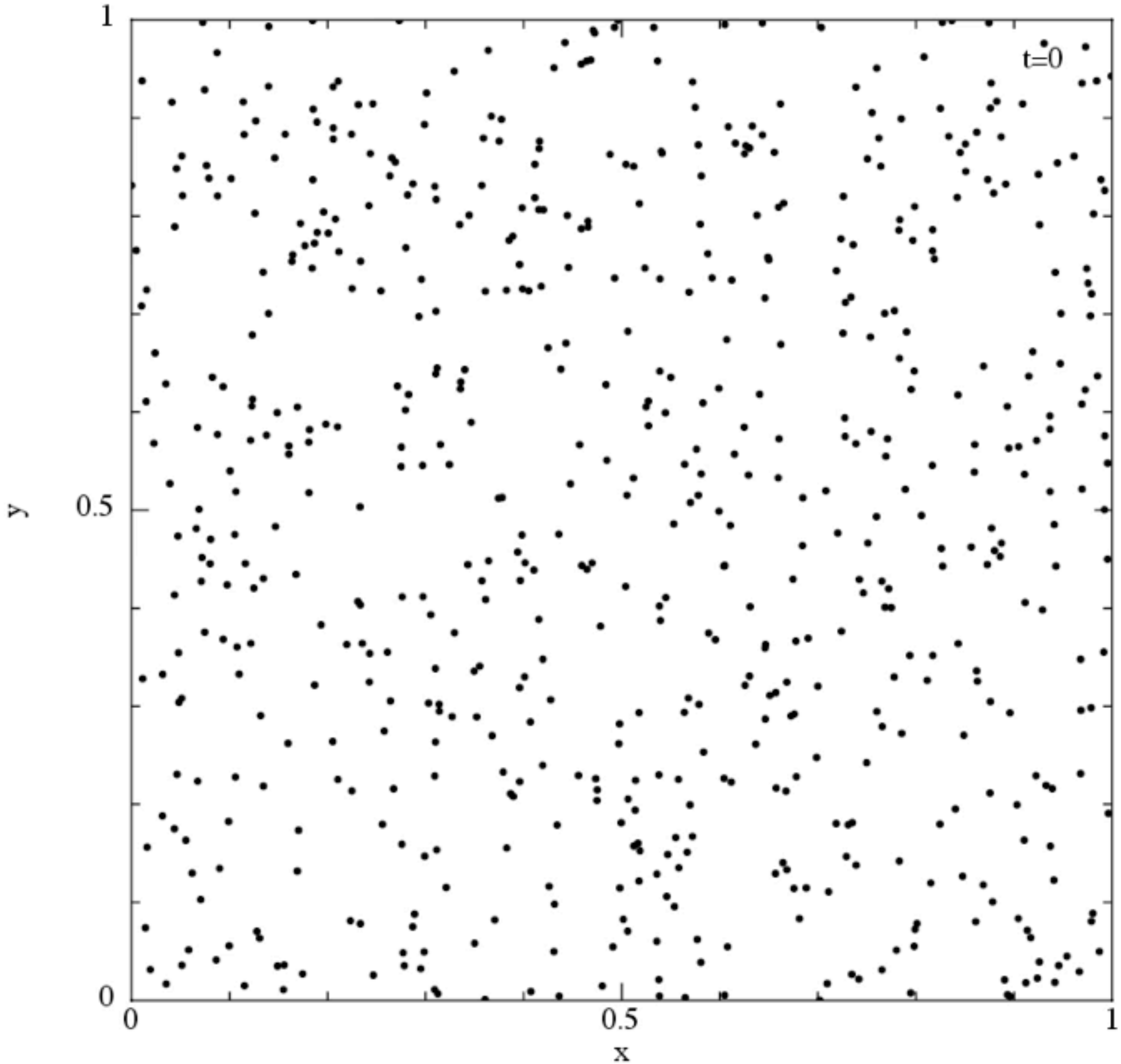
Price (2012)

The smoothing kernel



- If you want to use more neighbours, use a **higher order** kernel
- M_6 ("quintic" kernel) truncates at 3h (not 2h, as in M_4).
- Does **not** mean that the resolution is less!

Random particle positions initially

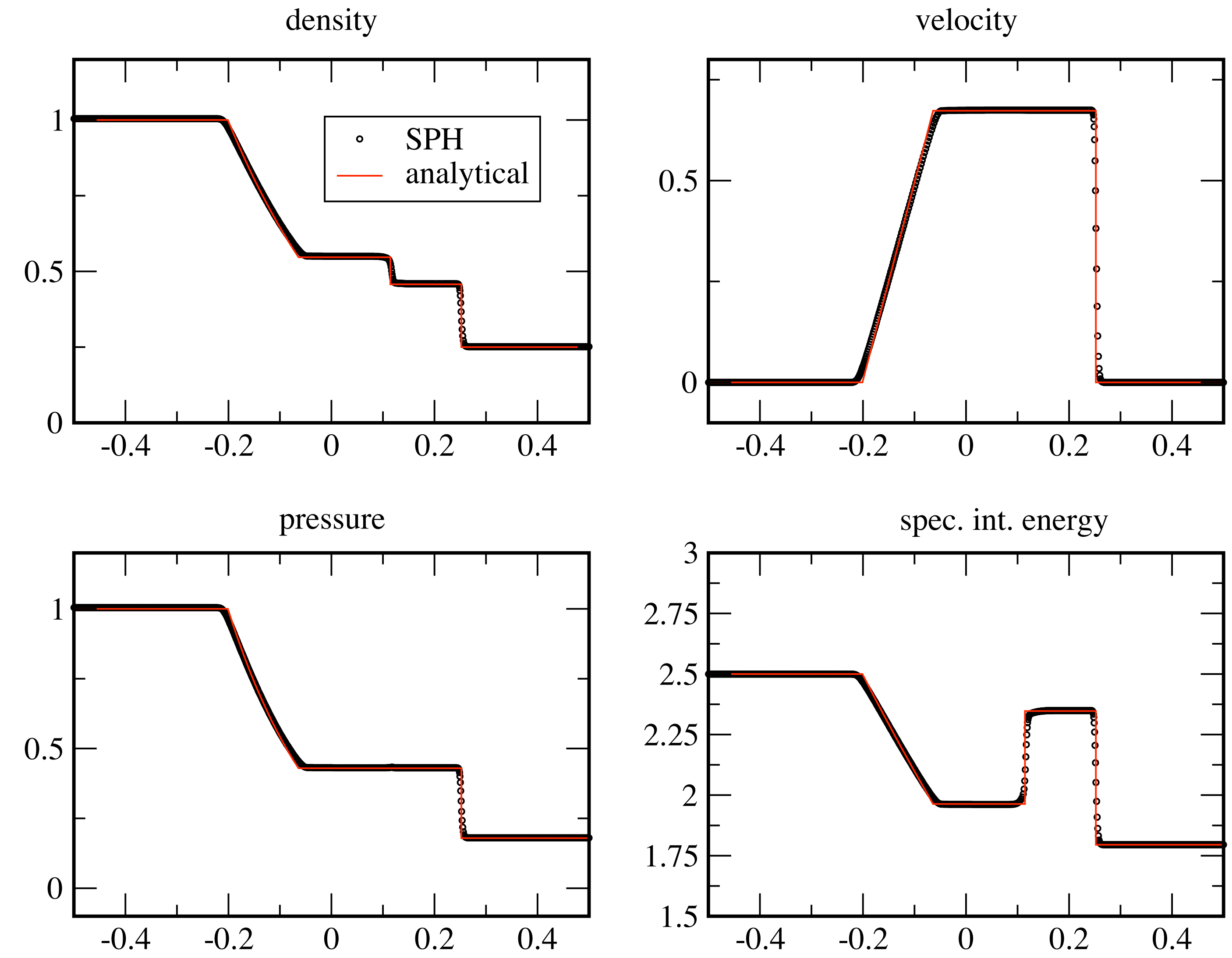


Price (2012)

Can SPH capture shocks?

In one dimension

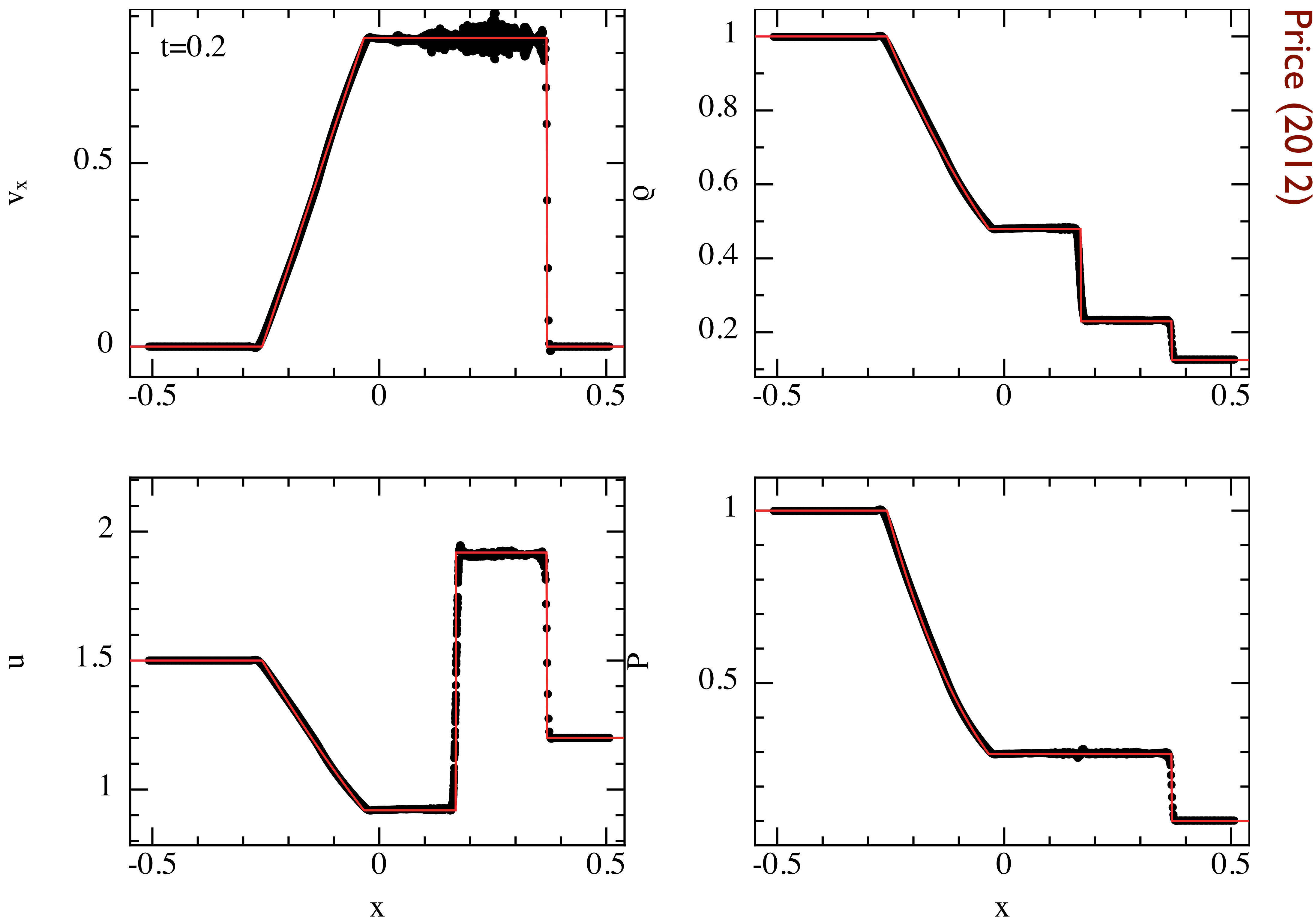
- Standard SPH uses artificial viscosity (AV) to treat shocks.
- Good match with the analytical solution.
- SPH smooths out the discontinuities to around a few h.
- Not as sharp as a high order grid code.



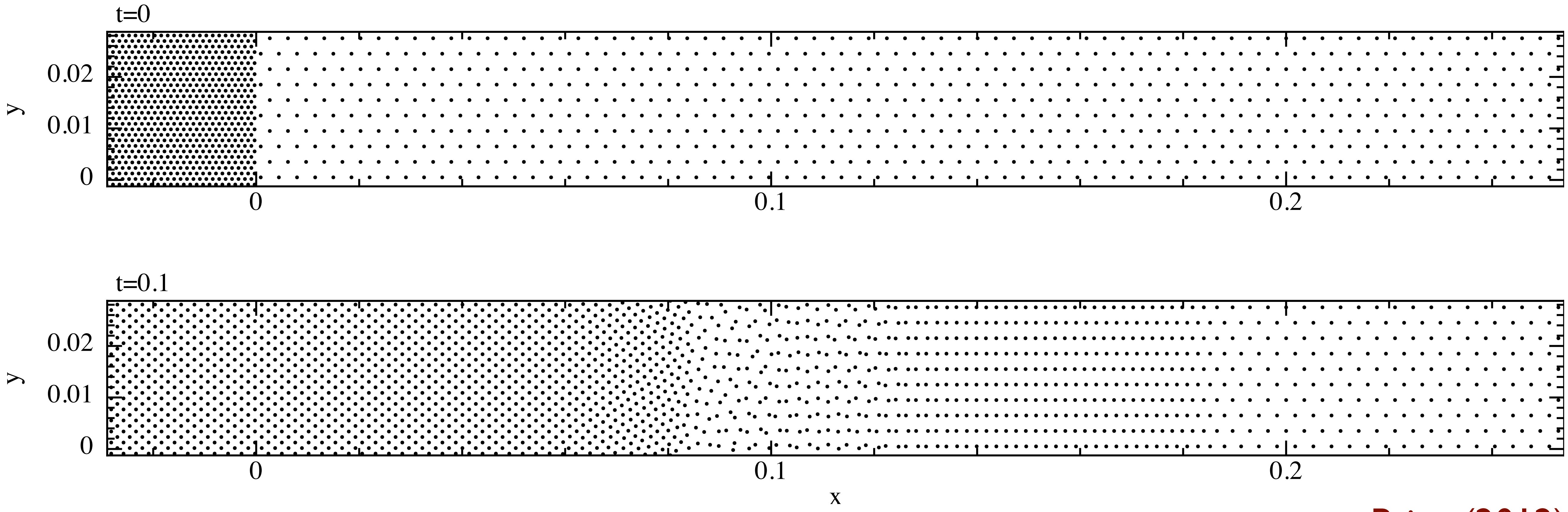
Rosswog (2009)

In higher dimensions

Sod shock (2D) M4 “cubic spline” kernel



Re-meshing...

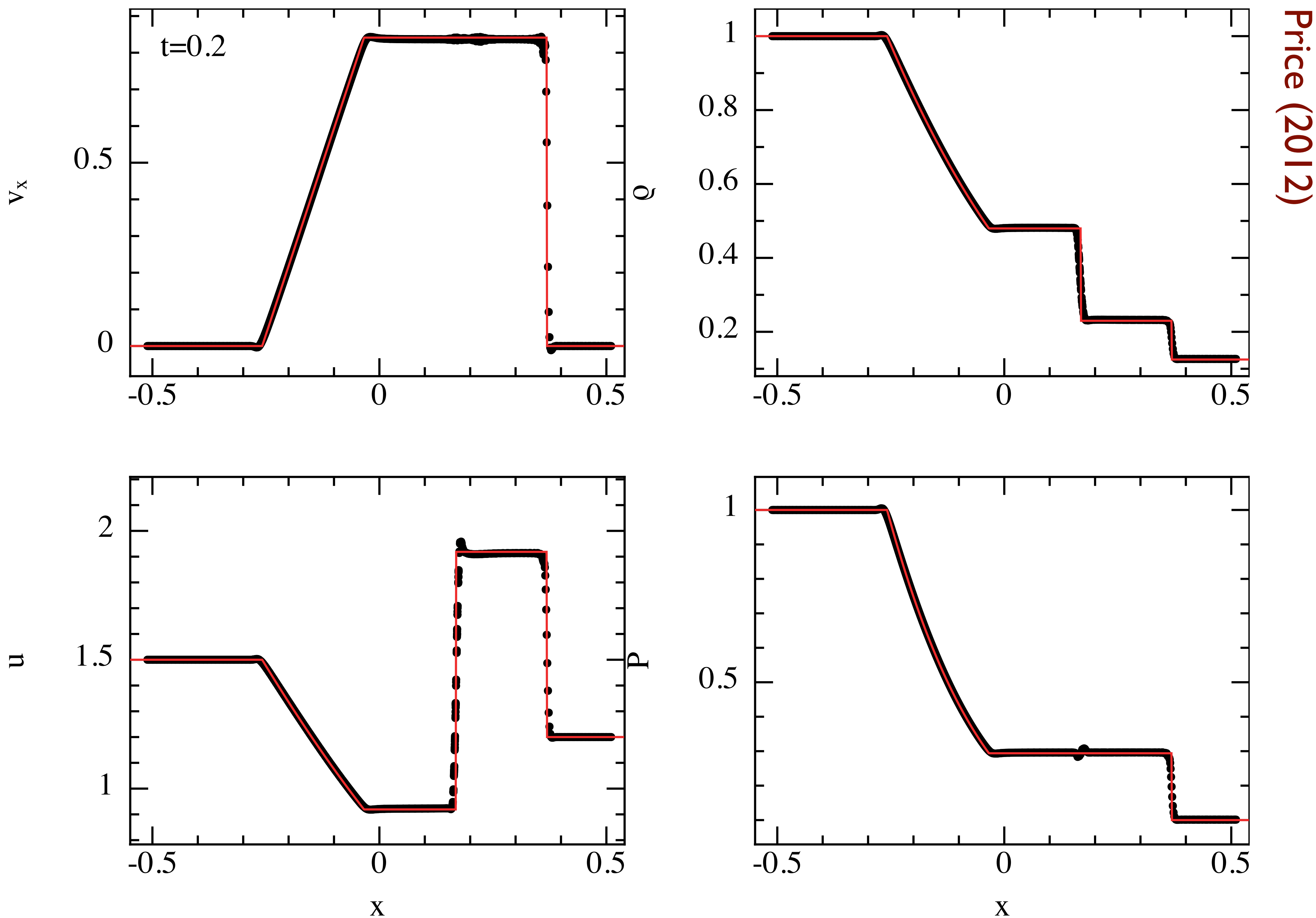


Price (2012)

- Re-meshing after the shock introduces errors in the velocity.
- You can't see this in 1D, since there's no free direction into which the particles can move.

Can we fix this?

Sod shock (2D) **M6 “quintic” kernel**



Second role of viscosity

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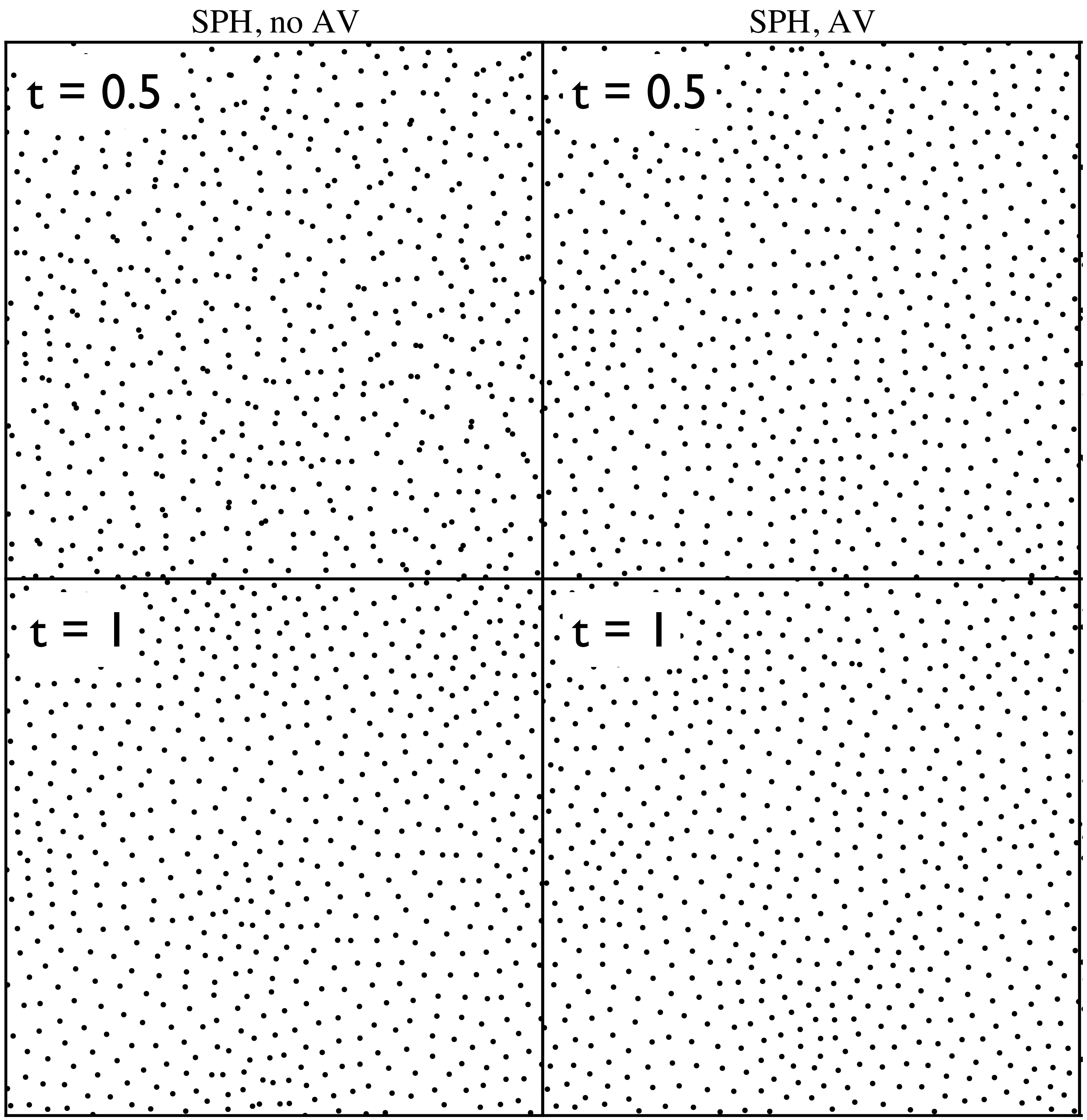
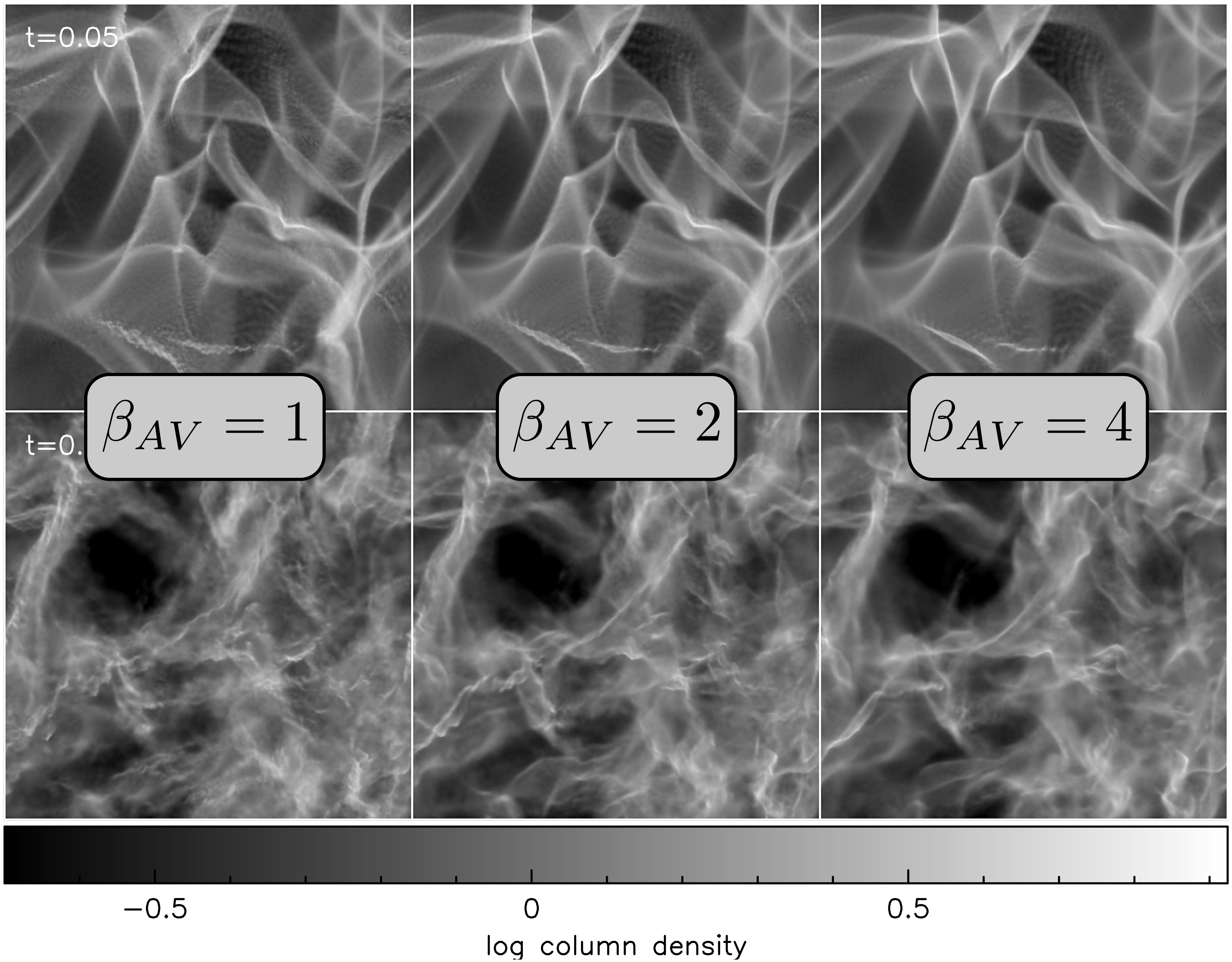
- Settling of a random particle distribution.
- AV helps to regularise the particle noise.



Price (2012)

Want lots of AV in strong shocks



Price & Federrath (2009)

Want AV to disappear when not needed

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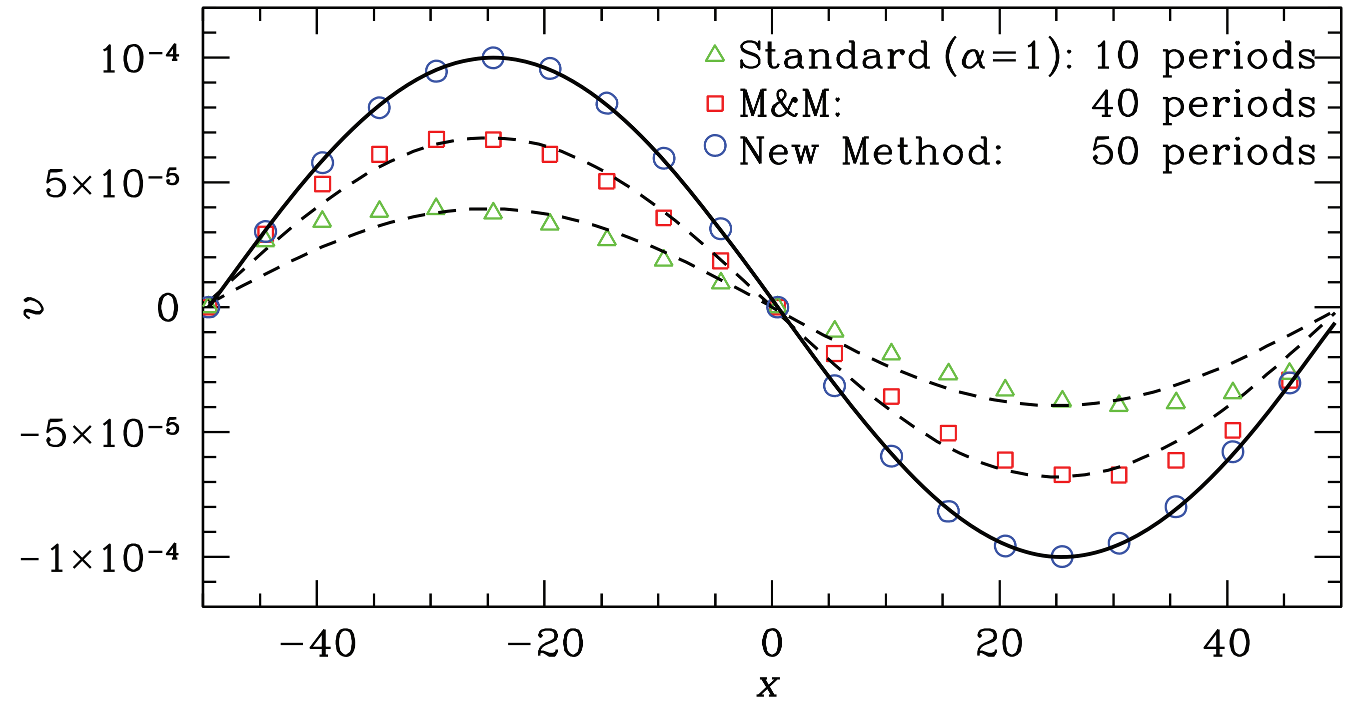
- Morris & Monaghan (1997) had a time-varying viscosity.
- Increases towards shocks, and decays after



Cullen & Dehnen (2010)
(Bauer & Springel 2011)

$$\dot{\alpha}_i = (\alpha_{\min} - \alpha_i)/\tau_i + S_i$$

$$S_i = \max\{-\nabla\cdot\boldsymbol{v}_i,\ 0\}$$

$$\tau_i = h_i/(2\ell c_i)$$

- Cullen & Dehnen (2010) improved on this dramatically (see also Read & Hayfield 2012).
- Have a 'inviscid' SPH away from where it is needed.

Cullen & Dehnen (2010)

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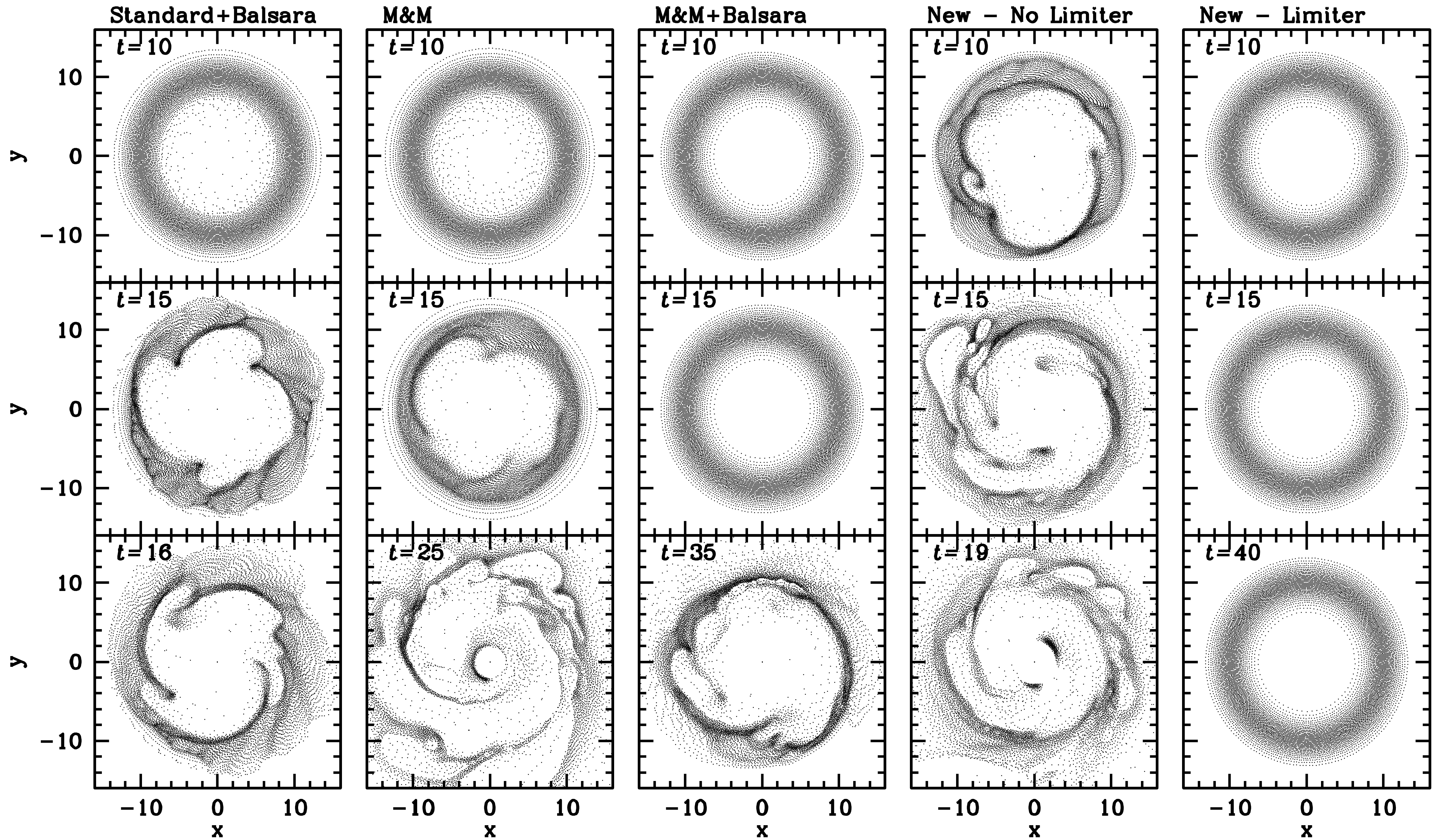

A brutal test...

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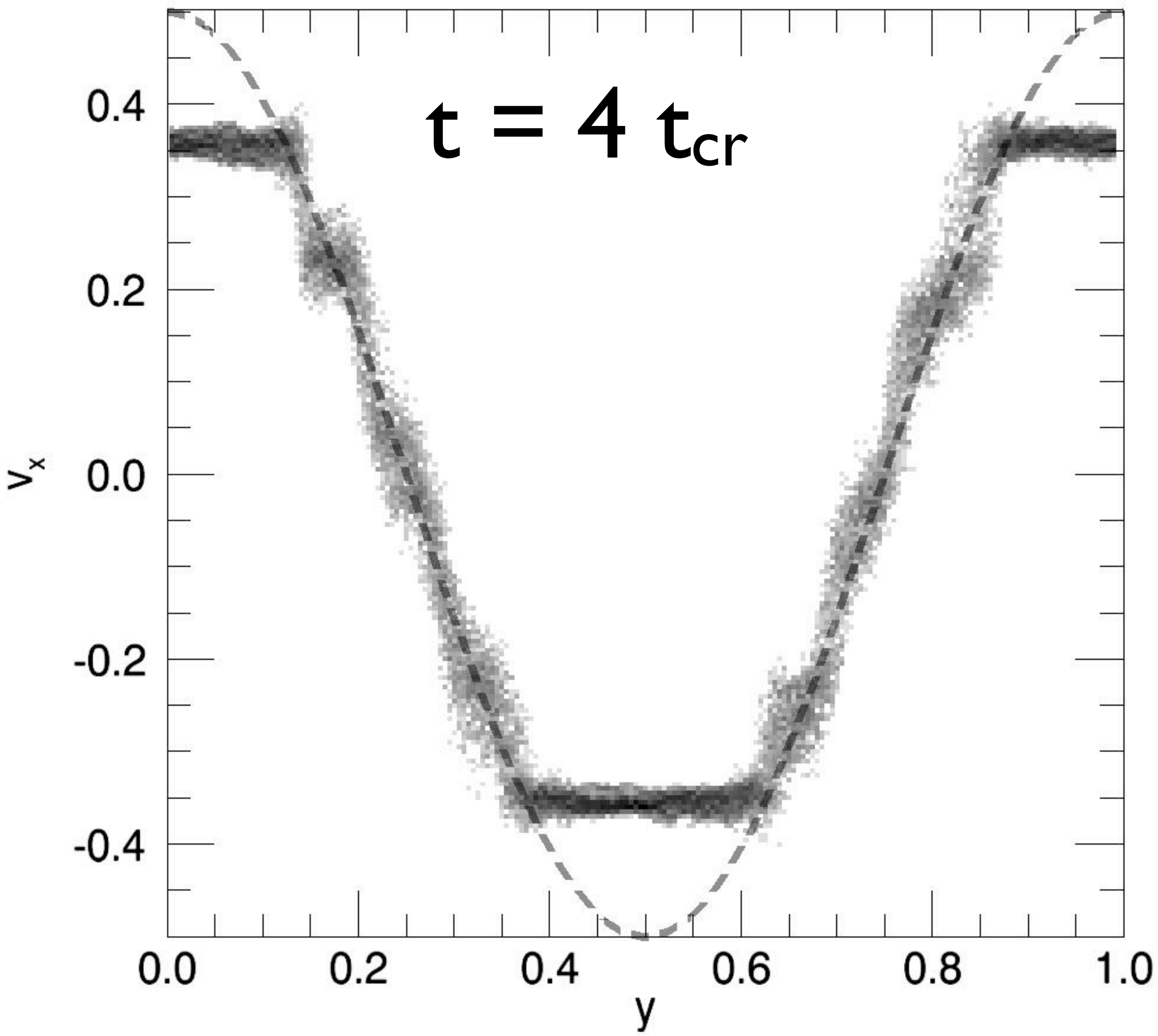
- Uniform periodic box with a shearing velocity:

$$v_x = A\, c_s \sin(2\pi y/L)$$

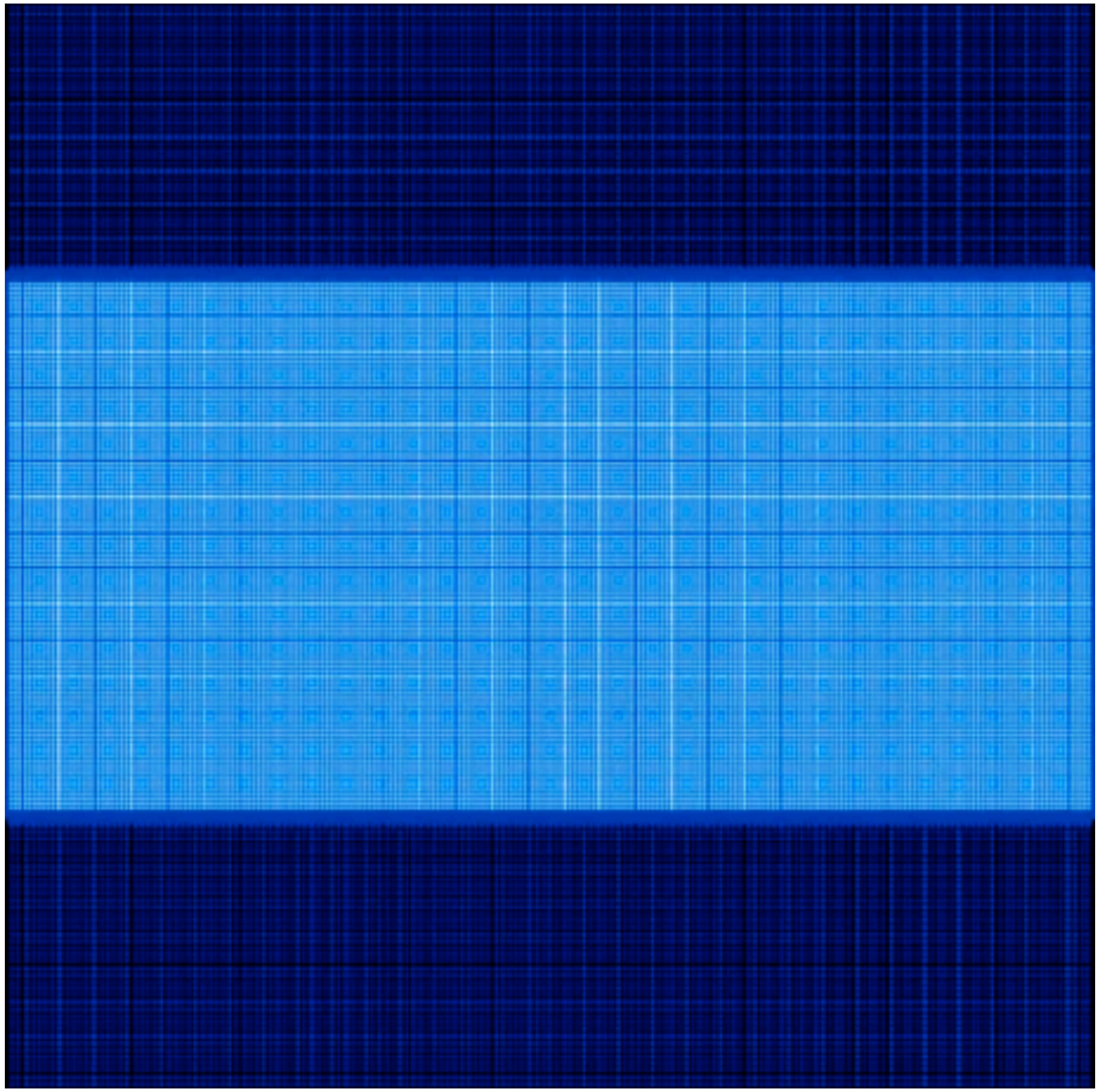
- Perfect for studying the viscosity, particle re-meshing, and the summation noise.



Abel (2012)

Resolving instabilities: KH

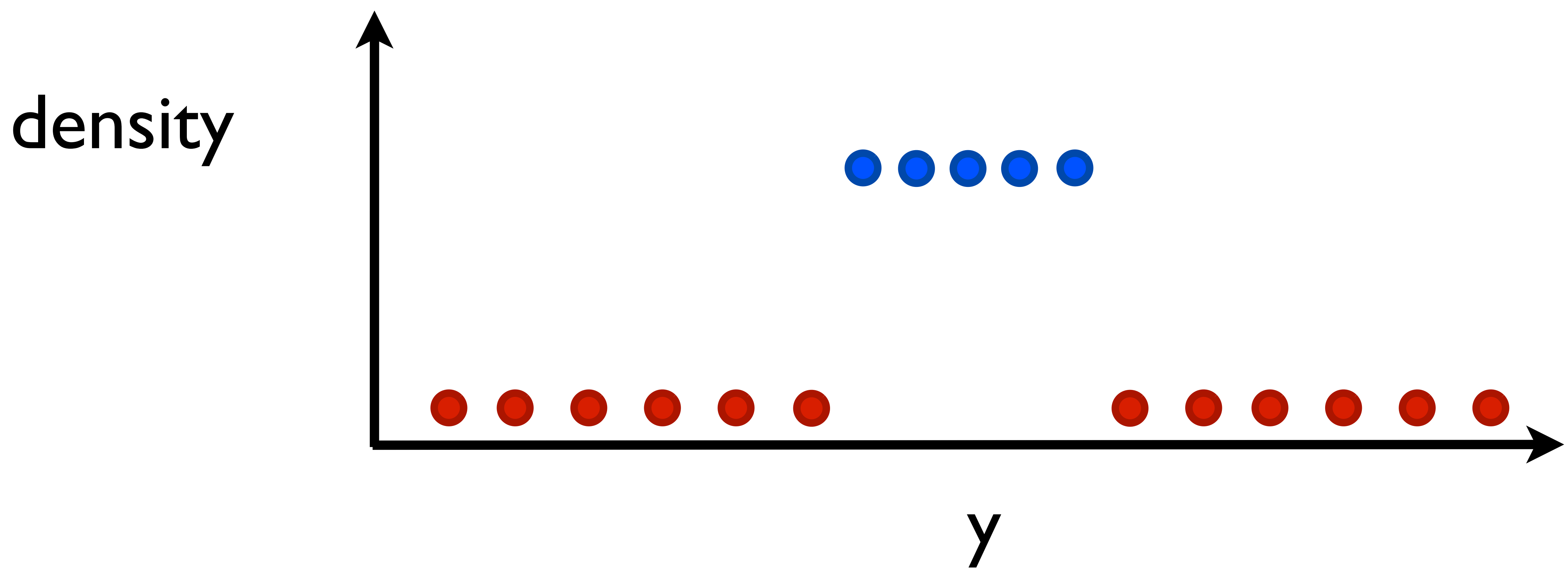
- For small amplitude perturbations the rolls don't grow.
- Looks as if the two fluid layers are pushing each other apart.
- What's going on?



Hopkins (2013)

What is the problem with the KH?

The ICs think you've got:



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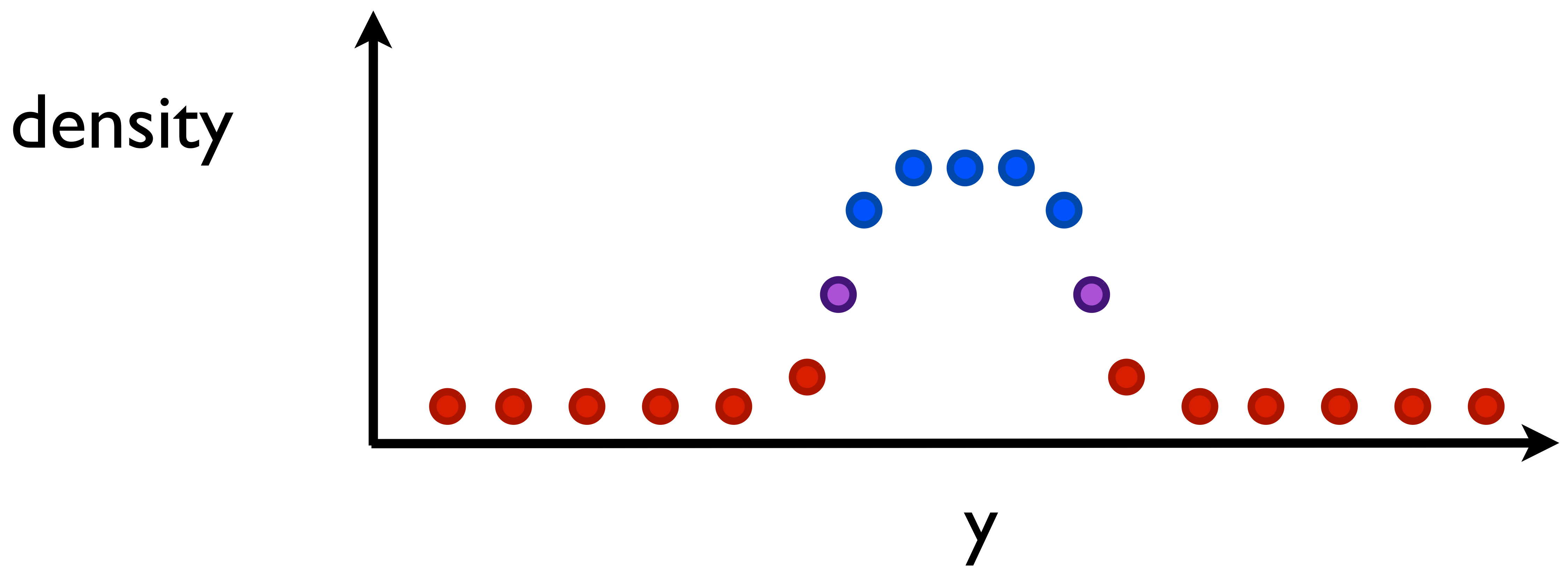
The ICs you've actually got:



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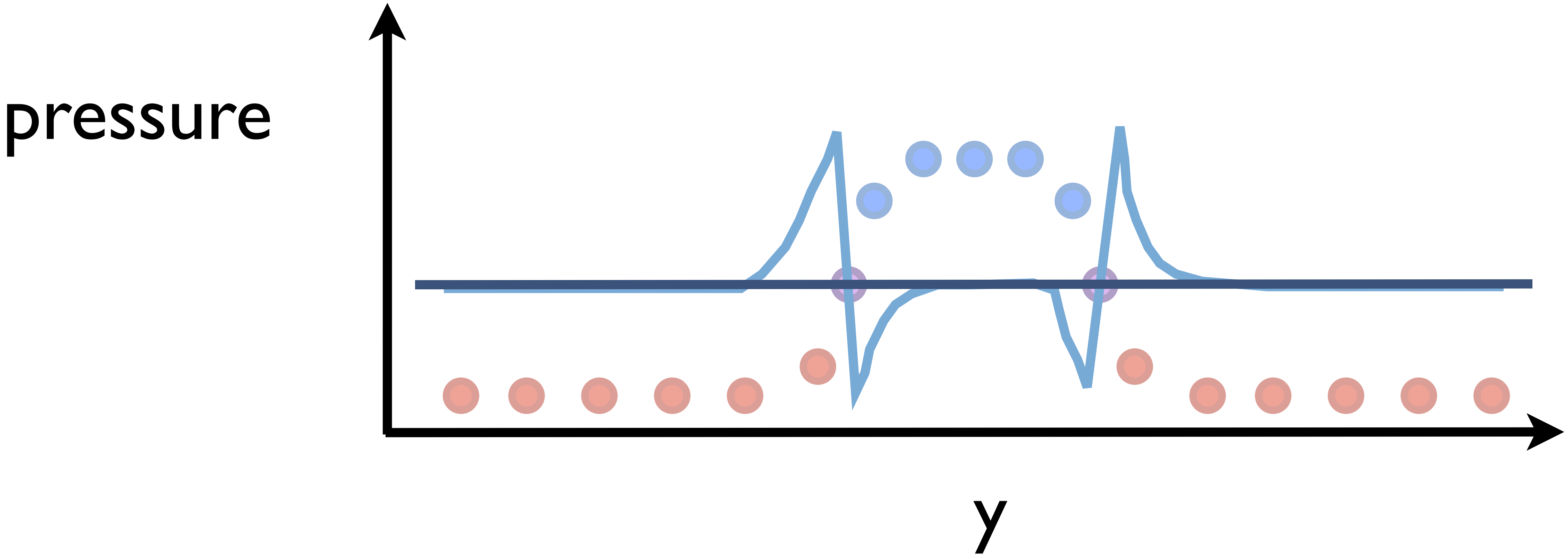

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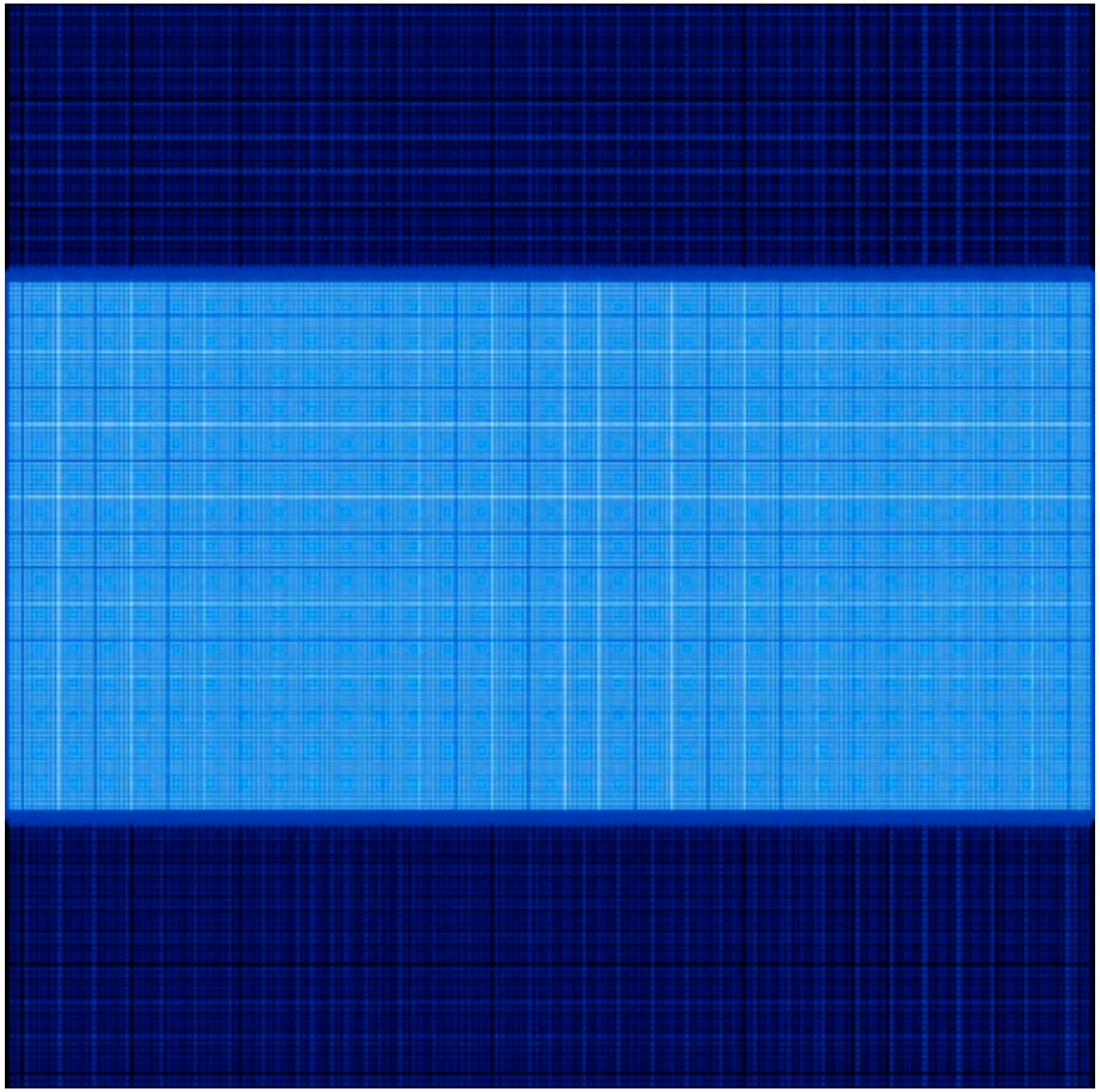


Hopkins (2013)

Solving the mixing problem

A cure:

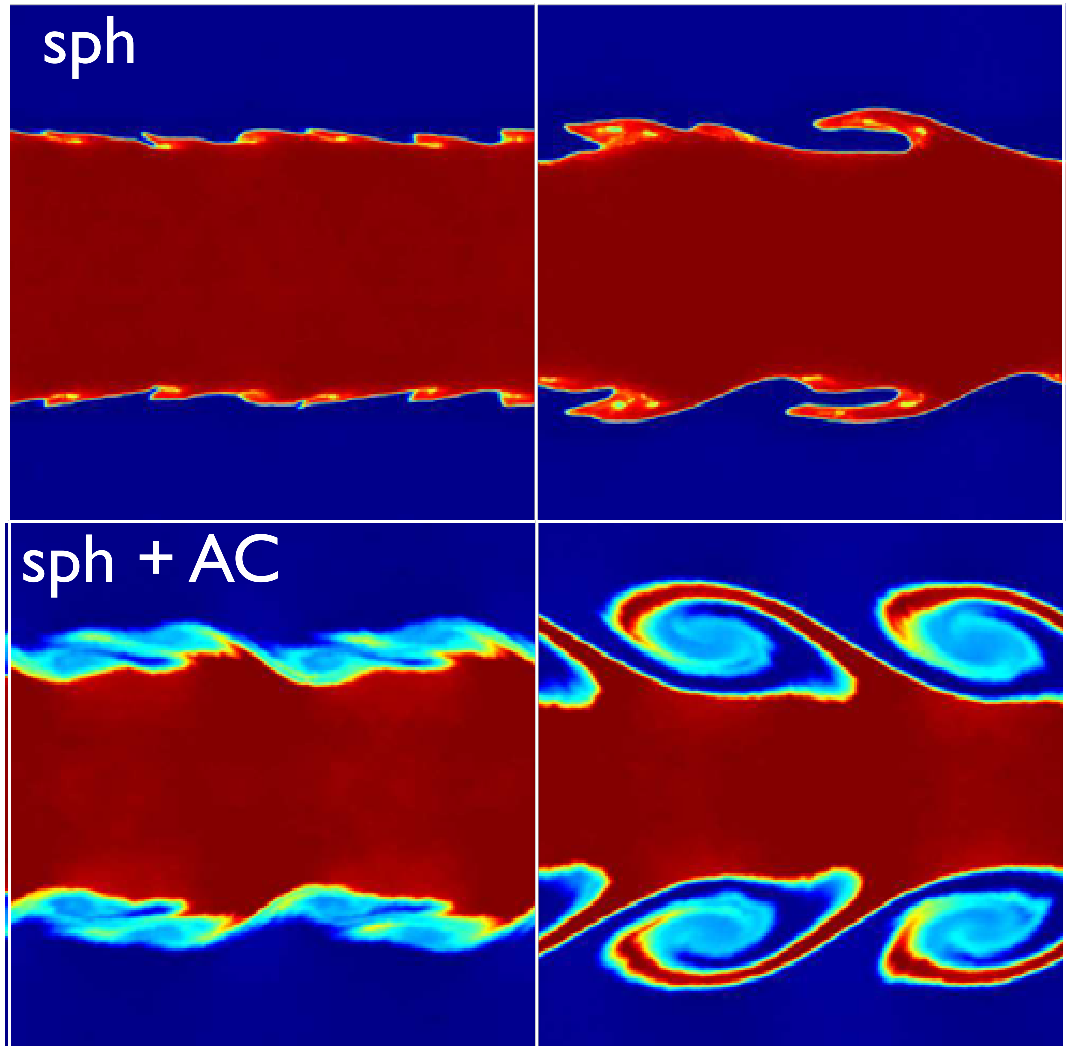
- Introduce a conductivity term (Price 2008; Wadsley).

$$\left(\frac{\mathrm{d}A_i}{\mathrm{d}t}\right)_{\mathrm{diss}} = \sum_j m_j \frac{\alpha_A v_{\mathrm{sig}}}{\bar{\rho}_{ij}} (A_i - A_j)\hat{\mathbf{r}}_{ij} \cdot \nabla W_{ij}$$

$$v_{\mathrm{sig}}^u = \sqrt{\frac{|P_i - P_j|}{\bar{\rho}_{ij}}}$$

- Removes the pressure blip by fluxing thermal energy (entropy) between the two states.

- Acts to work against the pressure gradient.



Price (2008)

Solving the mixing problem

Prevent SPH from seeing the blip in the first place:

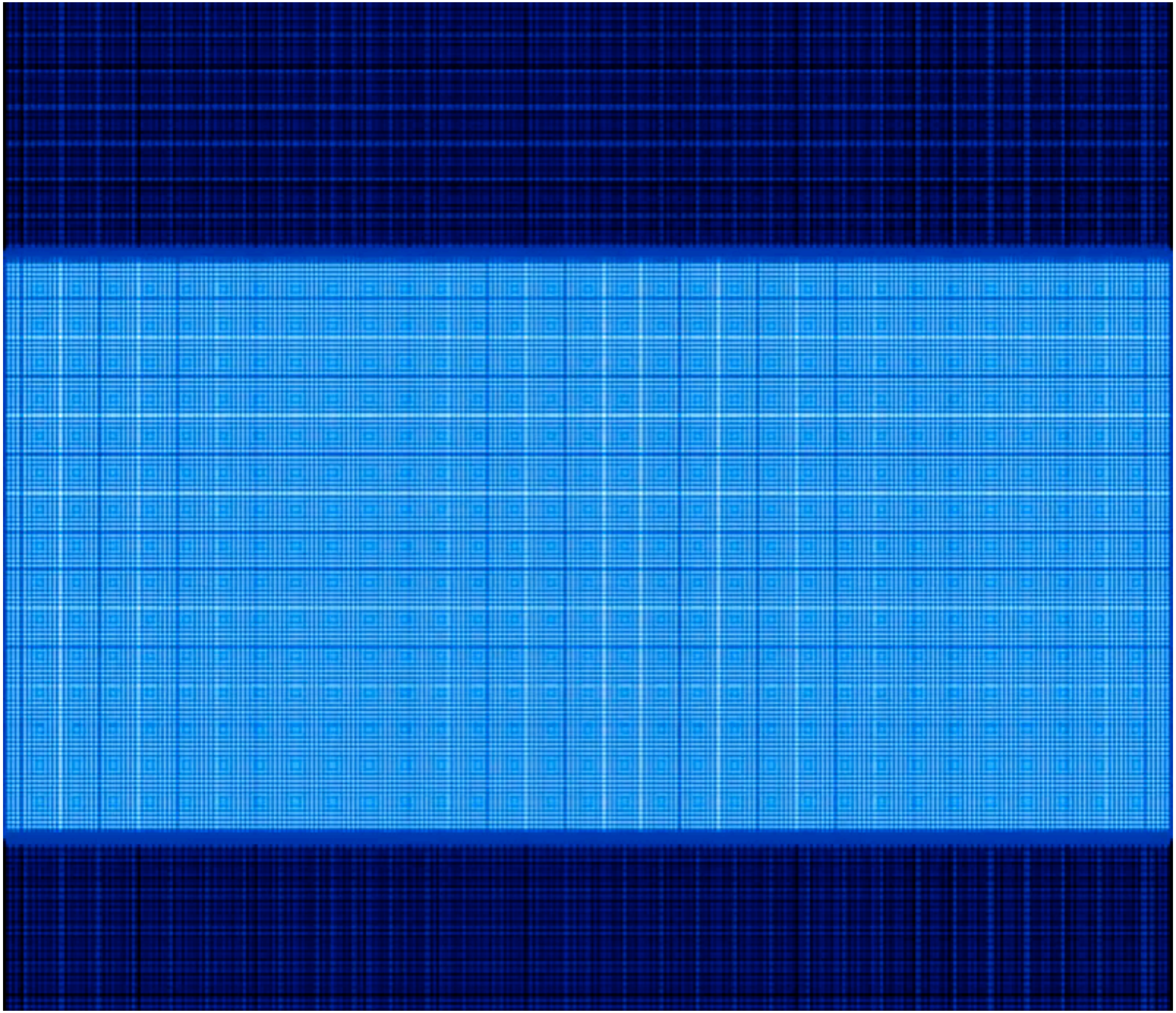
- Use a different definition of the density, which is derived from the pressure.
- Recast the momentum equation so that it is not as sensitive to the density.

Ritchie & Thomas:

$$\frac{d\mathbf{v}_i}{dt} = (1-\gamma)\sum_j m_j \left[\frac{u_j}{\langle \rho_i \rangle} \nabla W_{ij}(h_i) + \frac{u_i}{\langle \rho_j \rangle} \nabla W_{ij}(h_j) \right]$$

$$\langle \rho_i \rangle = \frac{\langle P_i \rangle}{(\gamma - 1) u_i} = \frac{\sum_j m_j u_j W_{ij}(h_i)}{u_i}$$

- Saitoh & Makino (2012) and Hopkins (2013) take a similar approach.
- Have a more conservative form of the equations than Ritchie and Thomas.



Hopkins (2013)

Is this a fair test?

- In many respects this is an 'unfair' test for SPH, as it starts with ICs that are alien to the formalism.
- When the density contrast is abrupt, grid codes also have problems converging.
- Primary roll displays secondary rolls -- seeded by grid noise.
- McNally et al. 2012 propose to smooth the ICs.
- SPH can handle this better (but AC or the pressure fix is still recommended.)

Tom Abel's 'fix'

Abel (2012)

- Tries to only do the particle forces when there **is** a pressure gradient.

$$\frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = -\sum_{j=1}^{N} m_j \left[f_j \frac{P_j - P_i}{\rho_j^2} \nabla_i \bar{W} \right]$$

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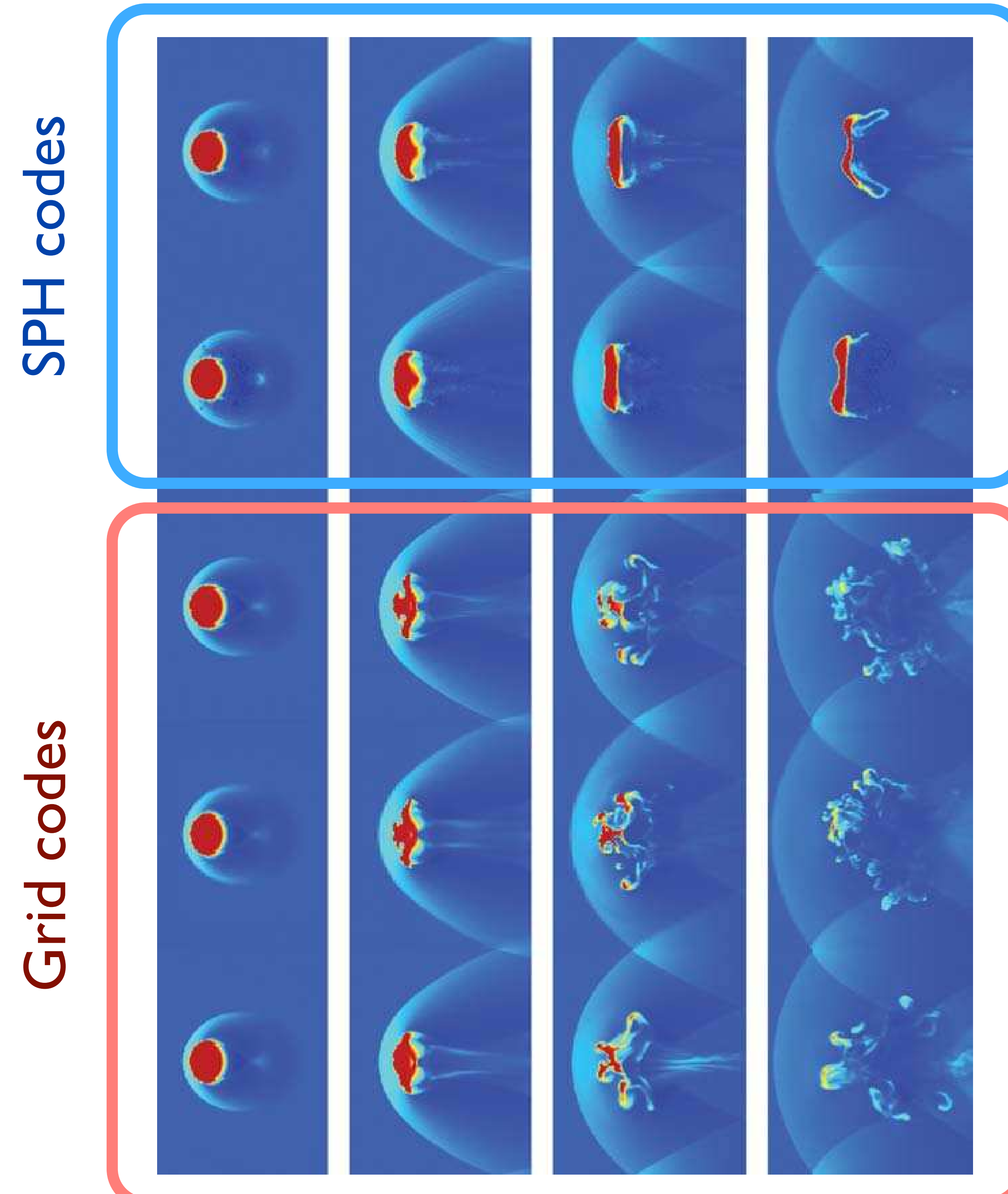
Do not use this fix

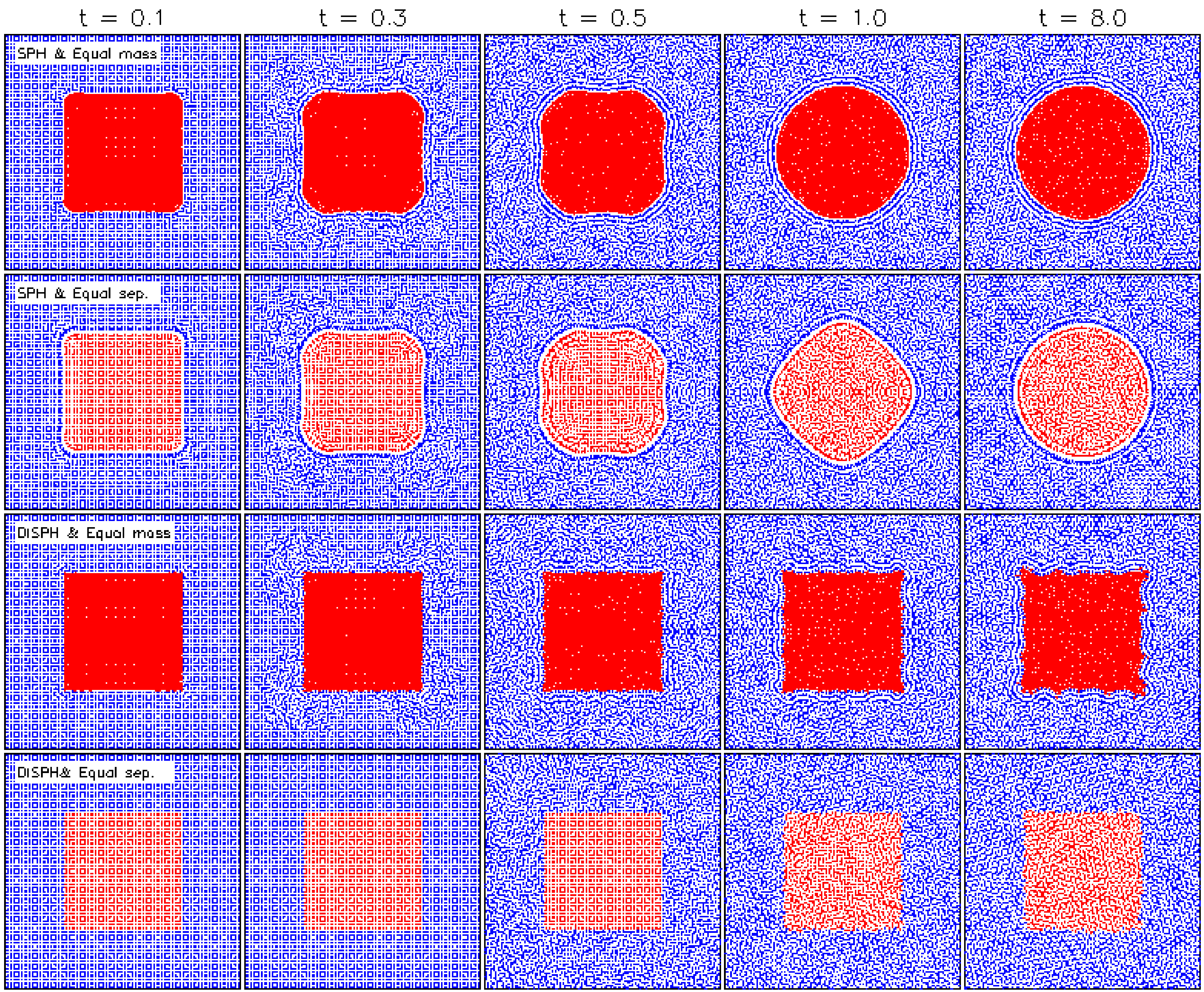
Similar problem (easy to set up!)

The 'blob' test:

- Dense sphere in pressure equilibrium with a low-density environment.
- Moves supersonically through the medium
- Grid code show that the blob is ripped apart.
- In SPH the blob survives for much longer.
- Conclusion: SPH is rubbish.



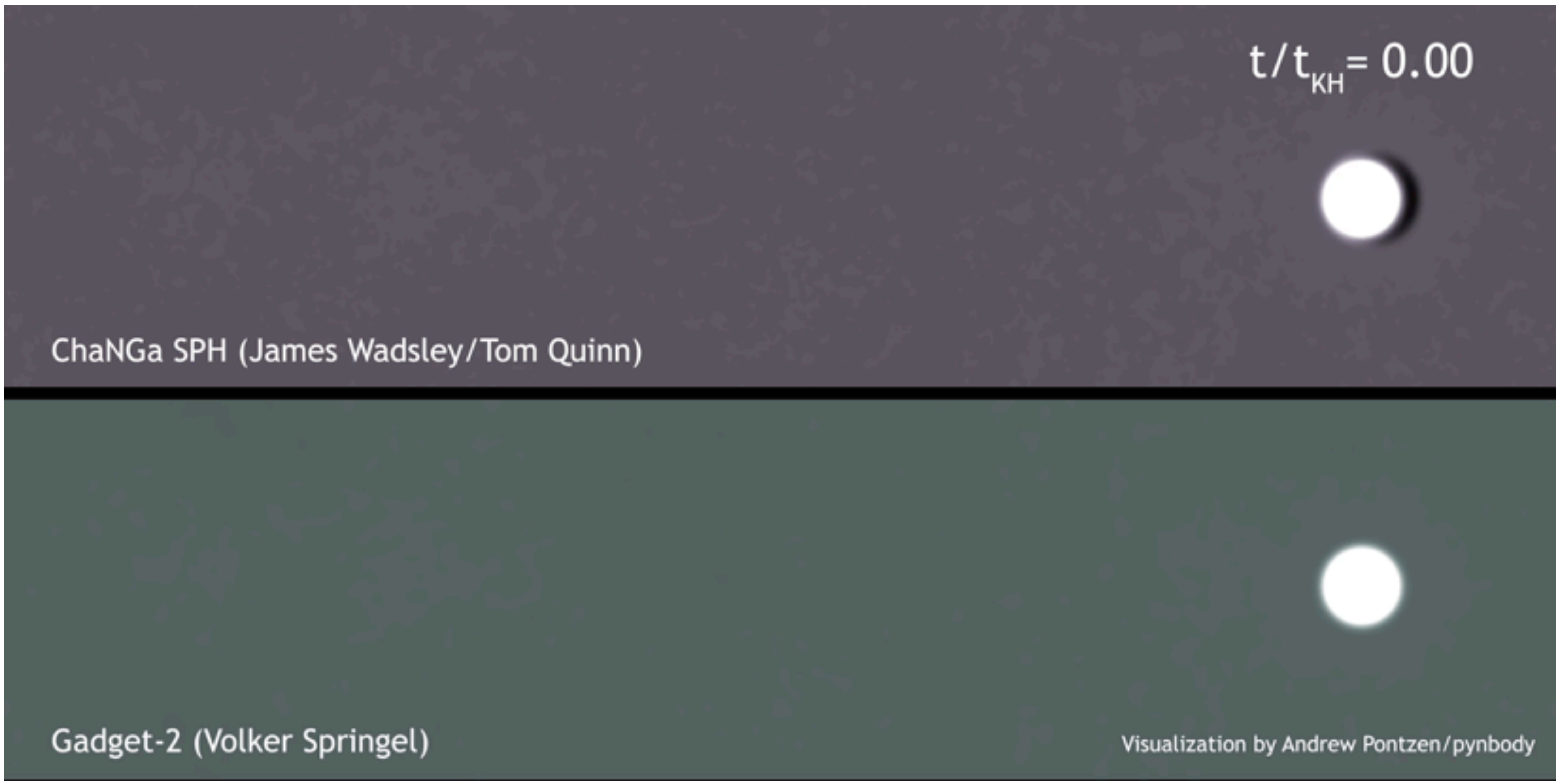
Agertz et al. (2007)



Saitoh & Makino (2012)

After a two line modification to the code...

The ‘blob’ test:



Fabio Governato

Conclusion

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... that doesn't mean that people have updated their code...