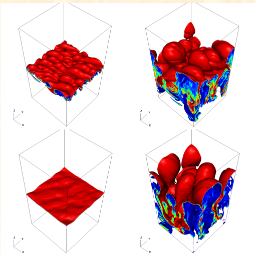
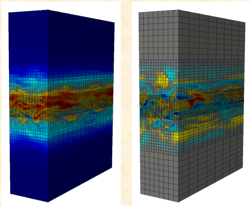
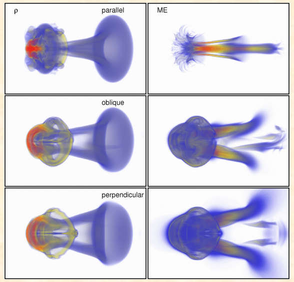
All about Athena



(Five lectures)





Jim Stone
Department of Astrophysical Sciences
Princeton University

Outline of lectures

Lecture 1. Introduction to basic algorithm

Lecture 2. Grids in grid codes

Lecture 3. Extra physics

Lecture 4. Radiation hydrodynamics

Lecture 5. Example applications; future developments

Outline for Lecture 3:

Additional Physics in grid codes.

- Optically-thin cooling
- Microscopic diffusion (viscosity, resistivity, conduction)
- Gravity
- Special relativity
- Dust particles

Adding more physics.

Adding more physics can require just small changes, or a complete re-write of the algorithm, depending on the physics.

Simple changes:
  Adding local source terms (e.g. cooling).
Moderate changes:
  Adding flux-divergence terms (e.g. viscosity, resistivity)
  Adding terms requiring elliptic solves (e.g. self-gravity)
Complete re-write:
  Adding new dynamical equations (e.g. special relativity, particles, radiation)

Adding simple source terms.

Simple source terms usually added via *operator splitting*.

1. Update flux divergence terms ignoring source terms
2. Update source term.

For Godunov methods, simple operator splitting:

1. formally makes scheme first-order in time
2. can lead to stability problems

Second-order can be achieved using multi-step methods (easy using van Leer unsplit integrator, or RK time stepping).

Stability issues can be addressed using implicit methods

Simple source term: Optically-thin cooling

Adds source terms to energy equation: $\frac{\partial E}{\partial t} + \nabla \cdot \mathbf{F}_E = -\rho^2 \Lambda(T) + \rho H$

Where Λ(T) is per-particle cooling rate, H is per particle heating rate.

Depending on cooling function, terms are usually nonlinear in T, and very stiff. *Forward Euler* differencing requires very small Δt

Better to use *Crank-Nicholson* (semi-implicit) differencing, where source terms are calculated at both current and advanced time (using E^n and E^{n+1}).

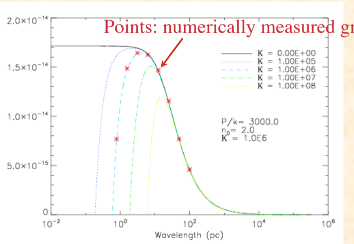
Not difficult to add cooling directly to integrator in Godunov methods by adding cooling term to calculation of L/R-states, and every partial update.

Warning: easy to add cooling, but makes physics of MHD much more complex. For example, need to add thermal conduction to be able to resolve Field length to get correct dynamics with cooling instability.

Moral: It takes work to really understand what is going on in both the physics and numerics.

Example: thermal instability. Adding heat conduction is crucial.

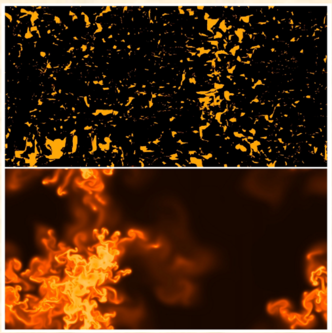
Points: numerically measured growth rate for exact eigenmode



Without conduction:

- do not get growth rate correct;
- too much small scale structure

Nonlinear saturation at 200 Myr



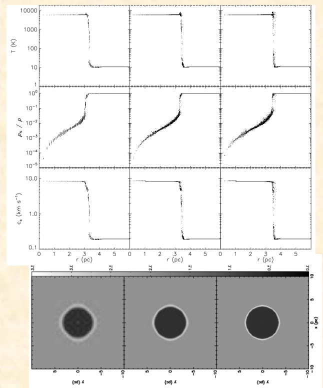
No conduction

With conduction

Example: overcooling at interfaces

Growth of spherical D-type ionization front in uniform medium. Plot subsample of points at some late time:

64^3 128^3 256^3



- Inevitably there are some points that lie between hot diffuse and cold dense phases.
- Cooling rates in these cells overestimated: must be limited.
- Alternative: need front tracking methods

Krumholz, Stone, & Gardiner 2007

Viscosity and thermal conduction

Momentum equation: $\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \mathbf{F}_{\rho \mathbf{v}} = \nabla \cdot (\nu \nabla \mathbf{v})$

Energy equation: $\frac{\partial E}{\partial t} + \nabla \cdot \mathbf{F}_E = \nabla \cdot (\kappa \nabla \mathbf{T}) + \nabla \cdot (\mathbf{v} \cdot (\nu \nabla \mathbf{v}))$

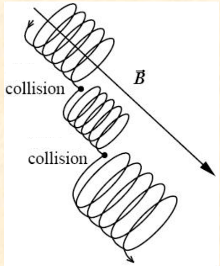
Both cases can be differenced using FTCS: $\frac{q_j^{n+1} - q_j^n}{\Delta t} = D\left(\frac{q_{j+1}^n - 2q_j^n + q_{j-1}^n}{\Delta t}\right)$

But stability constraint on FTCS for parabolic equations is very restrictive $\Delta t_D \leq (\Delta x)^2/4D$

Solution: (1) sub-stepping: take many steps at Δt_D for every MHD Δt.
(2) super-timestepping; size of sub-time steps varied.
(3) implicit differencing: $\frac{q_j^{n+1} - q_j^n}{\Delta t} = D\left(\frac{q_{j+1}^{n+1} - 2q_j^{n+1} + q_{j-1}^{n+1}}{\Delta t}\right)$

Latter leads to large sparse-banded matrices in 2D and 3D, which must be solved using, e.g. multigrid.

Anisotropic conduction and viscosity



In a magnetized, weakly collisional plasma the thermal conduction and viscous transport will be mainly along field lines. Produces *qualitative* change in the dynamics (magneto-thermal instability, heat-flux buoyancy instability, magneto-viscous instability).

Study through the inclusion of anisotropic viscous and heat fluxes.

$\mathbf{Q} = -\chi \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \nabla T$ Anisotropic heat flux (χ = conductivity)

$\mathbf{\Pi} = -3\eta \left(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{1}{3}\mathsf{I}\right)\left(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{1}{3}\mathsf{I}\right) : \nabla \mathbf{v}$ Anisotropic viscous stress tensor. "Braginskii viscosity"

Difference using FTCS with monotonic transverse temperature or velocity gradients (Parrish & Stone 2005; Sharma & Hammett 2007)
Can represent 1:1000 anisotropies in flux with any orientation of **B** on grid.

New dynamics in kinetic MHD: magneto-thermal instability

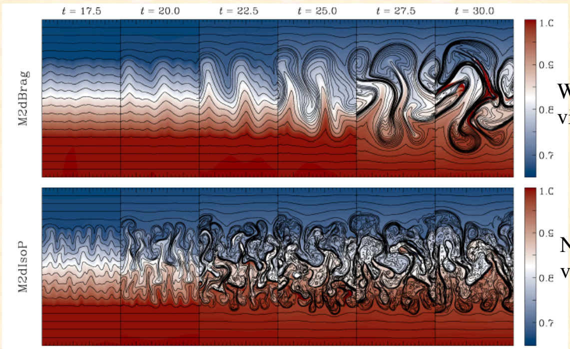
Balbus 2000; Parrish & Stone 2005; 2007

With anisotropic conduction, atmospheres with temperature decreasing upward are convectively unstable, regardless of entropy profile

cold

Increasing temperature

hot



With Braginskii viscosity

No Braginskii viscosity

Kunz et al. 2012

Colors = Temperature
Lines = B-field

Problems with Braginskii

Large pressure anisotropies in the plasma drive instabilities at microscopic (close to Larmor radius) scale.

When $P_{perp} << P_{para}$: firehose instability
$P_{perp} >> P_{para}$: mirror instability

But Braginskii gets the wrong growth rates for both. Moreover, fastest growth rate is near Larmor radius, which MHD simulations can never resolve.

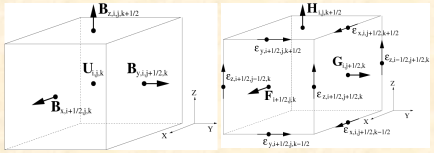
- Saturation of firehose and mirror at small scales can strongly affect MHD on large scales by tangling field and limiting P anisotropy
- Need sub-grid model for firehose and mirror at large β

Ohmic resistivity

Induction equation becomes: $\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J}) = 0$

J = current density

Over-riding concern is to keep div(B)=0. This suggests a CT differencing is required, using an "effective" EMF **E**=η**J** located at cell corners



Once again, time step constraint very restrictive: $\Delta t_\eta \leq (\Delta x)^2/4\eta$

Can use (1) sub-stepping, or (2) super-timestepping. Implicit CT differencing is complex.

Can be extended to ambipolar-diffusion and Hall regimes by appropriate definition of AD or Hall EMF.

MRI with ambipolar diffusion

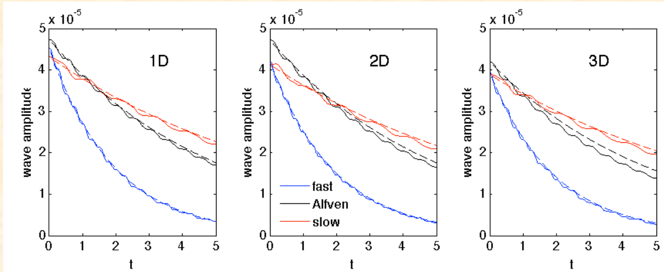
Generalized Ohm's Law:

Ohmic Hall AD

$$\mathbf{E} = \mathbf{V} \times \mathbf{B} - \frac{4\pi\eta \mathbf{J}}{c} - \frac{\mathbf{J} \times \mathbf{B}}{n_e e} + \frac{(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}}{c\gamma\rho_i\rho}$$

Implement both AD and Ohmic diffusion EMFs using CT.

Test: decay rate of linear waves.



dashed=analytic soln.
solid= numerical soln.

Gravity

With gravity, momentum and energy equations can be written as:

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \mathbf{F}_{\rho \mathbf{v}} + \nabla \cdot \mathbf{G} = 0 \qquad \frac{\partial E}{\partial t} + \nabla \cdot \mathbf{F}_E = \rho \mathbf{v} \cdot \mathbf{g}$$

Where **g**=gravitational accn, $\mathbf{G} = (\mathbf{gg} - g^2/2)/4\pi G$ gravitational stress tensor

For fixed gravitational potential (e.g. central star)

- Linear momentum is not conserved
- Total energy is conserved

So add source term to momentum equations using analytic form for acceleration, and add source term to total energy using mass fluxes and potential difference - conserves total energy exactly

For self gravity

Add source terms to momentum as divergence of gravitational stress tensor - conserves total momentum exactly. Add source terms to total energy using mass fluxes and potential difference.

Of course, must also solve Poisson's equation for the potential: use time average of Φ^n and Φ^{n+1} to ensure second order accuracy without solving PE twice per timestep.

Self-gravity: conserving momentum

Crucial to compute gravitational acceleration as a divergence of a flux rather than a source term in order to conserve momentum.

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \mathbf{F}_{\rho \mathbf{v}} + \nabla \cdot \mathbf{G} = 0$$

$$\mathbf{G} = (\mathbf{gg} - g^2/2)/4\pi G$$

Simple test: advection of self-gravitating hydrostatic sphere.

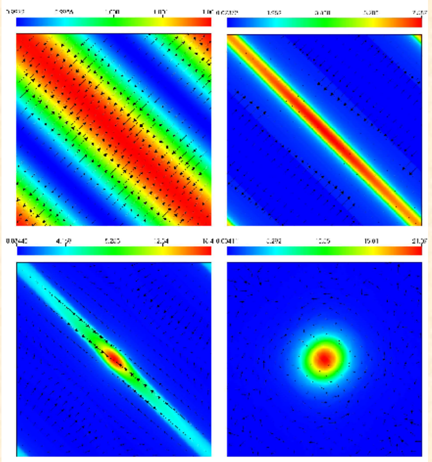
With source term: sphere comes to rest on mesh

With flux-divergence: sphere propagates at constant momentum

Self-gravity: conserving momentum

Another good test: gravitational instability of linear planar sound wave

With flux-divergence: total momentum conserved exactly.

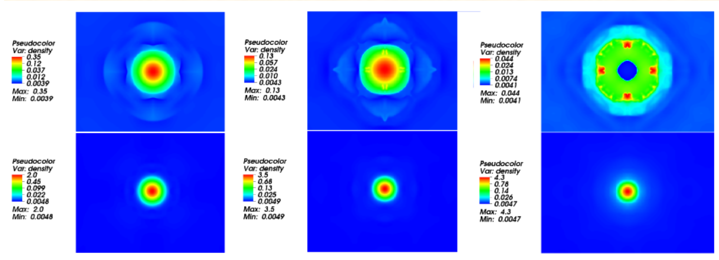


Self-gravity: conserving energy

With appropriate definition of the energy flux, energy equation can be cast in strictly conservative form (no source terms) with self-gravity.

One possible form: $\mathbf{F}_g = \frac{1}{8\pi G}\left(\phi\nabla\dot{\phi} - \dot{\phi}\nabla\phi\right) - \rho\mathbf{v}\phi$

Test, oscillations of γ=1.36, n=3 polytrope



Non-conservative algorithm

Conservative algorithm

Jiang et al. 2013

Special relativity

Such a substantial change to algorithm that it can be considered as writing a new solver rather than extending existing solver.

SR MHD equations can also be written in conservative form $\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$

But definition of conserved variables (and their fluxes) is more complicated:

$$\mathbf{U} = f(\mathbf{W}) = \begin{pmatrix} \gamma\rho \\ (w+b^2)\gamma^2 v_x - b_0 b_x \\ (w+b^2)\gamma^2 v_y - b_0 b_y \\ (w+b^2)\gamma^2 v_z - b_0 b_z \\ (w+b^2)\gamma^2 - P_t - b_0^2 \\ B_y \\ B_z \end{pmatrix}.$$

$b_0 = \gamma(\mathbf{B}\cdot\mathbf{v})$

$\mathbf{b} = \mathbf{B}/\gamma + \gamma(\mathbf{B}\cdot\mathbf{v})\mathbf{v}$

$w = \rho + [\Gamma/(\Gamma-1)]P$ (enthalpy)

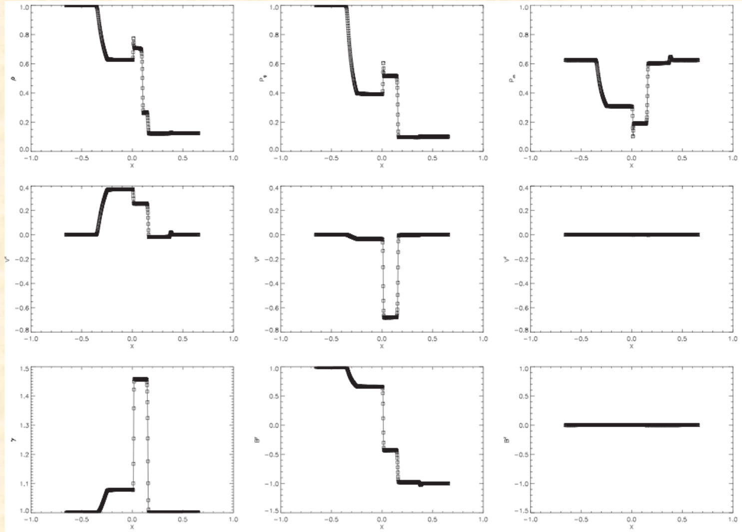
So overall integration algorithm remains the same

1. Reconstruction step
2. Compute fluxes with Riemann solver
3. van Leer unsplit integrator

But significant changes required in each step:

1. Conversion from conserved to primitive variables requires nonlinear root finding, we use method of Noble et al. (2006)
2. Relativistic Riemann solver required (HLL, HLLC, HLLD)
3. Use van Leer unsplit integrator since no characteristic decomposition needed in reconstruction step.

Relativistic MHD shock test

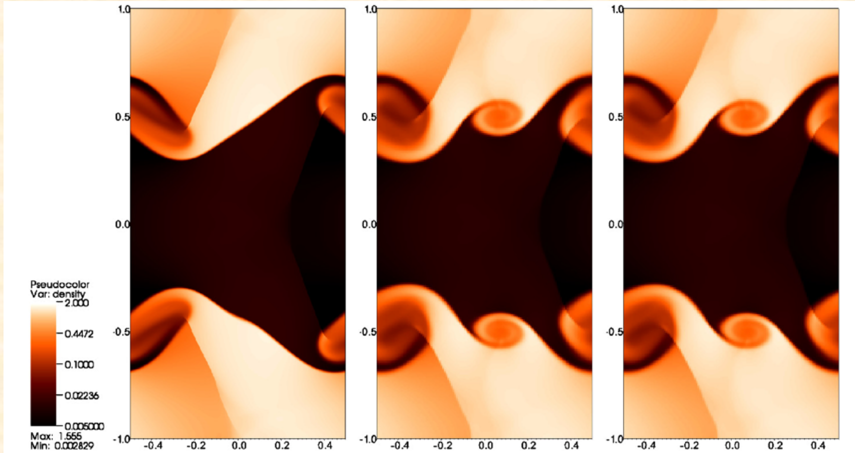


Line=1D
Pts=3D

Γ=2 Brio-Wu shocktube

Relativistic KH (γ=2.29)

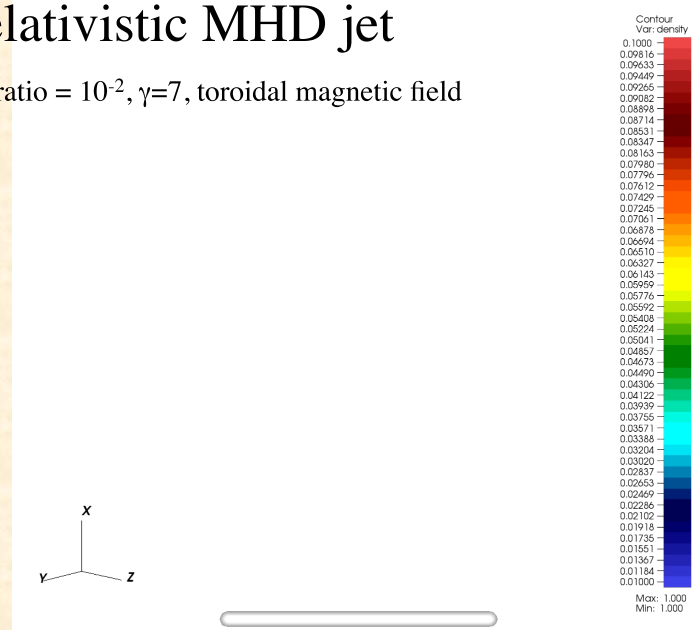
HLLE HLLC HLLD



HLLE: does not capture secondary rolls even at 2x resolution

Relativistic MHD jet

Density ratio = 10^{-2}, γ=7, toroidal magnetic field



Summary

- Adding extra physics requires care to ensure algorithms are stable and accurate, and any new length- or time-scales are resolved
- *Radiation hydrodynamics:* so complicated requires all of next lecture.