radiation transport, monte carlo
and supernova light curves

daniel kasen, UC Berkeley/LBNL
supernovae and the transient universe

optical light curve
days since explosion

optical spectrum
Wavelength (Angstroms)

2000 3000 4000 5000 6000 7000 8000
Ordinary core collapse supernovae
ordinary core collapse supernovae

Graph showing the relationship between peak luminosity (ergs/sec) and light curve duration (days) for ordinary core collapse supernovae.
ordinary core collapse supernovae

type Ia
supernova light curves
some basic physical scales
supernova light curves
some basic physical scales

assume (conservatively) blackbody emission at $T \sim 10^4$ K

$$L = 4\pi R^2 \sigma_{SB} T^4 \quad \rightarrow \quad R_{\text{sn}} \sim 10^{15} \text{ cm} \approx 10^4 R_\odot$$
supernova light curves
some basic physical scales

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if the remnant expanded to this radius over $\sim 20$ days

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the kinetic energy of the remnant is then (for $M \sim M_{\odot}$)

$$E \approx \frac{1}{2} M v^2 \approx 10^{51} \text{ ergs} \equiv 1 \text{ B}$$
stellar evolution ($>10^6$ years)

$\rho(r), T(r), A_i(r)$ at ignition/collapse
the computational problem

stellar evolution (>10^6 years)

\[ \rho(r), T(r), A_i(r) \text{ at ignition/collapse} \]

explosion (seconds/hours)

hydrodynamics, equation of state
nuclear burning, neutrino transport
the computational problem

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neutrinos
grav. waves
x-rays, $\gamma$-rays
the computational problem

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\[ \rho(x,y,z), v(x,y,z), T(x,y,z), A_i(x,y,z) \text{ in free expansion} \]

expanding ejecta (months)

photon transport
matter opacity
thermodynamics
radioactive decay

neutrinos
grav. waves
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neutrinos
grav. waves
x-rays, \( \gamma \)-rays

optical spectra
light curves
# how to explode a supernova

## simple description

<table>
<thead>
<tr>
<th>take a</th>
<th>white dwarf</th>
<th>helium star</th>
<th>red giant</th>
</tr>
</thead>
<tbody>
<tr>
<td>with a mass</td>
<td>$1.4 , M_{\odot}$</td>
<td>$\sim 5 , M_{\odot}$</td>
<td>$10-20 , M_{\odot}$</td>
</tr>
<tr>
<td>and a radius</td>
<td>$10^9 , \text{cm}$</td>
<td>$10^{11} , \text{cm}$</td>
<td>$10^{13} , \text{cm}$</td>
</tr>
<tr>
<td>dump in</td>
<td>~$10^{51} , \text{ergs}$</td>
<td><strong>hydro, burning, neutrinos, etc...</strong></td>
<td></td>
</tr>
<tr>
<td>get a</td>
<td>type Ia</td>
<td>type Ib/Ic</td>
<td>type II</td>
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</tbody>
</table>
Jet powered supernova

Sean Couch
jet
powered
supernova
sean
couch
energetics
energetics

immediately after the explosion (e.g., strong shock) total energy is split between kinetic energy and radiation

\[ E_{\text{kinetic}} \approx E_{\text{thermal}} \approx 10^{51} \text{ ergs} \]
energetics

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radiation energy dominates over gas thermal energy density
e.g., explode the sun, with \( E = 10^{51} \text{ ergs} \)

\[ \frac{\varepsilon_{\text{rad}}}{\varepsilon_{\text{gas}}} \approx \frac{aT^4}{\frac{3}{2}nkT} \approx 60 \left( \frac{T}{10^8 K} \right)^3 \left( \frac{1 \text{ g cm}^{-3}}{\rho} \right) \]
energetics

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but the radiation can’t escape because a star is opaque. The ejecta expands by a factor of \( 10^2 \text{-} 10^6 \) in radius before the density drops enough to become translucent
initial radiation from supernova explosions
shock breakout x-ray burst from a red super-giant

\( E_x = 10^{48} \) ergs

![Graph showing shock breakout and temperature vs radius, with temperature on the y-axis and radius in cm on the x-axis.]
adiabatic expansion

converts $E_{\text{thermal}}$ into $E_{\text{kinetic}}$ as the radiation does work
adiabatic expansion

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first law of thermodynamics (with no heat transfer)

$$\frac{\partial (\epsilon_{\text{rad}} V)}{\partial t} = -p_{\text{rad}} \frac{\partial V}{\partial t}$$
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use chain rule and $\epsilon_{\text{rad}} = 3p_{\text{rad}}$
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\dot{\epsilon}_{\text{rad}} V + \epsilon_{\text{rad}} \dot{V} = -\frac{\epsilon_{\text{rad}}}{3} \dot{V}
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$$\frac{\dot{\varepsilon}_{\text{rad}}}{\varepsilon_{\text{rad}}} = -\frac{4}{3} \frac{\dot{V}}{V}$$
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$$\frac{\dot{\epsilon}_{\text{rad}}}{\epsilon_{\text{rad}}} = -\frac{4}{3} \frac{\dot{V}}{V} \quad \rightarrow \quad \epsilon_{\text{rad}} \propto V^{-4/3} \propto R^{-4}$$
adiabatic expansion
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basic thermodynamics
adiabatic expansion

basic thermodynamics

$pV^\gamma = \text{Constant}$
adiabatic expansion

basic thermodynamics

\[ pV^\gamma = \text{Constant} \]

(with \( \gamma = 4/3 \) for radiation dominated)
adiabatic expansion

basic thermodynamics

\[ p V^\gamma = \text{Constant} \]

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now since

\[ \epsilon_{\text{rad}} = 3 \rho_{\text{rad}} = aT^4 \quad \text{and} \quad V \propto R^3 \]
adiabatic expansion

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the energy density drops as ejecta radius expands
adiabatic expansion

basic thermodynamics

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now since

\[ \epsilon_{\text{rad}} = 3p_{\text{rad}} = aT^4 \quad \text{and} \quad V \propto R^3 \]

the energy density drops as ejecta radius expands

\[ \epsilon_{\text{rad}} \propto R^{-4} \quad \text{or} \quad \epsilon_{\text{rad}} V = E_{\text{rad}} \propto R^{-1} \]
transition to free expansion
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right after the explosion shock, the sound speed $c_s$ is of order the expansion velocity, but $c_s$ drops with adiabatic cooling
transition to free expansion

right after the explosion shock, the sound speed $c_s$ is of order the expansion velocity, but $c_s$ drops with adiabatic cooling

$$c_s = \sqrt{\frac{\gamma p_{\text{rad}}}{\rho}} \propto R^{-1/2}$$
transition to free expansion

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pressure waves can't communicate forces faster than the ejecta expands, so hydrodynamics freezes out and fluid moves ballistically
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$$R(t) = vt + R_0$$
transition to free expansion

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pressure waves can’t communicate forces faster than the ejecta expands, so hydrodynamics freezes out and fluid moves ballistically
homologous expansion
self-similar ejecta structure expands over time

\[ R(t) = vt \]
\[ \rho(t) = \rho_0 (R_0/R)^3 \propto t^{-3} \]

rule of thumb:
to reach homology
run your hydrodynamics simulations until
\[ R_{\text{final}} > \sim 10 \ R_0 \]
better: \[ R_{\text{final}} \sim 100 \ R_0 \]

check, \[ E_{\text{thermal}} << E_{\text{kinetic}} \]
duration of the light curve
duration of the light curve

the \textit{diffusion time} of photons through the optically thick remnant

\[ t_d = \tau \left[ \frac{R}{c} \right] \]
duration of the light curve

diffusion time of photons through the optically thick remnant

\[ t_d = \tau \left[ \frac{R}{c} \right] = \kappa \rho R \left[ \frac{R}{c} \right] \]
duration of the light curve

the *diffusion time* of photons through the optically thick remnant

\[ t_d = \tau \left[ \frac{R}{c} \right] = \kappa \rho R \left[ \frac{R}{c} \right] \sim \frac{M \kappa}{Rc} \]

since

\[ \rho \sim \frac{M}{R^3} \]
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but since the remnant is expanding, \( R = vt \)

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solving for time (i.e., diffusion time \( \sim \) elapsed time)
duration of the light curve

the *diffusion time* of photons through the optically thick remnant

\[
    t_d = \tau \left( \frac{R}{c} \right) = \kappa \rho R \left( \frac{R}{c} \right) \sim \frac{M \kappa}{R \epsilon} \quad \text{since} \quad \rho \sim \frac{M}{R^3}
\]

but since the remnant is expanding, \( R = vt \)

\[
    t_d \sim \frac{M \kappa}{(vt) \epsilon} \quad \text{solving for time (i.e., diffusion time \( \sim \) elapsed time)}
\]

\[
    t_d \sim \left[ \frac{M \kappa}{vc} \right]^{1/2}
\]

e.g., arnett (1979)
diffusion in an expanding medium

arnett 1979, 1980, 1982

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diffusion in an expanding medium

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or substituting:  \[ v = \sqrt{2E/M} \]
diffusion in an expanding medium
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or substituting:
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gives the scaling relation for light curve duration

\[ t_d \approx 29 \text{ days} \ M_{1,\odot}^{1/2} \kappa_{0.4}^{1/2} E_{51}^{-1/4} \]
diffusion in an expanding medium

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mass often tends to be the dominating factor
opacity terminology
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for example
opacity terminology

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for example

\[ \sigma_t \simeq 0.6 \times 10^{-24} \text{ cm}^2 \text{ for thomson scattering} \]
\[ \kappa_{es} = \frac{x_{ion} \sigma_t}{m_a} \approx 0.4 \text{ for ionized hydrogen} \]
\[ \approx 0.007 \text{ for singly ionized iron} \]
## Sources of Supernova Opacity


<table>
<thead>
<tr>
<th>Process</th>
<th>Interaction/phenomenon</th>
<th>Wavelength</th>
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<tr>
<td>Thomson scattering</td>
<td>Interaction with free electrons</td>
<td>optical</td>
</tr>
<tr>
<td>Atomic lines</td>
<td>Scattering/absorption from doppler broadened lines</td>
<td>UV/optical</td>
</tr>
<tr>
<td>Bound-free</td>
<td>Photo-ionization of atoms</td>
<td>UV</td>
</tr>
<tr>
<td>Free-free</td>
<td>Bremsstrahlung (free electron + nucleus)</td>
<td>infrared</td>
</tr>
</tbody>
</table>

All of these depend sensitively on the composition and **ionization state** of the ejecta!
bound-free electron scattering
free-free lines
solar composition
$T = 10^4$, $\rho = 10^{-13}$

Opacity (cm$^2$ g$^{-1}$)

Wavelength (Angstroms)
pure iron composition
$T = 10^4$, $\rho = 10^{-13}$
Local Thermodynamic Equilibrium (LTE)

\[ \frac{n_i}{n_j} = \frac{g_i}{g_j} \exp\left(-\frac{\Delta E}{kT}\right) \]

non-equilibrium (NLTE)

\[
\frac{\partial n_i}{\partial t} = \sum_{j \neq i} \left( n_j R_{ji} - n_i R_{ij} \right) + \sum_{j \neq i} \left( n_j C_{ji} - n_i C_{ij} \right) + \sum_{j \neq i} \left( n_j G_{ji} - n_i G_{ij} \right) = 0
\]

nxn matrix, where \( n \) = number of atomic levels (sparsity depends on number of transitions included)
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\(\text{n x n matrix, where n = number of atomic levels (sparsity depends on number of transitions included)}\)
2005ap, 2008es, scp06f6, ptf09cnd, 2006gy, 2007bi

ordinary core collapse supernovae

light curve duration (days)

peak luminosity (ergs/sec)

$10^{42}$

$10^{43}$

$10^{44}$

$10^{45}$

type Ia
More massive, opaque (longer diffusion time)
how to power a supernova light curve
how to power a supernova light curve

- thermal energy released in the explosion shock, nuclear burning
how to power a supernova light curve

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- radioactive decay of freshly synthesized isotopes: $^{56}$Ni ($^{52}$Fe, $^{48}$Cr, $^{44}$Ti, R-process)
how to power a supernova light curve

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- interaction of the ejecta with a dense surrounding medium
how to power a supernova light curve

- thermal energy released in the explosion shock, nuclear burning
- radioactive decay of freshly synthesized isotopes: $^{56}\text{Ni}$ ($^{52}\text{Fe}$, $^{48}\text{Cr}$, $^{44}\text{Ti}$, R-process)
- interaction of the ejecta with a dense surrounding medium
- energy injection from a rotating, highly magnetized neutron star (magnetar)
thermally powered supernovae (Type IIP)
luminosity of thermal light curve
energy deposited by the explosion
luminosity of thermal light curve
energy deposited by the explosion

the radiation energy drops in the expanding gas
and takes a diffusion time to escape

\[ E_{\text{rad}}(t) = E_0 \left( \frac{R_0}{R(t)} \right) \]

\[ t_d \sim \left[ \frac{M \kappa}{\nu c} \right]^{1/2} \]
luminosity of thermal light curve
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simple estimate of supernova luminosity
luminosity of thermal light curve
energy deposited by the explosion

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simple estimate of supernova luminosity

\[ L_{\text{sn}} = \frac{E_0}{t_d} \left[ \frac{R_0}{\nu t_d} \right] \approx 10^{41} \text{ ergs s}^{-1} R_{1,\odot} E_{51} \kappa_{0.4}^{-1} M_{1,\odot}^{-1} \]
luminosity of thermal light curve
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the radiation energy drops in the expanding gas

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simple estimate of supernova luminosity

\[ L_{\text{sn}} = \frac{E_0}{t_d} \left[ \frac{R_0}{\nu t_d} \right] \approx 10^{41} \text{ ergs s}^{-1} R_{1,\odot} E_{51} \kappa_{0.4}^{-1} M_{1,\odot}^{-1} \]

low radiative efficiency if initial radius is small!
for a bright thermally powered supernova, must have \( R_0 \gg R_{\text{sun}} \)
ordinary core collapse supernovae

more massive (longer diffusion time)
more massive (longer diffusion time)

more energetic, larger radius

ordinary core collapse supernovae

Type Ia
ordinary core collapse supernovae

more massive (longer diffusion time)

more energetic, larger radius

fixed E/M

$R = 10^2 R_{\text{sun}}$

$R = 10^3 R_{\text{sun}}$

$R = 10^4 R_{\text{sun}}$

peak luminosity (ergs/sec)

light curve duration (days)
Type IIP core collapse supernovae
explosion of red supergiant stars

\[ M = 15M_\odot \quad R_\ast \approx 10^{13} \text{ cm} \]

I-D models (vary explosion energy)

DK & woosley, 2009
recombination wave in Type II
p supernova

opacity from electron scattering drops as ejecta cool and become neutral

recombination at $T \sim 6000$ K
light curve scalings with recombination
gives a Type II plateau light curve
light curve scalings with recombination
gives a Type II plateau light curve

the photosphere forms at the recombination front

$$R_I^2 = \frac{L}{4\pi\sigma T_I^4} = \frac{E_0 R_0}{v t_d^2 \sigma T_I^4}$$
light curve scalings with recombination

gives a Type II plateau light curve

the photosphere forms at the recombination front

\[ R_1^2 = \frac{L}{4\pi\sigma T_1^4} = \frac{E_0 R_0}{\nu t_d^2 \sigma T_1^4} \]

where the recombination temperature \( T_1 = \sim 6000 \text{ K} \).
light curve scalings with recombination gives a Type II plateau light curve

the photosphere forms at the recombination front

$$R_{I}^{2} = \frac{L}{4\pi\sigma T_{I}^{4}} = \frac{E_{0}R_{0}}{vt_{d}^{2}\sigma T_{I}^{4}}$$

where the recombination temperature $T_{i} = \sim 6000$ K.

Using previous results for diffusion time:

$$t_{sn} \propto E^{-1/6} M_{ej}^{1/2} R_{0}^{1/6} \kappa^{1/6} T_{I}^{-2/3}$$

$$L_{sn} \propto E^{5/6} M_{ej}^{-1/2} R_{0}^{2/3} \kappa^{-1/3} T_{I}^{4/3}.$$  

see Popov (1993), DK & Woosley (2009)
radioactivity
powered supernovae
radioactively powered light curves

most important chain: $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$
radioactive $^{56}$Ni decay

$^{56}$Ni → $^{56}$Co  
8.8 days  
100% electron capture

$^{56}$Co → $^{56}$Fe  
113 days  
81% electron capture  
19% positron production

<table>
<thead>
<tr>
<th>Important Gamma-Ray Line for $^{56}$Ni and $^{56}$Co Decays</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{56}$Ni Decay</td>
</tr>
<tr>
<td>Energy (keV)</td>
</tr>
<tr>
<td>158............</td>
</tr>
<tr>
<td>270............</td>
</tr>
<tr>
<td>480............</td>
</tr>
<tr>
<td>750............</td>
</tr>
<tr>
<td>812............</td>
</tr>
<tr>
<td>1562............</td>
</tr>
</tbody>
</table>

milne et al. (2004)
gamma-ray deposition by compton scattering

since gamma-ray energies (MeV) are much greater than ionization potentials, all electrons (free + bound) contribute
gamma-ray deposition by compton scattering

since gamma-ray energies (MeV) are much greater than ionization potentials, all electrons (free + bound) contribute

change in photon wavelength

\[ \lambda_{\text{out}} = \lambda_{\text{in}} + \frac{\hbar}{m_e c} (1 - \cos \theta) \]

angle between incoming and outgoing photon directions
gamma-ray deposition by compton scattering

since gamma-ray energies (MeV) are much greater than ionization potentials, all electrons (free + bound) contribute

des change in photon wavelength

\[ \lambda_{out} = \lambda_{in} + \frac{h}{mec^2}(1 - \cos \theta) \]

change in photon energy from inelastic scattering

\[ E_{out} = E_{in} \left[1 + \frac{E_{in}}{mec^2(1 - \cos \theta)}\right]^{-1} \]
gamma-ray deposition by Compton scattering

since gamma-ray energies (MeV) are much greater than ionization potentials, all electrons (free + bound) contribute

change in photon wavelength

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change in photon energy from inelastic scattering

\[ E_{\text{out}} = E_{\text{in}} \left[ 1 + \frac{E_{\text{in}}}{m_e c^2} (1 - \cos \theta) \right]^{-1} \]

so an MeV (~2 m_e c^2) gamma-ray loses most of its energy after just a few Compton scatterings (then it gets photo-absorbed)
type Ia supernova light curves

- optical (thermalized gamma-rays)
- escaping gamma-ray
type Ia gamma-ray spectrum

40 days after explosion
radioactively powered light curves

most important chain: $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$
radioactive supernovae
light curve estimates
radioactive supernovae

light curve estimates

light curve duration given by standard diffusion time

\[ t_d \sim \left( \frac{M \kappa}{v c} \right)^{1/2} \]
radioactive supernovae
light curve estimates

light curve duration given by standard diffusion time

\[ t_d \sim \left[ \frac{M \kappa}{vc} \right]^{1/2} \]

luminosity estimate from radioactive energy deposition

\[ L_{ni} \approx \frac{M_{ni} \epsilon_{ni}}{t_{ni}} e^{-t_d / t_{ni}} \]
arnett’s law
arnett’s law

\[ \frac{\partial \left( \epsilon_{\text{rad}} V \right)}{\partial t} = -p \frac{\partial V}{\partial t} - L_{\text{diff}} + L_\gamma \]
arnett’s law

\[ \frac{\partial(\epsilon_{\text{rad}} V)}{\partial t} = -p \frac{\partial V}{\partial t} - L_{\text{diff}} + L_\gamma \]

use \( V \propto t^3 \), \( p_{\text{rad}} = \epsilon_{\text{rad}}/3 \)

\[ \frac{1}{t} \frac{\partial(\epsilon_{\text{rad}} V t)}{\partial t} = -L_{\text{diff}} + L_\gamma \]
arnett’s law

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\frac{\partial (\epsilon_{\text{rad}} V)}{\partial t} = -p \frac{\partial V}{\partial t} - L_{\text{diff}} + L_{\gamma}
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arnett’s law

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\[ \frac{1}{t} \frac{\partial (\epsilon_{\text{rad}} V t)}{\partial t} = -L_{\text{diff}} + L_{\gamma} \]

assume diffusion approximation for photon loses

\[ L_{\text{diff}} = 4\pi r^2 \frac{c}{3\kappa \rho} \frac{\partial \epsilon_{\text{rad}}}{\partial r} \approx 4\pi r^2 \frac{c}{3\kappa \rho} \frac{\epsilon_{\text{rad}}}{r} \]
arnett’s law

\[
\frac{\partial (\varepsilon_{\text{rad}} V)}{\partial t} = -p \frac{\partial V}{\partial t} - L_{\text{diff}} + L_{\gamma}
\]

use \( V \propto t^3 \), \( p_{\text{rad}} = \varepsilon_{\text{rad}}/3 \)

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\frac{1}{t} \frac{\partial (\varepsilon_{\text{rad}} V t)}{\partial t} = -L_{\text{diff}} + L_{\gamma}
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\]

find

\[
L_{\text{diff}} = \frac{\varepsilon_{\text{rad}} V t}{t_d^2}
\]
ordinary core collapse supernovae

- 2005ap
- 2008es
- scp06f6
- ptf09cnd
- 2006gy
- 2007bi

light curve duration (days)

peak luminosity (ergs/sec)
ordinary core collapse supernovae

more massive (longer diffusion time)
more radioactive

ordinary core collapse supernovae

more massive (longer diffusion time)
more radioactive

ordinary core collapse supernovae

more massive (longer diffusion time)
\[ M_{\text{Ni}} = M_{\text{ej}} \]

more radioactive

\[ M_{\text{Ni}} = 0.1 M_{\text{ej}} \]

ordinary core collapse supernovae

more massive (longer diffusion time)
pulsations and interaction

eta carinae
“tamped” supernova models

interacting supernovae

second ejection

slow moving shell of debris from first ejection
tamped supernova models
colliding shell toy model
density
velocity
density

velocity
colliding shell
toy model
interacting supernovae

simple estimate of peak luminosity
interacting supernovae

simple estimate of peak luminosity

peak luminosity for shocked debris at shell radius

\[ L_{\text{sn}} \approx \frac{E_{\text{sn}}}{t_d} \left[ \frac{R_{\text{sh}}}{R_{\text{sn}}} \right] \sim 10^{45} \text{ ergs s}^{-1} \left[ \frac{R_{\text{sh}}}{10^4 R_\odot} \right] \]
interacting supernovae

simple estimate of peak luminosity

peak luminosity for shocked debris at shell radius

\[ L_{\text{sn}} \approx \frac{E_{\text{sn}}}{t_d} \left[ \frac{R_{\text{sh}}}{R_{\text{sn}}} \right] \sim 10^{45} \text{ ergs s}^{-1} \left[ \frac{R_{\text{sh}}}{10^4 R_{\odot}} \right] \]

to reach the highest luminosities, shell must be at radius

\[ R_{\text{sh}} \approx 10^4 R_{\odot} \approx 10^{15} \text{ cm} \]
interacting supernovae

simple estimate of peak luminosity

peak luminosity for shocked debris at shell radius

\[ L_{sn} \approx \frac{E_{sn}}{t_d} \left[ \frac{R_{sh}}{R_{sn}} \right] \approx 10^{45} \text{ ergs s}^{-1} \left[ \frac{R_{sh}}{10^4 R_{\odot}} \right] \]

to reach the highest luminosities, shell must be at radius

\[ R_{sh} \approx 10^4 R_{\odot} \approx 10^{15} \text{ cm} \]

time between pulses of ejection

\[ t_{sh} = \frac{R_{sh}}{v_{sh}} = 2 \text{ years} \left[ \frac{100 \text{ km s}^{-1}}{v_{sh}} \right] \]
pulsational pair SNe
110 $M_{\text{sun}}$ star from woosley et al., 2007

first pulse
$M \sim 25 \, M_{\text{sun}}$
$E \sim 10^{50}$ ergs

second pulse
$M \sim 5 \, M_{\text{sun}}$
$E \sim 6 \times 10^{50}$ ergs
The figure shows a graph of peak luminosity (ergs/sec) versus light curve duration (days). The graph distinguishes between ordinary core collapse supernovae and type Ia supernovae. The ordinary core collapse supernovae are indicated by the green shaded region, while type Ia supernovae are indicated by the red shaded region.

The graph includes data points for supernovae labeled 2005ap, 2006gy, 2007bi, 2008es, scp06f6, and ptf09cnd. The lines $M_{Ni} = M_{ej}$ and $M_{Ni} = 0.1 M_{ej}$ are also plotted on the graph, indicating different mass relations for nickel and ejected mass.
$M_{Ni} = M_{ej}$

$M_{Ni} = 0.1 M_{ej}$

ordinary core collapse supernovae

light curve duration (days)

peak luminosity (ergs/sec)

2005ap
2008es

2007bi

2006gy interaction?

pair sn?

ptf09cnd

scp06f6
power from neutron star spindown

crab nebula
B \sim 5 \times 10^{12} \text{ g}
neutron star spindown

~10% of neutron stars are born as magnetars, with $B \sim 10^{14} - 10^{15}$ g

rotational energy

$$E_{\text{rot}} = \frac{1}{2} I_{\text{ns}} \Omega^2 = 2 \times 10^{50} \text{ ergs} \left(\frac{P}{10 \text{ ms}}\right)^{-2}$$

spindown timescale

$$t_m = \frac{6 I_{\text{ns}} c^3}{B^2 R_{\text{ns}}^6 \Omega^2} = 1.3 \text{ yrs} \left(\frac{B}{10^{14} \text{ g}}\right)^{-2} \left(\frac{P}{10 \text{ ms}}\right)^2$$
light curves from magnetars

roughly

\[ L \sim \frac{E_m}{t_d} \left( \frac{t_m}{t_d} \right) \]

high radiative efficiency when \( B,P \) give \( t_m \sim t_d \)

better (for \( l = 2 \))

\[ L_{\text{peak}} \approx \frac{E_m t_m}{t_d^2} \left[ \ln \left( 1 + \frac{t_d}{t_m} \right) - \frac{t_d}{t_d + t_m} \right] \]

\[ t_{\text{peak}} = t_m \left( \left[ \frac{E_m}{L_{\text{peak}} t_m} \right]^{1/2} - 1 \right) \]

kasen&bildsten (2010)
bolometric magnetar models
$M_{Ni} = M_{ej}$

$magnetar theoretical maximum \sim 2 \times 10^{45}$ ergs/s

$P = 1 \text{ ms}$

$P = 5 \text{ ms}$
Monte Carlo and Numerical Radiation Transport
light curve computation

\[ \rho(x,y,z), v(x,y,z), T(x,y,z), A_i(x,y,z) \]

from hydro explosion

calculate matter opacity/emissivity

advance time step

update matter state (temperature, density, ionization from thermodynamics)

solve radiation transport equation for optical photons

determine radioactive energy deposition (gamma-ray transport)

\[ \rho(x,y,z), v(x,y,z), T(x,y,z), A_i(x,y,z) \]

from hydro explosion
radiation transfer equation

\[ \frac{dI}{ds} = -\chi I + \eta + \int d\Omega \, R(\hat{n}, \hat{n}') I \]

where:

- \( I(x, y, z, \lambda, \theta, \phi) \): radiation specific intensity
- \( \eta(x, y, z, \lambda) \): matter emissivity
- \( \chi(x, y, z, \lambda) \): matter extinction coefficient

a 6 dimensional integro-differential equation
coupled through microphysics to matter energy equation
transport methods in astrophysics

grey flux limited diffusion
ignore $\theta, \phi, \lambda$ dependence, solve diffusion equation for “seeping” radiation fluid

multi-group flux limited diffusion (MGFLD)
ignore $\theta, \phi$, keep $\lambda$ dependence, solve diffusion equation

ray tracing
follow individual trajectories; ignore scattering and diffusive terms

implicit monte carlo transport
mixed-frame stochastic particle propagation; retains the full angle, wavelength, & polarization information

variable Eddington tensor
solve moments of the radiation transport equation with closure relation
$S_n$ methods, etc....
2-D shadow problem
multi-angle transport (monte carlo)
2-D shadow problem
multi-angle transport (monte carlo)
2-D shadow problem
diffusion approximation (DD monte carlo)
2-D shadow problem
diffusion approximation (DD monte carlo)
special relativistic transport in 1-D radiating flows

e.g., mihalas&mihalas

\[
\frac{dI}{ds} = -\chi I + \eta + \int d\Omega \ R(\hat{n}, \hat{n}') I
\]
special relativistic transport in 1-D radiating flows
e.g., mihalas&mihalas

\[
\gamma(1 + \beta \mu) \frac{\partial I_v}{\partial t} + \gamma(\mu + \beta) \frac{\partial I_v}{\partial r}
+ \frac{\partial}{\partial \mu} \left\{ \gamma(1 - \mu^2) \left[ \frac{1 + \beta \mu}{r} - \gamma^2(\mu + \beta) \frac{\partial \beta}{\partial r} \right] I_v \right\} - \frac{\partial}{\partial v} \left\{ \gamma v \left[ \frac{\beta(1 - \mu^2)}{r} \right] I_v \right\}
+ \gamma^2 \mu(\mu + \beta) \frac{\partial \beta}{\partial r} + \gamma^2 \mu(1 + \beta \mu) \frac{\partial \beta}{\partial t} I_v \right\}
+ \gamma \left\{ \frac{2\mu + \beta(3 - \mu^2)}{r} + \gamma^2(1 + \mu^2 + 2\beta \mu) \frac{\partial \beta}{\partial r} \right\} I_v = \eta_v - \chi_v I_v . \tag{1}
\]
monte carlo transport

ulam
calculating pi at the bar

\[ P_{\text{in}} = \frac{\pi R^2}{4R^2} = \frac{\pi}{4} \]

Signal to noise goes like \( N^{-1/2} \)
Need to throw \( N = 10,000 \) darts to get \( \pi \) to two significant digits
Monte Carlo Transport
Monte Carlo Transport
monte carlo transport

each particle has a position vector \((x,y,z)\)
a direction vector \((D_x, D_y, D_z)\), an energy, wavelength.
Evolution is sampled from appropriate probability distributions
monte carlo transport

each particle has a position vector \((x, y, z)\)
a direction vector \((D_x, D_y, D_z)\), an energy, wavelength.
Evolution is sampled from appropriate probability distributions

probability of traveling a distance \(x\) before scattering

\[ P = \exp(-\tau) = \exp(-\kappa \rho x) = R \]

where \(R\) is a random number sampled uniformly between \((0, 1]\)
monte carlo transport

each particle has a position vector \((x,y,z)\)
a direction vector \((D_x, D_y, D_z)\), an energy, wavelength.
Evolution is sampled from appropriate probability distributions

probability of traveling a distance \(x\) before scattering

\[
P = \exp(-\tau) = \exp(-\kappa \rho x) = R
\]

where \(R\) is a random number sampled uniformly between \((0, 1]\)

solve for \(x\) (distance traveled before scattering)

\[
x = - (\kappa \rho)^{-1} \log(R)
\]
rejection method for compton scattering

\[
\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2} \frac{E_{out}^2}{E_{in}^2} \left[ \frac{E_{out}}{E_{in}} + \frac{E_{in}}{E_{out}^2} - 1 + \cos^2 \theta \right]
\]

(normalized differential cross-section)

scattering angle (degrees)
$e = 0.1 \text{ m}_e \text{c}^2$

$e = 1 \text{ m}_e \text{c}^2$

$e = 10 \text{ m}_e \text{c}^2$

Rejection method for Compton scattering

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2} \frac{E_{\text{out}}^2}{E_{\text{in}}^2} \left[ \frac{E_{\text{out}}}{E_{\text{in}}} + \frac{E_{\text{in}}}{E_{\text{out}}^2} - 1 + \cos^2 \theta \right]$$

$R_1 = 80$
\[ \frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2} \frac{E_{\text{out}}^2}{E_{\text{in}}^2} \left[ \frac{E_{\text{out}}}{E_{\text{in}}} + \frac{E_{\text{in}}}{E_{\text{out}}^2} - 1 + \cos^2 \theta \right] \]

R\text{\textsubscript{1}} = 80

rejection method for Compton scattering

R\text{\textsubscript{2}} = 0.63

e = 0.1 \text{\textsubscript{m}c}^2

e = 1 \text{\textsubscript{m}c}^2

e = 10 \text{\textsubscript{m}c}^2

normalized differential cross-section vs. scattering angle (degrees)
\[ \sigma_{KN} = \frac{1}{2} E_{in}^2 \left[ \frac{E_{out}}{E_{in}} + \frac{E_{in}}{E_{out}} - 1 + \cos^2 \theta \right] \]

Normalized differential cross-section vs. scattering angle (degrees) for different electron energies:

- \( e = 0.1 \, m_e c^2 \)
- \( e = 1 \, m_e c^2 \)
- \( e = 10 \, m_e c^2 \)

Rejection method for Compton scattering:

- \( R_1 = 80 \)
- \( R_2 = 0.63 \)
The rejection method for Compton scattering yields the differential cross-section:

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2} \frac{E_{out}^2}{E_{in}^2} \left[ \frac{E_{out}}{E_{in}} + \frac{E_{in}}{E_{out}^2} - 1 + \cos^2 \theta \right]$$

with parameters $R_1 = 25$, $R_1 = 80$, and $R_2 = 0.63$. The graph shows the normalized differential cross-section as a function of the scattering angle (degrees) for different energies $e = 0.1 \ m_e c^2$, $e = 1 \ m_e c^2$, and $e = 10 \ m_e c^2$. The $d\sigma_{KN}$ values are plotted for these energies, demonstrating the impact of varying energy on the scattering cross-section.
rejection method for compton scattering

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2} \frac{E_{out}^2}{E_{in}^2} \left[ \frac{E_{out}}{E_{in}} + \frac{E_{in}}{E_{out}^2} - 1 + \cos^2 \theta \right]$$
special relativistic transport in 1-D radiating flows

\[ \frac{dI}{ds} = -\chi I + \eta + \int d\Omega \ R(\hat{n}, \hat{n}') I \]
special relativistic transport in 1-D radiating flows

\[ \gamma(1 + \beta \mu) \frac{\partial I_v}{\partial t} + \gamma(\mu + \beta) \frac{\partial I_v}{\partial r} \]

\[ + \frac{\partial}{\partial \mu} \left\{ \gamma(1 - \mu^2) \left[ \frac{1 + \beta \mu}{r} - \gamma^2(\mu + \beta) \frac{\partial \beta}{\partial r} \right] \right\} - \frac{\partial}{\partial v} \left\{ \gamma v \left[ \frac{\beta(1 - \mu^2)}{r} \right] \right\} \]

\[ + \gamma^2 \mu(\mu + \beta) \frac{\partial \beta}{\partial r} + \gamma^2 \mu(1 + \beta \mu) \frac{\partial \beta}{\partial t} \right\} \]

\[ + \gamma \left\{ \frac{2\mu + \beta(3 - \mu^2)}{r} + \gamma^2(1 + \mu^2 + 2\beta \mu) \frac{\partial \beta}{\partial r} \right\} \]

\[ + \gamma^2 [2\mu + \beta(1 + \mu^2)] \frac{\partial \beta}{\partial t} \right\} I_v = \eta_v - \chi_v I_v . \quad (1) \]
mixed frame monte carlo transport

opacities/emissivities calculated in the comoving frame
monte carlo particles propagated in the observer frame
lorentz transformation photon four vector at scattering events

\[
\nu_0 = \gamma \nu (1 - d \cdot v/c)
\]

\[
\chi = \gamma \chi_0 (1 - d \cdot v/c)
\]

\[
d_0 = \left( d - \frac{\gamma v}{c} \left[ 1 - \frac{\gamma d \cdot v/c}{\gamma + 1} \right] \right) \left[ \gamma (1 - d \cdot v/c) \right]^{-1}
\]

lorentz transformations

automatically accounts for all aberration, advection,
doppler shifts, and adiabatic loses to all orders of v/c

general relativistic effects (geodesic tracking) can also be included
e.g., Dolence et al., (2009), Dexter et al., (2009)
implicit monte carlo methods
fleck and cummings 1971

\[ G_0^0 = \left[ \frac{1}{V\Delta t} \sum_i \epsilon_0 l_i \chi_0(\nu_0) \right] - \chi_0 a T_g^4 \]
\[ G_0^i = \frac{1}{cV\Delta t} \sum_i \epsilon_0 l_i \chi_0(\nu_0) d_0^i \]

momentum four-force vector (i.e., radiative heating/cooling, radiative acceleration)
timescale for matter/radiation coupling

\[ t_{RM} \sim \frac{l_p n k T}{c a T^4} \ll t_{dyn}, t_{diff} \]

implicit methods: particle absorption/re-emission (i.e., creation/destruction) is replaced by “effective scattering”
For highly asymmetric 3D radiative flows, some zones may be under (over)-sampled by monte carlo particles.

**strategies**
- pressure tensor methods
- russian roulette
- particle splitting/killing
- directionally biased emission
- replicate heavily loaded zones

(Nathan Roth, UCB)
population control and load balancing

For highly asymmetric 3D radiative flows, some zones may be under (over)-sampled by monte carlo particles.

**strategies**
- pressure tensor methods
- russian roulette
- particle splitting/killing
- directionally biased emission
- replicate heavily loaded zones

black hole accretion disk
(Nathan Roth, UCB)
discrete diffusion monte carlo

For regions of high opacity, monte carlo is very inefficient. Instead, sample from the diffusion approximation:

\[ F = -\frac{c}{3\chi} \nabla E_{\text{rad}} \]

jump probabilities

\[
\begin{align*}
    P_L &= \frac{c}{3\chi_L \Delta x} \frac{\Delta t}{\Delta x} \\
    P_R &= \frac{c}{3\chi_R \Delta x} \frac{\Delta t}{\Delta x} \\
    P_{\text{abs}} &= c\chi_{\text{abs}} \Delta t \\
    P_{\text{stay}} &= 1 \\
    \text{norm} &= \left[ 1 + P_R + P_L + P_{\text{abs}} \right]^{-1}
\end{align*}
\]
Monte Carlo parallelization strategies using hybrid MPI/open MP, run on 10,000-100,000 cores using Cray XE6 (Hopper @ NERSC), Cray XT5 (Jaguar @ ORNL) Blue Gene/P (Intrepid @ ALCF)

**Full Replication**

Each core holds entire model and propagates particles independently; MPI all reduce of radiation/matter coupling terms after each time step. Memory limited (2D, low resolution 3D).

**Domain Decomposed**

Spatial grid partitioned over cores; particles leaving local domain communicated via MPI to neighbors.

**Hybrid**

Use OpenMP threading to do additional particles on shared memory node, can fully replicate certain domains on additional nodes to extend scaling and manage load balancing.
weak scaling: 2D transport calculation
full replication -- embarrassingly parallel

SEDONA 4.0 weak scaling for 4K photons/CPU on BG/P (VN mode)

Run time (min)

Number of CPUs (divided by 1024)
Domain decomposed Monte Carlo transport using hybrid MPI/openMP BoxLib AMR framework on Hopper XE6 (NERSC). Two twelve-core AMD "Mangy-Cours" (4 NUMA "nodes" of 6 cores) with 2.1 GHz processors per node, resulting in 49,152 cores (2048 nodes). The weak scaling tests were performed with 3-D unigrid, constant density.

- Total particles: $1.8 \times 10^{11}$
- Total cells: $4.5 \times 10^{7}$
- Wavelength points: 10,000
- Total memory: 65 TB