Eternal inflation and the multiverse

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Outline

1. Everlasting inflation and the structure of an eternally inflating multiverse

2. How does inflation arise from non-inflation?

3. Does inflation start?

I. Inflation to everlasting inflation

\[ V(\phi) \]

\[ \phi \]
Everlasting bubbly inflation

- Multiple minima $\rightarrow$ vacuum transitions.
Everlasting bubbly inflation

- (Multiple minima) + (slow transitions) = eternal inflation
Everlasting bubbly inflation

Inflating bulk endures:

Inflating fraction:

\[ f_{\text{inf}} = \exp \left[ -\lambda \frac{4\pi (t - t_0)}{3H^3} \right] \]

Inflating volume:

\[ v_{\text{inf}} \propto \exp(3Ht) f_{\text{inf}} \propto \exp\left[ (3 - 4\pi \lambda/3H^3) Ht \right] \]

*Grows for small \( \lambda \)*!
Everlasting bubbly inflation

Inflating bulk endures:

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Inflating volume:
\[ v_{\text{inf}} \propto \exp(3Ht) f_{\text{inf}} \propto \exp\left[ (3 - \frac{4\pi \lambda}{3H^3}) Ht \right] \]

The remaining inflating region approaches a fractal\(^*\) of dimension 3-4\(\pi\lambda\)

\(^*\) Q: What does the ‘B’. In Benoit B. Mandelbroit stand for?
A: Benoit B. Mandelbroit.
Everlasting bubbly inflation

- (Multiple minima) +
  (slow transitions)
  = eternal inflation
de Sitter space(time)
Everlasting bubbly inflation

- (Multiple minima) + (slow transitions) = eternal inflation
Everlasting bubbly inflation = infinite spacetime

- Spacetime has infinite proper volume.

- There are future-directed timelike worldlines of infinite proper length that stay in the inflating phase.

- The bubble distribution in the inflating background becomes stationary.
Other versions of everlasting inflation

Topological eternal inflation

\[ V(\phi) \]

Vilenkin 94
Other versions of everlasting inflation

Stochastic eternal inflation

Note: these are exactly the same fluctuations that lead to the fluctuations in the CMB.
Is the universe spatially infinite?

• *This can depend on the foliation of spacetime*
Infinite uniform spaces inside bubbles

• bubbly inflation: each (isolated) bubble has open FRW cosmology inside.

\[ V(\phi) \]

\[ \phi_F \quad \phi_W \quad \phi_T \]

T=const. slice → infinite negatively curved, homogenous space.

Slices of constant \( \phi \)

False vacuum \( \phi=\phi_F \)

Nucleation event

Bubble wall \( (\phi = \phi_W) \)

True vacuum \( \phi=\phi_F \)

t=const. slice → space with expanding finite-size bubble

Coleman & DeLuccia 1980
Infinite uniform spaces inside bubbles that collide

- Bubble self-collisions: merge into homogeneous slices (const. field lines are hyperbolas)
Infinite uniform spaces inside bubbles that collide

- Two different bubbles: some perturbation, then return to homogeneity.
Infinite uniform spaces inside bubbles that collide

\[ \xi = \text{const.} \]

Fractal with infinite grey volume

Poincare disk
Constant-field surfaces in other eternal models

- Open, topological, stochastic eternal inflation have same conformal structure.
- Note that this is true even if the ‘initial’ configuration of the universe is a closed universe.
Global structure of (everlasting) inflation.

- Observational and theoretical cosmology strongly suggests inflation*
- Inflation is ‘generically’ everlasting (or ‘future eternal’).
- The structure of eternal inflation can in some sense be thought of as an eternally inflating ‘background’ of some phase within which ‘pockets’ or ‘bubbles’ of another phase form. (These pockets may then be the background for more pockets, etc.) The overall structure is fractal.
- Everlasting inflation implies:
  - Infinite spacetime
  - Infinite time in (infinitely many) some places
  - (Unboundedly many) infinite homogeneous spaces.

*Most significant rival is probably cyclic model, with many similar properties.
II. How does an inflationary phase start?

1. Easy: inflation from a high-energy, near-uniform background

2. Understood: inflation from a higher-energy inflation phase

3. Hard: inflation from a lower-energy inflation phase
How does an inflationary phase start from a high-energy, near-uniform background?

Requirement: violate SEC → \textbf{Vacuum energy} dominates over:
- thermal energy of other fields
- kinetic energy of inflaton
- ‘curvature’
How does an inflationary phase start from a higher-energy vacuum phase?
How does an inflationary phase start from a high-energy, near-uniform background?

Coleman-DeLuccia

- False-vacuum dS background
- Small true vacuum bubble nucleates and expands

\[ V(\varphi) \]

\[ V_f \]

\[ V_t \]
Coleman-DeLuccia tunneling

$\phi \rightarrow S$

Nucleation surface
How does an inflationary phase start from a lower-energy vacuum phase?

I. Baby universe creation (a la Farhi, Guth & Guven)

- True-vacuum background
- Tiny bubbles of false vacuum nucleate.
- One tunnels through BH horizon, creating baby universe across Einstein-Rosen wormhole.

\[ V(\varphi) \]

\[ V_f \quad V_t \]

Farhi, Guth & Guven 90
How does an inflationary phase start from a lower-energy vacuum phase?

II. “Reverse” Coleman-DeLuccia (Lee-Weinberg)

- True-vacuum dS background
- Enormous, trans-horizon bubble of false vacuum nucleates, vainly accelerates into false vacuum.

\[ V(\varphi) \]

\[ V_f \]

\[ V_t \]
Two tunnels to inflation

(See AA & Johnson 06)
How does an inflationary phase start from a lower-energy vacuum phase?

II. “Reverse” Coleman-DeLuccia (Aguirre, Carroll, Johnson)

- Looks a lot like a downward fluctuation in entropy: $P \sim e^{-\Delta S}$

- Result (ACJ 2011): Evolution* from equilibrium￠ to a chosen macrostate $A$ is† the time-reverseΨ of the evolution from $A$’s time reverse to equilibrium.

* That is, the evolution of the probability distribution over macrostates.

￠ Or metastable equilibrium that is attained more quickly than, but does not decay more quickly than, the typical time it takes to fluctuate $A$.

† Under assumptions of a unitary time evolution and democracy of microstates.

Ψ Where this is the involution under which the theory is symmetric, and includes time-reversal.

Zen Even if it seems weird.
How does an inflationary phase start from a lower-energy vacuum phase?

II. “Reverse” Coleman-DeLuccia
III. Did inflation start?

Even in eternal inflation, it is often assumed that there is a singular, pre-inflation epoch.  *Need there be?*

Key classic big-bang singularities theorems generally do not apply (assume Strong energy condition).

Several theorems proven that eternally inflating space-times must contain singularities (not all past null or timelike geodesics are complete):

- Requiring only conditions that ensure future-eternal inflation, and the weak energy condition (Borde & Vilenkin 1996).
- Requiring only that a certain “locally measured Hubble constant” $H > H_{\text{min}} \geq 0$ (Borde, Guth & Vilenkin 2001).

**Note:** singularity theorems indicate geodesic incompleteness of the spacetime region over which the theorem’s assumptions hold.
de Sitter space redux
Construct 1: Steady-State eternal inflation

Strategy: make state
approached by semi-
 eternal inflation exact.
Steady-State eternal inflation

Strategy: make state approached by semi-eternal inflation exact.

- Flat spatial sections.
Steady-State eternal inflation

Strategy: make state *approached* by semi-eternal inflation *exact*.

- Flat spatial sections.
- Consider bubbles formed between $t_0$ and $t$. 
Steady-State eternal inflation

Strategy: make state *approached* by semi-eternal inflation *exact*.

- Flat spatial sections.
- Consider bubbles formed between $t_0$ and $t$. 
Steady-State eternal inflation

Strategy: make state *approached* by semi-eternal inflation *exact*.

- Flat spatial sections.
- Consider bubbles formed between $t_0$ and $t$.
- Send $t_0 \to -\infty$.
- Inflation endures.
Steady-State eternal inflation

- Globally time-symmetric, locally time-asymmetric
Steady-State eternal inflation

There seems to be no basic problem (AA & Gratton 2002, 2003; also Vilenkin 13)

Like any theory describing a physical system, this model has:

a) Dynamics (stochastic bubble formation).

b) “Boundary” conditions. These can be posed as:
   i) Inflaton field in false vacuum on an infinite null surface $J^-$.  
   ii) Other (classical) fields are at minima on $J^-$.  
   iii) Weyl curvature = 0 on $J^-$.  

Underlying symmetry principle: the “Perfect Cosmological Principle”
Construct 2: the bi-evolving universe

Version 1

- Low-entropy boundary condition surface B.
- Entropy increases away from this surface

AA & Gratton 02+03
Carroll & Chen 04
AA 07
Construct 2: the bi-evolving universe

Version 2

- ‘generic’ condition on a spacelike surface.
- Entropy increases away from this surface.

Carroll & Chen 04
Carroll book

AA & Gratton 02+03

from Vilenkin ’13
Construct 2: the bi-evolving universe

Version 2

- How do higher-energy regions form?
  - 2a: Baby universes
  - 2b: Fluctuations

from Vilenkin ’13
Some Open Questions

• Where does the inflationary potential come from?
  • Model-building
  • String theory landscape (Aguirre; Susskind?)

• How can we test everlasting inflation, (Johnson?) and/or do cosmology in everlasting inflation? (Aguirre)

• How exactly do we think of the arrow of time, and ‘past hypothesis’? (Carroll)

• What do we make of the infinities? (Aguirre)