Disk fragmentation and Numerical Resolution

References:

Helled et al upcoming PPVI review
Durisen et al 2006 PPV review
Meru & Bate 2011, 2012
Lodato & Clarke 2011
Paardekooper 2012

Apologies to references I missed

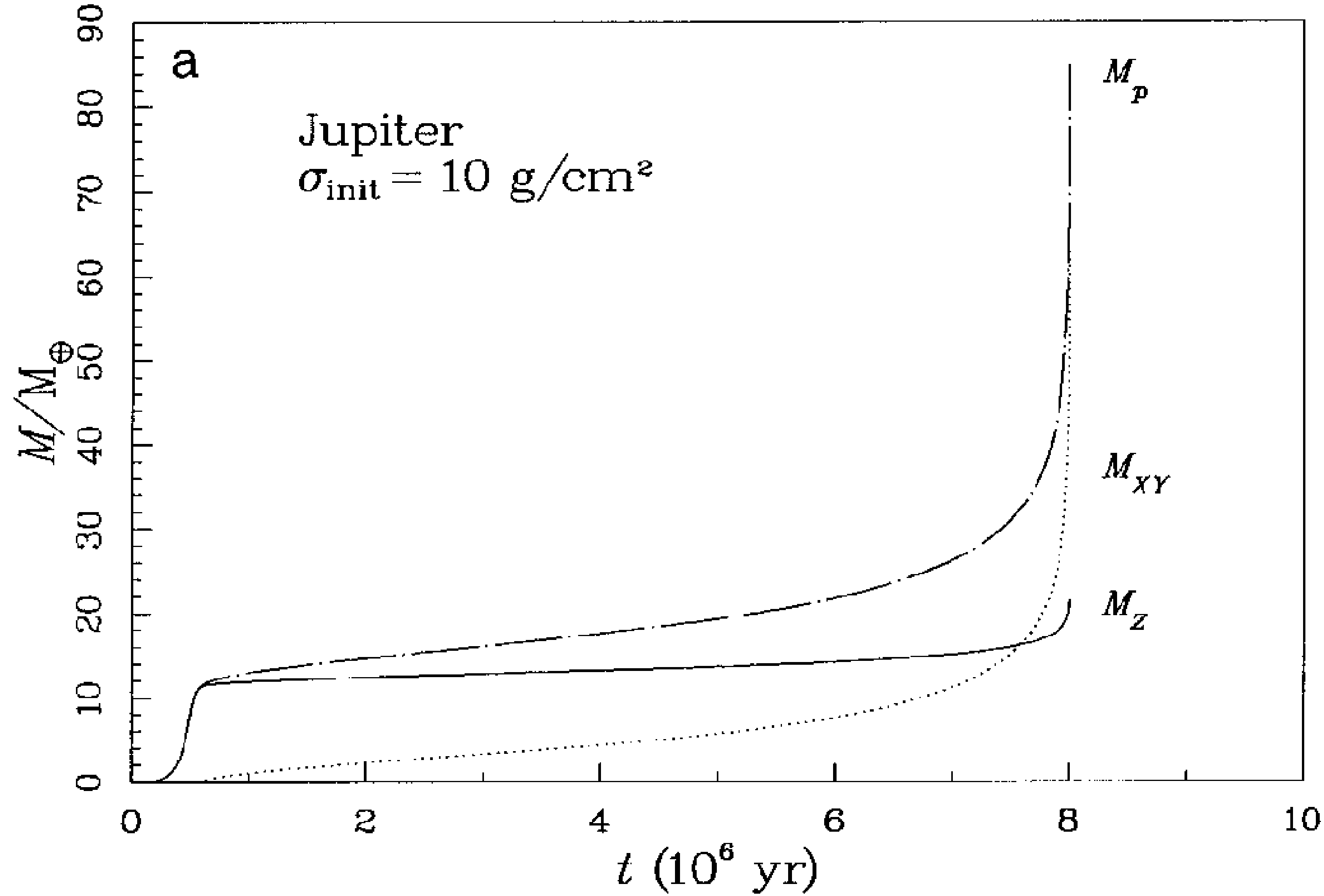
Outline

- Motivation: GI and planet formation
  - Standard Planet formation theory and its problems
- Theoretical background
  - Toomre Q
  - Disk fragmentation
- Cooling condition for fragmentation
- Numerical simulations and convergence

Stages of planet formation

- Initial: Collapse of cloud into a protostellar core and a flattened rotating disk.
- Early: Sedimentation of grains to form condensation sites for planetesimals.
- Middle: Growth of planetesimals into protoplanets through binary collision and gravitational interaction.
- Late: Final assembly to planets and cleansing of remaining planetesimals. Outer planets accrete remaining gas.

Core accretion timescales



Several Myr required to form Jupiter in a “near minimum” mass protosolar nebula.

Pollack et al. 1996

Accretion Problems

- Disk lifetimes (0.1 – few Myr) longer than growth timescale
- Disk-planet interaction causes migration in less than .1 Myr
- Core mass of Jupiter may be small
- Planets at large distances
  - Even Uranus/Neptune are problems
  - Fomalhaut b?

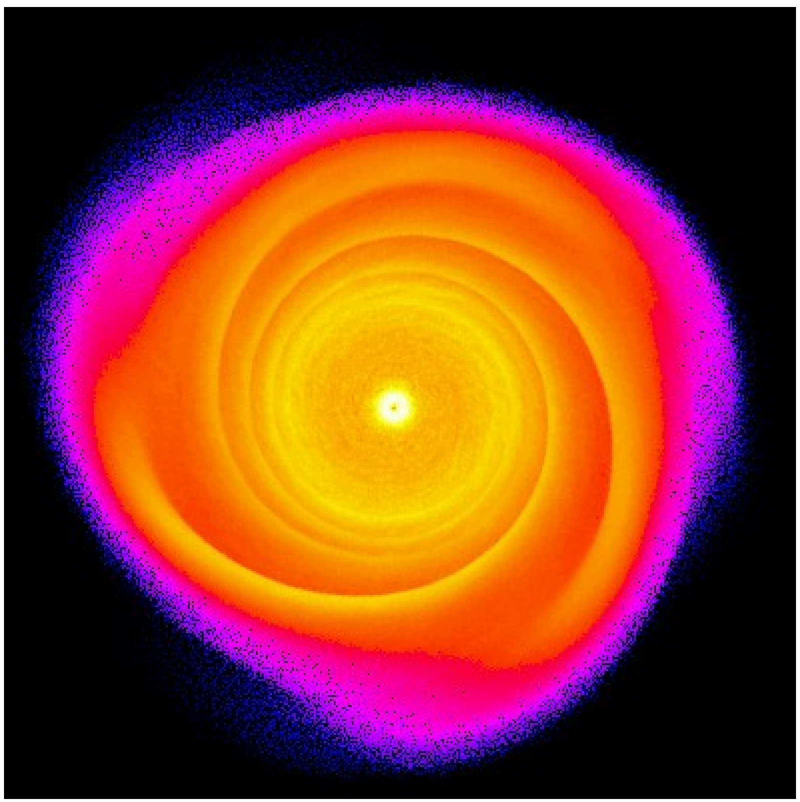
Protoplanetary Disk Stability

- Local criterion: $Q = c_S\Omega/\pi\Sigma G < 1$
  - Critical wavelength: $\lambda_{crit} = 4\pi^2 G\Sigma/\Omega^2$
- $1 < Q < 2$ allows global non-axisymmetric instabilities. These spiral arms could fragment.
- ``Minimum" protosolar disk ($M \sim 0.01\ M_S$) gives $Q > 2$ everywhere.

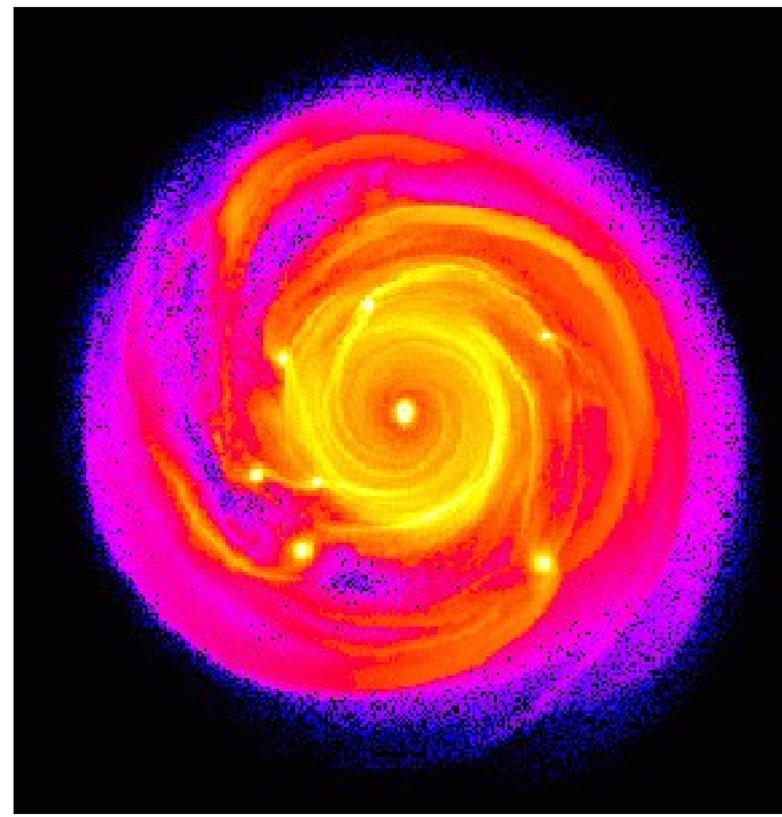
➔ Need a more massive disk for low Q.

Disk Evolution vs Q

1 million particles, locally isothermal eq. of state , R=20 AU



Q ~ 1.7



Q ~ 1.4

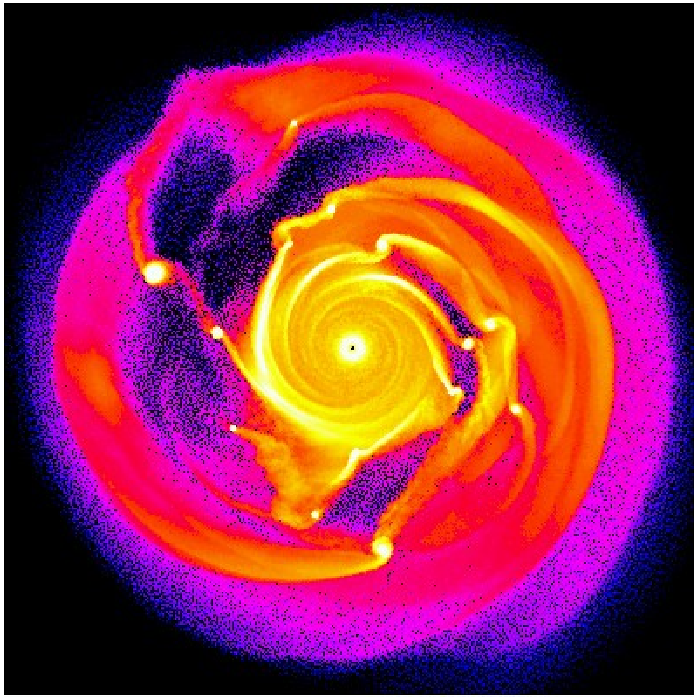
T=350 yr

T_{orb} (10 AU) = 28 years

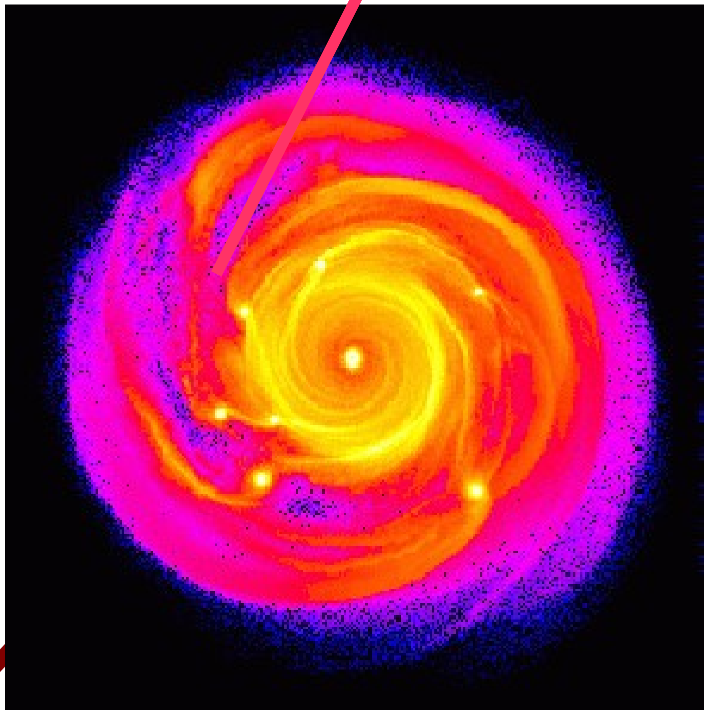
Adiabatic versus Isothermal

Adiabatic EOS ($\gamma = 1.4$): cooling only by decompression, heating by compression + artificial viscosity (shocks)

$\rho_{max} \sim 10^{-5}$ gm/cm^3



T=350 yr



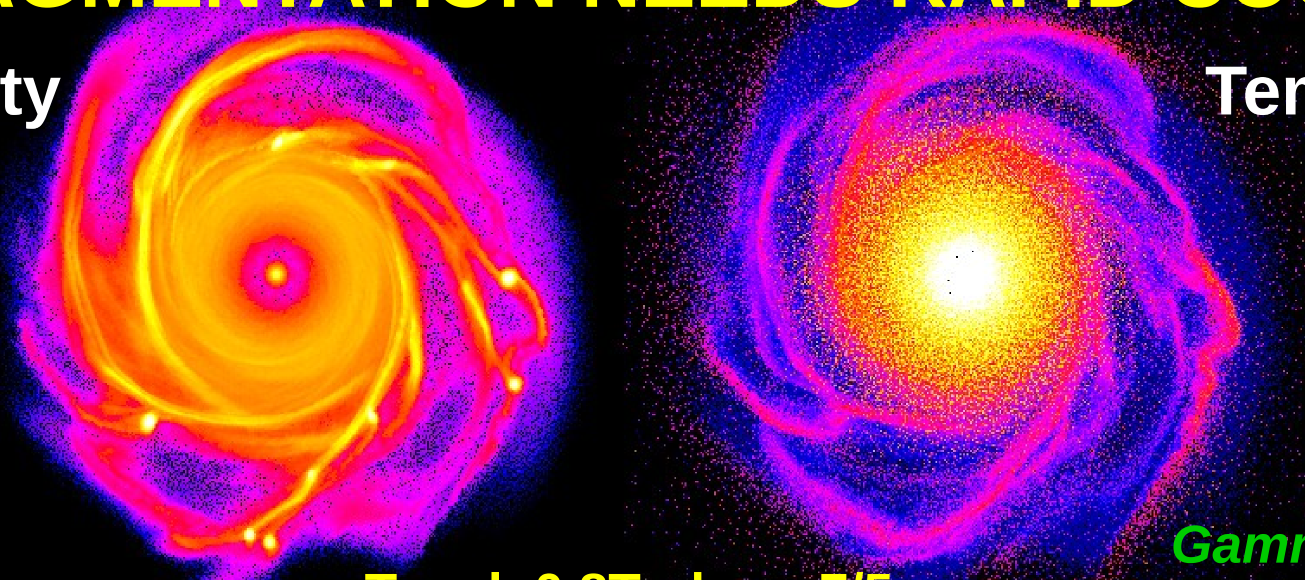
T=350 yr

EOS switched to adiabatic when local density becomes > 10 times higher than the initial value.

Long-lived clumps occur whether EOS changed or not

FRAGMENTATION NEEDS RAPID COOLING

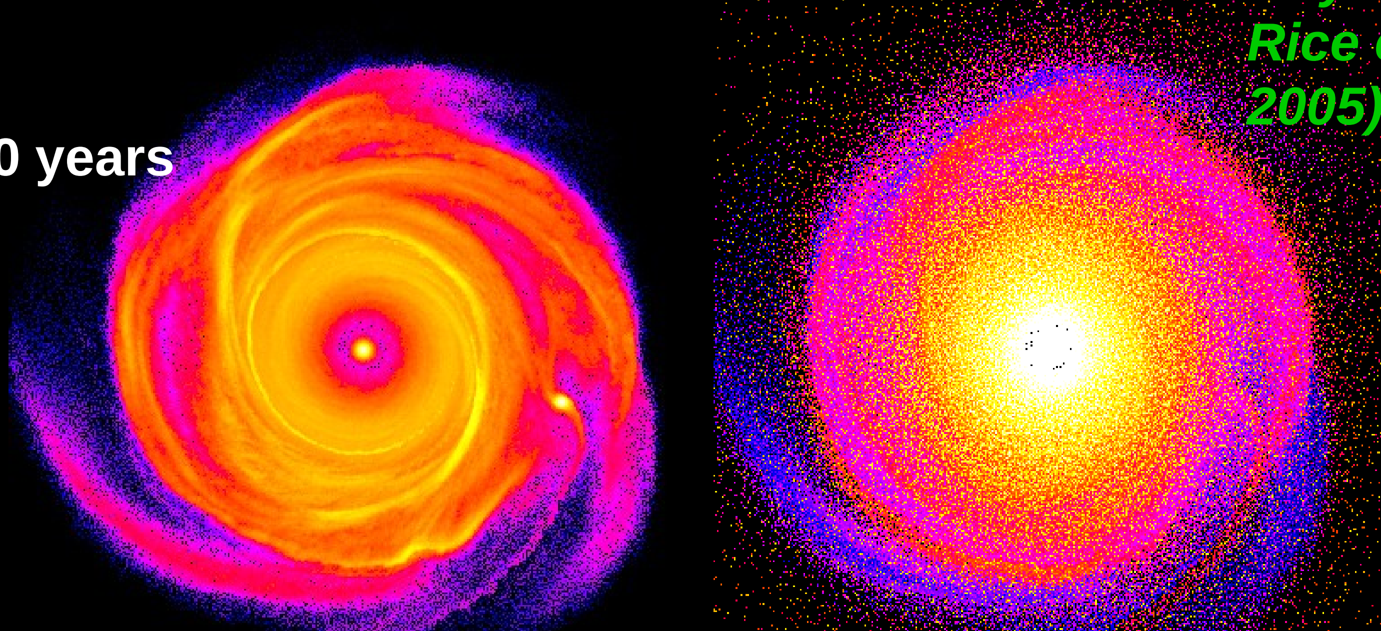
Density **Temperature**



Tcool=0.8Torb; γ=7/5

Gammie 2001;
Mayer et al . (2003, 2005),
Rice et al. (2002, 2003, 2005), Mejia et al. 2005

T=300 years



Tcool=1.4 Torb; γ=7/5

Snapshots of sims with different Tcool, all after ~ 10 Torb at (10 AU) ~ 300 years

Cooling time and fragmentation

- Cooling time/fragmentation condition: $\Omega t_{\rm cool} < \xi,$
- But cooling time is given by:

$$t_{\rm cool} \approx \frac{\Sigma c_s^2}{\gamma - 1} \frac{f(\tau)}{2\sigma T^4}, \qquad f(\tau) = \tau + \frac{1}{\tau},$$

- Hence we have two constraints on c_s:

$$\left[\Sigma \frac{f(\tau)}{\zeta} \frac{\Omega}{\sigma} \left(\frac{k}{\mu}\right)^4\right]^{1/6} \leq c_s \leq \pi Q_0 \frac{G\Sigma}{\Omega},$$

where $\zeta = 2\,\xi(\gamma-1)$

Rafikov (2005)

Constraints on disk density and temperature

- Minimum Density greater than:

$$\Sigma_{\rm inf} \equiv \Omega^{7/5}(\pi G Q_0)^{-6/5}\left[\frac{1}{\zeta\sigma}\left(\frac{k}{\mu}\right)^4\right]^{1/5}$$

- Minimum Temperature greater than:

$$T_{\rm inf} \equiv \Omega^{4/5}(\zeta\pi Q_0 G\sigma)^{-2/5}\left(\frac{k}{\mu}\right)^{3/5}$$

- This is > 220 K at 10 AU: much hotter than measured disk T
- But: Q_0 and ζ are determined by numerical experiments

Viability of Fragmentation

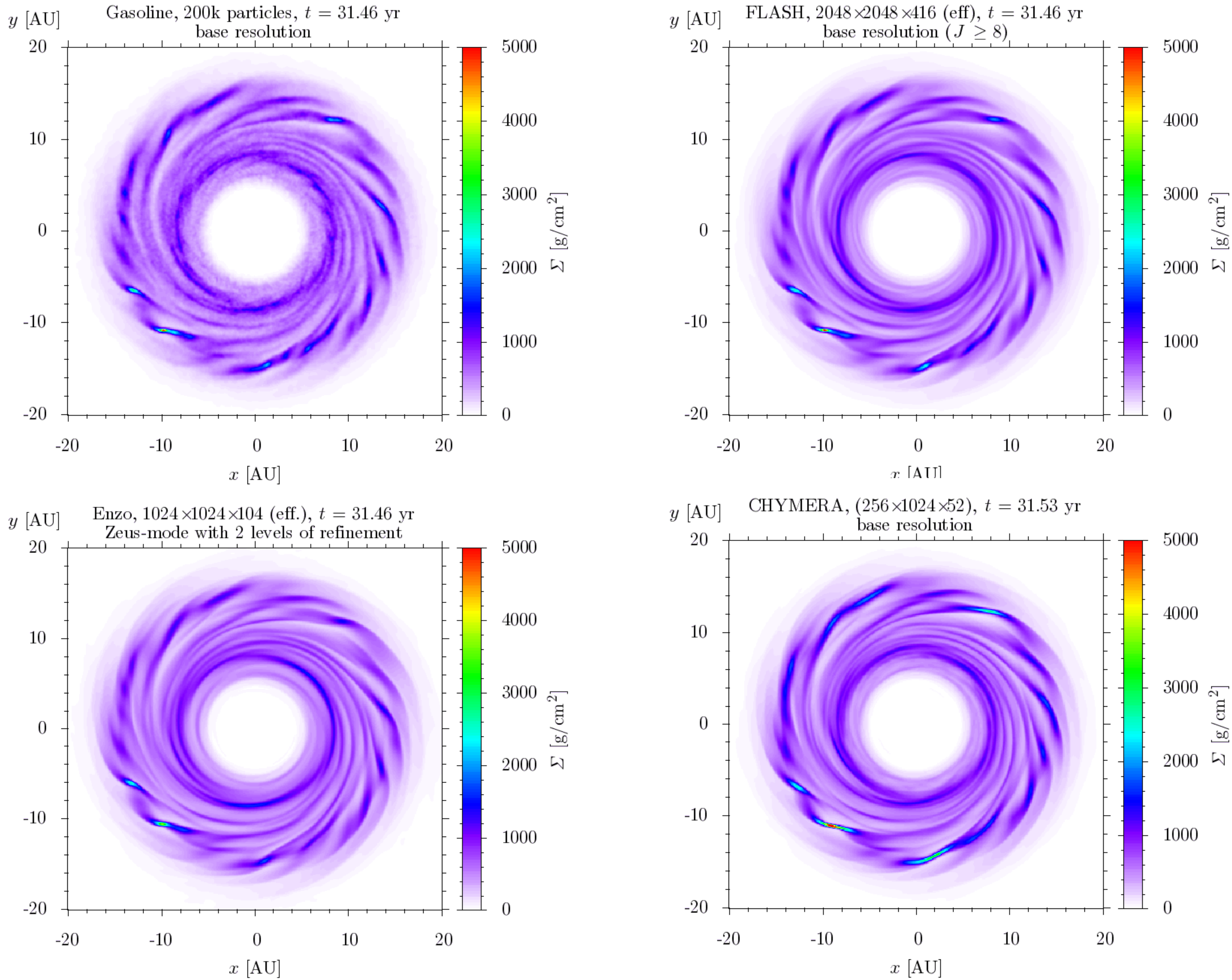
- Metallicity?
- Terrestrial Planets?
- Small bodies?
- Has to be ubiquitous!

Can the clumps survive and collapse?

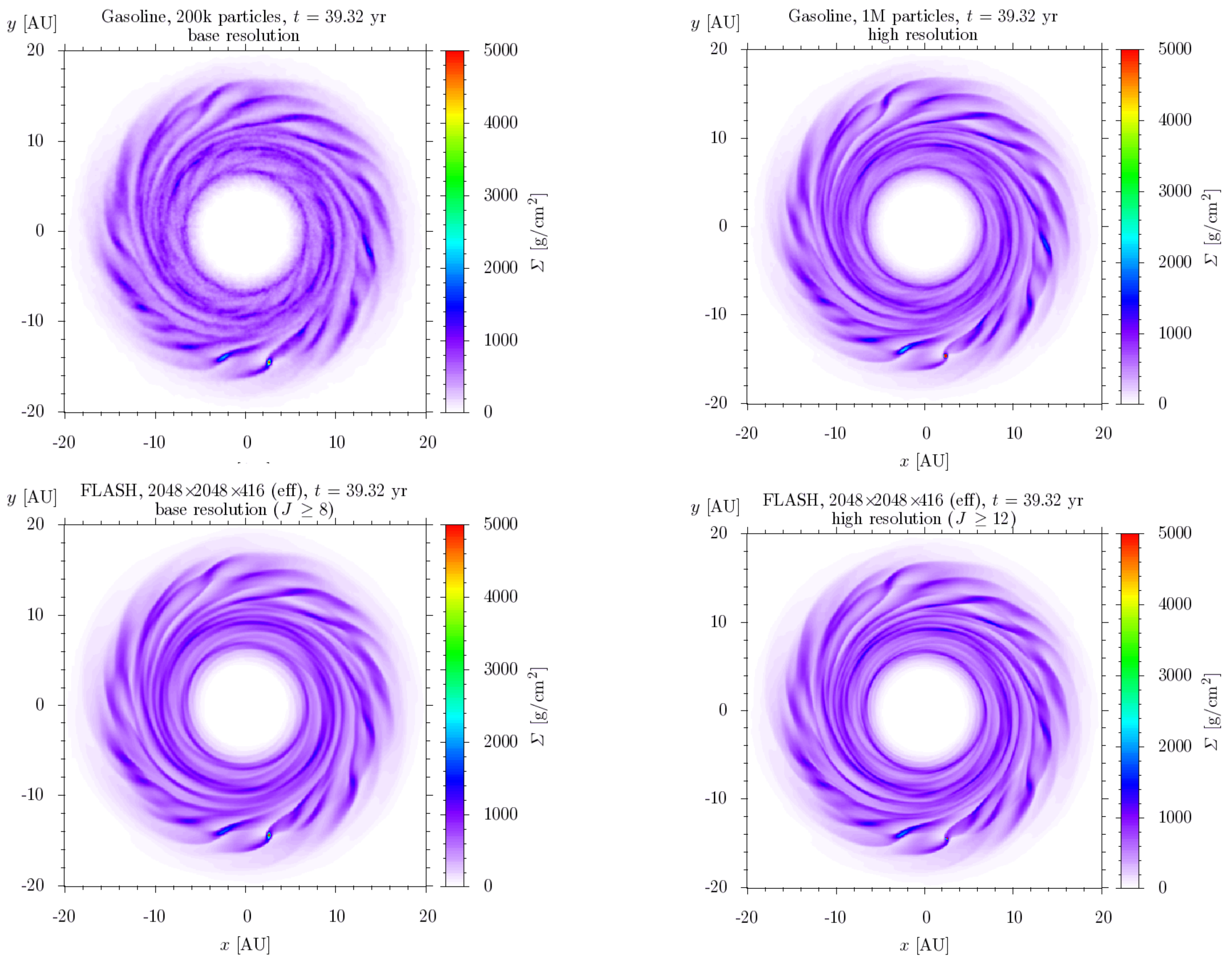
- Numerics:
  - simulations limited by spatial resolution
  - Also need to resolve large dynamic range in densities.
  - Numerical issues can enhance/damp fragmention.
- Thermodynamics/radiation physics
  - General agreement with simple Equations of State
  - Hard to model

Wengen tests

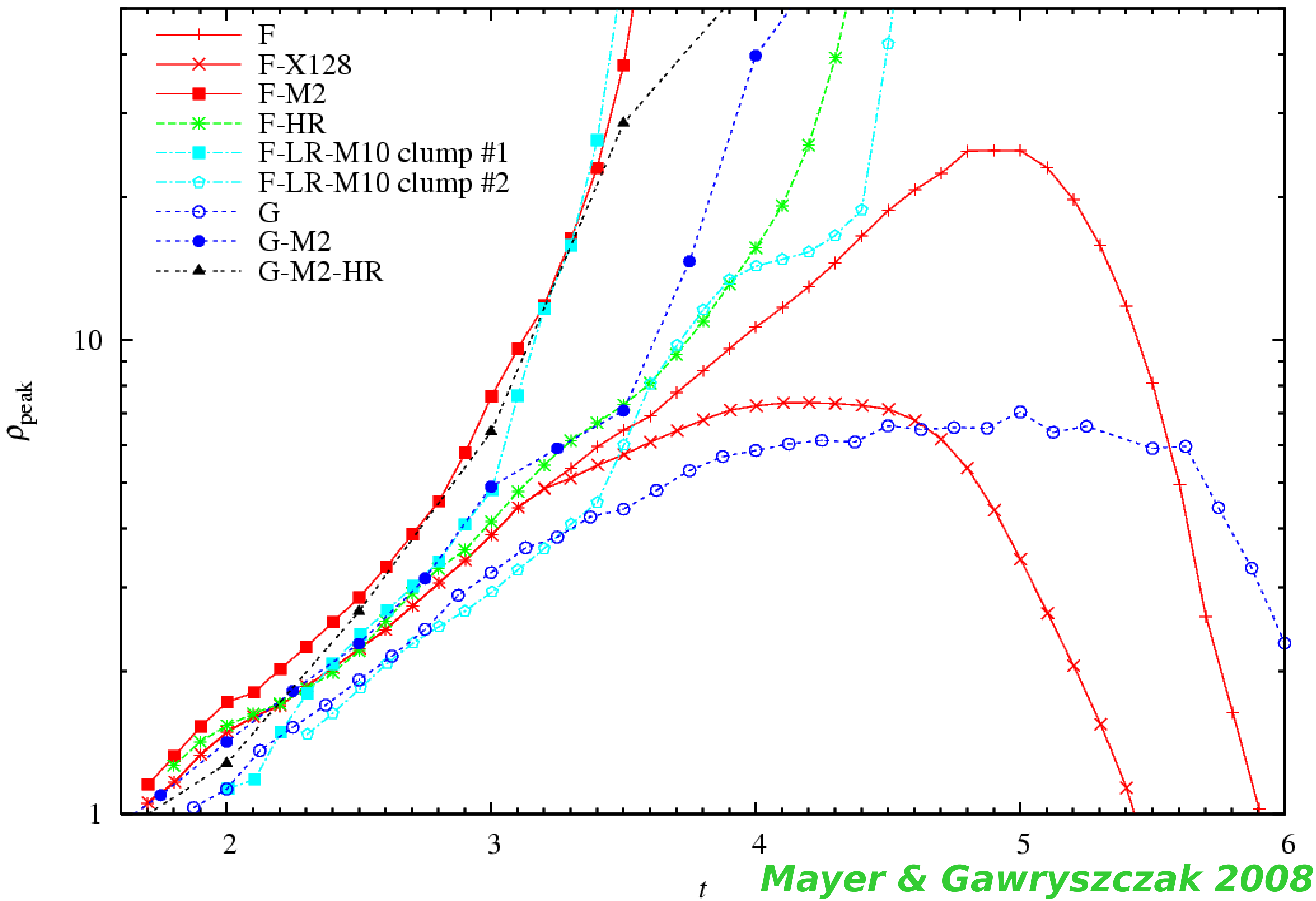
- Aim: test fragmentation of self-gravitating gas disk with different numerical techniques
- Strategy: SAME initial conditions for both SPH and grid codes: interpolate particles onto grid
  - Even grid codes start with “particle noise”
- IC: Q ~ 1 (marginally unstable), evolved with simple equation of state (isothermal)


Gasoline, 200k particles, t = 31.46 yr
base resolution
FLASH, 2048×2048×416 (eff), t = 31.46 yr
base resolution (J ≥ 8)
Enzo, 1024×1024×104 (eff.), t = 31.46 yr
Zeus-mode with 2 levels of refinement
CHYMERA, (256×1024×52), t = 31.53 yr
base resolution
y [AU]
x [AU]
Σ [g/cm2]
20
10
0
-10
-20
0
1000
2000
3000
4000
5000

Once resolution high enough to avoid spurious fragmentation clumps denser and longer lasting as resolution is further increased



Density evolution of first clump in AMR (FLASH) and SPH (GASOLINE)



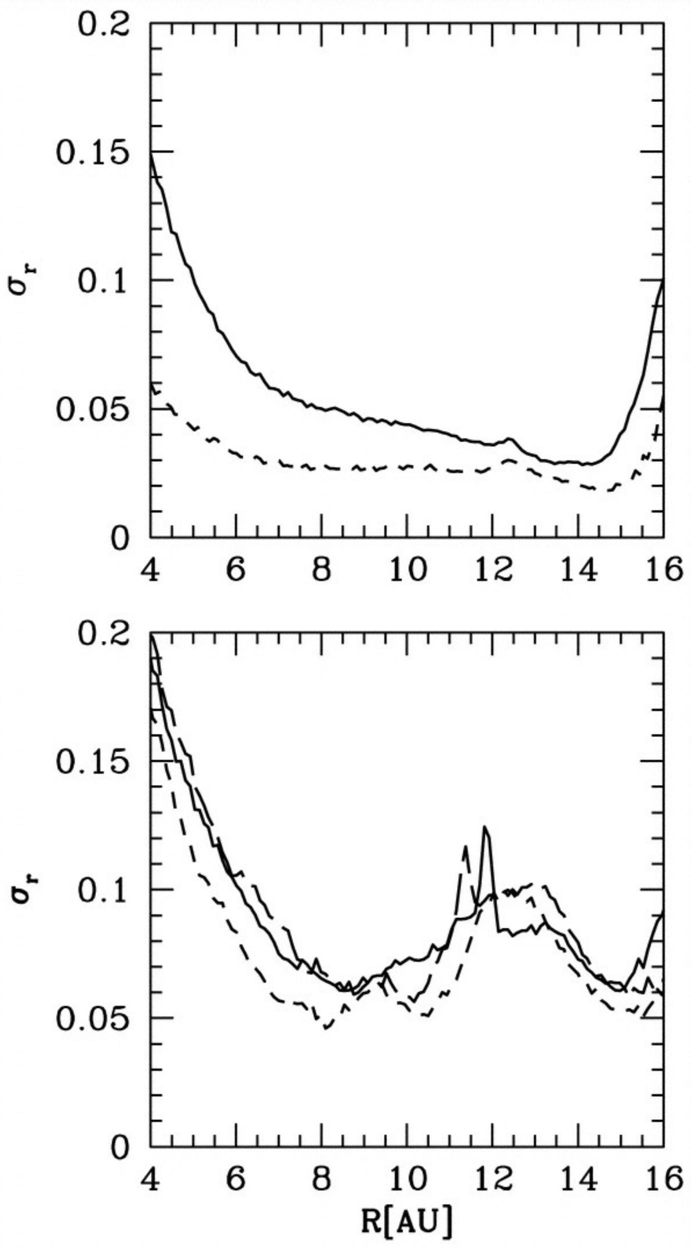
Mayer & Gawryszczak 2008

Wengen (preliminary) conclusions

- AMR and SPH converge at high enough resolution
  - This may be problem dependent
- In this case we need to resolve:
  - Jeans length
  - Disk scale height with 12 cells
- And this is with a simple EOS

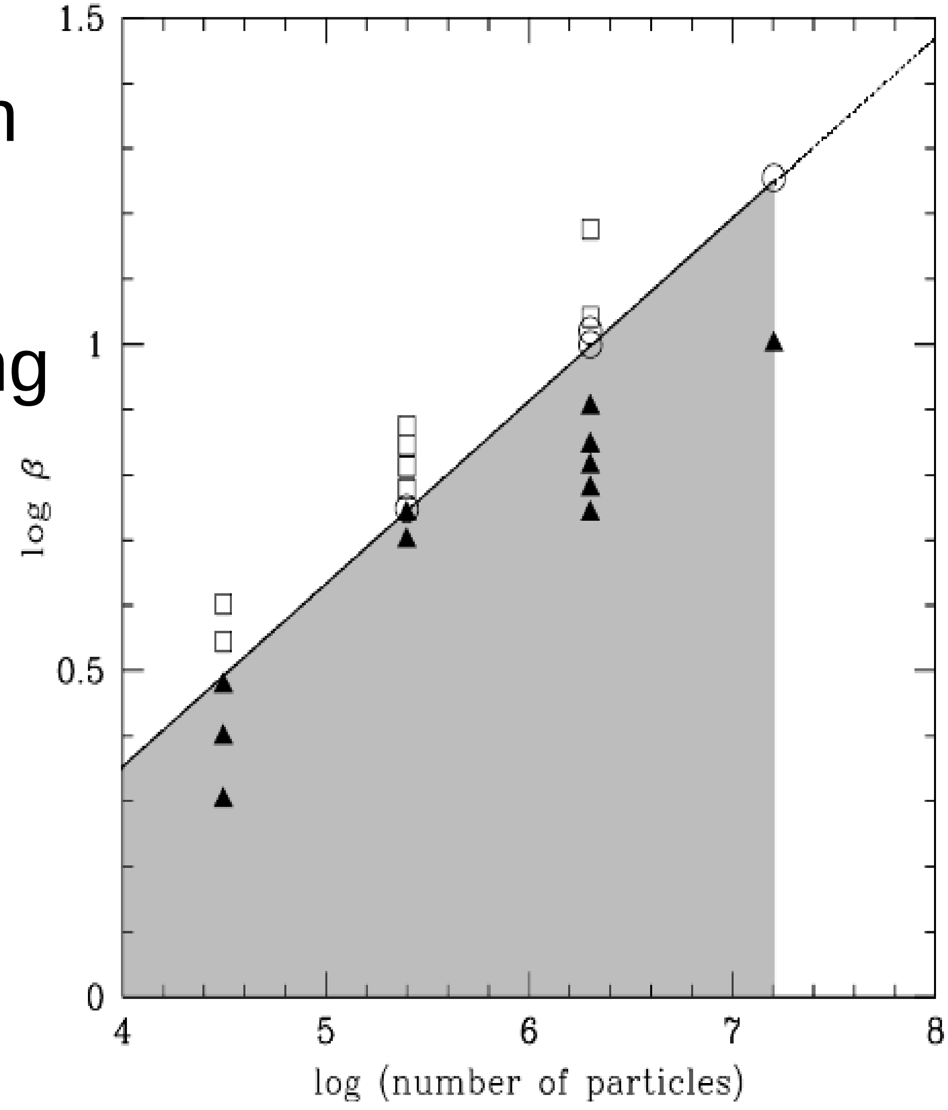
Artificial viscosity and fragmentation

- Mayer et al, 2004:
  - lowering AV does not make stable disk unstable
  - Raising AV makes unstable disk stable
- Pickett & Durisen 2007:
  - AV preserves clumps once formed
  - Raising AV stabilizes disk
- How does AV enter into critical cooling rate?



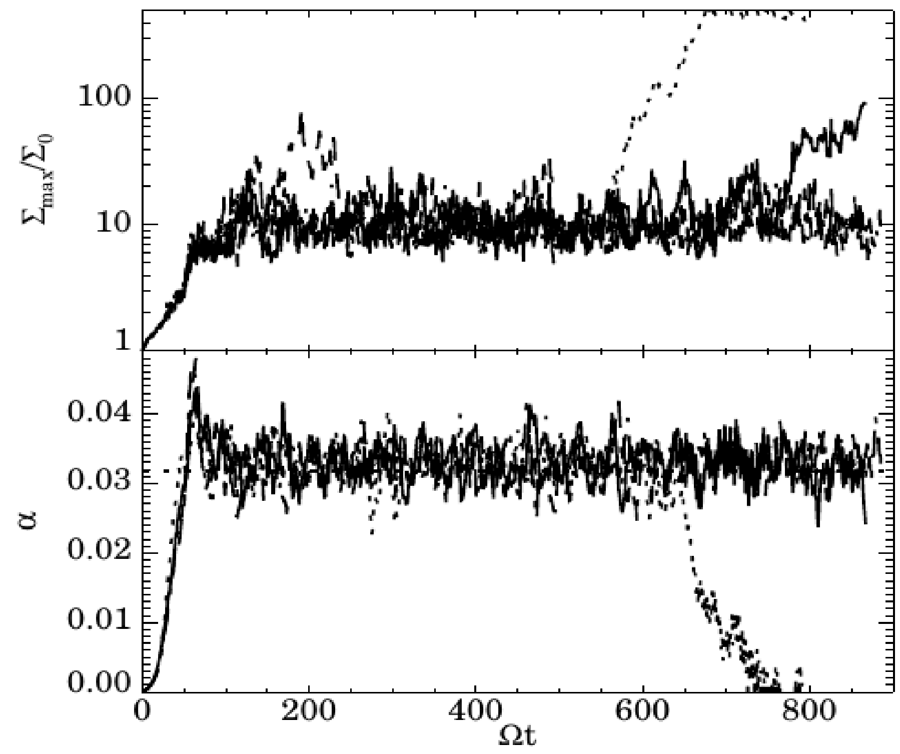
Artificial viscosity and cooling times

- β depends on resolution
  Meru & Bate 2011,
  Lodato & Clarke
- Also seen in 2D shearing sheet (Paardekooper 2012)



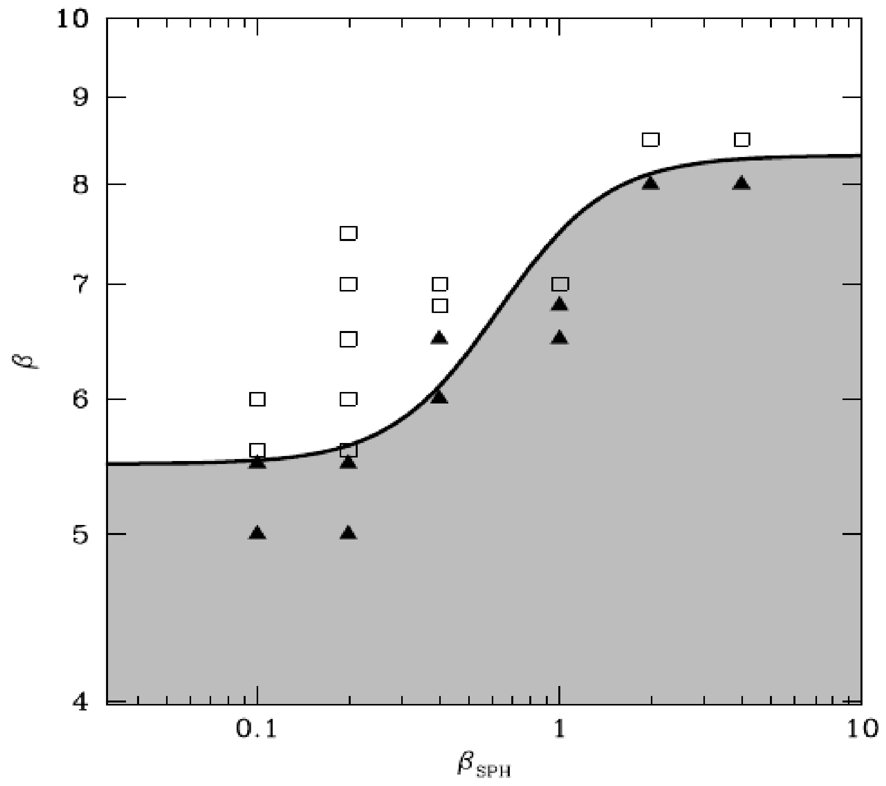
Stochastic process?

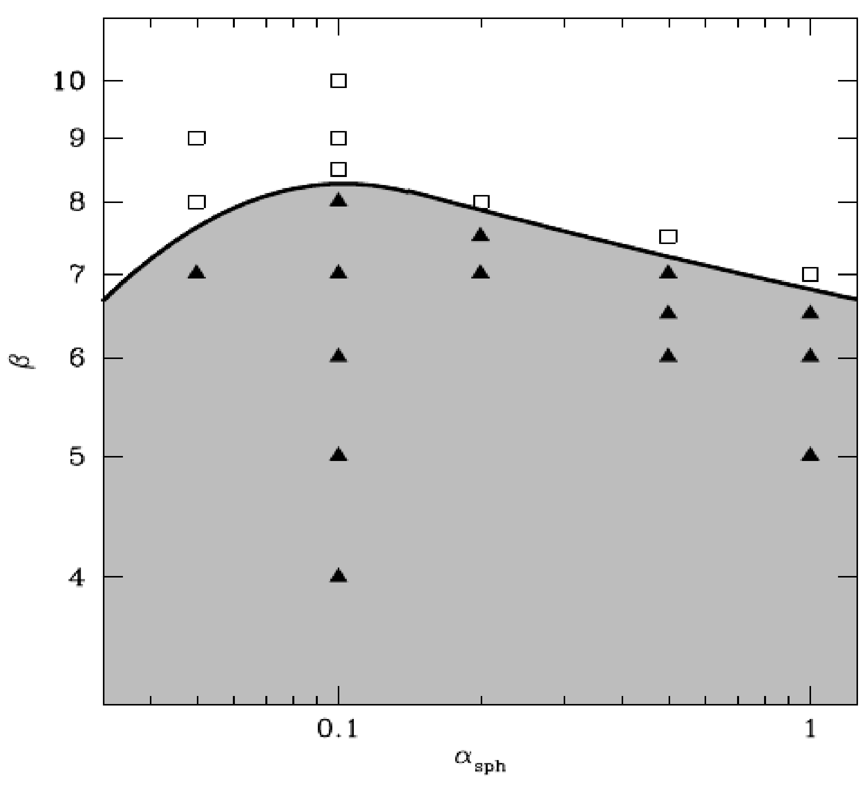
- At low cooling rates fragmentation occurs
- But it takes a while
- No critical cooling rate?



Paardekooper, 2012

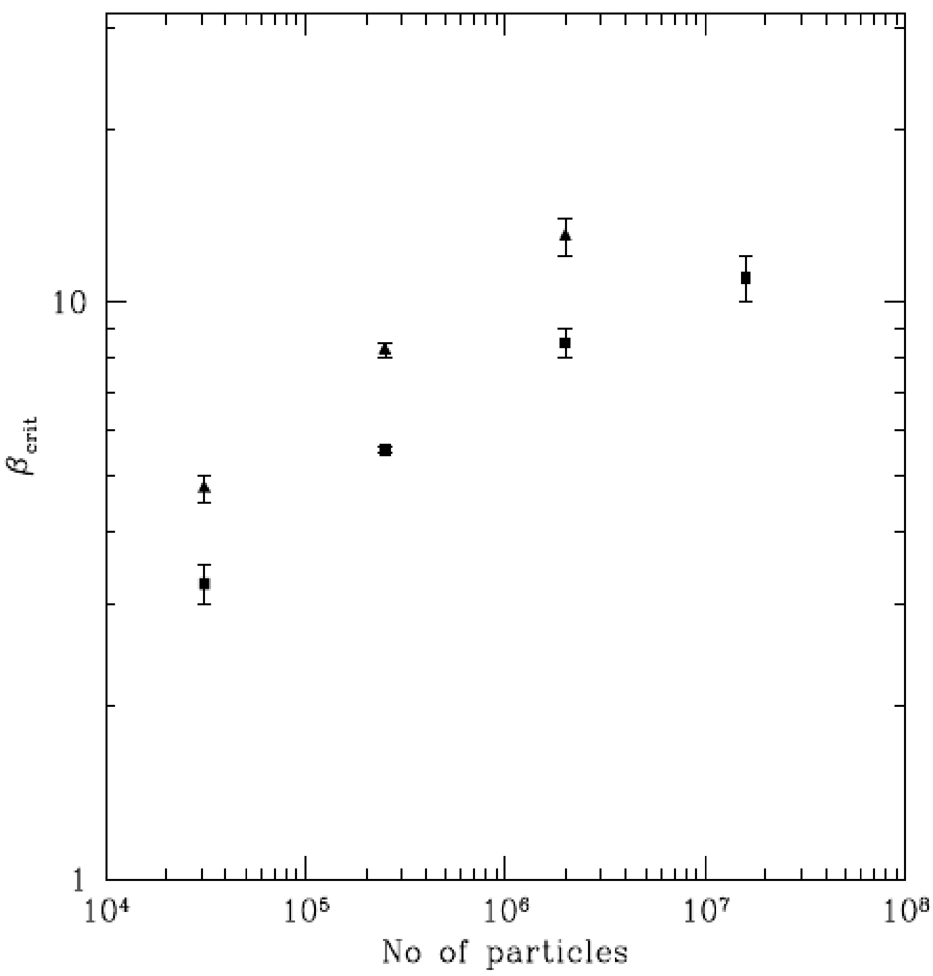
Artificial viscosity and fragmentation



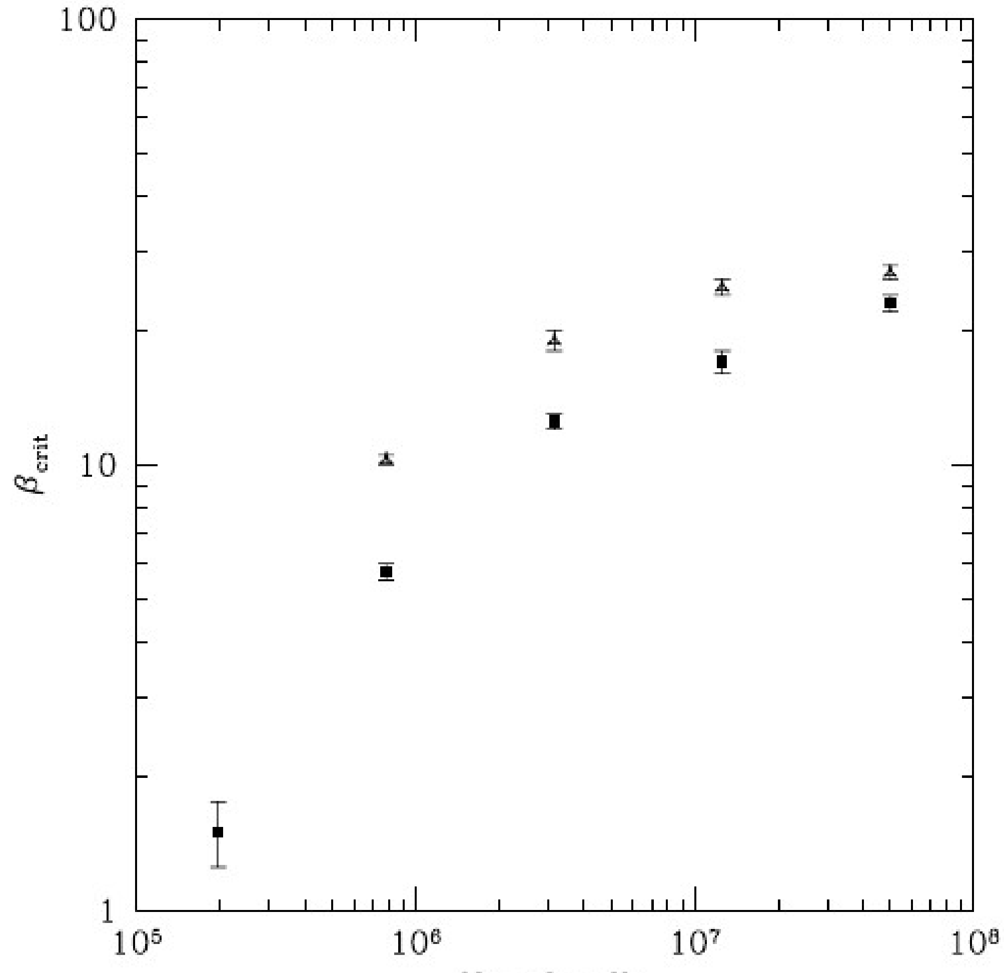


Meru & Bate 2012

Convergence and AV



SPH



Grid

Improving SPH convergence

- Better artificial viscosity (e.g. Cullen & Dehnen, 2010)
- Limit viscosity in rotating flows (Balsara)
- Higher order kernel (Dehnen & Aly 2012)
- Heating is local:
  - Weak shocks generate entropy locally
  - Global cooling might not be a good model

Improvements to EOS

- Recall energy equation:

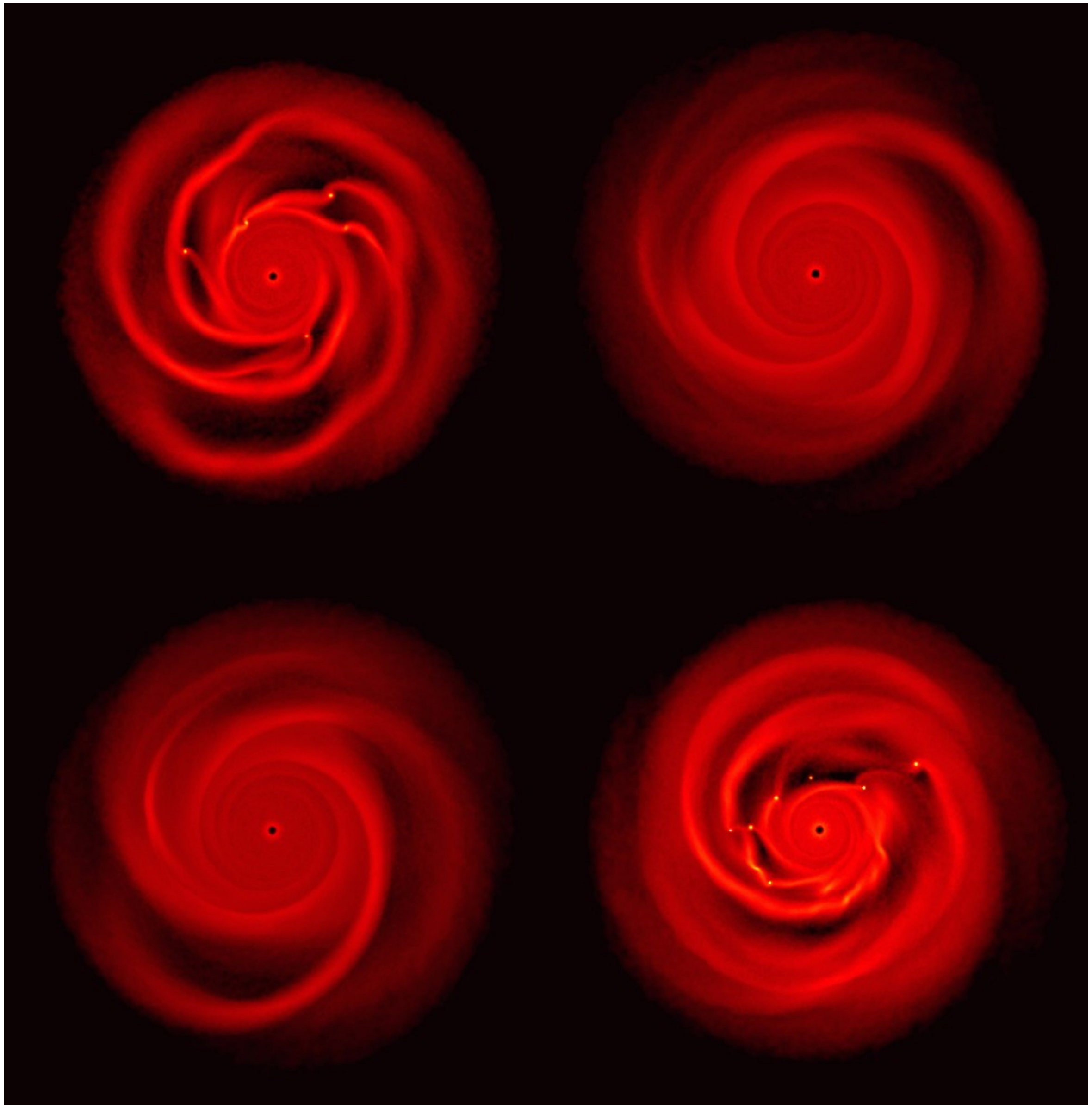
$$\frac{d\ u_i}{dt} = \frac{P_i}{\rho_i^2} \sum_{j=1}^{n} m_j \vec{v}_{ij} \cdot \nabla_i W_{ij} \qquad \text{- ???}$$

- Optically thin: scales as ρ^2, not ρ.
- Diffusion: only for very optically thick
- Flux limited diffusion

Flux limited diffusion

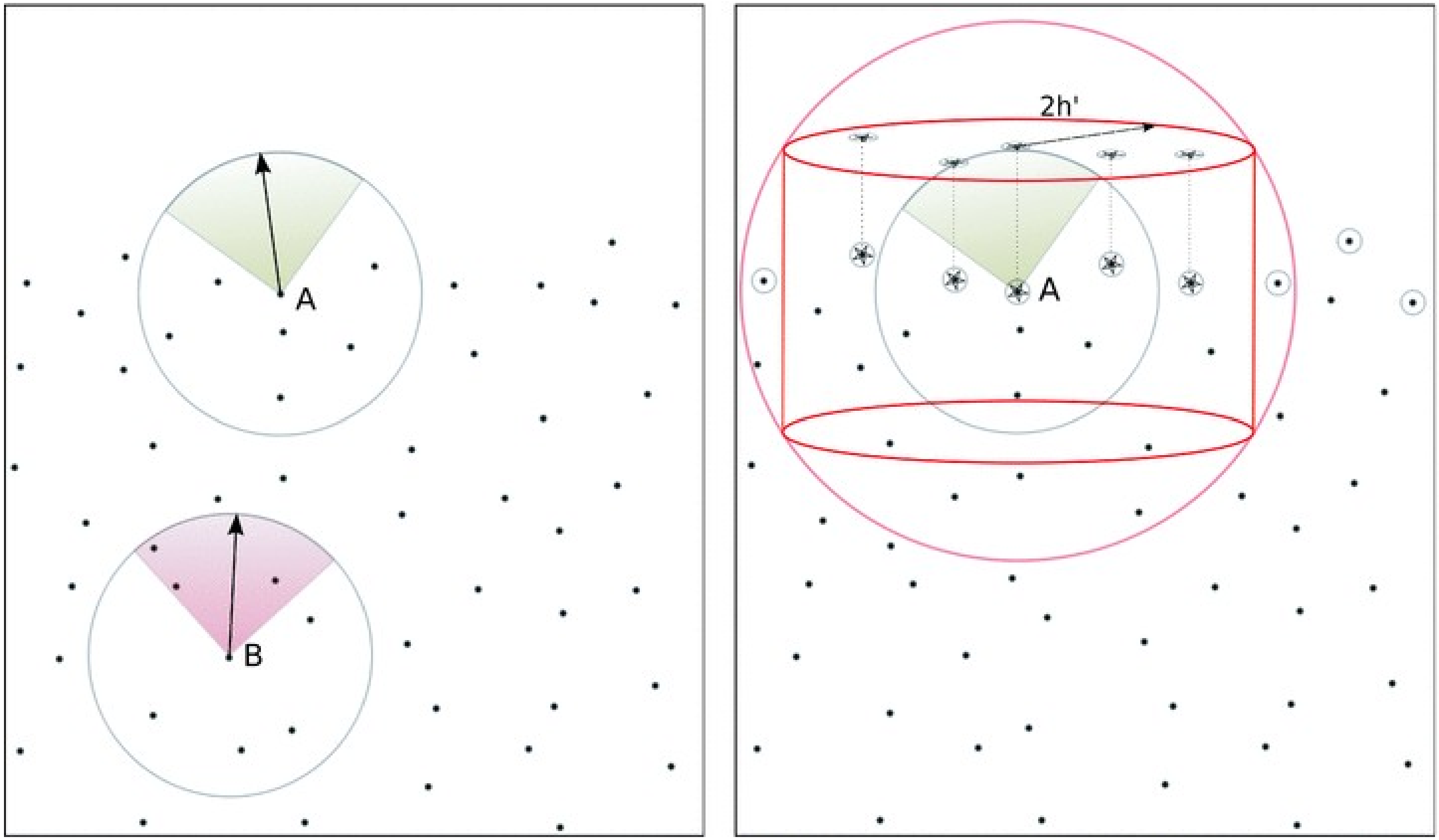
- Energy Flux: $$\mathbf{F} = -\frac{c\lambda}{\rho\kappa}\nabla U_r$$
- Flux limiter: $$\lambda = \lambda(R) = \frac{2+R}{6+3R+R^2}$$
- Where
$$R \equiv \frac{|\nabla U_r|}{U_r \rho\kappa} = \frac{4|\nabla T|}{T\rho\kappa}.$$
- Boundary particles radiate:
$$\dot{U}_a = f_a S \sigma T_a^4 / m_a$$

Fragmentation depends on boundary parameters



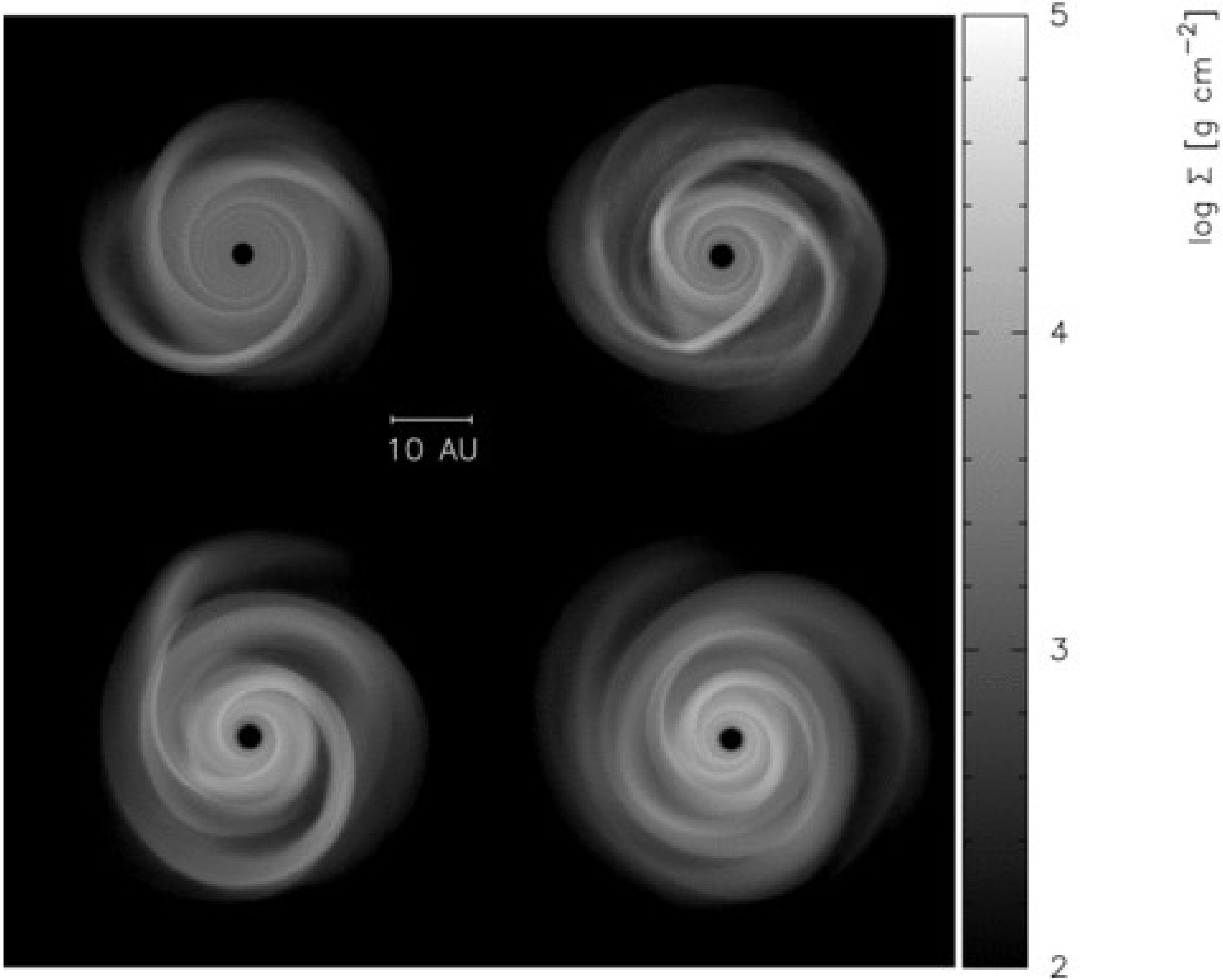
Mayer et al
2007

More accurate boundaries



Rodgers & Wadsley 2011

No Fragmentation



Summary

- Resolution matters
- Understanding numerical stability matters
- Equation of State matters
- Other things I haven't considered probably matter:
  - MHD
  - Streaming instabilities
  - ...