A (necessarily incomplete) review of Protostellar Accretion Disks

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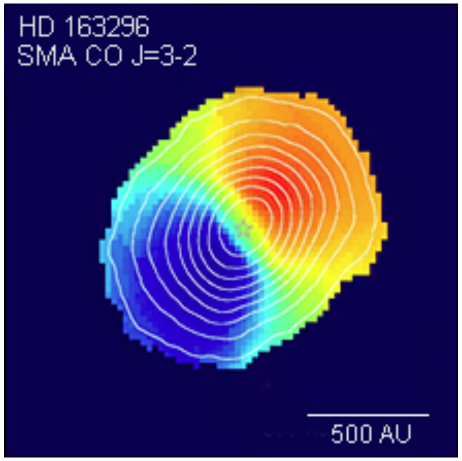
Outline

- **Observational overview (brief!)**
- **Viscous Disk Theory: The good, the bad and the ugly**
- **Sources of Angular Momentum Transport**
    - **Magneto-rotational Instability**
    - **Self Gravity**
    - **Disk Winds**
- **Disk phenomenology**
- **Numerical Techniques**

References:
Pringle 1981
Lynden-Bell & Pringle 1975
Frank, King & Raine (2002, textbook)
Lin & Papaloizou, 1995
Balbus & Hawley 2003

Observational “Facts”

- Protostellar disks live ~3-5 Myr
  - inferred from fraction of stars with infrared excess as a function of cluster age
- Typical disk sizes are ~100-1000 AU
  - measured in sub-mm, scattered light, SEDs
  - Consistent with core velocity gradients
- Disks are mostly neutral and cold (compared to compact accretion disks)
- Typical measured disk masses are .001 - .01 $M_\odot$ (when/if we can measure it!)
  - observe warm dust, assume dust-gas ratio and infer grain size distribution



Credit: A. Isella

Refs: Andrews et al, 2009, 2011, Hillenbrand et al 1998, Calvet et al 2005, D’alessio et al 1998, Gammie 1996 Goodman et al 1993 ...

Accretion Disks: *Angular Momentum Transport Machines*

- Disks are responsible for funneling material onto the star, so disk material must lose both energy and angular momentum

- Disks are often described via vertically integrated, viscous fluid equations: the *thin disk* approximation:

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\Sigma r v_r) = S_\Sigma,$$

mass conservation

$$v_r \frac{d}{dr}(r^2\Omega) = \frac{1}{r\Sigma}\frac{d}{dr}(r^2\langle T_{r\phi}\rangle) + \frac{S_\Sigma j}{\Sigma} + \Lambda$$

momentum conservation

$$\langle T_{r\phi}\rangle = \langle \nu \rangle \Sigma r \frac{d\Omega}{dr} \qquad \langle \nu \rangle = \frac{\int_{-\infty}^{\infty} \nu \rho dz}{\int_{-\infty}^{\infty} \rho dz}$$

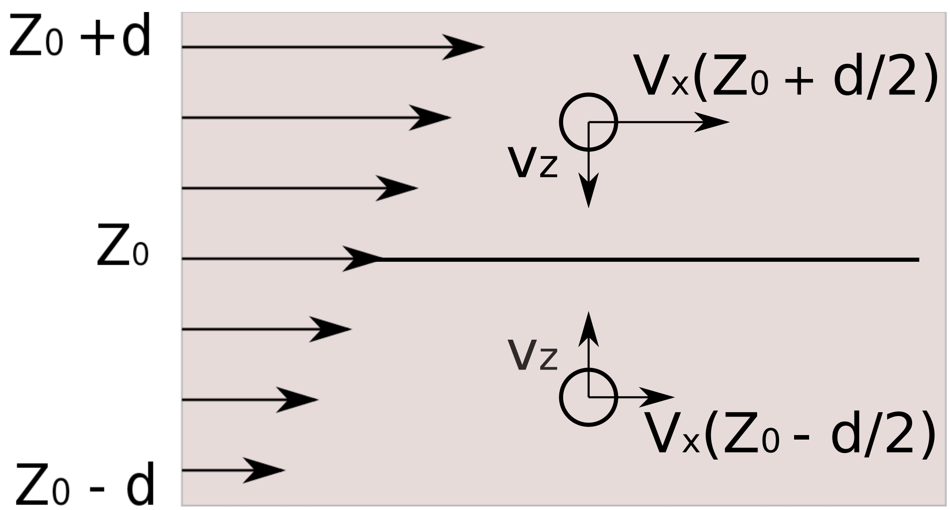
integrated viscous tensor

$$\frac{\partial \Sigma}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left[3r^{1/2}\frac{\partial}{\partial r}(\Sigma\langle\nu\rangle r^{1/2}) - \frac{2S_\Sigma j}{\Omega} - \frac{2\Sigma\Lambda}{\Omega}\right] - S_\Sigma = 0$$

Diffusion Equation

Why do we talk about "viscosity"?

- Viscosity will transport momentum *orthogonal* to a shear flow:



Net angular momentum exchange:

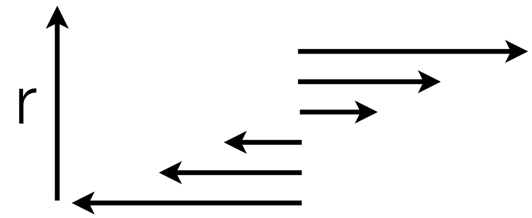
$$\delta l \approx \rho v_z (v_x(z_0 - d/2) - (v_x(z_0 + d/2))$$

viscosity works to remove shear

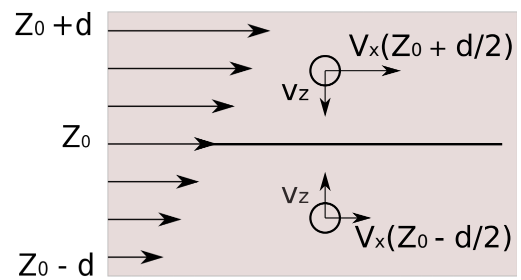
“The Bad”

Anomalous viscosity: Keplerian flow

If we naively exchange z for r.....



$$V_{\rm kep}(r_0 - d/2) > V_{\rm kep}(r_0 + d/2)$$



Angular momentum goes out!

But what if we conserve *angular momentum* across the surface?

$$j_{\rm Kep} = v \times r \propto r^{1/2}$$

angular momentum goes in?

You’re confused. It’s ok. It’s just an analogy

The Ugly

The viscous prescription seems convenient, but...

Recall that the Reynolds number is: $Re = \frac{rv}{c_s \lambda}$

$$\mathrm{Re} \approx 5 \times 10^{14} \left(\frac{M_*}{1 M_\odot}\right) \left(\frac{T}{400 K}\right)^{-1} \left(\frac{\Sigma}{5 \times 10^3 \mathrm{g\ cm^{-2}}}\right) \left(\frac{r_d}{1 \mathrm{AU}}\right) \left(\frac{\sigma}{10^{-15} \mathrm{cm}^2}\right)$$

molecular viscosity is irrelevant. But high Re means turbulence...

$$c_s \to v_{\mathrm{turb}}, \lambda \to \mathrm{l_{eddy}}$$

$$\nu = \alpha c_s H = \alpha \frac{c_s^2}{\Omega}$$

$$T_{r\phi} = \langle \nu \rangle \Sigma r \frac{d\Omega}{dr} \equiv \alpha \Sigma c_s^2 \left| \frac{\mathrm{dln}\Omega}{\mathrm{dlnR}} \right|$$

This is (sort of) the Shakura-Sunyaev effective viscosity

The Ugly, continued: α viscosity

- The parameterization oversimplifies the physics, and is often abused:
  - transport may not be constant
  - transport mechanisms don't necessarily correlate with local pressure
  - Numerical models "show" that it works well in many cases, but....
    - numerical diffusivity acts like viscosity

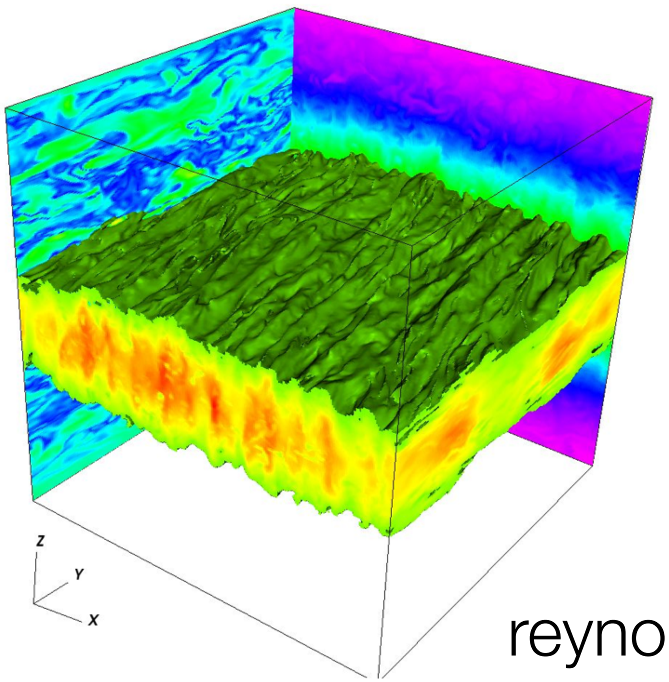
$$\frac{\partial \Sigma}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left[3r^{1/2}\frac{\partial}{\partial r}(\Sigma\langle\nu\rangle r^{1/2}) - \frac{2S_{\Sigma}j}{\Omega} - \frac{2\Sigma\Lambda}{\Omega}\right] - S_{\Sigma} = 0$$

$$\nu = \alpha c_s H = \alpha\frac{c_s^2}{\Omega}$$

“The good”

What really transports angular momentum

Davis et al 2010



- Positive correlations between radial and azimuthal velocities and maxwell stress produce outward angular momentum transport: this is like a positive kinematic viscosity
- A priori, it’s not clear why any microscopic process should produce such correlations in a disk
- Note that positive velocity correlations extract energy from the disk shear

reynolds

$$T_{r\phi} = \langle \Sigma(v_r v_\phi - u_{\mathrm{r,A}} u_{\phi,\mathrm{A}} \rangle \equiv \langle \nu \rangle \Sigma r \frac{d\Omega}{dr}$$

maxwell

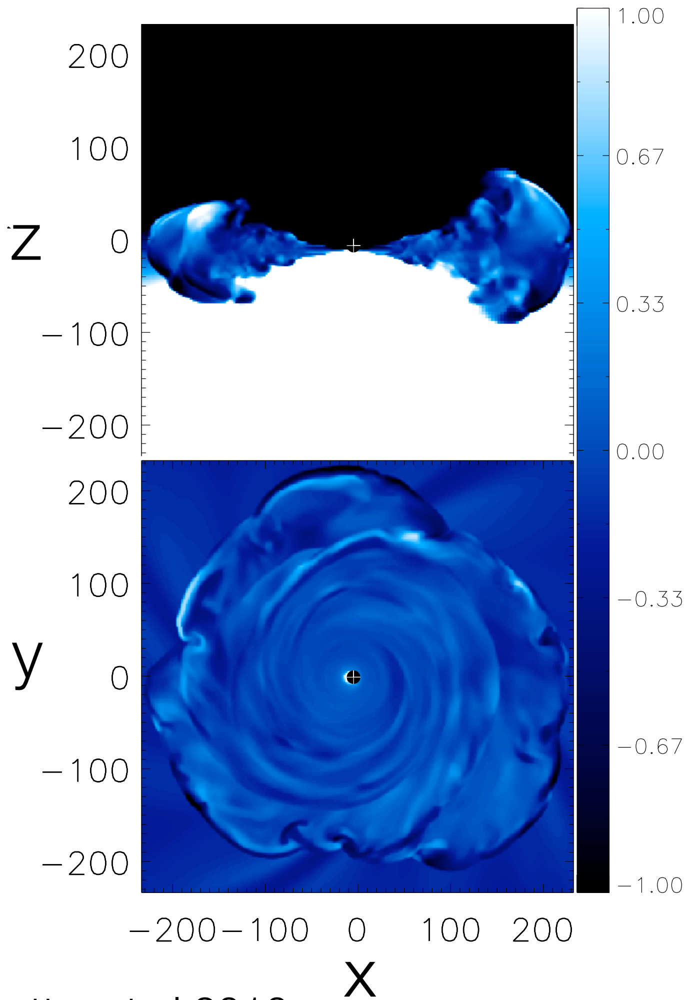
$$u_A = \frac{\vec{B}}{\sqrt{4\pi\rho}}$$

(angular momentum -> angular momentum flux)

$$\vec{v} \times \vec{r} \longrightarrow v_\phi v_r$$

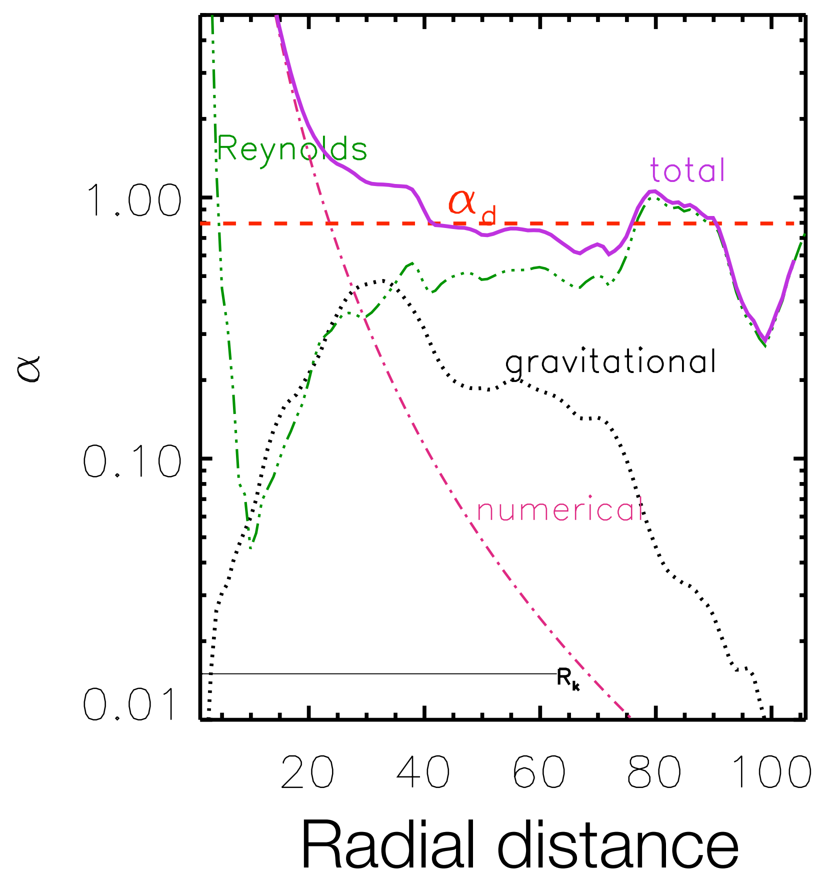
Balbus & Hawley 2003

Numerical Simulations: the good?



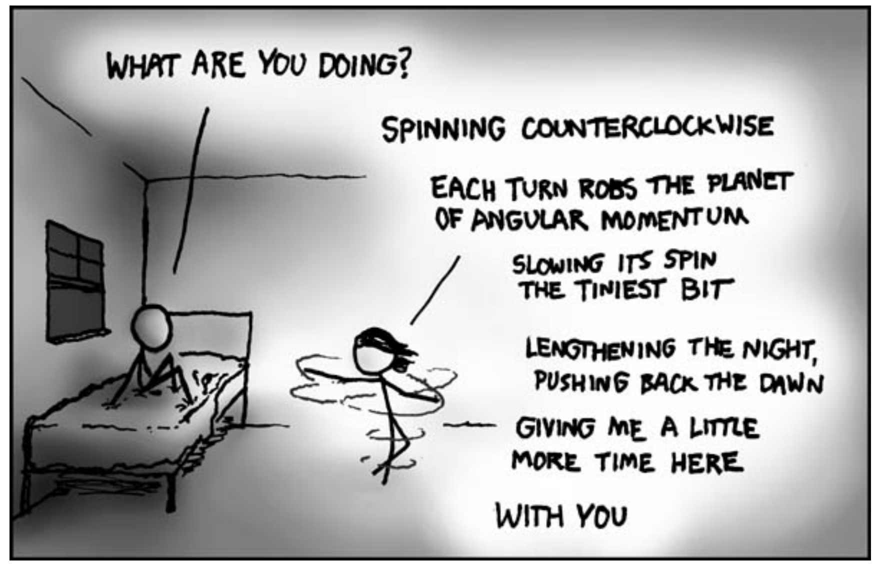
Kratter et al 2010

- Run numerical simulations of the relevant processes, and measure stresses, but...



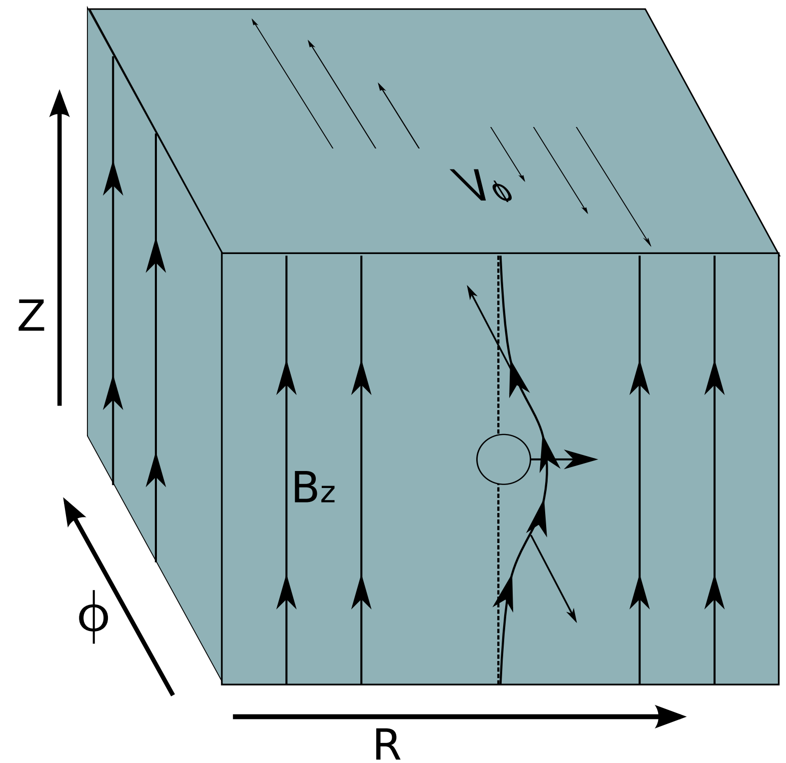
Sources of angular momentum transport

- Magneto-Rotational Instability
- Disk Self-Gravity
  - Spiral arms
  - Gravitoturbulence
- Disk Winds
- Convection ?
- Hydrodynamic turbulence ?



Magneto-Rotational Instabilty (MRI)

- Disk threaded by a weak B-field
- Perturb a fluid element in R
- Magnetic tension accelerates fluid element, which increases angular momentum, and causes it to continue to move outward, which further stresses the field line...
- *ideal MHD limit...*



MRI stability Criterion $$\frac{d}{dr}|v_\phi| > 0$$

Rayleigh Criterion $$\frac{d}{dr}|\Omega r^2| > 0$$

A weak field instability...

$$v_A/H < 3\Omega \rightarrow B < \sqrt{36\pi\Omega\Sigma c_s}$$

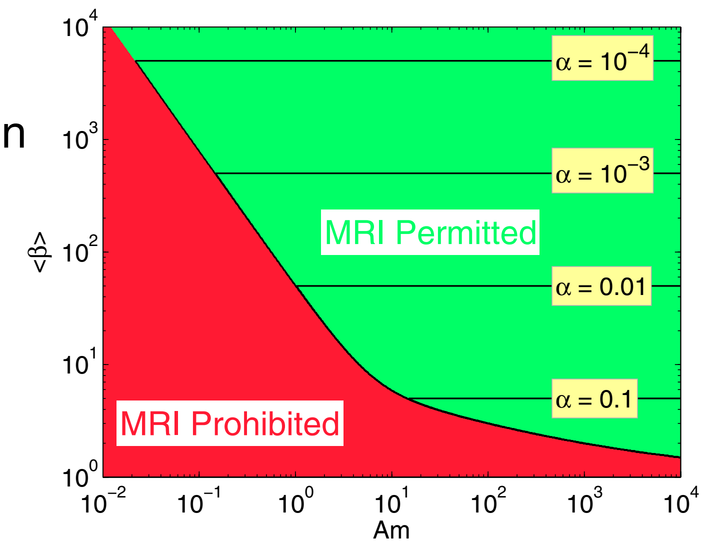
MRI Complications

- Non-ideal MHD effects are important
  - Disk are cold and neutral: ionization due to cosmic rays $\zeta \approx 10^{-17} s^{-1}$
    - dust properties (and chemistry) influence ionization
    - layered accretion and dead zones
      - ohmic dissipation
      - ambipolar diffusion

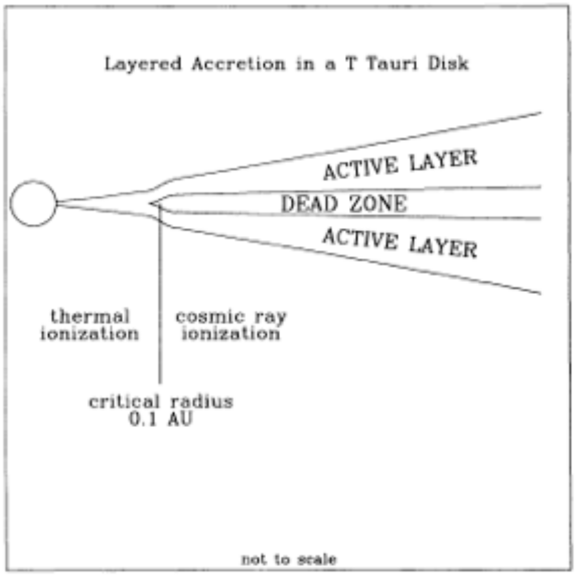
$$\mathrm{Am} = \frac{\gamma \rho_i}{\Omega}$$

ion-neutral collisions per dynamical time

$$\beta = \frac{P_{\mathrm{gas}}}{P_{\mathrm{B}}}$$



Bai & Stone 2011



Gammie 1996

Disk Self-Gravity

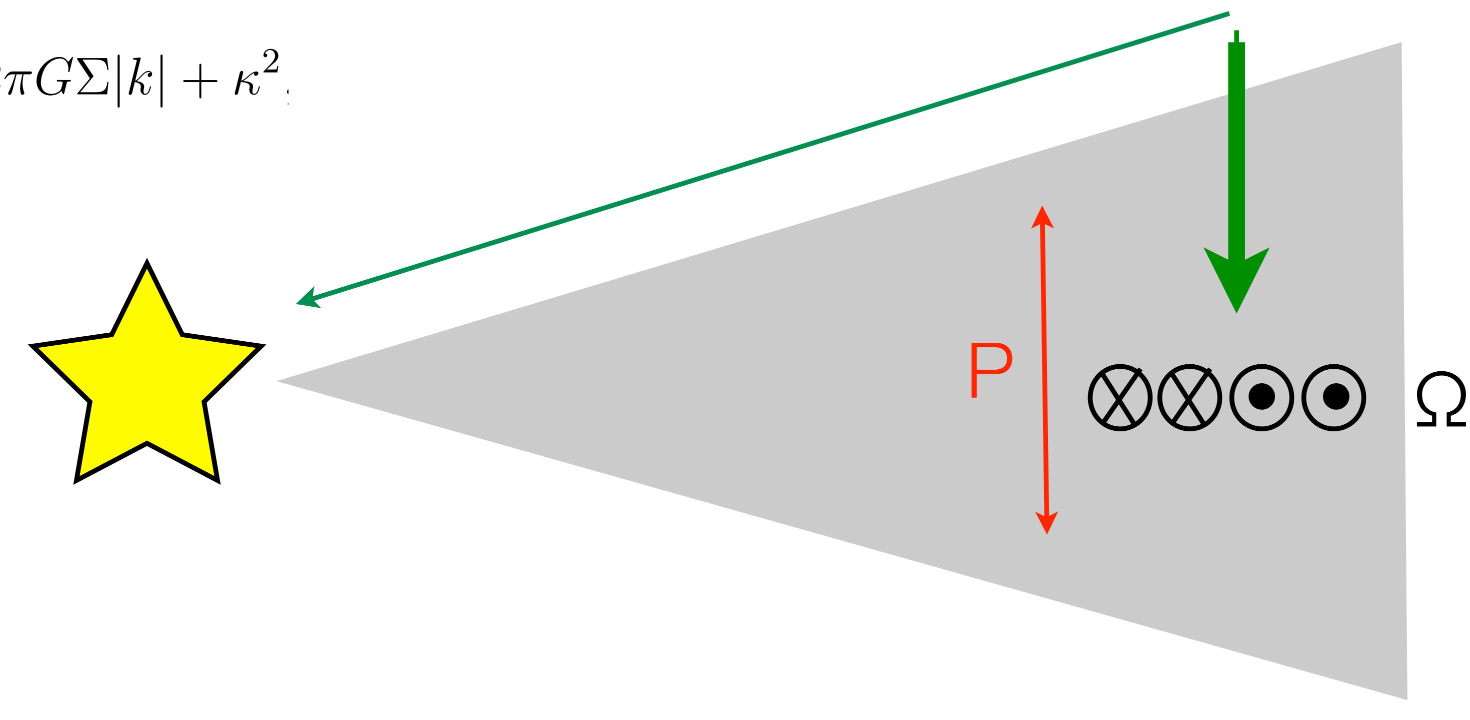
- Global spiral arms

$$Q = \frac{c_s \kappa}{\pi G \Sigma} \sim 1$$

- Local "turbulence"

$$\omega^2 = v_s^2 k^2 - 2\pi G \Sigma |k| + \kappa^2,$$

$$\lambda = 2\pi \frac{c_s^2}{\pi G \Sigma}$$



Toomre 1964

Lynden-Bell & Kalnajs 1972

Spiral Arm Torques

- Trailing spiral waves induce positive correlations between g_ϕ and g_r
- Can also induce velocity correlations
- Magnitude of transport can be derived from conserving wave action as waves cross co-rotation

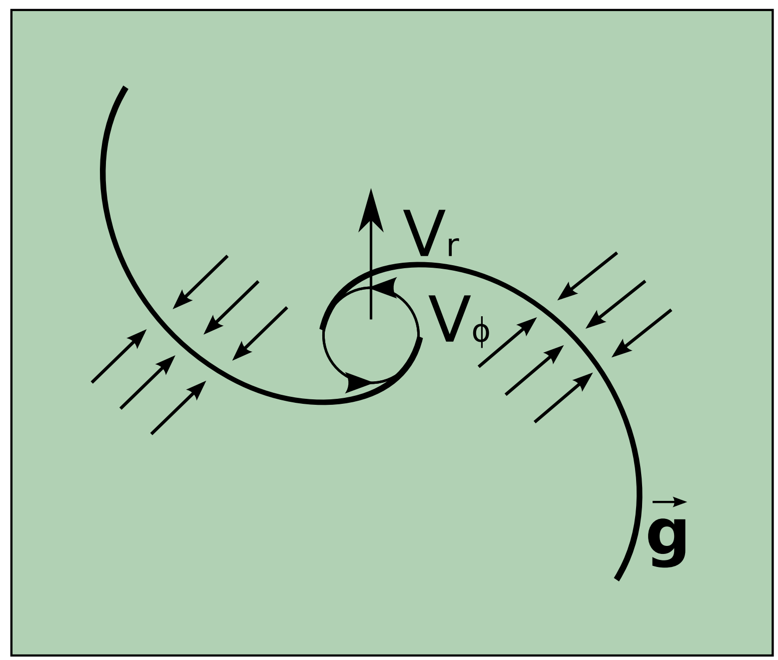
Force density

$$\rho \nabla \Psi = \frac{\nabla^2 \Psi \nabla \Psi}{4\pi G} = -\nabla \cdot T \quad \vec{g} = \nabla \Psi$$

$$T_{r\phi} = \int dz \frac{g_\phi g_r}{4\pi G}$$

acts like a maxwell stress

$$\alpha_{GI} \approx \frac{2}{3} m \frac{H}{R} \left(\frac{\eta}{Q^2} - \frac{1}{Q} \right) |\Delta|^2.$$



Gravito-turbulence

- In thin disks, GI can drive small scale turbulence (m increases, lengthscale decreases)
- If the disk is "viscously" heated, we can calculate the steady-state turbulent transport rate by assuming a balance between heating and cooling

Viscous Dissipation

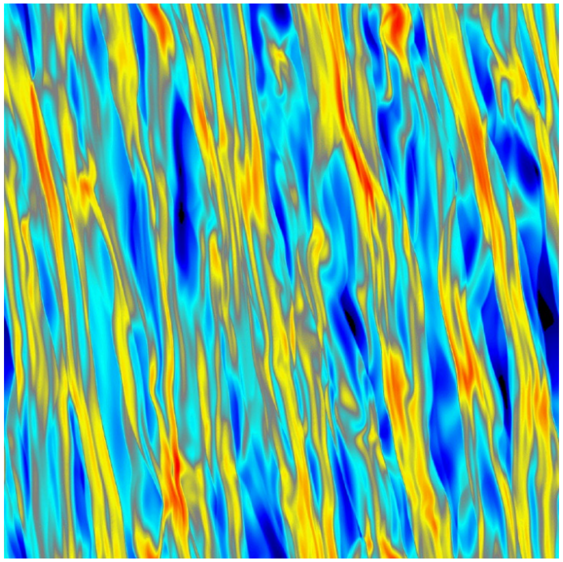
$$2F_v = \frac{3\dot{M}\Omega^2}{4\pi}$$

Radiative Cooling

$$F_r = \frac{8}{3\tau_R}\sigma T_d^4,$$

Accretion rate in steady state

$$\dot{M}_{\rm visc} = \frac{3\pi\alpha\Sigma c_s^2}{\Omega}.$$



Gammie 2001

Cooling Timescale

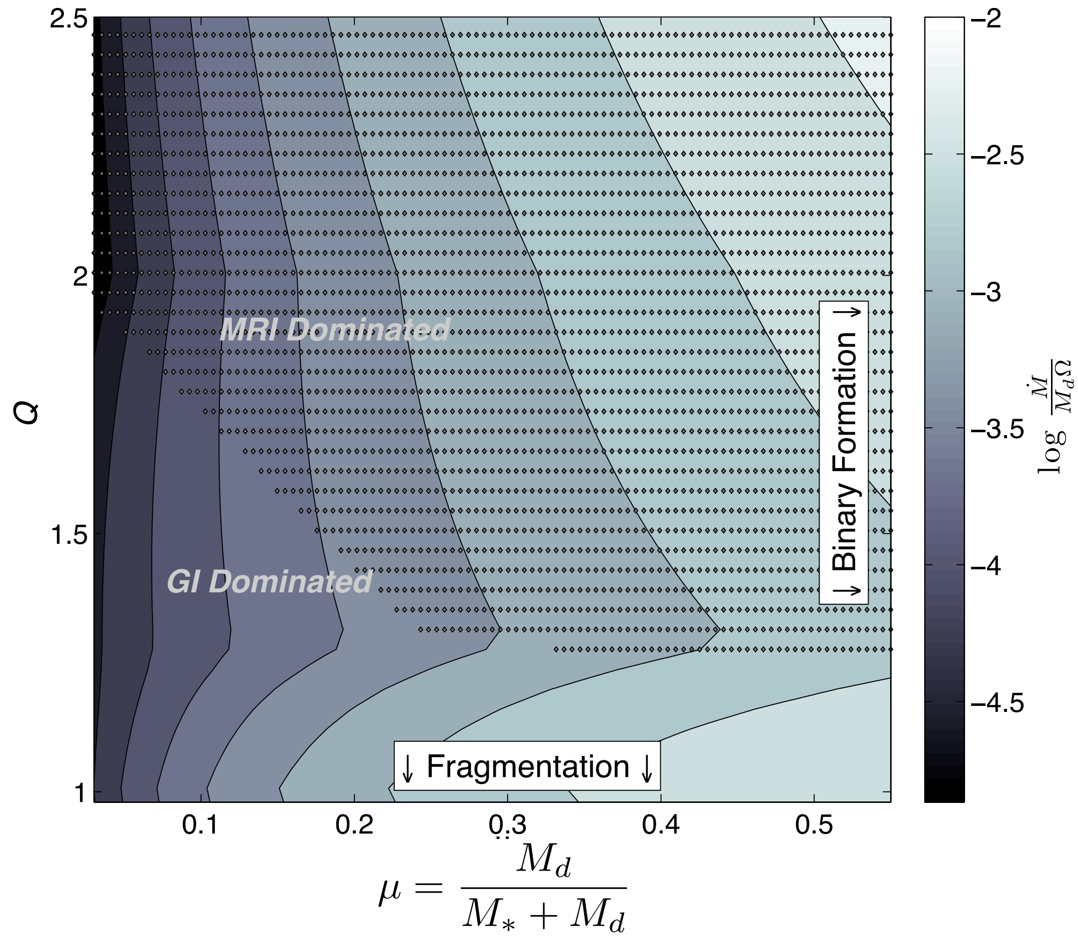
$$t_{\rm cool} = \frac{3\gamma\Sigma c_s^2}{32(\gamma-1)}\frac{f(\tau)}{\sigma T^4} \lesssim \zeta\Omega^{-1}.$$

Steady-State GI

$$\alpha = \frac{1}{\gamma(\gamma-1)}\frac{4}{9\Omega t_{\rm cool}}$$

Relevance of GI vs MRI

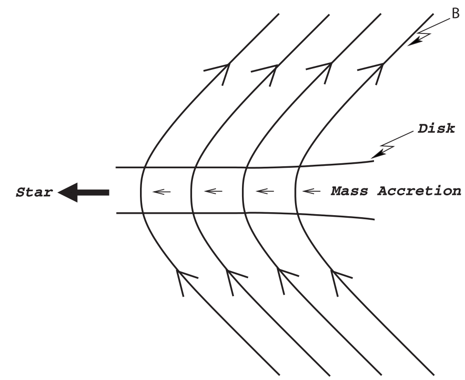
- Self-gravity likely dominates angular momentum transport for the first 1e4 -1e5 yrs
- more important for more massive stars
- MRI dominates for most disks in this diagram, and likely most disks that we observe



Kratter et al 2008

Disk Winds

Shu et al 2007



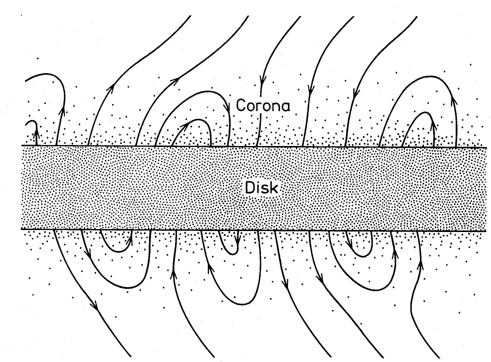
- Magnetic fields that thread the disk can also launch material, and transport angular momentum

- Requires special field geometry $\theta < 60$

- Requires large scale, well coupled field

$$\dot{\Sigma} = \rho c_s = \frac{|B_0||B_A|}{\mu_0 r_A \omega}$$

Blandford & Payne 1982



  - dragged in? what about ambipolar diffusion?

$$\dot{j} \propto r_A^2 \omega$$

disk winds are not the same as magnetized jets: different scales

Numerical Methods for Disk Studies

1D Viscous Models

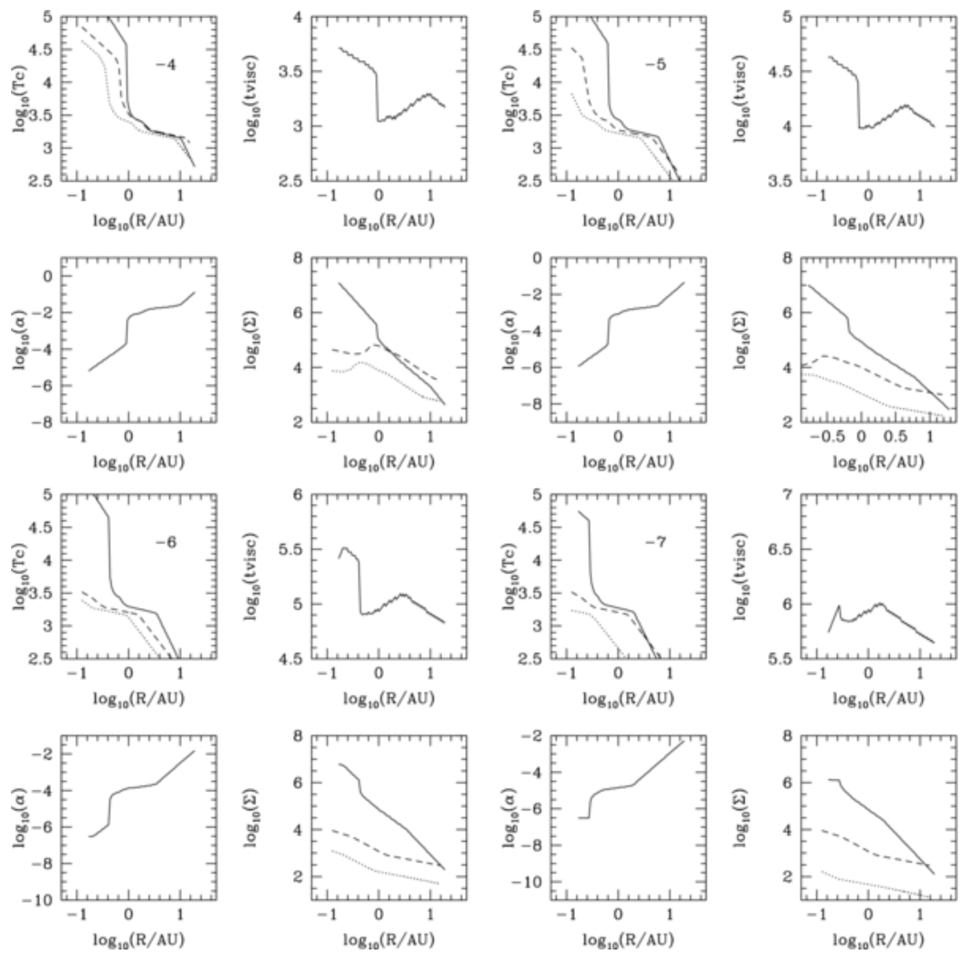
2D Shearing Sheets

3D Shearing Boxes

3D Global Models

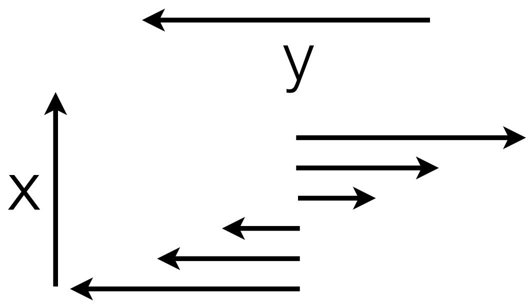
1D- Viscous Models

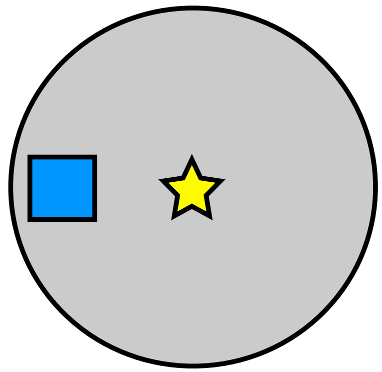
- Solve coupled diffusion and energy equations using some prescription for an effective viscosity
- Pros: fast, wide parameter study possible
- Cons: no real dynamics, mocked up transport



Zhu et al 2009

2D Shearing sheet approximation

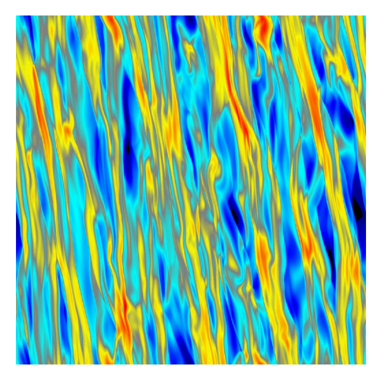




$$x = r - r_0$$
$$y = r_0(\phi - \Omega(r_0)t)$$
$$g_x = -\frac{GM_*}{r_0^2}(1 - 2x/r_0)$$

$$\frac{\partial u_x}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u_x} = 2\Omega_0 u_y + 3\Omega_0^2 x - \frac{1}{\rho}\frac{\partial P}{\partial x}$$
$$\frac{\partial u_y}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u_y} = -2\Omega_0 u_x - \frac{1}{\rho}\frac{\partial P}{\partial y}$$

- Simulate local 2D patch of the disk with approximate central potential and Coriolis force
- No curvature
- Shear-periodic boundary conditions: material leaves the box and re-enters, shifted

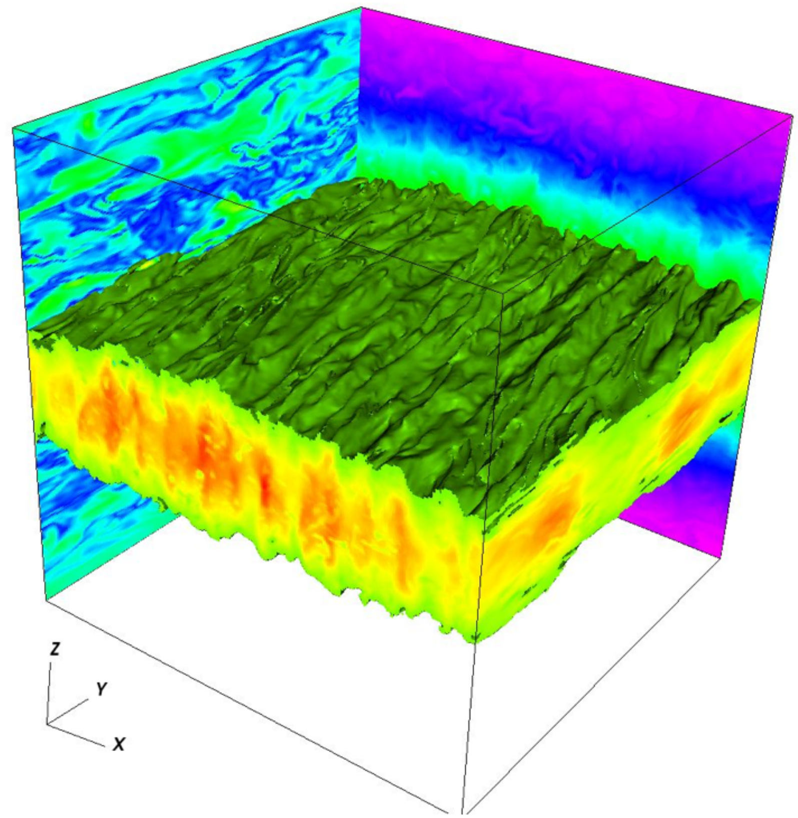


Goldreich & Tremaine 1978
Gammie 2001

3D Shearing box

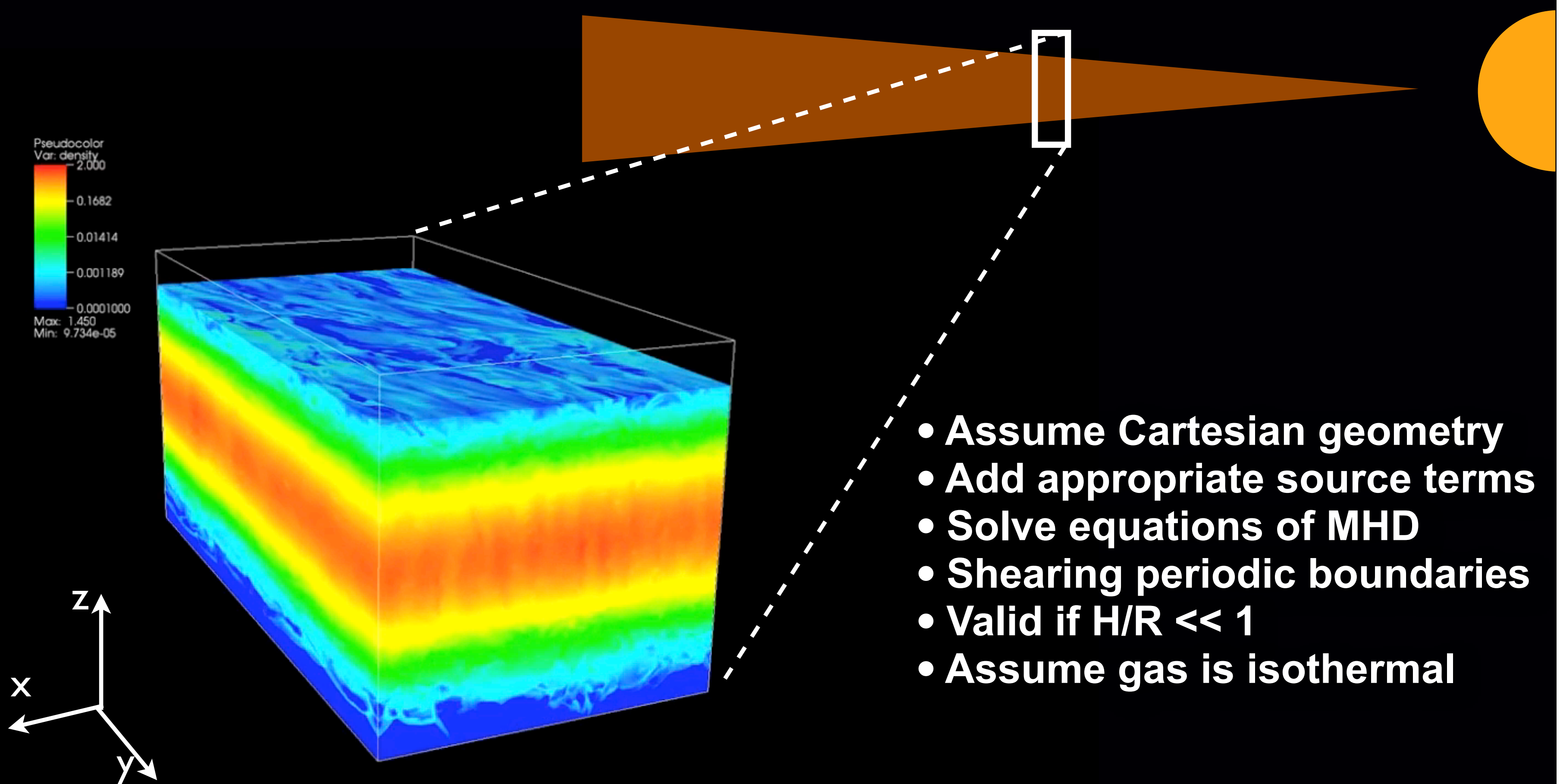
- Add z
- Unlike the shearing sheet, in 3D we begin to worry about the relative height to width of the box
- Computational expense becomes and issue
- “real” physics is easier to incorporate (MHD, self-gravity)

$$\frac{\partial u_z}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u_z} = -\frac{1}{\rho}\frac{\partial P}{\partial z}$$



Davis et al 2010

Local simulations: examine small co-rotating disk patch

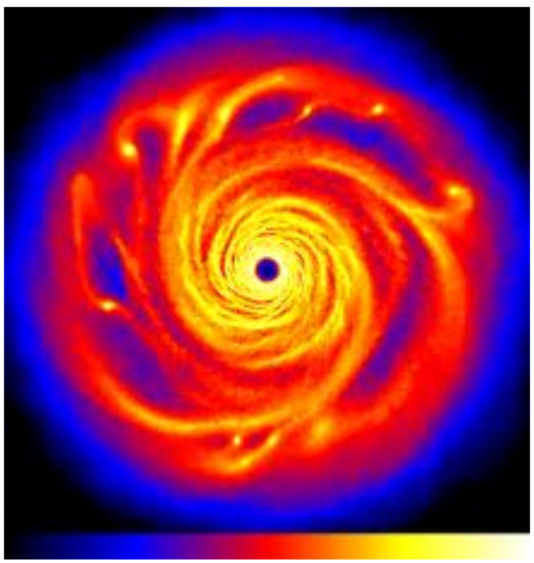


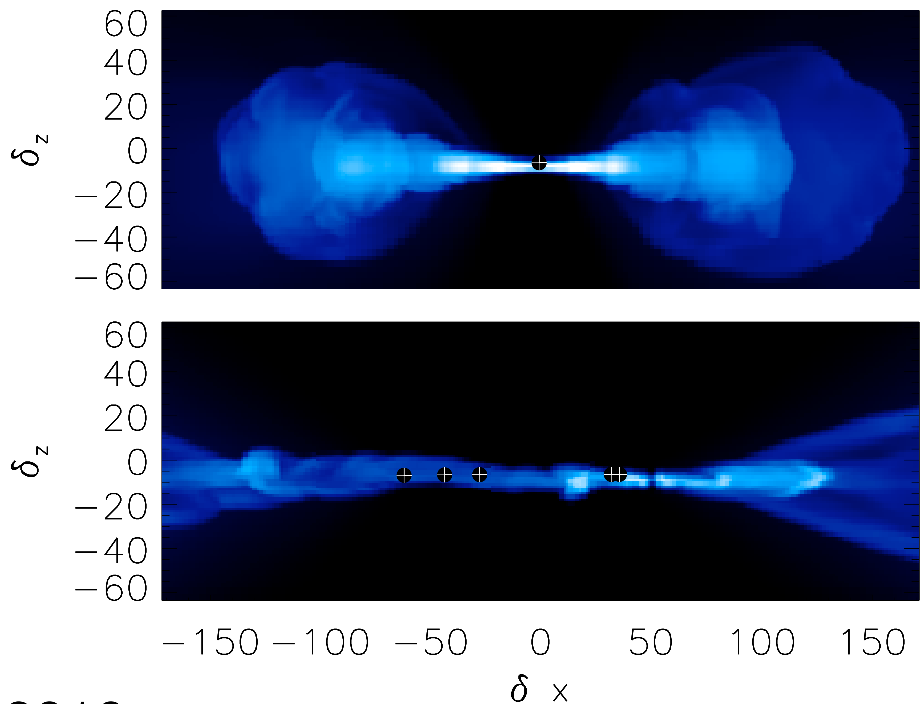
- Assume Cartesian geometry
- Add appropriate source terms
- Solve equations of MHD
- Shearing periodic boundaries
- Valid if H/R << 1
- Assume gas is isothermal

Courtesy of J. Simon

Full Global Disk

- "Brute force" technique: capture the whole disk, but resolution becomes difficult
- Possible to consider non-steady state configurations: isolated versus fed disks





Rice & Lodato 2005; Kratter et al 2010

Conclusions

- Viscous disk are convenient, but only an analogy. “alpha” viscosity is even more restrictive
- Current observations and theory suggest that in reality, the MRI and GI are likely responsible for angular momentum transport. Disk winds are also possible, but harder to model / measure
- Since all of these processes involve non-linear phenomena, large-scale numerical simulations + observations are our best hope
- Only talked about gas: particles matter, too